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Huang, X.; Nazarov, Yuli V.

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Supercurrents in Unidirectional Channels Originate from Information Transfer in the Opposite Direction: A Theoretical Prediction

Xiao-Li Huang and Yuli V. Nazarov
Kavli Institute of NanoScience, Delft University of Technology, Lorentzweg 1, 2628 CJ Delft, The Netherlands
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It has been thought that the long chiral edge channels cannot support any supercurrent between the superconducting electrodes. We show theoretically that the supercurrent can be mediated by a nonlocal interaction that facilitates a long-distance information transfer in the direction opposite of electron flow. We compute the supercurrent for several interaction models, including that of an external circuit.

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The proximity effect in normal metal-superconducting structures has been known for a long time [1,2] but still is a subject of intense theoretical and experimental research [3,4]. The most prominent manifestation of the proximity effect is a supercurrent flowing through a normal metal between distant superconducting electrodes. The interesting feature of the effect is that the induced superconducting correlations persist in a normal metal for long distances. The distance even diverges at energies close to Fermi level $\epsilon \equiv E - E_F \to 0$, $L \simeq v_F/|\epsilon|$ for ballistic structures with a typical electron velocity v_F [1].

In the quantum Hall (QH) regime, the conducting electrons are restricted to the quantized transport channels at the structure edge [5]. Importantly, these channels are chiral: the electrons propagate in one direction only. Superconducting leads connected to the edge modes may induce the proximity effect. Interestingly, the Andreev reflection phenomena [6,7] and the supercurrent in chiral channels [8–10] have been thoroughly investigated. Notably, it was shown that the supercurrent carried by a chiral channel requires the closing of the channel and is inversely proportional to the full perimeter of the QH sample. Therefore, there seems to be no current in the situation when this perimeter is macroscopically long, for instance, in the situation given in Fig. 1(a). An heuristic explanation is that the supercurrent is due to the bouncing of electrons and Andreev-reflected holes between the superconducting electrodes. In a chiral channel, both electrons and holes move in the same direction and no bouncing can occur unless a particle encircles the perimeter of the whole macroscopic sample. If there were transport channels propagating in the opposite direction, we would have a current of the scale ev_F/L, L being the distance between the superconducting electrodes. The absence of the supercurrent seems a simple but fundamental property of the chiral channels. It is not affected by local electron-electron interactions in the channel that can be easily be taken into account in a framework of a Luttinger-like model [11].

In this Letter, we show that the supercurrent in a chiral channel can be induced by a nonlocal interaction that potentially provides an information flow in the upstream direction, that is, opposite to the propagation direction of the electrons.

We compute the supercurrent for several interaction models and demonstrate that the current is limited by a typical information transfer rate. The effect persists in the ground state where no actual event of information transfer takes place: rather, the supercurrent indicates *potential* for such events. A transport mechanism based on information transfer is rather exotic for electrons, and its experimental observation would be rewarding. We consider a situation of special experimental relevance where the interaction is arranged by means of an external electric circuit.

The primary setup under consideration is shown in Fig. 1(a). Assuming the macroscopically large QH sample, we consider an infinite 1D chiral channel at the sample edge. For simplicity, we concentrate on a single spin-degenerate channel: adding more channels does not change the results qualitatively. Two superconducting electrodes separated by distance $L = x_2 - x_1$ are in contact with the channel, the contact length being $\ll L$. They are kept at the superconducting phase difference $\phi = \phi_2 - \phi_1$. We assume low energies at the scale of Landau level separation. In this limit, the electron states in the channel can be described by a simple linearized Hamiltonian,

$$H_0 = -iv_F \sum_\sigma \int dx \psi_\sigma^\dagger(x) \partial_x \psi_\sigma(x), \quad (1)$$

$\psi_\sigma(x)$ being the annihilation operator of an electron, with spin σ at point x. The normal-electron Green's function in the Matsubara representation is explicitly chiral,

$$G_\omega(x - x') = -\frac{i\,\mathrm{sgn}(\omega)}{v_F} e^{-\frac{\omega}{v_F}(x-x')} \theta(\omega(x - x')); \quad (2)$$

it extends to the right (left) for positive (negative) ω and is zero on the left (right). The lowest-order anomalous Green's function $F_\omega(x, x')$, induced by a superconducting pairing at point x_1 [Fig. 1(b)], encompasses the normal

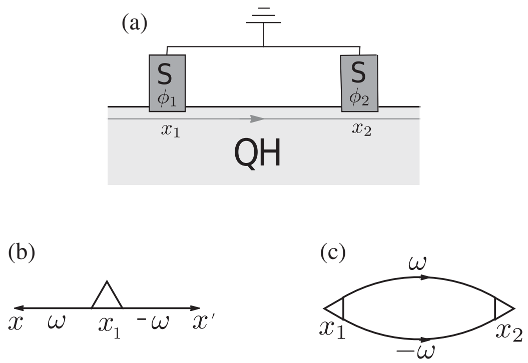

FIG. 1. (a) The basic setup: two superconducting leads biased at phase difference $\phi = \phi_2 - \phi_1$ are attached to the edge of a QH sample. (b) The lowest-order anomalous Green's function. (c) The noninteracting lowest-order contribution shown vanishes for chiral channels.

Green's functions at opposite frequencies $F_\omega(x, x') = \Delta(x_1)G_\omega(x, x_1)G_{-\omega}(x', x_1)$. This describes the superconducting correlations that are essentially nonlocal and, owing to chirality, vanish at the same point $x = x'$. The superconducting current is expressed through the phase-dependent energy correction. This one could emerge from the transfer of the superconducting correlations to the second contact $\simeq \sum_\omega F(x_2, x_2)\Delta(x_2)$ [Fig. 1(c)], yet it vanishes since the correlations vanish at the same point. The main point of this Letter is that a *nonlocal interaction* can change this. Let us consider a diagram shown in Fig. 2. Here, at positive ω, the correlations propagate from x_1 to the separated points $x > x_1$ and $x_1 > x'$. The nonlocal interaction between these distant points can flip the frequency sign of the electron line $\omega' < 0$ so the correlations move in opposite directions to meet at the point x_2. This works, provided $x' < x_1 < x_2 < x$. We see that the interaction should connect the region *to the left* of both electrodes, with the region *to the right of both*.

Let us proceed with the evaluation of the current. We start with noninteracting Green's functions in the system; those are easy to evaluate in all orders in pairing potential, which is incorporated in a form of a unitary transformation

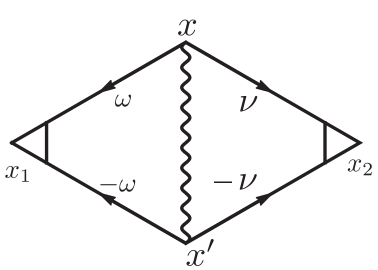

FIG. 2. Nonvanishing contribution to the superconducting current. The wavy line represents interaction. By virtue of chirality, the points x, x' are on opposite sides of both x_1 and x_2. This is only possible if $x' < x_1 < x_2 < x$ and ω and ν are of opposite signs.

in the Nambu space. This transformation relates the electron-hole amplitudes before and after the electrode and reads $\hat{U} = \exp[-i\int dx \hat{\Delta}(x)/v_F]$, $\hat{\Delta} \equiv (0, \Delta; \Delta^*, 0)$, where integration is taken in the vicinity of an electrode. Following [12], we can conveniently parameterize this matrix with the probability p of the Andreev electron-hole conversion at the corresponding contact $\hat{U}_{1,2} = (\sqrt{1-p_{1,2}}, -i\sqrt{p_{1,2}}e^{i\phi_{1,2}}, -i\sqrt{p_{1,2}}e^{-i\phi_{1,2}}, \sqrt{1-p_{1,2}})$. In the usual Nambu spinor formulation, for x and x', situated at opposite sides of both superconducting contacts,

$$G(i\omega, x, x') = -\frac{i}{v_F}e^{-\frac{\omega}{v_F}(x-x')}[\theta(\omega)\theta(x-x')\hat{U}_2\hat{U}_1 - \theta(-\omega)\theta(x'-x)\hat{U}_1^\dagger\hat{U}_2^\dagger]. \tag{3}$$

We evaluate the correction to the energy brought by interaction and differentiate it with respect to ϕ to obtain the current,

$$I(\phi) = -8eR\sqrt{p_1p_2(1-p_1)(1-p_2)}\sin\phi, \tag{4}$$

$$R \equiv \left(\frac{T}{v_F}\right)^2 \sum_{\omega,\omega'}\Theta(-\omega\omega')\int_{-\infty}^{x_1}dx\int_{x_2}^{\infty}dx', V(\omega-\omega'; x, x')e^{|\omega-\omega'|(x-x')/v_F}. \tag{5}$$

T is the temperature in energy units. We give most results at vanishing temperature. Here, we have not yet specified the form of interaction $V(\nu; x, x')$. We see that the current assumes a usual sinusoidal Josephson phase dependence and is proportional to $\sqrt{p_1p_2(1-p_1)(1-p_2)} < 1/4$, indicating that the current comes about the interference of two processes: (i) an electron propagation with Andreev reflection in superconductor 1 and no Andreev reflection in superconductor 2, and (ii) the propagation with no reflection in superconductor 2 and Andreev reflection in superconductor 2. All details of the contacts are incorporated in $p_{1,2}$, while the coefficient R characterizes the interaction in the setup. Further, we evaluate the coefficient R for various interaction setups and prove its relation to the rate of the upstream information transfer. For detailed derivation of (4) and (5), see Supplemental Material [13].

We start with a rather artificial but instructive setup. Let us consider a harmonic oscillator with eigenfrequency ω_b that is coupled to the edge states in two points $x_{3,4}$. The coupling is described with

$$H_I = [\alpha_3\hat{n}(x_3) + \alpha_4\hat{n}(x_4)](\hat{b} + \hat{b}^\dagger). \tag{6}$$

where $\hat{n}(x) \equiv \sum_\sigma \psi_\sigma^\dagger(x)\psi_\sigma(x)$. Owing to the coupling, the quanta of the oscillator can be absorbed by the edge states, with the rates $\Gamma_{3,4} = \alpha_{3,4}^2\varepsilon_b/v_F^2$. The oscillator provides a channel of upstream information exchange whereby an excitation at the point x_4 is absorbed and transferred to the upstream point x_3. The information flow rate through the oscillator is limited by the emission or absorption rates and thus, can be estimated as $\min(\Gamma_3, \Gamma_4)$.

Let us look at the superconducting properties, assuming $x_3 < x_1 < x_2 < x_4$. The oscillator provides an effective electron-electron interaction ($x > x'$),

$$V(x, x', \nu) = \frac{\alpha_3 \alpha_4 \omega_b}{\omega_b^2 + \nu^2} \delta(x - x_3)\delta(x' - x_4). \quad (7)$$

Making use of the relation (5) and integrating over the frequencies, we arrive at the coefficient R characterizing the current,

$$R = \frac{1}{2\pi} \sqrt{\Gamma_3 \Gamma_4} C, \quad (8)$$

where the dimensionless coefficient C in two opposite limits $\omega_b \ll v_F/L$ and $\omega_b \ll v_F/L$ is evaluated as $C = \ln(\omega_b L/v_F)$ and $C = (v_F/L\omega_b)^2$, respectively. We see that for $\omega_b \ll v_F/L$, the coefficient R is of the order of the information transfer rate. This correspondence is not exact, as seen from the different dependence on the Γ_3/Γ_4 ratio. This is not surprising since, distinct from information transfer, no real events are associated with the supercurrent that results from quantum interference. However, such correspondence is remarkable even on a qualitative level. As seen from (5), the relevant frequency window for supercurrent formation is limited by v_F/L. This explains suppression of R at $\omega_b \gg v_F/L$: the oscillator cannot efficiently transmit such low frequencies.

A QH sample is always mounted on a substrate. This makes electron-phonon interaction a default mechanism for a long-range upstream information flow: an electron-hole pair can be converted to a phonon that propagates upstream and is absorbed there. We describe the electron-phonon interaction by the Hamiltonian,

$$H_{e\text{-}ph} = A \sum_{\vec{q}} \frac{i\vec{q}}{\sqrt{2\rho\omega_{\vec{q}} V}} \int d\vec{x} e^{i\vec{q}\cdot\vec{x}} \hat{n}(\vec{x})(a_{\vec{q}} + a^{\dagger}_{-\vec{q}}). \quad (9)$$

Here, ρ is the density of the substrate material, $\mathcal{V}$ is the normalization volume, $\vec{q}$ is the phonon wave vector, and $\omega_{\vec{q}} = c|\vec{q}|$, with c being the sound velocity. For electrons in the edge channel, $\vec{x}$ is one-dimensional. This results in the following long-range electron-electron interaction:

$$V(x, x', \nu) = -\frac{A^2}{2\rho\mathcal{V}} \sum_{\vec{q}} \frac{\vec{q}^2}{\nu^2 + \omega_{\vec{q}}^2} e^{i\vec{q}\cdot(\vec{x}-\vec{x}')}. \quad (10)$$

Let us note the analogy with the previous setup: each phonon mode is, in fact, an oscillator that is coupled to the electrons both upstream and downstream of the superconducting contacts.

The strength of the interaction is convenient to express in terms of the electron relaxation rate, which is proportional to ϵ^3, ϵ being the electron energy above the Fermi level,

$$\Gamma(\epsilon) = \frac{A^2}{12\pi\rho v_F c^4} (\epsilon)^3 \equiv \frac{d\Gamma}{d\epsilon^3} \epsilon^3. \quad (11)$$

Integrating over all oscillators, we arrive at the superconducting current defined by

$$R = \frac{3}{16\pi^2} \frac{d\Gamma}{d\epsilon^3} \frac{c}{v_F} \left(\frac{c}{L}\right)^3. \quad (12)$$

The typical energies involved in the integration are of the order of inverse sound propagation time between the superconducting junctions c/L. To estimate the typical information transfer rate, let us consider electrons excited to these energies. The phonon information transfer rate is the number of relevant excitations times the relaxation rate of a single excitation $(d\Gamma/d\epsilon^3)(c/L)^3$. The relevant excitations are at the space scale $\simeq L$, so their number is $(c/v_F) \ll 1$. This reproduces R by order of value.

However, for realistic structures, the intrinsic electron-phonon effect is fairly small, albeit intrinsic. For typical GaAs parameters, $c/L = 10^{10}$ Hz, $c/v_F = 10^2$ [14], and we estimate $d\Gamma/d\epsilon^3 \simeq \omega_D^{-2} \simeq 5 \times 10^{-28}$ Hz^{-2}. All this gives $R \simeq 0.1$ Hz, and the corresponding current is truly unmeasurable. At low energies, the electron-electron interaction, that is, interaction with electricity fluctuations, is more important for relaxation than for the phonons [14]. However, it is not obvious that electron-electron interaction alone can provide the upstream information transfer required. For instance, the edge magnetoplasmons transfer information only downstream.

There is a simple way to circumvent this: one can embed the QH sample edge into an external electric circuit that will transfer the electric signals upstream (Fig. 3). This is the

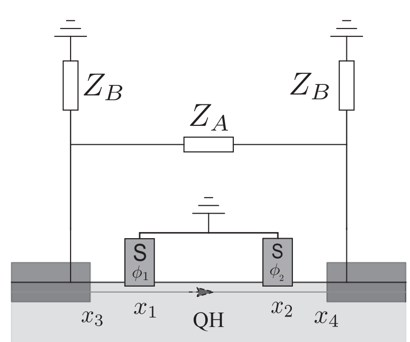

FIG. 3. The QH edge with superconducting contacts included into an external circuit characterized by (frequency-dependent) impedances $Z_{A,B}$. The superconducting current between the terminals 1,2 is proportional to the cross-impedance Z_{34}. Dark grey: metal contacts covering the QH structure.

last setup that we consider. It has advantages of tunability since the strength of the long-range interaction is determined by the circuit parameters. As we will see, it also provides large values of the supercurrent.

To describe the connection of the edge with an external circuit, we cover it with two metallic electrodes that are spread at $x < x_3$ and $x > x_4$, respectively, $(x_4 - x_3 = \tilde{L})$ and are characterized by fluctuating voltages $\hat{V}_{3,4}$. It is convenient to make a gauge transform introducing $\varphi_{a,b}(t) = e\int_{-\infty}^{t} d\tau \hat{V}_{ab}(\tau)$ that is a phase shift induced by the corresponding voltage. With this, the interaction with the external circuit is local, $H_\varphi = -v_F[\hat{\varphi}_3\hat{n}(x_3) - \hat{\varphi}_4\hat{n}(x_4)]$, and is similar to that in the setups considered. The effective interaction is expressed in terms of the correlator of the phases,

$$V(x, x', \nu) = \frac{v_F^2}{2}\langle\varphi_3(\nu)\varphi_4(-\nu)\rangle\delta(x - x_a)\delta(x' - x_b), \quad (13)$$

which is related to the frequency-dependent *cross impedance* $Z_{34}(\nu)$ between the leads 3 and 4, $\langle\varphi_3(\nu)\varphi_4(-\nu)\rangle = Z_{34}(\nu)/\nu$. For the circuit in Fig. 3, $Z_{34} = Z_B^2/(Z_A + 2Z_B)$. We obtain the current coefficient from (5)

$$R = \frac{e^2}{\pi^2}\int_0^\infty d\omega e^{-\frac{\omega}{v_F}\tilde{L}} Z_{34}(\omega). \quad (14)$$

A simple relation is obtained for a frequency-independent cross impedance,

$$R = \frac{Z_{34}}{\pi R_Q}\frac{v_F}{\tilde{L}}, \quad (15)$$

$R_Q \equiv \pi\hbar/e^2$ being the self-impedance of the QH sample edge. This also can be interpreted as a potential information transfer rate, given the bandwidth $v_F/\tilde{L}$ and the fraction of information transferred upstream, defined as the ratio of impedances Z_{34}/R_Q. Upon increasing the impedance of the external circuit to the values of the order R_Q, this fraction becomes of the order of 1, and the effect is maximized up to R of the order of the bandwidth. To give an example of a practical device, for $\tilde{L} = 1\ \mu$m, the bandwidth $v_F/\tilde{L} \simeq 10^{11}$ Hz. For typical high-frequency impedances, $Z \simeq 10^2\ \Omega$. and the corresponding supercurrent $\simeq$100 pA. The resistors used in nanocircuits can be in the $1 - 10\ k\Omega$ range, corresponding to the supercurrents $\simeq 0.1 - 1\ \mu$A.

In conclusion, we have demonstrated theoretically that a supercurrent can exist in long chiral channels. In distinction from all known mechanisms of a supercurrent, it essentially requires interaction. Moreover, it requires a special kind of nonlocal interaction that connects points that are downstream of the superconducting electrodes to the points upstream of those. This connection is not galvanic: it is not the charge that is transferred upstream but rather the information about the charge transfer. We argue that the maximum value of the supercurrent is associated with the rate of upstream information transfer, at least at the qualitative level. Even at this level, this relation is rather intriguing since the supercurrent is a property of the ground state where no process associated with information transfer can occur. This suggests that the supercurrent can probe the potential for information transfer without actually transferring the information. This may be useful in the context of defining quantum information flows [15,16]. On the practical side, this property of the supercurrent makes it feasible to check if in more complex QH states, all of the edge channels actually flow in the same direction [17]. It is feasible to observe the effect experimentally. The traditional difficulties of good contact between metals and 2D gas can be circumvented if utilizing the edge channels in graphene [18,19]. The best setup is likely the one with the external circuit, provided the impedances involved can be controlled on chip, proving the scaling predicted by (14). Here, we present the results at vanishing temperature. We expect the temperature to start playing a role at $k_B T \simeq v_F/L$ at the current to decrease exponentially, $R \propto \exp(-k_B T L/v_F)$, at $k_B T \simeq v_F/L$.

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