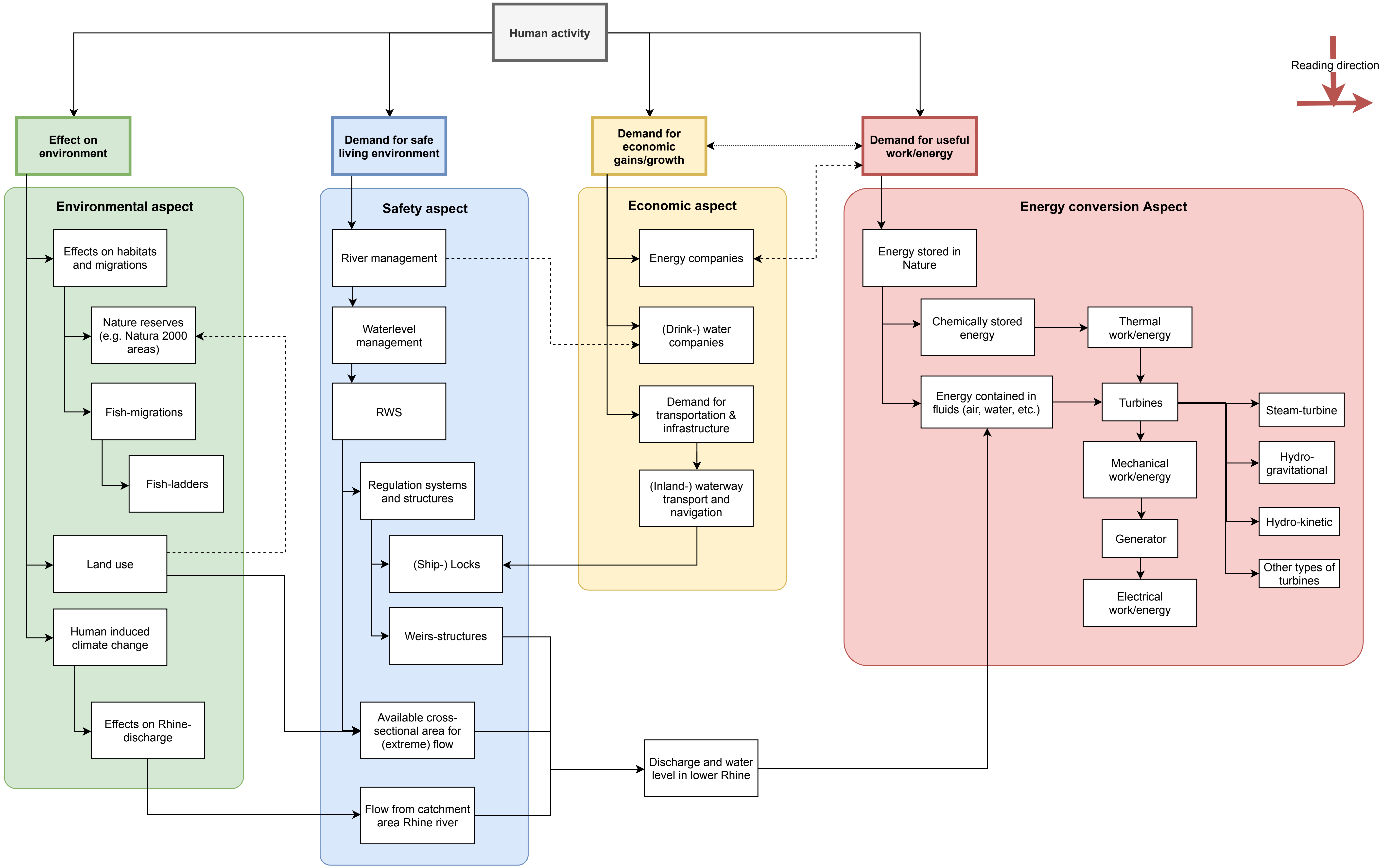
APPENDICES

APPENDIX 1 – MIND-MAP PROBLEM ANALYSIS

– see inserted pages behind this page –


Human activity
Reading direction
Effect on environment
Demand for safe living environment
Demand for economic gains/growth
Demand for useful work/energy
Environmental aspect
Effects on habitats and migrations
Nature reserves (e.g. Natura 2000 areas)
Fish-migrations
Fish-ladders
Land use
Human induced climate change
Effects on Rhine-discharge
Safety aspect
River management
Waterlevel management
RWS
Regulation systems and structures
(Ship-) Locks
Weirs-structures
Available cross-sectional area for (extreme) flow
Flow from catchment area Rhine river
Economic aspect
Energy companies
(Drink-) water companies
Demand for transportation & infrastructure
(Inland-) waterway transport and navigation
Discharge and water level in lower Rhine
Energy conversion Aspect
Energy stored in Nature
Chemically stored energy
Thermal work/energy
Energy contained in fluids (air, water, etc.)
Turbines
Steam-turbine
Hydro-gravitational
Hydro-kinetic
Other types of turbines
Mechanical work/energy
Generator
Electrical work/energy

APPENDIX 2 – LITERATURE REVIEW

HYDRO-POWER THEORY

Estimating the power and energy output of a turbine system requires some theory, which is introduced in this paragraph of the literature study.

Introduction of hydro-power-formulae and principles

Derived from the commonly known 1st (first) rule of thermodynamics, the general energy-equation for stationary flow through a restricted volume, with on side 1 an inflow point and on the other side 2 an outflow point, is:

$$\frac{d}{dt}E = P + W = \left(\frac{p_2}{\rho_2} + gz_2 + \frac{u_2^2}{2} + \tau_2\right)\rho_2 v_2 A_2 - \left(\frac{p_1}{\rho_1} + gz_1 + \frac{u_1^2}{2} + \tau_1\right)\rho_1 v_1 A_1 \qquad \textbf{(L 1)}$$

Where:

$\frac{d}{dt}E =$ Energy flux in Joules per second $[J/s]$ or Watt $[W]$

$P =$ Power exchange within the system in $[J/s]$ or $[Watts]$

$W =$ Heat exchange within the system also in $[J/s]$ or $[Watts]$

$p_i =$ Pressure at location "i" in the system in $[N/m2]$ or $[Pa]$

$\rho_i =$ Mass density of the fluid at location "i" in the system in $[kg/m3]$

$g =$ gravitational acceleration (in this report assumed to be constant with a value of 9,81) in $[m/s^2]$

$z_i =$ Elevation head in $[m]$ with respect to a pre-defined constant reference level

$u_i =$ Flow-velocity at location "i" in the system in $[m/s]$

$\tau_i =$ Fluid thermodynamic internal energy per unit mass at location "i" in the system in $[J/kg]$ (Note: $J = kg * m^2 * s^{-2}$)

A 2D visual representation of this function is given in **Figure L 1**.

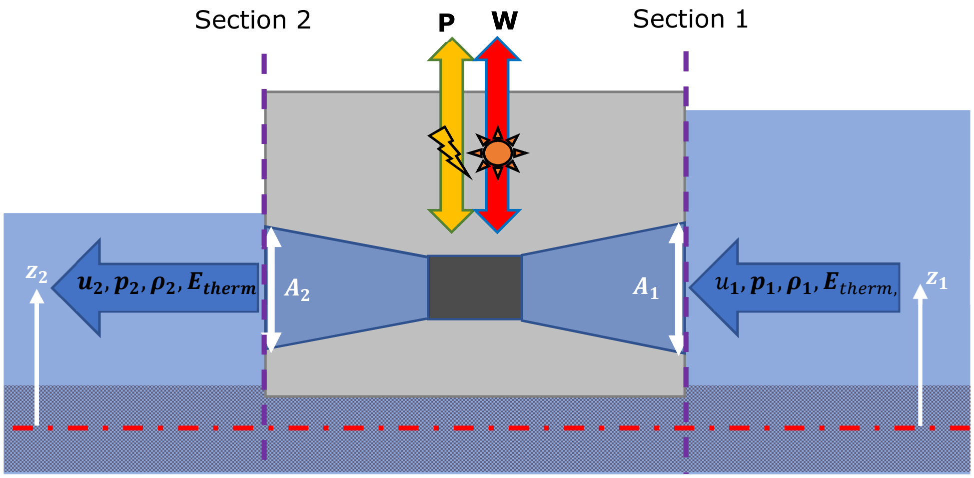


Figure L 1 *- Schematic cross-section of the system described in formula* ***(L 1)***

Formula **(L 1)** can be rewritten to **(L 2)** assuming that:

- the temperature of the water stays the same (i.e. $W = 0$ and $u1 = u2$);
- the fluid is incompressible (constant mass-density i.e. $\rho_1 = \rho_2$);
- there is no change in kinetic energy between the points before inflow and after outflow of the tube (i.e. $v_1 = v_2$);

- At a certain cross-section the cross-sectional area multiplied with the velocity equals discharge (i.e. $Q = v * A$);
- the energy flux through this system is only dependent on the loss of potential energy in terms of elevation.

See also **Figure L 2** on the next page. With mentioned assumptions:

$$\frac{d}{dt}E = \rho g \Delta H Q \qquad \textbf{(L 2)}$$

Where:

$Q =$ Discharge through the considered system in $[m^3/s]$

$\Delta H =$ Water-level-difference over the considered system in $[m]$

Where: Point 1 is the inflow point of the system as shown in **Figure L 2**

Point 2 the outflow point shown in the same figure.

The amount of mass flowing through the system per second being determined by ρQ, being accelerated by gravity g and travels in the direction of gravity (vertically downwards) over a distance ΔH.

The so called "hydro-power-equation" is derived from **(L 2)**, by introducing an efficiency-factor of the turbine as shown below:

$$P_t = \eta * \rho * g * \Delta H * Q \qquad \textbf{(L 3)}$$

Where:

$P_t =$ Power output of the turbine in $[kW]$

$\eta =$ efficiency factor of the considered system, dimensionless, $[0{,}0 \geq \eta \geq 1{,}0]$

Also important to define is Bernoulli's equation, where assuming no energy losses, at any point along a streamline the following holds:

$$H(x) = \frac{p(x)}{\rho g} + z(x) + \frac{(u(x))^2}{2g} = h(x) + h_v(x) = constant \qquad \textbf{(L 4)}$$

Where:

$H(x) =$ Energy-head in $[m]$

$\frac{p(x)}{\rho g} =$ Pressure-head in $[m]$

$z(x) =$ Elevation-head in $[m]$ with respect to a pre-defined constant reference level (see also **Figure L 1** and **Figure L 2**)

$h_v(x) = \frac{(u(x))^2}{2g} =$ Velocity-head in $[m]$

$h(x) =$ Hydraulic head, sum of pressure- and elevation-head, in $[m]$

i.e. $h(x) = \frac{p(x)}{\rho g} + z(x)$

When including losses between two points along the stream-line the Bernoulli-equations needs to be altered slightly, by adding a loss-term:

$$H(x_1) = h(x_1) + h_v(x_1) = h(x_2) + h_v(x_2) + \Delta H = H(x_2) + \Delta H \qquad \textbf{(L 5)}$$

Where:

$\Delta H =$ Water-level-difference over the considered system in $[m]$,

i.e. $\Delta H = H(x_2) - H(x_1)$

$x_i =$ Distance along a streamline in the flow of point "i" from a chosen reference point (where $x = 0m$) in $[m]$

Looking at a pipe system as indicated in **Figure L 2**, it can be seen that due to friction and expansion losses the energy head decreases over distance and, apart from that, the pressure decreases as flow-velocity increases. However, when the flow slows down, the part of the velocity head, that is not lost to turbulence, is converted into pressure again.

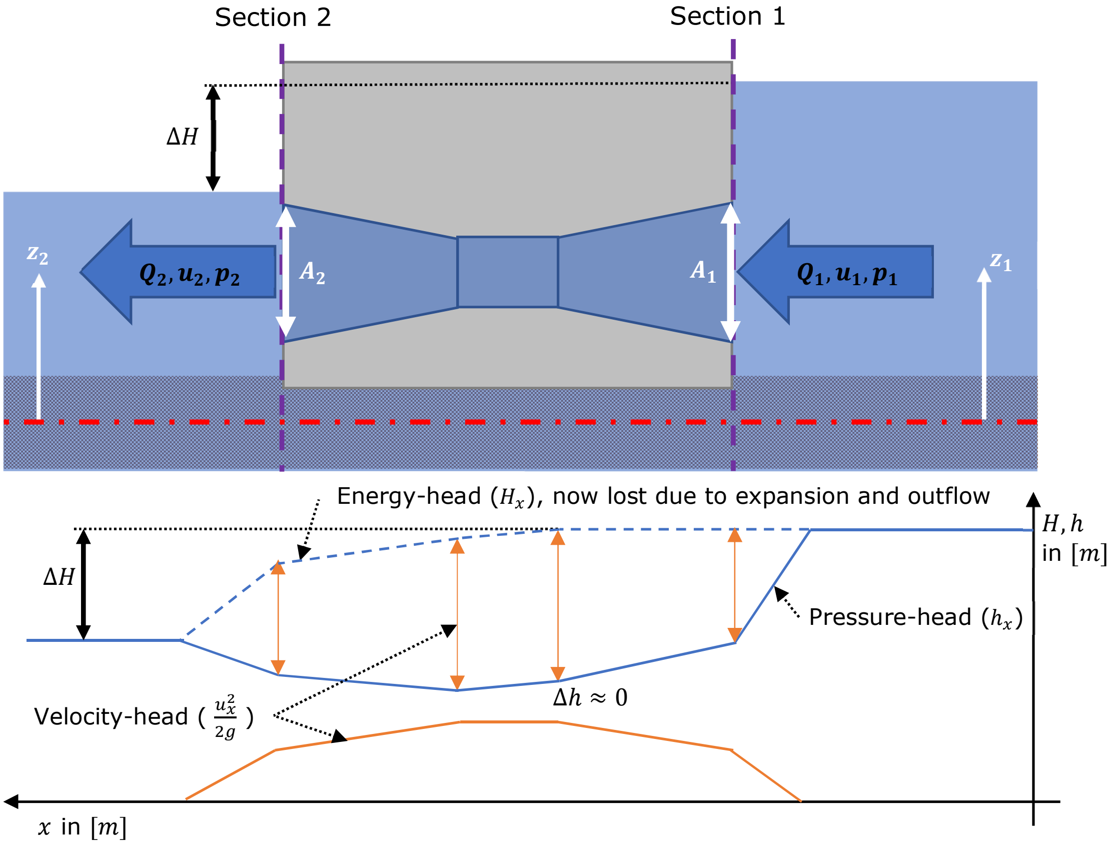


Figure L 2 - Schematic pipe-system without power-extraction. Wall-friction, contraction and inflow losses considered negligible. Turbine that is not extracting power.

Head-discharge-relation

For (low head run of river) hydro-power-plants there is a limit as to how much energy can be extracted. From **(L 2)** and **(L 3)** it becomes clear that the amount of energy for a hydro-power turbine is dependent on both the discharge and the head-difference over the turbine. However, the head difference and discharge that are available are different from what happens within the turbine with given dimensions and specifications. Therefore, the following relations are required:

$$\Delta H_t = f_1(Q_{ava}, \Delta H_{ava}) \quad \textbf{(L 6)}$$

$$Q_t = f_2(Q_{ava}, \Delta H_{ava}) \quad \textbf{(L 7)}$$

Where:

$\Delta H_t =$ Head difference over the turbine in $[m]$

$\mathrm{Q_t} =$ The discharge going through the turbine in $[m^3/s]$

$f(Q_{ava}, \Delta H_{ava}) =$ A function of Q_{ava} and ΔH_{ava} resulting in the head-difference over the turbine in $[m]$

$Q_{ava} =$ The available discharge in $[m^3/s]$

$\Delta H_{ava} =$ The available head difference in $[m]$

Both Q_{ava} and ΔH_{ava} are boundary conditions for the design of the turbine. Neither can be exceeded.

The discharge through the pipe-system is mainly determined by the head-difference and the effective discharge area. The effective discharge area is a product of the physical cross-sectional area of the pipe at a chosen location and a discharge coefficient μ, which traditionally corrects the area such that the contraction of the flow at an intake with a sharp angle is taken into account.

However, this discharge coefficient can also be used to sum all the friction and expansion loss effects of a pipe-system into one factor. The losses in the system determine how much discharge is let through and are how the geometry, besides the size of the opening, influence the discharge. In **paragraph "Loss-coefficients"** the considered losses are explained in more detail.

From Torricelli's law **(L 8)**, giving the exit velocity of a fluid from a orifice (hole or opening in an container) at a certain height below the fluid-surface, and assuming discharge is the product of discharge area and the average flow velocity ($Q = u * A$), formula **(L 9)** can be derived. This formula gives the discharge for a system that has (turbulent) flow through a pressurised (pipe) system.

$$u = \sqrt{2g * h} \quad \textbf{(L 8)}$$

Where:

$u =$ Fluid-velocity at the hole of the considered system in $[m/s]$

$h =$ Height of the fluid-column above the hole in $[m]$

$g =$ As defined before in **(L 1)**, gravitational acceleration constant in $[m/s^2]$

$$Q = \mu A * \sqrt{2g * \Delta H} \quad \textbf{(L 9)}$$

Where:

$Q = u * A$ Discharge through the considered system in $[m^3/s]$

$A =$ Physical cross-sectional-area through which the water flows in $[m^2]$

$\mu =$ Dimensionless discharge coefficient that scales the cross-sectional area A such that the correct discharge is found.

$\Delta H =$ As defined before in **(L 2)** , the water-level-difference over the considered (part of the) system in $[m]$

Loss-coefficients

Hydraulic losses, or also called head losses, are expressed such that they are proportional to the velocity head. They can be categorised in several ways. For instance if they occur locally or over a distance. Most important is that the sum of them determines the discharge through the whole system. Most common losses that occur in pipe-systems are: inflow, contraction, expansion, outflow and friction.

The head-loss in each part of the system can be determined by the related loss-coefficient ξ_i (Xi), that are being used in the design of sewage systems and other pipe-flow situations. These Xi factors are used in many literature references, among which Deltares's guide for designing and maintaining sewage-transport-systems [14], and more in depth in W.H. Hager's "Wastewater Hydraulics - chapter 2 Losses in flow" [15].

The head-loss scales quadratically with the flow-velocity, as can be seen below:

$$\Delta H_i = \xi_i \frac{u_i^2}{2g} = \xi_i \frac{Q_i^2}{2gA_i^2} \qquad \textbf{(L 10)}$$

Where:

$\Delta H_i =$ The head-loss in a particular part "i" of the system in $[m]$

$\xi_i =$ The dimensionless loss-coefficient that determines what fraction of the velocity head results in loss of energy head. (Note the energy-head is not necessarily reduced, but the losses scale with the velocity-head).

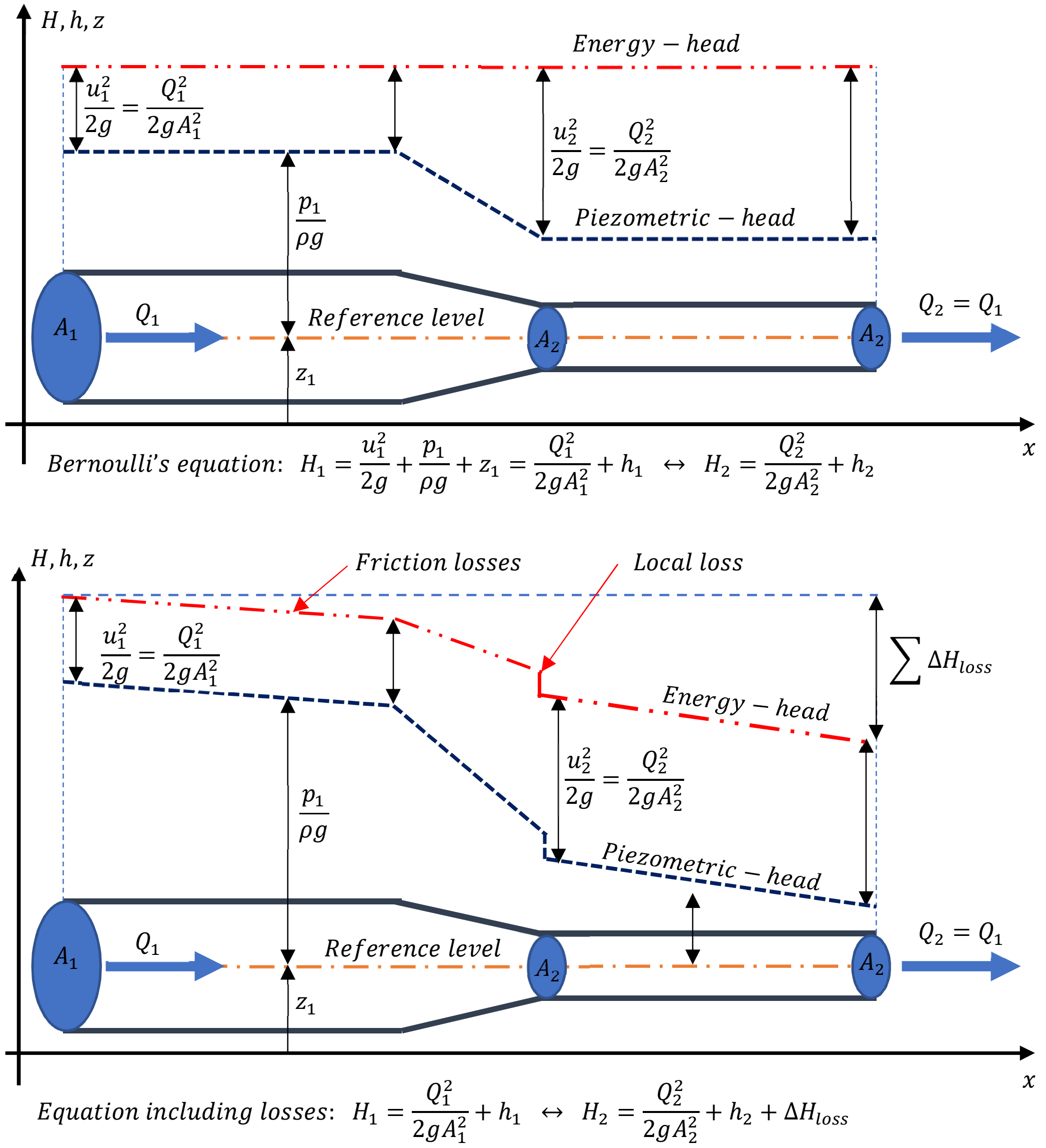
$u_i =$ The flow-velocity in $[m/s]$ at point "i" in the system that governs the losses, sometimes this is the velocity before, at or sometimes after the point of interest. Is Determined by discharge Q_i and cross-sectional area A_i at point "i"

Hager also references the book about hydraulic losses from I.E. Idel'cik [16], which is an extensive work that uses both theory and experimental data for determining loss-coefficients and has been reprinted and updated several times.

In the section below the relevant factors have been worked out further. The losses included in the hydraulic models for this thesis are:

1. Wall-friction losses
2. Expansion losses
3. Contraction losses
4. Inflow losses
5. Combining conduit losses (Y-junction)
6. Outflow losses
7. Trash-rack losses

From Bernoulli's equation it's possible to define "head-losses" as a function of velocity head. In this appendix the friction and local losses are defined that are used in the hydraulic model for the turbines.



$$Bernoulli's\ equation:\ H_1 = \frac{u_1^2}{2g} + \frac{p_1}{\rho g} + z_1 = \frac{Q_1^2}{2gA_1^2} + h_1 \ \leftrightarrow\ H_2 = \frac{Q_2^2}{2gA_2^2} + h_2$$

$$Equation\ including\ losses:\ H_1 = \frac{Q_1^2}{2gA_1^2} + h_1 \ \leftrightarrow\ H_2 = \frac{Q_2^2}{2gA_2^2} + h_2 + \Delta H_{loss}$$

Figure L 3 *- Bernoulli's equation and same equation with losses included.*

Wall-friction-losses

In civil engineering it's customary to define (wall-) friction-loss as the result of boundary layer development. Hager [15, p. 18] notes that this is not quite correct, but has become the popular expression. It is therefore decided to also use this, to keep calculations comparable with previous research.

A friction gradient S_f, a head-loss ΔH per unit length Δx, can be defined as follows:

$$S_f = \Delta H / \Delta x \qquad \textbf{(L 11)}$$

For wall friction the dimensionless Reynolds number, established by Osborne Reynolds (1842-1912), is quite important as it describes the flow regime. The regimes it describes are laminar, layered flow, and also turbulent flow for which the Reynolds number exceeds around 2300.

$$R = \frac{u * D}{\nu} \qquad \textbf{(L 12)}$$

Where:

R	The dimensionless Reynolds number
$u =$	The flow velocity in $[m/s]$ (for a circular pipe: $V = \frac{4Q}{\pi * D^2}$)
$D =$	The relevant or reference spatial size dimension in [m]. This could be the diameter of the pipe in case flow through or perpendicular around a pipe is considered.
$\nu =$	Kinematic viscosity in $[m^2/s]$
$\nu = \frac{\mu}{\rho}$	Where $\mu =$ dynamic viscosity in $[kg * m^{-1}s^{-2}] = [Pa]$ and $\rho =$ density of the fluid in $[kg/m^3]$

Regular water at a temperature of 20°C has the following properties:

Description	**Value and unit**
Dynamic viscosity	$\mu = 1{,}002\ mPa\ (= 1{,}002 * 10^{-3} Pa)$
Mass density	$\rho = 998{,}2 kg/m^3$
Resulting kinematic viscosity	$\nu = \frac{\mu}{\rho} = \frac{1{,}002 * 10^{-3} Pa}{998{,}2 kg/m^3} = 1{,}004 * 10^{-6} m^2/s$
Order of magnitude of diameter	$D = O(1m)$
Transition Reynolds nr.	$R = 2300$
Transition flow velocity	$u_{transition} = \frac{2300 * 1{,}004 * 10^{-6} m^2/s}{1m} = 2 * 10^{-1} m/s$

Table 35 - Properties of water at a temperature of 20°C

For flow situations in civil engineering/hydraulic structures the order of magnitude of dimensions in meters (so $D = O(1m)$), so the transition from laminar to turbulent is at around 2 mm/s. Thus in practise flow is always turbulent except when considering ground-water flows and flows through really small openings like porous materials.

Hager [15, p. 18] notes that Henry Darcy (1803-1858) and Julius Weisbach (1806-1871) found that the friction gradient increases more or less quadratically with velocity head and decreases about linearly with diameter. This led them to propose a friction gradient expression of the following form:

$$S_f = \frac{u^2}{2g} * \frac{f}{D} \qquad \textbf{(L 13)}$$

Where:

$\frac{V^2}{2g} =$	Velocity head in [m]
$f =$	Dimensionless friction factor, sometimes indicated (in German texts) with λ , which was assumed to be nearly constant
$D =$	Diameter of the considered pipe in [m]

Later it was found that the friction factor f actually depends largely on the Reynolds number and a dimensionless number called the “relative wall roughness”, which is a ratio between the wall roughness height and the diameter of the pipe (see **(L 14)**).

$$\varepsilon = k_s/D \tag{L 14}$$

Where:

$\varepsilon =$ Dimensionless, relative wall roughness.

$k_s =$ Wall roughness height of the pipe/conduit in [m]

The wall roughness height is defined such that it should create the same friction loss as produced by a Nikuradse's sand-roughened pipe. These pipes were pasted with a uniform diameter sand and, in order to find the friction factor f, measurements were taken to find the Reynolds number R and relative wall roughness ε by Johann Nikuradse (1894-1979) [15, p. 19]. The same experiment was then repeated with a series of irregular surfaces and then matched with the uniform surfaces. This way an equivalent wall-roughness-height k_s could be linked to various materials.

The experiment as such was suggested by Ludwig Prandtl (1875-1953) [15, p. 19], to overcome the problem of complex surface irregularity of pipe-surface materials.

For smooth pipes the Reynolds number is the governing factor and for rough surfaced pipes the ε dominates. Therefore, in 1937, Colebrook and White compared results for smooth pipes and rough pipes and thus came up with an universal relation for the friction factor (or as Hager refers to as: "… a universal law for the friction factor f…" [15, p. 19]).

$$\frac{1}{\sqrt{f}} = -2 * \log\left(\frac{\varepsilon}{3{,}71} + \frac{2{,}51}{R * \sqrt{f}}\right) \tag{L 15}$$

Where:

$R > 2300$ The Reynolds number

Rewriting:

$$f = \left(-2 * \log\left(\frac{k_s}{3{,}71 * D} + \frac{2{,}51}{R * \sqrt{f}}\right)\right)^{-2} \tag{L 16}$$

This is an implicit function and requires iterative solving to find factor f. There are many ways of finding it via other ways, most commonly known being the "Moody-diagram" (see also **Figure L 4**), but also simplified equations that only hold for a part of the domain of the universal relation.

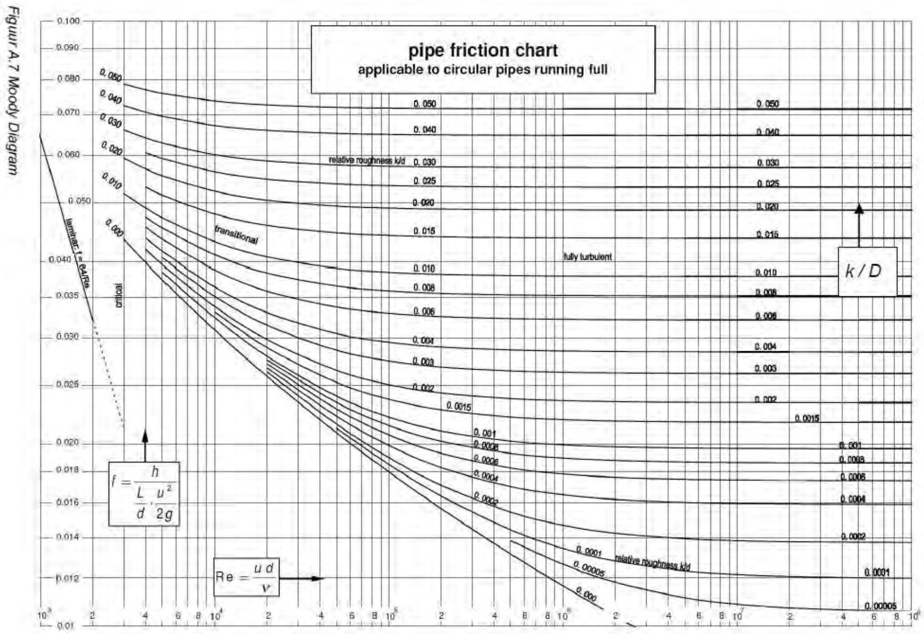


Figure L 4 - Moody diagram - Source: Deltares: [14]

Expansion-losses

The loss-coefficient for expanding flow when the angle $\delta = 90°$ only depends on the ratio of areas, but when the angle is shallower than $\delta < 30°$, then the angle plays a more significant role as Sinniger and Hager experimentally determined in 1989 [15, p. 36].

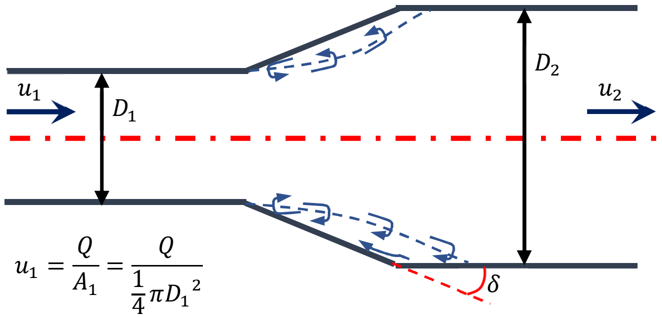


***Figure L 5** - Expansion losses for circular sections – The losses scale with the velocity in the narrow part, the v_1 indicated in the image – Source: [15]*

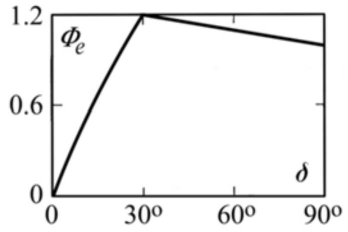


***Figure L 6** - Sinniger and Hager expressions for expansion loss correction of Borda-Carnot-expression - [15, p. 36]*

To combine both phenomena in one formula the expansion losses can be defined as follows:

$$\xi_e = \xi_{e90°} * \Phi_e(\delta)$$ **(L 17)**

With the so called "Borda-Carnot"-expression [15, p. 36]:

$$\xi_{e90°} = \frac{\Delta H_{12}}{\frac{{u_1}^2}{2g}} = \left[1 - \frac{A_1}{A_2}\right]^2$$ **(L 18)**

And with the "Sinniger and Hager 1989"-expressions [15, p. 36]:

$$\Phi_e(\delta) = \frac{\delta}{90°} + \sin(2\delta)\,, 0° \leq \delta \leq 30°$$ **(L 19)**

$$\Phi_e(\delta) = \frac{5}{4} - \frac{\delta}{360°},\ 30° \leq \delta \leq 90°$$ **(L 20)**

Where:

$\xi_{e90°} =$	The dimensionless loss-coefficient taking into account the increase in cross-sectional area
$A_1 =$	The cross-sectional area of the pipe before the expansion in $[m^2]$
$A_2 =$	The cross-sectional area of the pipe after the expansion in $[m^2]$
$\Phi_e(\delta) =$	The dimensionless factor taking into account the effect of the expansion angle visualised also in **Figure L 5**

Outflow losses

The outflow losses are actually a form of expansion losses, only in this case the area in the second cross-section is much larger than many pipe-diameters. If the area is considered infinite then the flow loses all its remaining velocity head ($\xi_{out} \rightarrow 1$). If some velocity is left in the flow after outflow (say for instance in a river, where there is a finite outflow area) the outflow losses can be considered as follows:

$$\xi_{out} = \frac{\Delta H_{12}}{\frac{{u_1}^2}{2g}} = \left[1 - \frac{A_1}{A_2}\right]^2 = \left[1 - \frac{Q * A_1}{Q * A_2}\right]^2 = \left[1 - \frac{u_2}{u_1}\right]^2$$ **(L 21)**

Contraction-losses

Flows that encounter a contracting geometry behave fundamentally different.

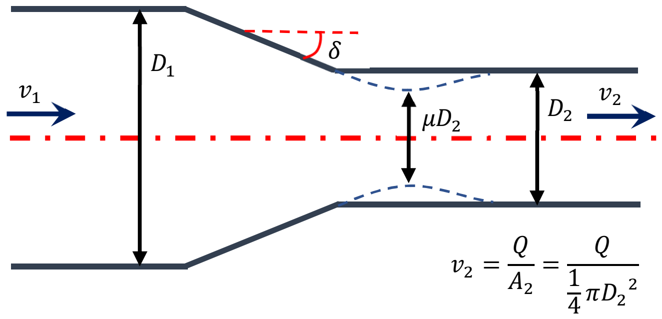


***Figure L 7** - Contraction losses for circular sections – The losses scale with the velocity in the narrow part, the v_2 indicated in the image – Source: [15, p. 37]*

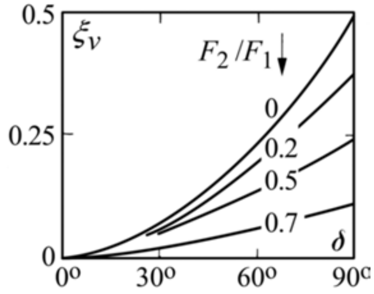


***Figure L 8** - Sinniger and Hager expressions for expansion loss correction of Borda-Carnot-expression - [15, p. 37]*

This is due to the difference in separation structure according to Hager [15, p. 37]. The difference is that an additional contraction μ occurs in the flow right after the geometric contraction ends, which leads to an expansion further down the pipe. The expansion is where the losses occur. The contracting flows themselves are actually nearly free of losses.

Again the function for losses have been made dependent on the ratio of cross-sectional areas and the angle δ. The empirical formula from the paper by Gardel (1962) [15, p. 37] gives the relationship:

$$\xi_c = \frac{\Delta H_{12}}{\frac{{u_2}^2}{2g}} = \frac{1}{2} * (1-\varphi)(\delta/90°)^{1{,}83*(1-\varphi)^{0{,}4}} \qquad \textbf{(L 22)}$$

Where:

$\varphi = A_2/A_1$ The area ratio of the tube after the contraction and the tube before the contraction. If there is a contraction, $\varphi < 1$

$\delta =$ The angle of contraction as indicated in **Figure L 7** in $[\,°\,]$

Inflow-losses for a conduit inlet

A conduit inlet is a special case of the contraction inlet where the area ratio is tending to zero as $A_1 \xrightarrow{goes\ to} \infty$. If the inflow edge is a hard 90° edge the losses are half the velocity head in the tube.

This can simply be derived by entering $\varphi = 0$ and $\delta = 90°$ into **(L 22)**:

$$\xi_{in,90°} = \frac{\Delta H_{12}}{\frac{{u_2}^2}{2g}} = \frac{1}{2} * (1)\left(\frac{\delta}{90°}\right)^{1{,}83*(1)^{0{,}4}} = \frac{1}{2}\left(\frac{90°}{90°}\right)^{1{,}83} = \frac{1}{2} \qquad \textbf{(L 23)}$$

In case the inlet has rounded edges (see **Figure L 9**) the loss-coefficient can be approximated by Hager's formula (2.30) [15, p. 38] based on data from Idel'cik [16].

$$\xi_{in,r} = \frac{1}{2} * \exp\left(-15 * \frac{r_v}{D}\right) \qquad \textbf{(L 24)}$$

Where:

r_v The radius of rounding in $[m]$ see also **Figure L 9**

$D =$ The diameter of the pipe right after the inlet in $[m]$

Because **(L 24)** is an descending exponential function there is a point where the loss-coefficient is approximately zero. Based on work from Knapp (1960) [15, p. 38] it was observed that when $\frac{r_v}{D} > 1/6$ the inflow losses are negligible.

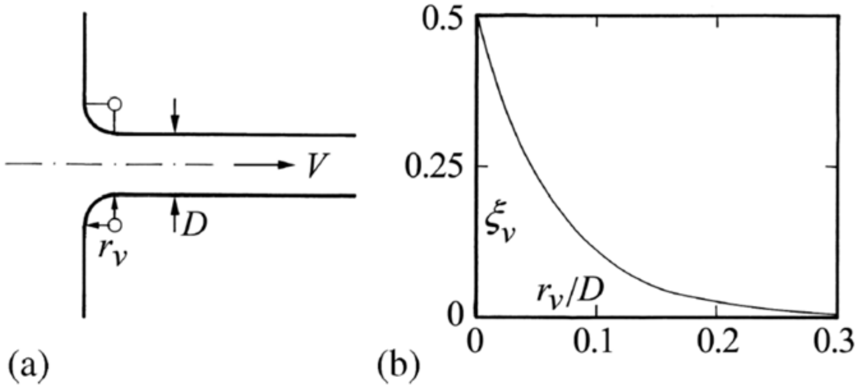
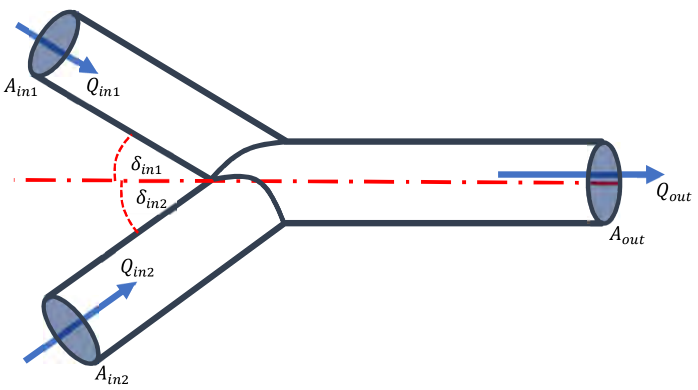


***Figure L 9** - Rounded inlet definition sketch - Source: [15, p. 38]*

Combining conduit junction (conflux)

This local loss occurs when two flows join into one flow, possibly with different angles, velocities and/or discharges.



$$r_{qd} = \frac{Q_{in1}}{Q_{out}}$$

$$r_A = \frac{A_{in1}}{A_{out}}$$

$$r_\delta = \frac{\delta_{in1}}{\delta_{in2}}$$

***Figure L 10** - Schematic representation of a combining (Y-)junction*

For such a situation it is useful to define a number of ratios between the two branches and the outflowing tube.

$$r_{qd} = \frac{Q_{in1}}{Q_{out}} \qquad \textbf{(L 25)}$$

$$continuity\text{:} \sum Q_{in} = Q_{out} \quad \rightarrow \quad So\ also\text{:} \quad Q_{in2} = Q_{out} * (1 - r_{qd}) \qquad \textbf{(L 26)}$$

$$r_{A1} = \frac{A_{in1}}{A_{out}} \qquad \textbf{(L 27)}$$

$$r_{A2} = \frac{A_{in2}}{A_{out}} \qquad \textbf{(L 28)}$$

Where:

Q_{in1}	The incoming discharge from branch nr. 1 in $[m^3/s]$
Q_{in2}	The incoming discharge from branch nr. 2 in $[m^3/s]$
Q_{out}	The outgoing discharge in $[m^3/s]$
$r_{qd} =$	Ratio between incoming discharge of branch 1 and outgoing discharge
A_{in1}	The cross-sectional area used by the incoming discharge from branch nr. 1 in $[m^2]$
A_{in2}	The cross-sectional area used by the incoming discharge from branch nr. 2 in $[m^2]$
A_{out}	The cross-sectional area used by the outgoing discharge in $[m^2]$
$r_{Ai} =$	Ratio between discharge area for incoming branch "i" and the discharge area for outgoing.

With these defined it is easier to define the loss-coefficients, which are defined as follows:

$$\xi_{in1} = \frac{H_{in1} - H_{out}}{\frac{u_{out}^2}{2g}} \quad ; \quad \xi_{in2} = \frac{H_{in2} - H_{out}}{\frac{u_{out}^2}{2g}} \tag{L 29}$$

Where:

u_{out}	Is the outgoing flow-velocity in $[m/s]$
$H_{ini} =$	Energy head 1 in $[m]$ just before the junction in the incoming branch "i"
$\xi_{in\ j} =$	Loss-coefficient of the flow going from branch "i" to the outflow branch.

For a conduit junction with sharp edges Vischer (1958) [15, p. 39] obtained the following relation by using basic relations of pressure distribution and the momentum equation:

$$\xi_{in1} = 1 - 2 * r_{A1}^{-1} * r_{qd} * \cos(\delta_1) - 2 * r_{A2}^{-1} * (1 - r_{qd})^2 * \cos(\delta_2) + (r_{A1}^{-1} * r_{qd})^2 \tag{L 30}$$

Where:

$\xi_{in\ j} =$	Loss-coefficient of the flow going from branch "i" to the outflow branch.

$$\xi_{in2} = 1 - 2 * r_{A1}^{-1} * r_{qd} * \cos(\delta_1) - 2 * r_{A2}^{-1} * (1 - r_{qd})^2 * \cos(\delta_2) + \left(r_{A2}^{-1} * (1 - r_{qd})\right)^2 \tag{L 31}$$

Where:

$\xi_{in\ j} =$	Loss-coefficient of the flow going from branch "i" to the outflow branch.

From Idel'cik [16] for an angle of 15°:

$$\xi_{in1} = 1 + (r_{A1}^{-1} * r_{qd})^2 - 2 * r_{A2}^{-1} * (1 - r_{qd})^2 - 1{,}94 * r_{A1}^{-1} * r_{qd} \tag{L 32}$$

Where:

$\xi_{in\ j} =$	Loss-coefficient of the flow going from branch "i" to the outflow branch.

$$\xi_{in2} = 1 + \left(r_{A2}^{-1} * (1 - r_{qd})\right)^2 - 2 * r_{A2}^{-1} * (1 - r_{qd})^2 - 1{,}94 * r_{A1}^{-1} * r_{qd} + K_s \tag{L 33}$$

Where:

$\xi_{in\ j} =$	Loss-coefficient of the flow going from branch "i" to the outflow branch.
$K_s =$	A coefficient based on table from Idel'cik [16]

Trash-rack-losses

As a general formula the trash rack loss coefficient is defined as:

$$\xi_{tr} = \beta_{tr} * \zeta_{tr} * c_{tr} * \sin(\delta_{tr}) \qquad \textbf{(L 34)}$$

Where:

$\beta_{tr} =$	the "rack" coefficient, which is a kind of shape coefficient
$\zeta_{tr} =$	the gap coefficient, also related to geometry
$c_{tr} =$	the "cleaning method" coefficient, determined by the method with which the rack will be cleaned.
$\delta_{tr} =$	the inclination angle of the rack.

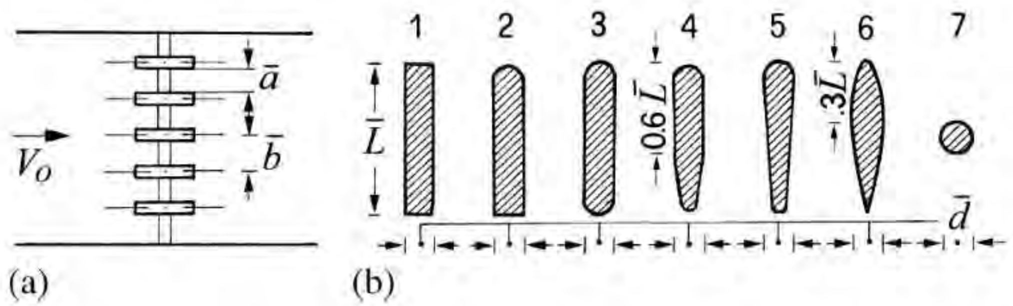


Fig. 2.18 (**a**) Plan of the rack bars, (**b**) Types of rack bars

Table 2.7 Rack factor β_{Re} in relation to the rack geometry shown in Fig. 2.18

Type	1	2	3	4	5	6	7
β_{Re}	1	0.76	0.76	0.43	0.37	0.30	0.74

Figure L 11 – *Top: fig. 2.18 from Hager 2010 [15, p. 45] (a) relevant dimensions for the ζ-shape coefficient (b) with the β-shape coefficient.*
Bottom: table 2.7 from Hager 2010 [15, p. 44], with values for β-shape coefficient related to the number indicated in 2.18(b) above it.

For a trash rack that is clean a c_{tr} value of 1 can be used. When the rack is mechanically cleaned it lies between 1,1 and 1,3 and when it is manually cleaned between 1,5 and 2,0. An simplification of **(L 34)** can be made when the L/d ratio (see **Figure L 11**) is about 5 and the gap ratio a/b>0,5 (slender bars), then according to Idel'cik [16]:

$$\xi_{tr} = \beta_{tr} * c_{tr} * \frac{7}{3} * \left[\frac{b}{a} - 1\right]^{4/3} * \sin(\delta_{tr}) \qquad \textbf{(L 35)}$$

Where:

$\zeta_{tr} = \frac{7}{3} * \left[\frac{b}{a} - 1\right]^{4/3}$

$a =$	the centre to centre distance of the bars
$b =$	the distance (gap) in between the bars

The flow velocity used is the one just before the trash rack. The relevant cross-section is there.

Sum of energy-head-losses in the system

In the case of a system with elements in series, like the turbine system as shown in **Figure L 2**, the sum of these losses ΔH_i must equal the total head difference over the system. With that in mind the following equations can be formed:

$$\Delta H_{sys} = \sum_{i=1}^{N} \Delta H_i = \sum_{i=1}^{N} \xi_i \frac{Q_i^2}{2gA_i^2} = \frac{Q^2}{2g} \sum_{i=1}^{N} \frac{\xi_i}{A_i^2} \qquad \textbf{(L 36)}$$

Where:

$\Delta H_{sys} =$	The head-difference over the entire system, the total available head-difference in $[m]$
$\sum_{i=1}^{N} \Delta H_i =$	The sum of all the head-losses due to elements "1" up to and including element "N"

Due to continuity ($Q_{in} = Q_{out}$, because flow of mass is equal, and due to density remaining constant also volume V must be equal: $m_{in} = m_{out}\ (=)\ V_{in} = V_{out}$), the discharge in the entire system is the same for each entry ($Q_i = Q_t$). Therefore, writing out the sum and solving for Q_t, gives the relation for Q_t in terms of ΔH_{sys}.

For shorter writing the equivalent loss coefficient "ξ_{eq}" and the quadratic resistance coefficient "C" are defined as:

$$\xi_{eq} = A_t^2 \sum_{i=1}^{N} \frac{\xi_i}{A_i^2} \qquad \textbf{(L 37)}$$

$$C = \frac{1}{2g} \sum_{i=1}^{N} \frac{\xi_i}{A_i^2} = \frac{\xi_{eq}}{2g * A_t^2} = \frac{\Delta H_{sys}}{Q_t^2} \qquad \textbf{(L 38)}$$

Where:

$\xi_{eq} =$	The equivalent loss coefficient.
$C =$	The Quadratic resistance coefficient in $[s^2/m^5]$

Note that most ξ_i values are dependent on the diameter and geometry of the turbine and thus ξ_{eq} too. Assuming ξ_{eq} being constant for all diameters is therefore not correct.

Energy extraction

When involving energy extraction, not all of the changes in head are due to friction or turbulence any more. A "head-difference" caused by the turbine is introduced. The discharge for all terms is the same Q and gravitational acceleration g is also assumed constant. The sum off head-differences is then:

$$\Delta H_{sys} = \sum_{i=1}^{M} \Delta H_i = \Delta H_t + Q_t^2 C \qquad \textbf{(L 39)}$$

Where:

$\Delta H_{sys} =$	The head-difference over the entire system, the total available head-difference in $[m]$
$\sum_{i=1}^{M} \Delta H_i =$	The sum of all the head-differences due to elements "1" up to and including element "M"
$Q_t^2 * C =$	The sum of all the head losses, sometimes also referred to as "minor-losses", caused by friction, turbulence and the like, in $[m]$.
$\Delta H_t =$	The head-difference over the turbine in $[m]$

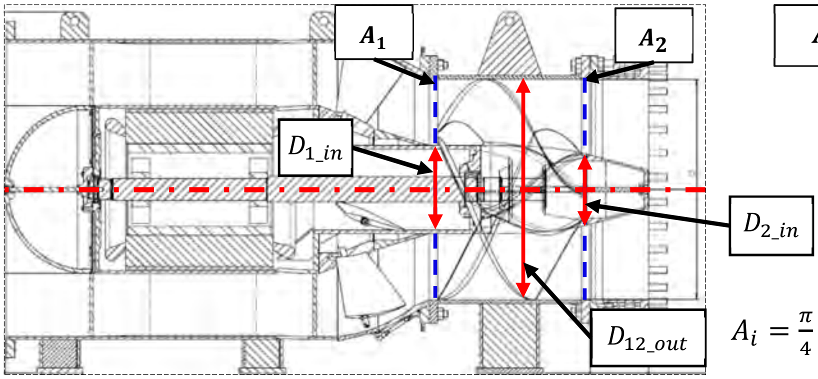
What this head difference over the turbine ΔH_t is, actually determines how much power will be produced and is also the input for the head-difference in the hydro power formula **(L 3)**.

The head over the turbine is determined by turbo-machinery theory, which will be explained in more detail in **paragraph 4.2.4**. The design and configuration of the turbine has some influence on how much head-difference the turbine creates, so for now it is enough to accept that this head-difference has a value smaller than the head-difference over the structure.

In **Figure L 13** a simplified pipe-system with turbine is schematised. In this schematisation some assumptions have been made:

3. The cross-sectional area A_1 before (upstream) and after A_2 (downstream) of the turbine blades are assumed equal ($\boldsymbol{A_1 = A_2}$)
4. With the assumptions above, just looking at the hydraulic-head and velocity head before and after the turbine and ignoring internal workings of the turbine with interaction with the rotor blades, the head-difference over the turbine is equal to the hydraulic-head-difference (i.e. $\Delta H = \Delta H_t = h_1 - h_2 = \Delta h$, see also formulas in **Figure L 12**). The useable head over the turbine is <u>not dependent on change in velocity head</u>.

This isn't such a strange assumption for reaction turbines considering designs like shown in **Figure L 12**. Even if the area's aren't exactly the same (e.g. $A_2 > A_1$), the head-difference that this geometry creates is not taken by the turbine to create energy (i.e. $\Delta H = \Delta H_A + \Delta H_t$, where ΔH_A is due to the geometry and ΔH_t due to the turbine, see also formulas in **Figure L 12**).



$$\boldsymbol{A_1 = A_2}$$

$$A_i = \frac{\pi}{4}\left(D_{i\_in}^2 - D_{i\_out}^2\right)$$

$$H_1 = H_2 - \Delta H_{12} \quad \rightarrow \quad h_1 + \frac{Q_1^2}{2g*A_1^2} = h_2 + \frac{Q_2^2}{2g*A_2^2} - \Delta H_t \,,$$

Bernoulli's equation

$$h_1 + \frac{Q^2}{2g*A_1^2} = h_2 + \frac{Q^2}{2g*A_1^2} + \Delta H_A - \Delta H_t \,,$$

$$(with\ Q_1 = Q_2 = Q\ )$$

From that: $\Delta H_t = h_1 - h_2$ and $\Delta H_A = \frac{Q^2}{2g}\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right)$, thus: $\Delta H_{12} = \Delta H_t + \Delta H_A$

If $\boldsymbol{\Delta H_A = 0}$ $\boldsymbol{(A_1 = A_2)}$**,** **then:** $\boldsymbol{\Delta H = \Delta H_t = h_1 - h_2 = \Delta h}$

Figure L 12 *- Pentair Fairbanks Nijhuis fish-friendly-turbine and formulas regarding head-difference assumptions - source: image used with permission from Pentair Fairbanks Nijhuis*

With these assumptions made **Figure L 10** on the next page can shows what happens with the Energy head, velocity head and hydraulic head for both the situation when energy is extracted and when it is not.

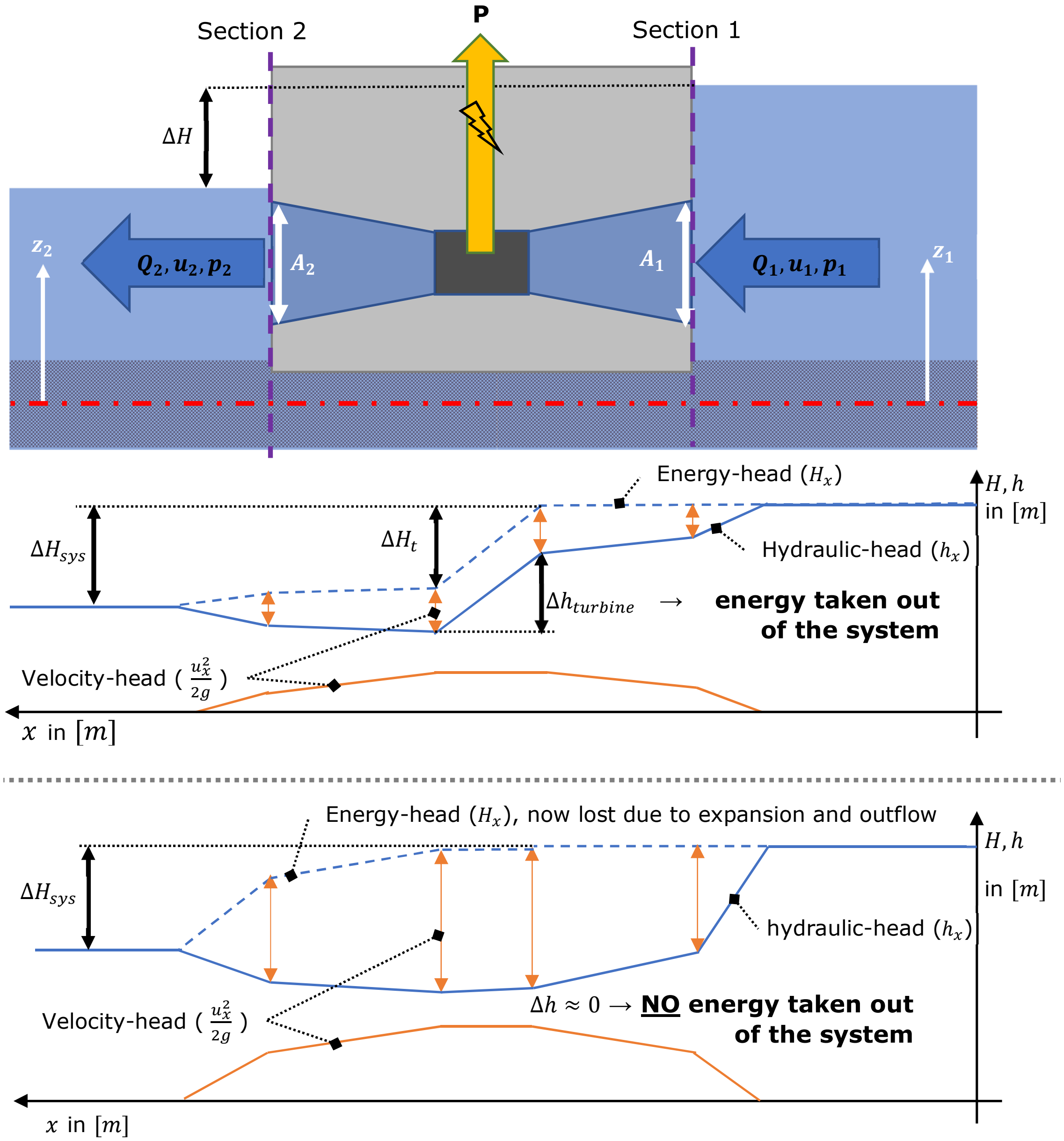


***Figure L 13** - Simplified, ideal hydropower system. Wall friction, inflow and contraction losses considered negligible.*

Comparing the situation schematised in the bottom graph of **Figure L 10** (without energy extraction), it can be seen that the velocity head in this case is much higher than in the case of the top graph of **Figure L 10** (with extraction).

This is also the reason why, when the generator is not enacting any resistance/load on the rotor, the turbine will free-spin, which leads to a much higher rotational speed than when the turbine is experiencing load.

Turbine theory

A turbine is a machine that falls in the group called “turbo-machines”. The definition of which, according to Dixon 1998 [17], is as follows:

> *''We classify as turbomachines all those devices in which energy is transferred either to, or from, a continuously flowing fluid by the dynamic action of one or more moving blade rows.''* - Dixon, S.L. 1998 *[17]*

The origin of the word “turbo” or “turbinis” traces back to Latin to something that spins or whirls around. The “moving blade rows” that is referred to in the quote points at the “rotor” or “impeller” that changes the enthalpy[2] (internal energy) of the fluid flowing through this enclosed system. The change is done by the rotor either doing positive of negative work, i.e. adding or extracting energy from the flow. A turbine is the latter of these.

Due to assumptions done at **“Introduction of hydro-power-formulae and principles”** the change in enthalpy is directly related to changes in pressure of the fluid.

Specific speed definition

The specific speed, sometimes also referred to as the shape number, is an important parameter that defines the type/class of turbine and says something about its range of operation. The relation can be derived with dimensional analysis or from hydraulic scaling theory. Both derivations are given in **Appendix 3 – Turbo-machinery-theory**.

For the dimensional approach the Buckingham’s Pi theory is used. With Buckingham’s Pi theory 4 independent dimensionless groups can be formed. The 4th group can be discarded because it is equivalent to the Reynolds number. Flow in a turbine in practise is always turbulent and variations in the viscosity will not have a significant effect on the performance of the turbine. Two other terms can be combined, such that the diameter cancels out of the equation to form the power specific speed and derived specific speed relation as shown below:

$$N_{sp} = \frac{(\hat{P})^{\frac{1}{2}}}{(\psi)^{\frac{5}{4}}} = \frac{N * \left(\frac{P}{\rho}\right)^{\frac{1}{2}}}{(g\Delta H_t)^{\frac{5}{4}}} \xrightarrow[\eta\rho g\Delta HQ)]{Subs.(P=} \frac{N * \left(\frac{\eta * \rho * g * \Delta H_t * Q_t}{\rho}\right)^{\frac{1}{2}}}{(g\Delta H_t)^{\frac{5}{4}}} = N_s = \frac{N * (\eta * Q_t)^{\frac{1}{2}}}{(g\Delta H_t)^{\frac{3}{4}}} \qquad \textbf{(L 40)}$$

Where:
$\hat{P} = \frac{P}{\rho * N^3 * D^5}$ The dimensionless power coefficient
$\psi = \frac{g\Delta H}{(N * D)^2}$ The dimensionless head or energy transfer coefficient
$N_{sp} =$ The (power) specific speed in $[rpm]$ or “revs”
$\Delta H_t =$ The head-difference over the turbine in $[m]$
$Q_t =$ The discharge through the turbine in $[m^3/s]$
$P =$ Turbine power in $[kW]$
$N =$ Rotations speed of the turbine in $[rpm]$ or “revs”
$N_s =$ The specific speed in $[rpm]$ or “revs”
$\eta =$ The turbine efficiency in $[\%]$

The specific speed can be seen as the comparing two similar turbines where, for instance, one is a prototype and the other is the full-scale turbine. The dimensionless coefficients will stay the same for both.

---

[2] Enthalpy is the sum of the internal energy and the product of the volume and pressure of a fluid of an enclosed system. It has the (SI) unit of Joule.

When for instance the diameter changes with factor x and to keep the same power coefficient, the power will change with factor x to the power 5.

The prototype is defined in such a way that it will generate $1W$ of power at $1m$ of head and have a discharge of $1\ m^3/s$ (hence the name "specific", this means per unit discharge, power, and head).

A ratio between the prototype and full scale turbine can be seen as the speed ratio.

$$r_s = \frac{N}{N_s}\left(or\ r_{sp} = \frac{N}{N_{sp}}\right) \qquad \textbf{(L 41)}$$

Where:
$r_s =$ The speed ratio [dimensionless].

For reasons that will become apparent later it is most practical to define r_s as N over N_s.

Head-discharge relation from specific speed

Due to the fact that both head-difference and discharge are in this relation for specific speed, the function can be used to determine the head-difference over the turbine when combined with the head-losses in a pipe-system. The $\Delta H_{turbine}$ in formula **(L 39)** was still unknown as of now. Rewriting **(L 40)** a relation between the turbine discharge and head-difference is found:

$$\Delta H_t = \frac{(\eta * Q_t)^{\frac{2}{3}}}{g}\left(\frac{N}{N_s}\right)^{\frac{4}{3}} \qquad \textbf{(L 42)}$$

Where:
$\Delta H_t =$ The head-difference over the turbine in $[m]$

Combining above with **(L 39)**:

$$\Delta H_{sys} = \Delta H_t + Q_t^2 * C \xrightarrow{substitute} \Delta H_{sys} = (Q_t)^{\frac{2}{3}} * \frac{\eta^{\frac{2}{3}}}{g}\left(\frac{N}{N_s}\right)^{\frac{4}{3}} + Q_t^2 * C \qquad \textbf{(L 43)}$$

Where:
$\Delta H_{sys} =$ The head-difference over the entire system, the total available head-difference in $[m]$
$Q_t^2 * C =$ The sum of all the minor losses in the pipe-system in $[m]$.

To find the relation to be found suggested in **(L 7)** (i.e. $f_2(Q_{ava}, \Delta H_{ava})$), equation **(L 43)** needs to be solved for Q_t. This can be done numerically by means of Newton-Raphson method [18].

To find the root, each step of the following relation should converge to the answer:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \qquad \textbf{(L 44)}$$

Where:
$f(x_n) =$ Function that the root is being searched for that is dependent on variable x.
$f'(x_n) =$ The first derivative of $f(x_n)$ with respect to variable x.
$x_n =$ Either the initial guess of the root, or the value found in a previous iteration number "n".
$x_{n+1} =$ The new estimate of the root for the current iteration.

According to [18] the error reduces with:

$$\epsilon_{k+1} \approx {\epsilon_k}^2 * \frac{f''(x^*)}{2f'(x^*)} \qquad \textbf{(L 45)}$$

Where:

$x^* =$ The actual value of the root, meaning estimating the error with the found value of the root will give an approximate error.
$f'(x^*) =$ The first derivative of $f(x_n)$ with respect to variable x.
$f''(x^*) =$ The second derivative of $f(x_n)$ with respect to variable x.

$\epsilon_k =$ The error at the k-th step in the iteration process.
$\epsilon_{k+1} =$ The new estimate of the error.

So if there is convergence, it happens quadratically.

Assuming for the moment that $\eta = 100\%$ and applying **(L 44)** to equation **(L 43)** this is:

$$\Delta H_{sys} = Q_t^{\frac{2}{3}} \frac{1}{g} \left(\frac{N}{N_s}\right)^{\frac{4}{3}} + Q_t^2 C \xrightarrow{rewrite} f(Q_t) = Q_t^{\frac{2}{3}} \frac{1}{g} \left(\frac{N}{N_s}\right)^{\frac{4}{3}} + Q_t^2 C - \Delta H_{sys} \qquad \textbf{(L 46)}$$

Solve Q_t with Newton-Raphson for:
$f(Q_t) = 0$

$$\frac{d}{dQ_t} f(Q_t) = \frac{2}{3 * g * (Q_t)^{\frac{1}{3}}} * \left(\frac{N}{N_s}\right)^{\frac{4}{3}} + 2 * Q_t * C$$

Therefore the following will lead to the root:

$$Q_{n+1} = Q_n - \frac{\frac{(Q_n)^{\frac{2}{3}}}{g} \left(\frac{N}{N_s}\right)^{\frac{4}{3}} + \frac{(Q_n)^2}{2g} G - \Delta H_{sys}}{\frac{2}{3 * g * (Q_n)^{\frac{1}{3}}} * \left(\frac{N}{N_s}\right)^{\frac{4}{3}} + 2 * Q_n * C} \qquad \textbf{(L 47)}$$

Where:

$Q_n =$ Either the initial guess of the root (if n=0), or the value found in a previous iteration (n>0).
$Q_{n+1} =$ The new estimate of the root for the current iteration.

Often not more than 3 to 5 iterations are needed till the value of $f(Q_t)$ **(L 46)** is close enough to 0, (assumed to be the case when $\frac{f(Q)}{\Delta H_{sys}} \leq 10^{-3}$) will give the value of Q_t that belongs to the that particular combination of $\Delta H_{sys}, C,\ N$ and N_s.

In the hydraulic models in this thesis using the turbo machinery theory, this is written as a loop and gives a numerical function $Q_t(\Delta H_{sys}, C,\ N, N_s)$, that can be used in other formulae to plot graphs and calculate other quantities like power output or the head over the turbine.

To make the relation complete the discharge of the complex is limited to the available discharge:

$$Q_{plant} = \min(Q_{ava}, Q_t(\Delta H_{ava}, C, \quad N, N_s)) \qquad \textbf{(L 48)}$$

Where:

$Q_{plant} =$ The summed discharge of all turbines in the power house
$Q_{ava} =$ The available discharge
$Q_t(\Delta H_{ava}, C,\ N, N_s) =$ The discharge through the turbine for a particular available head difference $\Delta H_{sys} = \Delta H_{ava}$ over the system and geometry determined by $C,\ N, N_s$

Optimisation of power output and the Speed ratio r_s

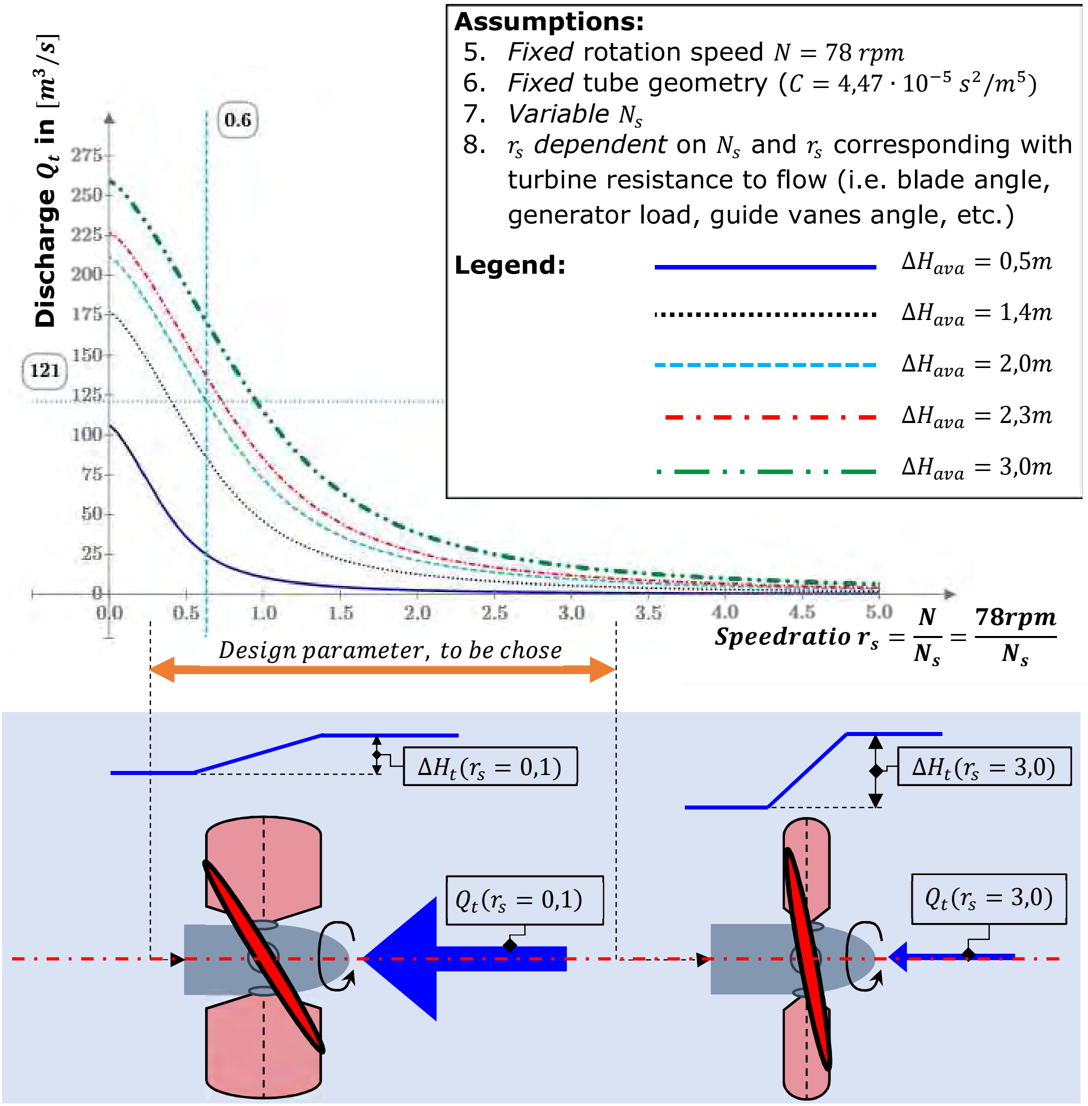
Optimising the power output isn't the main goal, however, to achieve the highest annual energy production the turbine should run on as high a power-output as possible for as long as possible, so knowing where actually the peak-power is found within certain ranges of the parameters is still useful.

The numerical function of $Q_t(\Delta H_{ava}\,, C,\ N, N_s)$ that is found before is now dependent on 4 variables, namely: $\Delta H_{ava}\,, C,\ N$ and N_s.

Earlier the speed ratio $r_s = \frac{N}{N_s}$ has been defined. Using this value instead of the two speeds reduces the amount of variables Q_t is dependent on to 3 of which:

- ΔH_{ava} is a boundary condition given by the flow regime,
- r_s is a design parameter or, in the case of a (double regulated) Kaplan turbine, can also be a regulating parameter
- C is another design parameter dependent on the chosen geometry

The discharge for a given head-difference ΔH_{ava} and quadratic resistance coefficient C can then be plotted as function of the speed-ratio r_s (see **Figure L 14**).



Assumptions:

5. *Fixed* rotation speed $N = 78\,rpm$
6. *Fixed* tube geometry ($C = 4{,}47 \cdot 10^{-5}\ s^2/m^5$)
7. *Variable* N_s
8. r_s *dependent* on N_s and r_s corresponding with turbine resistance to flow (i.e. blade angle, generator load, guide vanes angle, etc.)

Figure L 14 *- Discharge as function of speed ratio for a fixed C coefficient and available head*

This can also be done with the head-difference over the turbine, as equation **(L 42)** can be written with the speed ratio r_s as well:

$$\Delta H_t = \frac{(Q_t)^{\frac{2}{3}}}{g}(r_s)^{\frac{4}{3}} \qquad \textbf{(L 49)}$$

Important note 3

Assuming a **fixed rotation speed of the turbine "N"** (often also required for network frequency stability), the speed ratio functions as a measure for resistance of the turbine where a zero speed ratio equals no resistance of from the turbine:

$$r_s = 0 \quad \rightarrow \quad \Delta H_t(r_s = 0) = 0$$

For a given Kaplan turbine this means that when the rotor blade angle and guide vane angle is set to fully open as well as the generator resistance set to zero, the specific speed of the turbine theoretically goes to infinity. Thus:

$$as \quad r_s \rightarrow 0 \quad then \quad N_s \rightarrow \infty \quad and \quad \Delta H_t(r_s) \rightarrow 0$$

When r_s is exactly equal to 0 one can imagine the rotation speed N is also 0 and N_s is undefined at such a moment.

When speed ratio r_s increases, then so does the resistance of the turbine to the flow. Consequently N_s must decrease. For a given Kaplan turbine, a low specific speed corresponds with angles for the rotor blades and guide-vanes that are closed as much as possible without fully blocking flow and the generator also giving maximum resistance. Therefore:

$$as \quad r_s \rightarrow \infty \quad then \quad N_s \rightarrow 0 \quad and \quad \Delta H_t(r_s) \rightarrow \Delta H_{ava}$$

Technically the specific speed cannot really reach zero ($N_s \neq 0$), nor can the head losses be completely zero when there is flow. In practise it is:

$$as \quad r_s \rightarrow r_{s,max} \quad then \quad N_s \rightarrow N_{s,min} \quad and \quad \Delta H_t(r_s) \rightarrow \Delta H_{t,max}$$

Where:

$$\Delta H_{t,max} = \Delta H_{ava} - C * \left(Q_t(r_{s,max})\right)^2 \qquad \textbf{(L 50)}$$

$$r_{s,max} = \frac{g * (\Delta H_{t,max})^{3/4}}{Q^{1/2}} = \frac{g * \left[\Delta H_{sys} - C * \left(Q_t(r_{s,max})\right)^2\right]^{3/4}}{\left(Q_t(r_{s,max})\right)^{1/2}} \qquad \textbf{(L 51)}$$

$$N_{s,min} = \frac{N}{r_{s,max}} = \frac{N * \left(Q_t(r_{s,max})\right)^{1/2}}{g * \left[\Delta H_{sys} - C * \left(Q_t(r_{s,max})\right)^2\right]^{3/4}} \qquad \textbf{(L 52)}$$

In **Figure L 15** on the next page the turbine head (ΔH_t), discharge (Q_t) and power (P_t) have been plotted for various available head differences (ΔH_{ava}) and a chosen quadratic resistance coefficient (C), that is based on the system installed in Maurik[3] (value of C shown below). This shows the influence of the speed ratio as explained in **Important note 1** above.

$$C_{Maurik} = \frac{1}{2g}\sum_{i=1}^{N}\frac{\xi_i}{A_i^2} = \frac{\xi_{eq}}{2g * (A_t)^2} = \frac{0{,}102}{2 * 9{,}81 m/s^2 * (10{,}80 m^2)^2} \approx 4{,}47 * 10^{-5}\ s^2/m^5$$

---

[3] The more detailed characteristics are listed later in this chapter.

For a fixed rotation speed $N = 78rpm$ and discharge coefficient C:

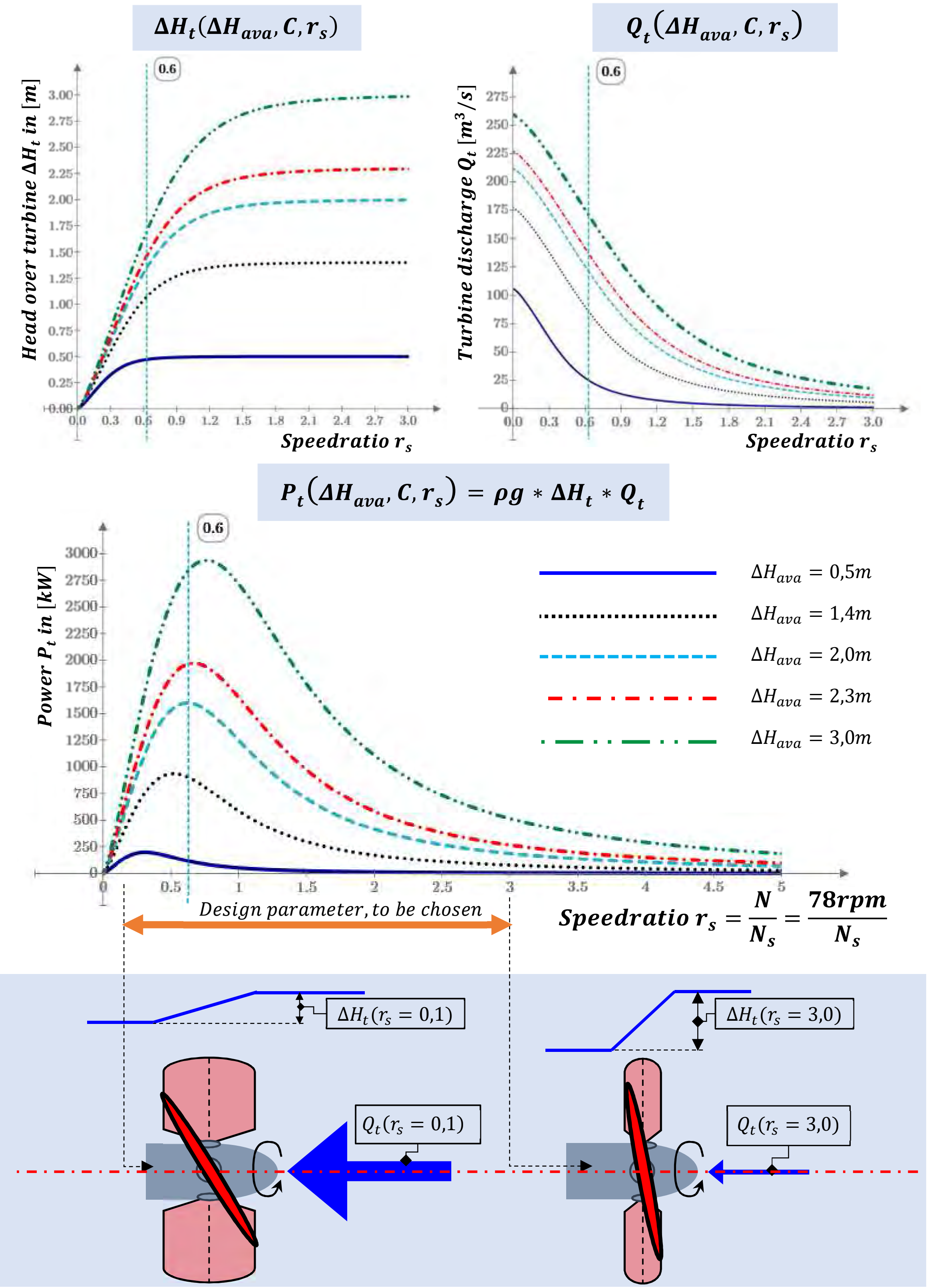


Figure L 15 *- Plot of head, discharge and power ratio against speed ratio $r_s = \frac{N}{N_s}$.*

The head-difference over the turbine ΔH_t can be normalised by the available head-difference over the structure ΔH_{ava} (see **(L 53)** below). The discharge through the turbine when loaded $Q_t(r_s > 0)$ can be normalised with the discharge that goes through the system when no load is applied (i.e. $Q_{NoLoad} = Q_t(r_s = 0)$, see also **Important note 1** and in the formula **(L 54)** below).

The power can be normalised by the product of the available head ΔH_{ava} and the unloaded discharge Q_{NoLoad}, which is equal to the total the energy flux when no load is applied or equivalently is the sum of the power taken by the turbine and the energy flux of the losses (i.e. $\frac{dE}{dt}_{NoLoad} = \rho g * Q_{NoLoad} * \Delta H_{ava} = \frac{dE_t}{dt} + \frac{dE_{loss}}{dt}$, see formula **(L 55)** below). All of these normalised values are shown in the relations below:

$$r_H(\Delta H_{ava}, C, r_s) = \frac{\Delta H_t(\Delta H_{ava}, C, r_s)}{\Delta H_{ava}} \qquad \textbf{(L 53)}$$

$$r_Q(\Delta H_{ava}, C, r_s) = \frac{Q(\Delta H_{ava}, C, r_s)}{Q(\Delta H_{ava}, C, 0)} = \frac{Q_t}{Q_{NoLoad}} \qquad \textbf{(L 54)}$$

$$r_P(\Delta H_{ava}, C, r_s) = \frac{P_t}{\max\left(\frac{d}{dt}E_{flow}\right)} = \frac{\rho g * Q_t * \Delta H_t(\Delta H_{ava}, C, r_s)}{\rho g * Q_{NoLoad} * \Delta H_{ava}} = r_H * r_Q \qquad \textbf{(L 55)}$$

Where:

$r_H(\Delta H_{ava}, C, r_s) =$	The dimensionless head ratio as function of available system head, quadratic resistance coefficient and speed ratio.
$r_Q(\Delta H_{ava}, C, r_s) =$	The dimensionless discharge ratio as function of available system head, quadratic resistance coefficient and speed ratio.
$Q_t =$	the loaded discharge (where the turbine gives resistance to the flow) in $[m^3/s]$
$Q_{NoLoad} =$	the unloaded discharge in $[m^3/s]$

Combining these factors with **Figure L 15** a plot of the normalised head, discharge and power can be made, shown in **Figure L 16** on the next page.

$\boldsymbol{Plotted\ against\ speedratio\ are\ for}$: ΔH_{ava} $= 0{,}50m$: $= 3{,}00m$:

$Head\ ratio\ r_H = \frac{\Delta H_t}{\Delta H_{ava}}$:	- - - -	―
$Discharge\ ratio\ r_Q = \frac{Q_t}{Q_{NoLoad}}$:	- - - -	―
$Power\ ratio\ r_P = r_H * r_Q$:	- - - -	―

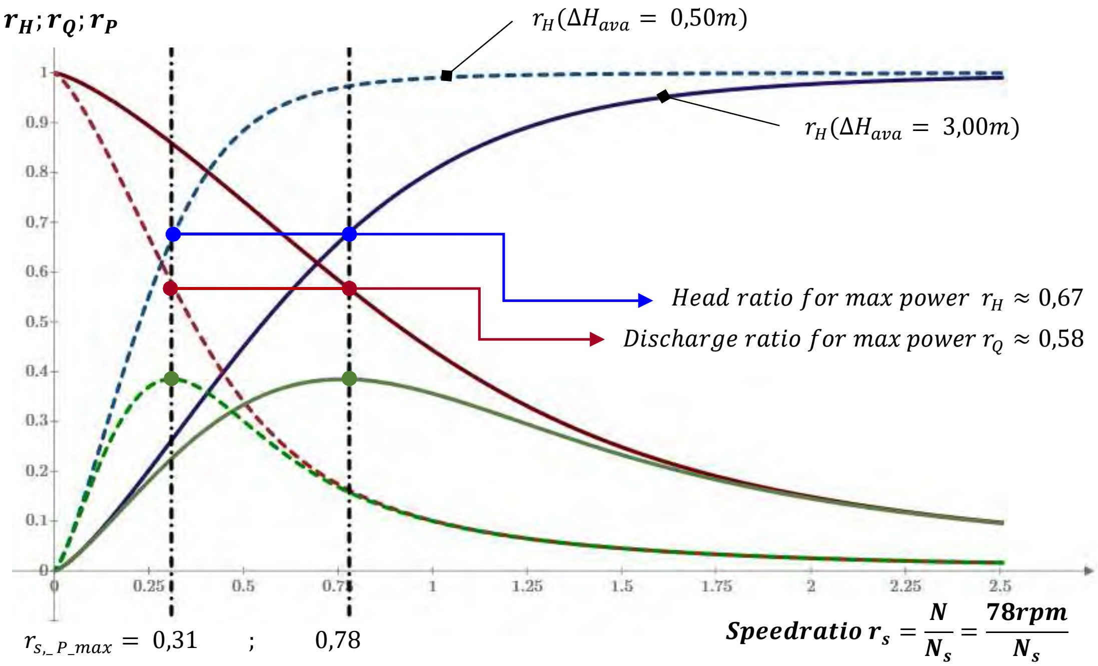


Figure L 16** - Plot of head, discharge and power ratio against speed ratio $r_s = \frac{N}{N_s}$. See **Appendix 3

Interestingly, the power output peaks at a certain value of r_s. This is because Q_t and ΔH_t have a more or less reciprocal relationship, where the discharge decreases with increasing turbine head. The maximum of the power P_t is P_{max}, consistently occurs when the turbine head is around $0{,}667 \approx 2/3$ of ΔH_{ava}. The speed ratio where this happens however, is different for each combination of system head and discharge coefficient C (see also **Figure L 16** above).

Important note 4

This head-ratio of $r_H = 2/3$ is only relevant when enough flow is available for the chosen diameter. If the available discharge is less than Q_{NoLoad} of the turbine, then the discharge curve has a point where it reaches its maximum and doesn't increase more if the speed ratio is reduced further. In other words r_Q will not reach 1,0 when r_s goes below the point where $Q_{ava} \leq Q_t(r_s)$ This has the effect of the location of maximum power in the graph is shifted to the right and no longer coincides with $r_H = 2/3$.

The maximum power output and related head ratio is therefore very much dependent on the available discharge and the size of the turbine. There is not one fixed head ratio that gives maximum power for all situations.

Examples

To illustrate the statement in the **Important note 2** two examples have been worked out below. First one uses a head ratio of 95% and the second one a head ratio of 2/3 for a situation where there is an available system head of 1,4m. The turbines are designed such that both generate the same amount of power of 130,4 kW.

Minimal loss example

For this example a head-ratio of $r_H = 0{,}95$ is chosen. In other words, the accepted losses will be 5% of the system head. This means, reading from **Figure 16**, that the discharge ratio lies around $r_Q = 0{,}224$ of the discharge through the system without turbine resistance.

Assuming a discharge through the turbine of $10m^3/s$ is desirable, that means that the discharge without turbine resistance would be: $Q_{NoLoad} = \frac{10}{0{,}224} \approx 44{,}65m^3/s$.

Q_{NoLoad} is a function of G and ΔH_{system}, namely:

$$\Delta H_{system} = G * \frac{(Q_{NoLoad})^2}{2g} \xrightarrow{rewrite} Q_{NoLoad} = \sqrt{\frac{2g * \Delta H_{system}}{G}} \qquad \textbf{(22)}$$

Or in other words:

$$G_d = \frac{2g * \Delta H_{system}}{(Q_{NoLoad})^2} \qquad \textbf{(23)}$$

Where:

$G_d =$ The design value for the resistance in $[m^{-4}]$

Assuming the design system head is $\Delta H_{system} = 1{,}4m$. The speed ratio then needs to be: $r_{S_{95\%}} = 2{,}17$. And the design value of the resistance must be $G_d = \frac{2g*\Delta H_{system}}{(Q_{NoLoad})^2} = 1{,}377 * 10^{-2}m^{-4}$

Because G is a summation of losses with different cross-sectional areas, some iterations will be necessary to find the turbine area. Once this is found the diameter can be calculated.

For this particular case, assuming : $\xi_{equivalent} = 0{,}212 = constant$

$$G_d = \frac{\xi_{equivalent}}{\left(A_{turbine_d}\right)^2} = \sum_{i=1}^{N} \frac{\xi_i}{A_i^2} = 1{,}377 * 10^{-2}\ m^{-4} \xrightarrow{rewrite} A_{turbine_d} = \sqrt{\frac{\xi_{equivalent}}{G_d}}$$

$$A_{turbine\_d} = \sqrt{\frac{0{,}212}{1{,}377 * 10^{-2}}} \approx 3{,}92m^2$$

Ignoring for the moment that actually the discharge area should actually be the blade area (i.e. area covered by the shaft diameter where the blades are attached to doesn't let through water), the diameter is as follows:

$$D_d = \sqrt{\frac{4 * A_{turbine\_d}}{\pi}} \approx 2{,}235m$$

The power delivered by the turbine, assuming for the moment $\eta = 100\%$, is then:

$$P_t = \eta * \rho * g * \Delta H * Q = 1000 * 9{,}81 * 1{,}33 * 10 = 130{,}4\ kW$$

To check the outcome:

By The head-losses with the active turbine discharge of $10m^3/s$, with a $G_d = 1{,}377 * 10^{-2} * m^{-4}$ are indeed 5% of the system-head:

$$\Delta H_{loss} = G_d * \frac{(Q_{turbine})^2}{2g} = 1{,}377 * 10^{-2} m^{-4} * \frac{\left(\frac{10m^3}{s}\right)^2}{2 * \frac{9{,}81m}{s^2}} = 0{,}07m \overset{equals}{\Longleftrightarrow} 1{,}4m * \frac{5}{100}$$

The head over the turbine with this discharge and a speed ratio of $r_{s_{95\%}} = 2{,}17$ is indeed:

$$\Delta H_{turbine} = \frac{(Q_{turbine})^{\frac{2}{3}}}{g} * \left(r_{s_{95\%}}\right)^{\frac{4}{3}} = \frac{(10)^{\frac{2}{3}}}{9{,}81} * (2{,}17)^{\frac{4}{3}} = 1{,}33m$$

So the sum of ΔH_{loss} and $\Delta H_{turbine}$ does indeed add up to $\Delta H_{system} = 1{,}33m + 0{,}07m = 1{,}4m$

Minimal size example

If the same power is to be produced with maximal hydraulic efficiency the head-ratio needs to be $r_H = \frac{2}{3} \approx 0{,}667$ or in other words, the losses will be 33,3% of the system head. This means that the discharge ratio lies around $r_Q = 0{,}577$ of the discharge through the system without turbine resistance.

With a power of 130,4kW the discharge through the turbine needs to be:

$$Q_{turbine} = \frac{P_t}{\eta * \rho * g * \Delta H} = \frac{130{,}4\ kW}{1000 * 9{,}81 * 1{,}4 * \frac{2}{3}} \approx 14{,}25m^3/s$$

And the discharge without the resistance is:

$$Q_{NoLoad} = \frac{14{,}25m^3/s}{0{,}577} \approx 24{,}71m^3/s$$

With the same design system head is $\Delta H_{system} = 1{,}4m$, using **(22)** and **(23)** the hydraulic resistance G is:

$$G_d = \frac{2g * \Delta H_{system}}{(Q_{NoLoad})^2} = 4{,}497 * 10^{-2}\ m^{-4}$$

The speed ratio for $\Delta H_{system} = 1{,}4m$, $r_H = 2/3$ and $G_d = 4{,}497 * 10^{-2}\ m^{-4}$ needs to be:

$$r_{s_{2/3}} = 1{,}39$$

Again for this case assuming : $\xi_{equivalent} = 0{,}212 = constant$

$$A_{turbine\_d} = \sqrt{\frac{\xi_{equivalent}}{G_d}} = \sqrt{\frac{0{,}212}{4{,}497 * 10^{-2}\ m^{-4}}} \approx 2{,}18m^2$$

$$D_d = \sqrt{\frac{4 * A_{turbine\_d}}{\pi}} \approx 1{,}67m$$

The power delivered by the turbine is the same only the diameter of the turbine is 25,5% smaller (i.e. 74,5% of the diameter of 95% head-ratio turbine).

To be complete and also check the outcome for this example:

The head-losses with the active turbine discharge of $Q_{turbine} = 14{,}25m^3/s$, with a value of $G_d = 4{,}497 \ * 10^{-2}\ m^{-4}$ are indeed 33,3% of the system-head:

$$\Delta H_{loss} = G_d * \frac{(Q_{turbine})^2}{2g} = 4{,}497 \ * 10^{-2}m^{-4} * \frac{\left(\frac{14{,}25m^3}{s}\right)^2}{2 * \frac{9{,}81m}{s^2}} \approx 0{,}47m \overset{about\ equal}{\Longleftrightarrow} 1{,}4m * \frac{1}{3} \approx 0{,}467m$$

The head over the turbine with this discharge and a speed ratio of $r_{s_{2/3}} = 1{,}39$ is indeed:

$$\Delta H_{turbine} = \frac{(Q_{turbine})^{\frac{2}{3}}}{g} * \left(r_{s_{\frac{2}{3}}}\right)^{\frac{4}{3}} = \frac{(14{,}25)^{\frac{2}{3}}}{9{,}81} * (1{,}39)^{\frac{4}{3}} = 0{,}93m$$

So the sum of ΔH_{loss} and $\Delta H_{turbine}$ does indeed add up to $\Delta H_{system} = 0{,}93m + 0{,}47m \approx 1{,}4m$

Comparison of the examples

Summarizing the calculation results in the table below:

Quantity	**Unit**	**Case 1: Minimal loss**	**Case 2: Minimal size**
r_h	$-$	95%	67%
ΔH_{sys}	m	1,40	1,40
$\Delta H_{turbine}$	m	1,33	0,93
ΔH_{loss}	m	0,07	0,47
$Q_{turbine}$	m^3/s	10,00	14,25
Q_{NoLoad}	m^3/s	44,65	24,71
r_Q	$-$	0,224	0,577
η	$-$	100%	100%
$\boldsymbol{P_t}$	$\boldsymbol{kW}$	**130,4**	**130,4**
$\boldsymbol{D_d}$	$\boldsymbol{m}$	**2,24**	**1,67**
$A_{turbine}$	m^2	3,92	2,18
r_s	rpm/rpm	2,17	1,39
ξ_{eq}	$-$	0,212	0,212
C_d	s^2/m^5	7,02*10^{-04}	2,29*10^{-03}

Table 36 *- Summarizing results of example calculations for two different head-ratios* r_h

A downside of the system in the last example is that the turbine will stop working when the system head is low, due to the fact that:

$$\Delta H_{system\_threshold} = r_H^{-1} * \Delta H_{Turbine\_threshold} \qquad \textbf{(24)}$$

Where:

$\Delta H_{Turbine\_threshold} =$	The minimum required head over the turbine for it to (start) work(ing) in $[m]$
$\Delta H_{system\_threshold} =$	The derived minimum system-head for which the turbine still works in $[m]$.

So for last example:

$$\Delta H_{system\_threshold} = 1{,}5 * \Delta H_{Turbine\_threshold}$$

Where the other system from the first example will work up till:

$\Delta H_{system\_threshold} = 1{,}053 * \Delta H_{Turbine\_threshold}$

On the other hand, the diameter of the turbine with the 2/3 head-ratio is a factor 74,5% smaller and the discharge area even a factor 55,6% smaller. Therefore it depends very much on the location and the hydraulic resources what is the most optimal choice of head-ratio.

If there is enough space, the hydraulic efficiency is highest when the highest head-ratio is chosen. This means that the largest possible diameter needs to be chosen, because then factor G is lowest and the head-losses smallest.

Cavitation limits

Cavitation is quite a complex phenomenon and will not be examined in too much detail. However, it has an influence on the design as it determines the depth at which the turbine needs to be installed.

In simple terms cavitation happens when the static pressure of the fluid is reduced to the vapour pressure for a certain temperature. In practise it is a more complex phenomenon and dependent on the physical state of the liquid. [17, p. 13]

Gasses dissolved come out of the solution when pressure goes down and create gas cavities. Interestingly, when no particles have been dissolved, a liquid can actually sustain negative pressures (tensile stresses), however this has only been achieved in laboratories and is not working practise for turbo-machinery.

The commonly used parameter is the available suction head, the "net positive suction head" (NPSH), although more commonly used for pumps, which is defined as:

$$\mathrm{H_{NPS}} = \frac{p_0 - p_v}{\rho g} \qquad \textbf{(L 56)}$$

Where:

$p_0 =$ The absolute stagnation pressure in $[Pa]$
$p_v =$ The vapour pressure in $[Pa]$
$\mathrm{H_{NPS}} =$ The NPSH in $[m]$

As Dixon noted:

> *"… To take into account the effects of cavitation, the performance laws of a hydraulic turbomachine should include the additional independent variable H_s." – Dixon [17, p. 14]*

Therefore, in the analysis should be included the suction specific speed (N_{ss}) and the efficiency as function of this N_{ss}:

$$N_{ss} = \frac{N * Q^{\frac{1}{2}}}{(g * H_{NPS})^{\frac{3}{4}}} \qquad \textbf{(L 57)}$$

Where:

$p_0 =$ The absolute stagnation pressure in $[Pa]$
$p_v =$ The vapour pressure in $[Pa]$
$\mathrm{H_{NPS}} =$ The NPSH in $[m]$

And:

$$\eta = f(\phi, N_{ss}) \qquad \textbf{(L 58)}$$

Where:

$\phi = \frac{Q}{ND^3} =$ The discharge/flow coefficient

Combining

$$N_{ss} = \frac{N * (\eta * Q)^{\frac{1}{2}}}{(g * H_{NPS})^{\frac{3}{4}}} \qquad \textbf{(L 59)}$$

Like the relation that was found in **(L 40)**.

Cavitation mostly determines the depth at which the turbine needs to be placed and is therefore important for the costs as excavation depth plays a huge role in the costs in civil engineering projects.

For horizontal axis turbines the reference height towards the tail-water is the turbine axis.

From the hydro-power theory the admissible head formula is used:

$$h_{s,adm} = h_{at} - h_{vap} - \sigma_{Ts} * h_f \qquad \textbf{(L 60)}$$

Where:

$h_{s,adm} =$ The admissible draft head in $[m]$. This is the minimal depth below the operational tailwater level.

$h_f =$ The usable fall head in [m]

$h_{at} =$ The atmospheric pressure head in [m] $h_{at} = \frac{p_{atm}}{\rho g}$ (at sea-level about 10m)

$h_{vap} =$ The vapour pressure head in [m] $h_v = \frac{p_v}{\rho g}$

$\sigma_{Ts} =$ Thoma's cavitation coefficient.

For Kaplan turbines a graph is available based on specific speed N_q. Also important to note is that the vapour pressure is temperature dependent. The vapour pressure head h_v is larger with higher temperature.

Temperature in $[°C]$	Vapour pressure head h_{vap} in $[m]$
0	0,062
10	0,125
20	0,238
30	0,433
40	0,752

***Table 37** - Vapour pressure head for different temperatures. - Source: Lecture slides Hydro-power engineering*

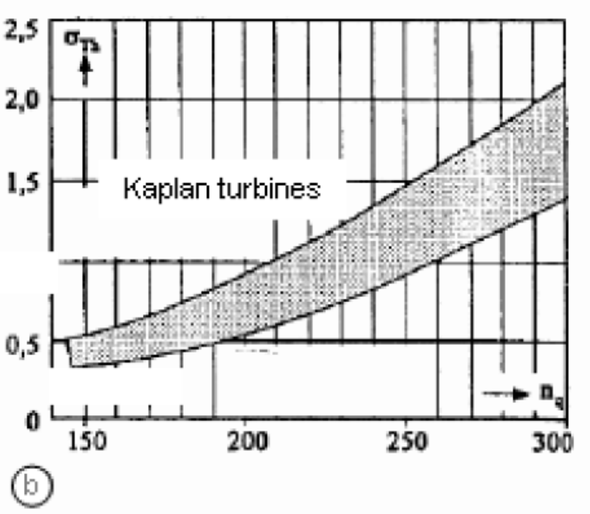


***Figure L 17** - Thoma's cavitation coefficient σ_{Ts} per specific speed N_q*

Annual Energy production

The annual energy production is a time integral of the power, as shown in **(L 61)**.

$$E_{annual} = \int_0^{t_{year}} P(t)\, dt \qquad \textbf{(L 61)}$$

Where:

$P(t) =$ The instantaneous power output of the considered system at time t in $[kW]$

The resource supply (discharge and head) for a hydro-power turbine is not always present and thus a turbine can often not run at full capacity the entire year. They have a considerable time of "down-time" or times where the turbine runs at reduced efficiency. In the energy production industry a common way to express the percentage of "uptime" is by the Capacity Factor.

$$CF = \frac{E_{annual}}{t_{year} * P_{rated}} = \frac{t_{full-load}}{t_{year}} \qquad \textbf{(L 62)}$$

Where:

$CF =$ Capacity factor in $[\%]$

$E_{annual} =$ The amount of energy produced in a year by the considered system in Joules $[J]$ or more commonly in kilo-Watt-hours $[kWh]$

$t_{year} =$ Amount of time in a year in $[s]$ or more commonly in $[hours]$, which is about 8760 hours.

$t_{full-load} =$ The equivalent amount of time in $[hours]$ the turbine runs at full capacity in a year. $\left(t_{full-load} = \frac{E_{annual}}{P_{rated}}\right)$

$P_{rated} =$ The rated (maximum) power-output of the considered system in Joules per second $[J/s]$ or more commonly in kilo-Watts $[kW]$

A global average capacity factor for hydro power, according to IPCC report on hydro-power of 2015, is 44% [10], based on a combined capacity of 926 GW (gigawatt) and combined annual energy production of 3,551*10^6 GWh/year (Gigawatt hours per year).

APPENDIX 3 – TURBO-MACHINERY-THEORY

– see inserted pages behind this page –

Turbo machinery theory - Dimensional analysis and performance laws

Introduction:
Definition of turbomachines:

"*We classify as turbomachines all those devices in which energy is transferred either to, or from, a continuously flowing fluid by the dynamic action of one or more moving blade rows." - Dixon, S.L. 1998*

The word *turbo* or *turbinis* is of Latin origin and implies that which spins or whirls around. Essentially, a rotating blade row, a rotor or an impeller changes the stagnation enthalpy of the fluid moving through it by either doing positive or negative work, depending upon the effect required of the machine. *These enthalpy changes are intimately linked with the pressure changes occurring simulataneously in the fluid.*

(Enthalpy is the internal energy plus the product of the volume and the pressure of a system, it has the SI unit of joule)

Dimensional analysis is a way to reduce a group of variables of a physical situation is reduced to a smaller number of dimensionless groups. It allows for experimentatal relations to be found relating the variables with each-other.

Dimensional analysis applied to turbo-machines can be used also for the following:
1) prediction of a prototype's performance from tests conducted on a scale model
2) determine the most suitable type of mahcine based only on efficiency, range of head, speed and flowrate.

Image source - Dixon, S.L. 1998

2 *Fluid Mechanics, Thermodynamics of Turbomachinery*

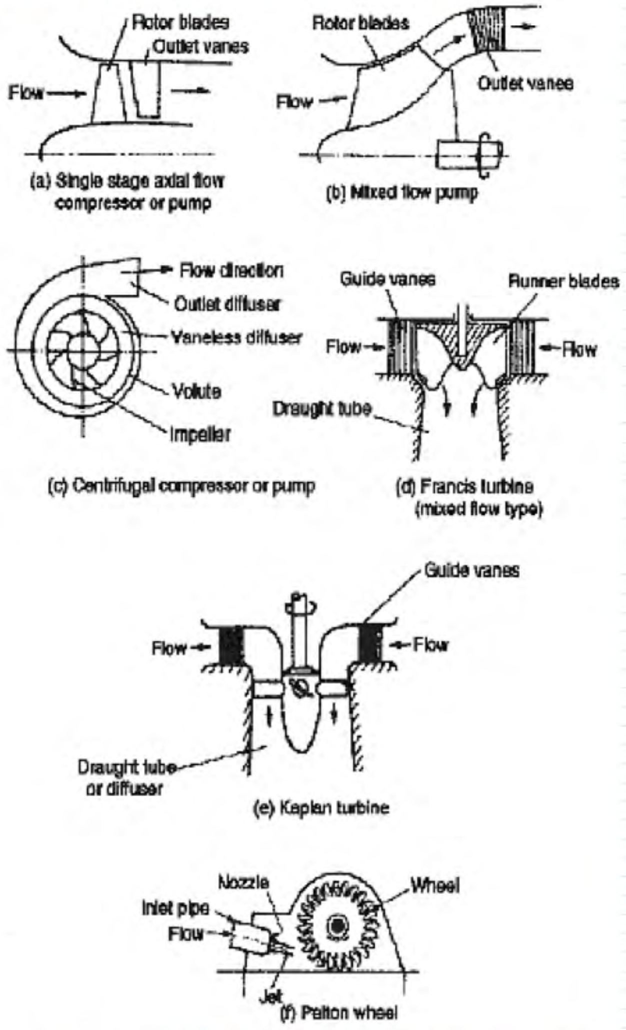


FIG. 1.1. Diagrammatic form of various types of turbomachine.

To apply the dimensional analysis to turbo machinery, consider a control surface, where on the interface at station (1) flow enters, and at station (2) leaves the control surface. Another interface with the control surface is the shaft where work is transmitted either to or from the control surface.

Control volume/surface:

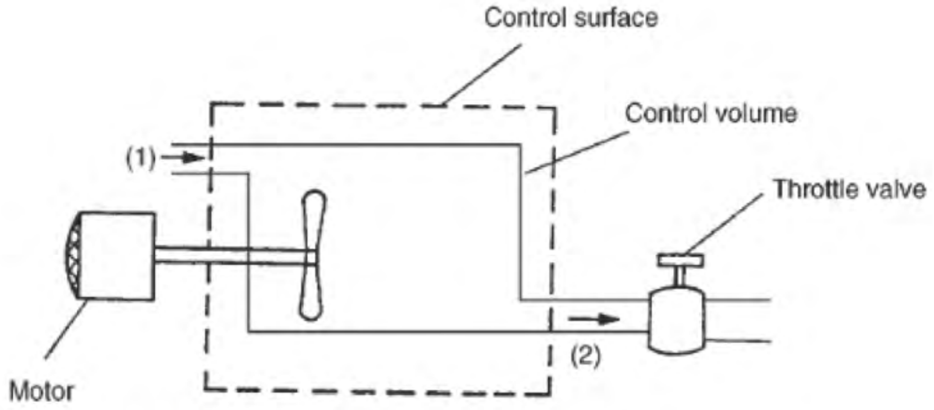


FIG. 1.2. Turbomachine considered as a control volume.

Image source - Dixon, S.L. 1998

Note:
- In figure 1.2 the Motor can also be generator.
- The throttle valve can adjust (reduce) the discharge independently of the turbomachine

Details of the flow inside are ignored for now and only features externally observed are considered, like:
- shaft speed/speed of rotation N,
- flow rate/discharge Q,
- torque on the shaft τ
- head H between station 1 and 2.

Two of these variables are chosen as *control variables.*

Also *fluid-properties* like density *ρ* and viscocity *μ* are important, they can change any of the above mentioned variables. For incompressible fluids, they differ per fluid, but don't in the flow.

Then there are the *geometric variables* that influence the performance of the turbo-machine. The machine can be charracterised by the impeller diameter D and all the other dimensions can be expressed as a ratio of this diameter e.g. l1/D, l2/D, etc.

The flow can be described with these 3 groups of variables: Control, fluid-properties and geometric variables.

Buckingham's Pi Theorem
Assumptions:
- Mechanical efficiency is the same
- the performance of a particular geometrically similar family of pumps/turbines ("Homologous series") may be expected to depend on:

Discharge $Q = \left[L^3 \cdot (T)^{-1}\right]$

Pressure change $\rho g \Delta H = \left[M \cdot L^{-1} \cdot (T)^{-2}\right]$

Power $P = \left[M \cdot L^2 \cdot (T)^{-3}\right]$

Rotor diameter $D = [L]$

Rotation rate $N = \left[(T)^{-1}\right]$

Fluid density $\rho = \left[M \cdot L^{-3}\right]$

Fluid viscosity $\mu = \left[M \cdot L^{-1} \cdot (T)^{-1}\right]$

From this we can conclude that there are **7 variables**, listed above, and **3 independent dimensions**, namely mass M, Length L and time T. This means that $N.\pi := 7 - 3 = 4$ independent, dimensionless groups can be made:

$$\Pi_1 = \frac{Q}{N \cdot D^3} \qquad \Pi_2 = \frac{g \Delta H}{(N \cdot D)^2} \qquad \Pi_3 = \frac{P}{\rho \cdot N^3 \cdot D^5} \qquad \Pi_4 = \frac{\rho \cdot N \cdot D^2}{\mu}$$

The fourth group can be recognised as being a form of Reynolds number.

i.e. $\Pi_4 = \dfrac{\rho N D^2}{\mu} \propto Re$

For fully turbulent flow, which is typically the operational range in turbines, the dependancy on molecular viscosity and thus on the Reynolds number is negligible, so this term is neglected.

What is left is a set of relations that are valid for geometrically similar pumps/ turbines, i.e. the ratios D/l1, D/l2, etc. are similar, but with different sizes D and speeds N:

$$\left(\frac{Q}{ND^3}\right)_1 = \left(\frac{Q}{ND^3}\right)_2$$

$$\left(\frac{g\Delta H}{(ND)^2}\right)_1 = \left(\frac{g\Delta H}{(ND)^2}\right)_2$$

$$\left(\frac{P}{\rho \cdot N^3 \cdot D^5}\right)_1 = \left(\frac{P}{\rho \cdot N^3 \cdot D^5}\right)_2$$

Where the indeces 1 and 2 indicated two such turbines/pumps.

Of these relations the first (Π_1) is called the *"Volumetric flow coefficient"* (Dixon, S.L. , 1998), which can be indicated with greek letter ϕ:

$$\phi = \frac{Q}{ND^3} \qquad \textbf{(1a)}$$

This coefficient is proportional to the velocity coefficient:

$$\phi \propto \frac{c.x}{U} \qquad \textbf{(1b)}$$

Where c.x is the average axial flow-velocity and U is the blade-tip-speed.

The second group (Π_2), is called the *"Energy transfer coefficient"* or *"Head-coefficient"*, which can be indicated with ψ:

$$\psi = \frac{g\Delta H}{(ND)^2} \qquad \textbf{(2)}$$

The last group to be considered (Π_3), is called the *"Power coefficient",* indicated by P^ circumflex, but here will be indicated with: $P.s$

$$P.s = \frac{P}{\rho \cdot N^3 \cdot D^5} \qquad \textbf{(3)}$$

The efficiency of both pumps and turbines can be described with these 3 relations as well:

$$\eta\_pump = \frac{\Pi_1 \cdot \Pi_2}{\Pi_3} = \frac{\phi \cdot \psi}{P.s} = \frac{\rho \cdot g \cdot Q \cdot \Delta H}{P} \qquad \textbf{(4a)}$$

and

$$\eta\_turbine = \frac{\Pi_3}{\Pi_1 \cdot \Pi_2} = \frac{P.s}{\phi \cdot \psi} = \frac{P}{\rho \cdot g \cdot Q \cdot \Delta H} \qquad \textbf{(4b)}$$

This also means that the power coefficient can be written as:

$$P.s = \eta\_turbine \cdot \psi \cdot \phi = \left(\frac{P}{\rho \cdot g \cdot Q \cdot H}\right) \cdot \left(\frac{Q}{ND^3}\right) \cdot \left(\frac{g\Delta H}{(ND)^2}\right) \qquad \textbf{(5)}$$

And the actual power as:

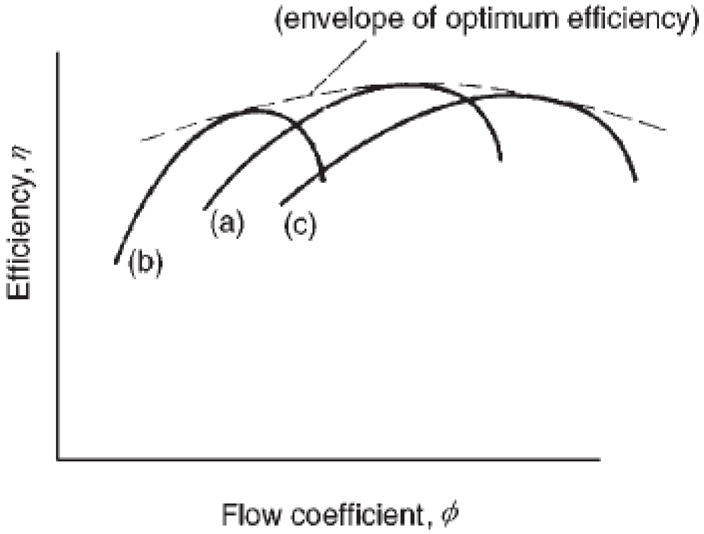
$$P = \eta\_turbine \cdot \psi \cdot \phi \cdot \rho \cdot N^3 \cdot D^5 \qquad \textbf{(6)}$$

Which, eliminating common terms, gives again the well known:

$$P = \eta\_turbine \cdot Q \cdot g\Delta H \cdot \rho = \eta\_turbine \cdot \rho g Q H$$

Variable geometry turbo-machines

Double regulated Kaplan turbines have a variable geometry, meaning the impeller/rotor blades and guide-vanes can be adjusted, to extend their range of high efficiency.
See image below.



In such a case the Head-coefficient also becomes a function of the blade angle, indicated with variable β. Each curve number k that has a different blade setting β gives one of the curves underneath the envelope shown in the image above and has a function of the form: $\eta = f_k(\phi, \beta)$

These curves all have a point $\eta = \eta_{max}$ for which the turbine has a unique value for the:

head coefficient	$\psi = \psi_1$
power coefficient	$P.s = P.s_1$
flow-coefficient	$\phi = \phi_1$

The envelope in the image above, gives all peak efficiencies $\eta_{max}\left(\phi_1\right)$ as a function of their corresponding ϕ_1 as a result of setting the blade angles to a specific β.

It is useful to define β as a function of discharge and head:
$\beta = f_1(\phi, \psi)$,

because then β can be eliminated to gain an expression for the efficiency in the following form:

$$\eta = f_2(\phi, \psi) = f_3\left(\frac{Q}{ND^3}, \frac{g\Delta H}{(ND)^2}\right)$$

This is why double regulated Kaplan turbines have a *Hill-chart* to either describe their power-output or efficiency, with on the axes discharge and head.

Specific speed

In all the previously defined expressions the Diameter of the turbine is present. To eliminate this variable for pumps often the following division is done to gain the expression for specific speed $N.s$:

$$N.s = \frac{\phi_1^{\frac{1}{2}}}{\psi_1^{\frac{3}{4}}} = \frac{\left(\frac{Q}{ND^3}\right)^{\frac{1}{2}}}{\left(\frac{g\Delta H}{(ND)^2}\right)^{\frac{3}{4}}} = \frac{N \cdot Q^{\frac{1}{2}}}{(g\Delta H)^{\frac{3}{4}}} \qquad (7)$$

This because the discharge and head are the desired results for pumps. For turbine the desired result is actually the power, so for that reason the power specific speed $N.sp$ is defined as follows:

$$N.sp = \frac{P.s_1^{\frac{1}{2}}}{\psi_1^{\frac{5}{4}}} = \frac{\left(\frac{P}{\rho \cdot N^3 \cdot D^5}\right)^{\frac{1}{2}}}{\left(\frac{g\Delta H}{(ND)^2}\right)^{\frac{5}{4}}} = \frac{N \cdot \left(\frac{P}{\rho}\right)^{\frac{1}{2}}}{(g\Delta H)^{\frac{5}{4}}} \qquad (8)$$

Note that both terms are dimensionless, except for the fact that they are multiplied with the rotational speed N. Thus the definitions of specific speed above have the same dimension as N, namely *revolutions per minute (rpm)*, or *1/s=Hz*, or *rad/sec*, but most commonly *rpm* is used.

If rad/s are used then formulae **(7)** and **(8)** are also written as follows:

$$\Omega.s = \frac{\Omega \cdot Q^{\frac{1}{2}}}{(g\Delta H)^{\frac{3}{4}}} \qquad (9)$$

and

$$\Omega.sp = \frac{\Omega \cdot \left(\frac{P}{\rho}\right)^{\frac{1}{2}}}{(gH)^{\frac{5}{4}}} \qquad (10)$$

$$\Omega.sp\_Maurik := \frac{\frac{78}{60} \cdot \frac{2\,\pi}{s} \cdot \left(\frac{2500\ kW}{1000\ kg \cdot m^{-3}}\right)^{\frac{1}{2}}}{\left(9.81\ m \cdot s^{-2} \cdot 3\ m\right)^{\frac{5}{4}}} = 5.958$$

In some literature the variables g and ρ are considered constant and therefore omitted, making the resutls for the specific speed dimensional. Then, what will be the value of the specific speed, depends greatly on what the input units are.

The ratio between the specific speed and the power specific speed actually has meaning, namely:

$$\frac{N.sp}{N.s} = \sqrt{\eta\_turbine}$$

The hydraulic efficiency was defined as **(4b)** and also equates to:

$$\eta\_turbine = \frac{P}{\rho \cdot g \cdot Q \cdot \Delta H} = \left(\frac{N.sp}{N.s}\right)^2$$

Specific speed from hydraulic scaling theory

Forgetting for the moment all the previously defined relations, geometric symilarity theory can also be used to derive the specific speed formula and gives some more insight where it's coming from.

Assumsion is that a turbine model that has been optimised (in controlled laboratry conditions) can be used to predict (optimal) performances of other turbines (assuming geometric symilarity).

From Bernoulli's theory the state of the fluid can be described at a point 1 just before the inflow point of the tube-system containing the turbine and an point 2 just behind the outflow point:

$$h_1+\frac{p_1}{\rho g}+\frac{{v_1}^2}{2\,g}=h_2+\frac{p_2}{\rho g}+\frac{{v_2}^2}{2\,g} \qquad \textbf{(S1)}$$

Where:

h_i the elevation above a reference level in $[m]$

p_i the pressure $[Pa]$

v_i the flow velocity in $[m \cdot s^{-1}]$

Assumed is that:

1) no pressure build up is happening at point 1 and point 2.

2) the flow-velocity at point 1 is $v_1=0\ m \cdot s^{-1}$

3) The reference level is at the downstream waterlevel $h_2=0\ m$

4) As such, the head difference is defined as: $\Delta H=h_1-h_2$

From that follows that:

$\Delta H=\frac{{v_2}^2}{2\,g}$ or for ease of writing: $\Delta H=\frac{v^2}{2\,g}$

Rewriting: $v=\sqrt{2\,g \cdot \Delta H}$ **(S2a)**

Assuming $Q=\mu A \cdot v$ --> $Q=\mu A \cdot \sqrt{2\,g \cdot \Delta H}$ **(S2b)**

Where μA is the effective discharge area in $[m^2]$.

Scaling factor x

Introducing scaling factor x, that is the ratio between a length dimension L' of the prototype and the length dimension L of the considered full scale installation like so:

$$x=\frac{L}{L'} \qquad \textbf{(S3)}$$

Then the ratio between areas scales with x^2 :

$$x^2=\frac{A}{A'} \qquad \textbf{(S4)}$$

And the ratio between the volumes scales with x^3 :

$$x^3=\frac{V}{V'} \qquad \textbf{(S5)}$$

With above defined scaling and assuming the factor μ doesn't change with the scaling, the discharge scales as follows:

$$\frac{Q}{Q'}=\frac{\mu}{\mu} \cdot \frac{A}{A'}\sqrt{\frac{2\,g \cdot \Delta H}{2\,g \cdot \Delta H'}}=x^2 \cdot \left(\frac{\Delta H}{\Delta H'}\right)^{\frac{1}{2}} \qquad \textbf{(S6)}$$

Defining the power of a turbine as:

$$P = \rho g \cdot Q \cdot \Delta H \cdot \eta \qquad \textbf{(S7)}$$

Using **(S7)** for the discharge scaling ratio, the power scales as follows:

$$\frac{P}{P'} = \frac{\rho g \cdot Q \cdot \Delta H \cdot \eta}{\rho g \cdot Q' \cdot \Delta H' \cdot \eta'} = \frac{\eta}{\eta'} \cdot \left(\frac{\Delta H}{\Delta H'}\right)^{\frac{3}{2}} \cdot x^2 \qquad \textbf{(S8)}$$

If it is assumed that the efficiency will be the same, then η and η' also fall out of the equation.
The flow-velocity scales as:

$$\frac{v}{v'} = \sqrt{\frac{2\ g \cdot \Delta H}{2\ g \cdot \Delta H'}} = \left(\frac{\Delta H}{\Delta H'}\right)^{\frac{1}{2}} \qquad \textbf{(S9)}$$

Assuming U is the blade-tip-speed, that it scales just like v and given that the radius of the turbine r is a length dimensions and thus also scales with x, then the angular velocity scales as follows:

$$\frac{n}{n'} = \frac{U \cdot (r)^{-1}}{U' \cdot (r')^{-1}} = \frac{U \cdot r'}{U' \cdot r} = \frac{1}{x} \left(\frac{\Delta H}{\Delta H'}\right)^{\frac{1}{2}} \qquad \textbf{(S10)}$$

Rewriting **(S10)** as x in terms of ΔH and n:

$$x = \frac{n'}{n} \left(\frac{\Delta H}{\Delta H'}\right)^{\frac{1}{2}} \qquad \textbf{(S11)}$$

Substituting **(S11)** into the power-scaling equation **(S7)**:

$$\frac{P}{P'} = \frac{\eta}{\eta'} \cdot \left(\frac{\Delta H}{\Delta H'}\right)^{\frac{3}{2}} \cdot \left(\frac{n'}{n} \left(\frac{\Delta H}{\Delta H'}\right)^{\frac{1}{2}}\right)^2 = \frac{\eta}{\eta'} \cdot \left(\frac{n'}{n}\right)^2 \left(\frac{\Delta H}{\Delta H'}\right)^{\frac{5}{2}} \qquad \textbf{(S12)}$$

Standardising this scaling formula to have a "Unit prototype" that has:
1) a prototype head of $\Delta H' = 1\ m$,
2) a prototype discharge of $Q' = 1\ m^3 \cdot s^{-1}$
3) generating a prototyp power of $P' = 1\ W$
The rotation speed the turbine prototype will have is $n' = n.s$ the specific speed.

Above applied to **(S12)**:

$$\frac{P}{1} = \frac{\eta}{\eta'} \cdot \left(\frac{n.s}{n}\right)^2 \left(\frac{\Delta H}{1}\right)^{\frac{5}{2}} \rightarrow P = \frac{\eta \cdot \Delta H^{\frac{5}{2}} \cdot n.s^2}{\eta' \cdot n^2}$$

making the (dimensional) specific speed as follows:

$$n.s = \frac{n \cdot \sqrt{\frac{\eta'}{\eta} \cdot P}}{\Delta H^{\frac{5}{4}}} \qquad \textbf{(S13)}$$

Using **(S7)** in **(S13)** the (dimensional) specific speed can also be writting as:

$$n.s = \frac{n \cdot \sqrt{\frac{\eta'}{\eta} \cdot \rho g \cdot Q \cdot \Delta H \cdot \eta}}{\Delta H^{\frac{5}{4}}} = \frac{n \cdot \sqrt{\eta' \cdot \rho g \cdot Q}}{\Delta H^{\frac{3}{4}}} \qquad \textbf{(S14)}$$

To make **(S13)** and (**S14)** dimensionless and use the rotation speed in $rad \cdot s^{-1}$:

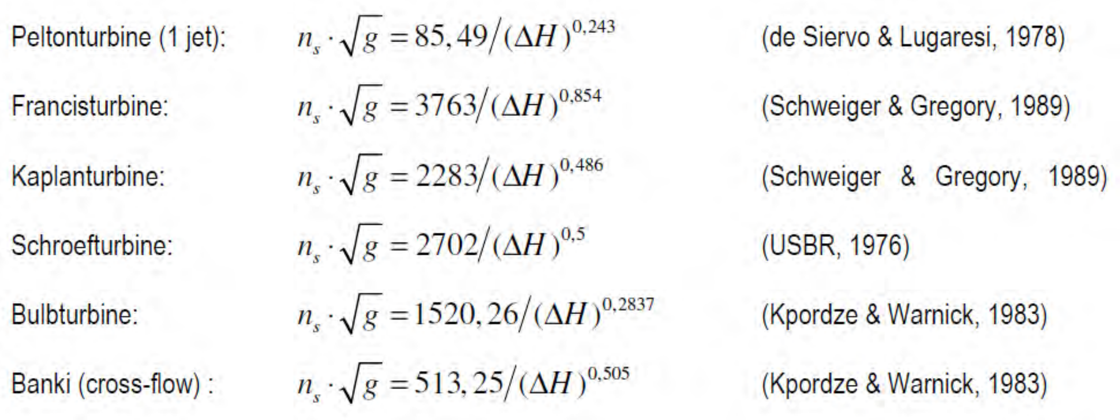
$$\Omega.s = \frac{\Omega \cdot \sqrt{\frac{\eta'}{\eta} \cdot \frac{P}{\rho}}}{(g \cdot \Delta H)^{\frac{5}{4}}}$$

and:

$$\Omega.s = \frac{\Omega \cdot \sqrt{\eta' \cdot Q}}{(g \cdot \Delta H)^{\frac{3}{4}}}$$

Turbine design
Chosing a specific speed depends on the turbine type as stated in the assumptions, the turbines should be geometrically symilar. Taken from:

Annerel, Sebastiaan (2008), *"Ontwikkeling van een ontwerpmethode voor axiale hydraulische turbines met sluitbare rotor."* , Universiteit Gent



Peltonturbine (1 jet):	$n_s \cdot \sqrt{g} = 85,49/(\Delta H)^{0,243}$	(de Siervo & Lugaresi, 1978)
Francisturbine:	$n_s \cdot \sqrt{g} = 3763/(\Delta H)^{0,854}$	(Schweiger & Gregory, 1989)
Kaplanturbine:	$n_s \cdot \sqrt{g} = 2283/(\Delta H)^{0,486}$	(Schweiger & Gregory, 1989)
Schroefturbine:	$n_s \cdot \sqrt{g} = 2702/(\Delta H)^{0,5}$	(USBR, 1976)
Bulbturbine:	$n_s \cdot \sqrt{g} = 1520,26/(\Delta H)^{0,2837}$	(Kpordze & Warnick, 1983)
Banki (cross-flow) :	$n_s \cdot \sqrt{g} = 513,25/(\Delta H)^{0,505}$	(Kpordze & Warnick, 1983)

Which are taken from:
PENCHE C.,1998, "*Layman's handbook on how to develop a small hydro site*", Commission of the European, Communities (Directorate General for Energy, DG XVII), 266 blz.;

So with a headdifference at driel between 0.5m and 2.3m that means the specific speed needs to be around:

$$n.s\_bulb(\Delta H) := \frac{1}{\sqrt{g}} \cdot \frac{1520.26}{(\Delta H)^{0.2837}} \cdot \frac{m^{\frac{250}{319}}}{s} \cdot rpm$$

$$n.s\_bulb(0.5\ m) = 590.96\ rpm$$

$$n.s\_bulb(2.3\ m) = 383.30\ rpm$$

Numeric check:

$$\frac{1}{\sqrt{9.81}} \cdot \frac{1520.26}{(0.5)^{0.2837}} = 590.861$$

$$n.s\_kaplan(\Delta H) := \frac{1}{\sqrt{g}} \cdot \frac{2283}{(\Delta H)^{0.486}} \cdot \frac{m^{\frac{493}{500}}}{s} \cdot rpm$$

$$n.s\_kaplan(0.5\ m) = 1021.05\ rpm$$

$$n.s\_kaplan(2.3\ m) = 486.35\ rpm$$

$$n.s\_screw(\Delta H) := \frac{1}{\sqrt{g}} \cdot \frac{2702}{(\Delta H)^{0.5}} \cdot \frac{m}{s} \cdot rpm$$

$$n.s\_screw(0.5\ m) = 1220.23\ rpm$$

$$n.s\_screw(2.3\ m) = 568.93\ rpm$$

$$n.s\_banki(\Delta H) := \frac{1}{\sqrt{g}} \cdot \frac{513.25}{(\Delta H)^{0.505}} \cdot \frac{m^{\frac{201}{200}}}{s} \cdot rpm$$

$$n.s\_banki(0.5\ m) = 232.59\ rpm$$

$$n.s\_banki(2.3\ m) = 107.62\ rpm$$

Analysis

Trying to find a head-discharge relationship:
N*D as function of Head-coefficient:

$$\psi = \frac{g\Delta H}{(N \cdot D)^2} \quad \text{-->} \quad N \cdot D = \sqrt{\frac{g\Delta H}{\psi}}$$

N*D as function of Flow-coefficient

$$\phi = \frac{Q}{N \cdot D^3} = \frac{Q}{N \cdot D \cdot D^2} \quad \text{-->} \quad N \cdot D = \frac{Q}{\phi \cdot D^2}$$

Equating these two, gives:

$$\frac{Q}{\phi \cdot D^2} = \sqrt{\frac{g\Delta H}{\psi}}$$

Then combining:

$$Q = \frac{\phi \cdot D^2}{\sqrt{\psi}} \cdot \sqrt{g\Delta H} \quad ; \qquad N.s = \frac{\phi_1^{\frac{1}{2}}}{\psi_1^{\frac{3}{4}}} \quad ; \qquad \psi_1 \cdot \left(\frac{\phi_1^{\frac{1}{2}}}{\psi_1^{\frac{3}{4}}} \right)^2 = \psi_1 \cdot N.s^2 \rightarrow \frac{\phi_1}{\sqrt{\psi_1}} = N.s^2 \cdot \psi_1$$

Gives:

$$Q = N.s^2 \cdot \psi_1 \cdot D^2 \cdot \sqrt{g\Delta H} \quad \text{-->} \quad Q = N.s^2 \cdot \frac{g\Delta H}{(ND)^2} \cdot D^2 \cdot \sqrt{g\Delta H}$$

$$Q = N.s^2 \cdot \frac{gH}{N^2} \cdot \sqrt{g\Delta H}$$

Gives the Discharge-head realtion from dimensional analysis/specific speed approach:

$$Q = \frac{N.s^2}{N^2} \cdot g\Delta H^{\frac{3}{2}} \quad \text{--> is same as deriving it from } N.s = \frac{N \cdot Q^{\frac{1}{2}}}{(g\Delta H)^{\frac{3}{4}}} \text{ -->} \quad Q = \frac{N.s^2}{N^2} \cdot g\Delta H^{\frac{3}{2}}$$

Rewriting to head as function of discharge:

$$\Delta H = \frac{1}{g} Q^{\frac{2}{3}} \cdot \left(\frac{N}{N.s} \right)^{\frac{4}{3}}$$

The noting that the total system head-difference is the sum of minor losses plus head-difference over turbine:

$$\Delta H\_sys = \sum_{i=1}^{K} \Delta H\_loss_i + \Delta H\_turbine = \frac{Q^2}{2\ g} \sum_{i=1}^{K} \frac{\xi_i}{A_i^{\ 2}} + \frac{Q^{\frac{2}{3}}}{g} \cdot \left(\frac{N}{N.s} \right)^{\frac{4}{3}} \qquad \textbf{(A1)}$$

Defining the quadratic discharge coefficient as:

$$C = \frac{1}{2\ g} \sum_{i=1}^{K} \frac{\xi_i}{A_i^{\ 2}} \qquad \textbf{(A2)}$$

This gives quite a strange polynomial to solve:

$$Q^2\ C + Q^{\frac{2}{3}} \cdot \frac{1}{g} \left(\frac{N}{N.s} \cdot \frac{1}{s} \right)^{\frac{4}{3}} - \Delta H\_sys = 0 \qquad \textbf{(A3)}$$

in the shape of:

$$Q^2\ c_1 + Q^{\frac{2}{3}} \cdot c_2 - c_3 = 0$$

Solving Q as function of system head numerically by Newton-Rapson method:

Note: if the discharge is "corrected" to be negative, one step in the opposite direction is made. This to prevent overshooting and the accompanying finding of complex valued head-differences in the turbine head (e.g. from the terms $Q^{\frac{1}{3}}$ and $Q^{\frac{2}{3}}$).

```
$Qsys(\Delta H\_sys, C, N, N.s) :=$
    $Q \leftarrow 1\ \frac{m^3}{s}$
    $D1 \leftarrow (\text{UnitsOf}(2 \cdot Q \cdot C))^{-1}$
    $D2 \leftarrow \left(\text{UnitsOf}\left(\frac{2 \cdot \left(\frac{N}{N.s} \cdot \frac{1}{s}\right)^{\frac{4}{3}}}{3 \cdot Q^{\frac{1}{3}} \cdot g}\right)\right)^{-1}$
    $DH \leftarrow \left(Q^2\ C + \frac{Q^{\frac{2}{3}}}{g} \cdot \left(\frac{N}{N.s} \cdot \frac{1}{s}\right)^{\frac{4}{3}} - \Delta H\_sys\right) \cdot \frac{1}{m}$
    while $\frac{|DH|\ m}{\Delta H\_sys} > 10^{-3}$
        $dDH\_dQ \leftarrow 2 \cdot Q \cdot C \cdot D1 + \frac{2}{3 \cdot Q^{\frac{1}{3}} \cdot g} \cdot D2 \cdot \left(\frac{N}{N.s} \cdot \frac{1}{s}\right)^{\frac{4}{3}}$
        $Q \leftarrow$ if $Q - \frac{DH}{dDH\_dQ} \cdot \frac{m^3}{s} < 0$
                $Q + \frac{DH}{dDH\_dQ} \cdot \frac{m^3}{s}$
            else
                $Q - \frac{DH}{dDH\_dQ} \cdot \frac{m^3}{s}$
        $DH \leftarrow \left(Q^2\ C + \frac{Q^{\frac{2}{3}}}{g} \cdot \left(\frac{N}{N.s} \cdot \frac{1}{s}\right)^{\frac{4}{3}} - \Delta H\_sys\right) \cdot \frac{1}{m}$
    return $Q$
```

To check Algorythm:

$\Delta H\_sys := 2\ \mathbf{m}$; $C := \frac{1}{2\ \mathbf{g}} \frac{.100}{\left(5\ \mathbf{m}^2\right)^2} = \left(2.039 \cdot 10^{-4}\right) \frac{\mathbf{s}^2}{\mathbf{m}^5}$;

$N := 78\ \mathbf{rpm}$; $N.s := 70\ \mathbf{rpm}$; $\frac{N}{N.s} = 1.114$;

Initial guess: $Q := 1\ \frac{\mathbf{m}^3}{\mathbf{s}}$

Difference between system head and calculated sum of losses + turbine head
(made Dimensionless) :

$$DH := \left(Q^2\ C + \frac{Q^{\frac{2}{3}}}{\mathbf{g}} \cdot \left(\frac{N}{N.s} \cdot \frac{1}{\mathbf{s}}\right)^{\frac{4}{3}} - \Delta H\_sys\right) \cdot \frac{1}{\mathbf{m}} = -1.882$$

Derivative of the sum of losses + turbine head (made Dimensionless) :

$$dDH\_dQ := 2 \cdot Q \cdot C \cdot \text{UnitsOf}\left(2\ Q \cdot C\right)^{-1} + \frac{2 \cdot \left(\frac{N}{N.s} \cdot \frac{1}{\mathbf{s}}\right)^{\frac{4}{3}}}{3 \cdot Q^{\frac{1}{3}} \cdot \mathbf{g}} \cdot \text{UnitsOf}\left(\frac{2 \cdot \left(\frac{N}{N.s} \cdot \frac{1}{\mathbf{s}}\right)^{\frac{4}{3}}}{3 \cdot Q^{\frac{1}{3}} \cdot \mathbf{g}}\right)^{-1} = 0.079$$

Discharge after first step:

$$Q2 := Q - \frac{DH}{dDH\_dQ} \cdot \frac{\mathbf{m}^3}{\mathbf{s}} = 24.841\ \frac{\mathbf{m}^3}{\mathbf{s}}$$

Running algorythm defined on the previous page, and thus running through all necessary steps, gives the following result:

$$Qsys\left(\Delta H\_sys, C, N, N.s\right) = 47.377\ \frac{\mathbf{m}^3}{\mathbf{s}}$$

Example figures for Maurik turbine positioned at Driel:

Water density:

$\rho \equiv 1000\ \frac{kg}{m^3}$

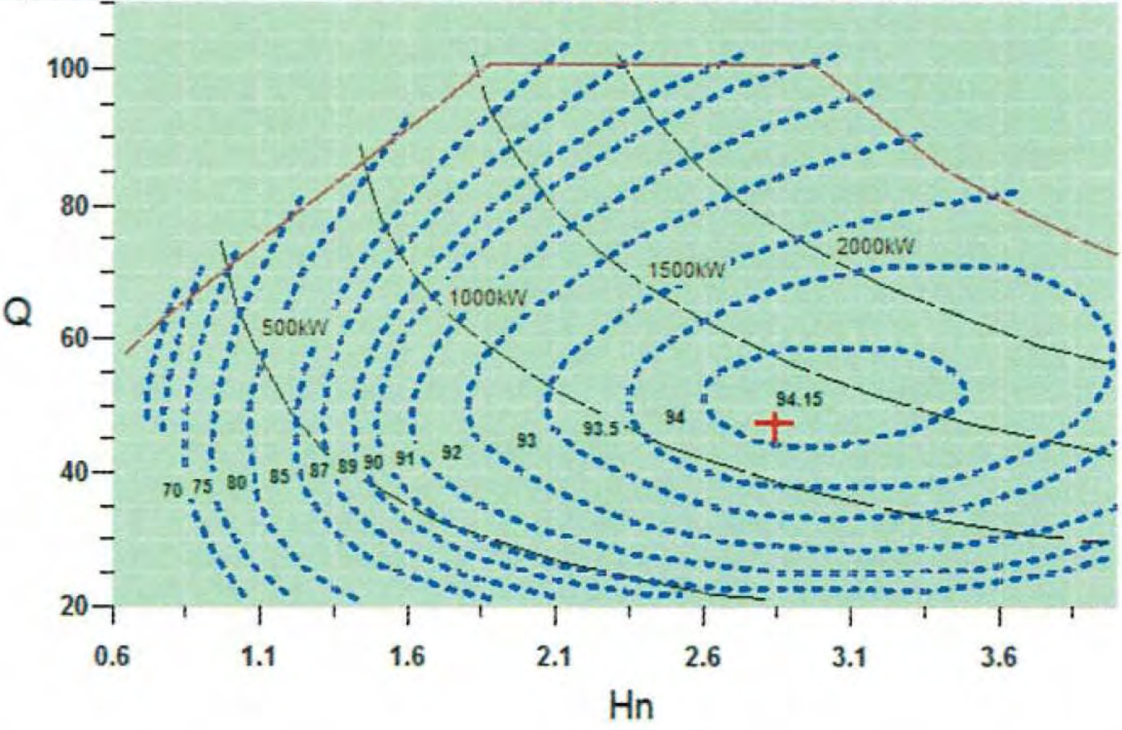
Geometry that present at Maurik:

$D.out\_M := 4\ m$	Outer diameter
$D.in\_M := 1.5\ m$	Inner diameter
$A.t\_M := \frac{\pi}{4}\left(D.out\_M^2 - D.in\_M^2\right) = 10.799\ m^2$	Flow-surface-area
$N\_Maurik := 78\ rpm$	Rotation speed turbine rotor

At Maurik on t=6th of June 11:25 the following was the case:

$Q\_M\_66 := 46.6\ m^3 \cdot s^{-1}$	Discharge through turbine
$\Delta H\_M\_T\_66 := 2.823\ m$	Head over turbine
$\Delta H\_M\_Sys\_66 := 2.92\ m$	Head over structure
$r\_h := \frac{\Delta H\_M\_T\_66}{\Delta H\_M\_Sys\_66} = 0.967$	Head-ratio
$C\_M := \frac{\left(\Delta H\_M\_Sys\_66 - \Delta H\_M\_T\_66\right)}{\left(Q\_M\_66\right)^2}$	Quadratic discharge coefficient.
$C\_M = \left(4.467 \cdot 10^{-5}\right)\ \frac{s^2}{m^5}$	
$\xi.eq\_M := C\_M \cdot 2 \cdot g \cdot A.t\_M^2 = 0.102$	Equivalent Xi-factor
$N.s\_Maurik := \frac{N\_Maurik \cdot Q\_M\_66^{\frac{1}{2}}}{\left(g \cdot \Delta H\_M\_T\_66\right)^{\frac{3}{4}}} = 44.118\ s \cdot rpm$	Specific speed Maurik at time t

Screenshot Hill-chart d.d. 06th of June 2019 at 11:25hr turbine nr. 3:



$$\Delta H\_M\_T\_check := \left(Q\_M\_66\right)^{\frac{2}{3}} \cdot \frac{1}{g}\left(\frac{N\_Maurik}{N.s\_Maurik}\right)^{\frac{4}{3}} = 2.823\ m$$

Plot H and Q in graph, define Q as x-axis:

$$Qx := 0\,\frac{m^3}{s}, 0.1\,\frac{m^3}{s} .. 200\,\frac{m^3}{s} = \begin{bmatrix} 0 \\ 0.1 \\ \vdots \end{bmatrix} \frac{m^3}{s}$$

System head from: Specific speed method:

$$Hsum1(Q, N.s, C) := Q^2\ C + Q^{\frac{2}{3}} \cdot \frac{1}{g} \cdot \left(\frac{N}{N.s} \cdot \frac{1}{s}\right)^{\frac{4}{3}}$$

Turbine head (specific speed method):

From factor f (linear relation) method:

$$Hsum2(Q, r\_h, C) := Q^2 \cdot \frac{C}{(1 - r\_h)}$$

$$H\_turbine2 = r\_h \cdot Hsum2(Q)$$

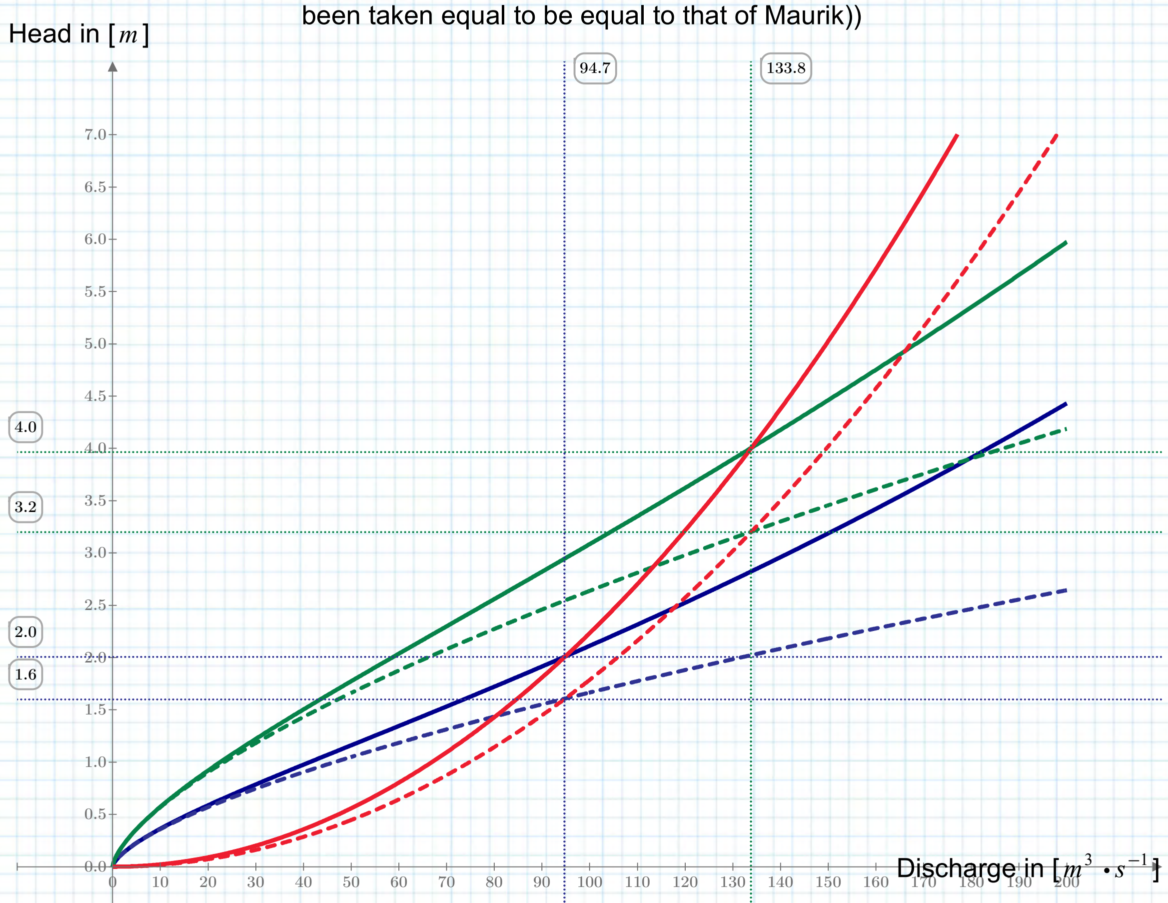
(=The linear relation)

$$H\_turbine1(Q, N.s) := Q^{\left(\frac{2}{3}\right)} \cdot \frac{1}{g} \cdot \left(\frac{N}{N.s} \cdot \frac{1}{s}\right)^{\frac{4}{3}}$$

$$H\_turbine2(Q, r\_h, C) := r\_h \cdot \left(Q^2\ \frac{C}{(1 - r\_h)}\right)$$

——— Head over system with (N.s= $96\ rpm$)
– – – Head over turbine with (N.s= $96\ rpm$)
——— Head over system with (N.s= $68\ rpm$)
– – – Head over turbine with N.s= $68\ rpm$)
——— Head over system with r_h fixed ratio of 0.8
– – – Head over turbine with r_h fixed ratio of 0.8

(For the purpose of this graph the quadratic discharge coefficient has been taken equal to be equal to that of Maurik))



Note: By changing the specific speed, the system head differences $Hsum1$ **and** $Hsum2$ **can be made to coincide for instance with the blue curve at the 2m.**
The two methods give the same result for both the system-head, discharge, but more importantly also for the <u>effective head over the turbine</u>.
Both can thus be used to desing the BEP (best efficiency point), but only one can be used to describe the head-discharge relation at lower head-differences once the turbine is chosen.

Showing difference in fixed headratio (red line)
and head-ratio determined by turbo-machinery theory (Green and blue line):

Fixed head ratio of 0.8:

Fixed head ratio of 0.9:

To show what the speed ratio does to the power output:

```
Px(Qx, N.s) := ‖ for i ∈ 0 .. rows(Qx) − 1
               ‖   ‖ $P_i \leftarrow g \cdot \rho \cdot Qx_i \cdot H\_turbine1(Qx_i, N.s)$
               ‖ P
```

$Px1 := Px(Qx, N.s)$ $Px8 := Px(Qx, .9 \cdot N.s)$

$Px2 := Px(Qx, 1.6 \cdot N.s)$ $Px9 := Px(Qx, .8 \cdot N.s)$

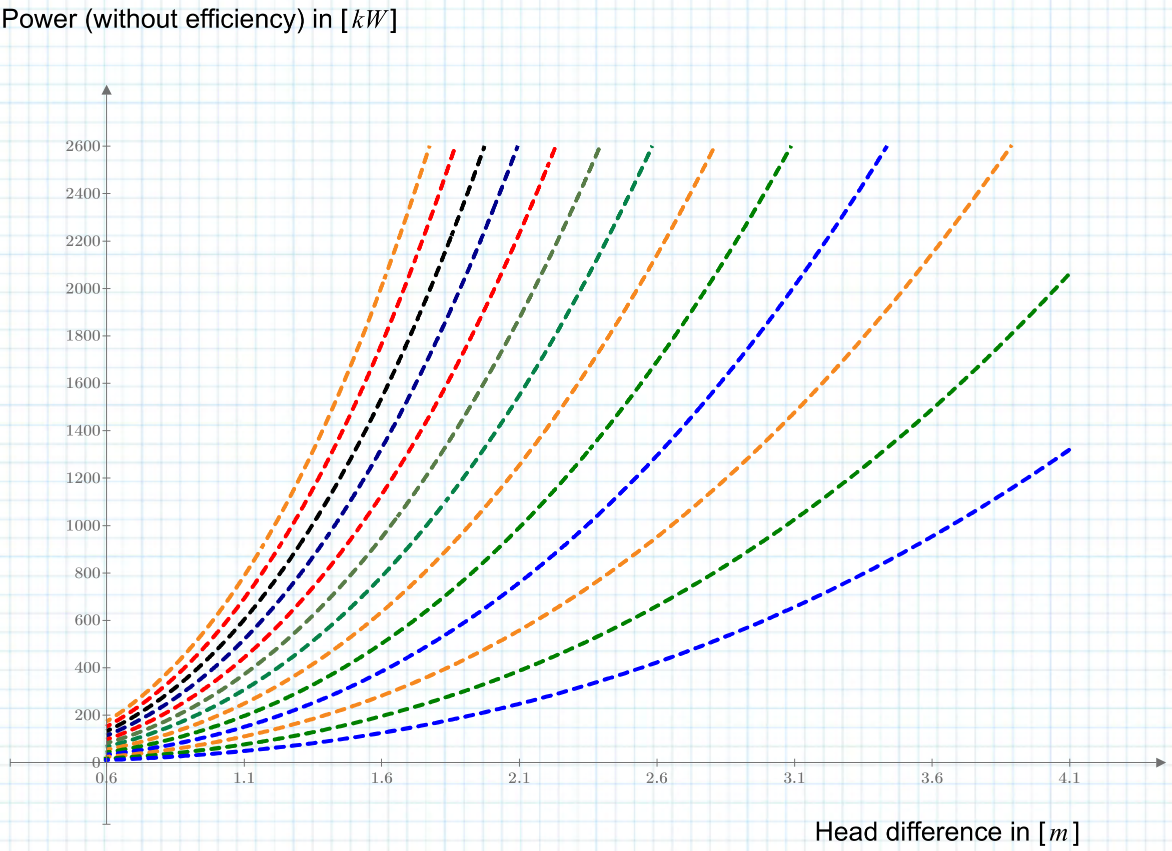
$Px3 := Px(Qx, 1.5 \cdot N.s)$ $Px10 := Px(Qx, .7 \cdot N.s)$

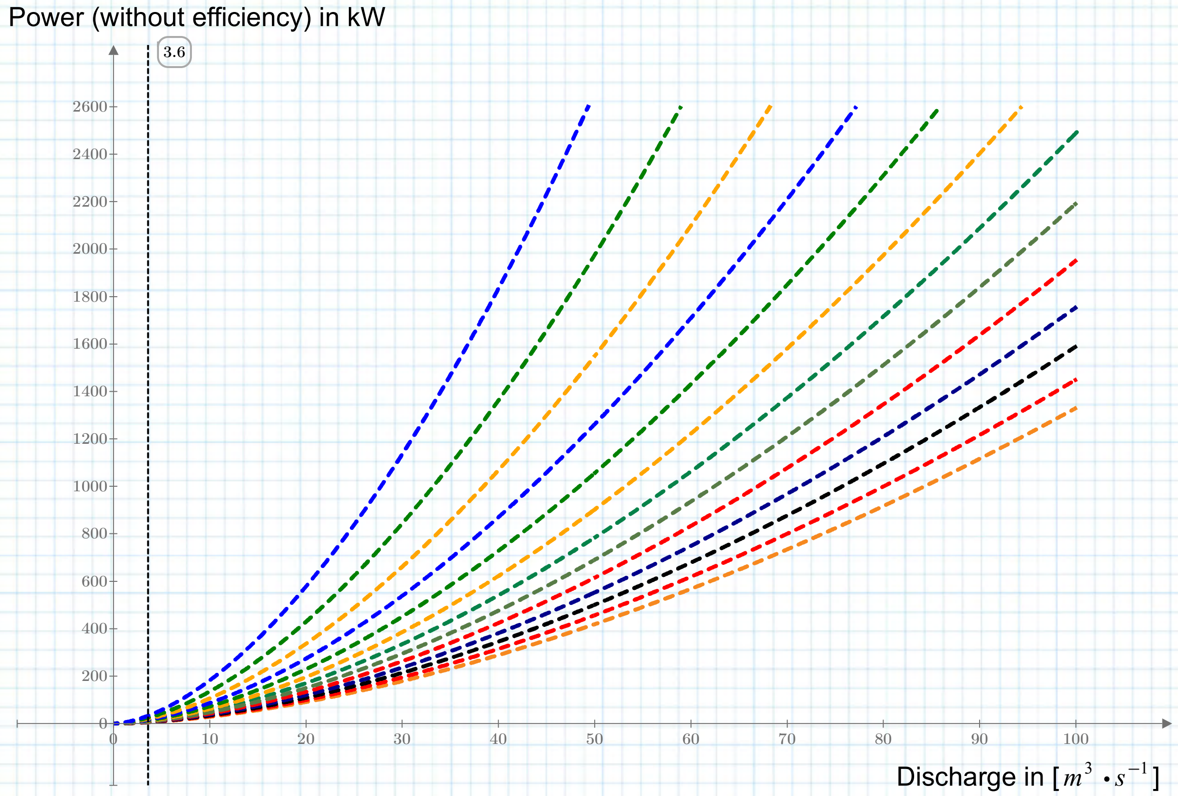
$Px4 := Px(Qx, 1.4 \cdot N.s)$ $Px11 := Px(Qx, .6 \cdot N.s)$

$Px5 := Px(Qx, 1.3 \cdot N.s)$ $Px12 := Px(Qx, .5 \cdot N.s)$

$Px6 := Px(Qx, 1.2 \cdot N.s)$ $Px13 := Px(Qx, .4 \cdot N.s)$

$Px7 := Px(Qx, 1.1\ N.s)$




Power (without efficiency) in kW
3.6
2600
2400
2200
2000
1800
1600
1400
1200
1000
800
600
400
200
0
0
10
20
30
40
50
60
70
80
90
100
Discharge in $[m^3 \cdot s^{-1}]$

Efficiency curves for plotting Power-isobars on Maurik Hill-chart:

$$Qx\_P := 20\ m^3 \cdot s^{-1}, 20.1\ \left(m^3 \cdot s^{-1}\right)..100 \cdot m^3 \cdot s^{-1} = \begin{bmatrix} 20 \\ \vdots \end{bmatrix} \frac{m^3}{s}$$

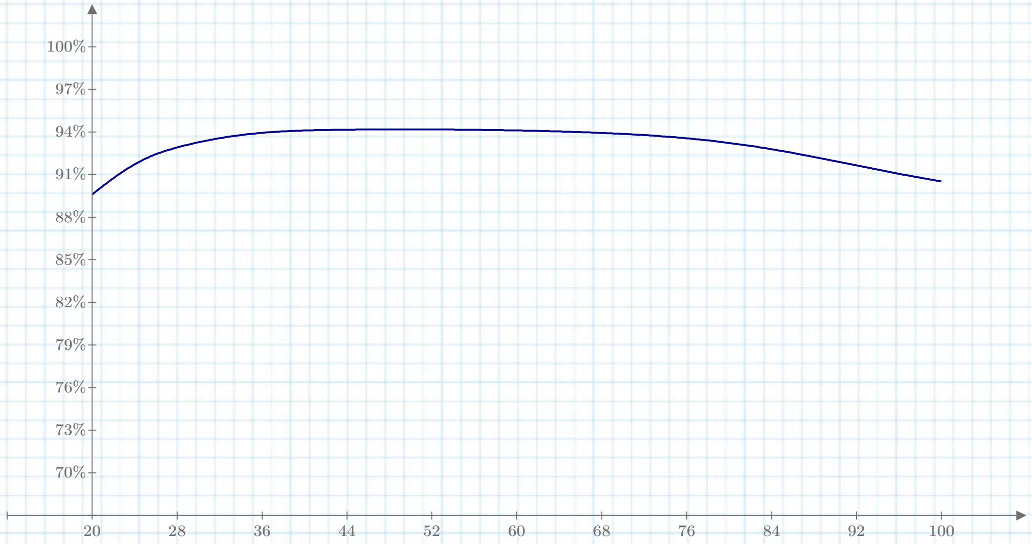
$$eff\_Q := \begin{bmatrix} 90 & 91 & 92 & 93 & 94 & 94.15 & 94.15 & 94 & 93 & 92 & 91 & 90 \\ 0 & 22.47 & 24.68 & 28.44 & 36.99 & 42.52 & 56.71 & 65.76 & 82.25 & 89.64 & 96.44 & 120 \end{bmatrix}^{T} = \begin{bmatrix} 90 & 0 \\ 91 & 22.47 \\ 92 & 24.68 \\ & \vdots \end{bmatrix}$$

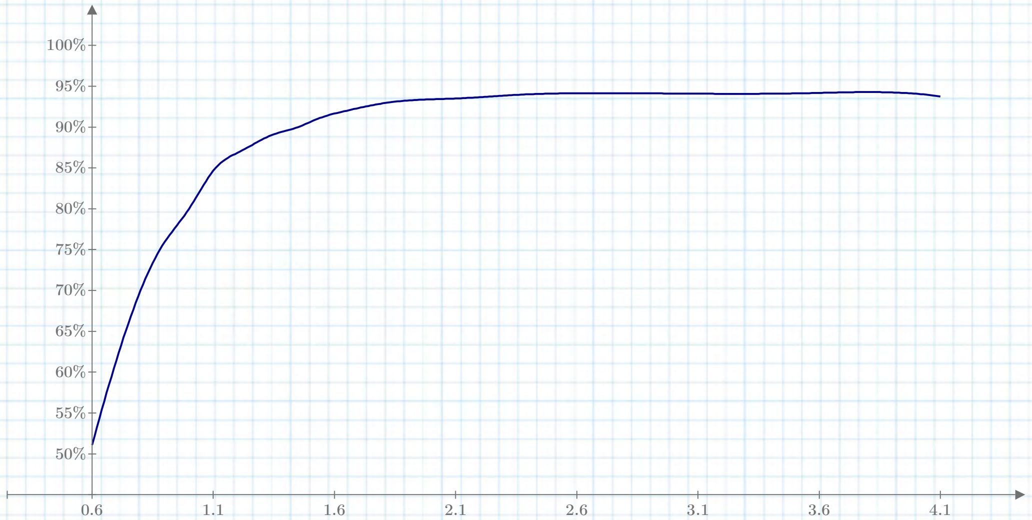
$$eff\_Q(Q) := \frac{1}{100} \cdot \text{interp}\left(\text{cspline}\left(eff\_Q^{\langle 1 \rangle}, eff\_Q^{\langle 0 \rangle}\right), eff\_Q^{\langle 1 \rangle}, eff\_Q^{\langle 0 \rangle}, Q\right)$$

$$eff\_H := \begin{bmatrix} 0 & 70 & 75 & 80 & 85 & 87 & 89 & 90 & 91 & 92 & 93 & 93.5 & 94 & 94.15 & 94.15 & 94 & 0 \\ 0 & .8 & 0.88 & 1 & 1.11 & 1.21 & 1.34 & 1.45 & 1.53 & 1.65 & 1.82 & 2.1 & 2.39 & 2.65 & 3.55 & 4.03 & 6 \end{bmatrix}^{T} = \begin{bmatrix} 0 & 0 \\ & \vdots \end{bmatrix}$$

$$eff\_H(H) := \frac{1}{94.15} \cdot \text{interp}\left(\text{cspline}\left(eff\_H^{\langle 1 \rangle}, eff\_H^{\langle 0 \rangle}\right), eff\_H^{\langle 1 \rangle}, eff\_H^{\langle 0 \rangle}, H\right)$$

$$Qxc := 20, 20 + \frac{(100-20)}{350}..100 = \begin{bmatrix} 20 \\ 20.229 \\ \vdots \end{bmatrix} \qquad Hxc := 0.6, 0.61..4.1 = \begin{bmatrix} 0.6 \\ 0.61 \\ \vdots \end{bmatrix}$$



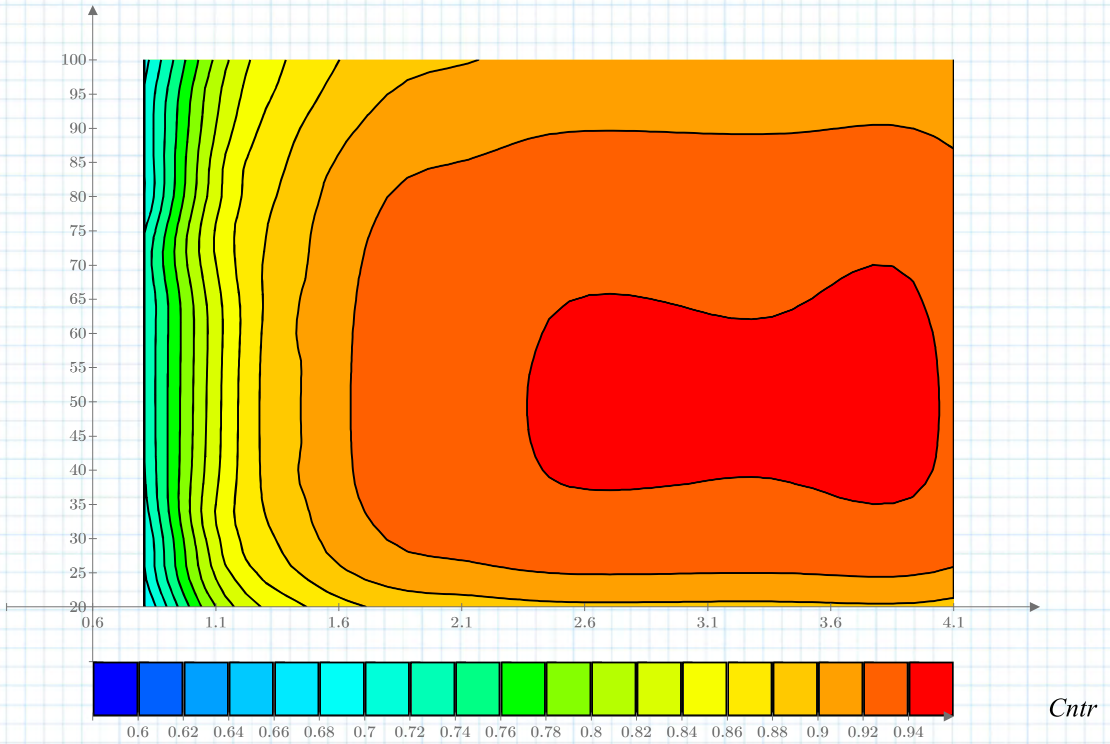


Efficiency curves:

$$eff\_M := \begin{bmatrix}94.15 & 94 & 93.5 & 93 & 92 & 91 & 90 & 89 & 87 & 85 & 80 & 75 & \dots\end{bmatrix}^{T} = \begin{bmatrix}94.15\\ \vdots\end{bmatrix}$$

```
Cntr := | n ← 0
        | d1 ← 100
        | d2 ← 500
        | for i ∈ 0 .. rows(eff_M) − 1
        |   for j ∈ 0 .. d1
        |     for k ∈ 0 .. d2
        |       xH ← 0.6 + j·$\frac{(4.1-0.6)}{d1}$
        |       xQ ← 20 + k $\frac{(100-20)}{d2}$
        |       Eff ← eff_Q(xQ)·eff_H(xH)
        |       if $\left|Eff - \frac{eff\_M_i}{100}\right| < 10^{-3}$
        |         if n > 0
        |           C ← stack(C, [xH  xQ  Eff])
        |           n ← n + 1
        |         else if n = 0
        |           $C^{\langle 0 \rangle}$ ← [xH  xQ  Eff]
        |           n ← n + 1
        | C
```

$$= \begin{bmatrix}2.42 & 43.84 & 0.941\\ 2.42 & 44 & 0.941\\ & & \vdots\end{bmatrix}$$



Defining function for power-isobars

```
$P\_isobar(PL, Qx) :=$
    $P \leftarrow 0$
    $QH^{\langle 0 \rangle} \leftarrow [0\ \ 0]$
    $c \leftarrow 0$
    for $i \in 0 .. \text{rows}(Qx) - 1$
        $n \leftarrow 0$
        for $j \in 0 .. 1000$
            $N.s \leftarrow \left(20 + j \cdot \frac{(200-20)}{1000}\right) \mathbf{rpm}$
            $H \leftarrow H\_turbine1\left(Qx_i, N.s\right)$
            $Hs \leftarrow Hsum1\left(Qx_i, N.s, C\_M\right)$
            if $Hs < 4.5\ \mathbf{m} \land Hs \geq H \land Hs \cdot Qx_i^{-2} \cdot \frac{\mathbf{s}^{-2}}{\mathbf{m}^{-5}} > 2.2 \cdot 10^{-4}$
                $P \leftarrow eff\_Q\left(Qx_i \cdot \mathbf{m}^{-3} \cdot \mathbf{s}\right) \cdot eff\_H\left(H \cdot \mathbf{m}^{-1}\right) \cdot \rho \cdot \mathbf{g} \cdot Qx_i \cdot H \cdot \mathbf{kW}^{-1}$
                if $\left(\frac{\left|P - PL \cdot \mathbf{kW}^{-1}\right|}{PL \cdot \mathbf{kW}^{-1}} < 0.05\right) \land (n = 0)$
                    $QH_{i-c,0} \leftarrow Qx_i \cdot \mathbf{m}^{-3} \cdot \mathbf{s}^{1}$
                    $QH_{i-c,1} \leftarrow H\_turbine1\left(Qx_i, N.s\right) \cdot \mathbf{m}^{-1}$
                    $N.sd_{i-c} \leftarrow N.s \cdot \mathbf{rpm}^{-1}$
                    $n \leftarrow 1$
                    break
            if $j = 1000 \land n = 0$
                $c \leftarrow c + 1$
    $QH1 \leftarrow \text{augment}(QH, N.sd)$
    return $QH1$
```

$$P\_isobar250 := P\_isobar(250\ \mathbf{kW}, Qx\_P) = \begin{bmatrix} 20 & 1.539 & 45.56 \\ & & \vdots \end{bmatrix}$$

$$P\_isobar500 := P\_isobar(500\ \mathbf{kW}, Qx\_P) = \begin{bmatrix} 20 & 2.982 & 27.74 \\ & & \vdots \end{bmatrix}$$

$$P\_isobar1000 := P\_isobar(2 \cdot 500\ \mathbf{kW}, Qx\_P) = \begin{bmatrix} 24.6 & 4.472 & 22.7 \\ & & \vdots \end{bmatrix}$$

$$P\_isobar1500 := P\_isobar(3 \cdot 500\ \mathbf{kW}, Qx\_P) = \begin{bmatrix} 36.2 & 4.429 & 27.74 \\ & & \vdots \end{bmatrix}$$

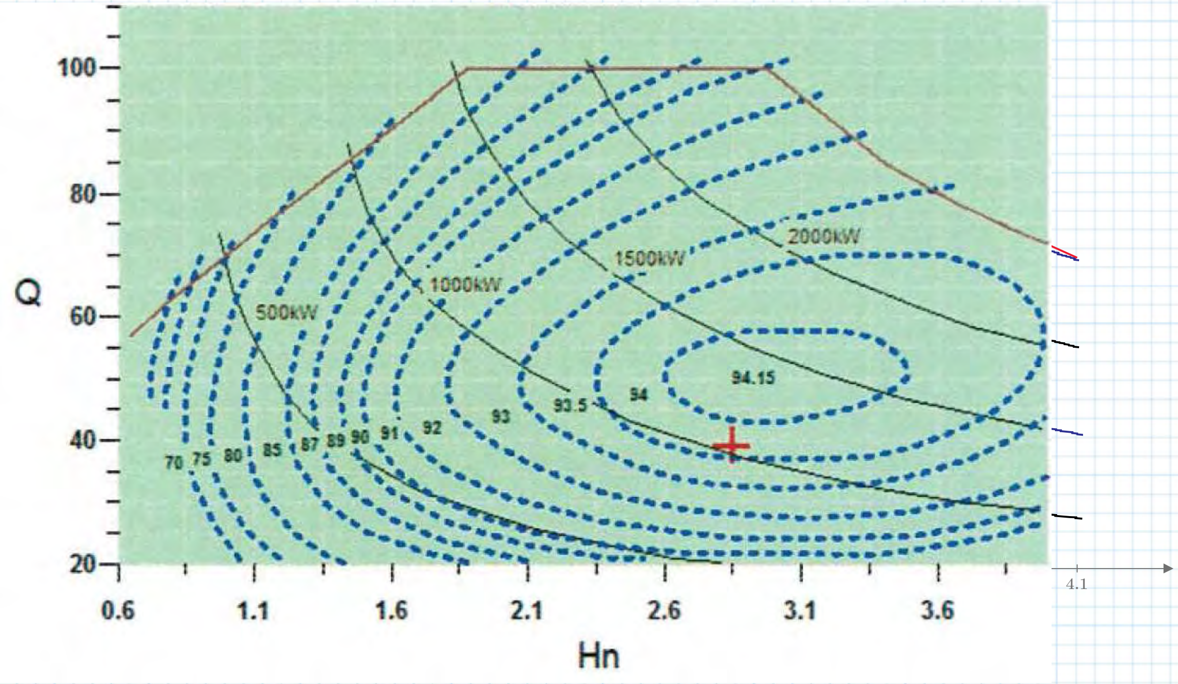
$$P\_isobar2000 := P\_isobar(4 \cdot 500\ \mathbf{kW}, Qx\_P) = \begin{bmatrix} 48.3 & 4.392 & 32.24 \\ & & \vdots \end{bmatrix}$$

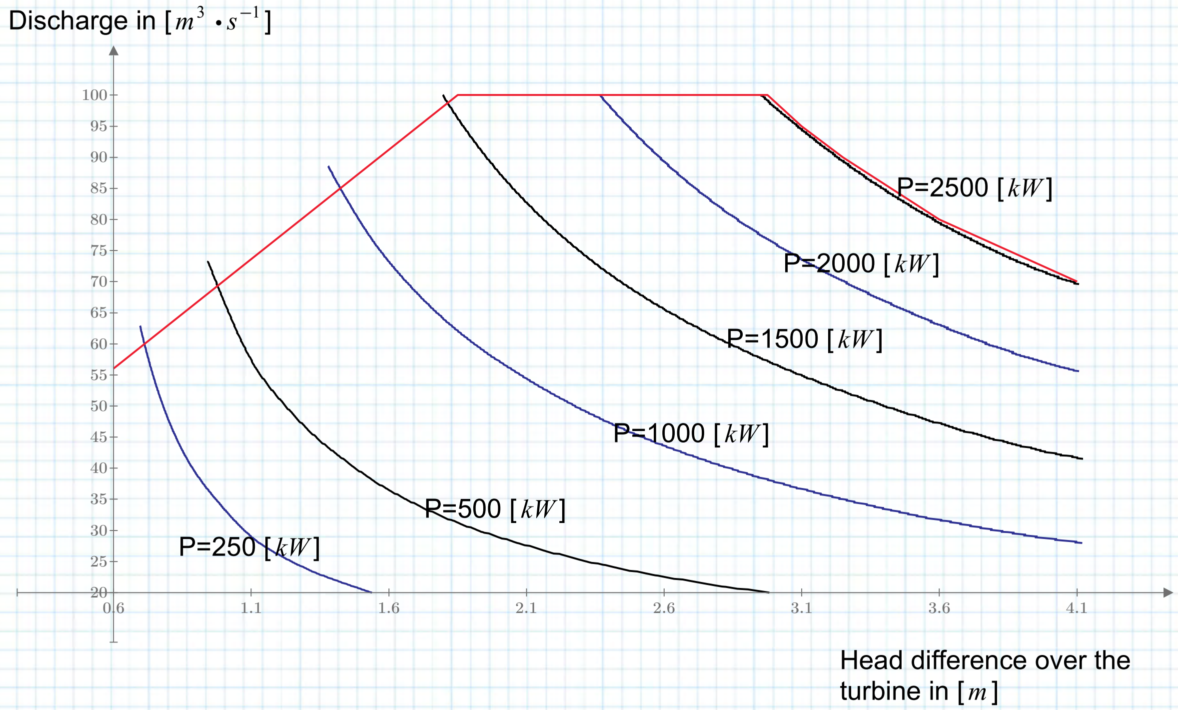
$$P\_isobar2500 := P\_isobar(5 \cdot 500\ \mathbf{kW}, Qx\_P) = \begin{bmatrix} 60.8 & 4.33 & 36.56 \\ & & \vdots \end{bmatrix}$$

Defining points that limit the power output in Maurik Hill-chart:

$$QH\_limit := \begin{bmatrix} 56 & 100 & 100 & 95 & 90 & 80 & 70 \\ 0.6 & 1.85 & 2.975 & 3.1 & 3.25 & 3.6 & 4.1 \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} 56 & 0.6 \\ 100 & 1.85 \\ 100 & 2.975 \\ 95 & 3.1 \\ 90 & 3.25 \\ 80 & 3.6 \\ 70 & 4.1 \end{bmatrix}$$

Comparing image with reverse engineered power isobars





Why is there an inclined cut of at the top left of the graph?

Specific speeds max/min:

$\max\left(P\_isobar250^{\langle 2\rangle}\right)=146.54$ $min\left(P\_isobar250^{\langle 2\rangle}\right)=45.56$

$\max\left(P\_isobar500^{\langle 2\rangle}\right)=125.84$ $min\left(P\_isobar500^{\langle 2\rangle}\right)=27.74$

$\max\left(P\_isobar1000^{\langle 2\rangle}\right)=104.06$ $min\left(P\_isobar1000^{\langle 2\rangle}\right)=22.7$

$\max\left(P\_isobar1500^{\langle 2\rangle}\right)=90.74$ $min\left(P\_isobar1500^{\langle 2\rangle}\right)=27.74$

$\max\left(P\_isobar2000^{\langle 2\rangle}\right)=73.82$ $min\left(P\_isobar2000^{\langle 2\rangle}\right)=32.24$

$\max\left(P\_isobar2500^{\langle 2\rangle}\right)=62.48$ $min\left(P\_isobar2500^{\langle 2\rangle}\right)=36.56$

$9.81 \cdot 85$

$9.81 \cdot 10$

Discharge, head over turbine and N.s

On the limit:

$P\_isobar250^{\widehat{402}}=\left[60.2\ \ 0.71\ \ 141.14\right]$

$P\_isobar500^{\widehat{500}}=\left[70\ \ 0.972\ \ 120.26\right]$

$P\_isobar1000^{\widehat{607}}=\left[85.3\ \ 1.42\ \ 99.92\right]$

$P\_isobar1500^{\widehat{638}}=\left[100\ \ 1.796\ \ 90.74\right]$

$P\_isobar2000^{\widehat{517}}=\left[100\ \ 2.364\ \ 73.82\right]$

Past limit:

$P\_isobar250^{\boxed{\widehat{725}}}=?$

$P\_isobar500^{\boxed{\widehat{800}}}=?$

$P\_isobar1000^{\boxed{\widehat{754}}}=?$

$Hsum1\left(60.2\cdot\mathbf{m}^{3}\cdot\mathbf{s}^{-1},140.96\ \mathbf{rpm},C\_M\right)=0.873\ \mathbf{m}$

$Hsum1\left(70\cdot\mathbf{m}^{3}\cdot\mathbf{s}^{-1},120.26\ \mathbf{rpm},C\_M\right)=1.191\ \mathbf{m}$

$Hsum1\left(85.3\cdot\mathbf{m}^{3}\cdot\mathbf{s}^{-1},99.92\ \mathbf{rpm},C\_M\right)=1.745\ \mathbf{m}$

$Hsum1\left(100\cdot\mathbf{m}^{3}\cdot\mathbf{s}^{-1},90.74\ \mathbf{rpm},C\_M\right)=2.242\ \mathbf{m}$

$Hsum1\left(100\cdot\mathbf{m}^{3}\cdot\mathbf{s}^{-1},73.82\ \mathbf{rpm},C\_M\right)=2.811\ \mathbf{m}$

$Hsum1\left(92.5\cdot\mathbf{m}^{3}\cdot\mathbf{s}^{-1},200\ \mathbf{rpm},C\_M\right)=0.976\ \mathbf{m}$

$Hsum1\left(100\cdot\mathbf{m}^{3}\cdot\mathbf{s}^{-1},166.88\ \mathbf{rpm},C\_M\right)=1.244\ \mathbf{m}$

$Hsum1\left(100\cdot\mathbf{m}^{3}\cdot\mathbf{s}^{-1},117.92\ \mathbf{rpm},C\_M\right)=1.713\ \mathbf{m}$

There seems to be a limit on the Quadratic discharge coefficient, so the limit on the left is a geometrical constraint. The geometry, eventhough its variable due to the double regulation, imposes a limit on the amount of water that can pass through the turbine with a certain head.

$$C\_P250(nr):=\frac{Hsum1\left(P\_isobar250_{nr,0}\cdot\mathbf{m}^{3}\cdot\mathbf{s}^{-1},P\_isobar250_{nr,2}\ \mathbf{rpm},C\_M\right)}{\left(P\_isobar250_{nr,0}\cdot\left(\mathbf{m}^{3}\cdot\mathbf{s}^{-1}\right)\right)^{2}}$$

$$C\_P500(nr):=\frac{Hsum1\left(P\_isobar500_{nr,0}\cdot\mathbf{m}^{3}\cdot\mathbf{s}^{-1},P\_isobar500_{nr,2}\ \mathbf{rpm},C\_M\right)}{\left(P\_isobar500_{nr,0}\cdot\left(\mathbf{m}^{3}\cdot\mathbf{s}^{-1}\right)\right)^{2}}$$

$$C\_P1000(nr):=\frac{Hsum1\left(P\_isobar1000_{nr,0}\cdot\mathbf{m}^{3}\cdot\mathbf{s}^{-1},P\_isobar1000_{nr,2}\ \mathbf{rpm},C\_M\right)}{\left(P\_isobar1000_{nr,0}\cdot\left(\mathbf{m}^{3}\cdot\mathbf{s}^{-1}\right)\right)^{2}}$$

$$C\_P1500(nr):=\frac{Hsum1\left(P\_isobar1500_{nr,0}\cdot\mathbf{m}^{3}\cdot\mathbf{s}^{-1},P\_isobar1500_{nr,2}\ \mathbf{rpm},C\_M\right)}{\left(P\_isobar1500_{nr,0}\cdot\left(\mathbf{m}^{3}\cdot\mathbf{s}^{-1}\right)\right)^{2}}$$

$$C\_P2000(nr):=\frac{Hsum1\left(P\_isobar2000_{nr,0}\cdot\mathbf{m}^{3}\cdot\mathbf{s}^{-1},P\_isobar2000_{nr,2}\ \mathbf{rpm},C\_M\right)}{\left(P\_isobar2000_{nr,0}\cdot\left(\mathbf{m}^{3}\cdot\mathbf{s}^{-1}\right)\right)^{2}}$$

$$C\_P2500(nr):=\frac{Hsum1\left(P\_isobar2500_{nr,0}\cdot\mathbf{m}^{3}\cdot\mathbf{s}^{-1},P\_isobar2500_{nr,2}\ \mathbf{rpm},C\_M\right)}{\left(P\_isobar2500_{nr,0}\cdot\left(\mathbf{m}^{3}\cdot\mathbf{s}^{-1}\right)\right)^{2}}$$

Limit to quadratic discharge coefficient seems to be around $C\_limit := 2.4 \cdot 10^{-4}\ \frac{s^2}{m^5}$

$$C\_P250(402) = (2.407 \cdot 10^{-4})\ \frac{s^2}{m^5}$$

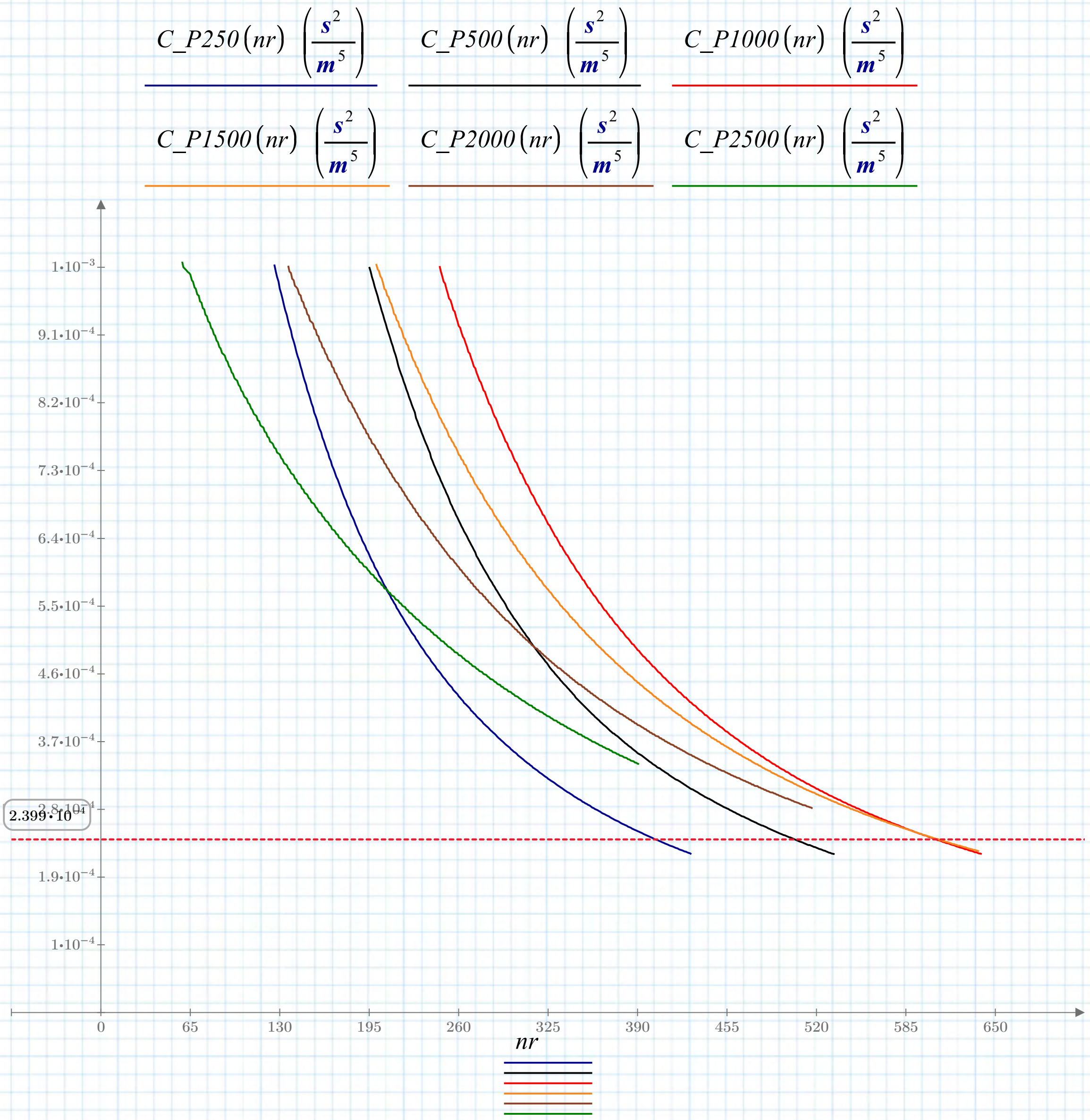
$$C\_P500(500) = (2.431 \cdot 10^{-4})\ \frac{s^2}{m^5}$$

$$C\_P1000(607) = (2.399 \cdot 10^{-4})\ \frac{s^2}{m^5}$$

$$C\_P1500(638) = (2.242 \cdot 10^{-4})\ \frac{s^2}{m^5}$$

Plot of Quadratic discharge coefficient:

$nr := 0, 1 .. 650$



Check from here on downwards...

Plotting the ratio between H_turbine and H_system:

$$Qx3 := 1 \cdot m^3 \cdot s^{-1}, 2 \cdot m^3 \cdot s^{-1} .. 250 \cdot m^3 \cdot s^{-1} = \begin{bmatrix} 1 \\ \vdots \end{bmatrix} \frac{m^3}{s}$$

```
Q_insc(N.s) :=
    Q ← Qx3
    a ← 99999999
    c ← 0
    for i ∈ 0 .. rows(Q) − 1
        b ← | H_turbine1(Q_i, N.s) / Hsum1(Q_i, N.s, C_M) − H_turbine2(Q_i, 0.8, C_M) / Hsum2(Q_i, 0.8, C_M) |
        if b < a
            a ← b
            c ← i
    Q_c
```

$$Q\_insc(N.s) = 130 \ \frac{m^3}{s}$$

$$Q\_insc(45\ rpm) = 202 \ \frac{m^3}{s}$$

Head-ratio r_H

$$Hsum1(Q\_insc(N.s), N.s, C\_M) = 3.778\ m$$

$$Hsum2(Q\_insc(N.s), 0.8, C\_M) = 3.774\ m$$

$$H\_turbine1(Q\_insc(N.s), N.s) = 3.023\ m$$

$$H\_turbine2(Q\_insc(N.s), 0.8, C\_M) = 3.02\ m$$

$$Hsum1(Q\_insc(45\ rpm), 45\ rpm, C\_M) = 9.132\ m$$

$$Hsum2(Q\_insc(45\ rpm), 0.8, C\_M) = 9.113\ m$$

$$H\_turbine1(Q\_insc(45\ rpm), 45\ rpm) = 7.31\ m$$

$$H\_turbine2(Q\_insc(45\ rpm), 0.8, C\_M) = 7.291\ m$$

Define Head over turbine $\Delta H\_turbine$ as a function of the speed ratio $rs := \frac{N}{N.s}$ and system head ΔH_sys:

$$\Delta H\_turbine(rs, C, \Delta H\_sys) = \Delta H\_sys - (Qsys\_rs(\Delta H\_sys, C, rs))^{\frac{2}{3}}\ C$$

So for that the system discharge is needed as function of system head and speedratio:

```
$Qsys\_rs(\Delta H\_sys, C, rs) :=$
    $Q \leftarrow 1 \frac{m^3}{s}$
    $D1 \leftarrow (\text{UnitsOf}(2\ Q \cdot C))^{-1}$
    $D2 \leftarrow \left(\text{UnitsOf}\left(\frac{2 \cdot \left(rs \cdot \frac{1}{s}\right)^{\frac{4}{3}}}{3 \cdot Q^{\frac{1}{3}} \cdot g}\right)\right)^{-1}$
    $DH \leftarrow \left(Q^2 \cdot C + \frac{Q^{\frac{2}{3}}}{g} \cdot \left(rs \cdot \frac{1}{s}\right)^{\frac{4}{3}} - \Delta H\_sys\right) \cdot \frac{1}{m}$
    while $\frac{|DH|\ m}{\Delta H\_sys} > 10^{-3}$
        $dDH\_dQ \leftarrow 2 \cdot Q \cdot C \cdot D1 + \frac{2}{3 \cdot Q^{\frac{1}{3}} \cdot g} \cdot D2 \cdot \left(rs \cdot \frac{1}{s}\right)^{\frac{4}{3}}$
        $Q \leftarrow$ if $Q - \frac{DH}{dDH\_dQ} \cdot \frac{m^3}{s} < 0$
                $Q + \frac{DH}{dDH\_dQ} \cdot \frac{m^3}{s}$
            else
                $Q - \frac{DH}{dDH\_dQ} \cdot \frac{m^3}{s}$
        $DH \leftarrow \left(Q^2 \cdot C + \frac{Q^{\frac{2}{3}}}{g} \cdot \left(rs \cdot \frac{1}{s}\right)^{\frac{4}{3}} - \Delta H\_sys\right) \cdot \frac{1}{m}$
    return $Q$
```

Define the function for the turbine head, now that the discharge in it is defined:

$$\Delta H\_turbine(\Delta H\_sys, C, rs) := \frac{(Qsys\_rs(\Delta H\_sys, C, rs))^{\frac{2}{3}}}{g} \cdot \left(rs \cdot \frac{1}{s}\right)^{\frac{4}{3}}$$

Define effective/normalised head ratio $H.r$ to be able to compare different system heads.

$$r\_H(\Delta H\_sys, C, rs) := \frac{\Delta H\_turbine(\Delta H\_sys, C, rs)}{\Delta H\_sys}$$

Define normalised discharge ratio $Q.r$ to be able to compare different system heads. The maximum discharge is when no power is extracted and the speed ratio is $rs = 0$, so that is what the discharge is normalised with:

$$r\_Q(\Delta H\_sys, C, rs) := \frac{Qsys\_rs(\Delta H\_sys, C, rs)}{Qsys\_rs(\Delta H\_sys, C, 0)}$$

Define normalised power ratio $P.r$ to be able to compare different system heads. It is normalised with the energy flux when the turbine is not running (i.e. $rs = 0$) and the system head. Hence the product of $Q.r$ and $H.r$. factors like ρ and g are left out, as they cross-out anyway.

$$r\_P(\Delta H\_sys, C, rs) := r\_Q(\Delta H\_sys, C, rs) \cdot r\_H(\Delta H\_sys, C, rs)$$

Maximum power can be achieved when $P.r$ is maximised:

```
$rsmax(\Delta H\_sys, C, rs) :=$
    $a \leftarrow 0$
    for $i \in 0 .. \mathrm{rows}(rs) - 1$
        $b \leftarrow \left( \frac{\Delta H\_turbine(\Delta H\_sys, C, rs_i)}{\Delta H\_sys} \cdot \frac{Qsys\_rs(\Delta H\_sys, C, rs_i)}{Qsys\_rs(\Delta H\_sys, C, 0)} \right)$
        if $a < b$
            $c \leftarrow i$
            $a \leftarrow b$
    $rs_c$
```

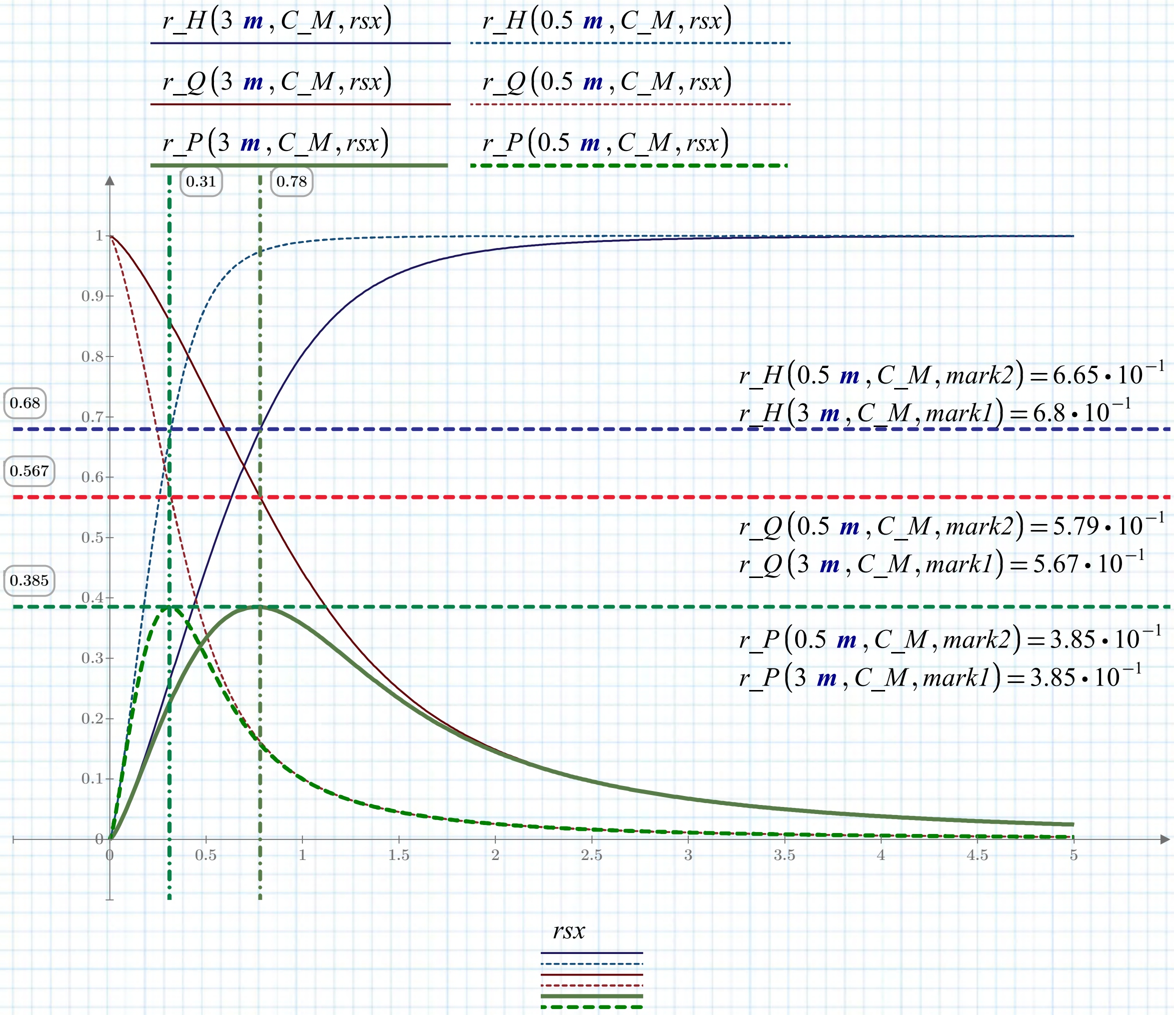
Define speedratio range:

$$rsx := 0, 0.01 .. 5 \quad rs := 0, 0.01 .. 5 = \begin{bmatrix} 0 \\ 0.01 \\ \vdots \end{bmatrix}$$

Approximate outer ranges head-differences Driel:

$$mark1 := rsmax(3\ \mathbf{m}, C\_M, rs) = 0.78$$

$$mark2 := rsmax(0.5\ \mathbf{m}, C\_M, rs) = 0.31$$



$$r\_H(0.5\ \mathbf{m}, C\_M, mark2) = 6.65 \cdot 10^{-1}$$

$$r\_H(3\ \mathbf{m}, C\_M, mark1) = 6.8 \cdot 10^{-1}$$

$$r\_Q(0.5\ \mathbf{m}, C\_M, mark2) = 5.79 \cdot 10^{-1}$$

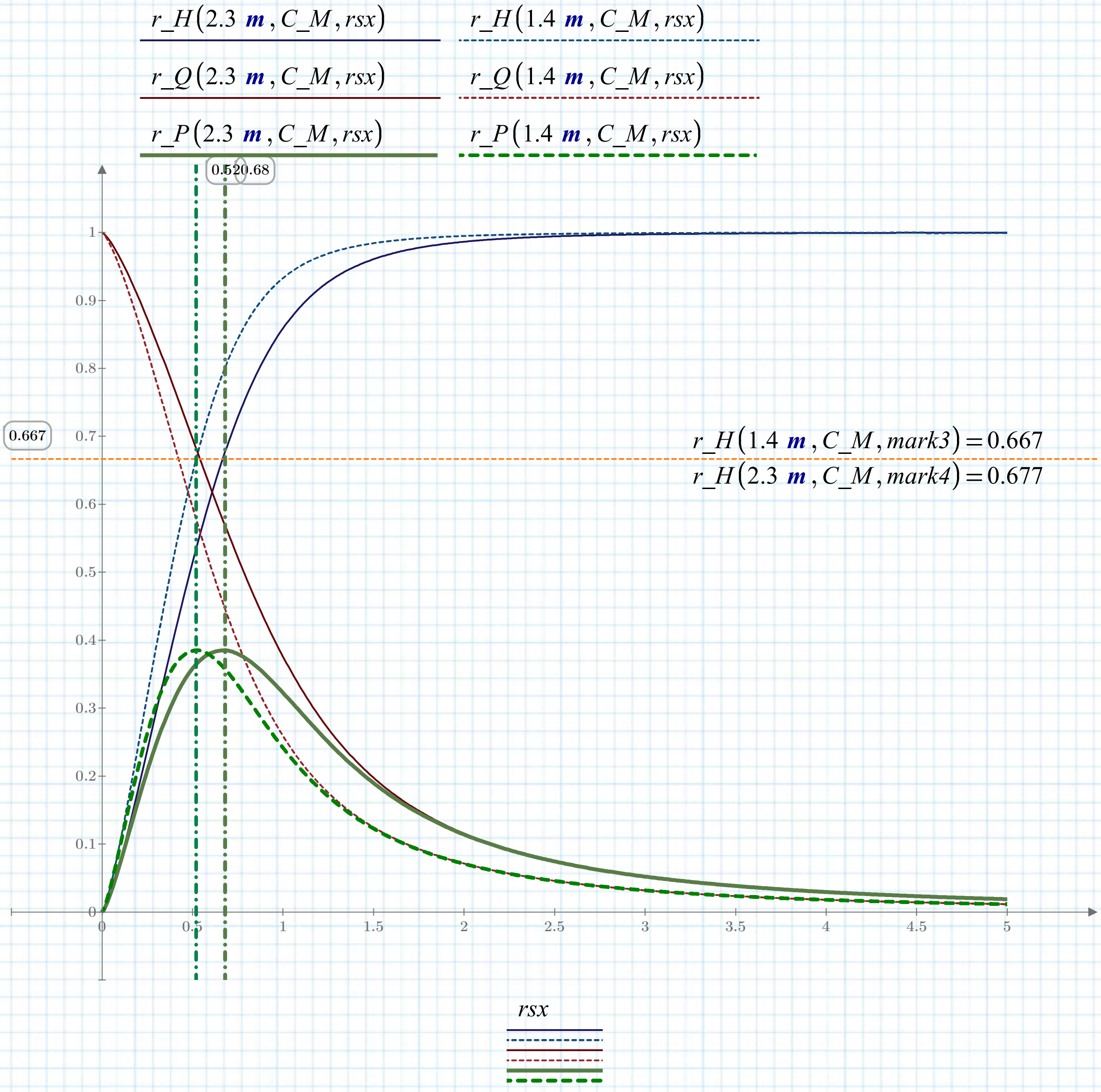
$$r\_Q(3\ \mathbf{m}, C\_M, mark1) = 5.67 \cdot 10^{-1}$$

$$r\_P(0.5\ \mathbf{m}, C\_M, mark2) = 3.85 \cdot 10^{-1}$$

$$r\_P(3\ \mathbf{m}, C\_M, mark1) = 3.85 \cdot 10^{-1}$$

rsmax for Hsys=Average head at Driel $mark3 := rsmax(1.4\ \mathbf{m}, C\_M, rs) = 0.52$

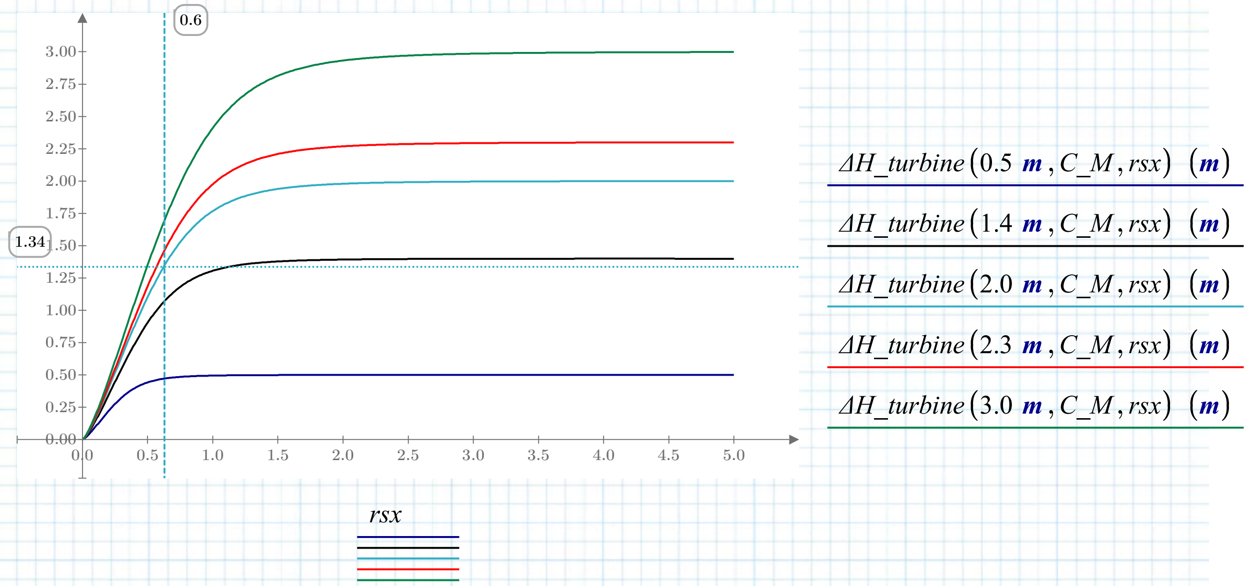
rsmax for Hsys=About Maximum head at Driel $mark4 := rsmax(2.3\ \mathbf{m}, C\_M, rs) = 0.68$



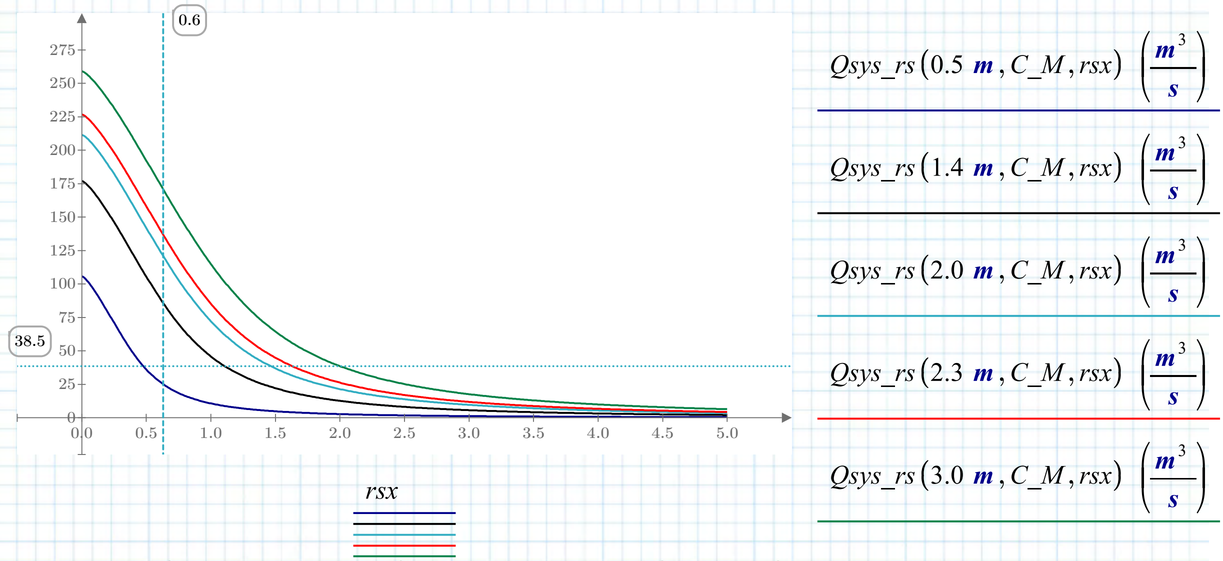
$r\_H(1.4\ \mathbf{m}, C\_M, mark3) = 0.667$

$r\_H(2.3\ \mathbf{m}, C\_M, mark4) = 0.677$

Dimensional (i.e. non-normalised) Head-difference as function of speed-ratio:

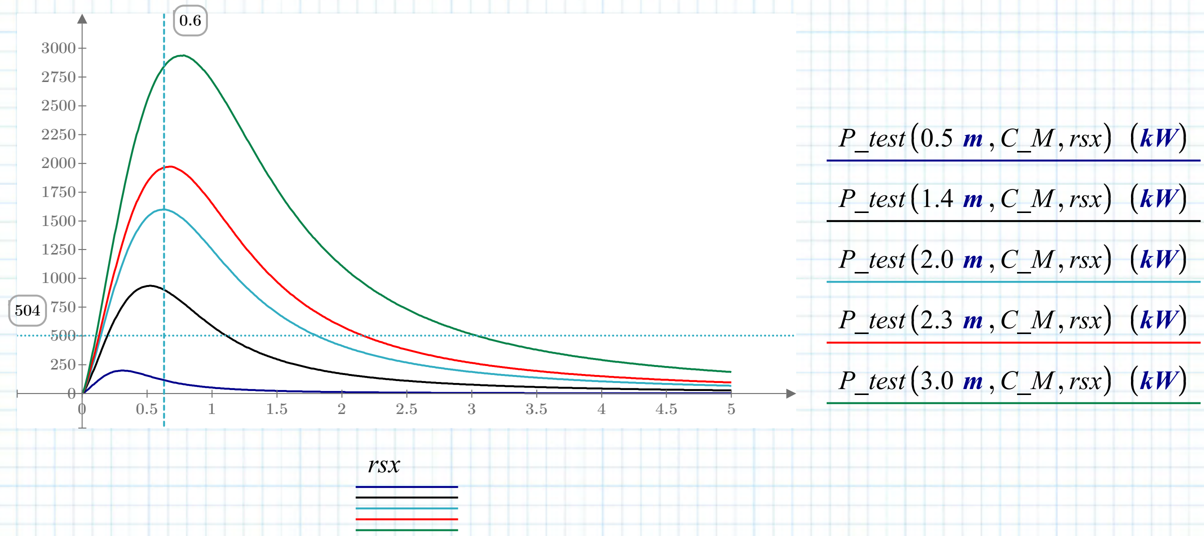


Dimensional (i.e. non-normalised) Discharge as function of speed-ratio:



Dimensional (i.e. non-normalised) Maximum Power as function of speed-ratio:

$$P\_test(\Delta H\_sys, G, rs) := \rho \cdot g \cdot \Delta H\_turbine(\Delta H\_sys, G, rs) \cdot Qsys\_rs(\Delta H\_sys, G, rs)$$



Plotting the speedratio value for which power is largest:

$\Delta H\_sys1 := 0.05\ \mathbf{m}, 0.1\ \mathbf{m} .. 4\ \mathbf{m}$ $mark5 := rsmax(2\ \mathbf{m}, C\_M, rs) = 0.63$

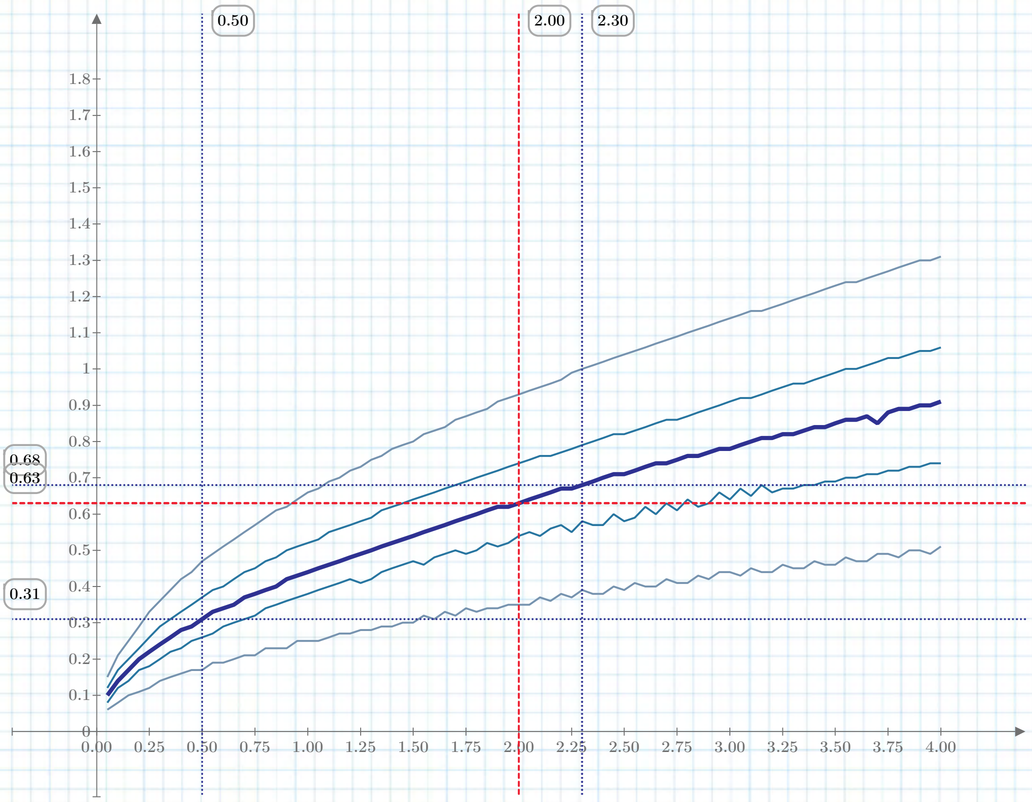
$rsmax(\Delta H\_sys1, 0.1 \cdot C\_M, rs)$

$rsmax(\Delta H\_sys1, 0.5 \cdot C\_M, rs)$

$rsmax(\Delta H\_sys1, C\_M, rs)$

$rsmax(\Delta H\_sys1, 2.0 \cdot C\_M, rs)$

$rsmax(\Delta H\_sys1, 5.0 \cdot C\_M, rs)$

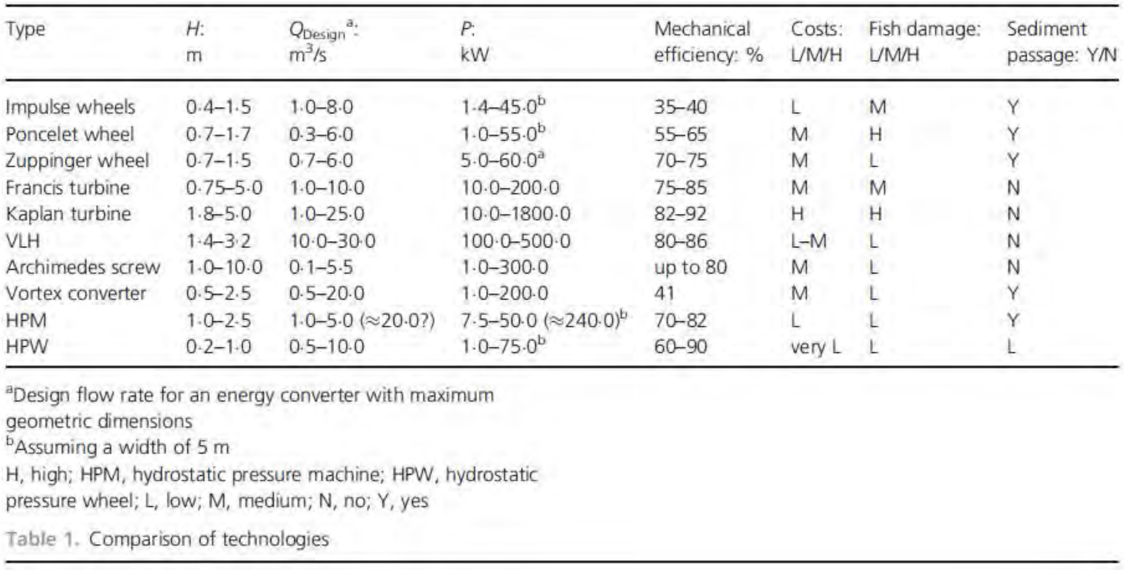


$\Delta H\_sys1\ (\mathbf{m})$

Note:
- this curve is for a specific Quadratic discharge coefficient, which is now taken as a equal to Maurik.
- The specific speed shouldn't change while running, as it defines a certain turbine (except when the turbine is a doulbe regulated Kaplan, because then the geometry changes when changing the angles of the rotor-blades and guid-vanes), however the running speed N can be changed if the turbine is a variable speed turbine. and then the optimal point can be maintained.

APPENDIX 4 – EXISTING TURBINE TECHNOLOGIES

The institution of Civil Engineers (ICE) published a paper about most suitable technologies for “Very Low Head” (VLH) situations. One of their results was the following table:



Type	H: m	Q_{Design}^{a}: m^{3}/s	P: kW	Mechanical efficiency: %	Costs: L/M/H	Fish damage: L/M/H	Sediment passage: Y/N
Impulse wheels	0·4–1·5	1·0–8·0	1·4–45·0^{b}	35–40	L	M	Y
Poncelet wheel	0·7–1·7	0·3–6·0	1·0–55·0^{b}	55–65	M	H	Y
Zuppinger wheel	0·7–1·5	0·7–6·0	5·0–60·0^{a}	70–75	M	L	Y
Francis turbine	0·75–5·0	1·0–10·0	10·0–200·0	75–85	M	M	N
Kaplan turbine	1·8–5·0	1·0–25·0	10·0–1800·0	82–92	H	H	N
VLH	1·4–3·2	10·0–30·0	100·0–500·0	80–86	L–M	L	N
Archimedes screw	1·0–10·0	0·1–5·5	1·0–300·0	up to 80	M	L	N
Vortex converter	0·5–2·5	0·5–20·0	1·0–200·0	41	M	L	Y
HPM	1·0–2·5	1·0–5·0 (≈20·0?)	7·5–50·0 (≈240·0)^{b}	70–82	L	L	Y
HPW	0·2–1·0	0·5–10·0	1·0–75·0^{b}	60–90	very L	L	L

[a] Design flow rate for an energy converter with maximum geometric dimensions

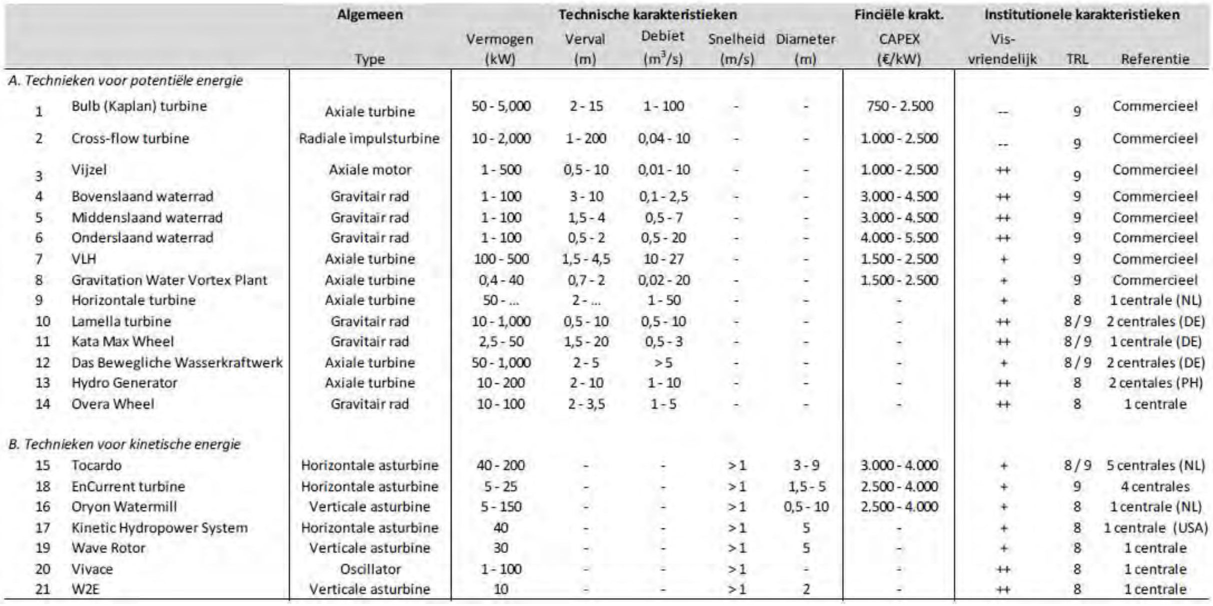
[b] Assuming a width of 5 m

H, high; HPM, hydrostatic pressure machine; HPW, hydrostatic pressure wheel; L, low; M, medium; N, no; Y, yes

Table 1. Comparison of technologies

Figure 74 *- Table with comparison of technologies for VHL hydro-power with head less than 2,5 meters. - Source: [43]*

However, the turbine and pump manufacturer Pentair Fairbanks Nijhuis (PFN) has turbines that can start at 0,3m head difference, making the workable range much larger than indicated in **Figure 74**. RH-DHV has instead formed the following table:



		Algemeen	Technische karakteristieken					Finciële krakt.	Institutionele karakteristieken		
		Type	Vermogen (kW)	Verval (m)	Debiet (m^{3}/s)	Snelheid (m/s)	Diameter (m)	CAPEX (€/kW)	Vis-vriendelijk	TRL	Referentie
A. Technieken voor potentiële energie											
1	Bulb (Kaplan) turbine	Axiale turbine	50 - 5,000	2 - 15	1 - 100	-	-	750 - 2.500	--	9	Commercieel
2	Cross-flow turbine	Radiale impulsturbine	10 - 2,000	1 - 200	0,04 - 10	-	-	1.000 - 2.500	--	9	Commercieel
3	Vijzel	Axiale motor	1 - 500	0,5 - 10	0,01 - 10	-	-	1.000 - 2.500	++	9	Commercieel
4	Bovenslaand waterrad	Gravitair rad	1 - 100	3 - 10	0,1 - 2,5	-	-	3.000 - 4.500	++	9	Commercieel
5	Middenslaand waterrad	Gravitair rad	1 - 100	1,5 - 4	0,5 - 7	-	-	3.000 - 4.500	++	9	Commercieel
6	Onderslaand waterrad	Gravitair rad	1 - 100	0,5 - 2	0,5 - 20	-	-	4.000 - 5.500	++	9	Commercieel
7	VLH	Axiale turbine	100 - 500	1,5 - 4,5	10 - 27	-	-	1.500 - 2.500	+	9	Commercieel
8	Gravitation Water Vortex Plant	Axiale turbine	0,4 - 40	0,7 - 2	0,02 - 20	-	-	1.500 - 2.500	+	9	Commercieel
9	Horizontale turbine	Axiale turbine	50 - ...	2 - ...	1 - 50	-	-	-	+	8	1 centrale (NL)
10	Lamella turbine	Gravitair rad	10 - 1,000	0,5 - 10	0,5 - 10	-	-	-	++	8 / 9	2 centrales (DE)
11	Kata Max Wheel	Gravitair rad	2,5 - 50	1,5 - 20	0,5 - 3	-	-	-	++	8 / 9	1 centrale (DE)
12	Das Bewegliche Wasserkraftwerk	Axiale turbine	50 - 1,000	2 - 5	> 5	-	-	-	+	8 / 9	2 centrales (DE)
13	Hydro Generator	Axiale turbine	10 - 200	2 - 10	1 - 10	-	-	-	++	8	2 centales (PH)
14	Overa Wheel	Gravitair rad	10 - 100	2 - 3,5	1 - 5	-	-	-	++	8	1 centrale
B. Technieken voor kinetische energie											
15	Tocardo	Horizontale asturbine	40 - 200	-	-	> 1	3 - 9	3.000 - 4.000	+	8 / 9	5 centrales (NL)
18	EnCurrent turbine	Horizontale asturbine	5 - 25	-	-	> 1	1,5 - 5	2.500 - 4.000	+	9	4 centrales
16	Oryon Watermill	Verticale asturbine	5 - 150	-	-	> 1	0,5 - 10	2.500 - 4.000	+	8	1 centrale (NL)
17	Kinetic Hydropower System	Horizontale asturbine	40	-	-	> 1	5	-	+	8	1 centrale (USA)
19	Wave Rotor	Verticale asturbine	30	-	-	> 1	5	-	+	8	1 centrale
20	Vivace	Oscillator	1 - 100	-	-	> 1	-	-	++	8	1 centrale
21	W2E	Verticale asturbine	10	-	-	> 1	2	-	++	8	1 centrale

Figure 75 - *Table with technologies from RH-DHV report about hydropower in Gelderland. Source:*

Kaplan

The Kaplan turbine is essentially a screw turbine with regulable guide vanes and when double regulable also the rotor blade angle or rotation speed can be changed. Kaplan Turbines can be installed in several ways. The axis always in the direction of flow wherever it is installed, but still the axis can be vertical or horizontal depending on the pipe-system connecting it with the water supply and outflow area.

The horizontal axis Kaplan turbines are often called Bulb turbines. An example if this variation is shown in **Figure 76**. The classical vertical axis Kaplan is shown in **Figure 77**

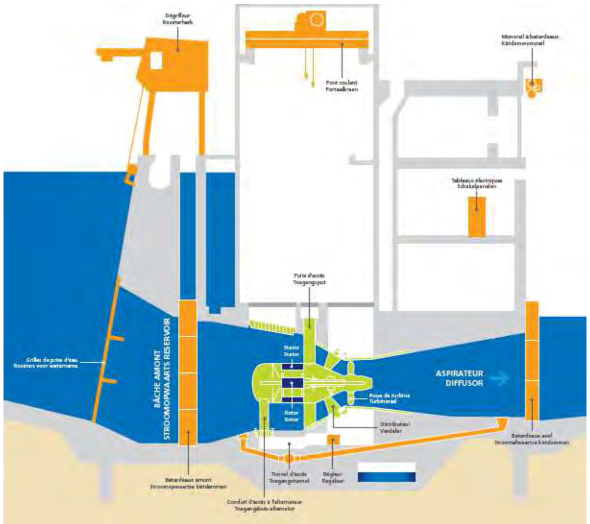


Figure 76 - Kaplan bulb turbine Lixhe and Andenne in the Meuse/Maas river in Belgium. Source: EDF [44]

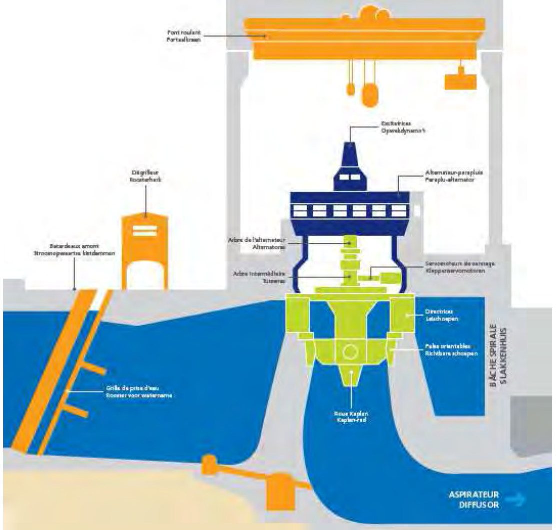


Figure 77 – Vertical axis Kaplan turbine in Ivot-ramet and Monsin the Meuse/Maas river in Belgium. Source: EDF [44]

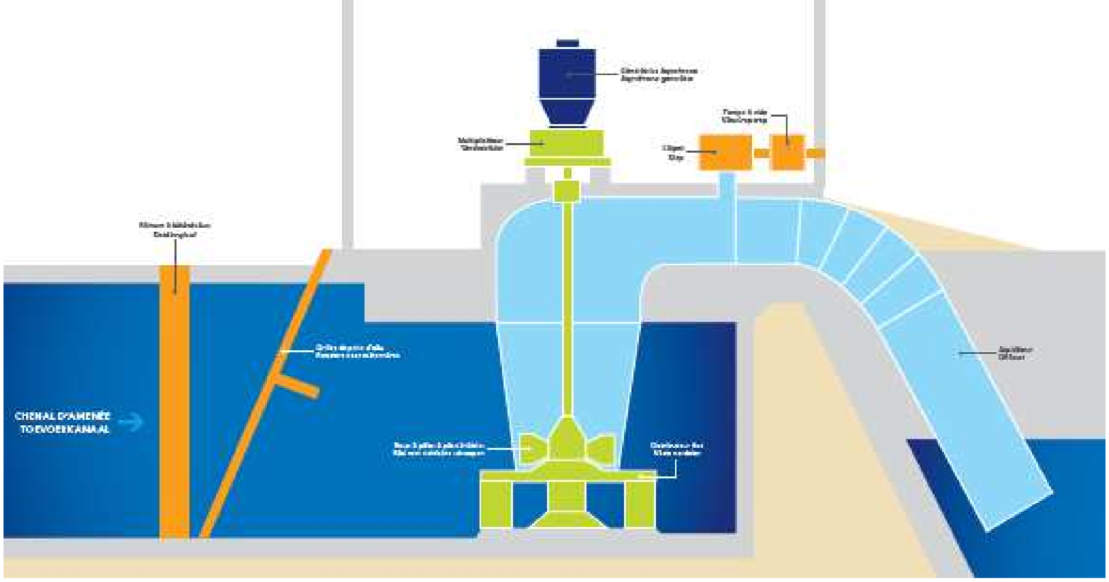


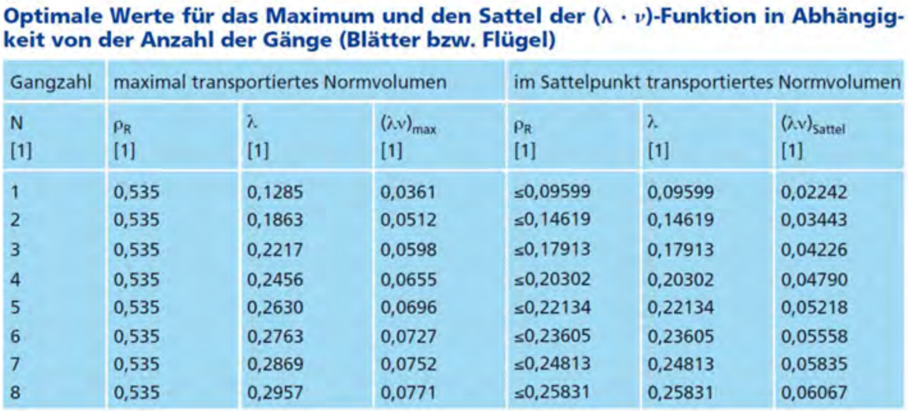
Figure 78 - Vertical axis Kaplan with siphon at Floriffoux - source: EDF [44]

Fish mortality of the Kaplan turbine depends on factor like the gap-size, blade angle and rotation speed. The fishfriendly turbine from Pentair can bring the mortality down to 2%, but existing have a mortality rate of about 13% [45].

Archimedes Screw Turbine (AST)

This ancient technology has, in the last few decades, found new application in low-head and low discharge combination hydro-power. Obviously named after the Greek physicist, Mathematician and inventor, Archimedes of Syracuse. Though some evidence suggests that the technology might have been used before his time as well [46]. In history the screws have mostly been used as pumps [47], but one of the first uses as turbine was in the 1990s. Karl-August Radlik (1997) [48] is the first to patent a Archimedes screw to be used as a turbine, but Karel Brada (1999) [49] is often cited as being the first to determine its efficiency to be around 80%.

D.M. Nuernbergk wrote several books and papers on Archimedes screw turbines among which [50], where he gives optimal values for non-dimensional parameters to achieve maximum flow-rate (see **Figure 79**) and a clear cross-section (see **Figure 80**).



Optimale Werte für das Maximum und den Sattel der (λ · ν)-Funktion in Abhängigkeit von der Anzahl der Gänge (Blätter bzw. Flügel)

Gangzahl	maximal transportiertes Normvolumen			im Sattelpunkt transportiertes Normvolumen		
N [1]	ρ_R [1]	λ [1]	$(\lambda.\nu)_{max}$ [1]	ρ_R [1]	λ [1]	$(\lambda.\nu)_{Sattel}$ [1]
1	0,535	0,1285	0,0361	≤0,09599	0,09599	0,02242
2	0,535	0,1863	0,0512	≤0,14619	0,14619	0,03443
3	0,535	0,2217	0,0598	≤0,17913	0,17913	0,04226
4	0,535	0,2456	0,0655	≤0,20302	0,20302	0,04790
5	0,535	0,2630	0,0696	≤0,22134	0,22134	0,05218
6	0,535	0,2763	0,0727	≤0,23605	0,23605	0,05558
7	0,535	0,2869	0,0752	≤0,24813	0,24813	0,05835
8	0,535	0,2957	0,0771	≤0,25831	0,25831	0,06067

***Figure 79** - Optimal values for maximum and saddle-point for (λ* v)-function, which is the normalized volume per rotation, depending on the inclination of the screw. In this: λ is the inclination ratio or normalized inclination, ρ is the radius ratio (inner over outer radius), v is the normalized volume and N is the number of blades and thus number of channels in the screw. From Nuernbergk and Rorres, 2014 [50]*

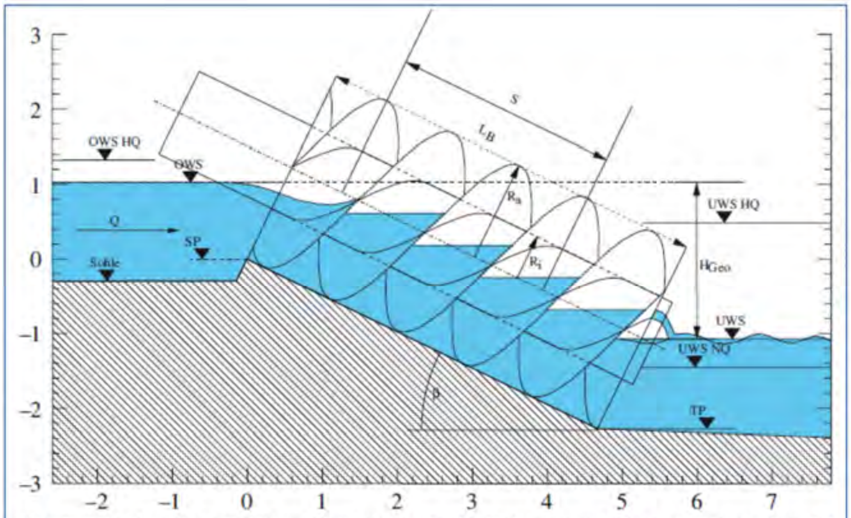


Abb. 1: Innere und äußere Parameter einer Archimedischen Schnecke am Beispiel einer dreigängigen Wasserkraftschnecke. Die inneren Parameter der Archimedischen Schnecke sind der innere bzw. äußere Radius R_i sowie R_a, die Anzahl der Blätter N und die Gangsteigung S. Die äußeren und damit standortabhängigen Parameter sind das Gefälle H_{Geo}, der Durchfluss Q, die Länge der Schnecke L_B und die Anstellung der Schnecke β.

***Figure 80** - Cross-sectional drawing of the Archimedes screw from Nuernbergk and Rorres, 2014 [50].*
Translation: "Internal and external parameters of an Archimedean screw using the example of a three bladed hydro-dynamic screw. The inner parameters of the Archimedean screw are the inner

and outer radius R_i and R_a , N is the number of blades, and S the Pitch-length. The outer, and thus location dependent parameters are the head-difference H_{geo} , the flow-rate Q , the length of the screw L_B and the inclination of the screw β."

Nuernbergk and Rorres [50] defined the following dimensionless parametres:

Radia ratio,

$$\rho = \frac{R_i}{R_o} \qquad \textbf{(25)}$$

where:

$R_o = R_a =$ the outer radius in $[m]$

$R_i =$ the inner radius in $[m]$

Note: Both are shown in **Figure 80**

The inclination ratio:

$$\lambda = \frac{S * \tan(\beta)}{2 * \pi * R_o} \qquad \textbf{(26)}$$

where:

$S =$ the pitch length in $[m]$

$\beta =$ the angle of inclination of the screw shown in **Figure 80**

The normalized volume or volume ratio,

$$\nu = \frac{V_U}{\pi * R_o * S} \qquad \textbf{(27)}$$

where:

$V_U =$ the volume of one complete turn in $[m^3/s]$

Nuernbergk and Rorres also note that the effective or working discharge is lower than the total discharge due to leakages through the gap and over the central axis. This is expressed in **(28)** below

$$Q_{total} = Q_{L,S} + Q_{L,ZR} + Q_W + Q_{L,R} + Q_{L,T} \qquad \textbf{(28)}$$

Where:

$Q_{L,S} =$ the leakage flow through the gap between the through and the blades

$Q_{L,RZ} =$ the leakage flow over the central pipe/axis

$Q_W =$ the working volume of the screw that is used for power-production

$Q_{L,R} =$ leakage through adhesion of water to the blades

$Q_{L,T} =$ leakage if there is no baffle present on the screw. A baffle is a protective plate over the screw to prevent water that reached escape velocity from "flying" out.

Nuernbergk and Rorres then argue that when the adhesion leakage (water sticking like glue to the surface of the steel) is neglected (as it is often very small) and the screw is operated at optimum filling rate, such that both.

$Q_{L,R} = 0m^3/s$ and $Q_{L,RZ} = 0\,m^3/s$

The assumption about the filling rate in practice is perhaps a bit dubious, but not much research or experimentation has been done into the effect of filling rate.

When also a baffle is installed to make $Q_{L,T} = \frac{0m^3}{s}$, the only remaining discharge that needs to be determined is the $Q_{L,S}$ and Q_W.

Nuernbergk and Rorres note that Muysken [51] found the following relation for leakage flow:

$$Q_{L,S} = 7{,}07 * s_{sp} * R_o^{1,5} = [m^3/s] \tag{29}$$

Where:

s_{sp} is the gap size in $[m]$

R_o is again the outer radius of the screw and is entered in this empirical formula in meters.

Note-bene: This means that the 7,07 needs to have dimension of $[m^{0,5}/s]$.

The working flow or effective flow Q_W has been found by J. Weisbach [52] with the following formula:

$$Q_W = \frac{n}{60} * V_U = \left[\frac{m^3}{s}\right] \tag{30}$$

Where:

$\frac{n}{60} = [Hz] = [s^{-1}] =$ the rotation frequency and n is the RPM (rotations per minute).

$V_U =$ the volume being transported by one full rotation of the screw

Muysken [51] also found the maximum value of n for ASTs:

$$n_{max} \leq \frac{50}{(2 * R_o)^{2/3}} = [min^{-1}] \tag{31}$$

From Lashofer and Willinger:

> *"Due to their robust design (fig. 9) and low investment costs hydropower screws become more and more attractive for low head power production application."*
>
> – A. Lashofer and R. Willinger, 2014

Defined are: screw diameter D, inclination angle β, number of blades N and pitch S.

Description of energy transfer for hydropower screw is quite difficult. Method using non-dimensional parameters from [53] the theoretical torque of the hydro-power screw is:

$$M_{th} = \eta_h * \rho * g * H * \frac{\pi * D^3}{8} * \alpha$$

Where:

$\eta_h \approx 0.85$, a typical value for hydraulic efficiency according to [54]

$\alpha = \frac{(\lambda * \nu)}{\tan(\beta)}$, a nondimensional parameter including:

$\lambda = \frac{S * \tan(\beta)}{D * \pi}$, the pitch ratio

$\nu = \frac{4 * V_S * N}{D^2 * \pi * S}$, the volume ratio

Where:

$V_s =$ is the volume of one bucket.

The theoretical power of the screw is:

$$P_{th} = M_{th} * 2 * \pi * n = \eta_h * \rho * g * H * \frac{\pi^2 * D^3}{4} * \alpha * n$$

And the volume flow-rate can be calculated by:

$$\dot{V} = \alpha * \frac{\pi^2 D^3}{4} * n$$

According to [53] typical values for the nondimensional parameters for hydropower screws are:

$\lambda * \nu = 0{,}04 - 0{,}08$; $\beta = 20° - 30°$; $\alpha = 0{,}07 - 0{,}22$

In literature there are some empirical rules, among which from [53], that give an estimate of the maximum rotational speed of the screw.

Lashofer and Willinger then use a simple approach:

> *"When the rotor is turned, the blade lifts up some amount of water due to the no-slip condition between fluid and blade surface. When the rotational speed is high, this water will be ejected due to centrifugal forces and as a result, this volume is lost for the energy transfer process. The individual blade is then modeled as a flat plate which moves upward in vertical direction with velocity U. This velocity can be interpreted as the circumferential speed of the screw."* – A. Lashofer and R. Willinger, 2014

Pump enhanced AST

Downside of the regular AST is that the effectiveness of its operation is very sensitive to and dependent on the intake water-level (the intake being on the upstream side for turbines). If a pump is installed to keep the water-level constant at the intake, the turbine can theoretically always run at full capacity (see also **Figure 81**).

This pump should only be active when the water-level-difference is large enough to create enough power to overcome the required power for the pump as a lower limit.

As upper-limit the pump should not be active when the water-level is sufficient for the turbine to work efficiently enough without it.

While installing a pump to the system introduces more losses, if this can make the turbine run for more time in the year, it may give more energy output per year at least compared to the regular AST. Question remains of course how it compares to Kaplan turbines that inherently have a broader range of operation.

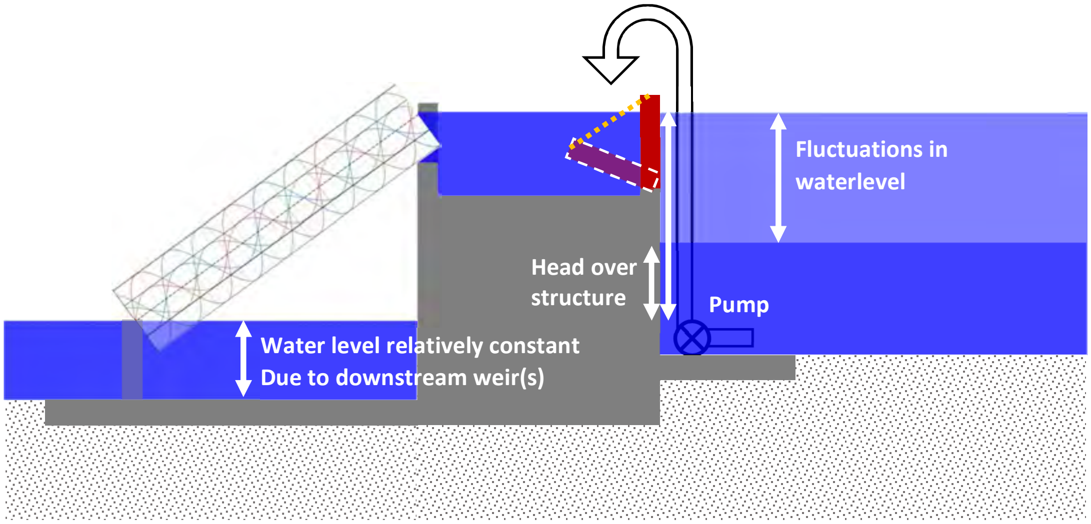


Figure 81 *- Working principle pump-enhanced AST – When water-level is lower than intake reservoir the pump elevates the water to the reservoir. When the water can enter the reservoir by itself, the pump is deactivated. Theoretically the turbine runs at optimal efficiency, but this is paid for by powering the pump. – Source: own work*

In theory the minimum required head-difference required for such a principle to be economical even in terms of energy is as follows:

$$P_{pump} = \frac{\rho g Q * (H_{t,\_d} - H_{up})}{\eta_{pump}} = \frac{\rho g Q * \Delta H_{raise}}{\eta_{pump}} =$$

$$P_t = \eta_t * \rho g Q * \left(H_{t,\_d} - H_{down}\right) = \eta_t * \rho g Q * \Delta H_{drop}$$

$$P_t > P_{pump} \quad \rightarrow \quad \frac{\Delta H_{drop}}{\Delta H_{raise}} > \frac{1}{\eta_{pump} * \eta_{turbine}}$$

$$\frac{\Delta H_{drop}}{\Delta H_{raise}} = \frac{\left(H_{t,d} - H_{down}\right)}{\left(H_{t,d} - H_{up}\right)} = \frac{\Delta H_{drop}}{\left(\left(H_{t,d} - H_{down}\right) - \left(H_{up} - H_{down}\right)\right)} =$$

$$\frac{\Delta H_{drop}}{\Delta H_{drop} - \Delta H_{sys}} > \frac{1}{\eta_{pump} * \eta_t} \qquad (32)$$

Where:

$\Delta H_{drop} =$ The fall-height in $[m]$ for the water going through the turbine

$\Delta H_{raise} =$ The height in $[m]$ the pump needs to raise the water to get to the upper level of the turbine

Assuming the installed pump has a slightly better efficiency than the AST, say 90% vs 80% and the system head difference is 2m then the maximum drop height for the turbine is:

$$\Delta H_{drop} < \Delta H_{sys} * \frac{\frac{1}{\eta_{pump} * \eta_t}}{\left(\frac{1}{\eta_{pump} * \eta_t} - 1\right)} = 2{,}0 * \frac{\frac{1}{0{,}9 * 0{,}8}}{\left(\frac{1}{0{,}9 * 0{,}8} - 1\right)} = 7{,}14m$$

The resulting power is:

$$P_t - P_{pump} = \eta_t * \rho g Q * \Delta H_{drop} - \frac{\rho g Q * \Delta H_{raise}}{\eta_{pump}} = \rho g Q * \left(\eta_t * \Delta H_{drop} - \frac{\Delta H_{raise}}{\eta_{pump}}\right)$$

$$= \rho g Q * \left(\eta_t * \Delta H_{drop} - \frac{\Delta H_{drop} - \Delta H_{sys}}{\eta_{pump}} \right)$$

$$= \rho g Q * \left(\frac{\Delta H_{sys}}{\eta_{pump}} - \left(\frac{1}{\eta_{pump}} - \eta_t \right) * \Delta H_{drop} \right)$$

Meaning that the system-head is being increased slightly by the pump efficiency, but reduced by a percentage of the drop-height. This percentage of the drop-height is larger than the gain by the pump-efficiency.

In the example of 2m head, 80% turbine efficiency and 90% pump efficiency, the head-differences is being reduced by a 31% of the drop-height, but only increased by 11% by the turbine efficiency. The resulting head for a drop-height of 3m is 1,29m, that is a net reduction of 36%.

In terms of power this “enhancement” is therefore never a good idea. Only if the total amount of energy produced in a year can be increased, will it be lucrative.

Clearly this, enhancement is only viable when there is a larger head-difference and a lot of fluctuation. The variations of the Nederrijn can be taken by an AST by taking a large enough diameter screw.

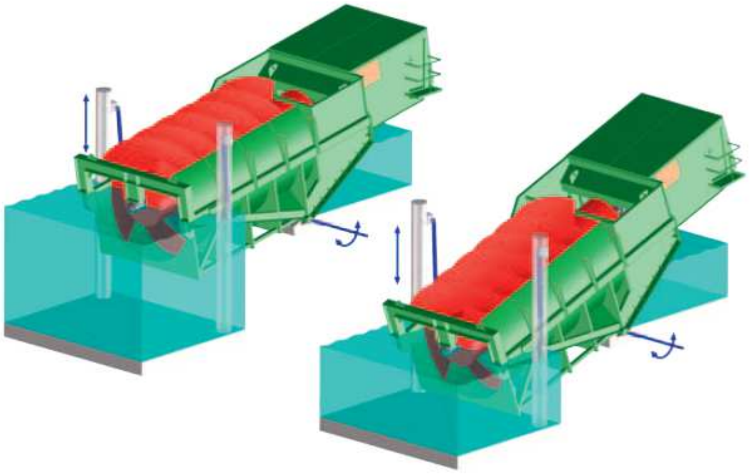


Figure 82 *- Adjustable inclination and height AST - Source: Landustrie [39, p. 7]*

A better option would perhaps be the Adjustable height AST. The company “Landustrie” has developed such a concept.

However, as noted in the literature review the inclination angle also has an influence on the efficiency and not many of these adjustable turbines have been installed as of yet. Landustrie notes that also for this type, it’s only beneficial for locations where water levels varies considerably.

Waterwheel

Together with the AST perhaps the oldest of hydro-power-technologies. According to Dr. Gerald Müller [55] waterwheels have 4 major variants shown in the figures below:

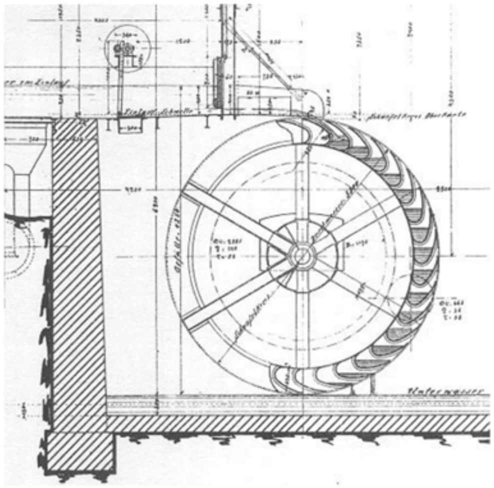


Figure 83 *– Overshot wheel – Water enters the wheel from above. Water-level differences are commonly 2,5 - 10m – Flow rates between 0,1 and 0,2m^3/s per meter width. – source image: [56] – Source information: [57]*

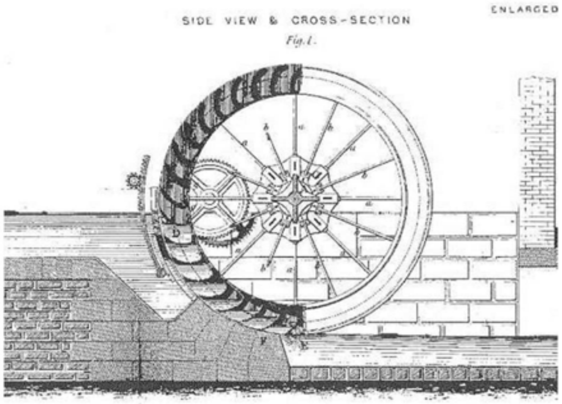


Figure 84 *– Breast-shot wheel – Water level upstream is about the height of the wheel's axis. Head-difference range from 1,5 - 4,0m and flow-rates between 0,35 and 0,65m^3/s per m width - source image: [58] – source info: [57]*

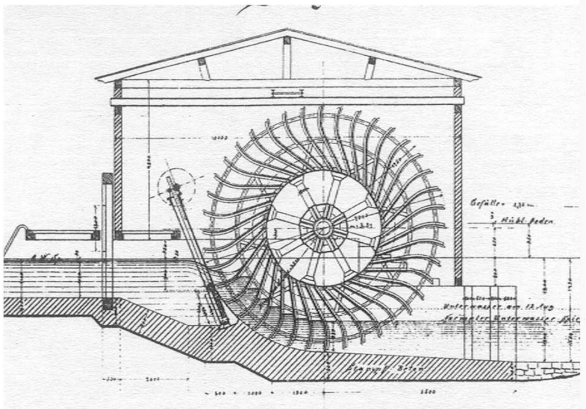


Figure 85 *– Undershot- (Zuppinger-)wheel – Water enters the wheel below its axis. Head-differences range from 0,5 - 2,5m and can handle discharges of 0,5 to 0,95m^3/s per m width – source image: [56] – source info: [57]*

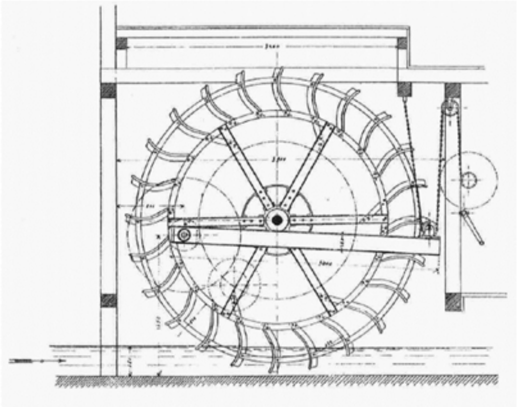
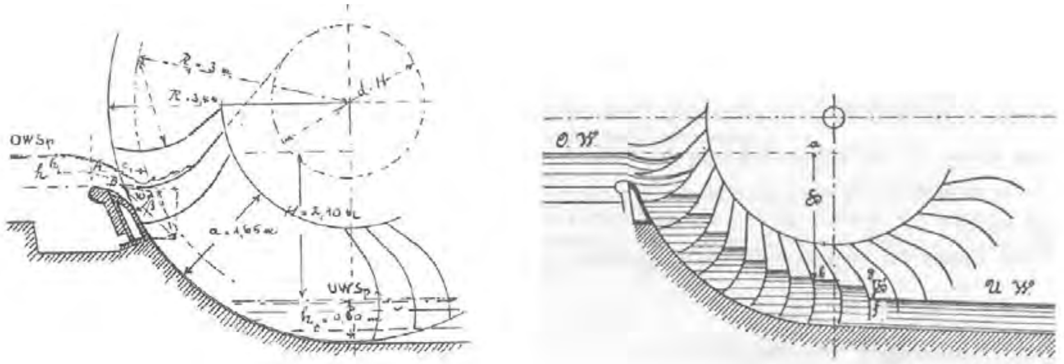


Figure 86 *– Stream-wheel – Use the principle of impulse. Kinetic energy from the water is employed to power the wheel. Unfortunately their efficiencies are too low to be used economically in large numbers - source: [56] – source info: [57]*

According to Gerald Müller water-wheels, contrary to popular belief, were not replaced by steam-engines in the industrial revolution, but further developed and evolved even till the beginning the 20th century and at that time in the order of ten thousands of waterwheels were in operation in Europe in [57].

Well-designed waterwheels can, according to Müller, reach an efficiency of 71-76% (undershot) to 85-90% (overshot). Also these turbines require a relatively fixed upstream and down-stream water-level.

For the Driel situation especially the Zuppinger wheel (first developed by the Swiss engineer Walter Zuppinger) may be the best solution regarding waterwheels. This wheel employs both kinetic and potential (height) energy and has the most efficient design currently known for undershot wheels due to its inflow and shape of the blades (backwards inclined). Due to the fact that it can deal with low head-situations (currently built waterwheels can handle 1,2-2,3m head-difference and can take up to 3m^3/s discharges), but still has a relatively high efficiency, makes this the most suitable variant within the waterwheels.



a. Side elevation and inflow b. Working principle

Fig. 8: Design principles of undershot or *Zuppinger*-wheels, Müller (1899)

Figure 87 *- Zuppinger wheel Inflow and working principles – Source: From: [57] which is referencing: [55]*

Müller [57] references measurements from the University of Stuttgart in Germany done in 1977 where a waterwheel built in 1886 an at the time 91 year old waterwheel that still reached an efficiency of 71% at maximum discharge. Which Müller noted to be remarkably high considering its condition and the materials used (wooden blades, bush bearing (metal cylinder around wooden axel), etc.) and the gaps at each side of the wall that had worn out during its use.

A large part of the costs of a waterwheel are due to the low rotation speed of waterwheels. The gear-box system to convert the motion to one acceptable for a generator takes up about 25-30% of the costs for undershot and 40-45% for overshot-wheels.

The payback period Müller references for the undershot wheel is 12-14 years with a life-expectancy of 30 years. This compared to 25-30 years payback period for Kaplan (according to Müller).

Oryon watermill

Is a vertical axis free flow watermill. It has lamellas that obstruct the flow when the arm is moving with the flow and let through most of the flow when the arm is moving against the flow.

Figure 88 - *Oryon watermill - Source: [1]*

However the manufacturer had already investigated the potentials at the Nederrijn and concluded the following:

Not viable in low flow-velocity situation of Nederrijn.

> "…
>
> *Drijvende versie hebben wij eerder getest in de Rijn bij Tolkamer (real life), bij Marin en met gevalideerd met CFD analyses.*
>
> *Gezien de relatief lage stroomsnelheden bij vrije stroming in Nederland, zien wij hier, gezien o.a. het lage rendement t.o.v. geforceerde stroming , weinig mogelijkheden, behalve als je dit in combinatie kan doen met geforceerde stroming, achter een stuw of ander kunstwerk.*
>
> *Ook inpassing van brede installaties heeft invloed op vaarwegen, vergunningen etc. etc.*
>
> *…"*

Translation:

> "…
>
> *Floating verions has been tested in the Rhine at TOlkamer (real-life test), at Marin test centre and validated with CFD analysis.*
>
> *Considering the low flow-velocities in the free-flow in the Netherlands and considering the low return on investment compared to channeled flow, we see little to no opportunities. Only if it is combined with forced/channeled flow behind a weir or other hydraulic structure.*
>
> *Then there is also combining the design with shipping and permits. Etc. etc.*
>
> *…" – Dolf Pasman, From Deepwater-energy (developers of Oryon Watermill)*

That, and the fact that very little information is available about the performance of this type of turbine (like efficiency, costs, etc.) this turbine type is not investigated further.

APPENDIX 5 – STAKEHOLDERS

In the table below the stake holders and their relation or interest in hydropower at Driel are listed.

Stakeholder	**Relation and/or interest with hydro-power at Driel**
Hevea initiatief	Wants to implement sustainable ideas in and around Heveadorp, specifically apply hydro-power at the weir of Driel
Dutch Goverment	Wants to please its electorate, by among other things: - promoting sustainable energy production (climate agreement) - to some extend promote ecological friendliness of any developments - Wants to maintain or increase economic value of existing and future assets - etc.
Rijkswaterstaat (RWS) - ministry of infrastructure and waterprotection	- Directed by Dutch government - Owner of the waterway - Responsible for discharge distribution and flood-protection in the Delta and main river-systems - Responsible for giving out (environmental) permits to run hydro-power-plants - Owner of the weir-complex of Driel
Shipping (ship-captains, companies dependent on shipping, etc.)	- Most important interest is a quick (shortest time as possible) transit through the weir-complex. - Also navigability through and in the area around the weir is in their interest.
Energy companies (NUON/Vattenfall, ESSENT, etc.)	- First and foremost, earning money with production of electricity (producing more power in an economic way is therefore in their interest); - Grid-stability (Any power-produced at Driel should not negatively affect this); - Producing power legally (with the relevant permits) to maintain good public image and for obvious moral reasons;
Nature, environmental and ecological protection agencies	- Maintain current habitat quantity (area) and quality; - wants to prevent or reduce impact due to any activities in or around important (reserved) areas;
Sport-fishing-community	- cares about the amount and quality of fish in fishing waters; - for hydropower specifically, wants the fish-mortality rate as low as possible (preferably 0%);
"Neighbours" i.e. citizens/people living near the weir of Driel	- Could potentially benefit from activities from Hevea Initiative; - Wants to keep to a minimum any hinderance and/or pollution; - During any construction works this can be noise, dust and dirt and during operation of the hydro-powerplant, mostly noise pollution;
Province of Gelderland	- Wants to, just like the Dutch government, promote production of sustainable energy; - Also cares about wishes from its respective electorate;

Municipality of Renkum	- wishes mostly the same as province; - electorate is a smaller group, so might have different wishes;
Municipality of Overbetuwe	- idem to municipality of Renkum
Koninklijke Nederlandse Heidemij Matschappij (KNHM)	- wants to gain/keep good public image; - Promotes sustainable and social development; - Helps Hevea Initiative in developing hydropower at Driel;
Arcadis NL	- interested in getting engineering work and gaining money; - wants to have good public image; - Helps KNHM in helping Hevea initiative with developing hydro-power at Driel

APPENDIX 6 – DATA QUALITY AND FILTERING METHOD

QUALITY OF THE DATA

Both the discharge and the water-levels had their own quirks concerning erroneous data. In the paragraphs below the quality of the data is reviewed.

Discharge data quality

In the table below a summary of the downloaded data:

Discharges data evaluation:	**Amount**	**% of total**	**Comment**
Number of measurements	1.387.382	100,00%	-
Measurements "inline"	1.387.057	99,98%	-
Shifted measurements (wrong column)	325	0,02%	No numerical correction has been done, just shifted back to the right column.
Corrupted (#Num!) , "not a number"-measurements	69.046	4,98%	These entries have been corrected to be the last measured value that didn't have an error.
Number of measurements exactly zero (=0 m^3/s)	1.831	0,13%	"Exactly", as in: within measurement precision.
Measurements smaller than zero (<0 m^3/s)	1.962	0,14%	For (hydro-power) design purposes, these values have been corrected to be 0 m^3/s.
Earliest measurement	01/01/1961		
Start of 20 min interval measurements	01/01/1990		From this point on daily averages have been determined from the gathered data and used for the analysis.
Last measurement	31/12/2018		

Table 38 Discharge Data evaluation, amount of errornous data and applied corrections. Interesting note: All of the "#Num!"-entries have been measured ***after*** *01-01-1990, the new system is likely a digital one.*

Some of the discharge values were non-sensible, like a discharge with the code "#Num!", which is MS Access version of "not a number". These data entries have been replaced with the last recorded value, to not have large jumps in the data that didn't actually occur. For example if somewhere in the data one value is missing between values of around a 1000 m^3/s and the missing value is replaced with 0m3/s there would be a unnecessarily large spike downwards. Such spikes are prevented with the used method.

In the year 1990 the way of measuring, but perhaps more importantly, the frequency of measuring changed. From this point on every 20 minutes instead of once a day a measurement was taken. Therefore after 1990 daily averages have been determined to have data of comparable level of detail.

Downside of this new system is the increased amount of erroneous, "empty" measurements that are recorded. However, this is compensated by the shear amount of measurements. Daily averages could still be determined despite the errors.

Because the missing data is only present in the part where every 20 minutes measurements have been taken, statistical analysis of the data is assumed to be valid.

Water-level-data quality

In the table below a summary of the downloaded data:

Waterlevel data evaluation:	**Amount**	**% of total**	**Comment**
Number of measurements	1.125.187	100,00%	-
Measurements "inline"	1.124.954	99,98%	-
Shifted measurements (wrong column)	233	0,02%	No numerical correction has been done, just shifted back to the right column.
Erroneous data: "99999m" measurements.	103	0,01%	These entries have been corrected to be the last measured value that didn't have an error. All of these have been measured in the end of October 1984, except for 3 values measured in mid-July of 1993. Apparently some kind of error occurred in these days.
Earliest measurement	01-11-1970 (01/01/1968)		Driel boven (upstream measuring point) has data starting from: 01-01-1968. Driel beneden (downstream measuring point) starts at: 01-11-1970. Therefore the head-difference data starts at this date.
Start of 1 hour interval measurements	01/01/1981		From this point on daily averages have been determined from the gathered data and used for the analysis.
Start of 10 min interval measurements	26/11/2013		Also here daily averages are determined. The measurements are done at a 10min interval.
Last measurement	31/12/2018		

Table 39 Data evaluation for water-level-measurements

Method of Loading and filtering the data:
To be able to load the data from the "waterinfo" website and use them in calculations some Access "tricks" can be useful. Below a step by step plan of how to load this data into Access. Once filtered, separated by time-period and saved in Access they can be loaded into Excel (Excel has a limit in rows that can be exceeded by Access).

The following steps have been performed:

1. Go to website with data:
    a. https://waterinfo.rws.nl/
    b. Select relevant data-type (discharge, waterlevel, etc.)
    c. Select relevant time-period (01-01-1900 till 31-12-2018 was taken)
    d. Select measuring stations (Driel in this case)
    e. Fill in e-mail address to receive download link to CSV-file
    f. Wait till link is sent
    g. Download csv-file and place in folder near Access database for ease of use
2. Open Microsoft Access and create a new database
    a. Save in folder where csv-files are placed
    b. Go to tab "External data"
    c. In ribbon "Import & link"
    d. --select--> New Data Source
    e. --select--> From file
    f. --select--> Text file
3. Menu opens ->
    a. --select--> [**Browse]** and select/open the downloaded csv-file
    b. --select--> 3rd option **"Link to the data source by creating a linked table"** (we don't want to import or append) and press [ok]
    c. --select--> "Delimited…" and press [Next >]
    d. --select--> "Semicolon"
    e. --tick box--> **"First Row Contains Field Names"** and press [Next>]
    f. Some of the data values are offset by 1 column for some reason (in the case of the Discharge data of Driel 375 values of the over 1 million entries). Therefore, it is useful to select the columns just left of fields that have an important data-type (like data or number values) and change them to data-type to the right (especially if the field left of it is not relevant for you), should one wish to rectify this offset. Press [Next>]
    g. One can rename the table here should one desire to… Change name and press [Finish]
4. Take a look at your newly imported table to see what it holds
5. In case of the discharge data of Driel the offset data-values had to be filtered out:
    a. Check for offset values by filtering the first, often empty column for non-empty values
    b. If any are found continue with the rest of this part, otherwise skip till @@@
    c. Create a Query at the "Create"-tab by pressing "Query-design"
    d. --select--> "Make table [*!]" to make a table with the values that aren't offset and give it a name e.g. "Q – inline data"
    e. Select the linked table that was just added
    f. --double click--> OR -- select in dropdown box --> any fields (columns) that are relevant and want in the new table (if one still has the linked table open one can see what each field actually holds)
    g. For Discharge data the column "Meetpunt_identificatie" was a field that could be used to filter out the shifted and unshifted values. -- enter --> = "[desired value or name]" in cell that is in the row of "criteria"

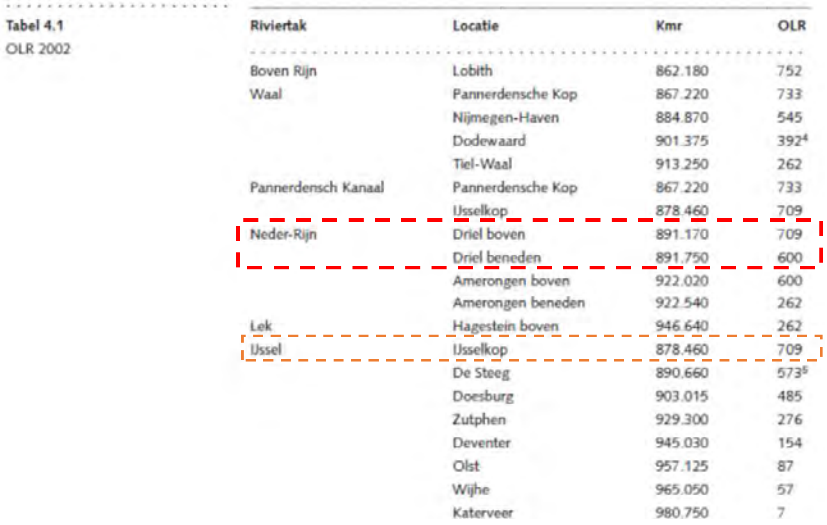
   h. Save the query as "Q1 – inline to table"
   i. Run query (a table will be created)
6. Should one want to correct shifted values:
   a. Copy the existing query
   b. -- Change --> in dropdown menu of table below: all the fields to the one above it (if its shifted to the left) or the one below it (if it is shifted to the right)
   c. Criterium can stay the same, because you changed the column (you want to check for the right names in the wrong column now)
   d. -- click --> Make Table [*!] -> change the name of the table to for instance "Q – shifted data"
   e. -- save --> query as "Q2 – shifted data to table"
   f. -- Run query --> and check if you got indeed the shifted data
7. Create new query
   a. -- Select --> your most recent table "Q – shifted data"
   b. -- Select --> Append [+!]
   c. -- In menu -->-- Select --> table name "Q - inline data" (so you will append the shifted data to the already inline table)
   d. -- Double click --> all the fields of the "Q - shifted data" table
   e. In the table below in the row "Append to" -- Select --> the fields that the shifted values correspond to (so if the time column was shifted to the data column, you select time in the field that says date in the top row)
   f. -- save --> query as "Q3 – Append reshifted to inline"
   g. -- run query –> now you have an a complete table with all your data in the right column
8. -- Copy --> the "Q – inline table"
   a. -- Paste -->
   b. -- Rename --> To for instance "Q – corrected data"
   c. -- select --> Structure only and press [ok]
   d. -- Open --> new table "Q – corrected data"
   e. -- Open --> design view
   f. -- Insert --> a row at the top and name it "ID"
   g. -- Select --> [autonumber] in the drop down menu in the second column
   h. -- save --> the table
   i. -- Insert --> new row underneath "ID" and call it "IDplus1"
   j. -- Select --> [calculated]
   k. -- double click --> "ID" and enter "+1" so that formula becomes: " [ID]+1" and press [ok]
   l. -- save --> the table and close it [x]
9. -- Create --> new query and select table "Q – inline data"
   a. -- Select --> type "Append [+!]"
   b. -- Select --> the new table "Q – corrected data"
   c. -- double click --> all columns of that table
   d. -- Save --> query as "Q4 – append to correct data"
   e. -- run --> query
10. -- Create --> new query (to change any value that is incorrect or strange to a previously measured value, i.e. this is the Is Null correction or =99999 correction)
   a. -- Select --> "Q – corrected data"
   b. -- Select --> "Q – corrected data" again (so that it is in there twice!)
   c. -- Link --> ID with IDplus1: ATTENTION:
      drag **"ID"** from **"Q – corrected data"** – to --> **"IDplus1"** from **"Q – corrected data_1"**
   d. -- Select --> type "Update [/!]"

e. -- Select --> in the drop-down menu in the table below (first column): “Numerieke waarde” (numerical value) or any value that you want to change.
f. -- enter --> in row: “Update to” this exact tekst: “[Q – corrected data_1].[Numeriekewaarde]”
g. -- enter --> in row: “criteria” the formula: “ >=99999” or “Is Null”
h. -- save --> query as: “Q5 – correct Is Null” or “Q5 – correct =99999”
i. -- run --> query

One can do other query’s where data is corrected, for instance if negative discharges are unwanted they can be updated in a similar fashion to 0 values.

Most important is loading the csv file and doing the first corrections like undoing the shift to the left.

Distance between measuring station:



Tabel 4.1
OLR 2002

Riviertak	Locatie	Kmr	OLR
Boven Rijn	Lobith	862.180	752
Waal	Pannerdensche Kop	867.220	733
	Nijmegen-Haven	884.870	545
	Dodewaard	901.375	392[4]
	Tiel-Waal	913.250	262
Pannerdensch Kanaal	Pannerdensche Kop	867.220	733
	IJsselkop	878.460	709
Neder-Rijn	Driel boven	891.170	709
	Driel beneden	891.750	600
	Amerongen boven	922.020	600
	Amerongen beneden	922.540	262
Lek	Hagestein boven	946.640	262
IJssel	IJsselkop	878.460	709
	De Steeg	890.660	573[5]
	Doesburg	903.015	485
	Zutphen	929.300	276
	Deventer	945.030	154
	Olst	957.125	87
	Wijhe	965.050	57
	Katerveer	980.750	7

Figure 89 *- OLR (“Overeengekomen Lage Rivierwaterstand”= agreed upon low river-water-level in cm+NAP) table 4.1. Kilometre marks of measurement locations along the Dutch river Delta. - Source:*

APPENDIX 7 – FLOW AND WATER-LEVEL DATA ANALYSIS

List of symbols in alphabetical order in the Mathcad sheet:

A.days	Vector with an array of numbers going from 1 till "N.days". Used to generate a series of days in a related Access file.
ALLDATA	Aggregated table/matrix with data from all added time-periods, in this case from 01-01-1970 up to and including 31-12-2018
CWD	Current Work Directory, used to always find the "Title.excel" file as long as both this file and the excel are in the same folder.
D.m	Measurement duration in years.
data	Within the f.PDF-function definition, this is the data that is used as input for the histrogram.
DATA7080	Matrix/table with Imported data for period of 01-01-1970 up to, but not including 01-01-1980. So 70 and 80 are used to refer to 1970 and 1980. For data of 1980 till 1990 the same format is used.
dH.bin	Bin-size for a histogram with input being the "ΔH" vector.
dL.Driel	Distance between the two waterlevel measuring stations around the weir of Driel
dP.bin	Bin-size for a histogram with input being the "P.0" vector.
dQ.bin	Bin-size for a histogram with input being the "Q" vector.
dQHP.bin	Bin-size for a histogram with input being the "QHP" vector.
dX.bin	Within the f.PDF-function definition, this is the bin-size for the histrogram.
f	Factor " f " which is defined as being the ratio between the pressure-head and the energy-head, where pressure-head is " f " times the energy-head.
f.PDF(dX.bin, data)	Function to create a table for a histogram from a data-set "data" with bin-sizes "dX.bin".
H.av.1	Yearly mean head-difference (mean head-difference over every year).
H.av.10	10-year mean Head-difference. Mean head-difference over a period of 10 years, calculated every year where there are 5 preceding and 5 subsequent measurement years.
H.av.30	30-year mean head-differences. Mean head-difference over a period of 30 years, calculated every year where there are 15 preceding and 15 subsequent measurement years.
H.desc2	A matrix with two columns, column 1: All head-difference measurments rearranged in descending order, column 2: Discharges rearranged in such a way that the corresponding head-differences are in descending order. Basically column 2 is the

	corresponding discharge for each head-difference in column 1.
h.down	Measurements of downstream waterlevels in meters with respect to NAP (Normaal Amsterdams Peil) ~sea-level
H.max.year	Maximum Head-difference of all measurements rounded upwards to a multiple of 0.1 meter.
h.up	Measurements of upstream waterlevels in meters with respect to NAP (Normaal Amsterdams Peil) ~sea-level
H1970	One row of the MHy Matrix. The first row with year number 1970. This has been done for all the measurement years to plot them.
incl.opengate	Inclination of the watersurface at the weir of Driel in high discharge, open gate conditions.
meanHy	The mean value of the head-difference over all measurement years for each day.
meanPy	The mean value of the Energy flux MPy over all measurement years for each day.
meanPy.ALLtime	All time mean value of energy flux. Mean over all measurements.
meanQday	The mean value of the discharge over all measurement years for each day.
MH10y	Head-difference measurements of the last 10 years, where each row is a year and each column is a day in the year to which that row belongs. First row is 2008, moving forward in time each row till 2018.
MH30y	Head-difference measurements of the last 30 yeasr, where each row is a year and each column is a day in the year to which that row belongs. First row is 1988, moving forward in time each row till 2018.
MHy	A Matrix with head-difference measurements, where each row is a measured year, starting with the first measurement year (1970) progressing in time with each row. Each column is a day in the measured year of that row.
MHy.max	Largest measured head-difference in the entire measurement series.
MPy	Matrix with daily average Energy flux present at Driel, product of discharge, head-difference, mass-density and gravitational acceleration. A Matrix with rows as measurment years and columns as days in a year.
MQ10y	Discharge measurements of the last 10 yeasr, where each row is a year and each column is a day in the year to which that row belongs. First row is 2008, moving forward in time each row till 2018.
MQ30y	Discharge measurements of the last 30 yeasr, where each row is a year and each column is a day in the year to which that row belongs. First row is 1988, moving forward in time each row till 2018.

MQy	A Matrix with discharge measurements, where each row is a measured year, starting with the first measurement year (1970) progressing in time with each row. Each column is a day in the measured year of that row.
MQy.max	The maximum, or highest value in the entire measurement series.
N.7080	Number of rows in the imported table/matrix "DATA7080".
N.ALL	Number of rows of table/matrix ALLDATA. Used in certain loops in this MC-sheet.
N.days	Input for "A.days" that is the maximum number.
N.Hbin	Number of bins in histogram "PDF.ΔH".
N.Pbin	Number of bins in the histogram "PDF.P0".
N.Q.bias	Number of bins that has biased information and were "cut out" to get the unbiased discharge distribution.
N.Qbin	Number of bins in histogram "PDF.Q".
N.QHPbin	Number of bins in the histogram "PDF.QHP".
Output1	Function for exporting discharge data "MQ10y" to an excel where it can be read by other programs or Mathcad sheets.
Output3	Function for exporting head-difference data "MQ10y" to an excel where it can be read by other programs or Mathcad sheets.
P(η, ΔH.min, Q.min, Q, ΔH)	Generic function for power output per day, where P is turbine power output in kilo-Watts, and has inputs η, ΔH.min, Q.min, Q, ΔH.
P.av.1	Yearly mean value of energy flux.
P.av.10	10-year mean value of energy flux. Mean over a period of 10 years, calculated every year where there are 5 preceding and 5 subsequent measurement years.
P.av.30	30-year mean value of energy flux. Mean over a period of 30 years, calculated every year where there are 15 preceding and 15 subsequent measurement years.
P.mean1	Mean value of the bins with lower than 975kW theoretically available power, of the distribution that contains the low power peak.
P.mean2	Mean value of the bins with higher than 975kW theoretically available power, of the distribution that contains the high power peak.
P.median1	Median value of the bins with lower than 975kW theoretically available power, of the distribution that contains the low power peak.
P.median2	Median value of the bins with higher than 975kW theoretically available power, of the distribution that contains the high power peak.
P.pdf.min975	Related power of the bins with lower than 975kW theoretically available power. This part of the distribution contains the low power peak.
P.pdf.plus975	Related power of the bins with higher than 975kW theoretically available power. This part of the distribution contains the high power peak.

P.thmax	Theoretical maximum power (turbine system 0)
P0.mean	Mean value of the complete distribution of the theoretically present Power.
P0.median	Median value of the complete distribution of the theoretically present Power.
p1eP	Bin number at which the cut is made to split the two peaks in the theoretically available power histogram
PDF.P.min975	Percentage of occurance for the bins with lower than 975kW theoretically available power. This part of the distribution contains the low power peak.
PDF.P.plus975	Percentage of occurance for the bins with higher than 975kW theoretically available power. This part of the distribution contains the high power peak.
PDF.P0	Histogram using f.PDF-function with input "P.0" vector and "dP.bin" as bin-size.
PDF.Q	Histogram using f.PDF-function with input "Q" vector and "dQ.bin" as bin-size.
PDF.QHP	Histogram using f.PDF-function with input "QHP" vector and "dQHP.bin" as bin-size.
PDF.QHP.p2	Percentage of occurance values of cut distribution to find 2nd peak of QHP (discharge-head-difference-product).
PDF.ΔH	Histogram using f.PDF-function with input "ΔH" vector and "dH.bin" as bin-size.
Q	Aggregated vector with all measured discharges over the entire measurement duration.
Q.av.1	Yearly mean discharge (mean discharge over every year).
Q.av.10	10-year mean discharge. Mean discharge over a period of 10 years, calculated every year where there are 5 preceding and 5 subsequent measurement years.
Q.av.30	30-year mean discharge. Mean discharge over a period of 30 years, calculated every year where there are 15 preceding and 15 subsequent measurement years.
Q.cut.unbs	Discharge at wish the unbiased distribution is cut off.
Q.desc2	A matrix with two columns, column 1: All discharge measurments rearranged in descending order, column 2: Waterlevel difference rearranged in such that the corresponding discharges are in descending order. Basically column 2 is the corresponding head for each discharge in column 1.
Q.low	An arbitrary low discharge chosen to indicate an area in the flow-duration graph that is reasonably flat.
Q.mean	Unbiased mean discharge.
Q.mean.biased	Mean discharge of the biased distribution. Biased as in the low-discharge peak is still included.
Q.median	Unbiased median discharge.

Q.median.biased	Median discharge of the biased distribution. Biased as in the low-discharge peak is still included.
Q.min	Minimum discharge through the turbine for it to produce power
Q.min0	Minimum discharge for turbine system 0
Q.open	Discharge for which the weir opens.
Q.y.max.rnd	Maximum discharge of all measurements rounded upwards to a multiple of 100 cubic meters per second.
Q1970	One row of the MQy Matrix. The first row with year number 1970. This has been done for all the measurement years to plot them.
Q7080	Measured discharges in time period from 01-01-1970 up to but not including 01-01-1980. For data of 1980 till 1990, 1990-2000, etc. the same format is used.
QHP	Vector with row by row Product of vectors "Q" and "ΔH", giving an indication of the amount of energy present.
QHP.desc	Matrix with 3 columns. Column 1: Discharge-Head-difference-product duration curve. All QHP values rearranged in descending order. Column 2: discharge corresponding to QHP value in column 1. Column 3: head-difference corresponding to QHP-value in column 1.
QHP.localmax	QHP value of local maximum or 2nd peak.
QHP.mean	Mean QHP (discharge-head-difference-product).
QHP.median	Median discharge-head-difference-product.
QHP.median	Median discharge-head-difference-product.
QHP.pdf.p2	QHP values of cut distribution to find 2nd peak.
SelPDF.Q.plus60	Percentage of occurance values of histrogram of the unbiased discharge data
SelPDF.ΔH.plus15cm	Histogram vector with percentages of occurance of only the high-head-part. The low-head-and-high-discharge-peak is removed from this vector.
SelQ.pdf.plus60	Related discharges of the histogram of the unbiased discharge data.
SelΔH.pdf.plus15cm	Histogram vector with Head-differences belonging to percentages of occurance in vector "SelPDF.ΔH.plus15cm".
Stdv.P0	Standard deviation of the complete distribution of the theoretically present Power.
Stdv.P0.min975	Standard deviation of the bins with lower than 975kW theoretically available power, of the distribution that contains the low power peak.
Stdv.P0.plus975	Standard deviation of the bins with higher than 975kW theoretically available power, of the distribution that contains the high power peak.
t	Time axis in days (only defined within the measurement period). Used to plot measurements in time-domain.
t.10yav	Time-axis on which to plot the 10 year averages "Q.av.10"

t.30yav	Time-axis on which to plot the 30 year averages "Q.av.30"
t.dayinyear	Exact number of days in a year.
t.my	Vector with measurement years.
Title.excel	File path to excel that serves as a data-link between Access database and Mathcad.
Var.P0	Variance of the complete distribution of the theoretically present Power.
Var.P0.min975	Variance of the bins with lower than 975kW theoretically available power, of the distribution that contains the low power peak.
Var.P0.plus975	Variance of the bins with higher than 975kW theoretically available power, of the distribution that contains the high power peak.
x.dayinyr	x-axis for plotting for plotting graphs that have values for each day in a year.
x.Driel.beneden	Kilometer-mark of the Neder-Rijn river (from OLR 2002) of the downstream waterlevel measurement station
x.Driel.boven	Kilometer-mark of the Neder-Rijn river (from OLR 2002) of the upstream waterlevel measurement station
x.percT	x-axis for duration curves going from 1/N.ALL percent to 100% with "N.ALL" minus 1 number of intervals.
x.percT.Qlow	Percentage of time the discharge "Q.low" is exceeded.
x.percT.Qopen	Percentage of time the discharge "Q.open" is exceeded.
ΔH	Agregated vector with all measured Head-differences over the entire measurement duration.
ΔH.mean	Unbiased mean of head-difference. Unbiased as in the low-head-and-high-discharge-peak has not been included in this mean vlaue.
ΔH.mean.biased	Mean value of the biased "PDF.ΔH" distribution. Biased as in there is a low-head-peak that is corrolated with high flow influencing the mean and median.
ΔH.median	Unbiased median of head-difference. Unbiased as in the low-head-and-high-discharge-peak has not been included in this median vlaue.
ΔH.median.biased	Median value of the biased "PDF.ΔH" distribution. Biased as in there is a low-head-peak that is corrolated with high flow influencing the mean and median.
ΔH.min	Minimum head-difference over the turbine for it to produce power
ΔH.min0	Minimum head-difference for turbine system 0
ΔH7080	Vector with Measured head-differences, i.e. waterlevel upstream minus waterlevel downstream, using data from "DATA7080".
η	Generic efficiency factor in percent of theoretical (maximum) power.
η.0	Efficiency of turbine system 0

ρ	Mass-density of water in kg per cubic meters.

- PDF-file output of MathCad sheet behind this page -

Discharge and Water-level data-analysis

List of Symbols (alphabetical):

$LOS := ListOfSymbolsABC^{\langle 0 \rangle}$

NOTE: For definitions/descriptions of list of symbols, please see the excel: "List of Symbols.xlsx"

$$\text{submatrix}(LOS, 1, 63, 0, 0) = \begin{bmatrix} \text{“ALLDATA”} \\ \text{“CWD”} \\ \text{“D.m”} \\ \text{“data”} \\ \text{“DATA7080”} \\ \text{“dH.bin”} \\ \text{“dL.Driel”} \\ \text{“dP.bin”} \\ \text{“dQ.bin”} \\ \text{“dQHP.bin”} \\ \text{“dX.bin”} \\ \text{“f.PDF(dX.bin, data)”} \\ \text{“H.av.1”} \\ \text{“H.av.10”} \\ \text{“H.av.30”} \\ \text{“H.desc2”} \\ \text{“h.down”} \\ \text{“H.max.year”} \\ \text{“h.up”} \\ \text{“H1970”} \\ \text{“incl.opengate”} \\ \text{“mavg\_H\_of\_Qd2”} \\ \text{“meanHy”} \\ \text{“meanPy”} \\ \text{“meanPy.ALLtime”} \\ \text{“meanQday”} \\ \text{“MH10y”} \\ \text{“MH30y”} \\ \text{“MHy”} \\ \text{“MHy.max”} \\ \text{“MPy”} \\ \text{“MQ10y”} \\ \text{“MQ30y”} \\ \text{“MQy”} \\ \text{“MQy.max”} \\ \text{“N.7080”} \\ \text{“N.ALL”} \\ \text{“N.days”} \\ \text{“N.Hbin”} \\ \text{“N.Pbin”} \\ \text{“N.Q.bias”} \\ \text{“N.Qbin”} \\ \text{“N.QHPbin”} \\ \text{“Output1”} \\ \text{“Output3”} \\ \text{“P(}\eta\text{, }\Delta\text{H.min, Q.min, Q, }\Delta\text{H)”} \\ \text{“P.av.1”} \\ \text{“P.av.10”} \\ \text{“P.av.30”} \\ \text{“P.mean1”} \\ \text{“P.mean2”} \\ \text{“P.mode1”} \\ \text{“P.mode2”} \\ \text{“P.pdf.lhs”} \\ \text{“P.pdf.rhs”} \\ \text{“P.thmax”} \\ \text{“P0.mean”} \\ \text{“P0.median”} \\ \text{“p1eP”} \\ \text{“p1eQHP”} \\ \text{“PDF.P.lhs”} \\ \text{“PDF.P.rhs”} \\ \text{“PDF.P0”} \end{bmatrix}$$

$$\text{submatrix}(LOS, 64, \text{rows}(LOS) - 1, 0, 0) = \begin{bmatrix} \text{“PDF.Q”} \\ \text{“PDF.QHP”} \\ \text{“PDF.QHP.p2”} \\ \text{“PDF.}\Delta\text{H”} \\ \text{“Q”} \\ \text{“Q.av.1”} \\ \text{“Q.av.10”} \\ \text{“Q.av.30”} \\ \text{“Q.cut.unbs”} \\ \text{“Q.desc2”} \\ \text{“Q.eco\_min”} \\ \text{“Q.low”} \\ \text{“Q.mean”} \\ \text{“Q.mean.biased”} \\ \text{“Q.min”} \\ \text{“Q.min0”} \\ \text{“Q.mode”} \\ \text{“Q.mode.biased”} \\ \text{“Q.open”} \\ \text{“Q.y.max.rnd”} \\ \text{“Q1970”} \\ \text{“Q7080”} \\ \text{“QHP”} \\ \text{“QHP.desc”} \\ \text{“QHP.localmax”} \\ \text{“QHP.mean”} \\ \text{“QHP.mode”} \\ \text{“QHP.pdf.p2”} \\ \text{“SelPDF.Q.plus60”} \\ \text{“SelPDF.}\Delta\text{H.plus15cm”} \\ \text{“SelQ.pdf.plus60”} \\ \text{“Sel}\Delta\text{H.pdf.plus15cm”} \\ \text{“Stdv.P0”} \\ \text{“Stdv.P0.lhs”} \\ \text{“Stdv.P0.lhs”} \\ \text{“t”} \\ \text{“t.10yav”} \\ \text{“t.30yav”} \\ \text{“t.dayinyear”} \\ \text{“t.my”} \\ \text{“t.Q\_eco\_min”} \\ \text{“t.Q\_low”} \\ \text{“t.Q\_open”} \\ \text{“Title.excel”} \\ \text{“Var.P0”} \\ \text{“Var.P0.lhs”} \\ \text{“Var.P0.rhs”} \\ \text{“x.dayinyr”} \\ \text{“x.Driel.beneden”} \\ \text{“x.Driel.boven”} \\ \text{“x.percT”} \\ \text{“x.percT.Q\_eco\_min”} \\ \text{“x.percT.Qlow”} \\ \text{“x.percT.Qopen”} \\ \text{“}\Delta\text{H”} \\ \text{“}\Delta\text{H.mean”} \\ \text{“}\Delta\text{H.min”} \\ \text{“}\Delta\text{H.min0”} \\ \text{“}\Delta\text{H.mode”} \\ \text{“}\Delta\text{H.mode.biased”} \\ \text{“}\Delta\text{H.mode.biased”} \\ \text{“}\Delta\text{H7080”} \\ \text{“}\eta\text{”} \\ \text{“}\eta\text{.0”} \\ \text{“}\rho\text{”} \end{bmatrix}$$

Interaction with Access and Excel:

For check for missing data/days a time line in days needs to be created:

$$N.days := \text{READEXCEL}\left(\text{concat}\left(\mathbf{CWD}, \text{“001 – INPUT–ACCESS – Days counter link.xlsx”}\right), \text{“Input!B1:B1”}\right)_{0,0} + 1 = 17897$$

$$A.days := 1, 2 .. N.days = \begin{bmatrix} 1 \\ \vdots \end{bmatrix}$$

$$writedays := \text{WRITEEXCEL}\left(\text{concat}\left(\mathbf{CWD}, \text{“001 – INPUT–ACCESS – Days counter link.xlsx”}\right), A.days, \text{“[2]”}\right)$$

Loading and preparing the data:

$$Title.excel := \text{“001 – INPUT–MATHCAD – Datalink – Driel.xlsx”}$$

$$\text{DATA7080} := \text{READEXCEL}\left(\text{concat}\left(\mathbf{CWD}, Title.excel\right), \text{“1970–80!A2:M3653”}, 0\right) \qquad N.7080 := \text{rows}\left(\text{DATA7080}\right) = 3652$$

$$\text{DATA8090} := \text{READEXCEL}\left(\text{concat}\left(\mathbf{CWD}, Title.excel\right), \text{“1980–90!A2:M3654”}, 0\right) \qquad N.8090 := \text{rows}\left(\text{DATA8090}\right) = 3653$$

$$\text{DATA9000} := \text{READEXCEL}\left(\text{concat}\left(\mathbf{CWD}, Title.excel\right), \text{“1990–00!A2:M3653”}, 0\right) \qquad N.9000 := \text{rows}\left(\text{DATA9000}\right) = 3652$$

$$\text{DATA0010} := \text{READEXCEL}\left(\text{concat}\left(\mathbf{CWD}, Title.excel\right), \text{“2000–10!A2:M3654”}, 0\right) \qquad N.0010 := \text{rows}\left(\text{DATA0010}\right) = 3653$$

$$\text{DATA1018} := \text{READEXCEL}\left(\text{concat}\left(\mathbf{CWD}, Title.excel\right), \text{“2010–18!A2:M3288”}, 0\right) \qquad N.1018 := \text{rows}\left(\text{DATA1018}\right) = 3287$$

$$ALLDATA := \text{stack}\left(\text{DATA7080}, \text{DATA8090}, \text{DATA9000}, \text{DATA0010}, \text{DATA1018}\right) \qquad N.ALL := \text{rows}\left(ALLDATA\right) = 17897$$

* CWD = "Current Work Directory".
(The related excel needs to be in the same folder as this Mathcad-sheet, to work together)

Input analysis:

$$\text{DATA7080} = \begin{bmatrix} 25569 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1970 & 1 \\ 25570 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1970 & 2 \\ 25571 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1970 & 3 \\ 25572 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1970 & 4 \\ 25573 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1970 & 5 \\ 25574 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1970 & 6 \\ 25575 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1970 & 7 \\ & & & & & & & & & & & & \vdots \end{bmatrix}$$

$$\text{DATA8090} = \begin{bmatrix} 29221 & \text{“Waterhoogte”} & \text{“Driel beneden”} & 790 & \text{“cm”} & \text{“Driel boven”} & 798 & \text{“cm”} & \text{“Q”} & 530 & \text{“m3/s”} & 1980 & 1 \\ 29222 & \text{“Waterhoogte”} & \text{“Driel beneden”} & 796 & \text{“cm”} & \text{“Driel boven”} & 803 & \text{“cm”} & \text{“Q”} & 539 & \text{“m3/s”} & 1980 & 2 \\ 29223 & \text{“Waterhoogte”} & \text{“Driel beneden”} & 793 & \text{“cm”} & \text{“Driel boven”} & 799 & \text{“cm”} & \text{“Q”} & 529 & \text{“m3/s”} & 1980 & 3 \\ 29224 & \text{“Waterhoogte”} & \text{“Driel beneden”} & 779 & \text{“cm”} & \text{“Driel boven”} & 786 & \text{“cm”} & \text{“Q”} & 501 & \text{“m3/s”} & 1980 & 4 \\ 29225 & \text{“Waterhoogte”} & \text{“Driel beneden”} & 762 & \text{“cm”} & \text{“Driel boven”} & 769 & \text{“cm”} & \text{“Q”} & 473 & \text{“m3/s”} & 1980 & 5 \\ 29226 & \text{“Waterhoogte”} & \text{“Driel beneden”} & 748 & \text{“cm”} & \text{“Driel boven”} & 781 & \text{“cm”} & \text{“Q”} & 445 & \text{“m3/s”} & 1980 & 6 \\ 29227 & \text{“Waterhoogte”} & \text{“Driel beneden”} & 741 & \text{“cm”} & \text{“Driel boven”} & 777 & \text{“cm”} & \text{“Q”} & 440 & \text{“m3/s”} & 1980 & 7 \\ & & & & & & & & & & & & \vdots \end{bmatrix}$$

$$\text{DATA9000} = \begin{bmatrix} 32874 & \text{“Waterhoogte”} & \text{“Driel beneden”} & 697 & \text{“cm”} & \text{“Driel boven”} & 742 & \text{“cm”} & \text{“Q”} & 355 & \text{“m3/s”} & 1990 & 1 \\ 32875 & \text{“Waterhoogte”} & \text{“Driel beneden”} & 668 & \text{“cm”} & \text{“Driel boven”} & 803 & \text{“cm”} & \text{“Q”} & 287 & \text{“m3/s”} & 1990 & 2 \\ 32876 & \text{“Waterhoogte”} & \text{“Driel beneden”} & 648 & \text{“cm”} & \text{“Driel boven”} & 811 & \text{“cm”} & \text{“Q”} & 232 & \text{“m3/s”} & 1990 & 3 \\ 32877 & \text{“Waterhoogte”} & \text{“Driel beneden”} & 631 & \text{“cm”} & \text{“Driel boven”} & 826 & \text{“cm”} & \text{“Q”} & 171 & \text{“m3/s”} & 1990 & 4 \\ 32878 & \text{“Waterhoogte”} & \text{“Driel beneden”} & 617 & \text{“cm”} & \text{“Driel boven”} & 834 & \text{“cm”} & \text{“Q”} & 119 & \text{“m3/s”} & 1990 & 5 \\ 32879 & \text{“Waterhoogte”} & \text{“Driel beneden”} & 605 & \text{“cm”} & \text{“Driel boven”} & 841 & \text{“cm”} & \text{“Q”} & 61 & \text{“m3/s”} & 1990 & 6 \\ 32880 & \text{“Waterhoogte”} & \text{“Driel beneden”} & 603 & \text{“cm”} & \text{“Driel boven”} & 840 & \text{“cm”} & \text{“Q”} & 40 & \text{“m3/s”} & 1990 & 7 \\ & & & & & & & & & & & & \vdots \end{bmatrix}$$

$$\text{DATA0010} = \begin{bmatrix} 36526 & \text{“Waterhoogte”} & \text{“Driel beneden”} & 1034 & \text{“cm”} & \text{“Driel boven”} & 1041 & \text{“cm”} & \text{“Q”} & 1111 & \text{“m3/s”} & 2000 & 1 \\ 36527 & \text{“Waterhoogte”} & \text{“Driel beneden”} & 1005 & \text{“cm”} & \text{“Driel boven”} & 1014 & \text{“cm”} & \text{“Q”} & 1061 & \text{“m3/s”} & 2000 & 2 \\ 36528 & \text{“Waterhoogte”} & \text{“Driel beneden”} & 963 & \text{“cm”} & \text{“Driel boven”} & 972 & \text{“cm”} & \text{“Q”} & 946 & \text{“m3/s”} & 2000 & 3 \\ 36529 & \text{“Waterhoogte”} & \text{“Driel beneden”} & 926 & \text{“cm”} & \text{“Driel boven”} & 933 & \text{“cm”} & \text{“Q”} & 847 & \text{“m3/s”} & 2000 & 4 \\ 36530 & \text{“Waterhoogte”} & \text{“Driel beneden”} & 894 & \text{“cm”} & \text{“Driel boven”} & 902 & \text{“cm”} & \text{“Q”} & 783 & \text{“m3/s”} & 2000 & 5 \\ 36531 & \text{“Waterhoogte”} & \text{“Driel beneden”} & 871 & \text{“cm”} & \text{“Driel boven”} & 879 & \text{“cm”} & \text{“Q”} & 731 & \text{“m3/s”} & 2000 & 6 \\ 36532 & \text{“Waterhoogte”} & \text{“Driel beneden”} & 850 & \text{“cm”} & \text{“Driel boven”} & 858 & \text{“cm”} & \text{“Q”} & 685 & \text{“m3/s”} & 2000 & 7 \\ & & & & & & & & & & & & \vdots \end{bmatrix}$$

$$\text{DATA1018} = \begin{bmatrix} 40179 & \text{“Waterhoogte”} & \text{“Driel beneden”} & 804 & \text{“cm”} & \text{“Driel boven”} & 813 & \text{“cm”} & \text{“Q”} & 600 & \text{“m3/s”} & 2010 & 1 \\ 40180 & \text{“Waterhoogte”} & \text{“Driel beneden”} & 842 & \text{“cm”} & \text{“Driel boven”} & 852 & \text{“cm”} & \text{“Q”} & 725 & \text{“m3/s”} & 2010 & 2 \\ 40181 & \text{“Waterhoogte”} & \text{“Driel beneden”} & 880 & \text{“cm”} & \text{“Driel boven”} & 890 & \text{“cm”} & \text{“Q”} & 834 & \text{“m3/s”} & 2010 & 3 \\ 40182 & \text{“Waterhoogte”} & \text{“Driel beneden”} & 901 & \text{“cm”} & \text{“Driel boven”} & 911 & \text{“cm”} & \text{“Q”} & 868 & \text{“m3/s”} & 2010 & 4 \\ 40183 & \text{“Waterhoogte”} & \text{“Driel beneden”} & 896 & \text{“cm”} & \text{“Driel boven”} & 906 & \text{“cm”} & \text{“Q”} & 812 & \text{“m3/s”} & 2010 & 5 \\ 40184 & \text{“Waterhoogte”} & \text{“Driel beneden”} & 868 & \text{“cm”} & \text{“Driel boven”} & 877 & \text{“cm”} & \text{“Q”} & 733 & \text{“m3/s”} & 2010 & 6 \\ 40185 & \text{“Waterhoogte”} & \text{“Driel beneden”} & 829 & \text{“cm”} & \text{“Driel boven”} & 837 & \text{“cm”} & \text{“Q”} & 637 & \text{“m3/s”} & 2010 & 7 \\ & & & & & & & & & & & & \vdots \end{bmatrix}$$

Head-difference over the Weir:
(Calculated from the data)

$$\Delta H7080 := \left\| \begin{array}{l} \text{for } i \in 0..N.7080-1 \\ \quad \left\| a_i \leftarrow \left| \mathrm{DATA7080}_{(i),6} - \mathrm{DATA7080}_{(i),3} \right| \right. \\ \text{return } a^{\mathrm{T}} \cdot cm \end{array} \right. = [0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ \dots]\ m$$

$$\Delta H8090 := \left\| \begin{array}{l} \text{for } i \in 0..N.8090-1 \\ \quad \left\| a_i \leftarrow \left| \mathrm{DATA8090}_{(i),6} - \mathrm{DATA8090}_{(i),3} \right| \right. \\ \text{return } a^{\mathrm{T}} \cdot cm \end{array} \right. = [0.08\ \ 0.07\ \ 0.06\ \ 0.07\ \ 0.07\ \ 0.33\ \ \dots]\ m$$

$$\Delta H9000 := \left\| \begin{array}{l} \text{for } i \in 0..N.9000-1 \\ \quad \left\| a_i \leftarrow \left| \mathrm{DATA9000}_{(i),6} - \mathrm{DATA9000}_{(i),3} \right| \right. \\ \text{return } a^{\mathrm{T}} \cdot cm \end{array} \right. = [0.45\ \ 1.35\ \ 1.63\ \ 1.95\ \ 2.17\ \ \dots]\ m$$

$$\Delta H0010 := \left\| \begin{array}{l} \text{for } i \in 0..N.0010-1 \\ \quad \left\| a_i \leftarrow \left| \mathrm{DATA0010}_{(i),6} - \mathrm{DATA0010}_{(i),3} \right| \right. \\ \text{return } a^{\mathrm{T}} \cdot cm \end{array} \right. = [0.07\ \ 0.09\ \ 0.09\ \ 0.07\ \ 0.08\ \ \dots]\ m$$

$$\Delta H1018 := \left\| \begin{array}{l} \text{for } i \in 0..N.1018-1 \\ \quad \left\| a_i \leftarrow \left| \mathrm{DATA1018}_{(i),6} - \mathrm{DATA1018}_{(i),3} \right| \right. \\ \text{return } a^{\mathrm{T}} \cdot cm \end{array} \right. = [0.09\ \ 0.1\ \ 0.1\ \ 0.1\ \ 0.1\ \ \dots]\ m$$

$$\Delta H := \mathrm{augment}(\Delta H7080, \Delta H8090, \Delta H9000, \Delta H0010, \Delta H1018) = [0\ \ 0\ \ 0\ \ 0\ \ 0\ \ \dots]\ m$$

Discharge (From data):

$$Q7080 := \left(\mathrm{DATA7080}^{\langle 9\rangle}\right)^{\mathrm{T}} \cdot m^3 \cdot s^{-1} = [0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ \dots]\ m^3 \cdot s^{-1}$$

$$Q8090 := \left(\mathrm{DATA8090}^{\langle 9\rangle}\right)^{\mathrm{T}} \cdot m^3 \cdot s^{-1} = [530\ \ 539\ \ 529\ \ 501\ \ 473\ \ 445\ \ 440\ \ 489\ \ 558\ \ 603\ \ \dots]\ m^3 \cdot s^{-1}$$

$$Q9000 := \left(\mathrm{DATA9000}^{\langle 9\rangle}\right)^{\mathrm{T}} \cdot m^3 \cdot s^{-1} = [355\ \ 287\ \ 232\ \ 171\ \ 119\ \ \dots]\ m^3 \cdot s^{-1}$$

$$Q0010 := \left(\mathrm{DATA0010}^{\langle 9\rangle}\right)^{\mathrm{T}} \cdot m^3 \cdot s^{-1} = [1111\ \ 1061\ \ 946\ \ 847\ \ 783\ \ \dots]\ m^3 \cdot s^{-1}$$

$$Q1018 := \left(\mathrm{DATA1018}^{\langle 9\rangle}\right)^{\mathrm{T}} \cdot m^3 \cdot s^{-1} = [600\ \ 725\ \ 834\ \ 868\ \ 812\ \ \dots]\ m^3 \cdot s^{-1}$$

$$Q := \mathrm{augment}(Q7080, Q8090, Q9000, Q0010, Q1018) = [0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ \dots]\ m^3 \cdot s^{-1}$$

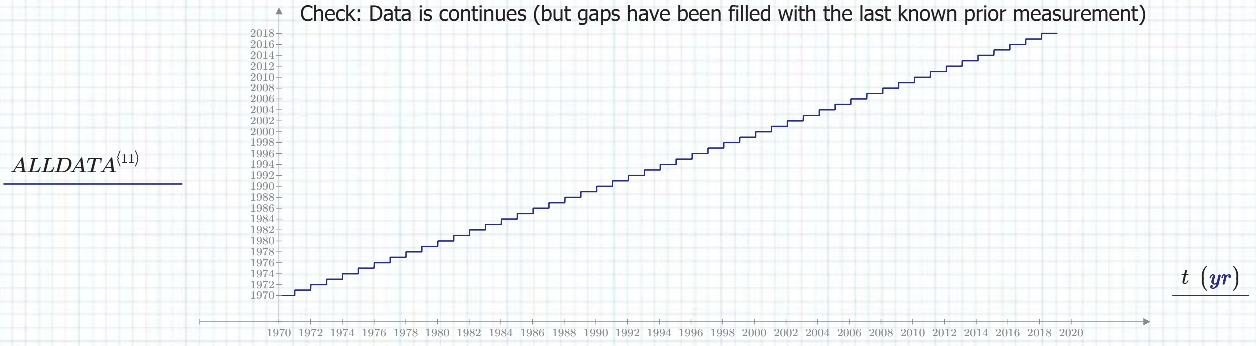
Time in a year:
(required to transform date from database value to year) $t.dayinyear := 365.256363004 \cdot \dfrac{day}{yr} = 365.2564\ \dfrac{day}{yr}$

Defining time-axis: $t := 1900\ yr + \left(\mathrm{stack}\left(\mathrm{DATA7080}^{\langle 0\rangle}, \mathrm{DATA8090}^{\langle 0\rangle}, \mathrm{DATA9000}^{\langle 0\rangle}, \mathrm{DATA0010}^{\langle 0\rangle}, \mathrm{DATA1018}^{\langle 0\rangle}\right) \cdot day\right)$

Density of water assumed to be: $\rho := 1000\ \dfrac{kg}{m^3}$

Measurments from: $D.m := \dfrac{\left(ALLDATA_{N.ALL-1,0} \cdot day - ALLDATA_{0,0} \cdot day\right)}{t.dayinyear} = 48.9957\ yr$ $D.m := \mathrm{ceil}\left(\dfrac{D.m}{yr}\right) yr = 49\ yr$

Measurement years: $t.my := 1970, 1971..2018 = \begin{bmatrix} 1970 \\ 1971 \\ \vdots \end{bmatrix}$


Check: Data is continues (but gaps have been filled with the last known prior measurement)

Defining the generic histrogram/porbability density function (pdf):

```
$f.PDF(dX.bin, data) :=$
    $Unit \leftarrow \text{UnitsOf}(dX.bin)$
    $dX.bin \leftarrow \frac{dX.bin}{\text{UnitsOf}(dX.bin)}$
    $data \leftarrow \frac{data}{\text{UnitsOf}(data)}$
    $N.bin \leftarrow \text{ceil}\left(\frac{\max(data)}{dX.bin}\right) + 1$
    for $i \in 0 .. N.bin$
        $n.X_i \leftarrow i$
    $N.data \leftarrow \text{rows}(data)$
    $B_{N.bin} \leftarrow 0$
    for $k \in 0 .. N.data - 1$
        $ind \leftarrow \text{floor}\left(\frac{data_k}{dX.bin}\right)$
        $B_{ind} \leftarrow B_{ind} + 1$
    $PDF.X \leftarrow \frac{B}{N.data}$
    $X.pdf \leftarrow \frac{dX.bin}{2} + n.X \cdot dX.bin$
    $A \leftarrow \text{augment}(PDF.X, X.pdf \cdot Unit)$
    return $A$
```

Explanation:

Input:

dX.bin = The bin-size in desired units for the histogram

data = Data set being converted to a pdf, needs to be a column-vector! (use transpose for row-vectors)

Clarification of program:

Unit = Unit that is being put into the function, output is given in SI version of this unit.

N.bin = Nr. of bins determined by the maximum value of the data-set and the bin-size

n.X = array going from 0 till N.bin with steps of 1

N.data = nr. of values of the data

Output:

A = Matrix with 2 columns and N.bin nr. of rows

1st column: y-axis values, the percentage of occurance in the data set of values in a certain bin.
So: [values in bin]/[total nr of values]

2nd column: x-axis values, the values that are the middle of each bin. So if the binsize is 20 units and there are 3 bins, then this will be a column with values: [10,30,50]
The values will thus be displayed at these locations at the x-axis.

Defining Generic Hydro-Power function:

```
$P(\eta, \Delta H.min, Q.min, Q, \Delta H) :=$
    $A^{\widehat{(N.ALL-1)}} \leftarrow 1$
    for $n \in 0 .. (N.ALL - 1)$
        $A^{\widehat{n}} \leftarrow$ if $\left(\left(\Delta H^{T}\right)_n \geq \Delta H.min\right) \wedge \left(\left(Q^{T}\right)_n \geq Q.min\right)$
                $\rho \cdot g \cdot \left(Q^{T}\right)_n \cdot \left(\Delta H^{T}\right)_n$
            else
                $0\ kW$
    return $A$
```

Turbine system 0 (theoretical maximum):

Efficiency coefficient system:
(for now assumed 1) $\eta.0 := 100\%$

Minimum head: $\Delta H.min0 := 0.0\ m$

Minimum discharge: $Q.min0 := 0\ m^{3} \cdot s^{-1}$

(The 3 variables above normally depend on <u>type of turbine</u>/ hydro-power-scheme)

Hydro-power-equation: $P.0 := P(\eta.0, \Delta H.min0, Q.min0, Q, \Delta H) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \end{bmatrix} kW$

Check calculation:
(because first few values are 0)

$Check.nr := 1000$

$\eta.0 \cdot \rho \cdot g \cdot \Delta H^{\langle Check.nr \rangle} \cdot Q^{\langle Check.nr \rangle} = 703.725\ kW$

$\Delta H^{\langle Check.nr \rangle} = [1.84]\ m$

$Q^{\langle Check.nr \rangle} = [39]\ m^{3} \cdot s^{-1}$

$P.0^{\widehat{Check.nr}} = [703.7252]\ kW$

Defining binsizes and calculating values for histograms with previously defined automatic Histogram function:

Head-difference PDF:

Binsize: $dH.bin := 0.05\ m$

Probability Density Values c.q. Histogram:

$$PDF.\Delta H := f.PDF\left(dH.bin, \Delta H^{T}\right) = \begin{bmatrix} 0.0308 & 0.025\ m \\ 0.2023 & 0.075\ m \\ 0.0427 & 0.125\ m \\ 0.0056 & 0.175\ m \\ 0.0083 & 0.225\ m \\ 0.0103 & 0.275\ m \\ & \vdots \end{bmatrix}$$

Check if sum is ammounts to 1: $\sum\left(PDF.\Delta H^{\langle 0\rangle}\right) = 1$

Discharge PDF:

Binsize: $dQ.bin := 10\ m^{3} \cdot s^{-1}$

Probability Density Values c.q. Histogram:

$$PDF.Q := f.PDF\left(dQ.bin, Q^{T}\right) = \begin{bmatrix} 0.0184 & 5\ \frac{m^{3}}{s} \\ 0.0072 & 15\ \frac{m^{3}}{s} \\ 0.1078 & 25\ \frac{m^{3}}{s} \\ 0.1193 & 35\ \frac{m^{3}}{s} \\ & \vdots \end{bmatrix}$$

Check if sum is ammounts to 1: $\sum\left(PDF.Q^{\langle 0\rangle}\right) = 1$

Calculation of Head-Discharge Product (QHP):

Q*H-product: $QHP := \overrightarrow{\Delta H^{T} \cdot Q^{T}}$

Binsize: $dQHP.bin := 10\ m^{3} \cdot s^{-1} \cdot 1\ m = 10\ \frac{m^{4}}{s}$

PDF function:

$$PDF.QHP := f.PDF\left(dQHP.bin, QHP\right) = \begin{bmatrix} 0.0238 & 5\ \frac{m^{4}}{s} \\ 0.0143 & 15\ \frac{m^{4}}{s} \\ 0.0467 & 25\ \frac{m^{4}}{s} \\ 0.085 & 35\ \frac{m^{4}}{s} \\ 0.0925 & 45\ \frac{m^{4}}{s} \\ 0.085 & 55\ \frac{m^{4}}{s} \\ & \vdots \end{bmatrix}$$

Check if sum is ammounts to 1: $\sum\left(PDF.QHP^{\langle 0\rangle}\right) = 1$

Power PDF (for theoretical maximum):

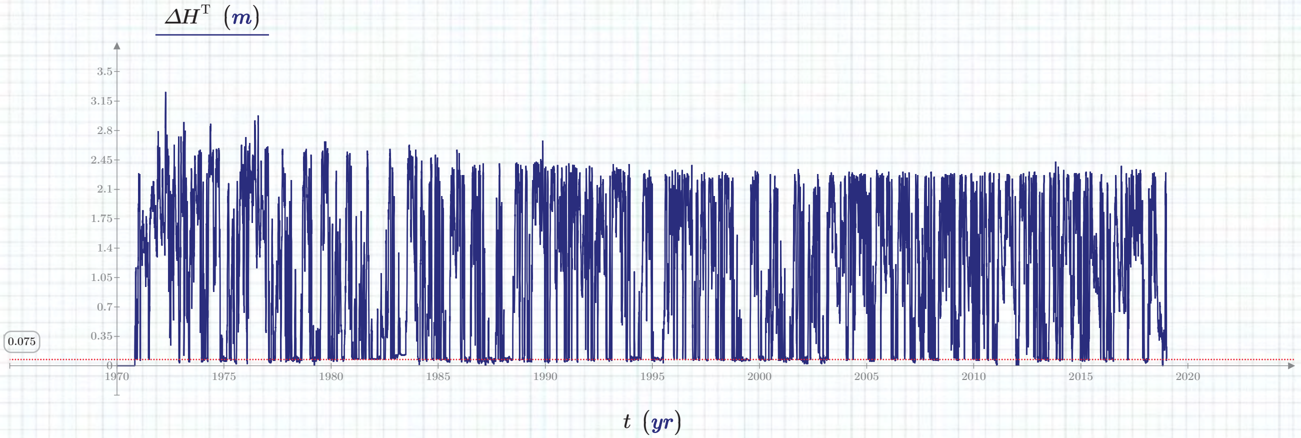
Binsize: $dP.bin := 50\ kW$

Probability Density Values c.q. Histogram:

$$PDF.P0 := f.PDF\left(dP.bin, P.0\right) = \begin{bmatrix} 0.0206 & 25000\ W \\ 0.0034 & 75000\ W \\ 0.0059 & 125000\ W \\ 0.0091 & 175000\ W \\ 0.0169 & 225000\ W \\ 0.0337 & 275000\ W \\ & \vdots \end{bmatrix}$$

Check if sum is ammounts to 1: $\sum\left(PDF.P0^{\langle 0\rangle}\right) = 1$

Head difference time series from 1970-2018:

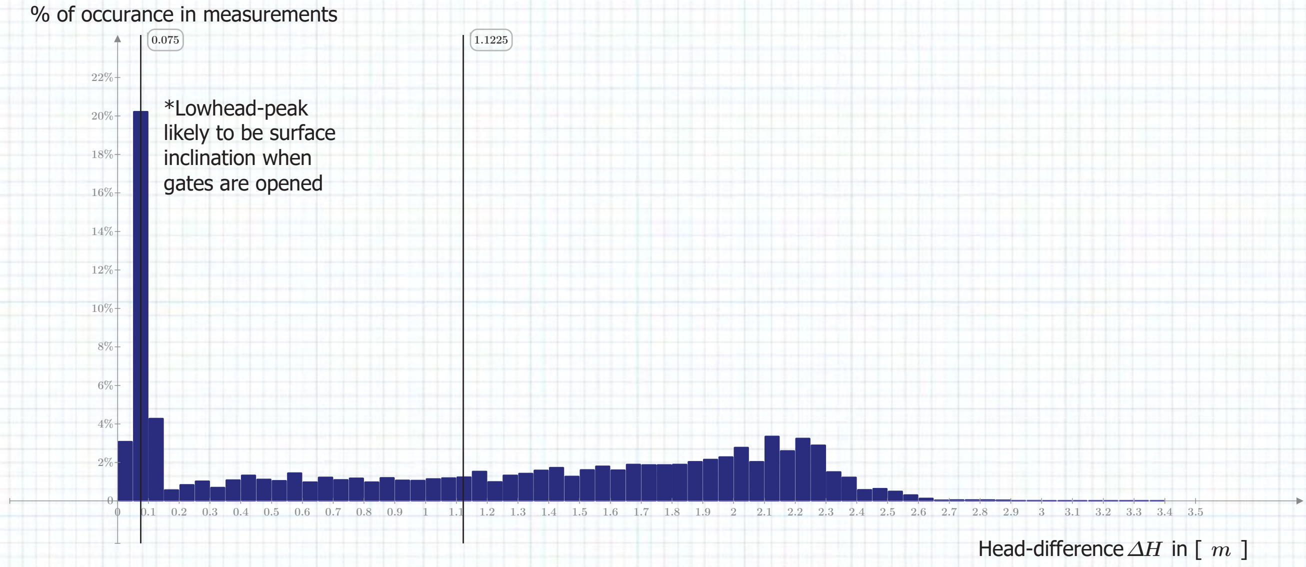


$N.Hbin := \mathrm{rows}\left(PDF.\Delta H\right)$

$\Delta H.mode.biased := \mathrm{lookup}\left(\max\left(PDF.\Delta H^{\langle 0\rangle}\right), PDF.\Delta H^{\langle 0\rangle}, PDF.\Delta H^{\langle 1\rangle}\right)_{0,0} = 0.075\ m$

$$\Delta H.mean.biased := \frac{\sum_{i=0}^{N.Hbin-1}\left(PDF.\Delta H^{\langle 0\rangle}\right)_i \cdot \left(PDF.\Delta H^{\langle 1\rangle}\right)_i}{\sum_{i=0}^{N.Hbin-1}\left(PDF.\Delta H^{\langle 0\rangle}\right)_i} = 1.1225\ m$$

Empirical PDF/histogram from measurements:



Kilometer marks:

$x.Driel.beneden := 891750\ m$

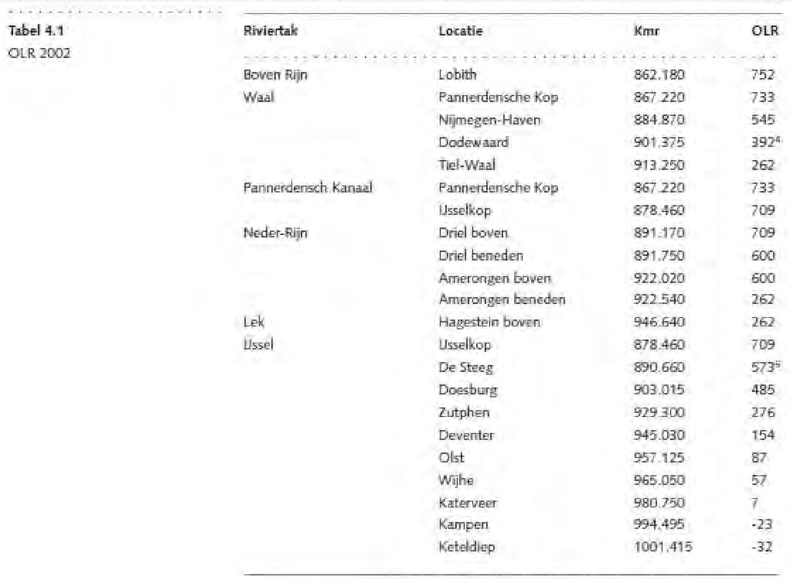
$x.Driel.boven := 891170\ m$

Distance between the two measuring stations:

$dL.Driel := x.Driel.beneden - x.Driel.boven = 580\ m$

Inclination of the water with open gates:

$$incl.opengate := \frac{\Delta H.mode.biased}{dL.Driel} = 1.2931 \cdot 10^{-4}$$



Tabel 4.1
OLR 2002

Riviertak	Locatie	Kmr	OLR
Boven Rijn	Lobith	862.180	752
Waal	Pannerdensche Kop	867.220	733
	Nijmegen-Haven	884.870	545
	Dodewaard	901.375	392^4
	Tiel-Waal	913.250	262
Pannerdensch Kanaal	Pannerdensche Kop	867.220	733
	IJsselkop	878.460	709
Neder-Rijn	Driel boven	891.170	709
	Driel beneden	891.750	600
	Amerongen boven	922.020	600
	Amerongen beneden	922.540	262
Lek	Hagestein boven	946.640	262
IJssel	IJsselkop	878.460	709
	De Steeg	890.660	573^5
	Doesburg	903.015	485
	Zutphen	929.300	276
	Deventer	945.030	154
	Olst	957.125	87
	Wijhe	965.050	57
	Katerveer	980.750	7
	Kampen	994.495	-23
	Keteldiep	1001.415	-32

Removing bias, Ignoring the first 3 bins and thus the low-head-peak:

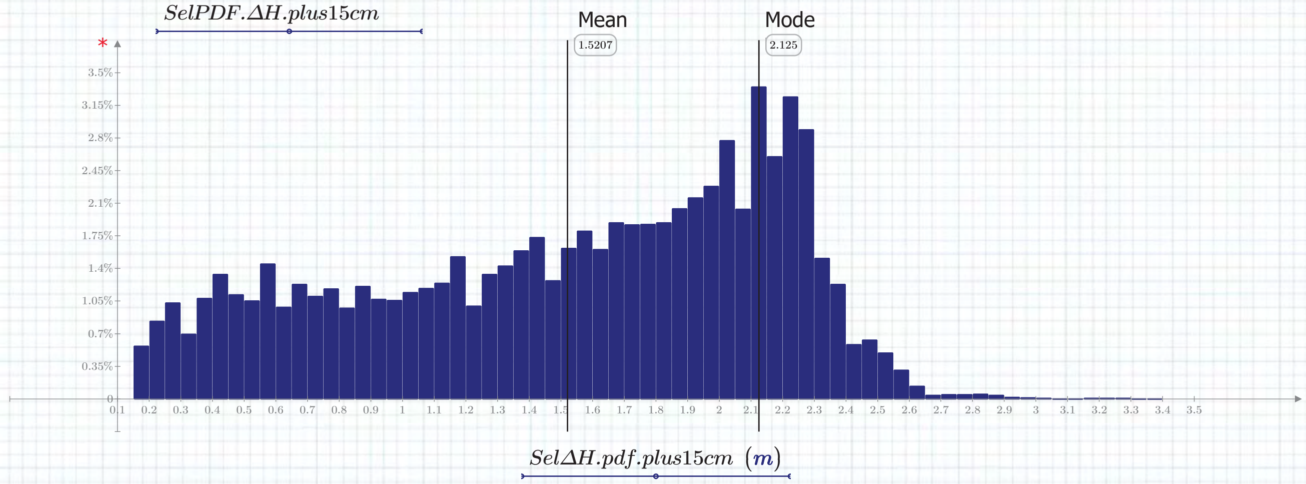
$$SelPDF.\Delta H.plus15cm := \text{submatrix}\left(PDF.\Delta H^{\langle 0\rangle}, 3, N.Hbin-1, 0, 0\right)$$

$$Sel\Delta H.pdf.plus15cm := \text{submatrix}\left(PDF.\Delta H^{\langle 1\rangle}, 3, N.Hbin-1, 0, 0\right)$$

$$\Delta H.mean := \frac{\sum_{i=0}^{N.Hbin-4} SelPDF.\Delta H.plus15cm_i \cdot Sel\Delta H.pdf.plus15cm_i}{\sum_{i=0}^{N.Hbin-4} SelPDF.\Delta H.plus15cm_i} = 1.52\ m$$

$$\Delta H.mode := \text{lookup}\left(\max\left(SelPDF.\Delta H.plus15cm\right), SelPDF.\Delta H.plus15cm, Sel\Delta H.pdf.plus15cm\right)_{0,0} = 2.13\ m$$

* please note: y-axis different scale as graph above!



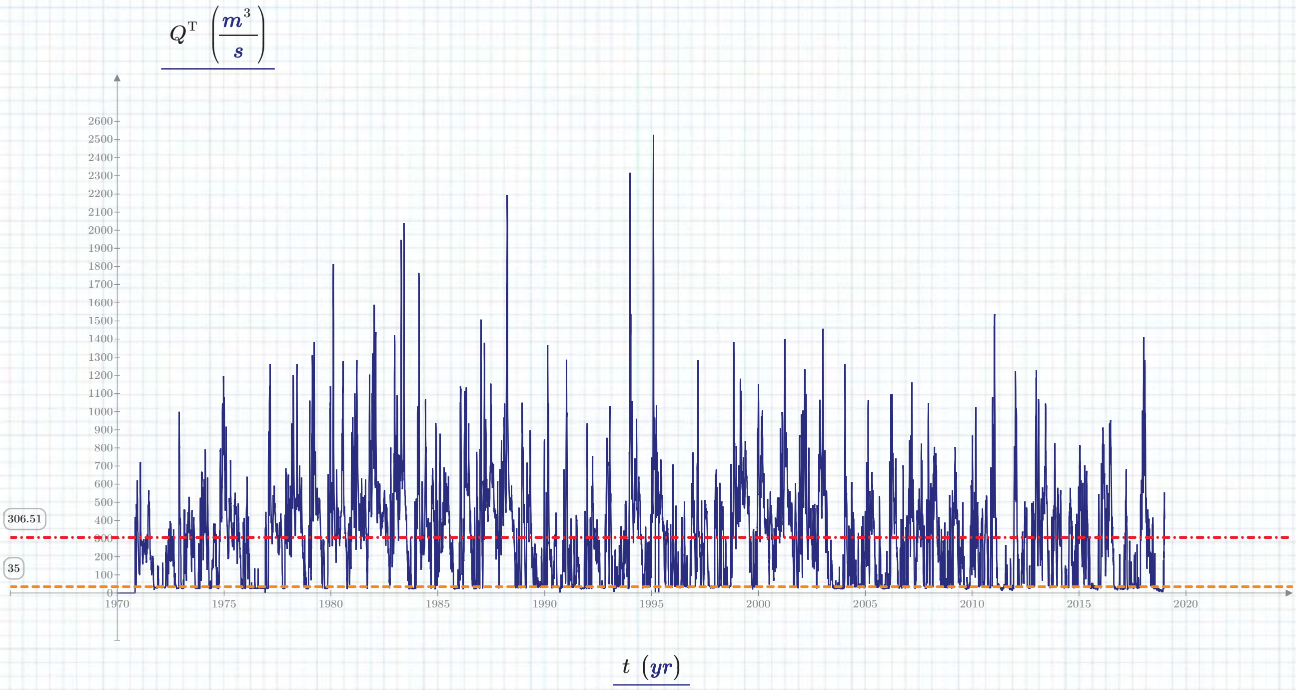
Discharge-time-series from 1970-2018:

Minimum discharge: 0-35m3/s

$$N.Qbin := \text{rows}\left(PDF.Q\right) = 255$$

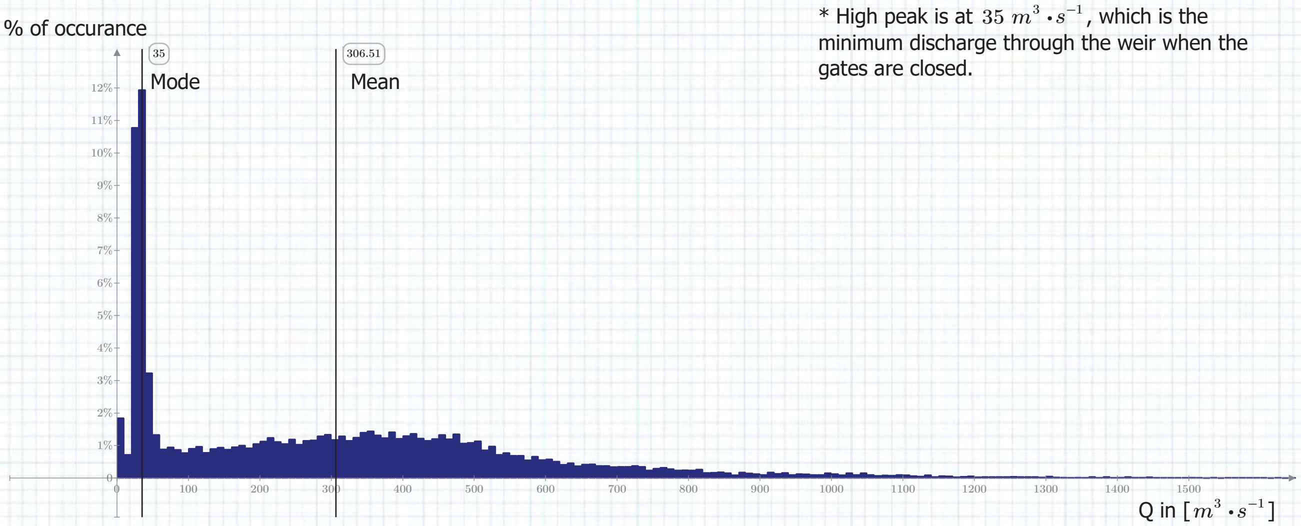
$$Q.mean.biased := \frac{\sum_{i=0}^{N.Qbin-1} \left(PDF.Q^{\langle 0\rangle}\right)_i \cdot \left(PDF.Q^{\langle 1\rangle}\right)_i}{\sum_{i=0}^{N.Qbin-1} \left(PDF.Q^{\langle 0\rangle}\right)_i} = 306.5098\ \frac{m^3}{s}$$

$$Q.mode.biased := \left(\text{lookup}\left(\max\left(PDF.Q^{\langle 0\rangle}\right), PDF.Q^{\langle 0\rangle}, PDF.Q^{\langle 1\rangle}\right)\right)_{0,0} = 35\ \frac{m^3}{s}$$



Empirical PDF/Histogram from discharge measurements:

* High peak is at $35\ m^3 \cdot s^{-1}$, which is the minimum discharge through the weir when the gates are closed.



Removing bias, Ignoring the first 2,5% of the bins, thus ignoring the low-discharge-spike:

$N.Q.bias := \mathrm{round}(N.Qbin \cdot 0.025)$ (Bias seems to occur at first 2,5% of the x-axis...)

$Q.cut.unbs := N.Q.bias \cdot dQ.bin \cdot s \cdot m^{-3} = 60$ (Which is at Q.cut amount of $m^3 \cdot s^{-1}$)

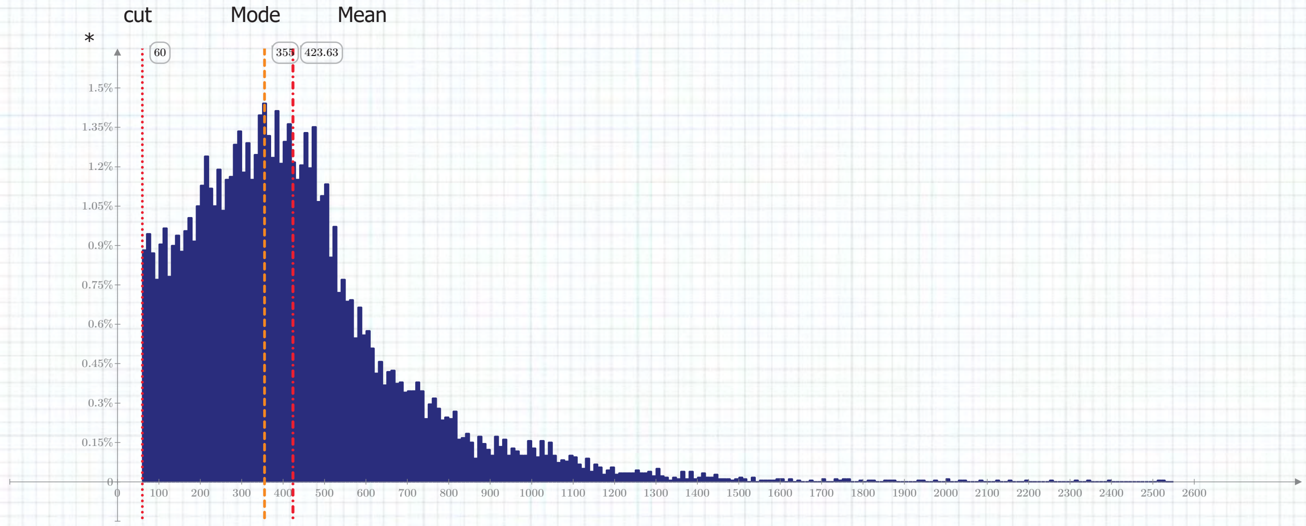
$SelPDF.Q.plus60 := \mathrm{submatrix}\left(PDF.Q^{\langle 0\rangle}, N.Q.bias, N.Qbin-1, 0, 0\right)$

$SelQ.pdf.plus60 := \mathrm{submatrix}\left(PDF.Q^{\langle 1\rangle}, N.Q.bias, N.Qbin-1, 0, 0\right) \cdot \frac{s}{m^3}$

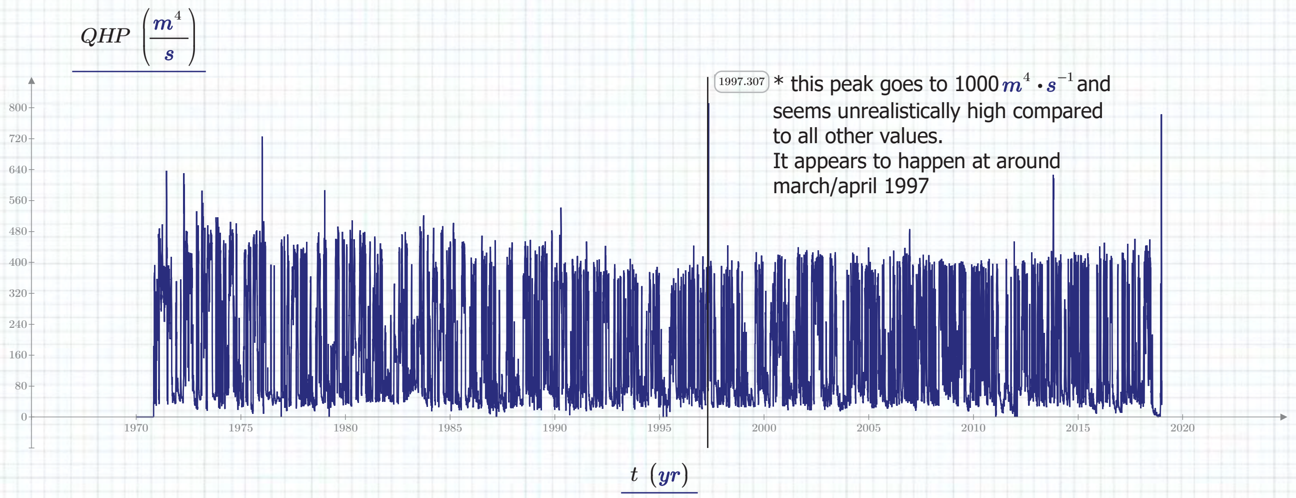
$$Q.mean := \frac{\sum_{i=0}^{N.Qbin-(N.Q.bias+1)} SelPDF.Q.plus60_i \cdot SelQ.pdf.plus60_i}{\sum_{i=0}^{N.Qbin-(N.Q.bias+1)} SelPDF.Q.plus60_i} = 423.6304$$

$Q.mode := \mathrm{lookup}\left(\max(SelPDF.Q.plus60), SelPDF.Q.plus60, SelQ.pdf.plus60\right)_{0,0} = 355$

*** please note: y-axis different scale than graph above!**



Product $Q \cdot \Delta H$ distribution:



* this peak goes to 1000 $m^4 \cdot s^{-1}$ and seems unrealistically high compared to all other values.
It appears to happen at around march/april 1997

$$N.QHPbin := \mathrm{rows}(PDF.QHP) = 97$$

Number of bins in the histogram

$$QHP.mean := \frac{\sum_{i=0}^{N.QHPbin-1} \left(PDF.QHP^{\langle 0\rangle}\right)_i \cdot \left(PDF.QHP^{\langle 1\rangle}\right)_i}{\sum_{i=0}^{N.QHPbin-1} \left(PDF.QHP^{\langle 0\rangle}\right)_i} = 163.3545\ \frac{m^4}{s}$$

Mean QHP value

$$QHP.mode := \mathrm{lookup}\left(\max\left(PDF.QHP^{\langle 0\rangle}\right), PDF.QHP^{\langle 0\rangle}, PDF.QHP^{\langle 1\rangle}\right)_{0,0} = 45\ \frac{m^4}{s}$$

Median QHP value

$$p1eQHP := 13 \qquad \left(PDF.QHP^{\langle 1\rangle}\right)_{p1eQHP} = 135\ \frac{m^4}{s}$$

Cut-off location to find second peak

$$PDF.QHP.p2 := \mathrm{submatrix}\left(PDF.QHP^{\langle 0\rangle}, p1eQHP, N.QHPbin - 1, 0, 0\right)$$

% of occurance of cut distribution to find 2nd peak

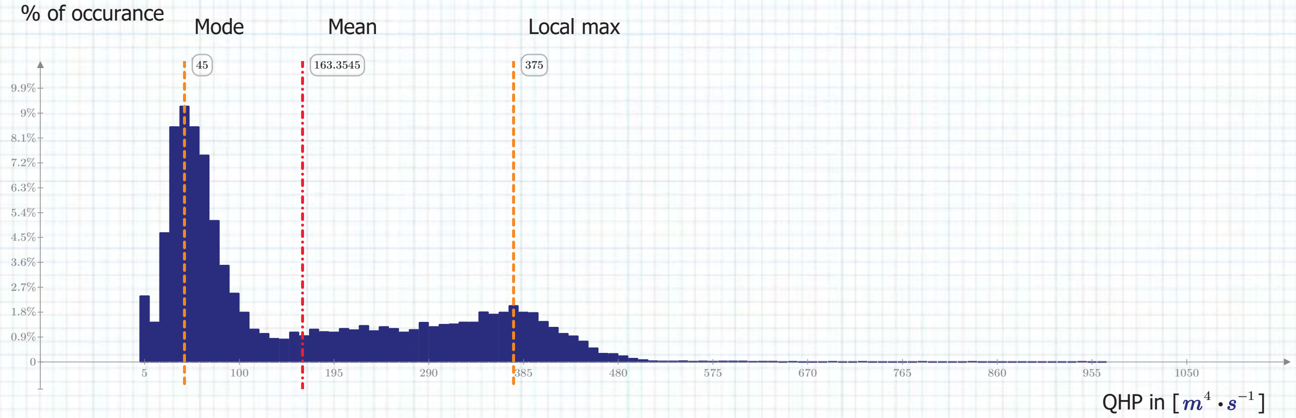
$$QHP.pdf.p2 := \mathrm{submatrix}\left(PDF.QHP^{\langle 1\rangle}, p1eQHP, N.QHPbin - 1, 0, 0\right)$$

QHP values of cut distribution to find 2nd peak

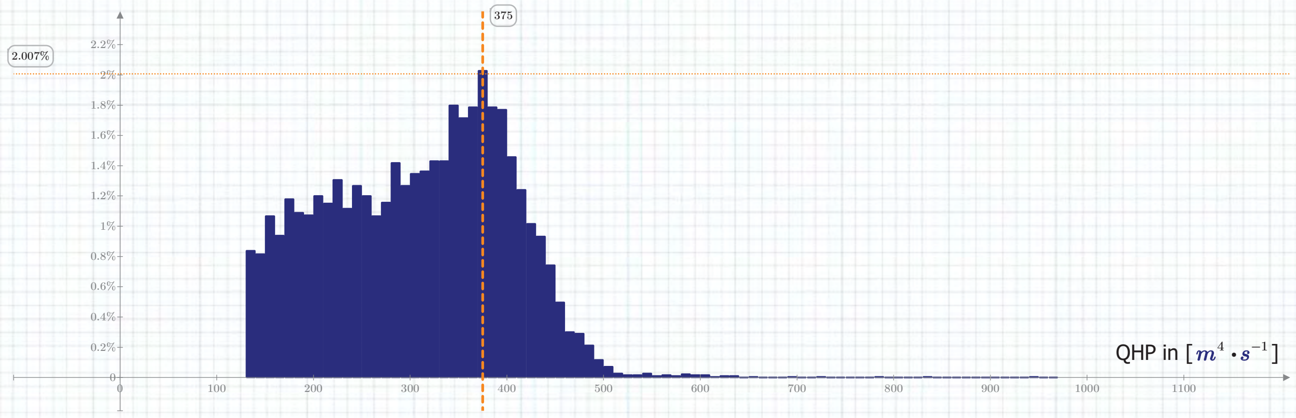
$$QHP.localmax := \mathrm{lookup}\left(\max\left(PDF.QHP.p2\right), PDF.QHP.p2, QHP.pdf.p2\right)_{0,0} = 375\ \frac{m^4}{s}$$

QHP value of 2nd peak

Empirical PDF/Histrogram QH-Product :



Removed low-discharge peak, from QH-curve:



- Page intentionally left blank -

Turbine system 0 (theoretical max):

$\eta.0 = 100\%$

$N.Pbin := \text{rows}(PDF.P0)$

$$P0.mean := \sum_{i=0}^{N.Pbin-1} \left(PDF.P0^{\langle 0 \rangle}\right)_i \cdot \left(PDF.P0^{\langle 1 \rangle}\right)_i = 1601.328\ kW$$

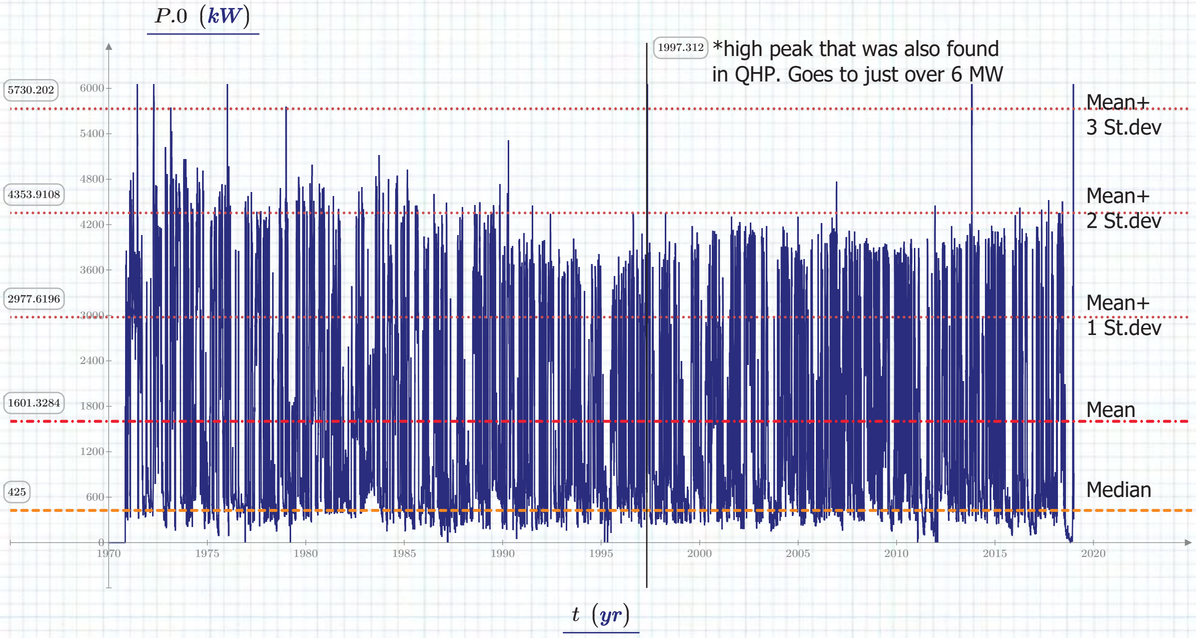
$$P0.median.biased := \text{lookup}\left(\max\left(PDF.P0^{\langle 0 \rangle}\right), PDF.P0^{\langle 0 \rangle}, PDF.P0^{\langle 1 \rangle}\right)_{0,0} = 425\ kW$$

$$Var.P0 := \sum_{i=0}^{N.Pbin-1} \left(PDF.P0^{\langle 0 \rangle}\right)_i \cdot \left(\left(PDF.P0^{\langle 1 \rangle}\right) - P0.mean\right)_i^2 = 1894177.4622\ kW^2$$

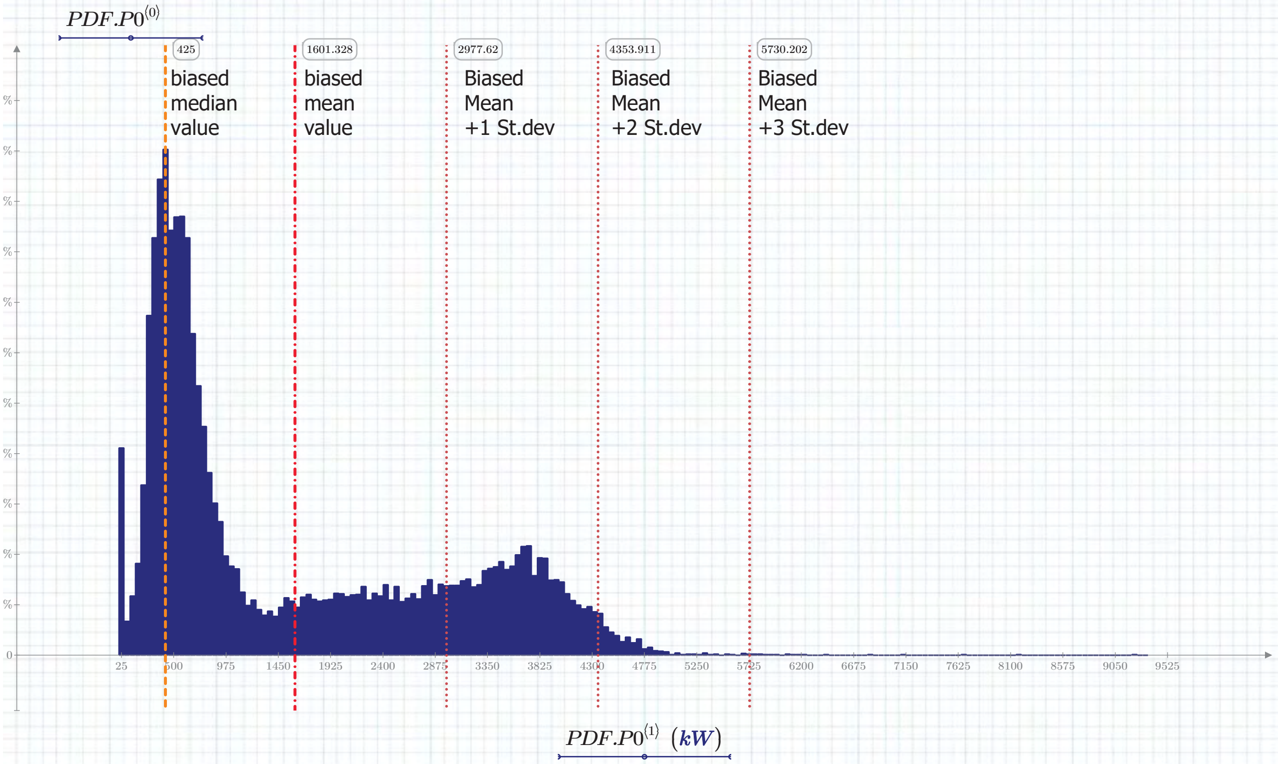
$$Stdv.P0 := \sqrt{Var.P0} = 1376.2912\ kW$$

$$\text{Stdev}(P.0) = 1376.7293\ kW$$

Potentially available power-time-series from 1970-2018:



Empirical PDF/Histogram for power production from measurements:



Splitting graph in low power and high power peak:

$p1eP := 25$ cut at $\left(PDF.P0^{\langle 1 \rangle}\right)_{p1eP} = 1275\ kW$ $\left(PDF.P0^{\langle 0 \rangle}\right)_{p1eP} = 0.453\%$

$$PDF.P.lhs := \text{submatrix}\left(PDF.P0^{\langle 0 \rangle}, 0, p1eP, 0, 0\right) = \begin{bmatrix} 0.0206 \\ \vdots \end{bmatrix}$$

$$P.pdf.lhs := \text{submatrix}\left(PDF.P0^{\langle 1 \rangle}, 0, p1eP, 0, 0\right) = \begin{bmatrix} 25 \\ \vdots \end{bmatrix} kW$$

$$P.mode1 := \text{lookup}\left(\max\left(PDF.P.lhs\right), PDF.P.lhs, P.pdf.lhs\right)_{0,0} = 425\ kW$$

$$P.mean1 := \frac{\sum_{i=0}^{\text{rows}(PDF.P.lhs)-1} \left(PDF.P.lhs\right)_i \cdot P.pdf.lhs_i}{\sum_{i=0}^{\text{rows}(PDF.P.lhs)-1} \left(PDF.P.lhs\right)_i} = 546.824\ kW$$

$$Var.P0.lhs := \sum_{i=0}^{\text{rows}(PDF.P.lhs)-1} \left(PDF.P.lhs\right)_i \cdot \left(\left(P.pdf.lhs\right) - P.mean1\right)_i^{\ 2} = 40216.9\ kW^2$$

$$Stdv.P0.lhs := \sqrt{Var.P0.lhs} = 200.54\ kW$$

To see the second peak:

$$PDF.P.rhs := \text{submatrix}\left(PDF.P0^{\langle 0 \rangle}, p1eP, \left(N.Pbin - 2\right), 0, 0\right)$$

$$P.pdf.rhs := \text{submatrix}\left(PDF.P0^{\langle 1 \rangle}, p1eP, N.Pbin - 2, 0, 0\right) = \begin{bmatrix} 1275 \\ \vdots \end{bmatrix} kW$$

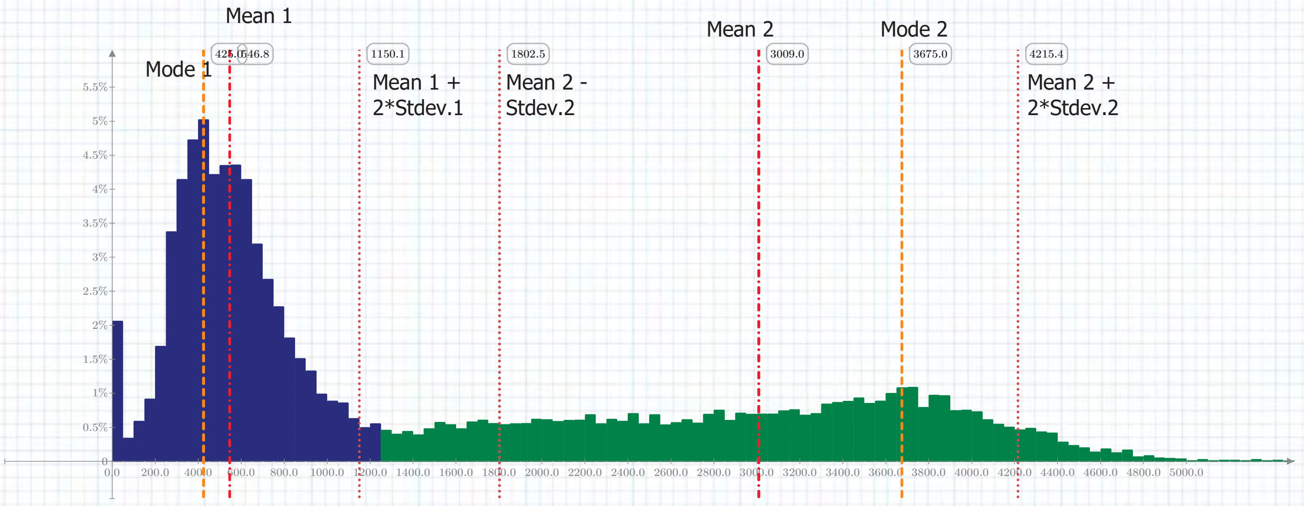
$$P.mean2 := \frac{\sum_{i=0}^{\text{rows}(PDF.P.rhs)-1} \left(PDF.P.rhs\right)_i \cdot P.pdf.rhs_i}{\sum_{i=0}^{\text{rows}(PDF.P.rhs)-1} \left(PDF.P.rhs\right)_i} = 3008.971\ kW$$

$$P.mode2 := \text{lookup}\left(\max\left(PDF.P.rhs\right), PDF.P.rhs, P.pdf.rhs\right)_{0,0} = 3675\ kW$$

$$Var.P0.rhs := \sum_{i=0}^{\text{rows}(PDF.P.rhs)-1} \left(PDF.P.rhs\right)_i \cdot \left(\left(P.pdf.rhs\right) - P.mean2\right)_i^{\ 2} = 363881.48\ kW^2$$

$$Stdv.P0.lhs := \sqrt{Var.P0.rhs} = 603.23\ kW$$

Plot with seperated pdfs:



Sorting Q- ΔH pairs on descending discharge Q:

Defining x-axis, percentage of occurance:

$$x.percT := \frac{1}{N.ALL}, \frac{2}{N.ALL} .. 1 = \begin{bmatrix} 0.006\% \\ 0.011\% \\ \vdots \end{bmatrix}$$

$$Q.desc2 := \text{reverse}\left(\text{csort}\left(\left(\text{augment}\left((Q)^{\mathrm{T}} \cdot s \cdot m^{-3}, \Delta H^{\mathrm{T}} \cdot m^{-1}\right)\right), 0\right)\right) = \begin{bmatrix} 2525 & 0.09 \\ 2514 & 0.09 \\ 2391 & 0.09 \\ 2348 & 0.08 \\ & \vdots \end{bmatrix}$$

Send to excel file:

$M2E.output :=$ "002 – OUTPUT–MATHCAD – Datalink QH–t–series for E–calc – v01.xlsx"

$Send.to.excel := \text{WRITEEXCEL}\left(\text{concat}\left(\mathbf{CWD}, M2E.output\right), \text{augment}\left(Q.desc2^{\langle 0 \rangle}, x.percT, Q.desc2^{\langle 1 \rangle}\right), \text{"FD curve!"}\right)$

Discharge at which the weir opens:

$Q.open := 440\ m^3 \cdot s^{-1}$

$$x.percT.Qopen := \text{mean}\left(\text{lookup}\left(\frac{Q.open}{m^3 \cdot s^{-1}}, Q.desc2^{\langle 0 \rangle}, x.percT\right)\right) = 27.69\%$$

$t.Q\_open := x.percT.Qopen\ yr = 101.1\ day$

$Q.low := 50\ m^3 \cdot s^{-1}$

$$x.percT.Qlow := \text{mean}\left(\text{lookup}\left(\frac{Q.low}{m^3 \cdot s^{-1}}, Q.desc2^{\langle 0 \rangle}, x.percT\right)\right) = 71.40\%$$

$t.Q\_low := (1 - x.percT.Qlow)\ yr = 104.5\ day$

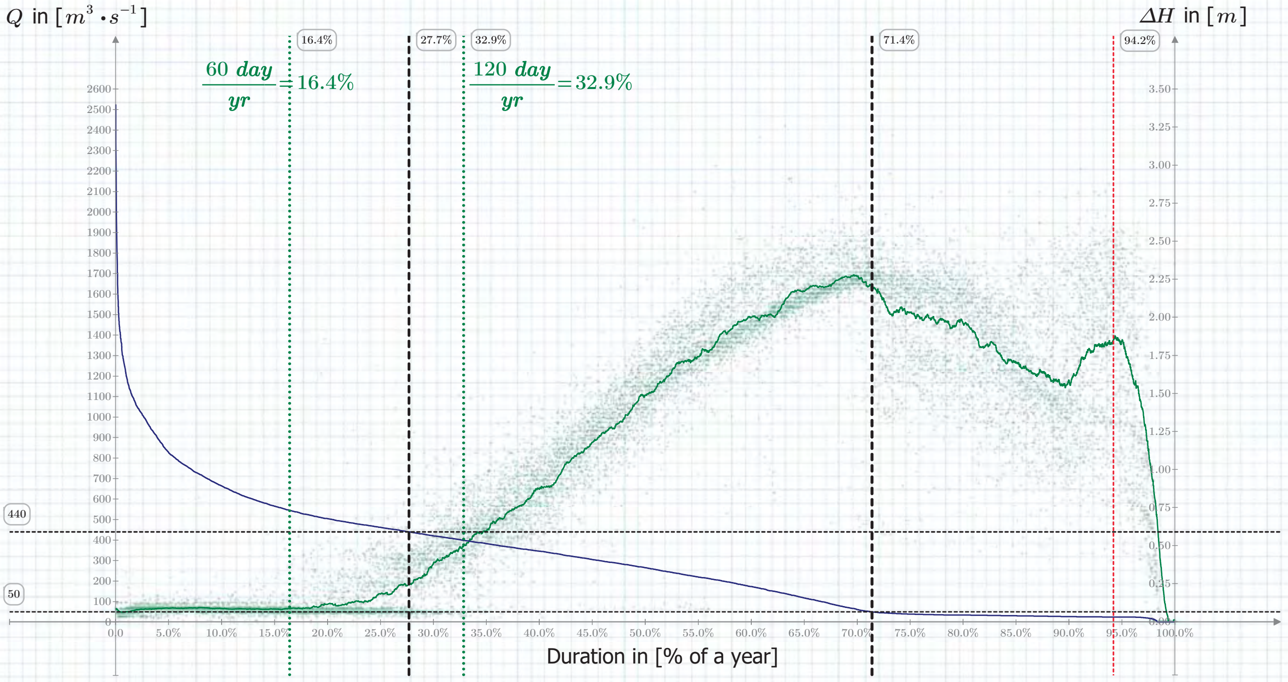
$Q.eco\_min := 25\ m^3 \cdot s^{-1}$

$$x.percT.Q\_eco\_min := \text{mean}\left(\text{lookup}\left(\frac{Q.eco\_min}{m^3 \cdot s^{-1}}, Q.desc2^{\langle 0 \rangle}, x.percT\right)\right) = 94.19\%$$

$t.Q\_eco\_min := (1 - x.percT.Q\_eco\_min)\ yr = 21.2\ day$

$$mavg\_H\_of\_Qd2 := \text{movavg}\left(Q.desc2^{\langle 1 \rangle}, 48 \cdot 4\right) = \begin{bmatrix} 0.09 \\ 0.09 \\ \vdots \end{bmatrix} \qquad 48 \cdot 4 = 192$$

Discharge Q (LHS-Vertical-axis) duration curve in % of time and related head-differences ΔH (RHS-Vertical-axis):



$t.Q\_open = 101.1\ day$ $t.Q\_low = 104.5\ day$ $t.Q\_eco\_min = 21.2\ day$

$\left(Q > 440\ m^3 \cdot s^{-1}\right)$ $\left(Q < 50\ m^3 \cdot s^{-1}\right)$ $\left(Q < 25\ m^3 \cdot s^{-1}\right)$

Legend:

Discharge Q in descending order: ———

Related head-differnences ΔH: ....

Moving average (per $48 \cdot 4 = 192$ points) of head-differnences ΔH: ———

Descending head-difference:

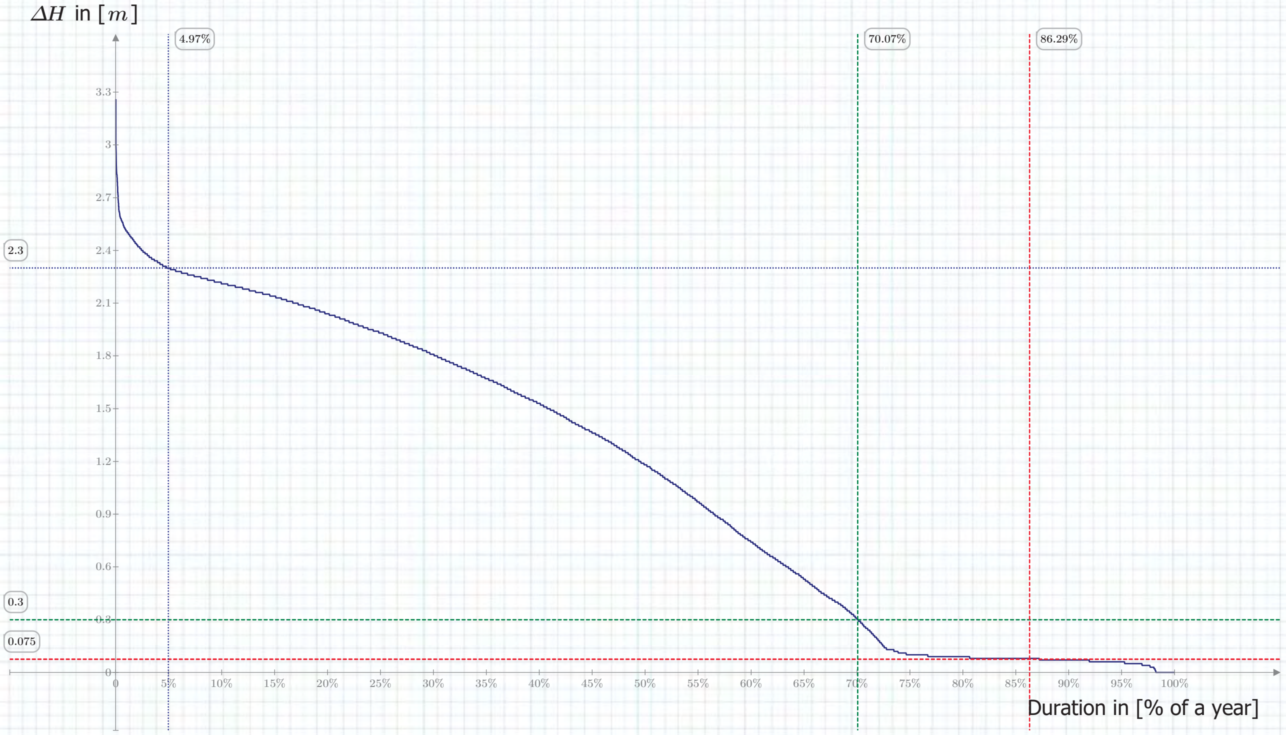
$$H.desc2 := \text{reverse}\left(\text{csort}\left(\left(\text{augment}\left(\Delta H^{\mathrm{T}} \cdot m^{-1}, (Q)^{\mathrm{T}} \cdot s \cdot m^{-3}\right)\right), 0\right)\right) = \begin{bmatrix} 3.26 & 140 \\ 3.2 & 43 \\ 3.19 & 165 \\ 3 & 207 \\ & \vdots \end{bmatrix}$$

Send to excel file:

$$Send.to.excel2 := \text{WRITEEXCEL}\left(\text{concat}\left(\mathbf{CWD}, M2E.output\right), \text{augment}\left(H.desc2^{\langle 0\rangle}, x.percT, H.desc2^{\langle 1\rangle}\right), \text{“HD curve!”}\right)$$

$$percT\_H0.3 := \text{mean}\left(\text{lookup}\left(0.3, H.desc2^{\langle 0\rangle}, x.percT\right)\right) = 70.1\%$$

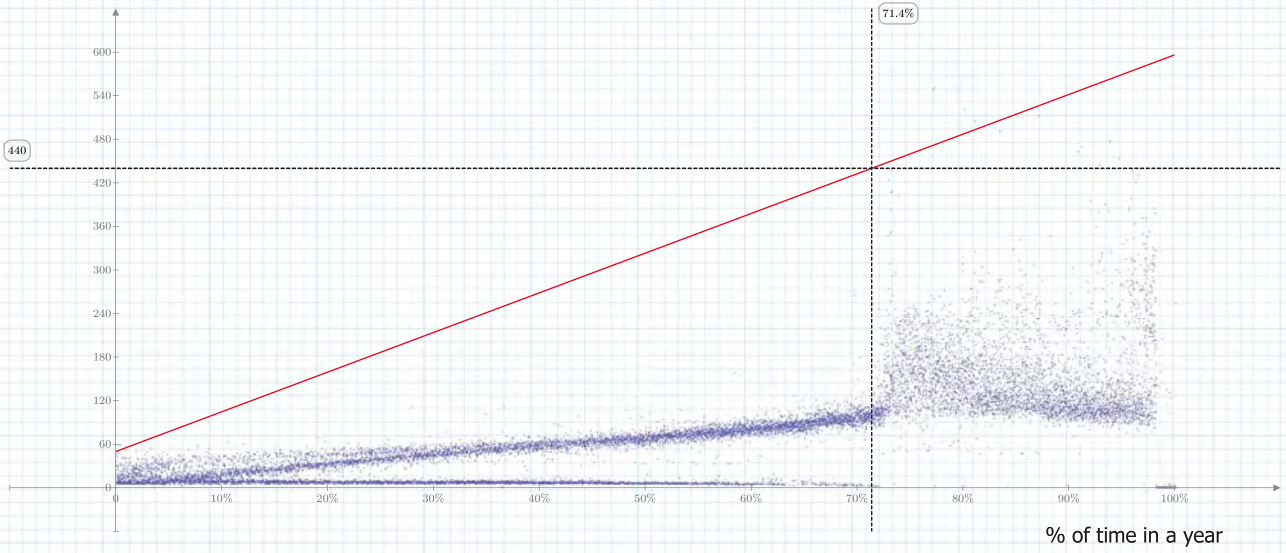
Head-difference-duration curve:



Related discharge Q for Head-difference-duration curve:

Eye-balled linear-fit: $b := 50$ $y2 := b + x.percT \cdot \frac{(440 - b)}{.714}$

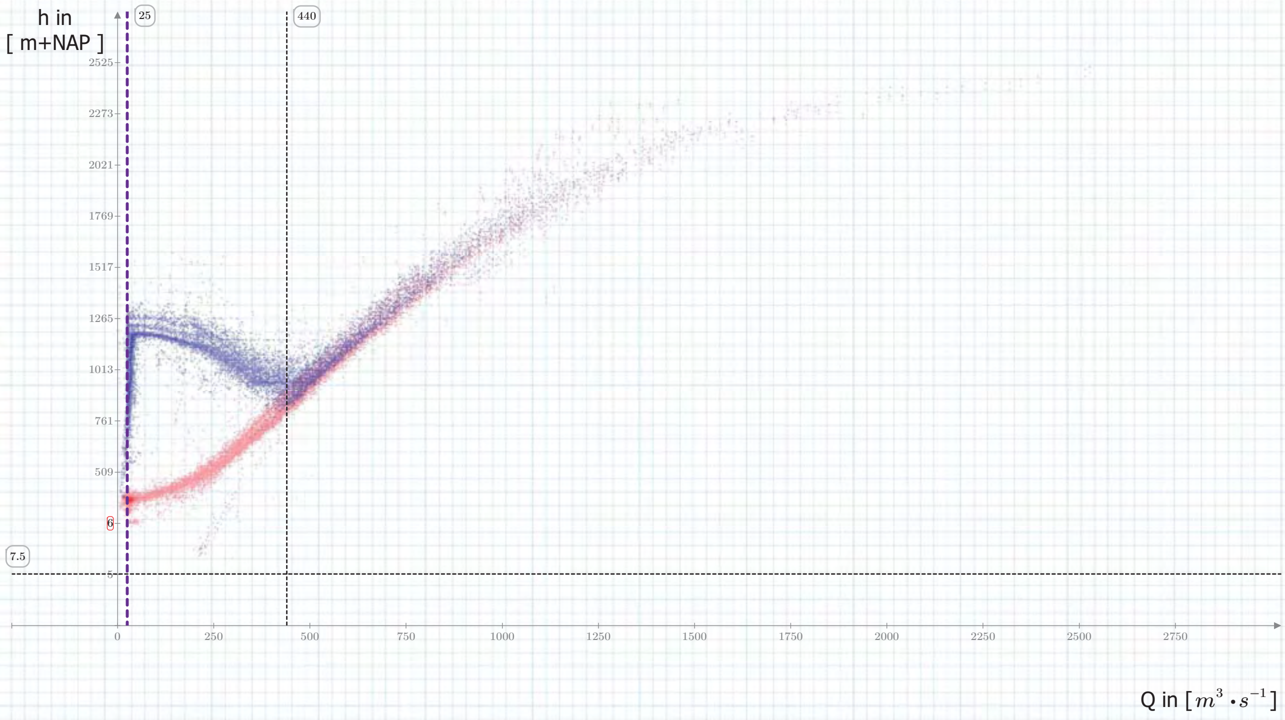
Q in $[m^3 \cdot s^{-1}]$



- Page intentionally left blank -

Empirical Q-h-relation derived from measurements from 1970-2018:

$h.down := ALLDATA^{\langle 3 \rangle}\ cm$ **(RED)** $h.up := ALLDATA^{\langle 6 \rangle}\ cm$ **(BLUE)**



Empirical Q- ΔH -relation:

Data per year (and averages over the years):

```
$MQy :=$
    $y \leftarrow 1970$
    $a \leftarrow 0$
    for $k \in 0..N.ALL-1$
        if $ALLDATA_{k,11} < y+1$
            $y \leftarrow y$
            $ind \leftarrow y-1970$
            $MQy_{ind,a} \leftarrow \left(Q^{T}\right)_k$
            $a \leftarrow ALLDATA_{k,12}-1$
        else if $ALLDATA_{k,11} \geq y+1$
            $y \leftarrow ALLDATA_{k,11}$
            $a \leftarrow ALLDATA_{k,12}-1$
            $ind \leftarrow y-1970$
            $MQy_{ind,a} \leftarrow \left(Q^{T}\right)_k$
    $MQy$
```

$$= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 161 & 160 & 153 & 140 & 121 & 120 & 113 & 118 & 114 & 110 & 107 \\ 34 & 34 & 34 & 32 & 33 & 33 & 33 & 32 & 32 & 32 & 31 \\ 27 & 26 & 26 & 26 & 26 & 26 & 26 & 25 & 25 & 27 & 26 \\ 351 & 303 & 279 & 256 & 216 & 180 & 218 & 229 & 241 & 286 & 331 \\ 1080 & 1054 & 983 & 889 & 796 & 724 & 697 & 690 & 674 & 684 & 697 \\ 26 & 27 & 29 & 117 & 195 & 217 & 275 & 268 & 221 & 216 & 218 \\ & & & & & & & & & & \ddots \end{bmatrix} \frac{m^3}{s}$$

$$MQy.max := \max(MQy) = 2525 \ \frac{m^3}{s}$$

```
$meanQday :=$
    for $i \in 0..\text{cols}(MQy)-1$
        $Qy^{\langle i \rangle} \leftarrow \dfrac{\sum_{n=1}^{\text{rows}(MQy^{\langle i \rangle})-1} \left(MQy_{n,i}\right)}{\text{rows}\left(MQy^{\langle i \rangle}\right)}$
    $Qy^{\langle \text{cols}(MQy)-1 \rangle} \leftarrow Qy^{\langle 0 \rangle}$
    $Qy$
```

$$= [454.8367 \ \ 461.3878 \ \ 466.5102 \ \ 474.9388 \ \ 476.9592 \ \ \ldots] \ \frac{m^3}{s}$$

Note: first year (1970) was incomplete and had a lot fo 0-values. This year is therefore excluded to calculated the mean values.

$$x.dayinyr := 0,1..\text{cols}(MQy)-1 = \begin{bmatrix} 0 \\ 1 \\ \vdots \end{bmatrix}$$

Averaged discharge over 48 years for every day in the year:

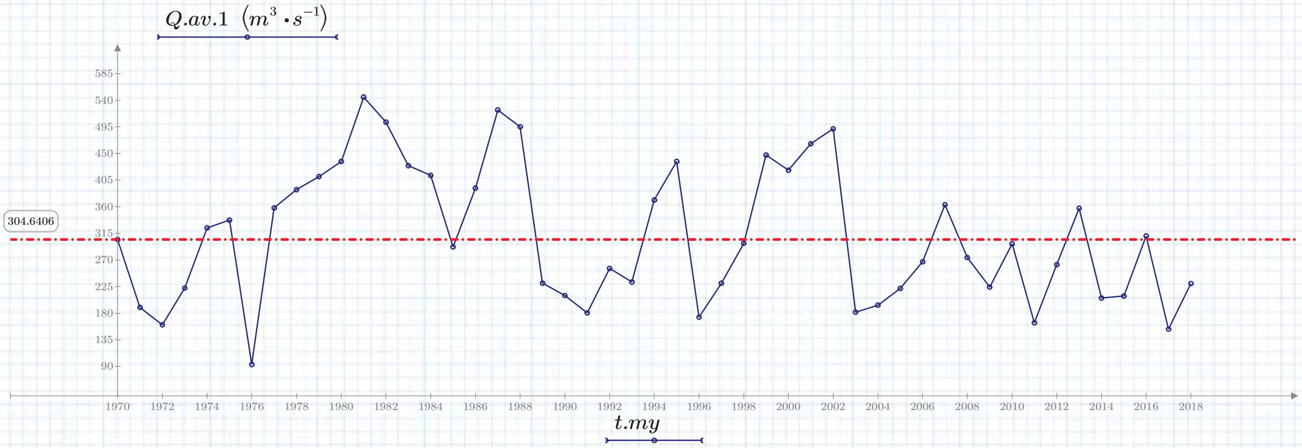
1 year average discharge:

```
$Q.av.1 :=$
    $Q.av_{0,0} \leftarrow \mathrm{mean}(meanQday)$
    for $i \in 1 .. \mathrm{rows}(MQy) - 1$
        $A \leftarrow \mathrm{submatrix}(MQy, i, i, 0, \mathrm{cols}(MQy) - 1)$
        $S \leftarrow \sum_{a=0}^{\mathrm{rows}(A)-1} \sum_{b=0}^{\mathrm{cols}(A)-1} A_{a,b}$
        $Q.av_{i,0} \leftarrow \frac{S}{(\mathrm{rows}(A)) \cdot (\mathrm{cols}(A))}$
    return $Q.av$
```

$$= \begin{bmatrix} 304.6406 \\ 189.8384 \\ 160.4 \\ \vdots \end{bmatrix} m^3 \cdot s^{-1}$$

Note:
Due to the first year missing a lot of data, the first year is set to the all-time mean.

$$t.my = \begin{bmatrix} 1970 \\ 1971 \\ 1972 \\ \vdots \end{bmatrix}$$



10 year average discharge:

```
$Q.av.10 :=$
    $Q.av_{0,0} \leftarrow$
        $A \leftarrow \mathrm{submatrix}(MQy, 1, 1+8, 0, \mathrm{cols}(MQy) - 1)$
        $S \leftarrow \sum_{a=0}^{\mathrm{rows}(A)-1} \sum_{b=0}^{\mathrm{cols}(A)-1} A_{a,b}$
        $\frac{S}{(\mathrm{rows}(A)) \cdot (\mathrm{cols}(A))}$
    for $i \in 1 .. \mathrm{rows}(MQy) - 10$
        $A \leftarrow \mathrm{submatrix}(MQy, i, i+9, 0, \mathrm{cols}(MQy) - 1)$
        $S \leftarrow \sum_{a=0}^{\mathrm{rows}(A)-1} \sum_{b=0}^{\mathrm{cols}(A)-1} A_{a,b}$
        $Q.av_{i,0} \leftarrow \frac{S}{(\mathrm{rows}(A)) \cdot (\mathrm{cols}(A))}$
    return $Q.av$
```

$$= \begin{bmatrix} 276.2058 \\ 292.2419 \\ 327.7882 \\ 362.0551 \\ \vdots \end{bmatrix} m^3 \cdot s^{-1}$$

Note:
Again first year (1970) is ingnored in the the first 10-year average due to large number of 0-values

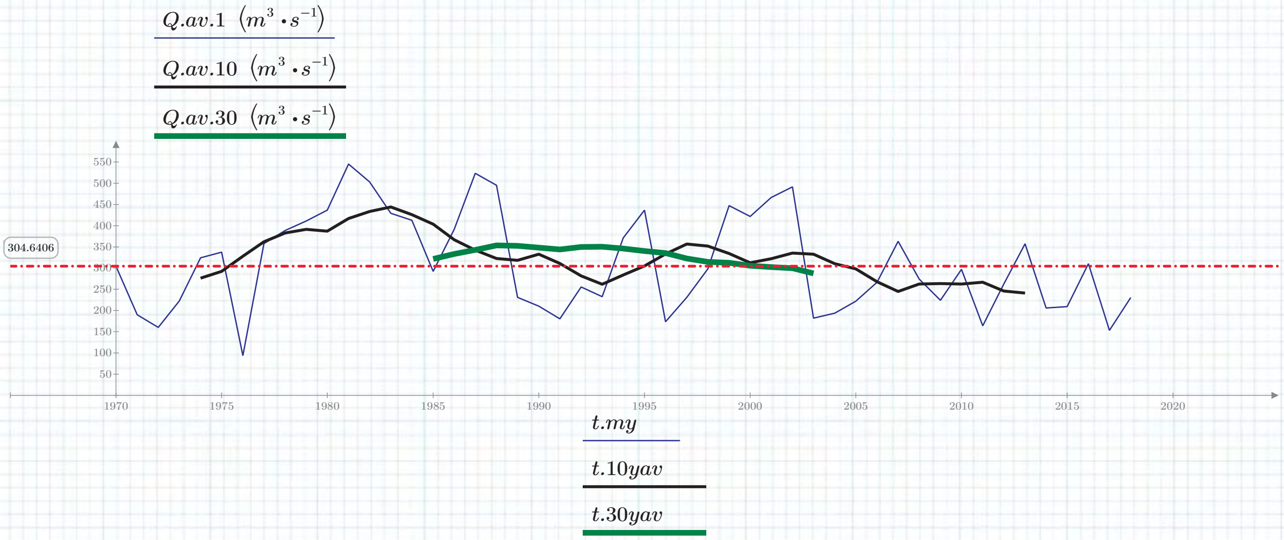
$$t.10yav := \left(1970 + 10 - \frac{10}{2} - 1\right), \left(1970 + 10 - \frac{10}{2}\right) .. \left(2018 - \frac{10}{2}\right) = \begin{bmatrix} 1974 \\ 1975 \\ 1976 \\ \vdots \end{bmatrix}$$

30 year average discharge:

```
Q.av.30 := | for i ∈ 0 .. rows(MQy) − 30
           |   | A ← submatrix(MQy, i, i+29, 0, cols(MQy) − 1)
           |   | S ← $\sum_{a=0}^{\mathrm{rows}(A)-1} \sum_{b=0}^{\mathrm{cols}(A)-1} A_{a,b}$
           |   | Q.av_{i,0} ← $\frac{S}{(\mathrm{rows}(A)) \cdot (\mathrm{cols}(A))}$
           | return Q.av
```

$$= \begin{bmatrix} 321.5437 \\ 333.4852 \\ 342.7066 \\ 353.7469 \\ \vdots \end{bmatrix} m^3 \cdot s^{-1}$$

$$t.30yav := \left(\frac{1970+1970+30}{2}\right), \left(\frac{1970+1970+30}{2}+1\right) .. \left(\frac{2018-30+2018}{2}\right) = \begin{bmatrix} 1985 \\ 1986 \\ \vdots \end{bmatrix}$$



Time series for last 10 years:

$$MQ10y := \mathrm{submatrix}(MQy, \mathrm{rows}(MQy)-10, \mathrm{rows}(MQy)-1, 0, \mathrm{cols}(MQy)-1) = \begin{bmatrix} 59 & 36 & 33 & \\ 725 & 834 & 868 & \\ 492 & 467 & 427 & \\ 534 & 581 & 679 & \\ 1026 & 937 & 847 & \\ 563 & 522 & 491 & \\ 287 & 272 & 291 & \\ 28 & 28 & 27 & \\ 22 & 20 & 20 & \\ 646 & 761 & 1024 & \cdots \end{bmatrix} \frac{m^3}{s}$$

Time series for last 30 years:

$$MQ30y := \mathrm{submatrix}(MQy, \mathrm{rows}(MQy)-30, \mathrm{rows}(MQy)-1, 0, \mathrm{cols}(MQy)-1) = \begin{bmatrix} 507 & 477 & 451 & \\ 287 & 232 & 171 & \\ 817 & 896 & 954 & \\ 450 & 404 & 344 & \\ 190 & 161 & 100 & \\ 1076 & 1245 & 1313 & \\ 748 & 747 & 731 & \\ & & & \ddots \end{bmatrix} \frac{m^3}{s}$$

Save in excel:

$$Output1 := \mathrm{WRITEEXCEL}\left(\mathrm{concat}(\mathbf{CWD}, M2E.output), \frac{MQ10y}{m^3 \cdot s^{-1}}, \text{“MQ10y!A1:NA10”}\right)$$

$$Output2 := \mathrm{WRITEEXCEL}\left(\mathrm{concat}(\mathbf{CWD}, M2E.output), \frac{MQ30y}{m^3 \cdot s^{-1}}, \text{“MQ30y!A1:NA30”}\right)$$

```
$MHy :=$
    $y \leftarrow 1970$
    $a \leftarrow 0$
    for $k \in 0..N.ALL-1$
        if $ALLDATA_{k,11} < y+1$
            $y \leftarrow y$
            $ind \leftarrow y-1970$
            $MHy_{ind,a} \leftarrow \left(\Delta H^{\mathrm{T}}\right)_k$
            $a \leftarrow ALLDATA_{k,12} - 1$
        else if $ALLDATA_{k,11} \geq y+1$
            $y \leftarrow ALLDATA_{k,11}$
            $a \leftarrow ALLDATA_{k,12} - 1$
            $ind \leftarrow y-1970$
            $MHy_{ind,a} \leftarrow \left(\Delta H^{\mathrm{T}}\right)_k$
    $MHy$
```

$$= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1.93 & 1.93 & 1.86 & 1.86 & 2.18 & 2.29 & 2.22 & 2.21 & 2.21 & 2.16 & 2.12 & 2.11 \\ 2.09 & 2.07 & 2.04 & 1.97 & 1.93 & 1.87 & 1.89 & 1.89 & 1.84 & 1.8 & 1.75 & 1.74 \\ 2.21 & 2.12 & 2.08 & 2.06 & 2.05 & 2.04 & 1.99 & 1.97 & 1.94 & 1.96 & 1.91 & 1.96 \\ 1.02 & 1.41 & 1.71 & 1.8 & 2.04 & 2.19 & 1.93 & 1.87 & 1.79 & 1.46 & 1.2 & 0.74 \\ 0.1 & 0.1 & 0.09 & 0.08 & 0.09 & 0.06 & 0.09 & 0.08 & 0.1 & 0.09 & 0.09 & 0.09 \\ 1.95 & 1.89 & 2.63 & 2.44 & 2.32 & 2.72 & 2.64 & 2.59 & 2.38 & 2.18 & 2.14 & 2.18 \\ & & & & & & & & & & & \ddots \end{bmatrix} m$$

$$MHy.max := \max(MHy) = 3.26\ m$$

```
$meanHy :=$
    for $i \in 0..\mathrm{cols}(MHy)-1$
        $Qy^{\langle i \rangle} \leftarrow \dfrac{\sum \left(MHy^{\langle i \rangle}\right)}{\mathrm{rows}\left(MHy^{\langle i \rangle}\right) - \sum_{n=0}^{\mathrm{rows}(MHy)-1} \left( \left( \dfrac{MHy}{m} \right)_{n,i} = 0 \right)}$
    return $Qy$
```

$$= \begin{bmatrix} 0.7738 & 0.79 & 0.8404 & 0.8306 & 0.8183 & 0.8079 & \dots \end{bmatrix} m$$

Note: 0-values have been removed from this calculations of the average, because the head-difference is never truely 0.

Averaged head-difference over 49 years for every day in the year:

1 year average Head-difference:

```
$H.av.1 :=$
    for $i \in 0..\text{rows}(MHy)-1$
        $A \leftarrow \text{submatrix}(MHy, i, i, 0, \text{cols}(MHy)-1)$
        $S \leftarrow \sum_{a=0}^{\text{rows}(A)-1} \sum_{b=0}^{\text{cols}(A)-1} A_{a,b}$
        $Q.av_{i,0} \leftarrow \frac{S}{(\text{rows}(A))\cdot(\text{cols}(A))}$
    return $Q.av$
```

$$= \begin{bmatrix} 0.0955 \\ 1.5942 \\ 1.6912 \\ 1.5783 \\ 1.3824 \\ \vdots \end{bmatrix} m$$

10 year average Head-difference:

```
$H.av.10 :=$
    for $i \in 0..\text{rows}(MHy)-10$
        $A \leftarrow \text{submatrix}(MHy, i, i+9, 0, \text{cols}(MHy)-1)$
        $S \leftarrow \sum_{a=0}^{\text{rows}(A)-1} \sum_{b=0}^{\text{cols}(A)-1} A_{a,b}$
        $Q.av_{i,0} \leftarrow \frac{S}{(\text{rows}(A))\cdot(\text{cols}(A))}$
    return $Q.av$
```

$$= \begin{bmatrix} 1.221 \\ 1.3037 \\ 1.1996 \\ 1.0881 \\ \vdots \end{bmatrix} m$$

30 year average Head-difference:

```
H.av.30 := | for i ∈ 0..rows(MHy) − 30
           |   | A ← submatrix(MHy, i, i+29, 0, cols(MHy) − 1)
           |   |         rows(A)−1  cols(A)−1
           |   | S ←       Σ          Σ        A
           |   |          a=0        b=0        a,b
           |   |                        S
           |   | Q.av     ← ─────────────────────────
           |   |     i,0    (rows(A)) · (cols(A))
           | return Q.av
```

$$= \begin{bmatrix} 1.1003 \\ 1.1205 \\ 1.0906 \\ 1.0601 \\ \vdots \end{bmatrix} \frac{s}{m^2} \cdot m^3 \cdot s^{-1}$$

Time series for last 10 years:

$$MH10y := \text{submatrix}(MHy, \text{rows}(MHy) - 10, \text{rows}(MHy) - 1, 0, \text{cols}(MHy) - 1) = \begin{bmatrix} 2.28 & 2.23 & 2.08 & 2.08 & 1.97 & \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.09 & \\ 0.06 & 0.16 & 0.42 & 0.44 & 0.5 & \\ 0.12 & 0.23 & 0.3 & 0.29 & 0.14 & \\ 0.04 & 0.08 & 0.09 & 0.08 & 0.08 & \\ 0.07 & 0.07 & 0.06 & 0.06 & 0.06 & \\ 1.35 & 1.54 & 1.41 & 1.13 & 0.6 & \\ 1.31 & 1.27 & 1.26 & 1.32 & 1.51 & \\ & & & & & \ddots \end{bmatrix} m$$

Time series for last 30 years:

$$MH30y := \text{submatrix}(MHy, \text{rows}(MHy) - 30, \text{rows}(MHy) - 1, 0, \text{cols}(MHy) - 1) = \begin{bmatrix} 0.05 & 0.05 & 0.04 & 0.29 & 0.77 & \\ 1.35 & 1.63 & 1.95 & 2.17 & 2.36 & \\ 0.07 & 0.08 & 0.08 & 0.08 & 0.07 & \\ 0.06 & 0.17 & 0.84 & 1.22 & 1.19 & \\ 1.72 & 1.81 & 2.1 & 2.16 & 2.16 & \\ 0.11 & 0.08 & 0.07 & 0.06 & 0.06 & \\ 0.09 & 0.08 & 0.08 & 0.08 & 0.07 & \\ 0.08 & 0.08 & 0.07 & 0.07 & 0.07 & \\ & & & & & \ddots \end{bmatrix} m$$

Save in excel:

$$Output3 := \text{WRITEEXCEL}\left(\text{concat}(\mathbf{CWD}, M2E.output), \frac{MH10y}{m}, \text{“MH10y!A1:NA10”}\right)$$

$$Output4 := \text{WRITEEXCEL}\left(\text{concat}(\mathbf{CWD}, M2E.output), \frac{MH30y}{m}, \text{“MH30y!A1:NA30”}\right)$$

Energy flux being lost throughout the years (per day) at the weir of Driel:

$$MPy := \rho \cdot g\ \overrightarrow{MQy \cdot MHy} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \\ 3047.2204 & 3028.2935 & 2790.7765 & 2553.6517 & 2586.7981 & 2694.8674 & \\ 696.8605 & 690.192 & 680.1892 & 618.2112 & 624.5855 & 605.1684 & \\ 585.1628 & 540.5425 & 530.3436 & 525.2442 & 522.6944 & 520.1447 & \\ 3510.9768 & 4189.6951 & 4678.6546 & 4518.9043 & 4321.2023 & 3865.7814 & \\ 1059.1182 & 1033.6209 & 867.5943 & 697.4489 & 702.5484 & 426.0009 & \\ 497.1972 & 500.4333 & 747.9532 & 2799.6024 & 4436.5285 & 5788.2771 & \\ 489.548 & 475.4264 & 548.1917 & 609.3852 & 637.4323 & 608.0123 & \\ 512.2994 & 440.7109 & 597.225 & 497.7856 & 363.1402 & 297.7299 & \\ 5758.9552 & 1198.3726 & 1351.3564 & 1153.262 & 906.1345 & 612.9156 & \\ 370.0049 & 311.2631 & 343.9192 & 324.6982 & 1440.1066 & 1553.3734 & \\ 638.4129 & 1207.3947 & 1475.9989 & 1129.7261 & 646.4544 & 757.0734 & \\ & & & & & & \ddots \end{bmatrix} \boldsymbol{kW}$$

Daily means over all years:

```
meanPy := for $i \in 0..\text{cols}(MPy)-1$
              $Qy^{\langle i \rangle} \leftarrow \dfrac{\sum \left(MPy^{\langle i \rangle}\right)}{\text{rows}\left(MPy^{\langle i \rangle}\right) - \sum_{n=0}^{\text{rows}(MHy)-1} \left( \left( \dfrac{MPy}{kW} \right)_{n,i} = 0 \right)}$
          return $Qy$
```

$$= \begin{bmatrix} 1361.2243 & 1329.841 & 1383.2974 & \dots \end{bmatrix} \boldsymbol{kW}$$

All time mean value:

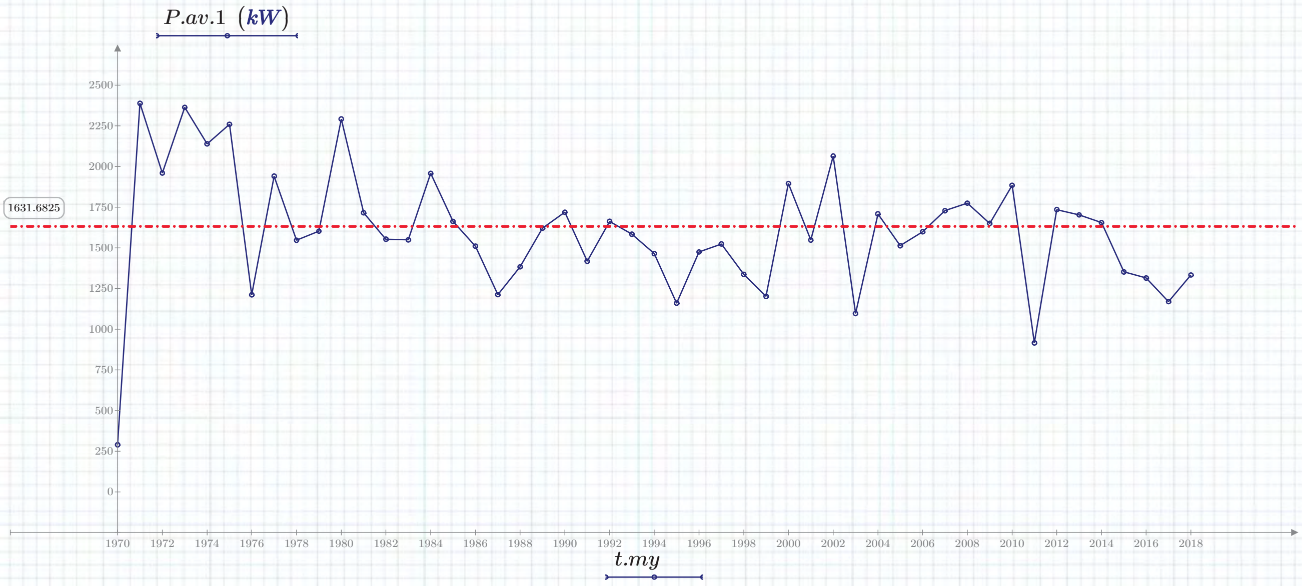
$$meanPy.ALLtime := \text{mean}(meanPy) = 1631.6825\ \boldsymbol{kW}$$

1 year average Energy flux being lost at Driel:

```
P.av.1 := for $i \in 0..\text{rows}(MPy)-1$
              $A \leftarrow \text{submatrix}(MPy, i, i, 0, \text{cols}(MPy)-1)$
              $S \leftarrow \sum_{a=0}^{\text{rows}(A)-1} \sum_{b=0}^{\text{cols}(A)-1} A_{a,b}$
              $Q.av \leftarrow \dfrac{S}{\ }$
```

$$= \begin{bmatrix} 289.2887 \\ 2388.0974 \\ 1959.5054 \\ 2363.566 \\ 2138.8752 \\ \vdots \end{bmatrix} \boldsymbol{kW}$$

```
    $_{i,0}$   $(\text{rows}(A))\cdot(\text{cols}(A))$
return $Q.av$
```



10 year average Energy flux being lost at Driel:

```
$P.av.10 :=$ for $i \in 0..\text{rows}(MPy)-10$
                 $A \leftarrow \text{submatrix}(MPy, i, i+9, 0, \text{cols}(MPy)-1)$
                 $S \leftarrow \sum_{a=0}^{\text{rows}(A)-1} \sum_{b=0}^{\text{cols}(A)-1} A_{a,b}$
                 $Q.av_{i,0} \leftarrow \frac{S}{(\text{rows}(A))\cdot(\text{cols}(A))}$
             return $Q.av$
```

$$= \begin{bmatrix} 1769.8484 \\ 1970.0871 \\ 1902.7851 \\ 1862.1058 \\ \vdots \end{bmatrix} \, kW$$

30 year average Energy flux being lost at Driel:

```
$P.av.30 :=$ for $i \in 0..\mathrm{rows}(MPy)-30$
                 $A \leftarrow \mathrm{submatrix}(MPy, i, i+29, 0, \mathrm{cols}(MPy)-1)$
                 $S \leftarrow \sum_{a=0}^{\mathrm{rows}(A)-1} \sum_{b=0}^{\mathrm{cols}(A)-1} A_{a,b}$
                 $Q.av_{i,0} \leftarrow \frac{S}{(\mathrm{rows}(A)) \cdot (\mathrm{cols}(A))}$
             return $Q.av$
```

$$= \begin{bmatrix} 1623.1245 \\ 1676.6485 \\ 1648.6416 \\ 1652.119 \\ \vdots \end{bmatrix} kW$$

Discharge series per year:

$Q.y.max.rnd := \mathrm{Ceil}\left(\max\left(MQy\right), 100\ m^3 \cdot s^{-1}\right) = 2600\ m^3 \cdot s^{-1}$

$Q1970 := \left(MQy^{\langle 0\rangle}\right)^{T} = \begin{bmatrix}0\\0\\0\\\vdots\end{bmatrix} \frac{m^3}{s}$ $Q1971 := \left(MQy^{\langle 1\rangle}\right)^{T} = \begin{bmatrix}161\\160\\153\\\vdots\end{bmatrix} \frac{m^3}{s}$ $Q1972 := \left(MQy^{\langle 2\rangle}\right)^{T} = \begin{bmatrix}34\\34\\34\\\vdots\end{bmatrix} \frac{m^3}{s}$

$Q1973 := \left(MQy^{\langle 3\rangle}\right)^{T} = \begin{bmatrix}27\\26\\26\\\vdots\end{bmatrix} \frac{m^3}{s}$ $Q1974 := \left(MQy^{\langle 4\rangle}\right)^{T} = \begin{bmatrix}351\\303\\279\\\vdots\end{bmatrix} \frac{m^3}{s}$ $Q1975 := \left(MQy^{\langle 5\rangle}\right)^{T} = \begin{bmatrix}1080\\1054\\983\\\vdots\end{bmatrix} \frac{m^3}{s}$

$Q1976 := \left(MQy^{\langle 6\rangle}\right)^{T} = \begin{bmatrix}26\\27\\29\\\vdots\end{bmatrix} \frac{m^3}{s}$ $Q1977 := \left(MQy^{\langle 7\rangle}\right)^{T} = \begin{bmatrix}24\\24\\26\\\vdots\end{bmatrix} \frac{m^3}{s}$ $Q1978 := \left(MQy^{\langle 8\rangle}\right)^{T} = \begin{bmatrix}653\\642\\609\\\vdots\end{bmatrix} \frac{m^3}{s}$

$Q1979 := \left(MQy^{\langle 9\rangle}\right)^{T} = \begin{bmatrix}725\\940\\1060\\\vdots\end{bmatrix} \frac{m^3}{s}$

$Q1980 := \left(MQy^{\langle 10\rangle}\right)^{T} = \begin{bmatrix}539\\529\\501\\\vdots\end{bmatrix} \frac{m^3}{s}$ $Q1981 := \left(MQy^{\langle 11\rangle}\right)^{T} = \begin{bmatrix}465\\456\\519\\\vdots\end{bmatrix} \frac{m^3}{s}$ $Q1982 := \left(MQy^{\langle 12\rangle}\right)^{T} = \begin{bmatrix}1045\\1321\\1409\\\vdots\end{bmatrix} \frac{m^3}{s}$

$Q1983 := \left(MQy^{\langle 13\rangle}\right)^{T} = \begin{bmatrix}622\\583\\555\\\vdots\end{bmatrix} \frac{m^3}{s}$ $Q1984 := \left(MQy^{\langle 14\rangle}\right)^{T} = \begin{bmatrix}371\\347\\355\\\vdots\end{bmatrix} \frac{m^3}{s}$ $Q1985 := \left(MQy^{\langle 15\rangle}\right)^{T} = \begin{bmatrix}230\\248\\215\\\vdots\end{bmatrix} \frac{m^3}{s}$

$Q1986 := \left(MQy^{\langle 16\rangle}\right)^{T} = \begin{bmatrix}273\\284\\257\\\vdots\end{bmatrix} \frac{m^3}{s}$ $Q1987 := \left(MQy^{\langle 17\rangle}\right)^{T} = \begin{bmatrix}1131\\1191\\1313\\\vdots\end{bmatrix} \frac{m^3}{s}$ $Q1988 := \left(MQy^{\langle 18\rangle}\right)^{T} = \begin{bmatrix}430\\419\\422\\\vdots\end{bmatrix} \frac{m^3}{s}$

$Q1989 := \left(MQy^{\langle 19\rangle}\right)^{T} = \begin{bmatrix}507\\477\\451\\\vdots\end{bmatrix} \frac{m^3}{s}$

$Q1990 := \left(MQy^{\langle 20\rangle}\right)^{T} = \begin{bmatrix}287\\232\\171\\\vdots\end{bmatrix} \frac{m^3}{s}$ $Q1991 := \left(MQy^{\langle 21\rangle}\right)^{T} = \begin{bmatrix}817\\896\\954\\\vdots\end{bmatrix} \frac{m^3}{s}$ $Q1992 := \left(MQy^{\langle 22\rangle}\right)^{T} = \begin{bmatrix}450\\404\\344\\\vdots\end{bmatrix} \frac{m^3}{s}$

$Q1993 := \left(MQy^{\langle 23\rangle}\right)^{T} = \begin{bmatrix}190\\161\\100\\\vdots\end{bmatrix} \frac{m^3}{s}$ $Q1994 := \left(MQy^{\langle 24\rangle}\right)^{T} = \begin{bmatrix}1076\\1245\\1313\\\vdots\end{bmatrix} \frac{m^3}{s}$ $Q1995 := \left(MQy^{\langle 25\rangle}\right)^{T} = \begin{bmatrix}748\\747\\731\\\vdots\end{bmatrix} \frac{m^3}{s}$

$Q1996 := \left(MQy^{\langle 26\rangle}\right)^{T} = \begin{bmatrix}583\\464\\428\\\vdots\end{bmatrix} \frac{m^3}{s}$ $Q1997 := \left(MQy^{\langle 27\rangle}\right)^{T} = \begin{bmatrix}346\\329\\324\\\vdots\end{bmatrix} \frac{m^3}{s}$ $Q1998 := \left(MQy^{\langle 28\rangle}\right)^{T} = \begin{bmatrix}543\\507\\495\\\vdots\end{bmatrix} \frac{m^3}{s}$

$Q1999 := \left(MQy^{\langle 29\rangle}\right)^{T} = \begin{bmatrix}421\\392\\378\\\vdots\end{bmatrix} \frac{m^3}{s}$

$Q2000 := \left(MQy^{\langle 30\rangle}\right)^{T} = \begin{bmatrix}1061\\946\\847\\\vdots\end{bmatrix} \frac{m^3}{s}$ $Q2001 := \left(MQy^{\langle 31\rangle}\right)^{T} = \begin{bmatrix}339\\311\\314\\\vdots\end{bmatrix} \frac{m^3}{s}$ $Q2002 := \left(MQy^{\langle 32\rangle}\right)^{T} = \begin{bmatrix}897\\984\\904\\\vdots\end{bmatrix} \frac{m^3}{s}$

$Q2003 := \left(MQy^{\langle 33\rangle}\right)^{T} = \begin{bmatrix}1019\\1043\\1116\\\vdots\end{bmatrix} \frac{m^3}{s}$ $Q2004 := \left(MQy^{\langle 34\rangle}\right)^{T} = \begin{bmatrix}29\\29\\27\\\vdots\end{bmatrix} \frac{m^3}{s}$ $Q2005 := \left(MQy^{\langle 35\rangle}\right)^{T} = \begin{bmatrix}309\\293\\290\\\vdots\end{bmatrix} \frac{m^3}{s}$

$Q2006 := \left(MQy^{\langle 36\rangle}\right)^{T} = \begin{bmatrix}26\\29\\40\\\vdots\end{bmatrix} \frac{m^3}{s}$ $Q2007 := \left(MQy^{\langle 37\rangle}\right)^{T} = \begin{bmatrix}33\\39\\146\\\vdots\end{bmatrix} \frac{m^3}{s}$ $Q2008 := \left(MQy^{\langle 38\rangle}\right)^{T} = \begin{bmatrix}37\\34\\34\\\vdots\end{bmatrix} \frac{m^3}{s}$

$$Q2009 := \left(MQy^{\langle 39 \rangle}\right)^{T} = \begin{bmatrix} 59 \\ 36 \\ 33 \\ \vdots \end{bmatrix} \frac{m^3}{s}$$

$$Q2010 := \left(MQy^{\langle 40 \rangle}\right)^{T} = \begin{bmatrix} 725 \\ 834 \\ 868 \\ \vdots \end{bmatrix} \frac{m^3}{s}$$

$$Q2011 := \left(MQy^{\langle 41 \rangle}\right)^{T} = \begin{bmatrix} 492 \\ 467 \\ 427 \\ \vdots \end{bmatrix} \frac{m^3}{s}$$

$$Q2012 := \left(MQy^{\langle 42 \rangle}\right)^{T} = \begin{bmatrix} 534 \\ 581 \\ 679 \\ \vdots \end{bmatrix} \frac{m^3}{s}$$

$$Q2013 := \left(MQy^{\langle 43 \rangle}\right)^{T} = \begin{bmatrix} 1026 \\ 937 \\ 847 \\ \vdots \end{bmatrix} \frac{m^3}{s}$$

$$Q2014 := \left(MQy^{\langle 44 \rangle}\right)^{T} = \begin{bmatrix} 563 \\ 522 \\ 491 \\ \vdots \end{bmatrix} \frac{m^3}{s}$$

$$Q2015 := \left(MQy^{\langle 45 \rangle}\right)^{T} = \begin{bmatrix} 287 \\ 272 \\ 291 \\ \vdots \end{bmatrix} \frac{m^3}{s}$$

$$Q2016 := \left(MQy^{\langle 46 \rangle}\right)^{T} = \begin{bmatrix} 28 \\ 28 \\ 27 \\ \vdots \end{bmatrix} \frac{m^3}{s}$$

$$Q2017 := \left(MQy^{\langle 47 \rangle}\right)^{T} = \begin{bmatrix} 22 \\ 20 \\ 20 \\ \vdots \end{bmatrix} \frac{m^3}{s}$$

$$Q2018 := \left(MQy^{\langle 48 \rangle}\right)^{T} = \begin{bmatrix} 646 \\ 761 \\ 1024 \\ \vdots \end{bmatrix} \frac{m^3}{s}$$

Head differences series per year:

$$H.max.year := \mathrm{Ceil}\left(\max\left(MHy\right), 0.1\ m\right) = 3.3\ \frac{s}{m^2} \cdot m^3 \cdot s^{-1}$$

$$H1970 := \left(MHy^{\langle 0 \rangle}\right)^{T} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \end{bmatrix} m$$

$$H1971 := \left(MHy^{\langle 1 \rangle}\right)^{T} = \begin{bmatrix} 1.93 \\ 1.93 \\ 1.86 \\ \vdots \end{bmatrix} m$$

$$H1972 := \left(MHy^{\langle 2 \rangle}\right)^{T} = \begin{bmatrix} 2.09 \\ 2.07 \\ 2.04 \\ \vdots \end{bmatrix} m$$

$$H1973 := \left(MHy^{\langle 3 \rangle}\right)^{T} = \begin{bmatrix} 2.21 \\ 2.12 \\ 2.08 \\ \vdots \end{bmatrix} m$$

$$H1974 := \left(MHy^{\langle 4 \rangle}\right)^{T} = \begin{bmatrix} 1.02 \\ 1.41 \\ 1.71 \\ \vdots \end{bmatrix} m$$

$$H1975 := \left(MHy^{\langle 5 \rangle}\right)^{T} = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.09 \\ \vdots \end{bmatrix} m$$

$$H1976 := \left(MHy^{\langle 6 \rangle}\right)^{T} = \begin{bmatrix} 1.95 \\ 1.89 \\ 2.63 \\ \vdots \end{bmatrix} m$$

$$H1977 := \left(MHy^{\langle 7 \rangle}\right)^{T} = \begin{bmatrix} 2.08 \\ 2.02 \\ 2.15 \\ \vdots \end{bmatrix} m$$

$$H1978 := \left(MHy^{\langle 8 \rangle}\right)^{T} = \begin{bmatrix} 0.08 \\ 0.07 \\ 0.1 \\ \vdots \end{bmatrix} m$$

$$H1979 := \left(MHy^{\langle 9 \rangle}\right)^{T} = \begin{bmatrix} 0.81 \\ 0.13 \\ 0.13 \\ \vdots \end{bmatrix} m$$

$$H1980 := \left(MHy^{\langle 10 \rangle}\right)^{T} = \begin{bmatrix} 0.07 \\ 0.06 \\ 0.07 \\ \vdots \end{bmatrix} m$$

$$H1981 := \left(MHy^{\langle 11 \rangle}\right)^{T} = \begin{bmatrix} 0.14 \\ 0.27 \\ 0.29 \\ \vdots \end{bmatrix} m$$

$$H1982 := \left(MHy^{\langle 12 \rangle}\right)^{T} = \begin{bmatrix} 0.08 \\ 0.08 \\ 0.08 \\ \vdots \end{bmatrix} m$$

$$H1983 := \left(MHy^{\langle 13 \rangle}\right)^{T} = \begin{bmatrix} 0.09 \\ 0.09 \\ 0.11 \\ \vdots \end{bmatrix} m$$

$$H1984 := \left(MHy^{\langle 14 \rangle}\right)^{T} = \begin{bmatrix} 0.62 \\ 0.72 \\ 0.74 \\ \vdots \end{bmatrix} m$$

$$H1985 := \left(MHy^{\langle 15 \rangle}\right)^{T} = \begin{bmatrix} 1.87 \\ 1.83 \\ 2.08 \\ \vdots \end{bmatrix} m$$

$$H1986 := \left(MHy^{\langle 16 \rangle}\right)^{T} = \begin{bmatrix} 1.51 \\ 1.42 \\ 1.57 \\ \vdots \end{bmatrix} m$$

$$H1987 := \left(MHy^{\langle 17 \rangle}\right)^{T} = \begin{bmatrix} 0.08 \\ 0.08 \\ 0.05 \\ \vdots \end{bmatrix} m$$

$$H1988 := \left(MHy^{\langle 18 \rangle}\right)^{T} = \begin{bmatrix} 0.21 \\ 0.37 \\ 0.24 \\ \vdots \end{bmatrix} m$$

$$H1989 := \left(MHy^{\langle 19 \rangle}\right)^{T} = \begin{bmatrix} 0.05 \\ 0.05 \\ 0.04 \\ \vdots \end{bmatrix} m$$

$$H1990 := \left(MHy^{\langle 20\rangle}\right)^{T} = \begin{bmatrix}1.35\\1.63\\1.95\\\vdots\end{bmatrix} m$$

$$H1991 := \left(MHy^{\langle 21\rangle}\right)^{T} = \begin{bmatrix}0.07\\0.08\\0.08\\\vdots\end{bmatrix} m$$

$$H1992 := \left(MHy^{\langle 22\rangle}\right)^{T} = \begin{bmatrix}0.06\\0.17\\0.84\\\vdots\end{bmatrix} m$$

$$H1993 := \left(MHy^{\langle 23\rangle}\right)^{T} = \begin{bmatrix}1.72\\1.81\\2.1\\\vdots\end{bmatrix} m$$

$$H1994 := \left(MHy^{\langle 24\rangle}\right)^{T} = \begin{bmatrix}0.11\\0.08\\0.07\\\vdots\end{bmatrix} m$$

$$H1995 := \left(MHy^{\langle 25\rangle}\right)^{T} = \begin{bmatrix}0.09\\0.08\\0.08\\\vdots\end{bmatrix} m$$

$$H1996 := \left(MHy^{\langle 26\rangle}\right)^{T} = \begin{bmatrix}0.08\\0.08\\0.07\\\vdots\end{bmatrix} m$$

$$H1997 := \left(MHy^{\langle 27\rangle}\right)^{T} = \begin{bmatrix}0.93\\0.68\\0.07\\\vdots\end{bmatrix} m$$

$$H1998 := \left(MHy^{\langle 28\rangle}\right)^{T} = \begin{bmatrix}0.08\\0.08\\0.07\\\vdots\end{bmatrix} m$$

$$H1999 := \left(MHy^{\langle 29\rangle}\right)^{T} = \begin{bmatrix}0.27\\0.53\\0.56\\\vdots\end{bmatrix} m$$

$$H2000 := \left(MHy^{\langle 30\rangle}\right)^{T} = \begin{bmatrix}0.09\\0.09\\0.07\\\vdots\end{bmatrix} m$$

$$H2001 := \left(MHy^{\langle 31\rangle}\right)^{T} = \begin{bmatrix}0.78\\1\\1.06\\\vdots\end{bmatrix} m$$

$$H2002 := \left(MHy^{\langle 32\rangle}\right)^{T} = \begin{bmatrix}0.14\\0.04\\0.05\\\vdots\end{bmatrix} m$$

$$H2003 := \left(MHy^{\langle 33\rangle}\right)^{T} = \begin{bmatrix}0.04\\0.05\\0.05\\\vdots\end{bmatrix} m$$

$$H2004 := \left(MHy^{\langle 34\rangle}\right)^{T} = \begin{bmatrix}2.04\\1.89\\1.74\\\vdots\end{bmatrix} m$$

$$H2005 := \left(MHy^{\langle 35\rangle}\right)^{T} = \begin{bmatrix}1.22\\1.3\\1.29\\\vdots\end{bmatrix} m$$

$$H2006 := \left(MHy^{\langle 36\rangle}\right)^{T} = \begin{bmatrix}1.61\\1.67\\2.07\\\vdots\end{bmatrix} m$$

$$H2007 := \left(MHy^{\langle 37\rangle}\right)^{T} = \begin{bmatrix}1.1\\1.24\\1.4\\\vdots\end{bmatrix} m$$

$$H2008 := \left(MHy^{\langle 38\rangle}\right)^{T} = \begin{bmatrix}2.3\\2.31\\2.25\\\vdots\end{bmatrix} m$$

$$H2009 := \left(MHy^{\langle 39\rangle}\right)^{T} = \begin{bmatrix}2.28\\2.23\\2.08\\\vdots\end{bmatrix} m$$

$$H2010 := \left(MHy^{\langle 40\rangle}\right)^{T} = \begin{bmatrix}0.1\\0.1\\0.1\\\vdots\end{bmatrix} m$$

$$H2011 := \left(MHy^{\langle 41\rangle}\right)^{T} = \begin{bmatrix}0.06\\0.16\\0.42\\\vdots\end{bmatrix} m$$

$$H2012 := \left(MHy^{\langle 42\rangle}\right)^{T} = \begin{bmatrix}0.12\\0.23\\0.3\\\vdots\end{bmatrix} m$$

$$H2013 := \left(MHy^{\langle 43\rangle}\right)^{T} = \begin{bmatrix}0.04\\0.08\\0.09\\\vdots\end{bmatrix} m$$

$$H2014 := \left(MHy^{\langle 44\rangle}\right)^{T} = \begin{bmatrix}0.07\\0.07\\0.06\\\vdots\end{bmatrix} m$$

$$H2015 := \left(MHy^{\langle 45\rangle}\right)^{T} = \begin{bmatrix}1.35\\1.54\\1.41\\\vdots\end{bmatrix} m$$

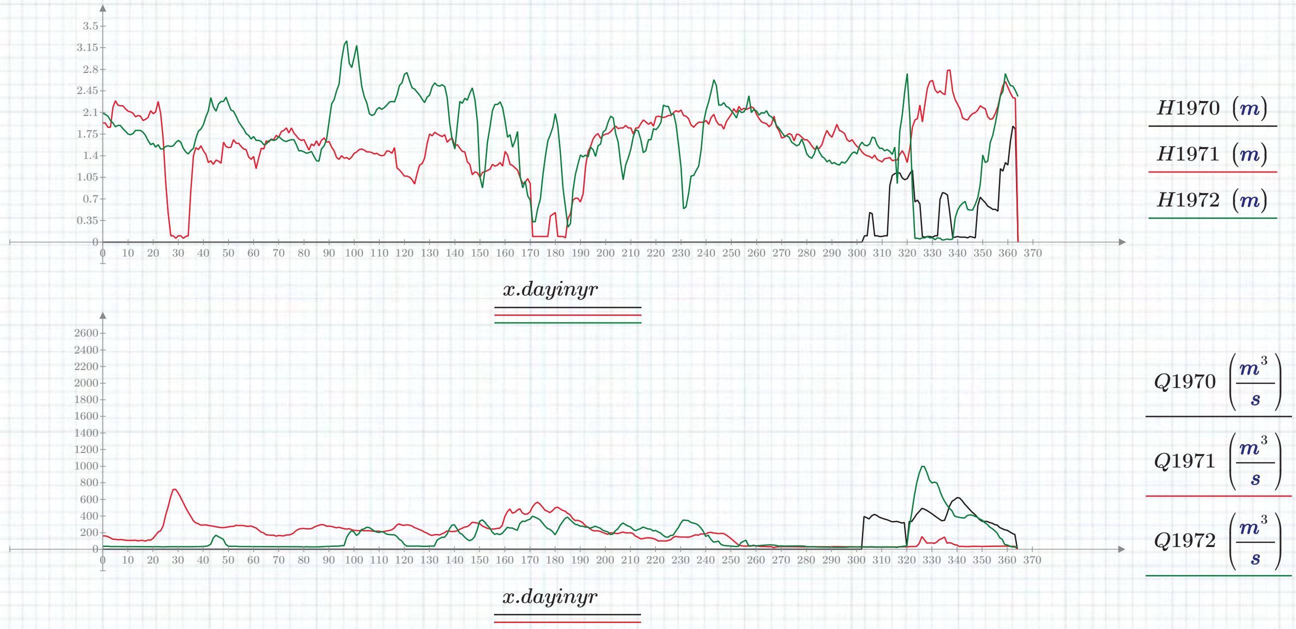
$$H2016 := \left(MHy^{\langle 46\rangle}\right)^{T} = \begin{bmatrix}1.31\\1.27\\1.26\\\vdots\end{bmatrix} m$$

$$H2017 := \left(MHy^{\langle 47\rangle}\right)^{T} = \begin{bmatrix}0.68\\0.65\\0.62\\\vdots\end{bmatrix} m$$

$$H2018 := \left(MHy^{\langle 48\rangle}\right)^{T} = \begin{bmatrix}0.06\\0.07\\0.07\\\vdots\end{bmatrix} m$$

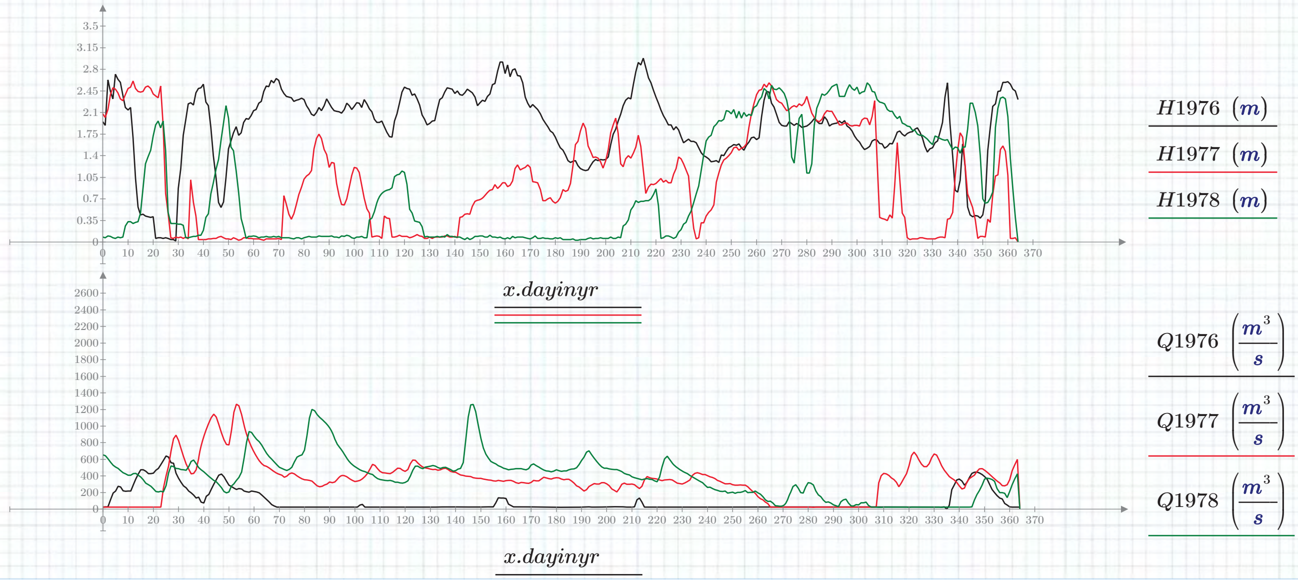
Plots of yearly data:

1970-1972

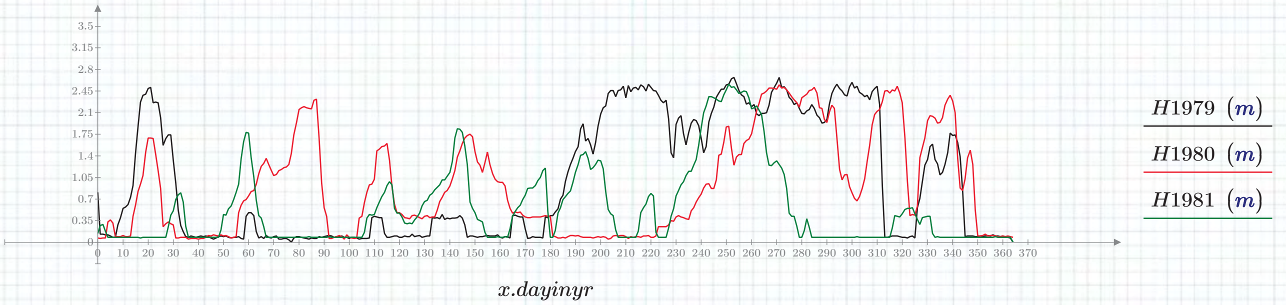


1973-1975

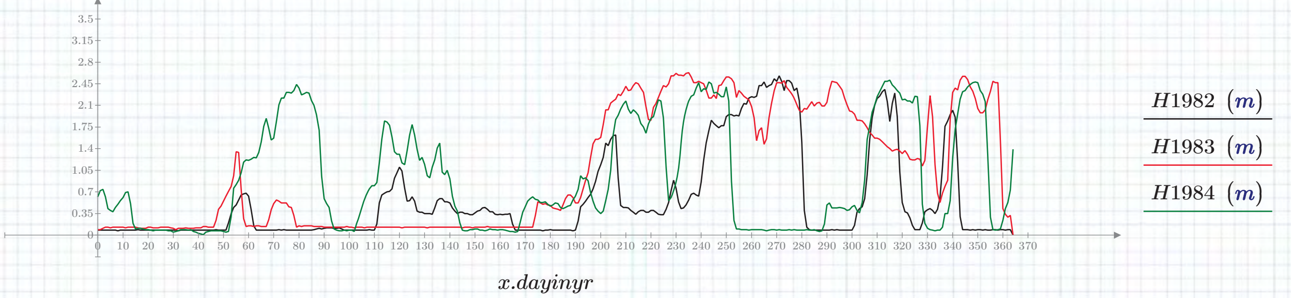
1976-1978

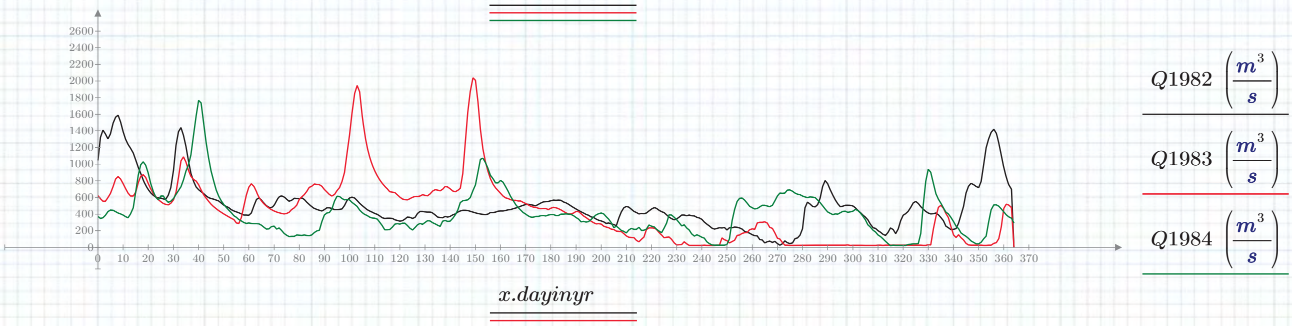


1979-1981

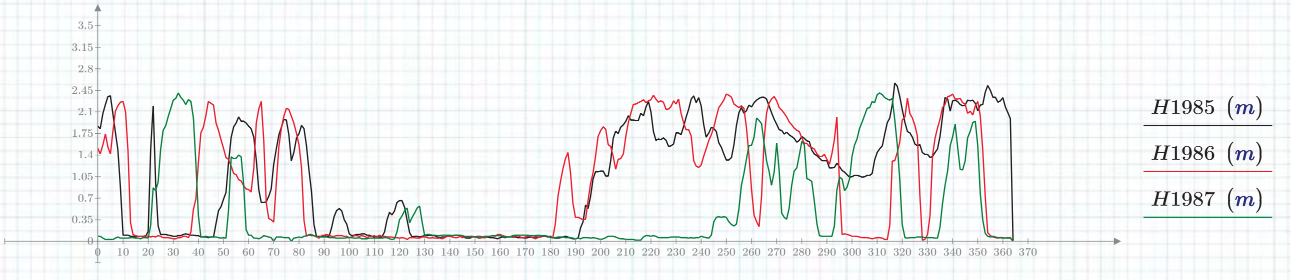


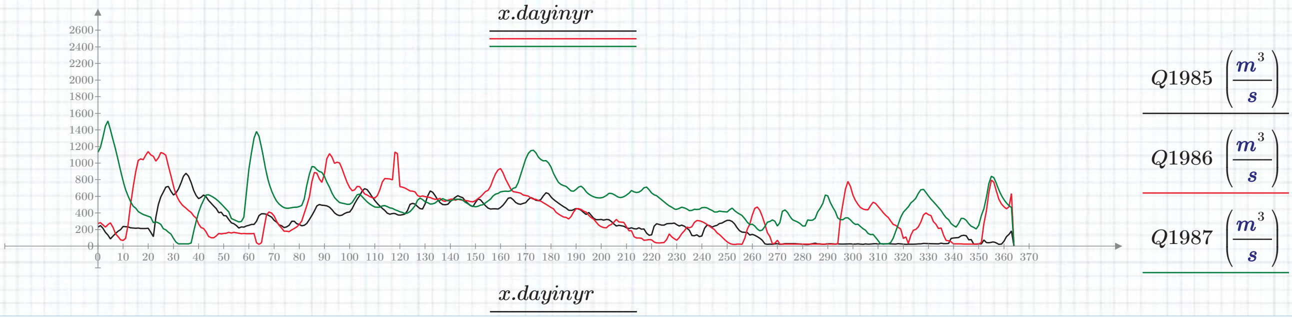
1982-1984



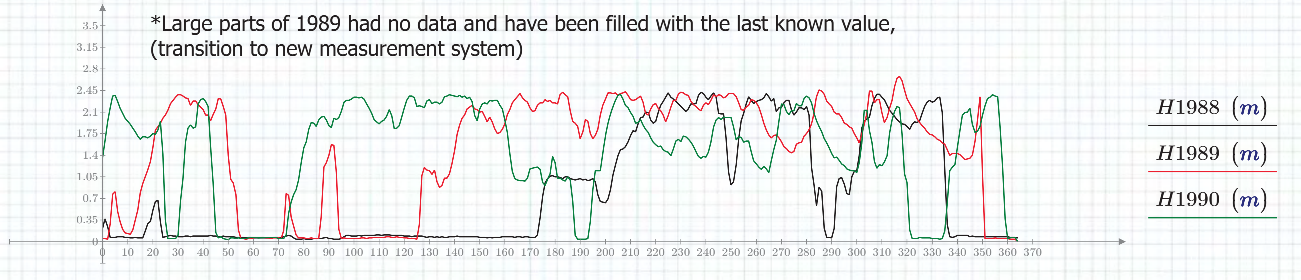


1985-1987

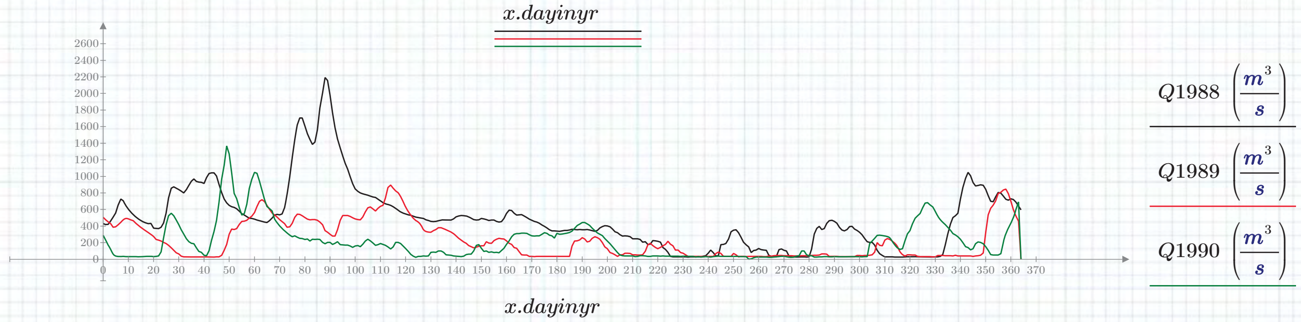




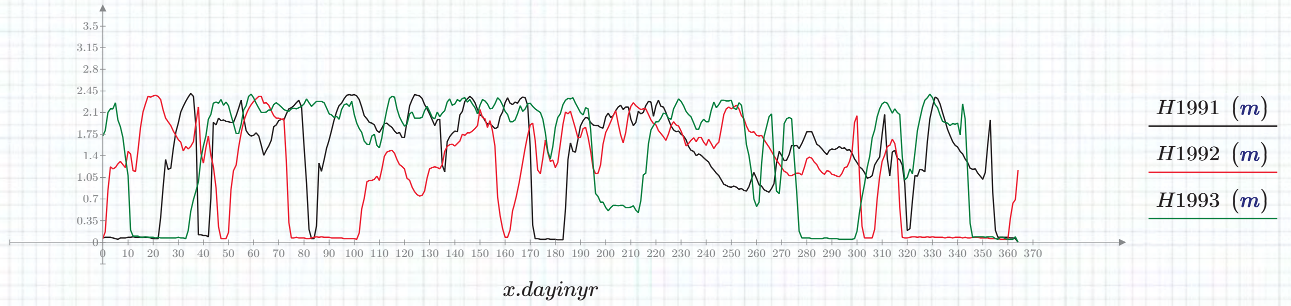
1988-1990

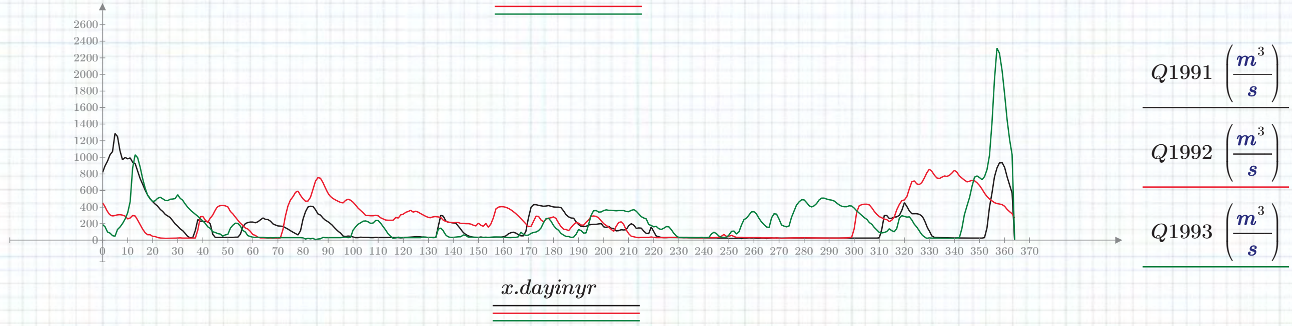


*Large parts of 1989 had no data and have been filled with the last known value, (transition to new measurement system)

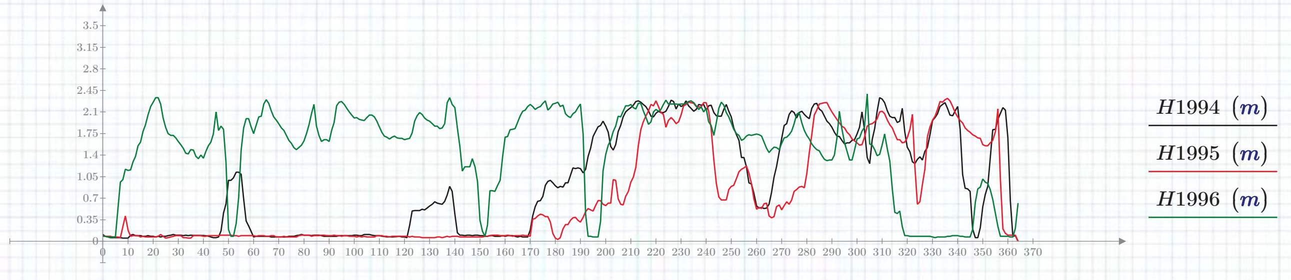


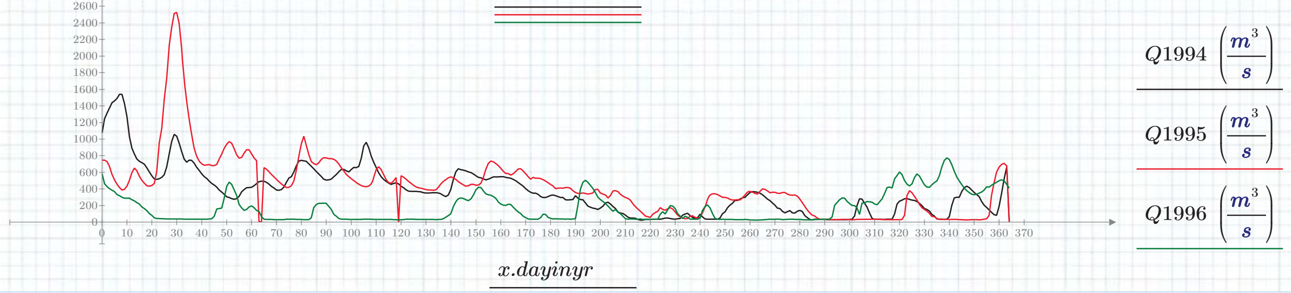
1991-1993



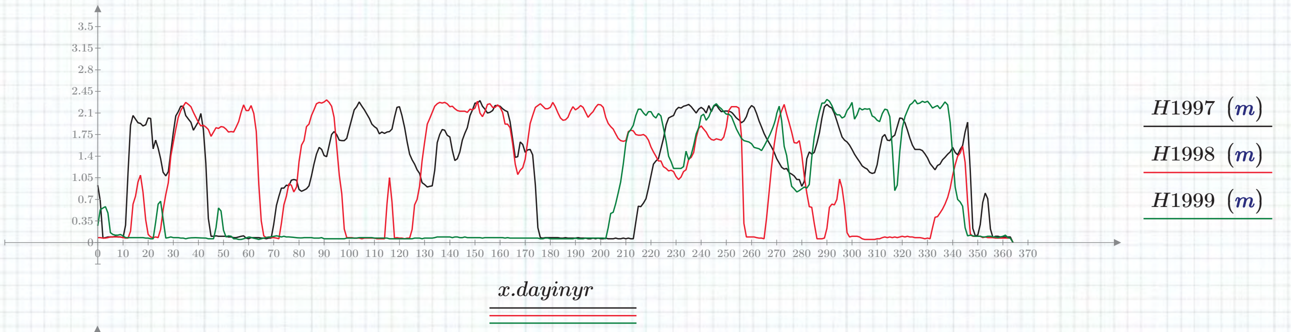


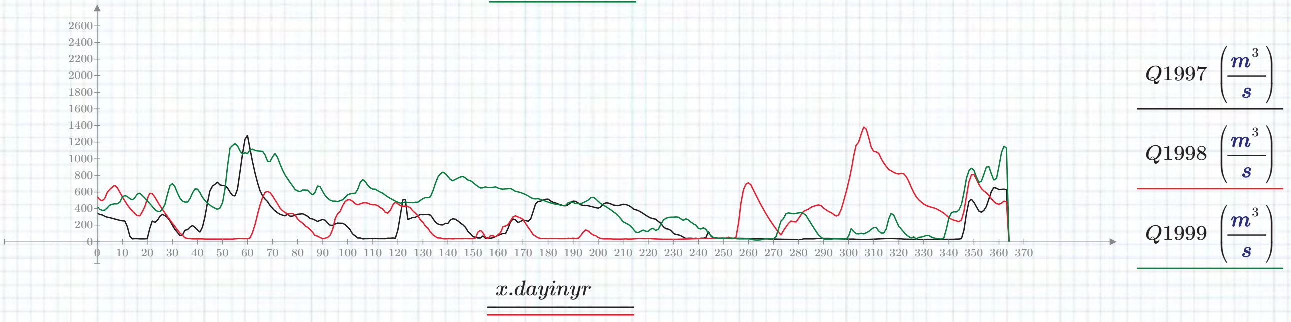
1994-1996



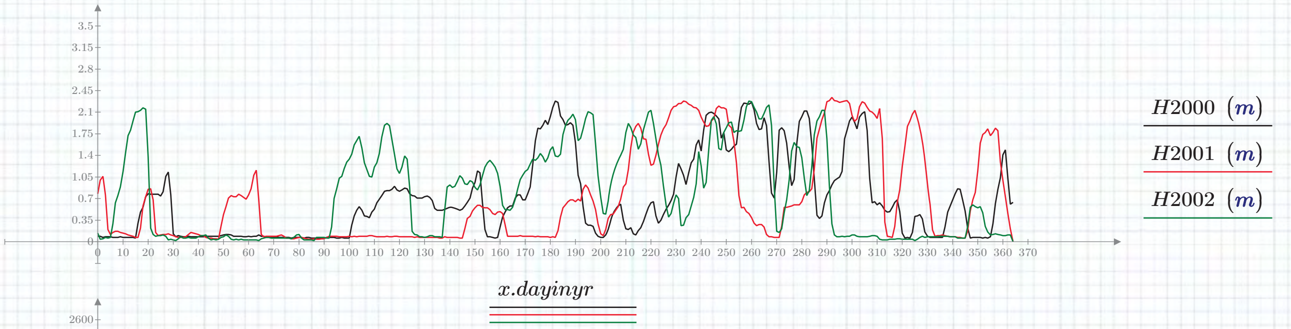


1997-1999

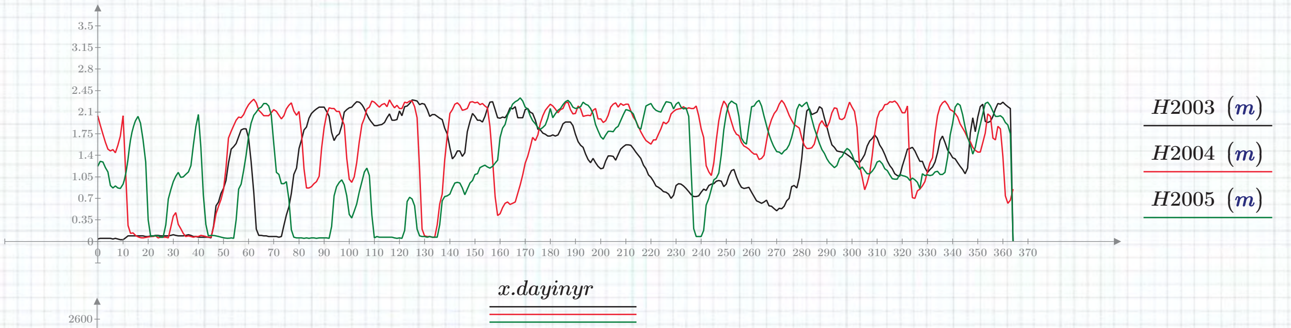


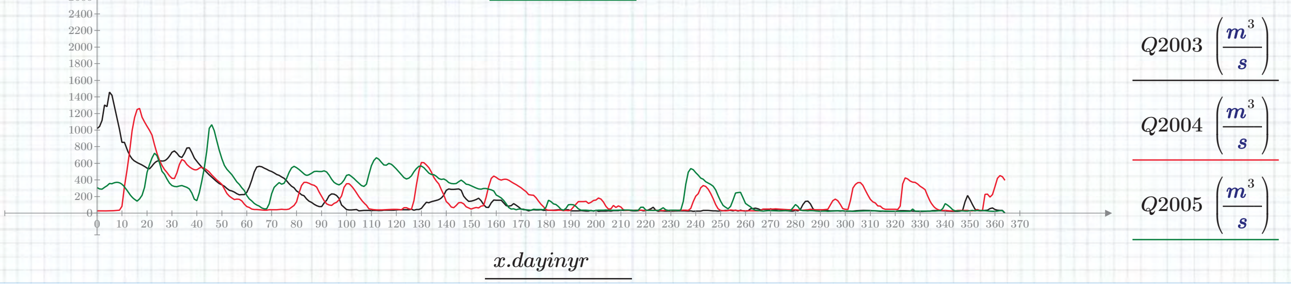


2000-2002

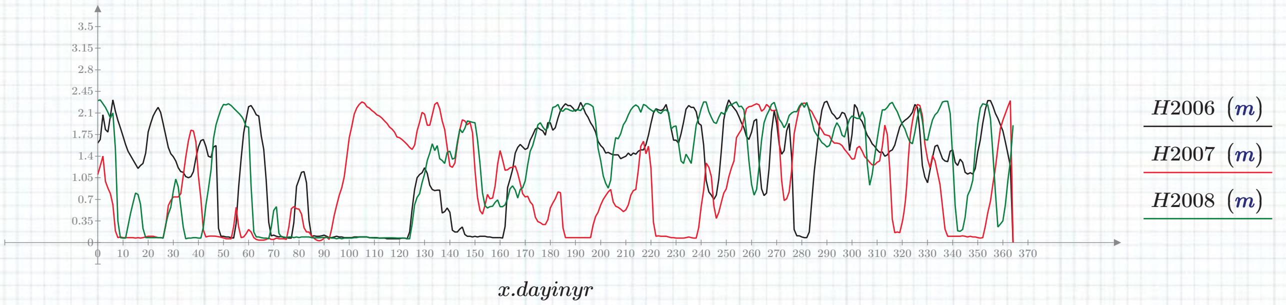


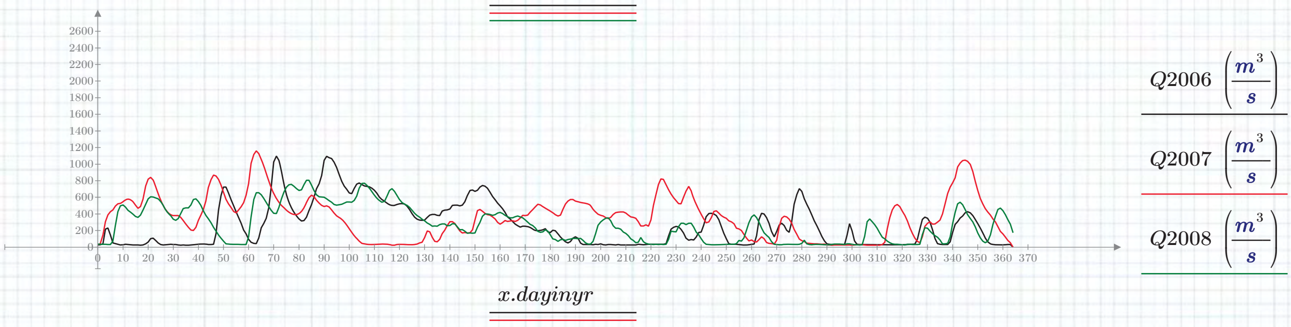
2003-2005



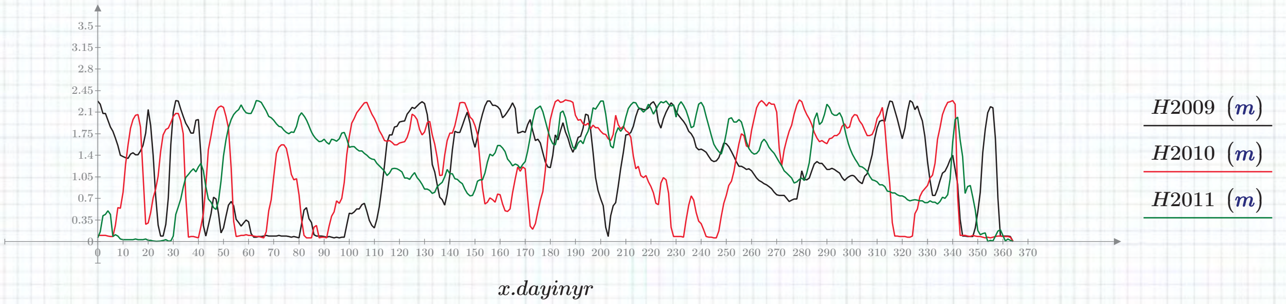


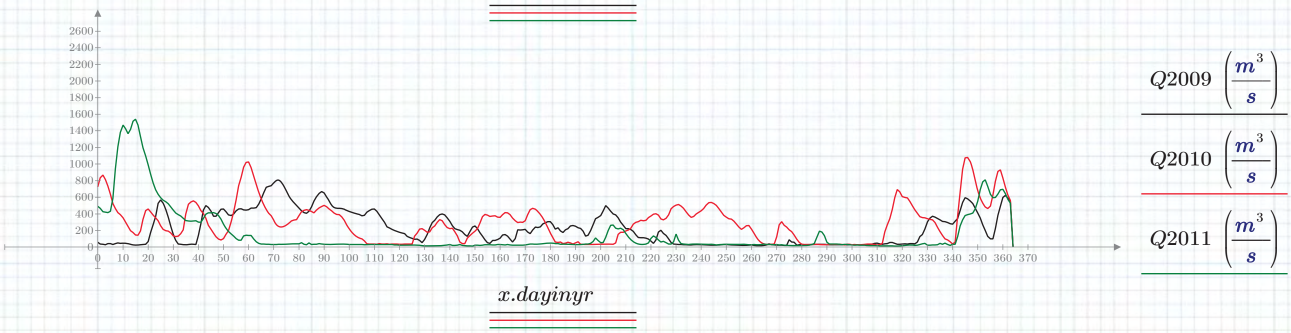
2006-2008



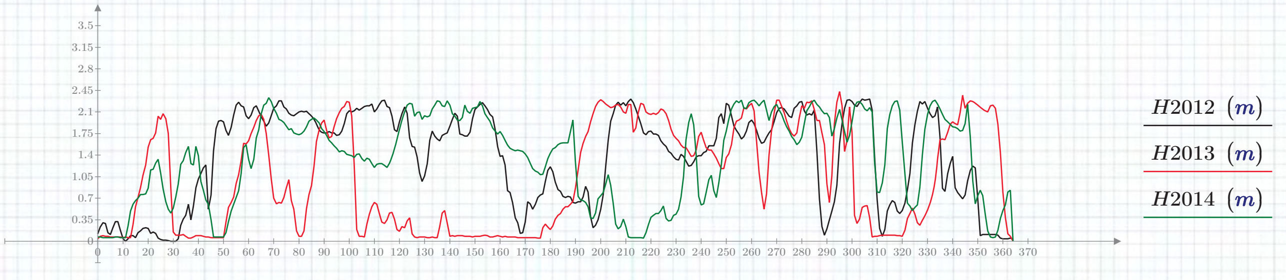


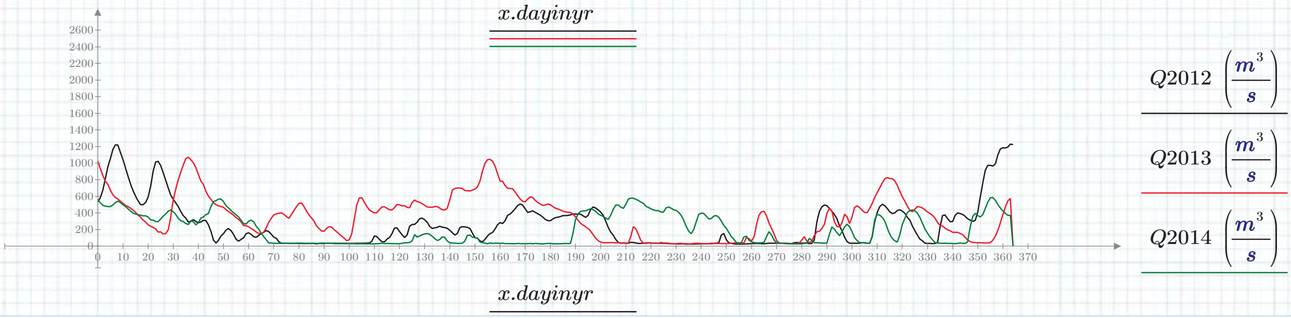
2009-2011





2012-2014

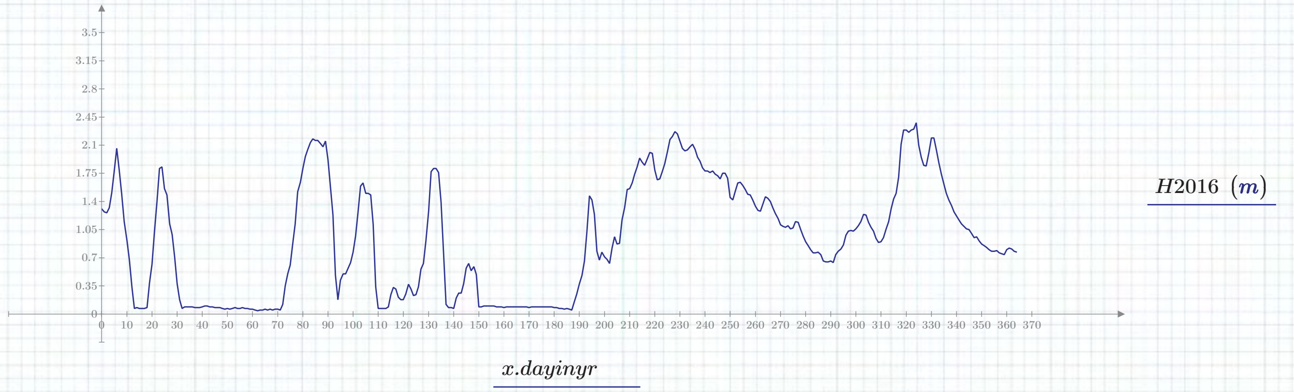


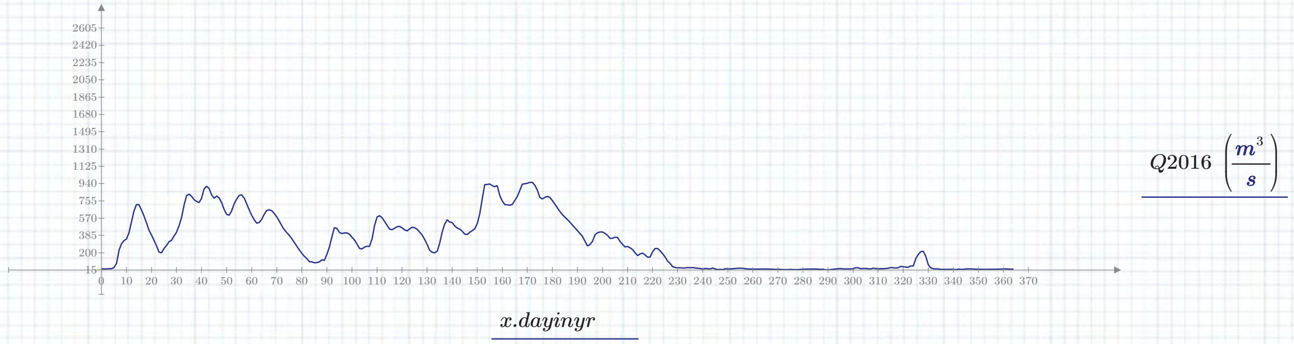


2015-2018

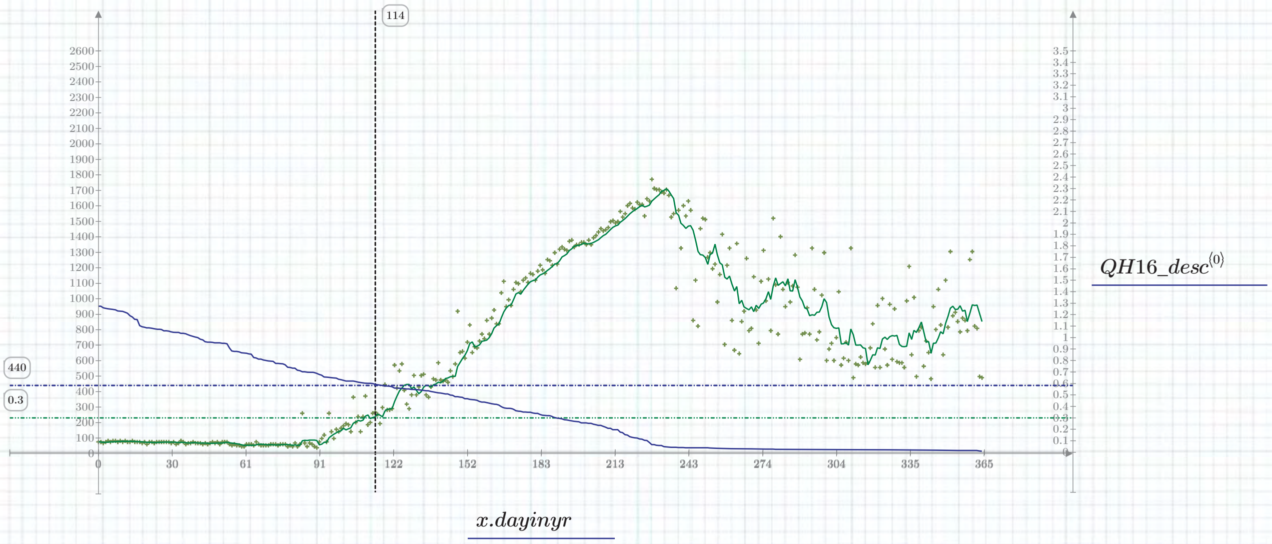
$Output5 := \text{WRITEEXCEL}\left(\text{concat}\left(\mathbf{CWD}, M2E.output\right), \frac{H2016}{m}, \text{“Q and dH 2016!A1:A365”}\right)$

$Output6 := \text{WRITEEXCEL}\left(\text{concat}\left(\mathbf{CWD}, M2E.output\right), \frac{Q2016}{m^{3} \cdot s^{-1}}, \text{“Q and dH 2016!B1:B365”}\right)$





$QH16\_desc := \mathrm{reverse}\left(\mathrm{csort}\left(\left(\mathrm{augment}\left(Q2016 \cdot s \cdot m^{-3}, H2016 \cdot m^{-1}\right)\right), 0\right)\right)$



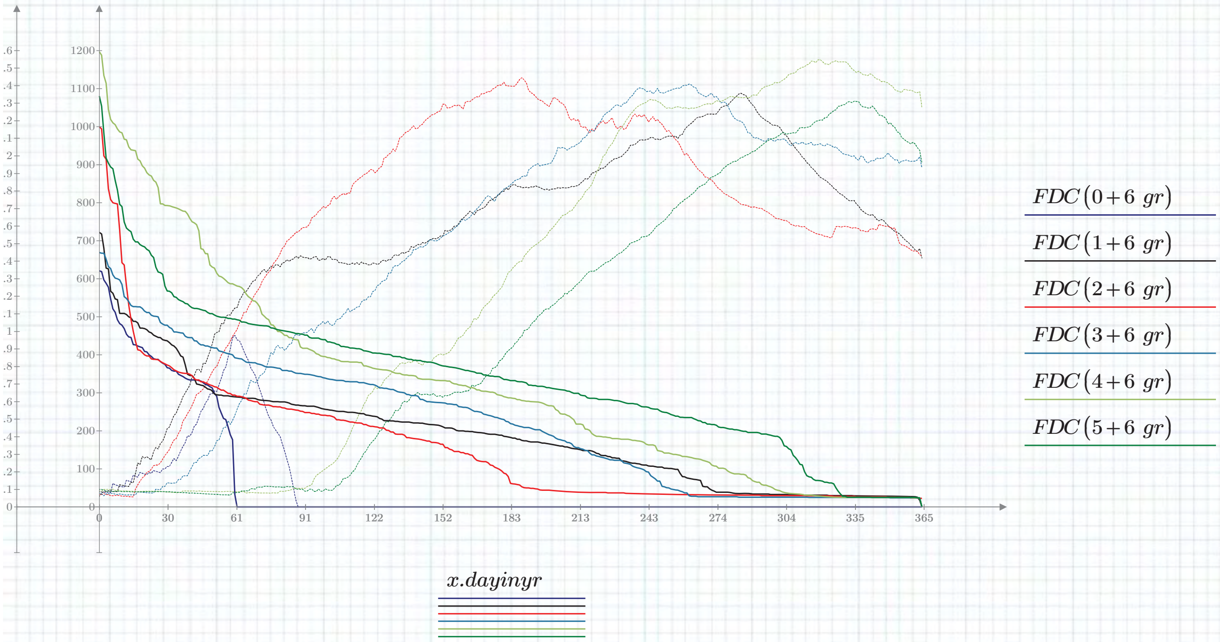
- Page intentionally left blank -

$FDC(choice) := \text{reverse}\left(\text{csort}\left(\left((MQy)^{\overset{\frown}{\underset{\smile}{choice}}}\right)^{T} \cdot m^{-3} \cdot s, 0\right)\right)$ $MQy^{\overset{\frown}{\underset{\smile}{35}}}$

$HFDC(choice) := \text{movavg}\left(\text{reverse}\left(\text{csort}\left(\left(\text{augment}\left(\left((MQy)^{\overset{\frown}{\underset{\smile}{choice}}}\right)^{T} \cdot m^{-3} \cdot s, \left((MHy)^{\overset{\frown}{\underset{\smile}{choice}}}\right)^{T} \cdot m^{-1}\right)\right), 0\right)\right)^{\langle 1\rangle}, 28\right)$

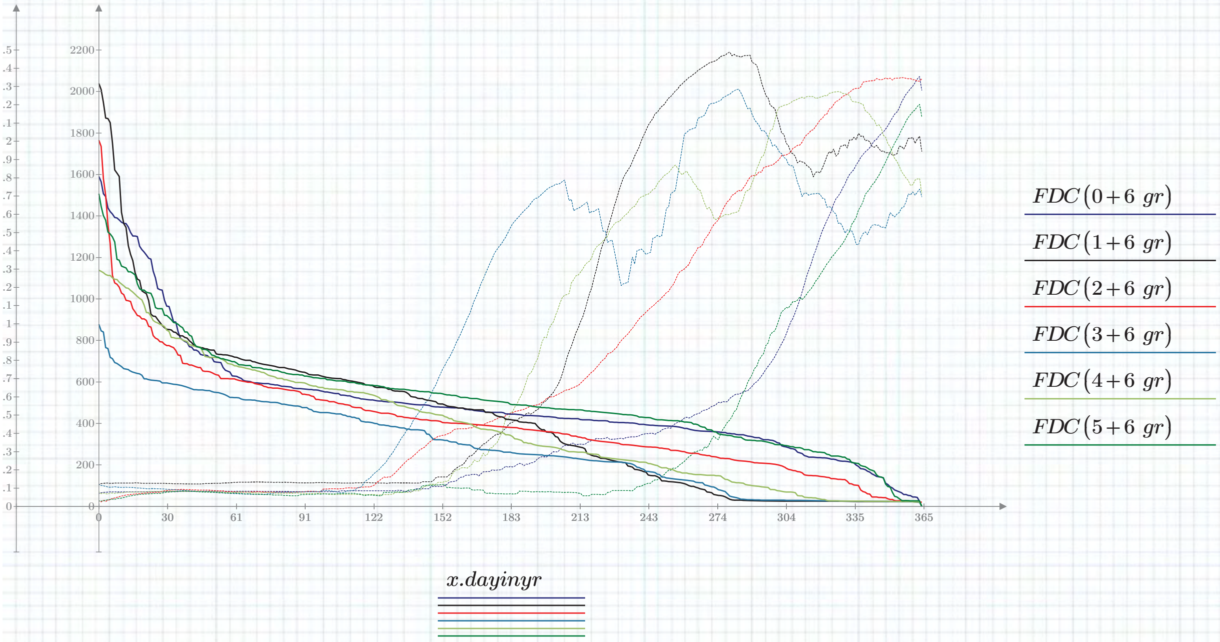
$gr := 0$

$choice := 0..5$



$gr := 1$

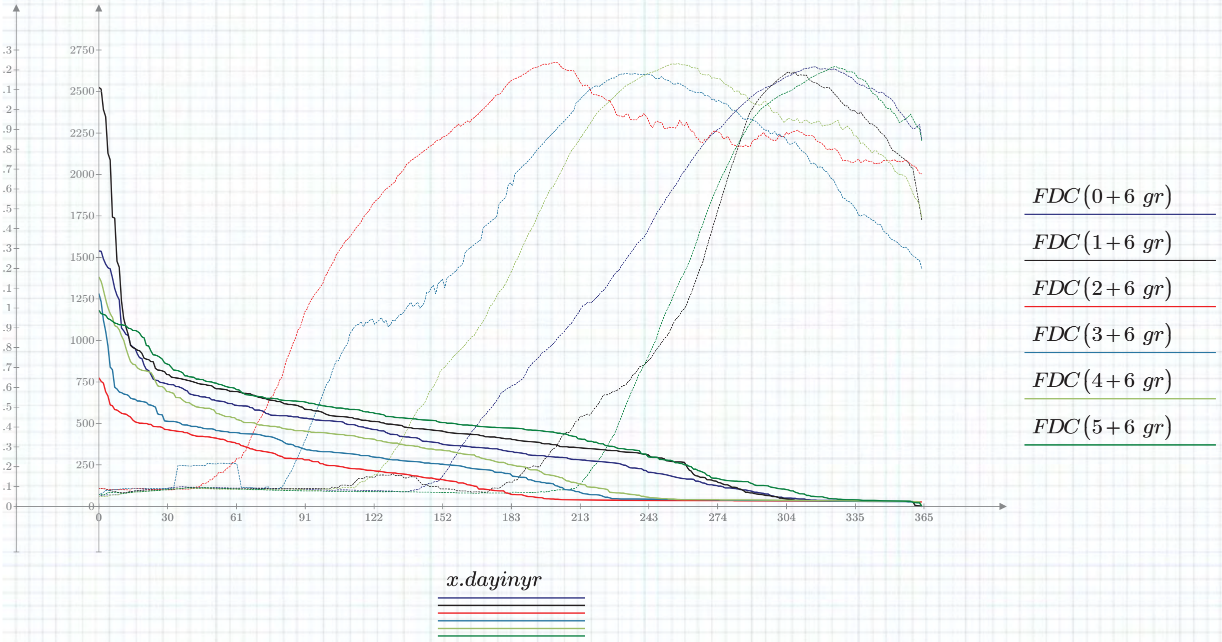
$gr := 2$


2200
2000
1800
1600
1400
1200
1000
800
600
400
200
0
0
30
61
91
122
152
183
213
243
274
304
335
365
x.dayinyr
FDC(0+6 gr)
FDC(1+6 gr)
FDC(2+6 gr)
FDC(3+6 gr)
FDC(4+6 gr)
FDC(5+6 gr)

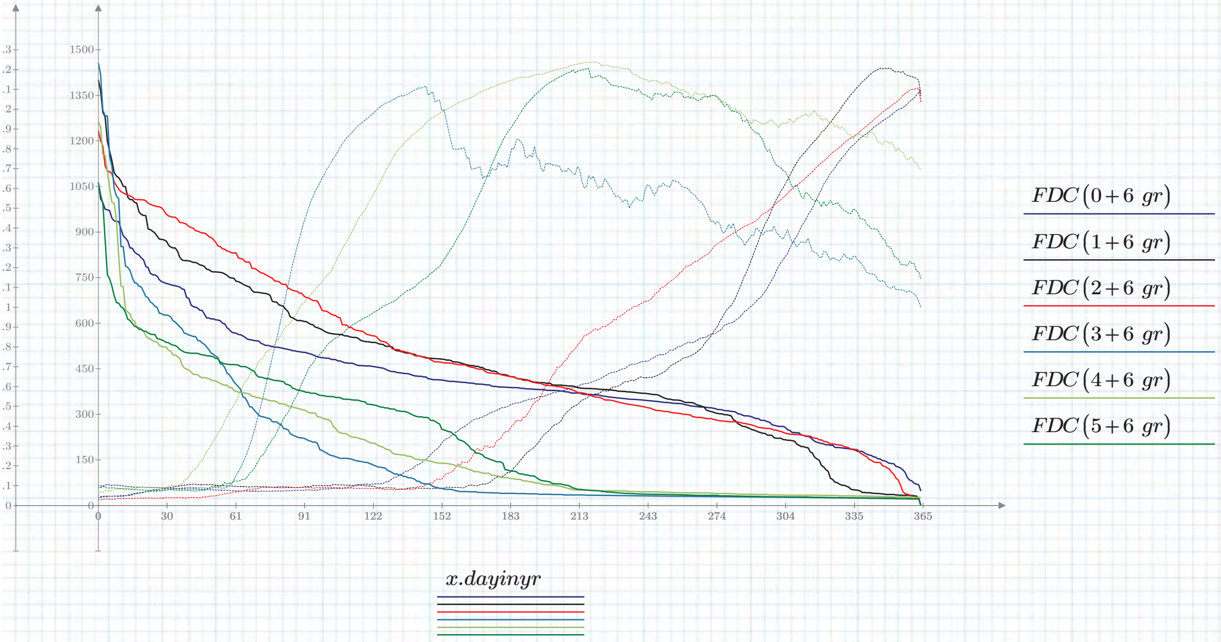
$gr := 3$


2500
2250
2000
1750
1500
1250
1000
750
500
250
0
0
30
61
91
122
152
183
213
243
274
304
335
365
x.dayinyr
FDC(0+6 gr)
FDC(1+6 gr)
FDC(2+6 gr)
FDC(3+6 gr)
FDC(4+6 gr)
FDC(5+6 gr)

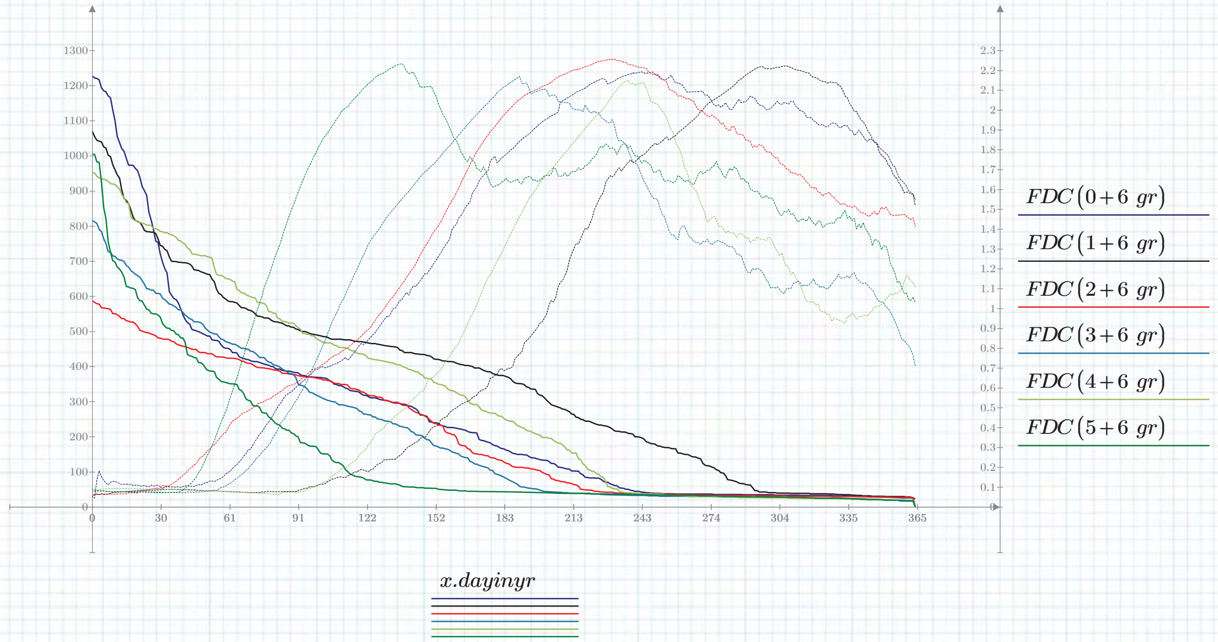
$gr := 4$


2750
2500
2250
2000
1750
1500
1250
1000
750
500
250
0
0
30
61
91
122
152
183
213
243
274
304
335
365
FDC(0+6 gr)
FDC(1+6 gr)
FDC(2+6 gr)
FDC(3+6 gr)
FDC(4+6 gr)
FDC(5+6 gr)
x.dayinyr

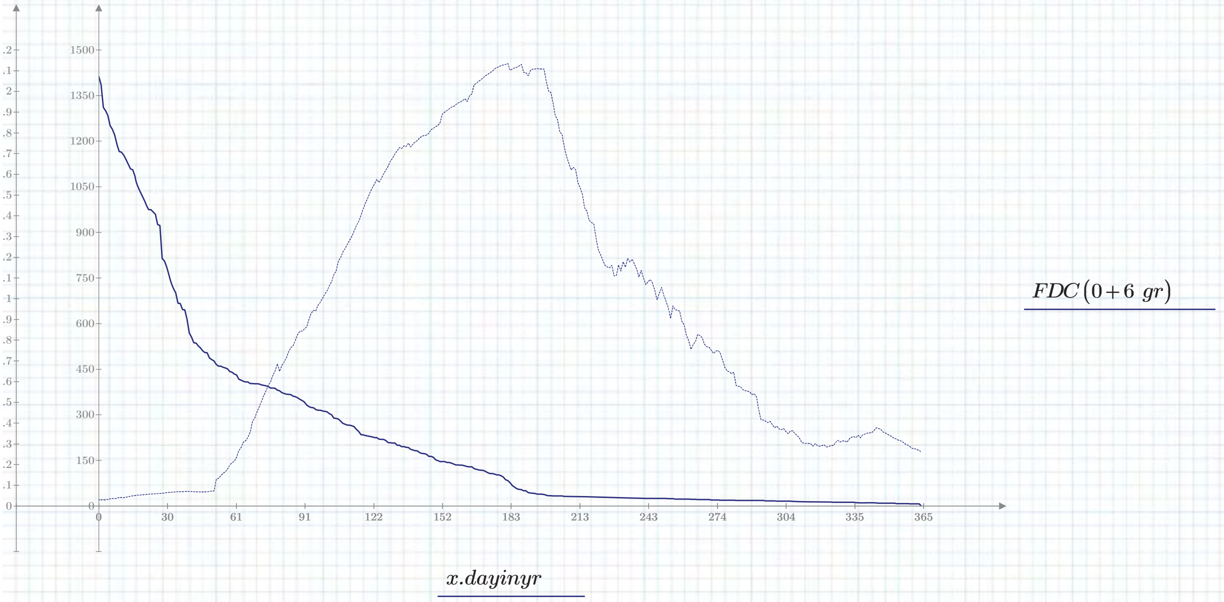
$gr := 5$

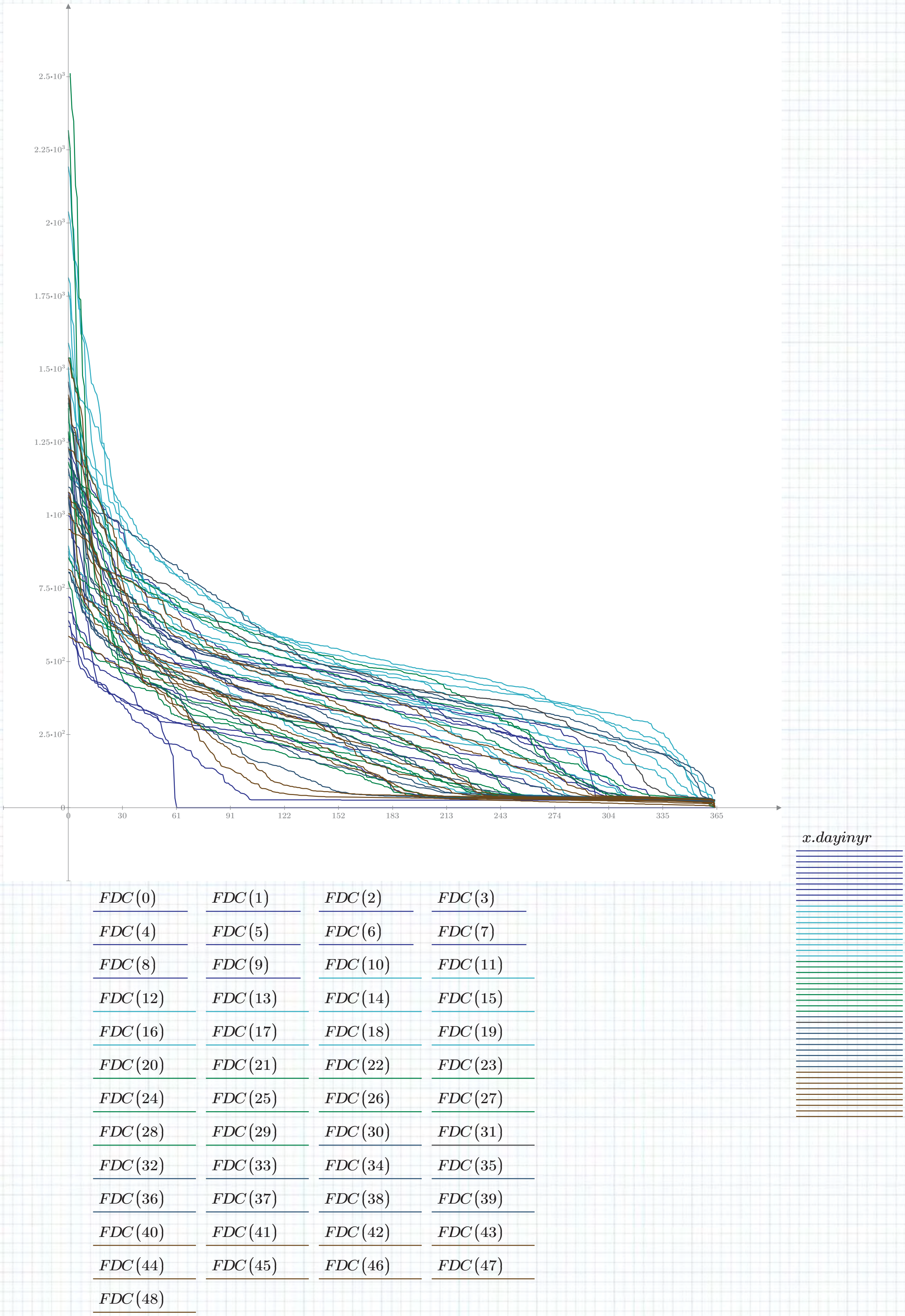

1500
1350
1200
1050
900
750
600
450
300
150
0
0
30
61
91
122
152
183
213
243
274
304
335
365
FDC(0+6 gr)
FDC(1+6 gr)
FDC(2+6 gr)
FDC(3+6 gr)
FDC(4+6 gr)
FDC(5+6 gr)
x.dayinyr

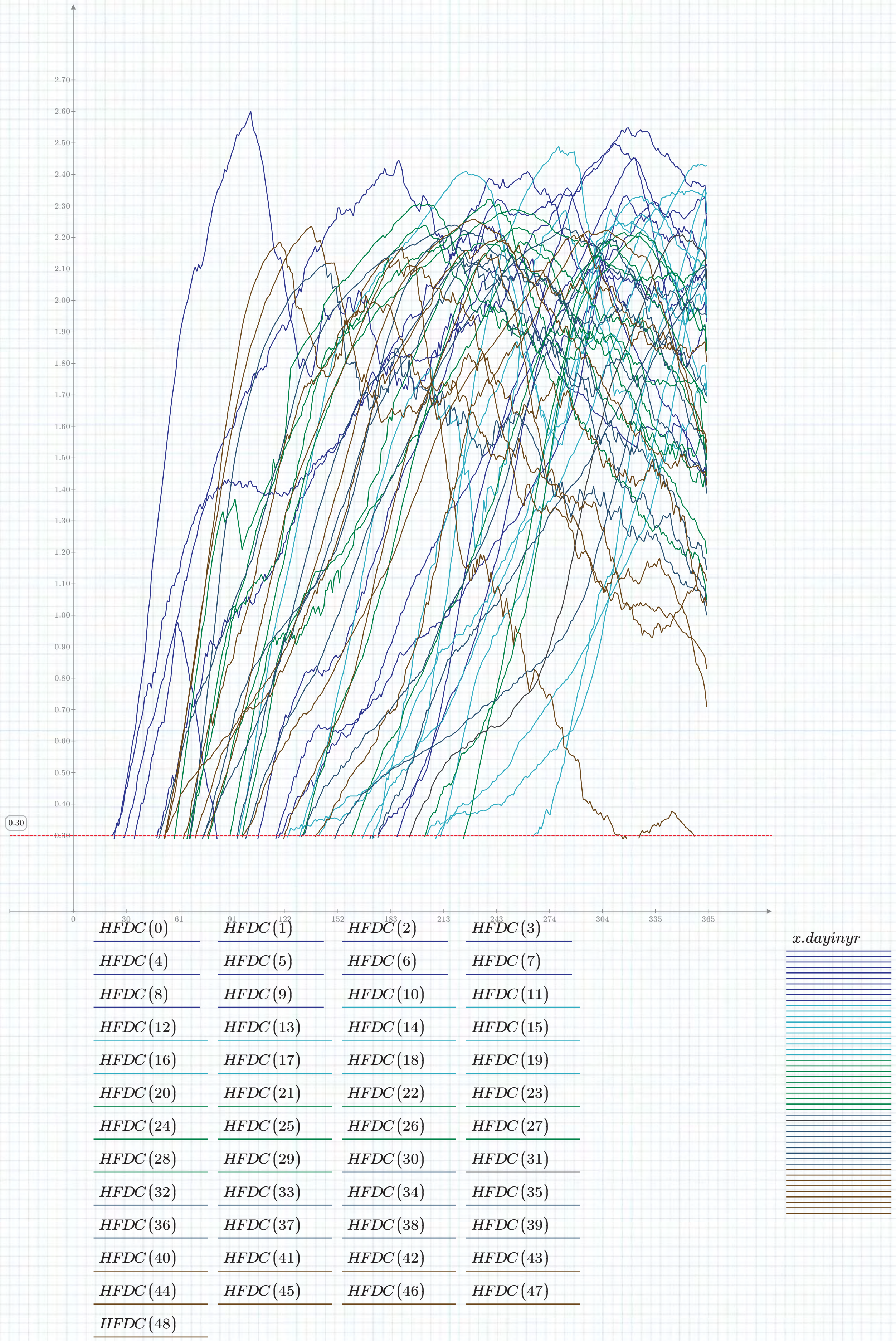
$gr := 7$

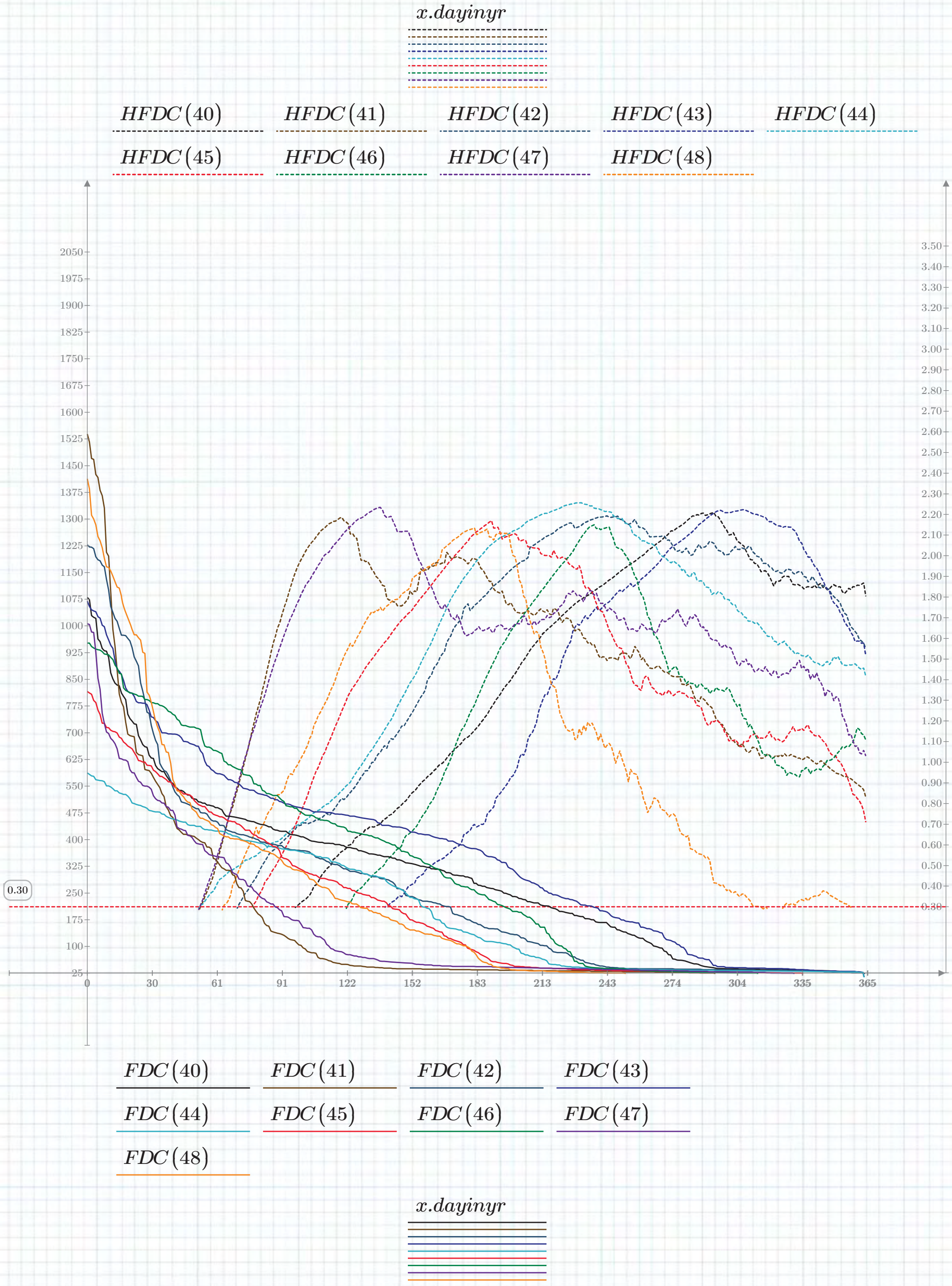

1300
1200
1100
1000
900
800
700
600
500
400
300
200
100
0
2.3
2.2
2.1
2
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
30
61
91
122
152
183
213
243
274
304
335
365
FDC(0+6 gr)
FDC(1+6 gr)
FDC(2+6 gr)
FDC(3+6 gr)
FDC(4+6 gr)
FDC(5+6 gr)
x.dayinyr

$gr := 8$


1500
1350
1200
1050
900
750
600
450
300
150
0
0
30
61
91
122
152
183
213
243
274
304
335
365
FDC(0+6 gr)
x.dayinyr


2.5·10³
2.25·10³
2·10³
1.75·10³
1.5·10³
1.25·10³
1·10³
7.5·10²
5·10²
2.5·10²
0
0
30
61
91
122
152
183
213
243
274
304
335
365
x.dayinyr
FDC(0)
FDC(1)
FDC(2)
FDC(3)
FDC(4)
FDC(5)
FDC(6)
FDC(7)
FDC(8)
FDC(9)
FDC(10)
FDC(11)
FDC(12)
FDC(13)
FDC(14)
FDC(15)
FDC(16)
FDC(17)
FDC(18)
FDC(19)
FDC(20)
FDC(21)
FDC(22)
FDC(23)
FDC(24)
FDC(25)
FDC(26)
FDC(27)
FDC(28)
FDC(29)
FDC(30)
FDC(31)
FDC(32)
FDC(33)
FDC(34)
FDC(35)
FDC(36)
FDC(37)
FDC(38)
FDC(39)
FDC(40)
FDC(41)
FDC(42)
FDC(43)
FDC(44)
FDC(45)
FDC(46)
FDC(47)
FDC(48)

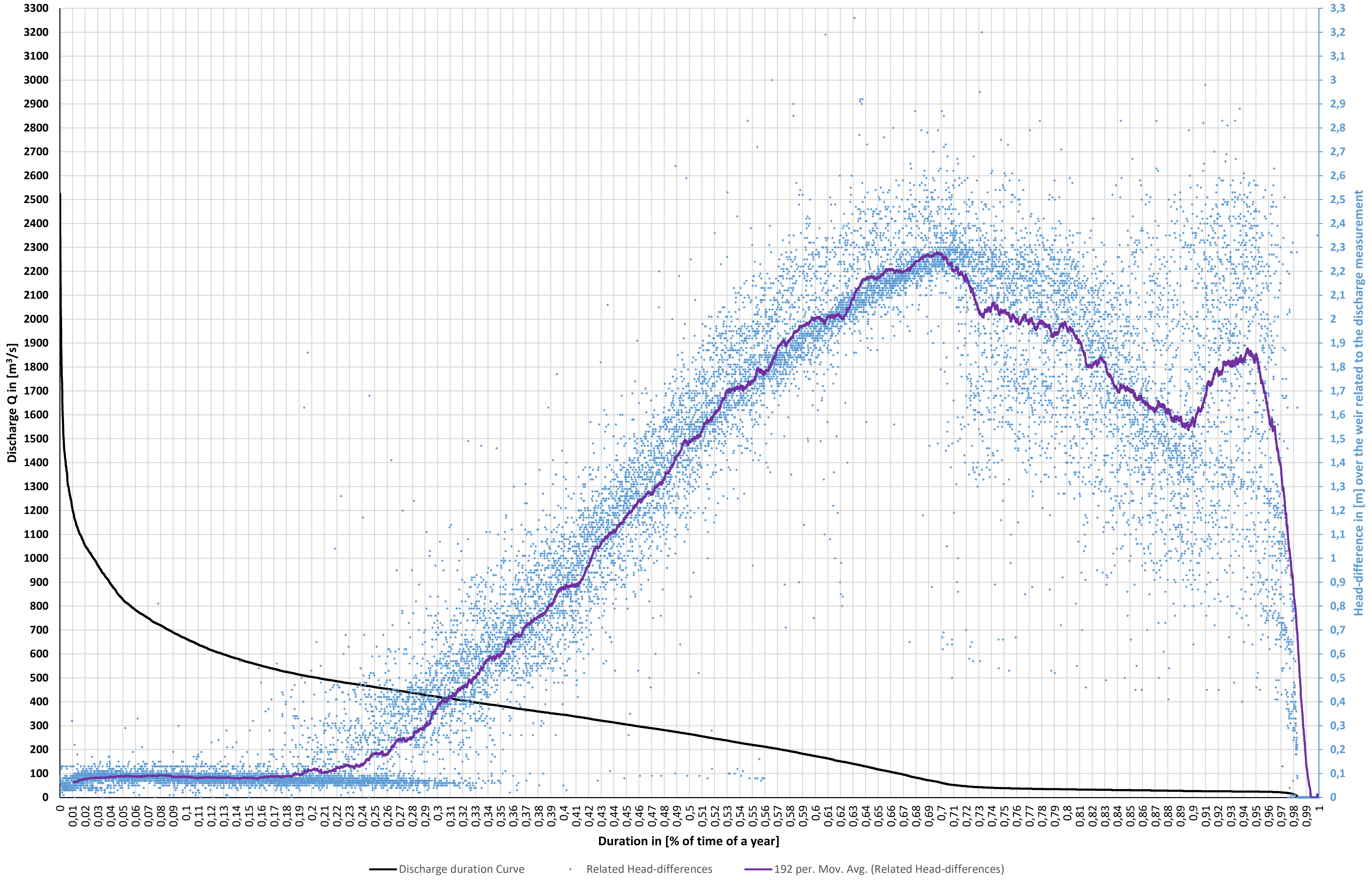

2.70
2.60
2.50
2.40
2.30
2.20
2.10
2.00
1.90
1.80
1.70
1.60
1.50
1.40
1.30
1.20
1.10
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.30
0
30
61
91
122
152
183
213
243
274
304
335
365
$HFDC(0)$
$HFDC(1)$
$HFDC(2)$
$HFDC(3)$
$HFDC(4)$
$HFDC(5)$
$HFDC(6)$
$HFDC(7)$
$HFDC(8)$
$HFDC(9)$
$HFDC(10)$
$HFDC(11)$
$HFDC(12)$
$HFDC(13)$
$HFDC(14)$
$HFDC(15)$
$HFDC(16)$
$HFDC(17)$
$HFDC(18)$
$HFDC(19)$
$HFDC(20)$
$HFDC(21)$
$HFDC(22)$
$HFDC(23)$
$HFDC(24)$
$HFDC(25)$
$HFDC(26)$
$HFDC(27)$
$HFDC(28)$
$HFDC(29)$
$HFDC(30)$
$HFDC(31)$
$HFDC(32)$
$HFDC(33)$
$HFDC(34)$
$HFDC(35)$
$HFDC(36)$
$HFDC(37)$
$HFDC(38)$
$HFDC(39)$
$HFDC(40)$
$HFDC(41)$
$HFDC(42)$
$HFDC(43)$
$HFDC(44)$
$HFDC(45)$
$HFDC(46)$
$HFDC(47)$
$HFDC(48)$
$x.dayinyr$


$x.dayinyr$
$HFDC(40)$
$HFDC(41)$
$HFDC(42)$
$HFDC(43)$
$HFDC(44)$
$HFDC(45)$
$HFDC(46)$
$HFDC(47)$
$HFDC(48)$
2050
1975
1900
1825
1750
1675
1600
1525
1450
1375
1300
1225
1150
1075
1000
925
850
775
700
625
550
475
400
325
250
175
100
25
3.50
3.40
3.30
3.20
3.10
3.00
2.90
2.80
2.70
2.60
2.50
2.40
2.30
2.20
2.10
2.00
1.90
1.80
1.70
1.60
1.50
1.40
1.30
1.20
1.10
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.30
0
30
61
91
122
152
183
213
243
274
304
335
365
$FDC(40)$
$FDC(41)$
$FDC(42)$
$FDC(43)$
$FDC(44)$
$FDC(45)$
$FDC(46)$
$FDC(47)$
$FDC(48)$
$x.dayinyr$

APPENDIX 8 – A3 DETAILED FLOW DURATION CURVE

– see inserted page(s) behind this page –

Discharge Duration Curve & related head-differences



APPENDIX 9 – NET PRESENT VALUE, LCOE AND IRR EXPLANATION

Both LCOE and IRR use the Net Present Value (NPV) equation:

$$NPV = \sum_{t=0}^{N} \frac{C_t}{(1+r)^t} \qquad (33)$$

Where:

C_t is the net cash inflow/outflow during a single period t

r is the discount rate or return rate that could be earned in alternative investments

t is the number of time periods in each step (usually years)

N is the total number of time periods

With the NPV in effect one is calculating what each cash flow (negative or positive) would be worth today (hence the term "Present Value"), where the discount rate is the power-factor to convert future cashflows to today's value, as future money is worth less than "today's money" due to effects like inflation. If the investment is done today it will have "today's value", but the money you earn next year will be transformed (reduced) by the return rate.

The comparison is made with an alternative investment that has a return-rate of "r". Imagine for instance an invest in the stock-market where the amount of money would increase at this rate or if it were put in a savings-account on a bank with that interest rate compared to this project. If the investment for the project has a higher return-rate than what it is compared with, then the project is the more profitable choice.

Summing all the cash-flows converted to Present Value gives the net result of the entire "endeavour", i.e. the NPV. For a hydro-power-plant the cash-flows that are usually involved often include, but are not limited to: an initial investment for building the power-plant and related parts, income from power production, costs for maintenance and running (e.g. manning) the power-plant.

Also, occasionally some larger parts may need to be replaced, which cause intermediate investments between the start of the life time till end of life-time. An example is given in **Figure 90**, the NPV is plotted for each year, but often one is only interested in the one for the entire life-time.

Figure 90 - Example of NPV curve

Input: Initial investment: €10.000,- ; r=0,12 (12%); inflation=0,04 (4%); cash-flow= €2000 (year 0); 10-year replacement investment = €2500 (at year 10: PV=1.191,5 and at year 20: NPV= 567,9); life-time = 25 years. NPV after 25 is now not above 0, so to find LCOE or IRR the variables need to be changed.

What both the LCOE and the IRR do, is take 1 parameter as a variable, which is the energy price for LCOE and the return rate for IRR. These variables are then solved for when the NPV is zero (0) at the end of the defined life-time. This also means that the IRR has a fixed energy price and LCOE has a fixed return rate, which need to be defined before solving for one of these variables.

In **Figure 91** the LCOE is found by taking a fixed r and changing the "cash-flow", which for the example turned out to be **€1499,30**. Say for instance the running costs are €1000,- and if the amount of produced energy is 30.000 kWh, this would result in an energy price of $\frac{€2499{,}3}{30.000 kWh} = 0{,}08331\ €/kWh$ or about 8 euro-cents as LCOE.

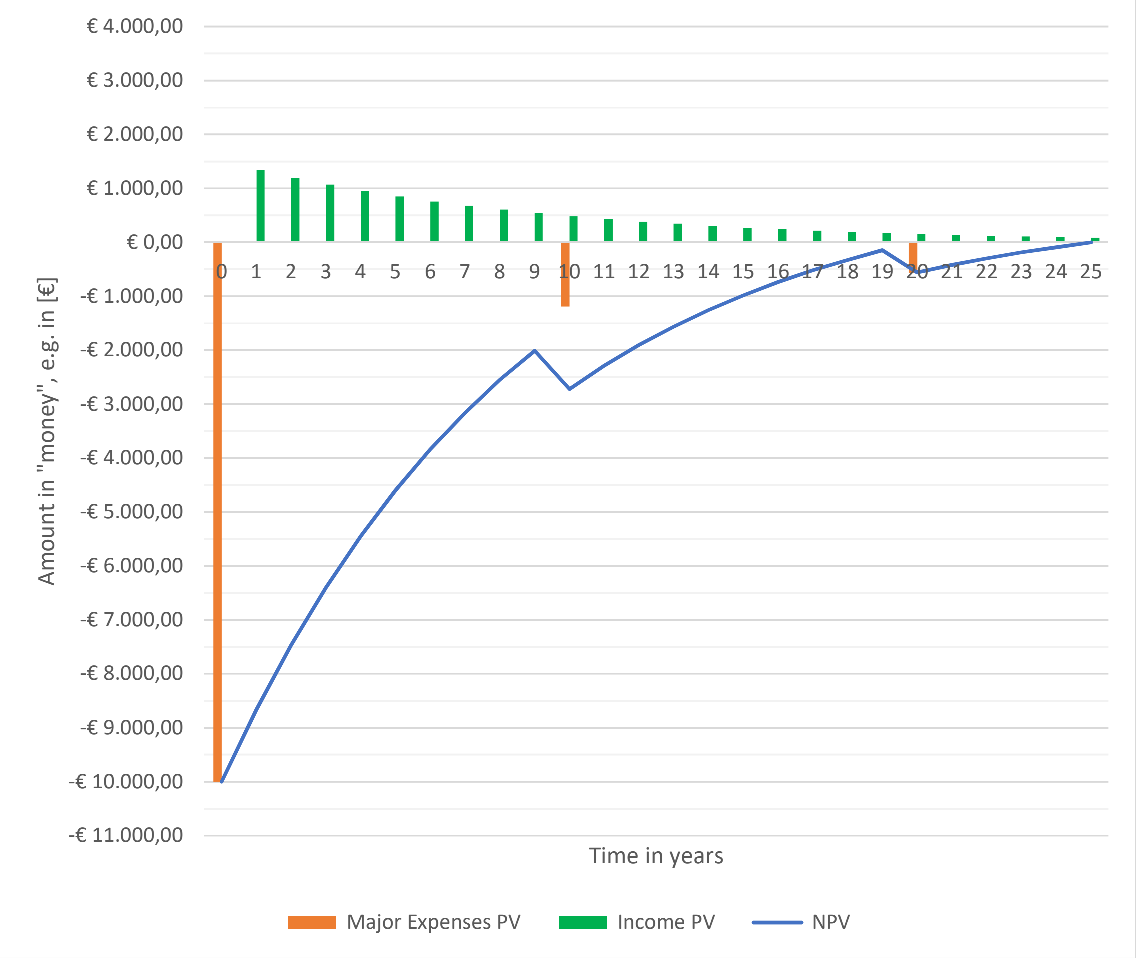


Figure 91 - Example of LCOE curve

Input:
Initial investment: €10.000,- ; rate of return is kept constant at r=0,12 (12%); inflation=0,04 (4%); 10-year replacement investment = €2500 (at year 0, so PV of investment at year 10: €1.191,5 and at year 20: €567,9); life-time = 25 years; Cash-flow has been solved and found to be a value of: €1499,3 (year 0) such that NPV of total project after 25 years is €0,00; Assuming running costs of €1000,- (making the required revenue = €2499,3/year at year 0) and energy production of 30.000 kWh, the energy price needs to LCOE=0,08331 €/kWh.

The IRR keeps the same “cash-flow” of €2000,- , but by changing rate of return “r” to about **r=0,18** also a NPV of 0 is found after 25 years as shown in **Figure 92**. From this it can be concluded that the example project has a high rate of return and is thus a good investment.

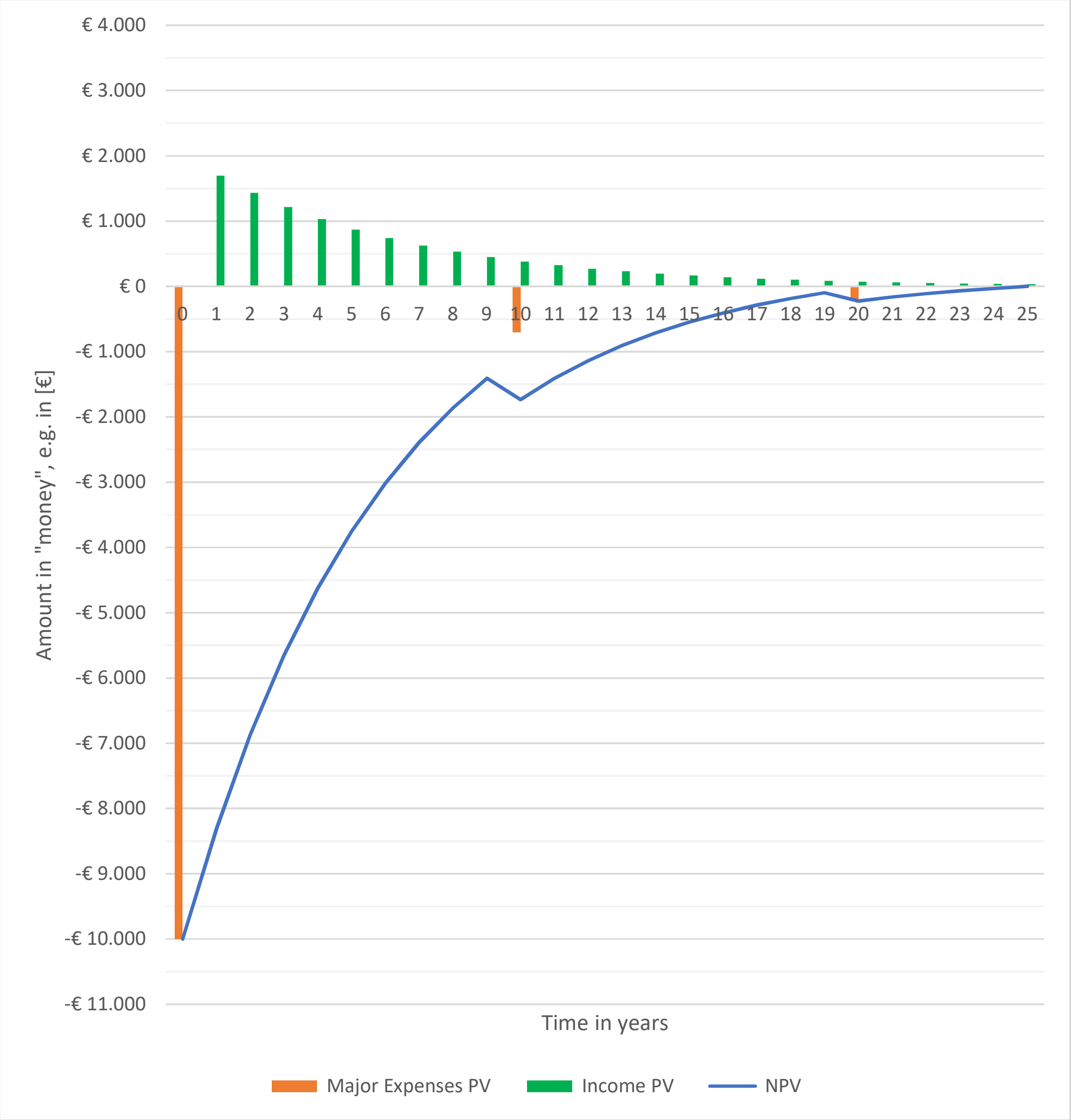
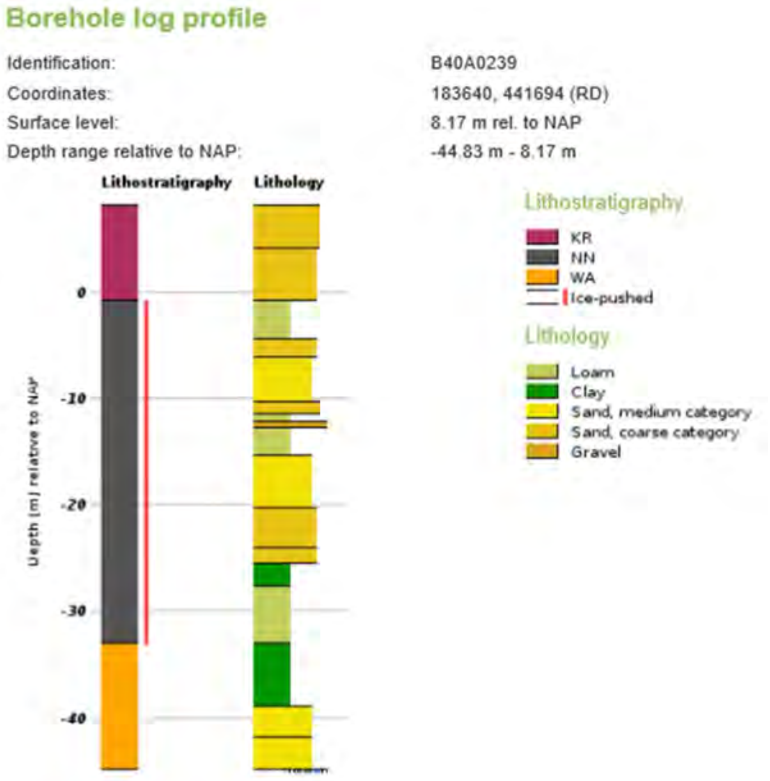


Figure 92 - Example of LCOE curve: Input: Initial investment: €10.000,- ; the cash-flow has been kept constant at a value of: €2000 (year 0); inflation=0,04 (4%); 10-year replacement investment = €2500 (at year 0, so PV of investment at year 10: €1.191,5 and at year 20: €567,9); life-time = 25 years; <u>Rate of return has been solved</u> and found to be r=0,18 (18%) such that NPV of total project after 25 is €0,00;

APPENDIX 10 – SUB-SOIL PROFILES AND INFO



Borehole log profile

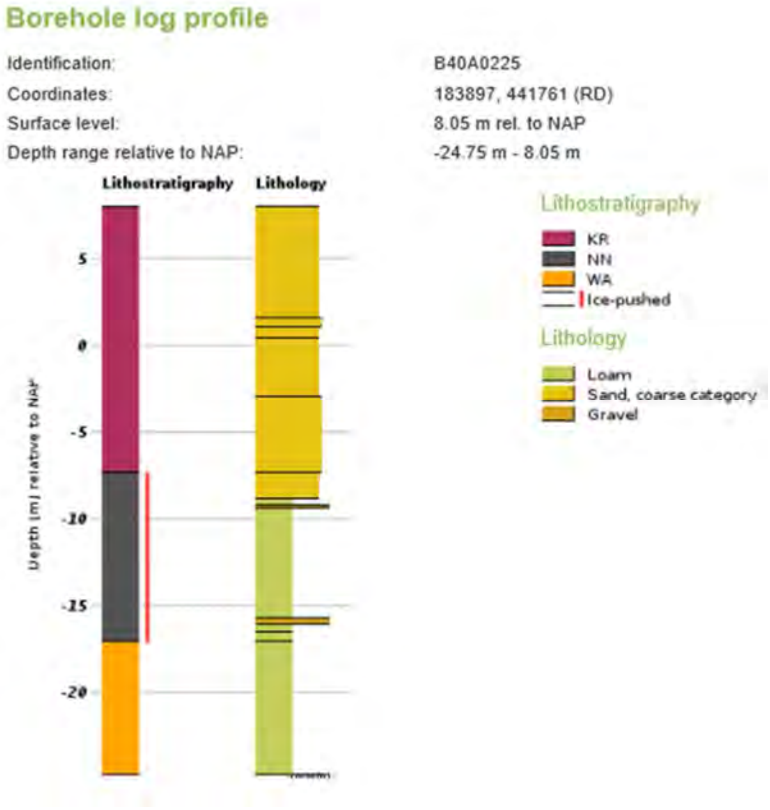
Identification: B40A0239

Coordinates: 183640, 441694 (RD)

Surface level: 8.17 m rel. to NAP

Depth range relative to NAP: -44.83 m - 8.17 m

Figure 93 - Bore-hole down-stream of the weir - source: Dino-loket [59]



Borehole log profile

Identification: B40A0225

Coordinates: 183897, 441761 (RD)

Surface level: 8.05 m rel. to NAP

Depth range relative to NAP: -24.75 m - 8.05 m

Figure 94 - Bore-hole up-stream of the weir - source: Dino-loket [59]

APPENDIX 11 – NATURE RESERVES AND PROTECTED AREAS

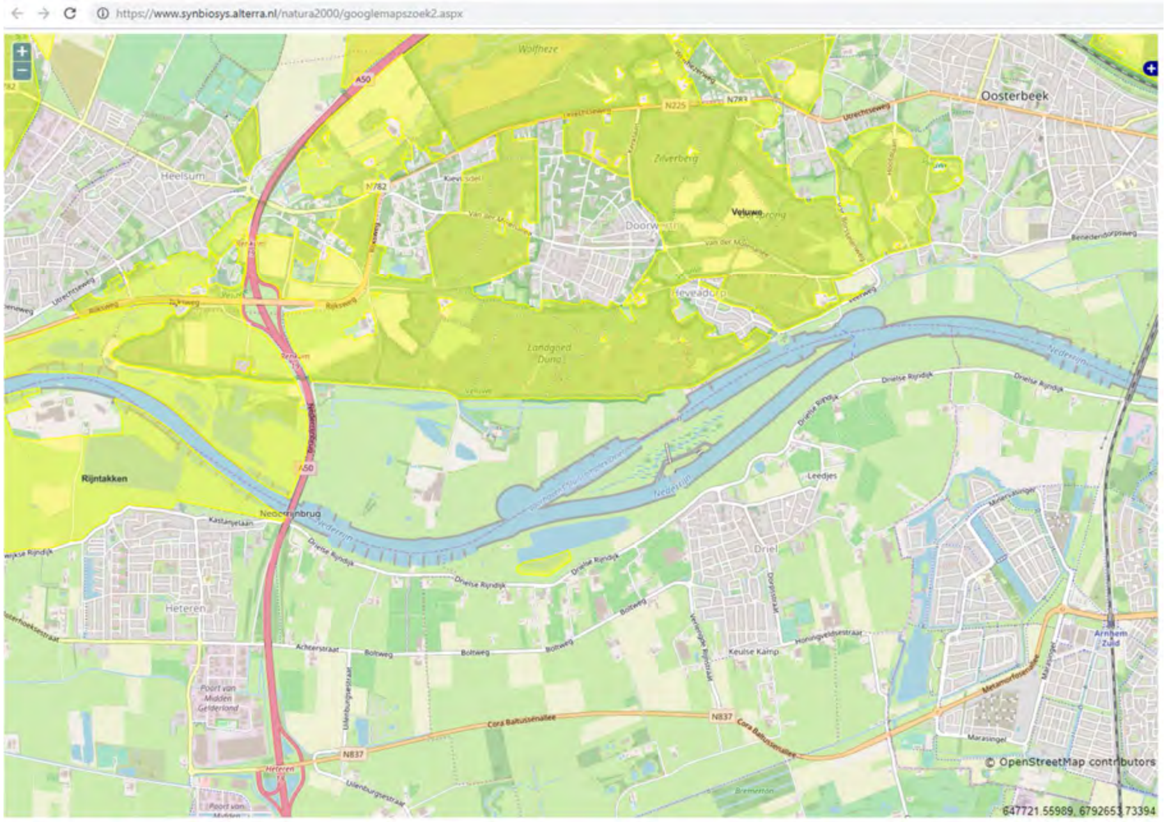


Figure 95 *- Natura2000 areas (marked in yellow) near the weir-complex Driel. These areas are protected by the Nature-protection act (Wet Natuurbescherming) which came into effect in 2017. From map-viewer website: [60] which is based on this map [61].*

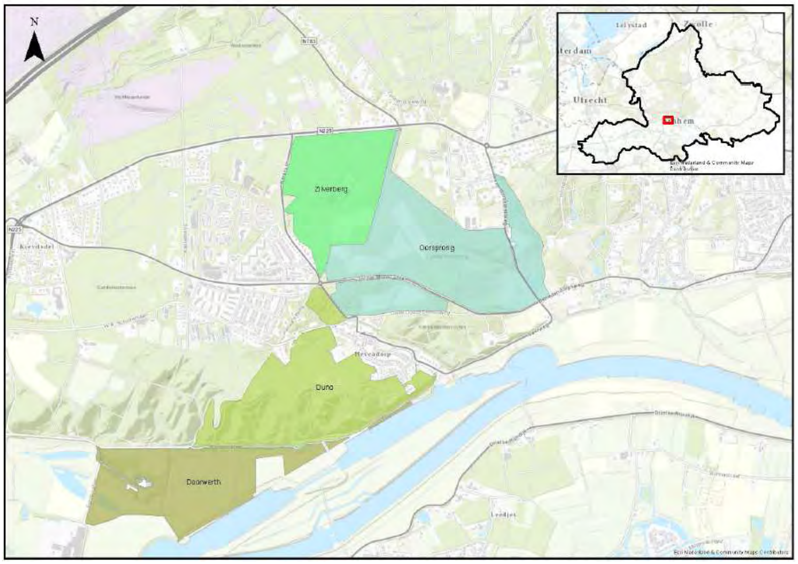


Figure 96 *- Areas owned by "Geldersch Landschap". Source: [34, p. 4]*

From top to bottom:
Zilverberg
Oorsprong
Duno
Doorwerth

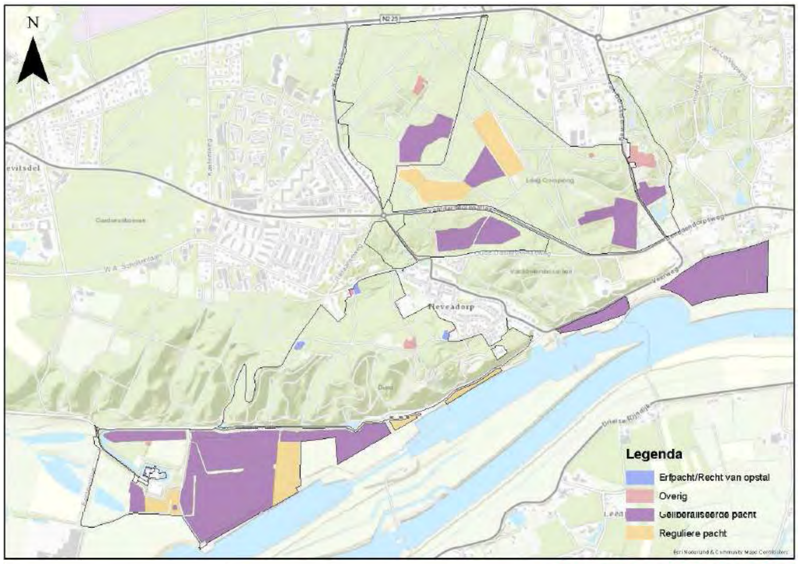


Figure 97 *– Running contracts with Geldersch Landschap.*
Source: [34, p. 20]

Legend:
Blue areas: Ground lease ("erfpacht" in Dutch);
Red areas: Other types of contracts;
Purple areas: Liberalised ground lease;
Orange areas: Regular lease.

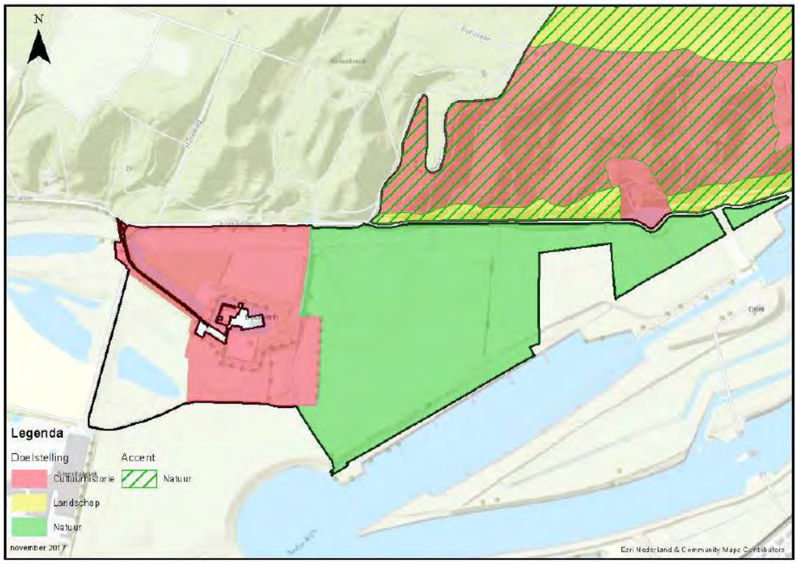
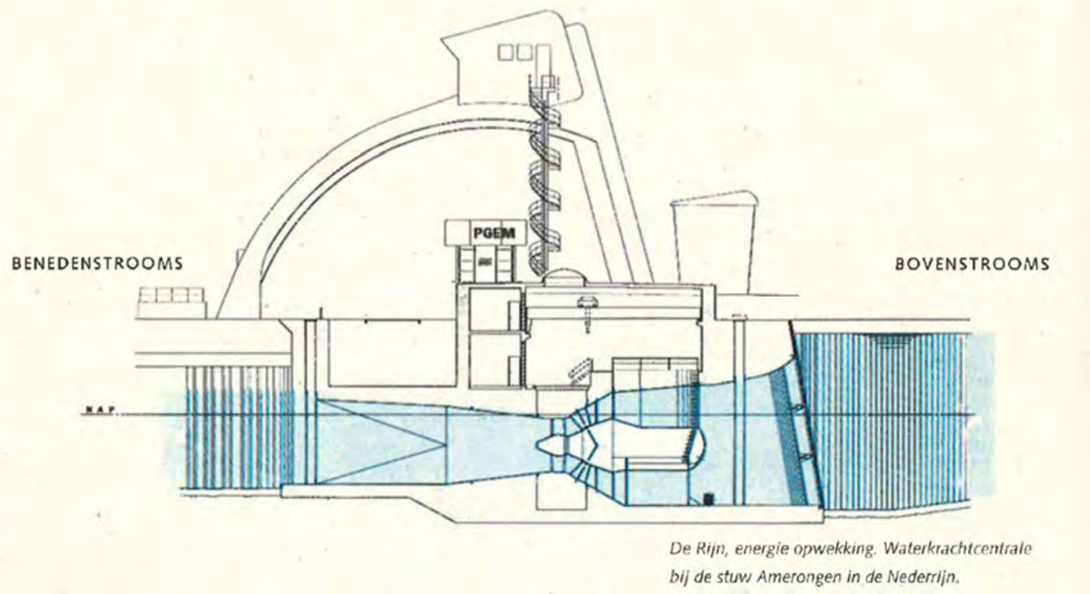


Figure 98 *- Goals and wishes for property of Geldersch Landschap.*
Source: [34, p. 24]

Per colour the goals are:
Green: Nature
Red: Cultural & historical
Yellow: Landscape
Green-hatched: combined with nature.

APPENDIX 12 – REVERSE ENGINEERED MAURIK POWER ISOBARS

The hydropower-plant near Maurik is the one that comes most close to the situation of Driel and is therefore one of the most interesting Hydro-power stations for to this research. Maurik is downstream of Driel and was built in 1988. A cross-section of the station is shown below.



De Rijn, energie opwekking. Waterkrachtcentrale bij de stuw Amerongen in de Nederrijn.

Figure 99 *- Cross-section Power-house Maurik. At the time owned and operated by PGEM, now NUON/Vattenfal - source: [19, p. 8]*

As can be seen, Maurik has horizontal axis Kaplan bulb turbines. Below a few parameters that have been gathered by visiting the power-plant:

Technical data:			
Description	**Quantity**	**Unit**	**Value**
Number of turbines	n_t	-	4
Turbine rotor diameter	D_t	m	4,0
Number of blades per rotor	n_b	-	3
Discharge per turbine	Q_t	m3/s	100
Average system discharge	Q_{sys_avg}	m3/s	250
Largest head-difference	ΔH_{max}	m	4,0
Average head-difference	ΔH_{avg}	m	3,5
Rated power of turbine	P_{t_rated}	kW	2.500
Turbine rotation speed	N_{rotor}	rpm	78,0
Output rotation speed gearbox	N_{gear_out}	rpm	750,0

Table 40 *- Performance data HPP Maurik - Source information: NUON*

From the output rotational speed the number of pole pairs can be derived. This must be 4 as:

$$N = \frac{f \cdot 60}{n_p} \rightarrow n_p = \frac{f \cdot 60}{N} = \frac{50 \cdot 60}{750} = 4$$

Also important to note is that the maximum head difference noted in **Table 4** doesn't coincide with the maximum discharge, but rather the opposite (maximum head with minimal discharge and vice versa).

Below performance data gathered from NUON is shown.

Description	**Quantity**	**Unit**	**Value**
Year of completion	-	-	1988
Initial investment	$Cost_{init}(1988)$	Mln. fl.	66,0
Current value of initial investment	$Cost_{init}(now)$	Mln. €	54,0 (120 mln. fl.)
Rated power Plant	$P_{rated,sys}$	kW	10.000
High average annual energy production	$E_{ann,high}$	GWh	25,0
Low average annual energy production	$E_{ann,low}$	GWh	20,0
High average full-load-hours per year	$t_{FL,high}$	hr	2.500
Low average full-load-hours per year	$t_{FL,low}$	hr	2.000
Capacity factor (high production)	CF_{high}	%	28,5%
Capacity factor (low production)	CF_{low}	%	22,8%

Table 41 *- Performance data HPP Maurik - Source: NUON. Guilder (fl.) in 1988 have been converted to current day (2019) value and currency using this source [20] and rounded to millions.*

Quite notable is that the capacity factor for Maurik lies much lower than the global average of 44%. This may be due to the shared function with the weir (shipping and power production).

Some screenshots from NUON showing the operator's screens were received, which were compared with flow data from Rijkswaterstaat:

description	**quantity**	**unit**	**MG1**	**MG2**	**MG3**	**MG4**	**Total**
Turbine discharge	Q	m^3/s	47,5	44,9	46,8	47,2	**186,4**
Net head over structure	ΔH	m	2,84	2,81	2,83	2,81	**-**
Output power	P	kW	1115	1137	1259	1243	**4754**
Load hours	t_{load}	hr	131.961	129.401	133.915	131.708	**526.985**
Energy produced	E	GWh	163,7	161,6	166,8	167,6	**660**

Table 42 *- Production figures at **11:25 hr on 6th of June 2019**. Note: MG is an abbreviation of the German word "Machinegruppe" = machine group indicating individual turbines. - Source: NUON*

The head difference is measured between the inflow and outflow pipe, so it's not exactly the head over the turbine. However the inflow losses are so small and most of the losses can be expected to happen at the outflow, that it is reasonable to assume this is the actual head over the turbine.

To compare the water-level differences and discharges from that same moment measured by RWS:

Measurements						**Average**
$t_{measurement} =$		**11:10**	**11:20**	**11:30**	**11:40**	**11:25**
H_up_Maurik	$m + NAP$	5,99	5,98	5,95	5,97	5,97
H_down_Maurik	$m + NAP$	3,06	3,05	3,06	3,04	3,05

dH_Maurik	$m + NAP$	2,93	2,93	2,89	2,93	2,92
Q_RWS	m^3/s	185,0	188,1	181,7	195,0	187,5

***Table 43** – Flow figures Maurik at 6th of June 2019 - Source: RWS [24]*

From that a resistance value can be estimated and also a head-ratio for this instant. The head-ratio is quite high, namely on average 96% the losses being on average 3,85%.

The specific speed is on average 44,1 rpm and with that the quadratic resistance coefficient for the turbine at this time is:

$$C_{Maurik} = \frac{(2{,}92m - 2{,}83m)}{\left(\frac{46{,}6m^3}{s}\right)^2} = 4{,}47 \cdot 10^{-5} \cdot \frac{s^2}{m^5}$$

However, this value can change, because the pitch of the guide-vanes and the rotor blades can be changed increasing or decreasing the resistance. The guide-vanes were opened on averaged over the 4 turbines about 70% and the rotor blades had a pitch between 42 and 52% (though with respect to what isn't clear).

Also within the screenshots was the hill-chart shown in **Figure 20**:

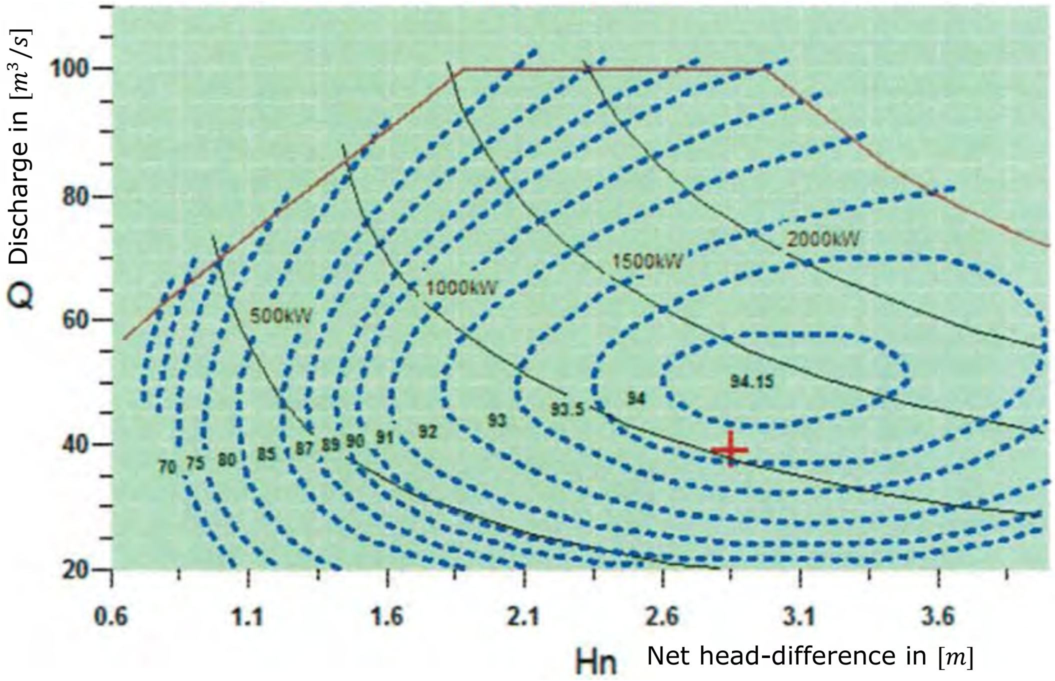


***Figure 100** - Hill-chart indicating power-output (black solid lines) and efficiency (blue dotted lines) of the Kaplan Bulb turbine in Maurik HPP. (Red cross indicating the position of the moment the screenshot was taken (6th of June 2019 around 11:25) - Source: NUON*

Using the turbine theory from **paragraph "Turbine theory"** the graph from **Figure 20** was reconstructed with a simplified efficiency curves shown in **Figure 101** and **Figure 102**. The Isobars of power-output match quite well at the extremes of the curve and only lag behind the actual curve slightly in the middle. This in a way also proves that the theory is indeed applicable for the situation in Maurik and similar situations.

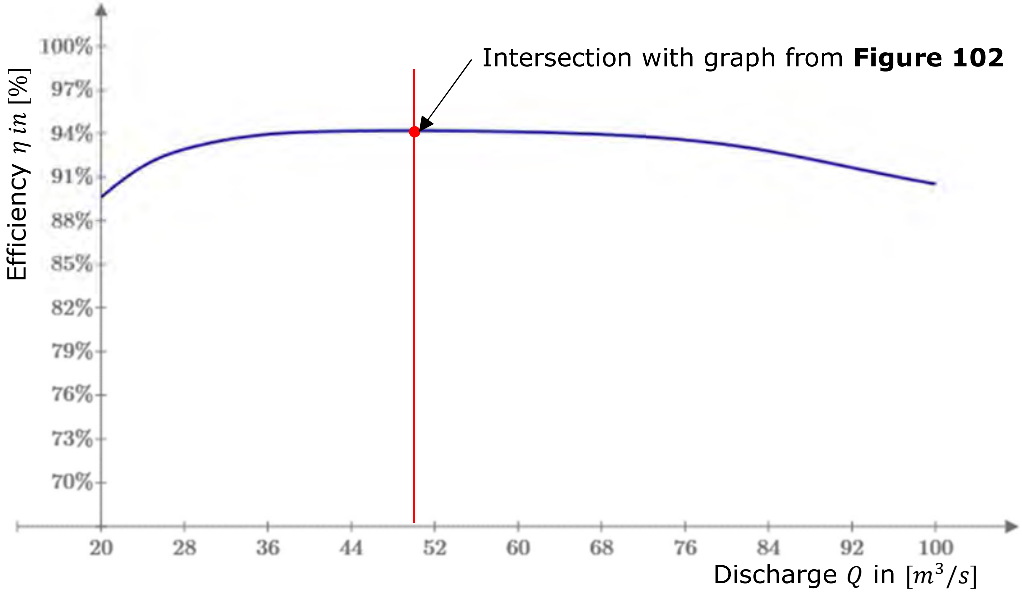


Figure 101 *- Approximate/estimated efficiency curve per discharge for Maurik HPP. Taken at a head-dffierence of 3,0m in the Hill-chart. - Used source: Hill chart from NUON*

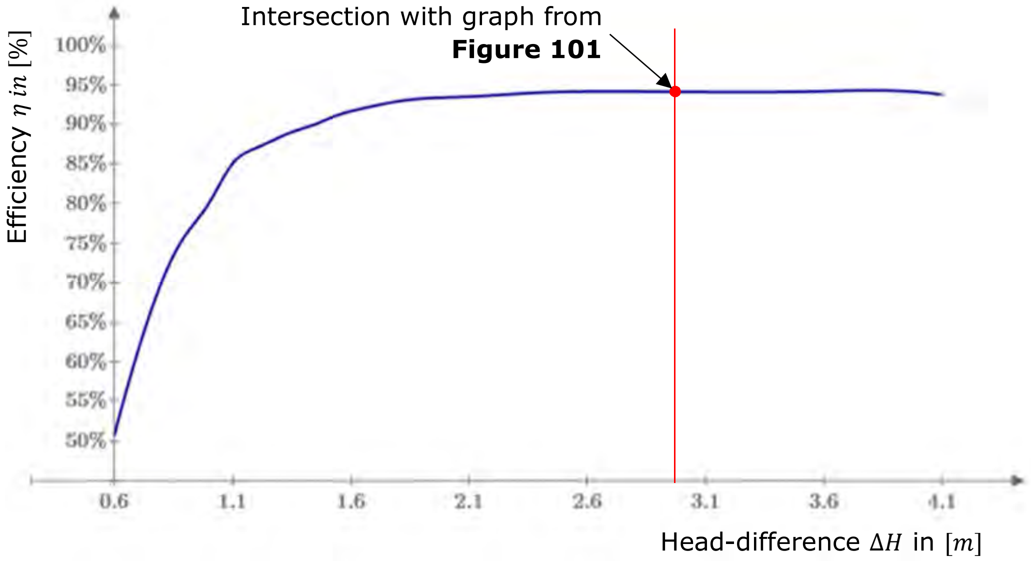


Figure 102 *- Approximate/estimated efficiency curve per head-difference over turbine for Maurik HPP. Taken at a discharge of 50m³/s in the Hill-chart - Used source: Hill chart from NUON*

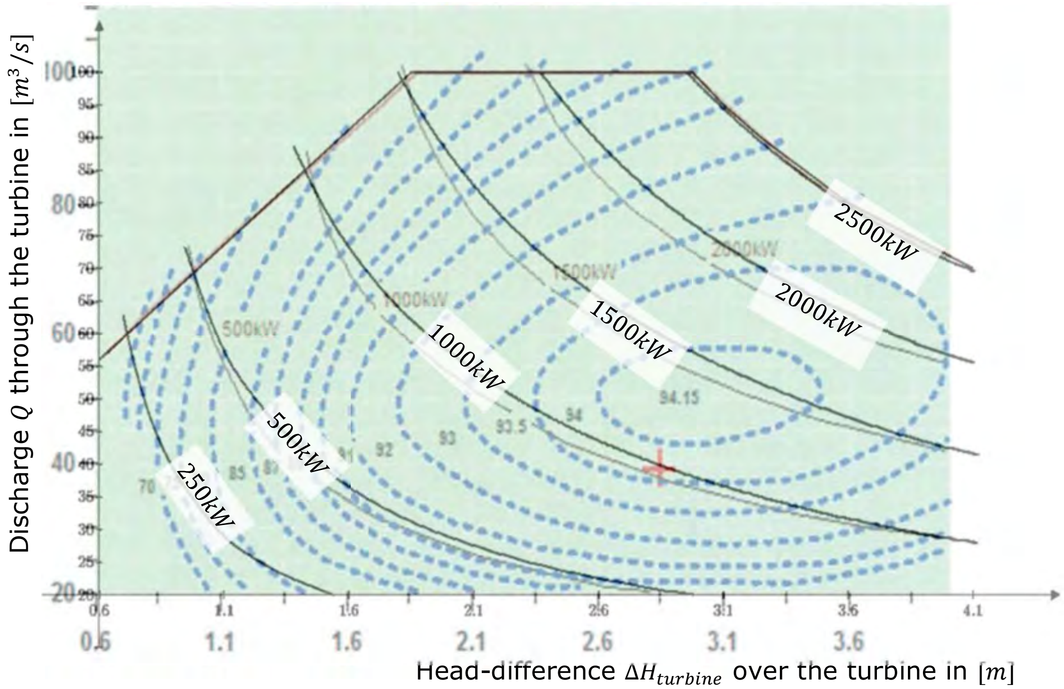
The shape of the graph in **Figure 101** in the calculation of the efficiency for the power of the Maurik turbine has been projected over all head-differences by dividing the maximum value of **Figure 101** (94,15%) and then multiplying it with the efficiency at any point in **Figure 102**.

For instance, with a head difference of 1,6m and a discharge of 24m³/s the combined efficiency would be:

$$\eta_{Maurik}(\Delta H, Q) = \eta(Q) \cdot \frac{\eta(\Delta H)}{\max(\eta(\Delta H))} = \eta(24\text{m}^3/\text{s}) \cdot \frac{\eta(1{,}6\text{m})}{94{,}15\%} = 90\% \cdot \frac{91\%}{94{,}15\%} \approx 87\%$$

This way the efficiency hill-chart somewhat resembles the one in **Figure 20**.

Using formula **(4 - 15)** makes it possible to reconstruct / reverse engineer the Hill-chart:



$$P_t = \eta_{Maurik}(Q, \Delta H_{turbine}) * \rho * g * \Delta H_{turbine} * Q$$

$$\Delta H_{turbine} = \frac{(\eta \cdot Q)^{\frac{2}{3}}}{g} \cdot \left(\frac{N}{N_s}\right)^{\frac{4}{3}} \quad ; \quad \Delta H_{system} = \Delta H_{turbine} + \frac{Q^2}{2g} \cdot C$$

Figure 103 - Reconstructed hill-chart using turbo-machinery theory and data from Maurik HPP

To create the graph in **Figure 103** the specific speed was set as a variable, because for a Kaplan turbine the "geometry" that a certain specific speed is linked to changes when for example the blade-angle or the guide-vane angle is changed. The specific speed ranges from

Note that the red line in **Figure 103** is cutting the graph off at the top is comprised of 3 lines. On the right side of the graph is the line representing the maximum power output of the turbine, i.e. 2500kW. On the top this is the maximum discharge through the turbine ($100m^3/s$) and on the left it is also a maximum discharge that is related to limits in the setting of the blade angles and the available head-difference. The minimum C-coefficient for the left part of the graph is about $2{,}2\ to\ 2{,}4 \cdot 10^{-4} \cdot s^2/m^5$.

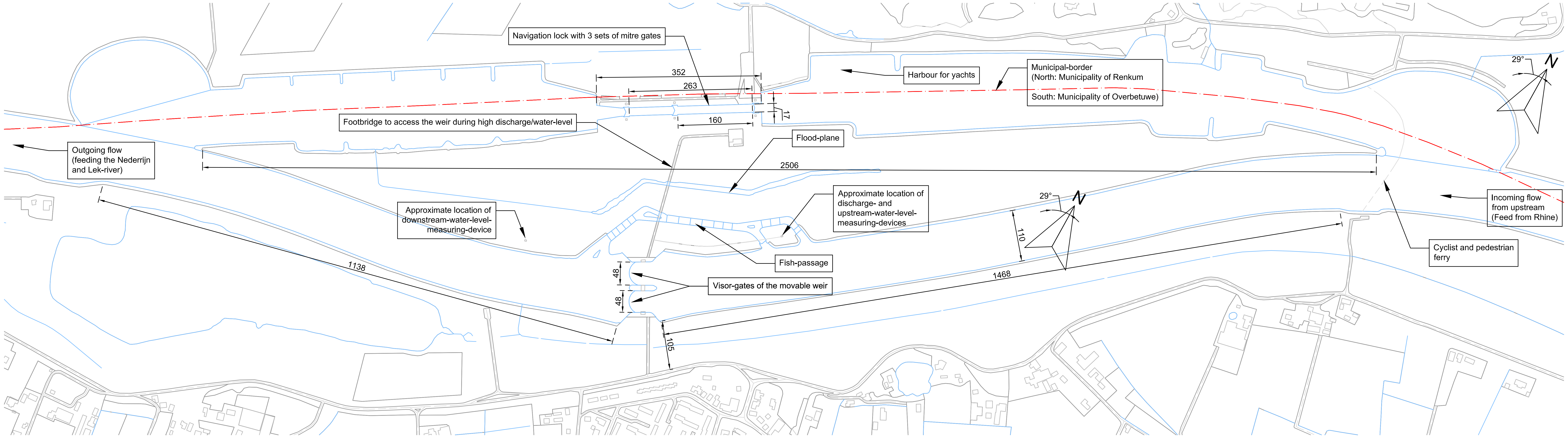
Limit location in graph	Limit
Left	$C \geq 2{,}2\ to\ 2{,}4 * 10^{-4} * s^2/m^5$
Centre	$Q \leq 100m^3/s$
Right	$P \leq 2500kW$

What can also been seen in this image is that the actual hill-chart that is projected under the reconstructed one. The difference with the actual chart is that the power curves are bended a bit more down-wards and to the left. This is likely due to errors in the approximation used for the efficiencies. Still the shape matches quite well, especially at the extremes, i.e. the borders of the graph.

APPENDIX 13 – LOCATION AND TOPOGRAPHY

– see inserted page(s) behind this page –

North-oriented Topography - Location of weir complex Driel in river Delta - Lower Rhine Bifurcations shown: Pannerdensch-kop and IJsel-Kop; Rivers shown: Waal river, Neder-Rijn river and IJssel river - Scale 1:50.000



Overview of weir complex Driel - view aligned to river axis at location of weir (North -29°) - Scale 1:5.000

Legend:

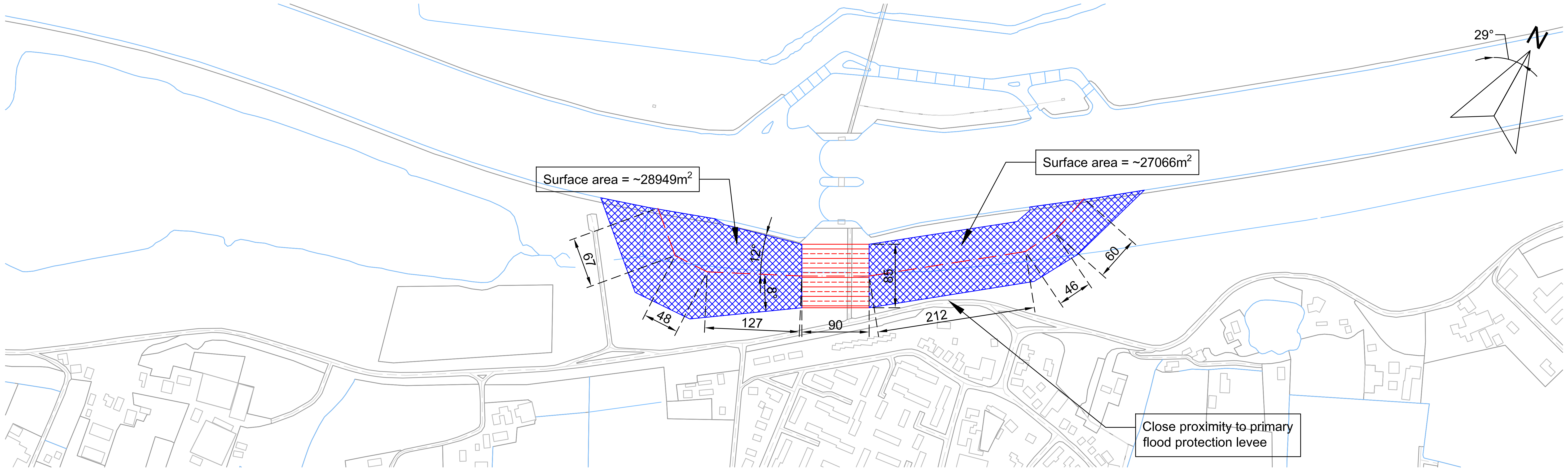
Borders (Country, Pronvincial and Municipality):	— · — · —
Buildings	———
Water	———
Terrain	———
Roads	- - - -
placeholder entry	— · — · —
placeholder entry	— · — · —

Drawing info:

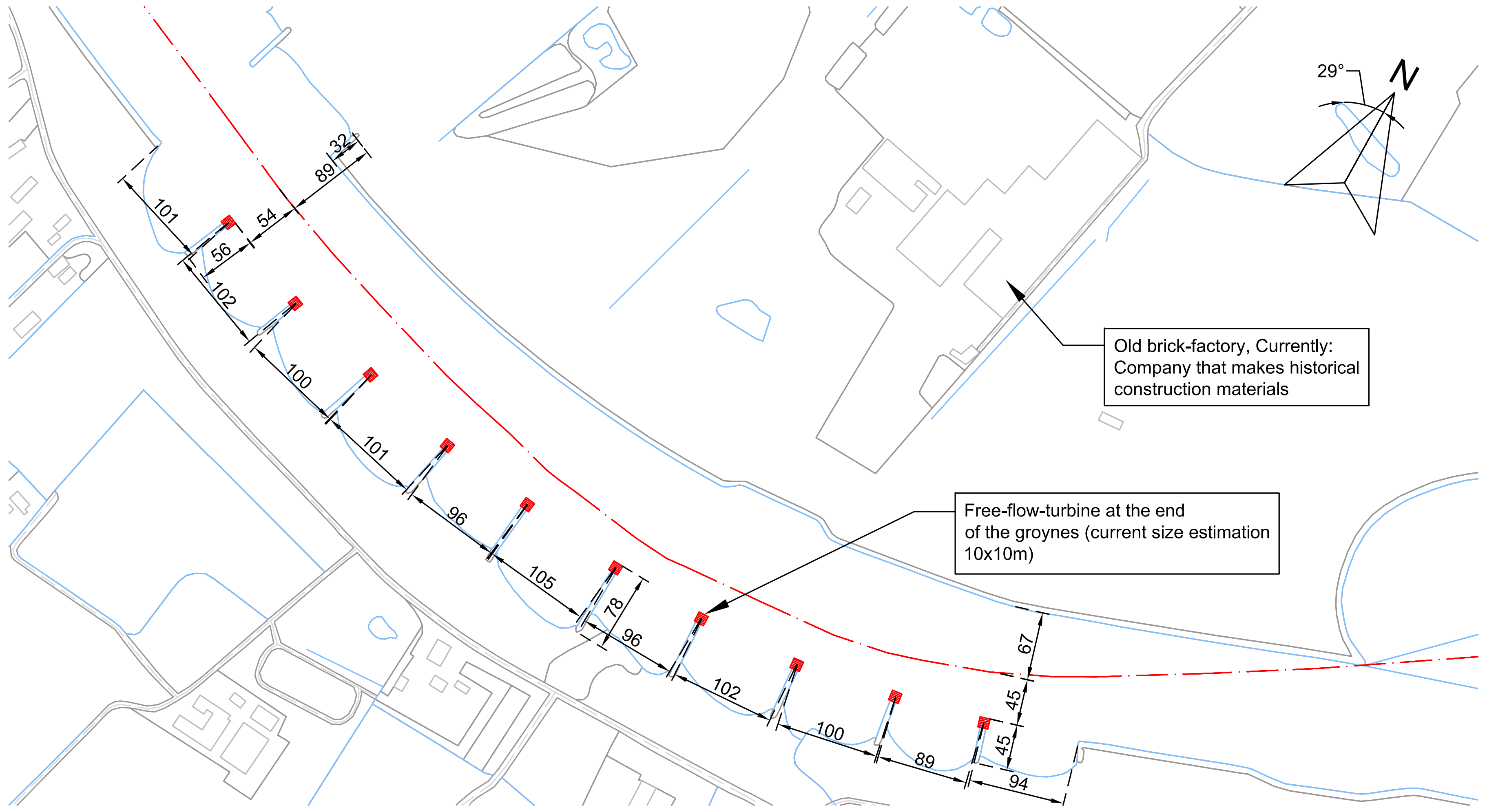
Project:	Master Thesis - Hydro-power at Driel
Title:	Location Driel
Sub title:	Topography
Last updated:	2019-03-29
Print date:	2019-04-02
Version:	0.1
Source topography:	"Basisregistratie topografie", Kadaster 2012 with CC licence. Sources set date: 01 Sept. 2012 & last edited: 27 May 2014 From: http://chmodarch.com/nlindwg/
Edited by:	ing. S.R. (Stefan) van Erp
Scale:	As indicated underneath each drawing-area
Dimensioning in:	[m] meters
Comments:	

APPENDIX 14 – HYDRO-POWER-SCHEME VARIANTS

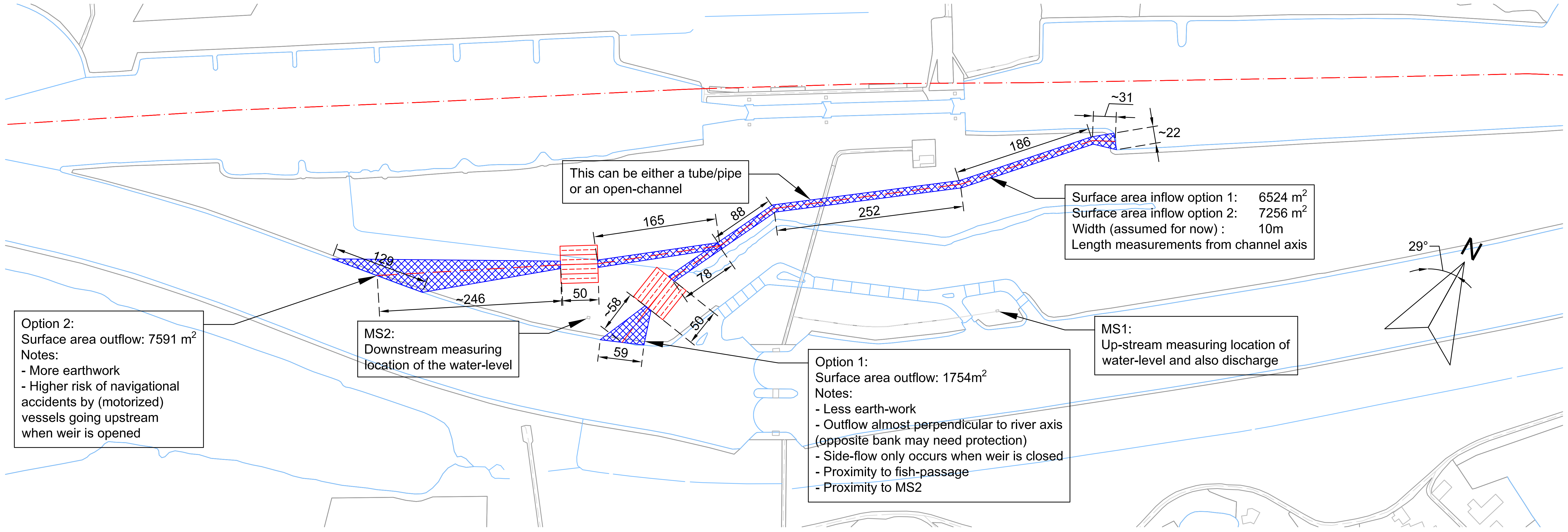
– see inserted page(s) behind this page –



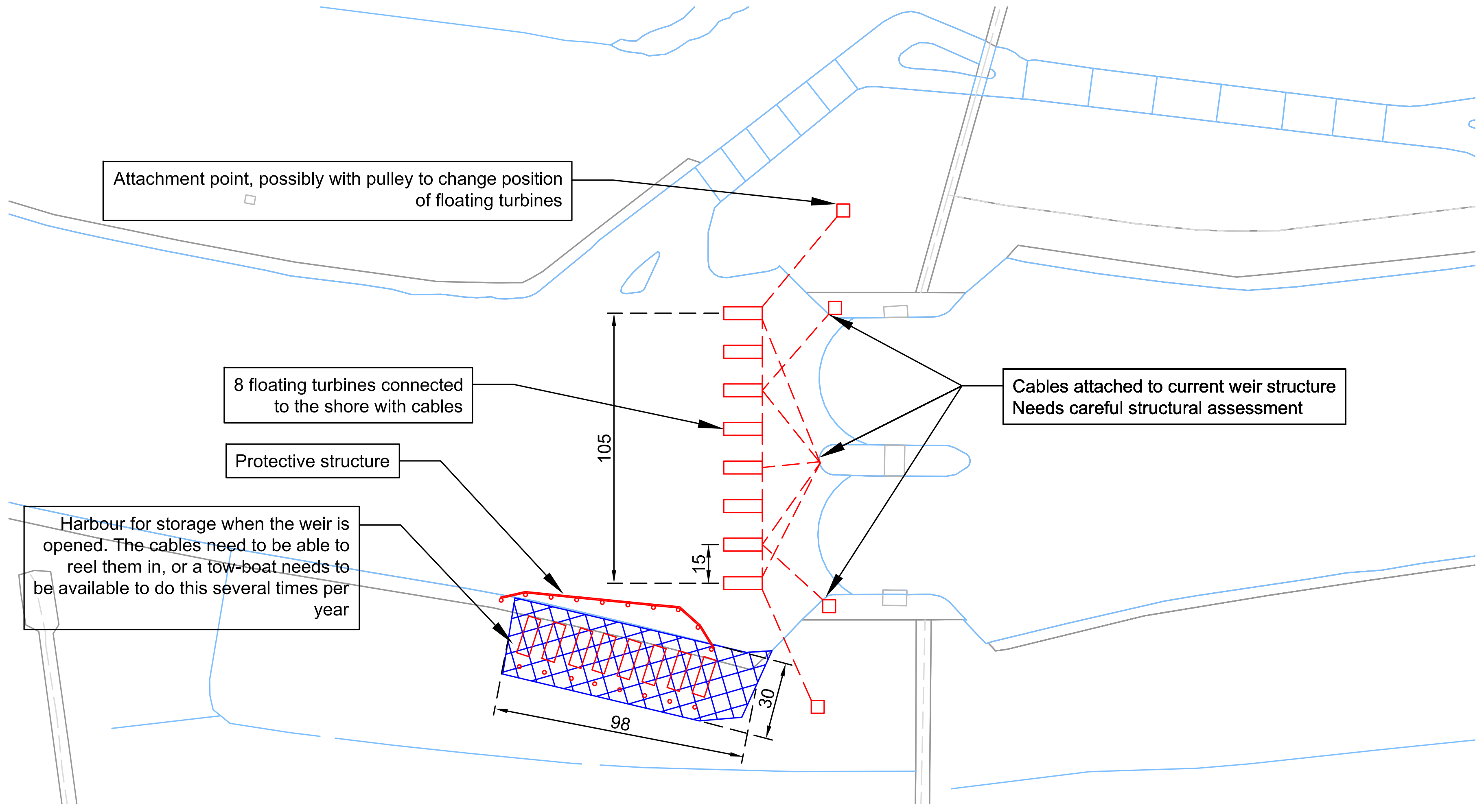
HPS Variant 1 - Bay-power-plant south of the weir



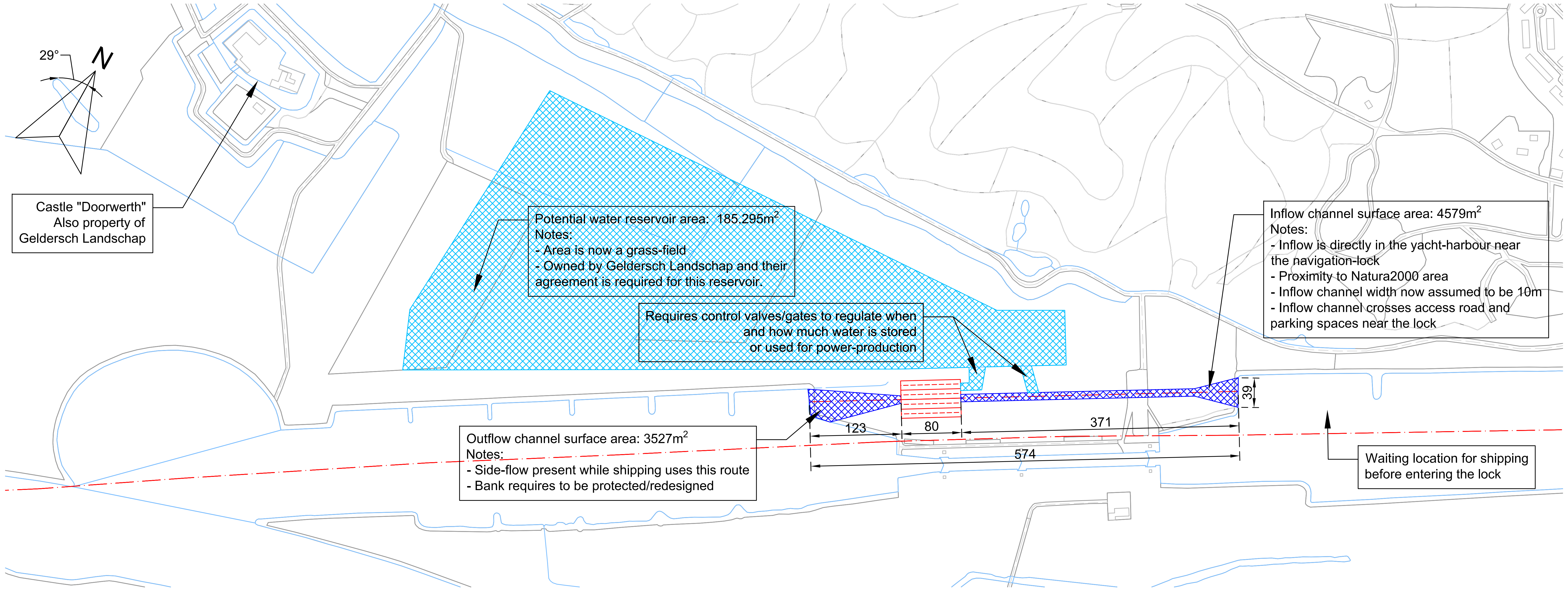
HPS Variant 4 - Free-flow-turbines on the groyne-ends - Scale 1:5.000



HPS Variant 2 - Penstocks through flood-plane, North of the weir, south of the navigation-lock



HPS Variant 5 - behind the weir outflow



HPS Variant 3 - Penstocks North of the navigation lock

Legend

Borders (Country, Provincial and Municipality):
Buildings
Water
Terrain
Roads
Structural parts of the hydro-power-scheme
New areas with water (reservoirs or channels)
Channel (central) axis
Potential reservoir areas

Drawing info

Project:	Master Thesis - Hydro-power at Driel
Title:	Weir of Driel
Subtitle:	Hydro-power-schemes
Last updated:	2019-06-17
Print date:	2019-06-17
Version:	0.1
Source topography:	"Basisregistratie topografie", Kadaster, 2012, with CC licence. Sources set date: 01 Sept. 2012 & last edited: 27 May 2014 From: http://chmodarch.com/nlindwg/
Drawn by:	ing. S.R. (Stefan) van Erp
Scale:	1:4.000 (unless otherwise indicated)
Dimensioning in:	[m] meters
Comments:	HPS = Hydro-power-scheme Please note: size of the new channel and structures indicative

APPENDIX 15 – OUTPUT ENERGY CALCULATION GENERIC TURBINE (0.)

– see inserted page(s) behind this page –

Generic turbine - method Hessel Voortman

Define:

$Nr\_t := 1, 2 .. 8 = \begin{bmatrix} 1 \\ \vdots \end{bmatrix}$ Number of turbine

$\rho := 998.7 \ kg \cdot m^{-3}$ Mass density of water

$\eta\_t := 0.9$ Efficiency of turbine

And take from Maurik:

$D.out\_M := 4 \ m$ Outer diameter

$D.in\_M := 1.5 \ m$ Inner diameter

$A.t\_M := \frac{\pi}{4} \left(D.out\_M^2 - D.in\_M^2 \right) = 10.799 \ m^2$ Flow-surface-area

At Maurik on t=6th of June 11:25 the following was the case:

$Q\_M\_66 := 46.6 \ m^3 \cdot s^{-1}$ Discharge through turbine

$\Delta H\_M\_T\_66 := 2.823 \ m$ Head over turbine

$\Delta H\_M\_Sys\_66 := 2.92 \ m$ Head over structure

$C\_M := \frac{\left(\Delta H\_M\_Sys\_66 - \Delta H\_M\_T\_66 \right)}{\left(Q\_M\_66 \right)^2}$ Quadratic discharge coefficient.

$C\_M = \left(4.467 \cdot 10^{-5} \right) \frac{s^2}{m^5}$

$\xi.eq\_M := C\_M \cdot 2 \cdot g \cdot A.t\_M^2 = 0.102$ Equivalent Xi-factor

Define:

For now define head-ratio as 2/3 to find the smallest system as approximation:

$r\_h := \frac{2}{3}$

Turbine discharge:

$$Q\_t \left(Q\_av, \Delta H\_av, r\_h, A\_t \right) := min \left(\sqrt{\frac{1 - r\_h}{\xi.eq\_M}} \cdot A\_t \cdot \sqrt{2 \cdot g \cdot \Delta H\_av}, Q\_av \right)$$

Head over turbine:

$$\Delta H\_t \left(\Delta H\_av, r\_h \right) := r\_h \cdot \Delta H\_av$$

Weir (lost) discharge:

$$Q\_w \left(Q\_av, \Delta H\_av, r\_h, A\_t \right) := Q\_av - Q\_t \left(Q\_av, \Delta H\_av, r\_h, A\_t \right)$$

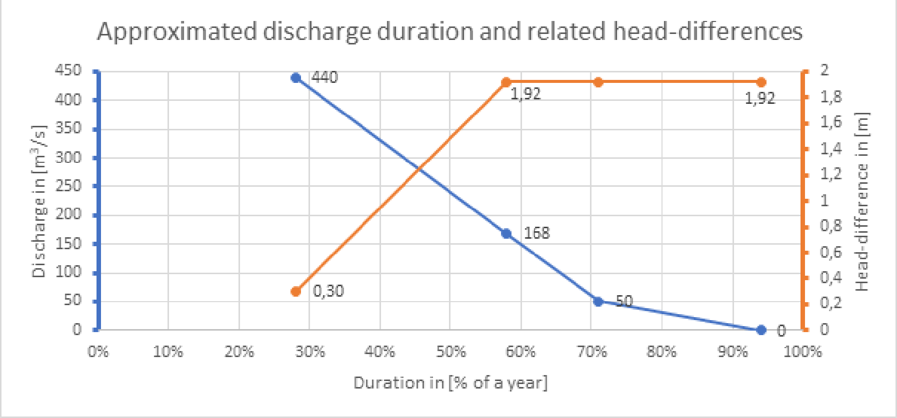
Turbine power:

$$P\_t \left(Q\_av, \Delta H\_av, r\_h, A\_t \right) := \eta\_t \cdot \rho \cdot g \cdot Q\_t \left(Q\_av, \Delta H\_av, r\_h, A\_t \right) \cdot \Delta H\_av \cdot r\_h$$

Design considerations:

- goal is to find number and size of turbines
- Turbine area is a continuous variable at this point, to be determined.

Using linearised flow-duration curve:

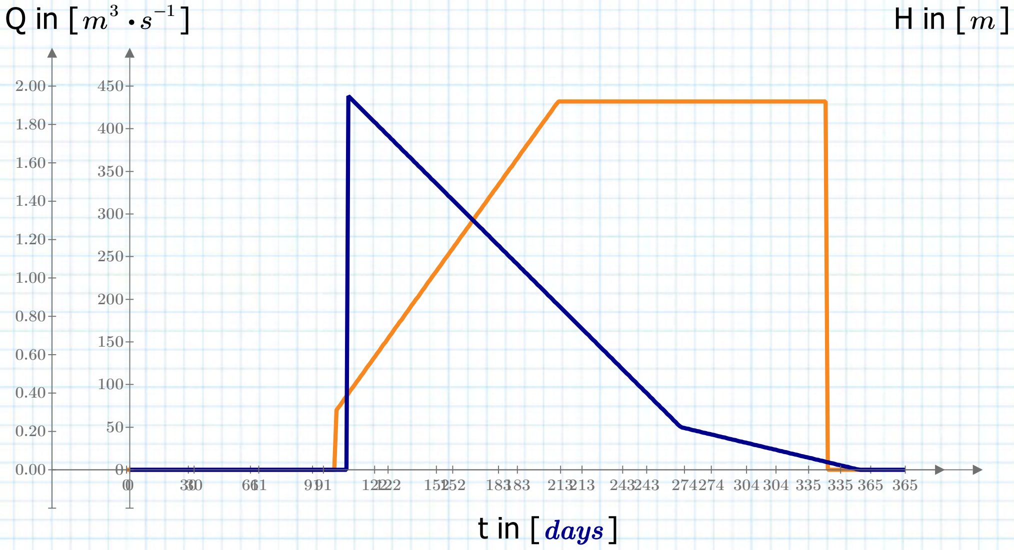


Duration of Q in % of year	28%	58%	71%	94%
$\boldsymbol{Q\ (m3/s)}$	440	168	50	0
$\boldsymbol{\Delta H (m)}$	0,30	1,92	1,92	1,92

$$Q\_av(t) := \left\| \begin{array}{l} \text{if } t \le 71\% \cdot yr \wedge t \ge 28\% \cdot yr \\ \quad \left\| \text{Q} \leftarrow \text{linterp}\left(\left(\left[28\% \cdot yr \ \ 58\% \cdot yr \ \ 71\% \cdot yr\right]^{\mathrm{T}}\right), \left[440 \ \ 168 \ \ 50\right]^{\mathrm{T}}, t\right) \right. \\ \text{else if } t < 94\% \cdot yr \wedge t \ge 71\% \cdot yr \\ \quad \left\| \text{Q} \leftarrow \text{linterp}\left(\left(\left[71\% \cdot yr \ \ 94\% \cdot yr\right]^{\mathrm{T}}\right), \left[50 \ \ 0\right]^{\mathrm{T}}, t\right) \right. \\ \text{return } \text{Q} \cdot m^{3} \cdot s^{-1} \end{array} \right.$$

$$\Delta H\_av(t) := \left\| \begin{array}{l} \text{if } t \le 58\% \cdot yr \wedge t \ge 28\% \cdot yr \\ \quad \left\| H \leftarrow \text{linterp}\left(\left(\left[28\% \cdot yr \ \ 58\% \cdot yr\right]^{\mathrm{T}}\right), \left[0.3 \ \ 1.92\right]^{\mathrm{T}}, t\right) \right. \\ \text{else if } t < 94\% \cdot yr \wedge t \ge 58\% \cdot yr \\ \quad \left\| H \leftarrow \text{linterp}\left(\left(\left[58\% \cdot yr \ \ 94\% \cdot yr\right]^{\mathrm{T}}\right), \left[1.92 \ \ 1.92\right]^{\mathrm{T}}, t\right) \right. \\ \text{return } H \cdot m \end{array} \right.$$

$$t := 0\ day, 1\ day .. 365\ day$$



Consider 3 desing points:

Define area range:

$A\_t := 0.25\ \mathbf{m}^2, 0.5\ \mathbf{m}^2 .. 250\ \mathbf{m}^2$

Define design-point 1:

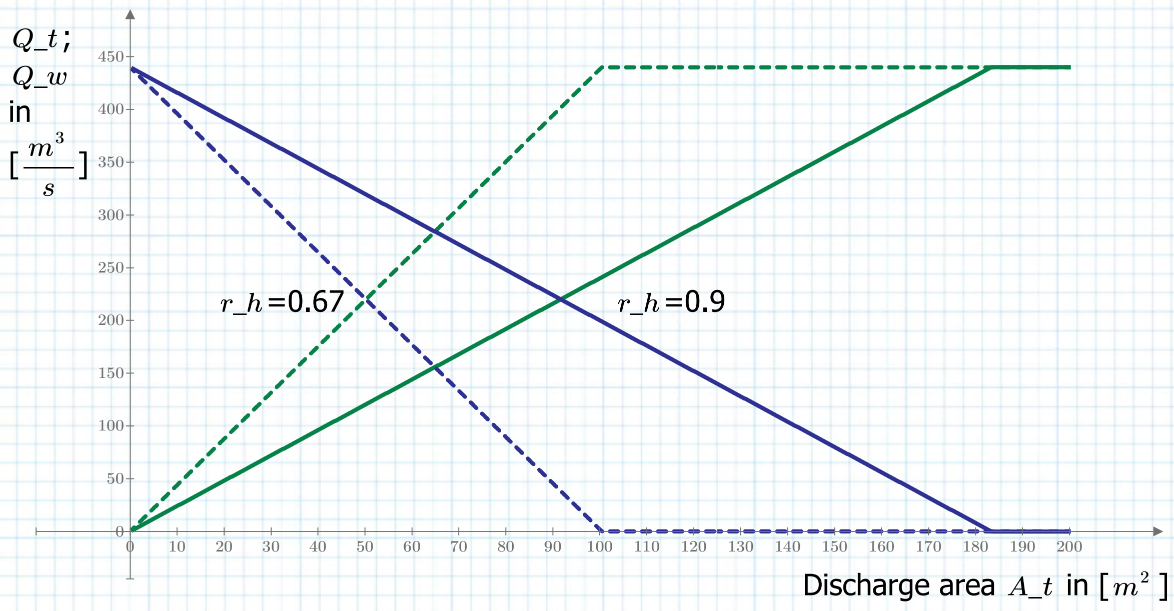
$t\_d1 := 28\% \cdot \mathbf{yr}$ ----> $\Delta H\_av(t\_d1) = 0.30\ \mathbf{m}$

$Q\_av(t\_d1) = 440\ m^3 \cdot s^{-1}$

Head ratio r_h: 0.90 ; 0.67

Discharge through turbine Q_t in $[m^3 \cdot s^{-1}]$ ———— ; - - - - - - - -

Discharge lost through weir Q_w in $[m^3 \cdot s^{-1}]$ ———— ; - - - - - - - -



Define design-point 2:

$t\_d2 := 58\% \cdot \mathbf{yr}$ ----> $\Delta H\_av(t\_d2) = 1.92\ \mathbf{m}$

$Q\_av(t\_d2) = 168\ m^3 \cdot s^{-1}$

Head ratio r_h: 0.90 ; 0.67

Discharge through turbine Q_t in $[m^3 \cdot s^{-1}]$ ———— ; - - - - - - - -

Discharge lost through weir Q_w in $[m^3 \cdot s^{-1}]$ ———— ; - - - - - - - -

Define design-point 3:

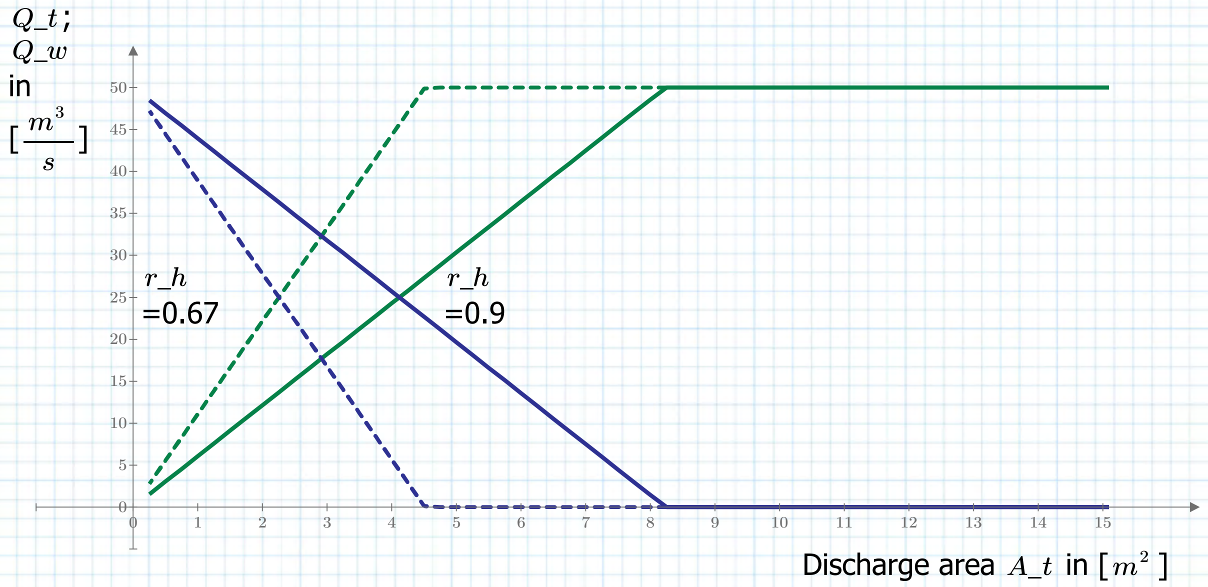
$t\_d3 := 71\% \cdot yr$ ----> $\Delta H\_av(t\_d3) = 1.92\ m$

$Q\_av(t\_d3) = 50\ m^3 \cdot s^{-1}$

Head ratio r_h: 0.90 ; 0.67

Discharge through turbine Q_t in $[m^3 \cdot s^{-1}]$ ———— ; – – – –

Discharge lost through weir Q_w in $[m^3 \cdot s^{-1}]$ ———— ; – – – –



Conclusion(s):
Clearly the area required to let through the same amount of discharge is larger for a higher head ratio r_h for all 3 design points.
However, for both head-ratio the same maximum discharge is achievable given sufficient discharge area.

Considering now the Power of the generic hydropower plant:

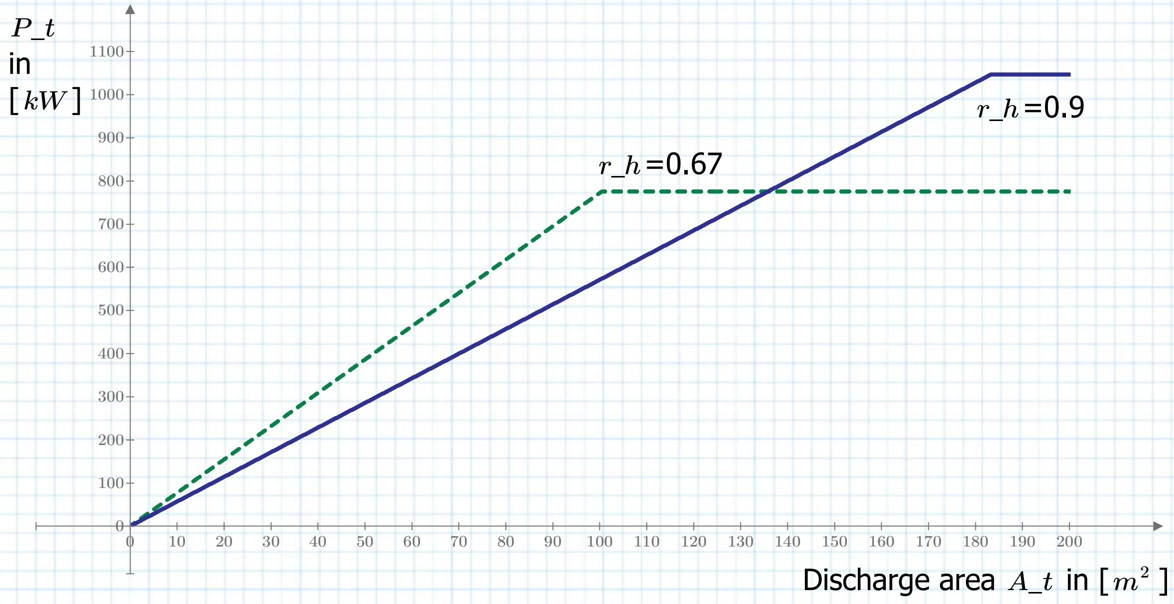
Define design-point 1:

$t\_d1 := 28\% \cdot yr$ ----> $\Delta H\_av(t\_d1) = 0.3\ m$

$Q\_av(t\_d1) = 440\ m^3 \cdot s^{-1}$

Head ratio r_h: 0.90 ; 0.67

Discharge through turbine P_t in $[kW]$ for ——— ; - - - - - - -



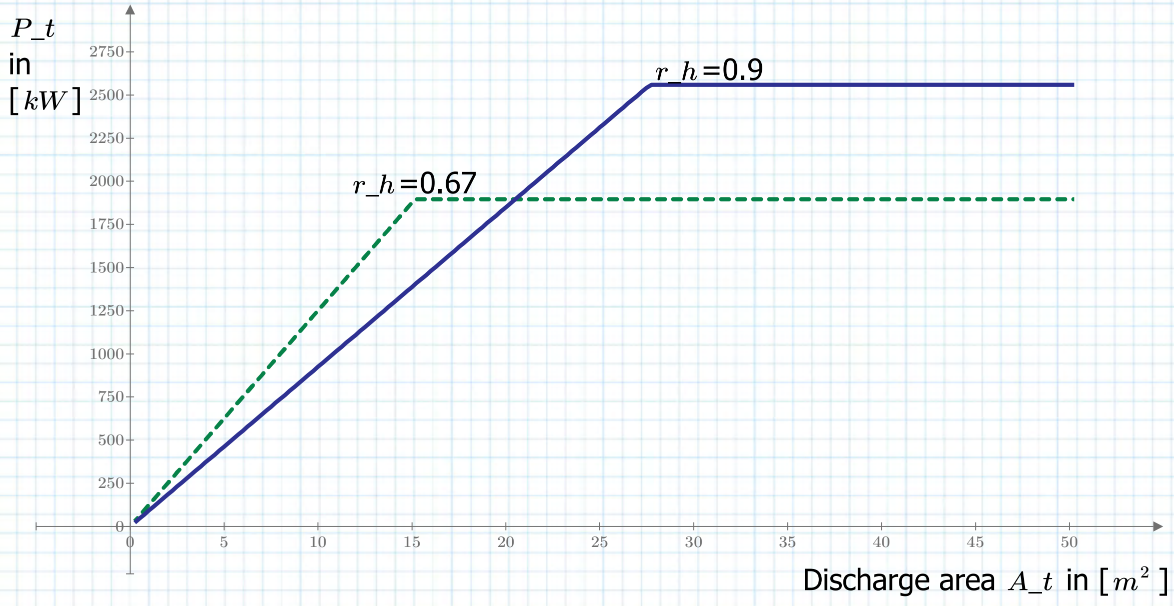
Define design-point 2:

$t\_d2 := 58\% \cdot yr$ ----> $\Delta H\_av(t\_d2) = 1.92\ m$

$Q\_av(t\_d2) = 168\ m^3 \cdot s^{-1}$ Head ratio r_h:

0.90 ; 0.67

Discharge through turbine P_t in $[kW]$ for ——— ; - - - - - - -



Define design-point 3:

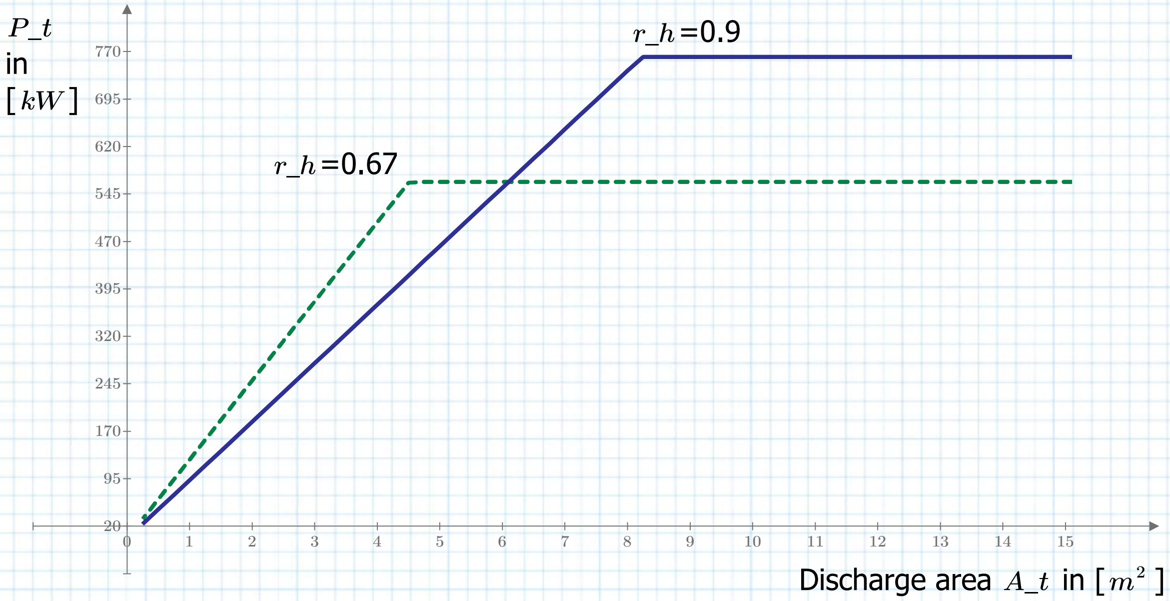
$t\_d3 := 71\% \cdot yr$ ----> $\Delta H\_av(t\_d3) = 1.92\ m$

$Q\_av(t\_d3) = 50\ \frac{m^3}{s}$

Head ratio r_h: 0.90 ; 0.67

Discharge through turbine Q_t in $[m^3 \cdot s^{-1}]$ ———— ; – – – –

Discharge lost through weir Q_w in $[m^3 \cdot s^{-1}]$ ———— ; – – – –



Conclusion(s):

Although the required the area required to let through the same amount of discharge is larger for a higher head ratio r_h, the power that is able to be generated is higher with a higher head-ratio again for all 3 design points.

This is due to the fact that given enough area, the same discharge can be let through, but with a higher head-ratio, the product of head an discharge in the end is larger.

Annual Energy in the particular case for Driel (using the liniarised discharge duration curve):

Theoretically speaking the energy produced would be:

$$E\_ann = \int_{t1=28\%\cdot yr}^{t2=94\%\cdot yr} P\_t\left(Q\_t\left(Q\_av(t),\Delta H\_av(t),r\_h,A\_t\right),\Delta H\_av(t),r\_h,A\_t\right)\,\mathrm{d}t$$

However due to the "conditional" nature of the defined $Q\_av(t)$ and $\Delta H\_av(t)$ mathcad can't solve this on its own. Therefore, a numerical integration approach is used:

Redefine time axis and discharge area to be a vector:

```
$tx :=$ | $a_{round\left(\frac{yr}{day}\right)-1} \leftarrow \text{round}\left(\frac{yr}{day}\right)$
        | for $k \in 0,1..\text{round}\left(\frac{yr}{day}\right)-1$
        |     | $a^{\langle k \rangle} \leftarrow (k+1)\cdot day$
        | return $a$
```

$$= \begin{bmatrix} 1 \\ \vdots \end{bmatrix} day$$

$$A\_t := 0.1\ m^2, 0.2\ m^2 .. 250\ m^2 = \begin{bmatrix} 0.1 \\ \vdots \end{bmatrix} m^2$$

Define integration limits:

$$t1 := \text{round}\left(28\%\cdot\frac{yr}{day}\right)\cdot day = 102\ day$$

$$t2 := \text{round}\left(94\%\cdot\frac{yr}{day}\right)\cdot day = 343\ day$$

```
$E\_a(r\_h) :=$ | $E_{rows(A\_t)-1,0} \leftarrow 0\ kW\cdot hr$
                | for $i \in 0..\text{rows}(A\_t)-1$
                |     | for $j \in 0..\text{rows}(tx)-1$
                |     |     | if $tx_j \ge t1 \wedge tx_j \le t2$
                |     |     |     | $P \leftarrow P\_t\left(Q\_t\left(Q\_av\left(tx_j\right),\Delta H\_av\left(tx_j\right),r\_h,A\_t_i\right),\Delta H\_av\left(tx_j\right),r\_h,A\_t_i\right)$
                |     |     |     | $E_{i,0} \leftarrow E_{i,0} + P\cdot 1\cdot day$
                | return $E$
```

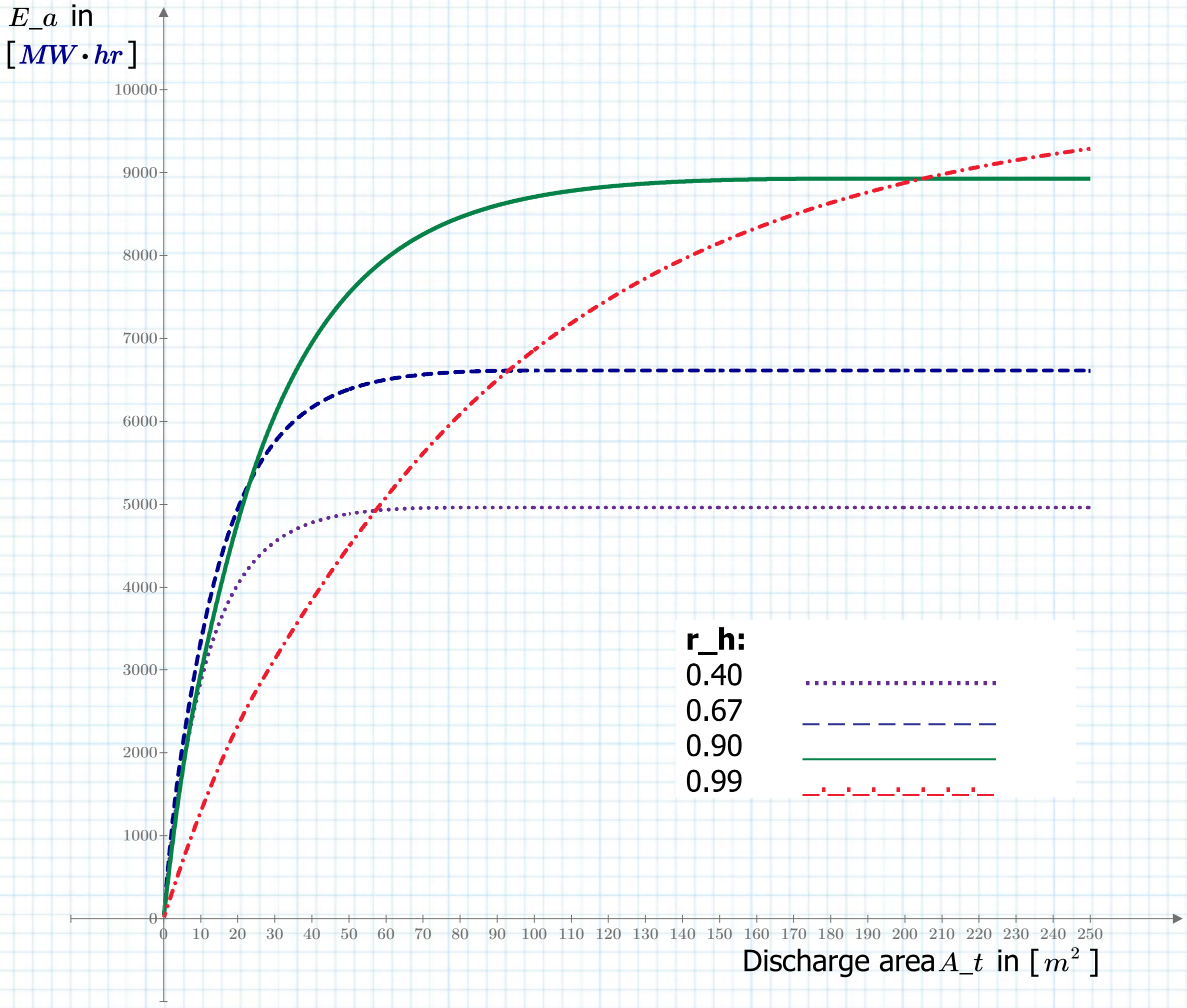
$$E\_a\_067 := E\_a\left(\frac{2}{3}\right) = \begin{bmatrix} 54.618 \\ \vdots \end{bmatrix} MW\cdot hr$$

$$E\_a\_090 := E\_a(0.9) = \begin{bmatrix} 40.482 \\ \vdots \end{bmatrix} MW\cdot hr$$

$$E\_a\_099 := E\_a(0.99) = \begin{bmatrix} 14.134 \\ \vdots \end{bmatrix} MW\cdot hr$$

$$E\_a\_040 := E\_a(0.5) = \begin{bmatrix} 50.125 \\ \vdots \end{bmatrix} MW\cdot hr$$

Plot of Energy per discharge area:



Conclusions:
- For a given head-ratio the energy production goes up with discharge area till a certain point, when it can take all the discharge that is available.
- For certain head-ratios the area goes to very large values. Maurik HPP doesn't have more than $A\_plant\_Maurik := 4 \cdot A.t\_M = 43.197\ m^2$, looking beyond lets say $50\ m^2$ is therefore not interesting. The costs of the powerplant will far exceed Maurik, but the production will be less than half.
- Now areas can be chosen and used for determining the number of turibnes and related dimensions.

Chosen design variants:

Discharge area where Energy curve of r_h=0.67 and r_h=0.90 are equal:

$A\_eql\_67is90 := \text{lookup}\left(0, (E\_a\_090 - E\_a\_067), A\_t, \text{“near”}\right)_{0,0} = 23.4\ m^2$

Discharge area of:

Design variant 1:	$A\_d\_1 := 10\ m^2$
Design variant 2:	$A\_d\_2 := A\_eql\_67is90 = 23.4\ m^2$
Design variant 3:	$A\_d\_3 := 35\ m^2$
Design variant "Copy of Maurik":	$A\_d\_com := 43.2\ m^2$
Design variant 4:	$A\_d\_4 := 50\ m^2$

Annual Energy production estimation for head ratio of 90%:

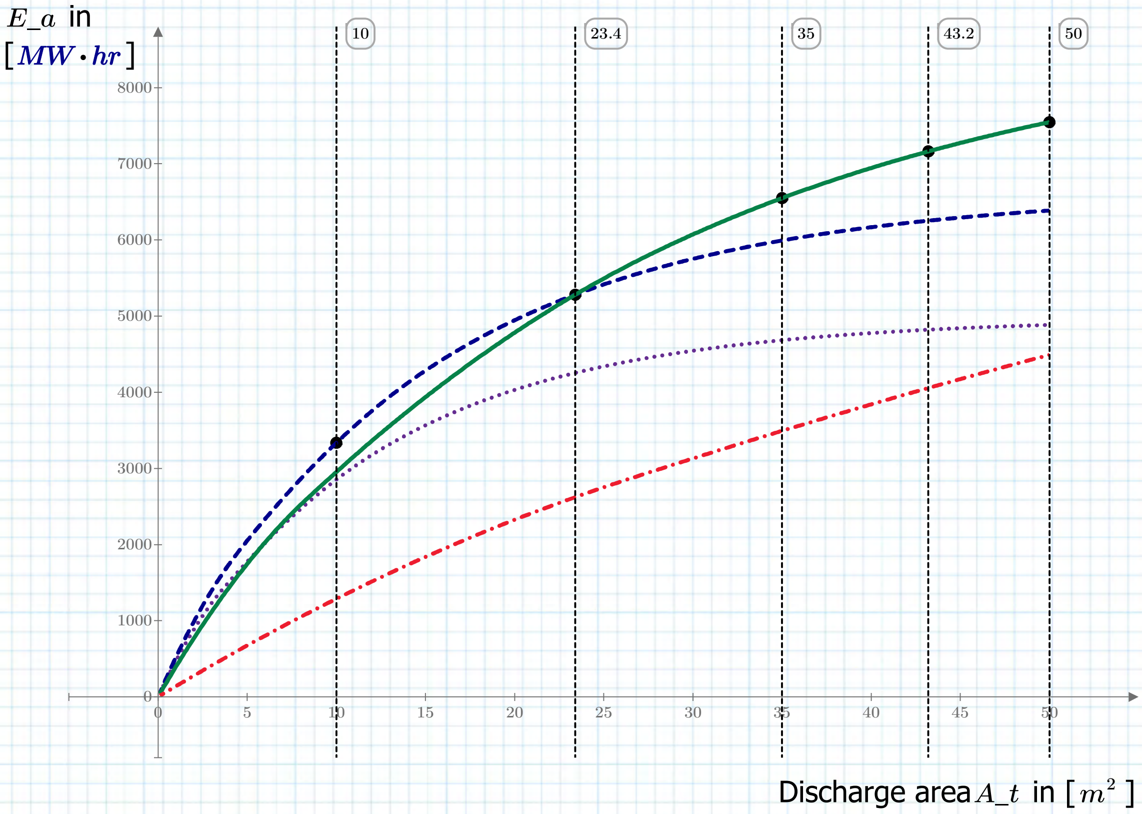
$E\_a\_d\_1 := \text{lookup}\left(A\_d\_1, A\_t, E\_a\_067, \text{“eq”}\right)_{0,0} = 3336\ MW \cdot hr$

$E\_a\_d\_2 := \text{lookup}\left(A\_d\_2, A\_t, E\_a\_090, \text{“eq”}\right)_{0,0} = 5281\ MW \cdot hr$

$E\_a\_d\_3 := \text{lookup}\left(A\_d\_3, A\_t, E\_a\_090, \text{“eq”}\right)_{0,0} = 6549\ MW \cdot hr$

$E\_a\_d\_com := \text{lookup}\left(A\_d\_com, A\_t, E\_a\_090, \text{“eq”}\right)_{0,0} = 7162\ MW \cdot hr$

$E\_a\_d\_4 := \text{lookup}\left(A\_d\_4, A\_t, E\_a\_090, \text{“eq”}\right)_{0,0} = 7546\ MW \cdot hr$



Compared to total available energy:

```
E_av := | for j ∈ 0 .. rows(tx) − 1
        |   if tx_j ≥ t1 ∧ tx_j ≤ t2
        |     | P ← ρ · g · Q_av(tx_j) · ΔH_av(tx_j)
        |     | E ← E + P · 1 · day
        | return E
                                              = 11022 MW · hr
```

Head-ratio optimisation:
With a chosen area we can see what the most optimal head ratio is for the 3 design points. First define range of head ratio:

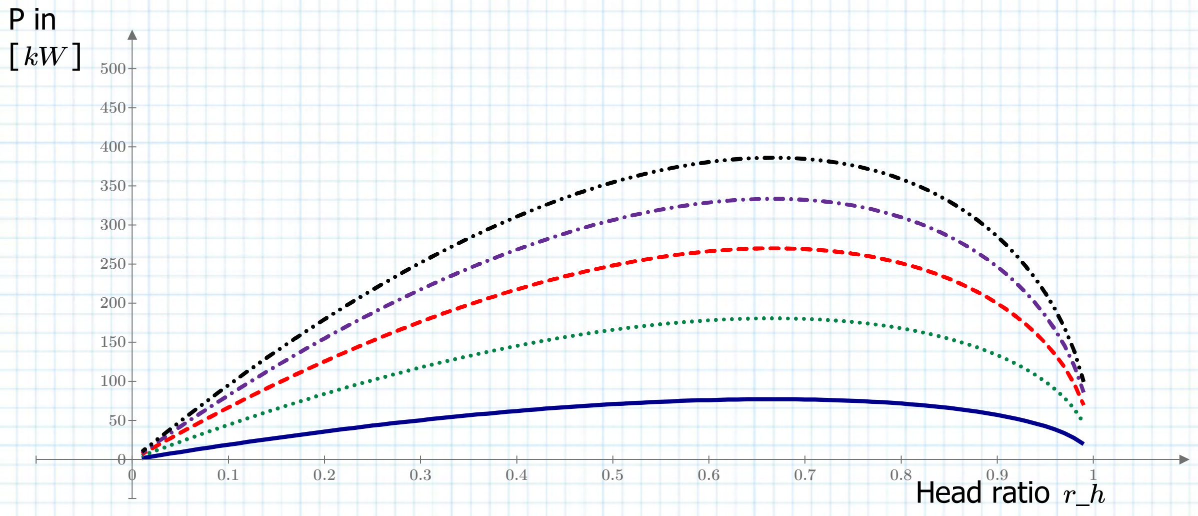
$r\_h\_d := 0.01, 0.02 .. 0.99$

Design-point 1:

$t\_d1 := 28\% \cdot yr$ ----> $\Delta H\_av(t\_d1) = 0.3\ m$

$Q\_av(t\_d1) = 440\ m^3 \cdot s^{-1}$

Power of DV1 P_t in $[kW]$ ————
Power of DV2 P_t in $[kW]$ ··········
Power of DV3 P_t in $[kW]$ – – – – –
Power of DVCOM P_t in $[kW]$ –·–·–·–
Power of DV4 P_t in $[kW]$ –··–··–··



Design-point 2:

$t\_d2 := 58\% \cdot yr$ ----> $\Delta H\_av(t\_d2) = 1.92\ m$

$Q\_av(t\_d2) = 168\ m^3 \cdot s^{-1}$

Power of DV1 P_t in $[kW]$ ————
Power of DV2 P_t in $[kW]$ ··········
Power of DV3 P_t in $[kW]$ – – – – –
Power of DVCOM P_t in $[kW]$ –·–·–·–
Power of DV4 P_t in $[kW]$ –··–··–··

Design-point 3:

$t\_d3 := 71\% \cdot yr$ ----> $\Delta H\_av(t\_d3) = 1.92\ m$

$$Q\_av(t\_d3) = 50\ \frac{m^3}{s}$$

Power of DV1 P_t in $[kW]$ ———

Power of DV2 P_t in $[kW]$ ┈┈┈

Power of DV3 P_t in $[kW]$ – – –

Power of DVCOM P_t in $[kW]$ –·–·–

Power of DV4 P_t in $[kW]$ –··–··–

Conclusions:

- It's clear that with a decreasing discharge it is better to take a higher head-ratio. When the available discharge is larger than a certain discharge the 2/3 value is again optimal for power output. This is something the power plant operator or system can aim for with the inclination of the blades.
- However the head-ratio of 1 can never be obtained. It is limited by the head-losses that will always occur, even at the lowest discharge.

Optimal value of head ratio r_h:

Define available discharge axis:

$$Q\_av2 := 0 \cdot m^3 \cdot s^{-1}, 1 \cdot m^3 \cdot s^{-1} .. 440 \cdot m^3 \cdot s^{-1} = \begin{bmatrix} 0 \\ \vdots \end{bmatrix} \frac{m^3}{s}$$

Define head as function of Q (same interpolation as used at the beginning of this sheet):

```
ΔH_av2(Q) :=  if $Q \le 440 \cdot m^3 \cdot s^{-1} \wedge Q \ge 50 \cdot m^3 \cdot s^{-1}$
                  $H \leftarrow \text{linterp}\left(\left(\left[50 \cdot m^3 \cdot s^{-1} \;\; 440 \cdot m^3 \cdot s^{-1}\right]^T\right), [1.92 \;\; 0.30]^T, Q\right)$
              else if $Q < 50 \cdot m^3 \cdot s^{-1} \wedge Q \ge 0 \cdot m^3 \cdot s^{-1}$
                  $H \leftarrow 1.92$
              return $H \cdot m$
```

Test:

$\Delta H\_av2\left(25 \cdot m^3 \cdot s^{-1}\right) = 1.92\ m$

$\Delta H\_av2\left(50 \cdot m^3 \cdot s^{-1}\right) = 1.92\ m$

$\Delta H\_av2\left(200 \cdot m^3 \cdot s^{-1}\right) = 1.297\ m$

$\Delta H\_av2\left(440 \cdot m^3 \cdot s^{-1}\right) = 0.3\ m$

For a certain discharge through the turbine there is a limit to the head ratio, due to the losses in the pipe system that scales with the velocity head, defined as:

$$\Delta H\_loss = \frac{Q^2}{2\ g \cdot A\_t^2} \cdot \xi.eq\_M$$

So if the entire available discharge goes through the system then the losses are:

$$\Delta H\_loss\_max = \frac{Q\_av^2}{2\ g \cdot A\_t^2} \cdot \xi.eq\_M$$

The head ratio is then:

$$r\_h\_Qmax = \frac{\Delta H\_av - \Delta H\_loss\_max}{\Delta H\_av} = 1 - \frac{\Delta H\_loss\_max}{\Delta H\_av} = 1 - \frac{\xi.eq\_M \cdot Q\_av^2}{2\ g \cdot A\_t^2 \cdot \Delta H\_av}$$

Defined as:

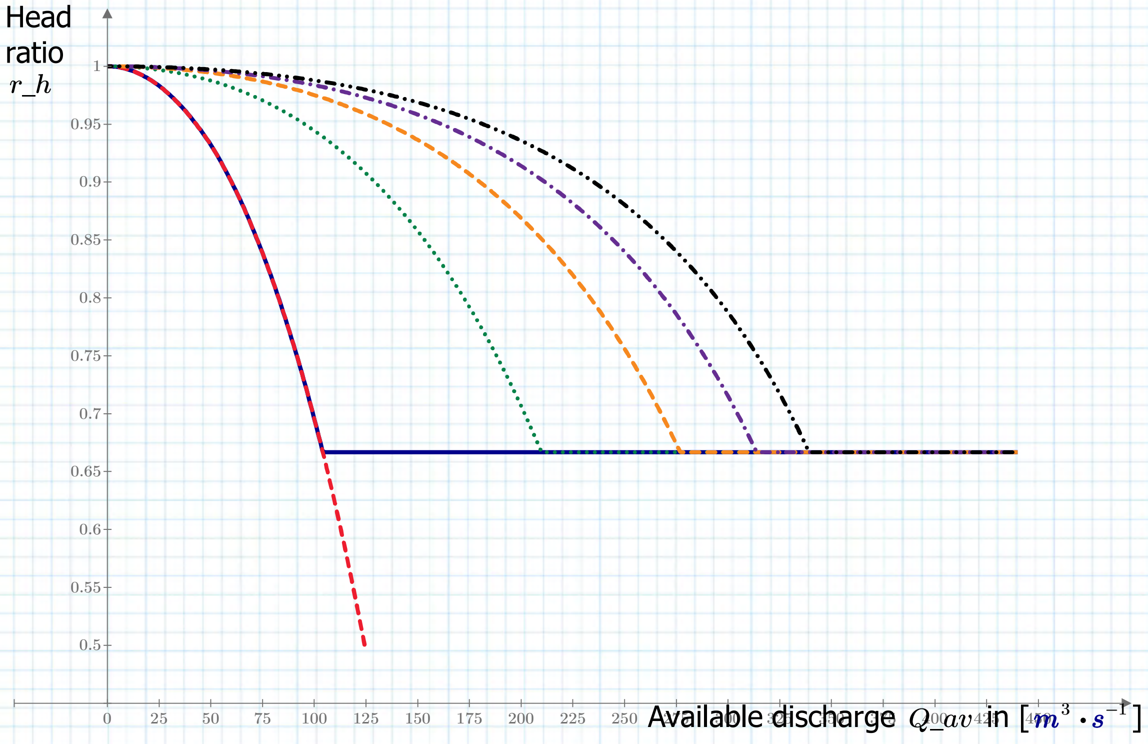
```
r_h_Qmax(Q_av, A_t) :=  if $\text{rows}(Q\_av) > 1$
                            for $i \in 0 .. \text{rows}(Q\_av) - 1$
                                $a_i \leftarrow 1 - \dfrac{\xi.eq\_M \cdot {Q\_av_i}^2}{2\ g \cdot A\_t^2 \cdot \Delta H\_av2\left(Q\_av_i\right)}$
                        else
                            $a \leftarrow 1 - \dfrac{\xi.eq\_M \cdot Q\_av^2}{2\ g \cdot A\_t^2 \cdot \Delta H\_av2(Q\_av)}$
                        return $a$
```

However at some point the head ratio goes lower than the 2/3 value, and for the energy production it should be equal or higher than that, so this is the point at which the resistance of the turbine should be increased such that it reaches a head ratio of 2/3 to get the highest product of head and discharge. The turbine gets more discharge than it can effectively handle.

As is defined on the next page:

Optimal head ratio:

```
$r\_h\_opt(Q\_av, A\_t) :=$ if rows$(Q\_av) > 1$
    for $i \in 0 .. \text{rows}(Q\_av) - 1$
        if $Q\_av_i < \sqrt{\frac{1-\frac{2}{3}}{\xi.eq\_M}} \cdot A\_t \cdot \sqrt{2 \cdot g \cdot \Delta H\_av2\left(Q\_av_i\right)}$
            $f_i \leftarrow 1 - \frac{\xi.eq\_M \cdot Q\_av_i^{\,2}}{A\_t^2 \cdot 2 \cdot g \cdot \Delta H\_av2\left(Q\_av_i\right)}$
        else
            $f_i \leftarrow \frac{2}{3}$
else
    if $Q\_av < \sqrt{\frac{1-\frac{2}{3}}{\xi.eq\_M}} \cdot A\_t \cdot \sqrt{2 \cdot g \cdot \Delta H\_av2(Q\_av)}$
        $f \leftarrow 1 - \frac{\xi.eq\_M \cdot Q\_av^2}{A\_t^2 \cdot 2 \cdot g \cdot \Delta H\_av2(Q\_av)}$
    else
        $f \leftarrow \frac{2}{3}$
return $f$
```



Energy production with optimal values:
Redefine energy calculations:

```
$E\_a\_opt(A\_t) :=$
    for $j \in 0..\mathrm{rows}(tx)-1$
        if $tx_j \ge t1 \wedge tx_j \le t2$
            $Qt \leftarrow Q\_t\left(Q\_av\left(tx_j\right), \Delta H\_av\left(tx_j\right), r\_h\_opt\left(Q\_av\left(tx_j\right), A\_t\right), A\_t\right)$
            $rh \leftarrow r\_h\_opt\left(Q\_av\left(tx_j\right), A\_t\right)$
            $dH \leftarrow \Delta H\_av\left(tx_j\right)$
            $P \leftarrow P\_t(Qt, dH, rh, A\_t)$
            $E \leftarrow E + P \cdot 1 \cdot day$
    return $E$
```

Previously:

$$E\_a\_d\_1 := \mathrm{lookup}(A\_d\_1, A\_t, E\_a\_067, \text{“eq”})_{0,0} = 3336\ MW \cdot hr$$

$$E\_a\_d\_2 := \mathrm{lookup}(A\_d\_2, A\_t, E\_a\_090, \text{“eq”})_{0,0} = 5281\ MW \cdot hr$$

$$E\_a\_d\_3 := \mathrm{lookup}(A\_d\_3, A\_t, E\_a\_090, \text{“eq”})_{0,0} = 6549\ MW \cdot hr$$

$$E\_a\_d\_com := \mathrm{lookup}(A\_d\_com, A\_t, E\_a\_090, \text{“eq”})_{0,0} = 7162\ MW \cdot hr$$

$$E\_a\_d\_4 := \mathrm{lookup}(A\_d\_4, A\_t, E\_a\_090, \text{“eq”})_{0,0} = 7546\ MW \cdot hr$$

Percentual improvement:

$$\frac{E\_a\_DV1\_opt - E\_a\_d\_1}{E\_a\_d\_1} = 10.457\%$$

$$\frac{E\_a\_DV2\_opt - E\_a\_d\_2}{E\_a\_d\_2} = 16.6\%$$

$$\frac{E\_a\_DV3\_opt - E\_a\_d\_3}{E\_a\_d\_3} = 12.379\%$$

$$\frac{E\_a\_DVcom\_opt - E\_a\_d\_com}{E\_a\_d\_com} = 10.672\%$$

$$\frac{E\_a\_DV4\_opt - E\_a\_d\_4}{E\_a\_d\_4} = 9.727\%$$

New values:

$$E\_a\_DV1\_opt := E\_a\_opt(A\_d\_1) = 3685\ MW \cdot hr$$

$$E\_a\_DV2\_opt := E\_a\_opt(A\_d\_2) = 6158\ MW \cdot hr$$

$$E\_a\_DV3\_opt := E\_a\_opt(A\_d\_3) = 7360\ MW \cdot hr$$

$$E\_a\_DVcom\_opt := E\_a\_opt(A\_d\_com) = 7926\ MW \cdot hr$$

$$E\_a\_DV4\_opt := E\_a\_opt(A\_d\_4) = 8280\ MW \cdot hr$$

$$\frac{E\_a\_DV1\_opt}{A\_d\_1} = 368.456\ \frac{MW \cdot hr}{m^2}$$

$$\frac{E\_a\_DV2\_opt}{A\_d\_2} = 263.147\ \frac{MW \cdot hr}{m^2}$$

$$\frac{E\_a\_DV3\_opt}{A\_d\_3} = 210.28\ \frac{MW \cdot hr}{m^2}$$

$$\frac{E\_a\_DVcom\_opt}{A\_d\_com} = 183.472\ \frac{MW \cdot hr}{m^2}$$

$$\frac{E\_a\_DV4\_opt}{A\_d\_4} = 165.607\ \frac{MW \cdot hr}{m^2}$$

Dimensioning and Discharge through power plant

The maximum discharge through the turbine is:

```
$Qtmax(A\_t) :=$
    $Qm \leftarrow Q\_t\left(Q\_av\left(tx_0\right), \Delta H\_av\left(tx_0\right), r\_h\_opt\left(Q\_av\left(tx_0\right), A\_t\right), A\_t\right)$
    for $j \in 0..\text{rows}(tx)-1$
        $Q \leftarrow Q\_t\left(Q\_av\left(tx_j\right), \Delta H\_av\left(tx_j\right), r\_h\_opt\left(Q\_av\left(tx_j\right), A\_t\right), A\_t\right)$
        if $Q > Qm$
            $Qm \leftarrow Q$
    $Qm$
```

Maximum:

$$Qtmax\_dv\_1 := Qtmax(A\_d\_1) = 110.84\ \frac{m^3}{s}$$

$$Qtmax\_dv\_2 := Qtmax(A\_d\_2) = 232.132\ \frac{m^3}{s}$$

$$Qtmax\_dv\_3 := Qtmax(A\_d\_3) = 298.767\ \frac{m^3}{s}$$

$$Qtmax\_dv\_com := Qtmax(A\_d\_com) = 333.571\ \frac{m^3}{s}$$

$$Qtmax\_dv\_4 := Qtmax(A\_d\_4) = 356.264\ \frac{m^3}{s}$$

When less than or equal to 15m3/s cannot be taken by the turbine, the amount of energy that is lost is less than 100 MW hours. Comparatively, if the same is done for 50m3/s, then almost 958 MW hours are lost. 15m3/s therefore seems a good cut-off discharge.
Assuming a turbine turns of at discharges below 20% of its maximum discharge, a minimum amount of turbines can be determined:

```
$n\_t\_min(Q\_20p) :=$
    $n \leftarrow 1$
    $Qmin \leftarrow Q\_20p$
    while $Qmin > 16\ m^3 \cdot s^{-1}$
        $n \leftarrow n + 1$
        $Qmin \leftarrow \text{round}\left(\frac{Q\_20p}{n \cdot m^3 \cdot s^{-1}}, 0\right) \cdot m^3 \cdot s^{-1}$
    return $n$
```

$$Q\_20p\_dv1 := 20\% \cdot Qtmax\_dv\_1 = 22.168\ \frac{m^3}{s} \quad \text{--->} \quad n\_t\_min(Q\_20p\_dv1) = 2$$

$$Q\_20p\_dv2 := 20\% \cdot Qtmax\_dv\_2 = 46.426\ \frac{m^3}{s} \quad \text{--->} \quad n\_t\_min(Q\_20p\_dv2) = 3$$

$$Q\_20p\_dv3 := 20\% \cdot Qtmax\_dv\_3 = 59.753\ \frac{m^3}{s} \quad \text{--->} \quad n\_t\_min(Q\_20p\_dv3) = 4$$

$$Q\_20p\_dvcom := 20\% \cdot Qtmax\_dv\_com = 66.714\ \frac{m^3}{s} \quad \text{--->} \quad n\_t\_min(Q\_20p\_dvcom) = 5$$

Although Maurik of course has 4 turbines, so $n\_t\_Maurik := 4$

$$Q\_20p\_dv4 := 20\% \cdot Qtmax\_dv\_4 = 71.253\ \frac{m^3}{s} \quad \text{--->} \quad n\_t\_min(Q\_20p\_dv4) = 5$$

Determining Diameter for Kaplan

The minimum discharges and discharge area per turbine then becomes:

$$\frac{Q\_20p\_dv1}{n\_t\_min(Q\_20p\_dv1)} = 11.084\ \frac{m^3}{s} \qquad A\_t\_d\_1 := \frac{A\_d\_1}{n\_t\_min(Q\_20p\_dv1)} = 5\ m^2$$

$$\frac{Q\_20p\_dv2}{n\_t\_min(Q\_20p\_dv2)} = 15.475\ \frac{m^3}{s} \qquad A\_t\_d\_2 := \frac{A\_d\_2}{n\_t\_min(Q\_20p\_dv2)} = 7.8\ m^2$$

$$\frac{Q\_20p\_dv3}{n\_t\_min(Q\_20p\_dv3)} = 14.938\ \frac{m^3}{s} \qquad A\_t\_d\_3 := \frac{A\_d\_3}{n\_t\_min(Q\_20p\_dv3)} = 8.75\ m^2$$

$$\frac{Q\_20p\_dvcom}{n\_t\_Maurik} = 16.679\ \frac{m^3}{s} \qquad A\_t\_d\_com := \frac{A\_d\_com}{n\_t\_Maurik} = 10.8\ m^2$$

$$\frac{Q\_20p\_dv4}{n\_t\_min(Q\_20p\_dv4)} = 14.251\ \frac{m^3}{s} \qquad A\_t\_d\_4 := \frac{A\_d\_4}{n\_t\_min(Q\_20p\_dv4)} = 10\ m^2$$

Defining formula for diameter:

$$D\_out(A\_t) := \sqrt{\frac{A\_t \cdot 4}{\pi \cdot \left(1 - \left(\frac{1.6}{4.0}\right)^2\right)}}$$

$$D\_out(A\_t\_d\_1) = 2.753\ m \qquad \text{Round}(D\_out(A\_t\_d\_1), 0.1\ m) = 2.8\ m$$

$$D\_out(A\_t\_d\_2) = 3.438\ m \qquad \text{Round}(D\_out(A\_t\_d\_2), 0.1\ m) = 3.4\ m$$

$$D\_out(A\_t\_d\_3) = 3.642\ m \qquad \text{Round}(D\_out(A\_t\_d\_3), 0.1\ m) = 3.6\ m$$

$$D\_out(A\_t\_d\_com) = 4.046\ m \qquad \text{Round}(D\_out(A\_t\_d\_com), 0.1\ m) = 4\ m$$

$$D\_out(A\_t\_d\_4) = 3.893\ m \qquad \text{Round}(D\_out(A\_t\_d\_4), 0.1\ m) = 3.9\ m$$

AST:

The AST and Kaplan Turbine don't have the same way of conveying discharge, so the discharge for the Kaplan can't be used to estimate the discharge for the AST. However, the total discharge for the Kaplan variants can be used estimate the number of turbines:

$$Q\_ast\_ref := 10\ m^3 \cdot s^{-1}$$

$$n\_ast\_dv\_1 := \text{Round}\left(\frac{Qtmax\_dv\_1}{Q\_ast\_ref}, 1\right) = 11$$

$$n\_ast\_dv\_2 := \text{Round}\left(\frac{Qtmax\_dv\_2}{Q\_ast\_ref}, 1\right) = 23$$

$$n\_ast\_dv\_3 := \text{Round}\left(\frac{Qtmax\_dv\_3}{Q\_ast\_ref}, 1\right) = 30$$

$$n\_ast\_dv\_com := \text{Round}\left(\frac{Qtmax\_dv\_com}{Q\_ast\_ref}, 1\right) = 33$$

$$n\_ast\_dv\_4 := \text{Round}\left(\frac{Qtmax\_dv\_4}{Q\_ast\_ref}, 1\right) = 36$$

Looking at reference years

File path:

$\mathit{FilePath} := \text{“C:\textbackslash Users\textbackslash vanerps6413\textbackslash OneDrive – ARCADIS\textbackslash 061 Flow and waterlevel data\textbackslash 01 D...}$

File name:

$\mathit{FileName} := \text{“002 – OUTPUT–MATHCAD – Datalink QH–t–series for E–calc – v01.xlsx”}$

Load data:

$\mathit{DatasetWtDrAv} := \mathrm{READEXCEL}\left(\mathrm{concat}\left(\mathit{FilePath}, \mathit{FileName}\right), \text{“Reference years!A2:F366”}, 0\right)$

Define data for each year:

$\mathit{Qwet} := \mathit{DatasetWtDrAv}^{\langle 0 \rangle} \cdot m^3 \cdot s^{-1}$

$\mathit{Qdry} := \mathit{DatasetWtDrAv}^{\langle 2 \rangle} \cdot m^3 \cdot s^{-1}$

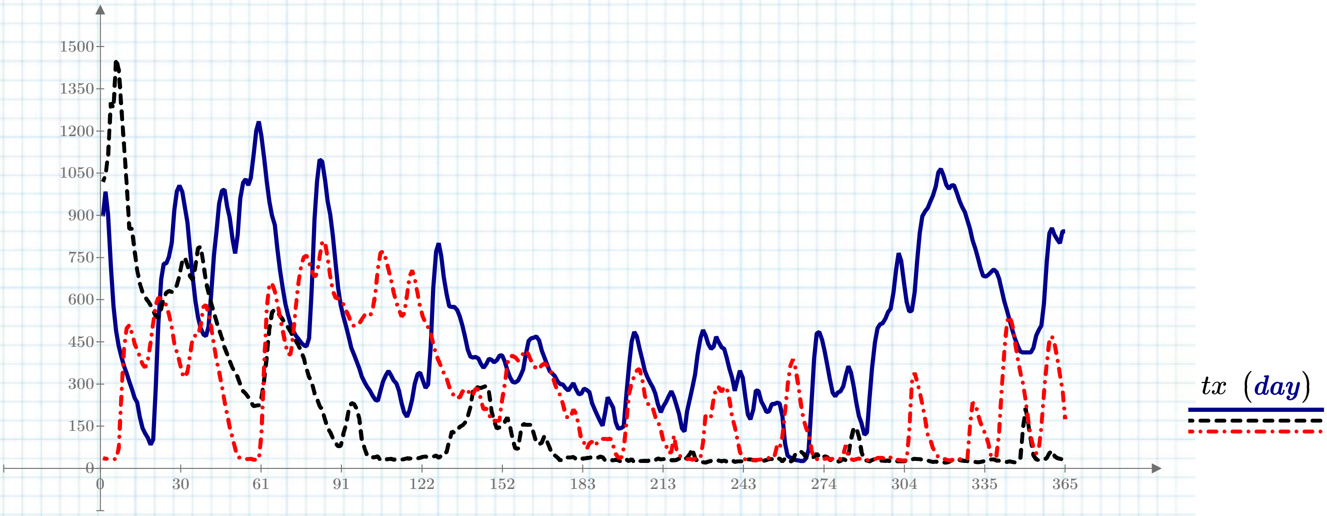
$\mathit{Qavg} := \mathit{DatasetWtDrAv}^{\langle 4 \rangle} \cdot m^3 \cdot s^{-1}$

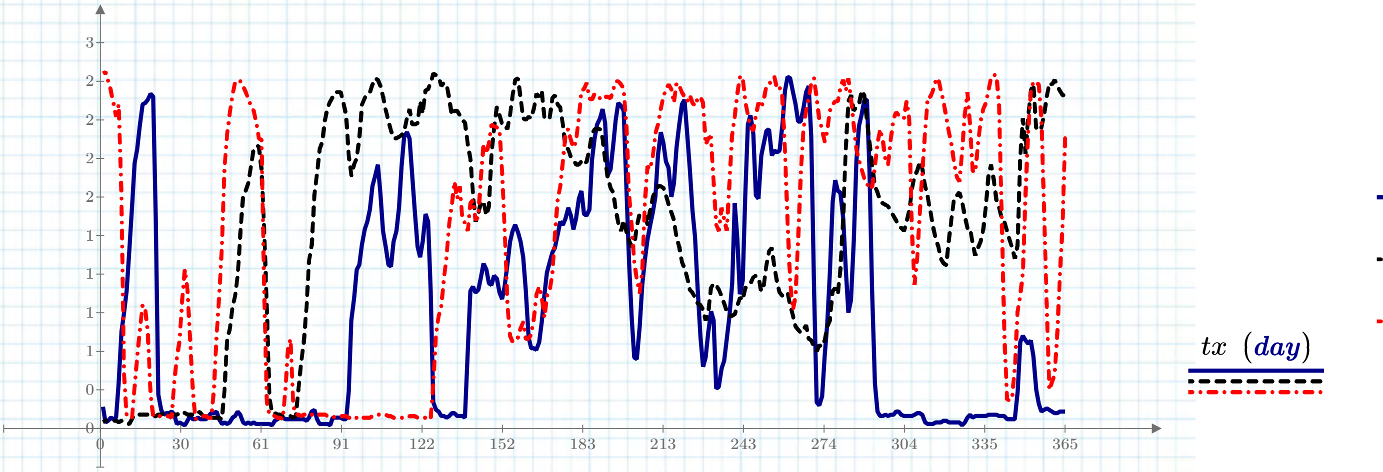
$\mathit{Hwet} := \mathit{DatasetWtDrAv}^{\langle 1 \rangle} \cdot m$

$\mathit{Hdry} := \mathit{DatasetWtDrAv}^{\langle 3 \rangle} \cdot m$

$\mathit{Havg} := \mathit{DatasetWtDrAv}^{\langle 5 \rangle} \cdot m$

Plot together to check data loading:





Power production with optimised head ratio:

```
$P\_t\_opt(A\_t, Q\_av, \Delta H\_av) :=$
    for $j \in 0 .. \text{rows}(tx) - 1$
        if $Q\_av_j - 25\ (m^3 \cdot s^{-1}) < 0 \cdot m^3 \cdot s^{-1}$
            $Q \leftarrow 0 \cdot m^3 \cdot s^{-1}$
        else
            $Q \leftarrow Q\_av_j - 25\ (m^3 \cdot s^{-1})$
        if $\Delta H\_av_j > 0.3\ m$
            $dH \leftarrow \Delta H\_av_j$
        else
            $dH \leftarrow 0\ m$
        $rh \leftarrow r\_h\_opt(Q, A\_t)$
        $Qt \leftarrow Q\_t(Q, dH, rh, A\_t)$
        $P_j \leftarrow P\_t(Qt, dH, rh, A\_t)$
        $E \leftarrow E + P_j \cdot 1 \cdot day$
    return $P$
```

For the wet year for each design variant:

$P\_t\_wet\_dv\_1 := P\_t\_opt(A\_d\_1, Qwet, Hwet)$
$P\_t\_wet\_dv\_2 := P\_t\_opt(A\_d\_2, Qwet, Hwet)$
$P\_t\_wet\_dv\_3 := P\_t\_opt(A\_d\_3, Qwet, Hwet)$
$P\_t\_wet\_dv\_com := P\_t\_opt(A\_d\_com, Qwet, Hwet)$
$P\_t\_wet\_dv\_4 := P\_t\_opt(A\_d\_4, Qwet, Hwet)$

$P\_t\_wet := \text{augment}(P\_t\_wet\_dv\_1, P\_t\_wet\_dv\_2, P\_t\_wet\_dv\_3, P\_t\_wet\_dv\_com, P\_t\_wet\_\ldots$

For the dry year:

$P\_t\_dry\_dv\_1 := P\_t\_opt(A\_d\_1, Qdry, Hdry)$
$P\_t\_dry\_dv\_2 := P\_t\_opt(A\_d\_2, Qdry, Hdry)$
$P\_t\_dry\_dv\_3 := P\_t\_opt(A\_d\_3, Qdry, Hdry)$
$P\_t\_dry\_dv\_com := P\_t\_opt(A\_d\_com, Qdry, Hdry)$
$P\_t\_dry\_dv\_4 := P\_t\_opt(A\_d\_4, Qdry, Hdry)$

$P\_t\_dry := \text{augment}(P\_t\_dry\_dv\_1, P\_t\_dry\_dv\_2, P\_t\_dry\_dv\_3, P\_t\_dry\_dv\_com, P\_t\_dry\_\ldots$

And for the average yeaer:

$P\_t\_avg\_dv\_1 := P\_t\_opt(A\_d\_1, Qavg, Havg)$
$P\_t\_avg\_dv\_2 := P\_t\_opt(A\_d\_2, Qavg, Havg)$
$P\_t\_avg\_dv\_3 := P\_t\_opt(A\_d\_3, Qavg, Havg)$
$P\_t\_avg\_dv\_com := P\_t\_opt(A\_d\_com, Qavg, Havg)$
$P\_t\_avg\_dv\_4 := P\_t\_opt(A\_d\_4, Qavg, Havg)$

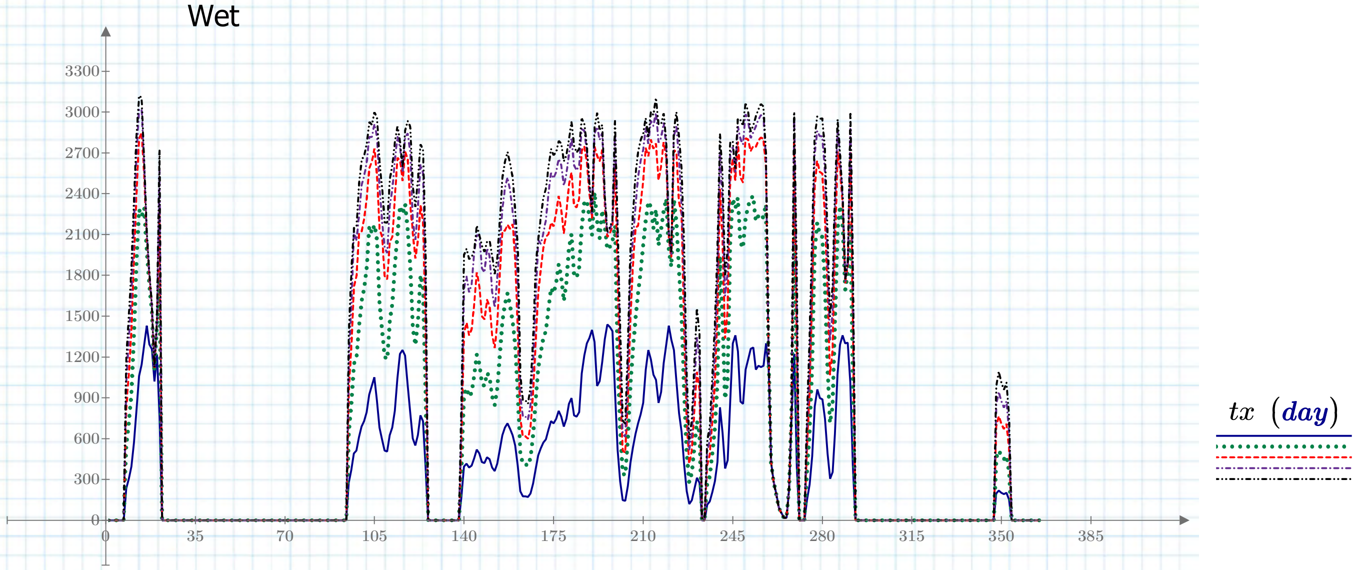
$P\_t\_avg := \text{augment}(P\_t\_avg\_dv\_1, P\_t\_avg\_dv\_2, P\_t\_avg\_dv\_3, P\_t\_avg\_dv\_com, P\_t\_avg\_\ldots$

Export to excel:

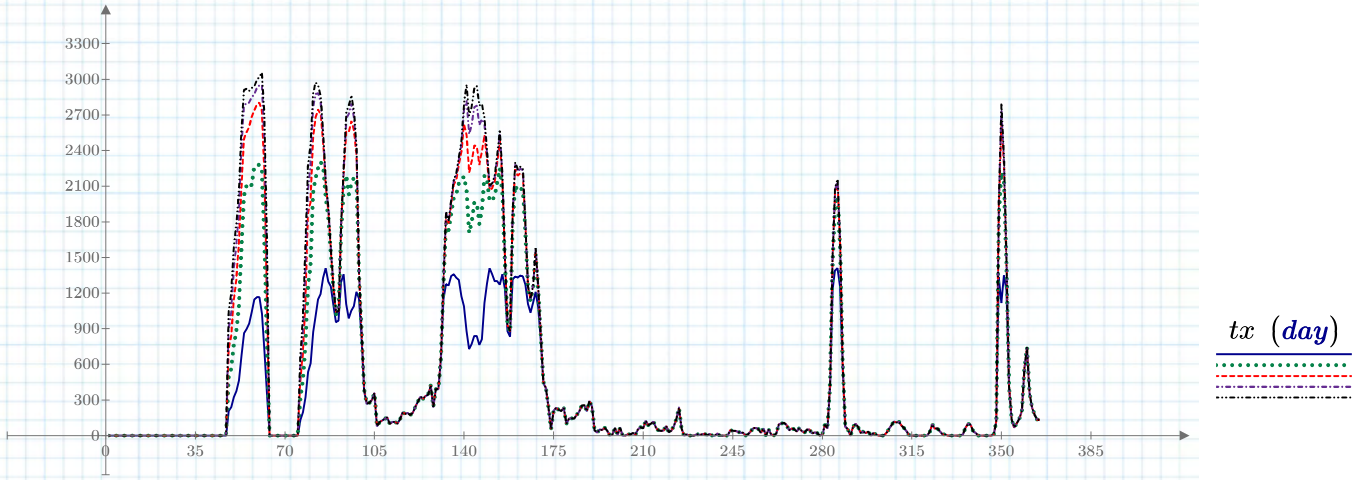
$$Output\_wet := \text{WRITEEXCEL}\left(\text{concat}(FilePath, FileName), \frac{P\_t\_wet}{kW}, \text{“Reference years!K2:O366”}\right)$$
$$Output\_dry := \text{WRITEEXCEL}\left(\text{concat}(FilePath, FileName), \frac{P\_t\_dry}{kW}, \text{“Reference years!Q2:U366”}\right)$$
$$Output\_avg := \text{WRITEEXCEL}\left(\text{concat}(FilePath, FileName), \frac{P\_t\_avg}{kW}, \text{“Reference years!W2:AA366”}\right)$$

Plots of power per design variant:

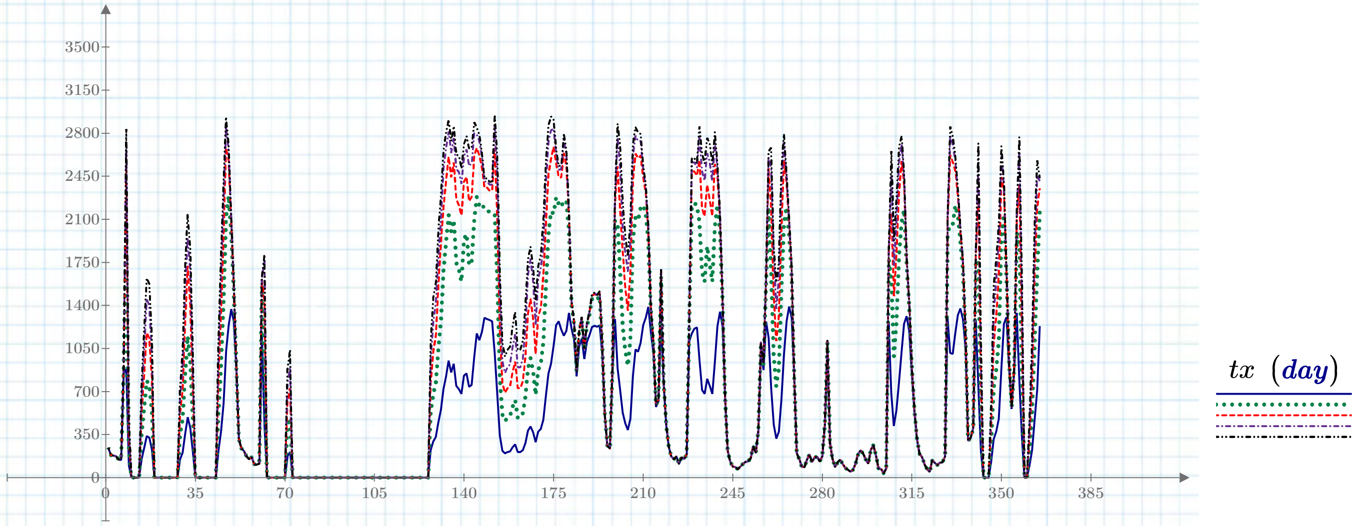
Wet



Dry



Average



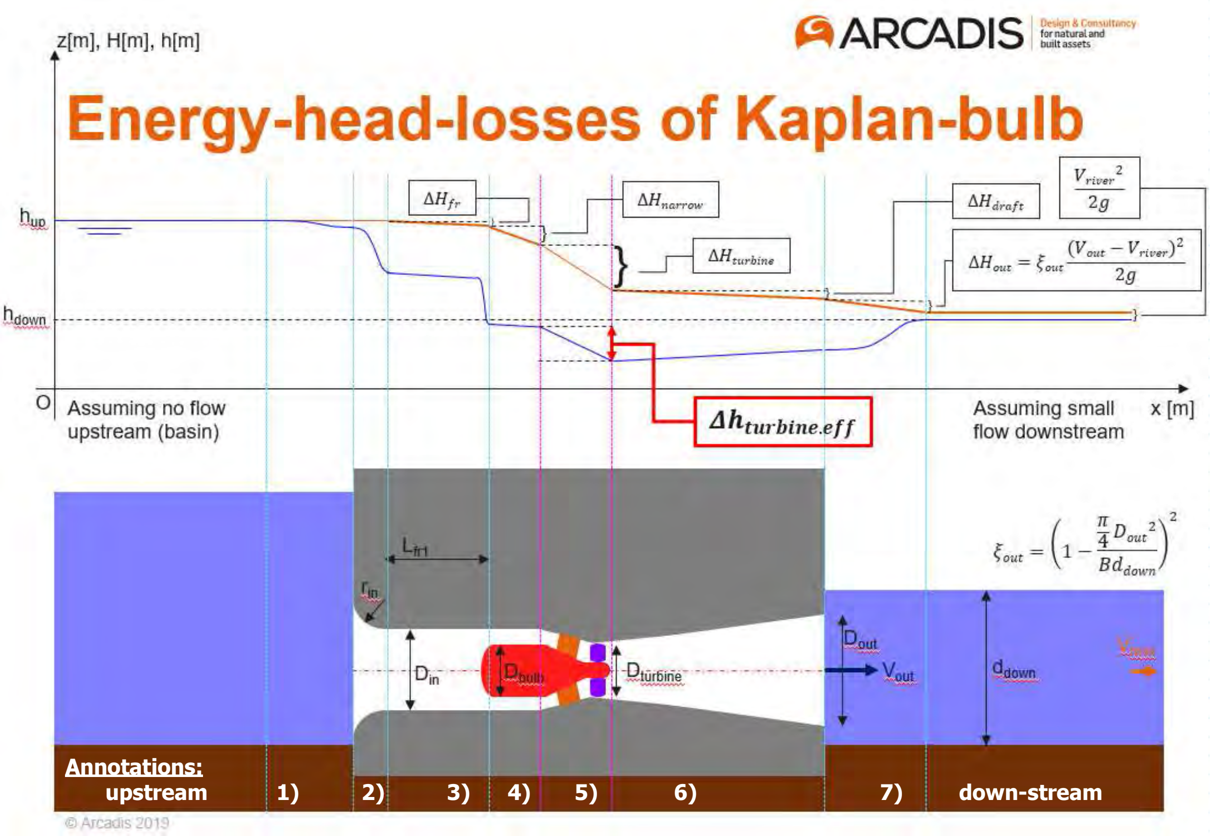
APPENDIX 16 – HYDRAULIC MODEL KAPLAN-BULB

– see inserted page(s) behind this page –

Hydraulic model Regular Kaplan Bulb turbines

Physical constants:

$\rho := 998.2\ kg \cdot m^{-3}$	Density of water at 20°C (assumed constant)
$g = 9.81\ m \cdot s^{-2}$	Gravitational acceleration (assumed constant)
$\mu := 1.002 \cdot 10^{-3} \cdot \frac{kg}{m \cdot s} = 0\ \frac{kg}{m \cdot s}$	Dynamic viscosity at 20°C is 1,002 mPa = 1,002*10-3 Pa
$\nu := \frac{\mu}{\rho} = 0\ \frac{m^2}{s}$	(Kinematic viscosity also assumed constant, value for temperature of 20°C)
$z.axis := 3\ m$ +NAP	Reference level



Discharge Q is equal everywhere, so cross-sectional area A determines the flow-velocity u and the sum of ΔH cannot be larger than the present head.

Geometry of the turbines

Diameters of the design variants 0 till 4 (5 variants, where variant 0 is a copy of Maurik):

$$D\_t\_dv := \begin{bmatrix} 4.0 & 2.8 & 3.4 & 3.6 & 3.9 \end{bmatrix}^T\ m$$

Number of turbines:

$$n\_t := \begin{bmatrix} 4 & 2 & 3 & 4 & 5 \end{bmatrix}^T$$

Diameter of rotor shaft/attachement of rotor blades

$$D\_in := \frac{1.6}{4.0} \cdot D\_t\_dv \qquad D\_in^T = \begin{bmatrix} 1.6 & 1.12 & 1.36 & 1.44 & 1.56 \end{bmatrix}\ m$$

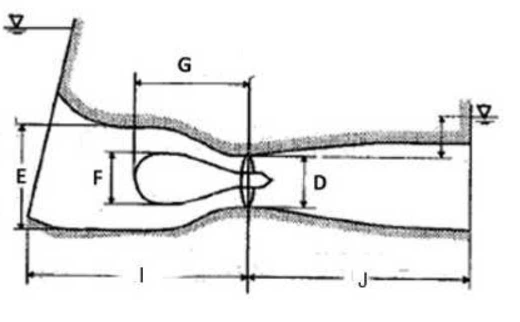
Geometry of the turbines (continued)
Discharge area. Cross-sectional area for flow:

```
A_t := for $i \in 0 .. \text{rows}(D\_t\_dv) - 1$
           $A_i \leftarrow \frac{\pi}{4} \cdot \left(D\_t\_dv_i^2 - D\_in_i^2\right)$
       return $A$
```

$$A\_t^{T} = \begin{bmatrix} 10.56 & 5.17 & 7.63 & 8.55 & 10.03 \end{bmatrix} \, m^2$$

In principle the resistance should be as low as possible so the size should be the largest value. The location will limit this, but also the design is not fixed in any way except for the turbine diameter. Hence:

Size: 0 to 100% within the given margins as shown in image 1b):

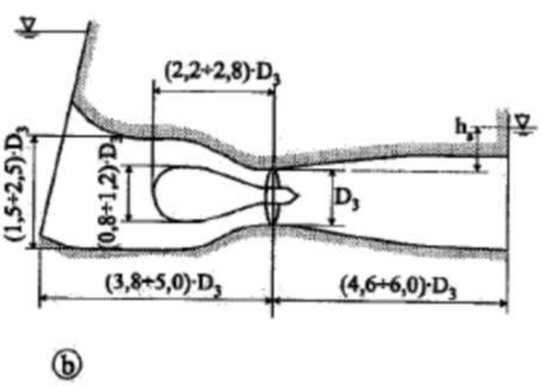


1a) Bulb turbine dimensions

$Size := 100$
Diameter of the inflow opening:
$D\_infl := \left(1.5 + \frac{Size}{100} \cdot (1)\right) \cdot D\_t\_dv$

Diameter of the bulb:
$D\_bulb := \left(0.8 + \frac{Size}{100} \cdot (0.4)\right) \cdot D\_t\_dv$

Length of the inflow till rotor blades
$L\_in := \left(3.8 + \frac{Size}{100} \cdot (5 - 3.8)\right) \cdot D\_t\_dv$



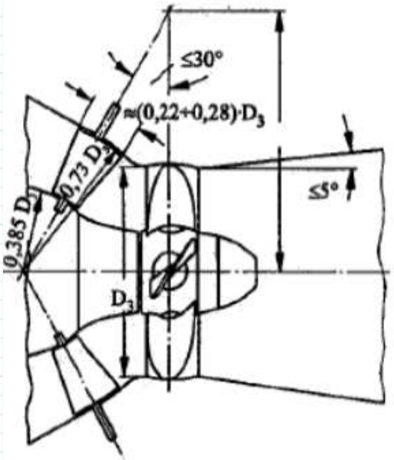
1b) Bulb turbine dimension ranges

Length of the outflow starting from the rotor blades:
$L\_out := \left(4.6 + \frac{Size}{100} \cdot (6 - 4.6)\right) \cdot D\_t\_dv$

Length of the bulb till the rotor blades
$L\_bulb := \left(2.2 + 0.6 \cdot \frac{Size}{100}\right) \cdot D\_t\_dv$

Radius of the inflow rounding:
$r\_in := \text{Round}\left(0.3 \; D\_infl \cdot m^{-1}, 0.1\right) \cdot m$

To reduce inflow losses to a minimum the radius is 0.3 times the diameter of the inflow pipe and rounded to a multiple of 10cm to have practical values.



1c) detail dimension ranges

Draft tube expansion angle:
$\beta\_draft := 5°$
Having the steepest possible angle give the lowest outflow losses, but makes the expansion losses larger.

Diameter of outflow:
$D\_outfl := D\_t\_dv + 2 \cdot \tan(\beta\_draft) \cdot L\_out$

To summerize the geometry:

$L\_total := L\_in + L\_out$

$GEO := \mathrm{augment}\left(D\_t\_dv, D\_in, D\_infl, r\_in, D\_bulb, D\_outfl, L\_in, L\_out, L\_bulb, L\_total\right)$

Collumns show:

$$GEO = \begin{array}{cccccccccc|c} \text{D\_out} & \text{D\_in} & \text{D\_infl} & \text{r\_in} & \text{D\_blb} & \text{D\_outfl} & \text{L\_infl} & \text{L\_out} & \text{L\_blb} & \text{L\_total} & \\ 4.00 & 1.60 & 10.00 & 3.00 & 4.80 & 8.20 & 20.00 & 24.00 & 11.20 & 44.00 & \text{DV0} \\ 2.80 & 1.12 & 7.00 & 2.10 & 3.36 & 5.74 & 14.00 & 16.80 & 7.84 & 30.80 & \text{DV1} \\ 3.40 & 1.36 & 8.50 & 2.60 & 4.08 & 6.97 & 17.00 & 20.40 & 9.52 & 37.40 & \text{DV2} \\ 3.60 & 1.44 & 9.00 & 2.70 & 4.32 & 7.38 & 18.00 & 21.60 & 10.08 & 39.60 & \text{DV3} \\ 3.90 & 1.56 & 9.75 & 2.90 & 4.68 & 7.99 & 19.50 & 23.40 & 10.92 & 42.90 & \text{DV4} \end{array} \; m \qquad n\_t = \begin{bmatrix} 4 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$$

Land use:

$A\_landuse := \overrightarrow{n\_t \cdot D\_infl \cdot L\_total}$

$A\_landuse^{\mathrm{T}} = \begin{bmatrix} 1760 & 431.2 & 953.7 & 1425.6 & 2091.38 \end{bmatrix} m^2$

$B\_landuse := \overrightarrow{1\ m + n\_t \cdot (1\ m + D\_infl)}$

$B\_landuse^{\mathrm{T}} = \begin{bmatrix} 45 & 17 & 29.5 & 41 & 54.75 \end{bmatrix} m$

Reristance of the flow

To determine the quadratic discharge coefficient all the loss coefficients and their related cross-sectional area need to be determined, as the **Quadratic Discharge Coefficient (QDC)** is defined as:

$$C = \frac{1}{2\ g} \cdot \sum_{i=0}^{N} \frac{\xi_i}{{A_i}^2}$$

So the ξ's and A's need to be determined.
They are calculated for:
1. trash rack
2. inflow
3. friction inflow pipe
4. contraction
5. friction bulb
6. turbine
7. expansion draft tube
8. friction draft turbe
9. outflow

They occur at "x-locations":

```
$x :=$ for $j \in 0..8$
    $x_{0,j} \leftarrow j$
    for $i \in 0..\mathrm{rows}(n\_t) - 1$
        $x_{i+1,0} \leftarrow -2\ m$
        $x_{i+1,1} \leftarrow 0\ m$
        $x_{i+1,2} \leftarrow r\_in_i$
        $x_{i+1,3} \leftarrow L\_in_i - L\_bulb_i$
        $x_{i+1,4} \leftarrow L\_in_i - L\_bulb_i + \frac{D\_bulb_i}{2}$
        $x_{i+1,5} \leftarrow L\_in_i - \frac{D\_t\_dv_i}{4}$
        $x_{i+1,6} \leftarrow L\_in_i$
        $x_{i+1,7} \leftarrow L\_in_i + L\_out_i$
        $x_{i+1,8} \leftarrow L\_in_i + L\_out_i + 2\ m$
  return $x$
```

$$x = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ -2\ m & 0\ m & 3\ m & 8.8\ m & 11.2\ m & 19\ m & 20\ m & 44\ m & 46\ m \\ -2\ m & 0\ m & 2.1\ m & 6.16\ m & 7.84\ m & 13.3\ m & 14\ m & 30.8\ m & 32.8\ m \\ -2\ m & 0\ m & 2.6\ m & 7.48\ m & 9.52\ m & 16.15\ m & 17\ m & 37.4\ m & 39.4\ m \\ -2\ m & 0\ m & 2.7\ m & 7.92\ m & 10.08\ m & 17.1\ m & 18\ m & 39.6\ m & 41.6\ m \\ -2\ m & 0\ m & 2.9\ m & 8.58\ m & 10.92\ m & 18.53\ m & 19.5\ m & 42.9\ m & 44.9\ m \end{bmatrix}$$

Trash rack (section 1):

As general formula the trash rack loss coefficient is defined as:
$\xi\_tr = \beta\_tr \cdot \zeta\_tr \cdot c\_tr \cdot sin(\delta\_tr)$

Where:

$\beta\_tr =$	the "rack" coefficient, which is actually more a shape coefficient
$\zeta\_tr =$	the gap geometry coefficient
$c\_tr =$	the "cleaning method" coefficient, determined by how the rack will be cleaned
$\delta\_tr =$	the inclination angle of the rack

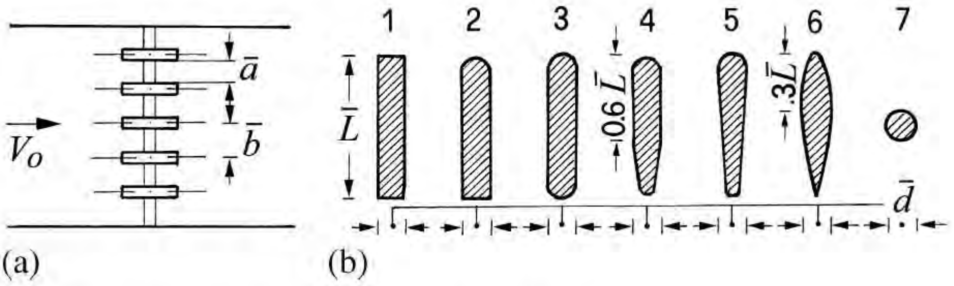


Fig. 2.18 (**a**) Plan of the rack bars, (**b**) Types of rack bars

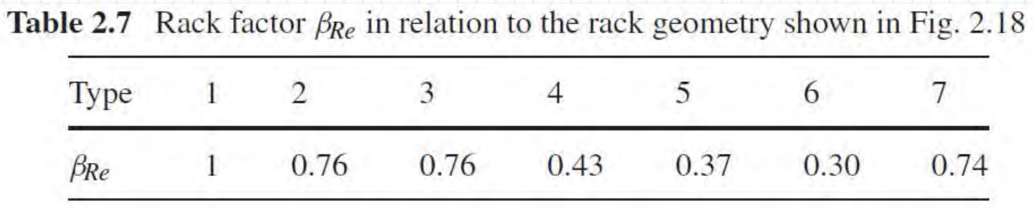


Table 2.7 Rack factor β_{Re} in relation to the rack geometry shown in Fig. 2.18

Type	1	2	3	4	5	6	7
β_{Re}	1	0.76	0.76	0.43	0.37	0.30	0.74

Figure and table above are from pages 45 and 44 (chapter 2) respectively.
[1] Source: W.H. Hager, Wastewater Hydraulics - Theory and Practice, 2nd ed., DOI 10.1007/978-3-642-11383-3_2, Springer-Verlag, Berlin Heidelberg 2010

The following assumptions are made regarding the trash-rack:

1. Rounded bars are used (type 3 from the previous page) with a L/d ratio of about 5.
$\beta\_tr := 0.76$
2. The gap ratio a/b is taken to be larger than 0.5.
3. If for example a/b=0.8 and the gaps are 1cm then the centre to centre distance is 1.25cm and the bar thickness needs to be 0.25cm and L=1.25cm. Comparing with images from existing trash-racks, these seem realistic values. thus:
$r\_ab := 0.8$
3. Assumptions 1 and 2 mean that Idel'cik's simplified formula can be used:

$$\xi\_tr = \frac{7}{3} \beta\_tr \cdot \left(\frac{b}{a} - 1\right)^{\frac{4}{3}} \cdot c\_tr \cdot sin(\delta\_tr)$$

4. Cleaning will be done mechanically, which means c_tr lies between 1.1 and 1.3, of which the average is assumed to be a reasonable value to estimate the losses with.
$c\_tr := 1.2$
5. The inclination will be taken as 10° from vertical.
$\delta\_tr := 90° - 10°$

Therefore, the loss-coefficient is:

$$\xi\_tr := \frac{7}{3} \beta\_tr \cdot \left(\frac{1}{r\_ab} - 1\right)^{\frac{4}{3}} \cdot c\_tr \cdot \sin(\delta\_tr) = 0.33$$

Trash rack (section 1) (continued):

The flow velocity through the trash rack determines the actual head losses. So for a variable discharge the cross sectional area is required. However, the discharge area is dependent on the waterlevel as there is free surface flow when the water goes through the trash rack.

For the moment the assumption is made that a area with a width of the inflow diameter plus twice the inflow rounding radius and a height of the inflow diameter is the flow area for the trash rack. Due to the bars the area is reduced by factor r_ab.

Related cross-sectional area:

$$A\_tr := r\_ab\ \overrightarrow{(D\_infl)\cdot(2\ r\_in + D\_infl)}$$

$$A\_tr^{\mathrm{T}} = \left[128\ \ 62.72\ \ 93.16\ \ 103.68\ \ 121.29\right]\ m^2$$

Contribution to quadratic discharge coefficient:

$$C\_1 := \frac{\xi\_tr}{2\ g\cdot\overrightarrow{A\_tr\cdot A\_tr}}$$

$$C\_1^{\mathrm{T}} = \left[1.03\cdot10^{-6}\ \ 4.28\cdot10^{-6}\ \ 1.94\cdot10^{-6}\ \ 1.57\cdot10^{-6}\ \ 1.14\cdot10^{-6}\right]\ \frac{s^2}{m^5}$$

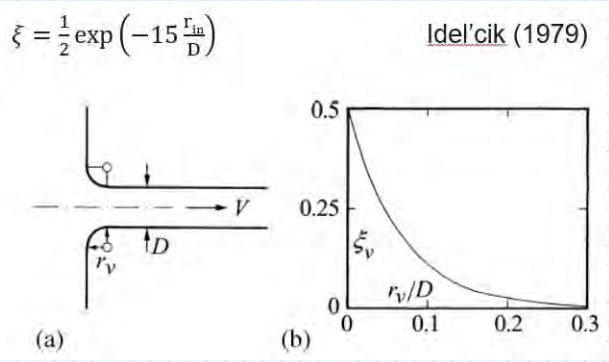
Collecting the contributions:
$C\_dv := C\_1$

Inflow (section 2):

Shape of inflow:
"Rounded with large radius"

Xi-loss coefficient:

```
ξ_in :=  for i ∈ 0 .. rows(D_t_dv) − 1
             $X_i \leftarrow \frac{1}{2}\cdot\exp\left(-15\cdot\frac{r\_in_i}{D\_infl_i}\right)$
         return X
```



$$\xi\_in^{\mathrm{T}} = \left[5.55\cdot10^{-3}\ \ 5.55\cdot10^{-3}\ \ 5.09\cdot10^{-3}\ \ 5.55\cdot10^{-3}\ \ 5.77\cdot10^{-3}\right]$$

Related cross-sectional area:

$$A\_infl := \frac{\pi}{4}\cdot\overrightarrow{D\_infl\cdot D\_infl}$$

$$A\_infl^{\mathrm{T}} = \left[78.54\ \ 38.48\ \ 56.75\ \ 63.62\ \ 74.66\right]\ m^2$$

Contribution to quadratic discharge coefficient:

$$C\_2 := \frac{\xi\_in}{2\ g\cdot\overrightarrow{A\_infl\cdot A\_infl}}$$

$$C\_2^{\mathrm{T}} = \left[4.59\cdot10^{-8}\ \ 1.91\cdot10^{-7}\ \ 8.05\cdot10^{-8}\ \ 7\cdot10^{-8}\ \ 5.28\cdot10^{-8}\right]\ \frac{s^2}{m^5}$$

Collecting the contributions:
$C\_dv := \mathrm{augment}(C\_dv, C\_2)$

Inlettube wall friction (section 3)
Wall friction is dependent on the Reynolds number, which is dependent on the flow velocity. This differs for each available discharge and head. However, in the generic turbine model the maximum discharge has been determined. These can be used to estimate the wall friction loss coefficient.

From the model of the generic turbine (total discharge of the plant):

$$Q\_p\_max\_g := \begin{bmatrix} 333.571 & 110.84 & 232.132 & 298.767 & 356.264 \end{bmatrix}^{\mathrm{T}} \cdot m^3 \cdot s^{-1}$$

The lowest discharge that will go through the turbine is about 20% of that:

$$Q\_p\_20pc := 20\% \cdot Q\_p\_max\_g$$

$$Q\_p\_20pc^{\mathrm{T}} = \begin{bmatrix} 66.71 & 22.17 & 46.43 & 59.75 & 71.25 \end{bmatrix} m^3 \cdot s^{-1}$$

So per turbine this is:

$$p2t(X) := \left\| \begin{array}{l} \text{for } i \in 0 \,..\, \text{rows}(X)-1 \\ \quad \left\| \; Y_i \leftarrow \dfrac{X_i}{n\_t_i} \right. \\ \text{return } Y \end{array} \right.$$

$$Q\_t\_max\_g := p2t(Q\_p\_max\_g)$$

$$Q\_t\_max\_g^{\mathrm{T}} = \begin{bmatrix} 83.39 & 55.42 & 77.38 & 74.69 & 71.25 \end{bmatrix} m^3 \cdot s^{-1}$$

$$Q\_t\_20pc := p2t(Q\_p\_20pc)$$

$$Q\_t\_20pc^{\mathrm{T}} = \begin{bmatrix} 16.68 & 11.08 & 15.48 & 14.94 & 14.25 \end{bmatrix} m^3 \cdot s^{-1}$$

The relevant cross-sectional areas for this part of the system are equal to that of the inflow, so flow velocities are:

$$u(Q, A) := \left\| \begin{array}{l} \text{for } i \in 0 \,..\, \text{rows}(Q)-1 \\ \quad \left\| \; Y_i \leftarrow \dfrac{Q_i}{A_i} \right. \\ \text{return } Y \end{array} \right.$$

$$u\_infl\_max := u(Q\_t\_max\_g, A\_infl) \qquad u\_infl\_max^{\mathrm{T}} = \begin{bmatrix} 1.06 & 1.44 & 1.36 & 1.17 & 0.95 \end{bmatrix} m \cdot s^{-1}$$

$$u\_infl\_min := u(Q\_t\_20pc, A\_infl) \qquad u\_infl\_min^{\mathrm{T}} = \begin{bmatrix} 0.21 & 0.29 & 0.27 & 0.23 & 0.19 \end{bmatrix} m \cdot s^{-1}$$

Making the Reynolds numbers:

$$Re(u, D) := \frac{\overrightarrow{u \cdot D}}{\nu} \qquad Re\_infl\_max := Re(u\_infl\_max, D\_infl)$$

$$Re\_infl\_max^{\mathrm{T}} = \begin{bmatrix} 1.06 \cdot 10^7 & 1 \cdot 10^7 & 1.15 \cdot 10^7 & 1.05 \cdot 10^7 & 9.27 \cdot 10^6 \end{bmatrix}$$

$$Re\_infl\_min := Re(u\_infl\_min, D\_infl)$$

$$Re\_infl\_min^{\mathrm{T}} = \begin{bmatrix} 2.12 \cdot 10^6 & 2.01 \cdot 10^6 & 2.31 \cdot 10^6 & 2.11 \cdot 10^6 & 1.85 \cdot 10^6 \end{bmatrix}$$

Relative roughness:
$k\_c := 1\ mm$ (smooth concrete tube)

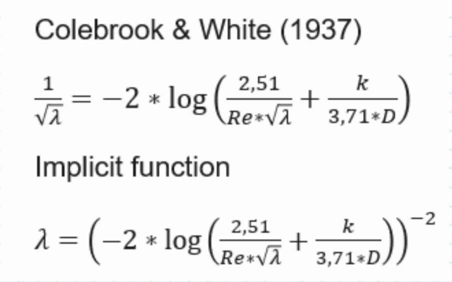
Length over which friction acts:

$$L\_fr\_infl := L\_in - L\_bulb \qquad L\_fr\_infl^{\mathrm{T}} = \begin{bmatrix} 8.8 & 6.16 & 7.48 & 7.92 & 8.58 \end{bmatrix} m$$

... continue on next page

Inlettube wall friction (section 3) (continued)

Colebrook & White (1937) friction factor:


Colebrook & White (1937)

$$\frac{1}{\sqrt{\lambda}} = -2 * \log\left(\frac{2{,}51}{Re*\sqrt{\lambda}} + \frac{k}{3{,}71*D}\right)$$

Implicit function

$$\lambda = \left(-2 * \log\left(\frac{2{,}51}{Re*\sqrt{\lambda}} + \frac{k}{3{,}71*D}\right)\right)^{-2}$$

```
$\lambda\_fr(D, k, Re) :=$
    for $i \in 0 .. \text{rows}(D) - 1$
        $a_i \leftarrow 0.010$
        $count \leftarrow 0$
        $b_i \leftarrow \left(-2 \cdot \log\left(\left(\frac{2.51}{Re_i \cdot \sqrt[2]{a_i}} + \frac{k}{3.71 \cdot D_i}\right), 10\right)\right)^{-2}$
        while $\left\| a_i - b_i \right\| > 0.1\%$
            $count \leftarrow count + 1$
            $a_i \leftarrow b_i$
            $b_i \leftarrow \left(-2 \cdot \log\left(\left(\frac{2.51}{Re_i \cdot \sqrt[2]{a_i}} + \frac{k}{3.71 \cdot D_i}\right), 10\right)\right)^{-2}$
    return $b$
```

Colebrook & White (1937) friction factors:

$\lambda\_fr\_umax := \lambda\_fr(D\_infl, k\_c, Re\_infl\_max)$

$\lambda\_fr\_umax^{T} = \left[1.22 \cdot 10^{-2} \;\; 1.3 \cdot 10^{-2} \;\; 1.25 \cdot 10^{-2} \;\; 1.24 \cdot 10^{-2} \;\; 1.22 \cdot 10^{-2}\right]$

$\lambda\_fr\_umin := \lambda\_fr(D\_infl, k\_c, Re\_infl\_min)$

$\lambda\_fr\_umin^{T} = \left[1.28 \cdot 10^{-2} \;\; 1.35 \cdot 10^{-2} \;\; 1.3 \cdot 10^{-2} \;\; 1.3 \cdot 10^{-2} \;\; 1.29 \cdot 10^{-2}\right]$

Loss coefficients using Darcy-Weisbach (1845):

$\xi\_in\_fr\_umax := \overrightarrow{\lambda\_fr\_umax \cdot L\_fr\_infl \cdot \frac{1}{D\_infl}}$

$\xi\_in\_fr\_umax^{T} = \left[1.07 \cdot 10^{-2} \;\; 1.14 \cdot 10^{-2} \;\; 1.1 \cdot 10^{-2} \;\; 1.09 \cdot 10^{-2} \;\; 1.08 \cdot 10^{-2}\right]$

$\xi\_in\_fr\_umin := \overrightarrow{\lambda\_fr\_umin \cdot L\_fr\_infl \cdot \frac{1}{D\_infl}}$

$\xi\_in\_fr\_umin^{T} = \left[1.12 \cdot 10^{-2} \;\; 1.19 \cdot 10^{-2} \;\; 1.14 \cdot 10^{-2} \;\; 1.14 \cdot 10^{-2} \;\; 1.13 \cdot 10^{-2}\right]$

```
$\xi\_in\_fr\_avg :=$
    for $i \in 0 .. \text{rows}(n\_t) - 1$
        $a_i \leftarrow \xi\_in\_fr\_umax_i + \xi\_in\_fr\_umin_i$
    return $0.5 \cdot a$
```

$\xi\_in\_fr\_avg^{T} = \left[1.1 \cdot 10^{-2} \;\; 1.16 \cdot 10^{-2} \;\; 1.12 \cdot 10^{-2} \;\; 1.11 \cdot 10^{-2} \;\; 1.11 \cdot 10^{-2}\right]$

Check error of head-losses for average value:

$\Delta H(u, \xi) := \frac{\overrightarrow{\xi \cdot u \cdot u}}{2\,g}$

$\Delta H(u\_infl\_max, \xi\_in\_fr\_umax)^{T} = \left[6.15 \cdot 10^{-4} \;\; 1.21 \cdot 10^{-3} \;\; 1.04 \cdot 10^{-3} \;\; 7.66 \cdot 10^{-4} \;\; 5 \cdot 10^{-4}\right] m$

$\Delta H(u\_infl\_max, \xi\_in\_fr\_avg)^{T} = \left[6.3 \cdot 10^{-4} \;\; 1.23 \cdot 10^{-3} \;\; 1.06 \cdot 10^{-3} \;\; 7.83 \cdot 10^{-4} \;\; 5.13 \cdot 10^{-4}\right] m$

$\Delta H(u\_infl\_min, \xi\_in\_fr\_umin)^{T} = \left[2.58 \cdot 10^{-5} \;\; 5.01 \cdot 10^{-5} \;\; 4.34 \cdot 10^{-5} \;\; 3.2 \cdot 10^{-5} \;\; 2.11 \cdot 10^{-5}\right] m$

$\Delta H(u\_infl\_min, \xi\_in\_fr\_avg)^{T} = \left[2.52 \cdot 10^{-5} \;\; 4.92 \cdot 10^{-5} \;\; 4.25 \cdot 10^{-5} \;\; 3.13 \cdot 10^{-5} \;\; 2.05 \cdot 10^{-5}\right] m$

Inlettube wall friction (section 3) (continued)

Error in using the average instead of the actual loss-coefficient:

$$error\_max := \Delta H\left(u\_infl\_max, \xi\_in\_fr\_umax\right) - \Delta H\left(u\_infl\_max, \xi\_in\_fr\_avg\right)$$

$$error\_max^{\mathrm{T}} = \left[-1.53\cdot10^{-5}\;\; -2.34\cdot10^{-5}\;\; -2.11\cdot10^{-5}\;\; -1.76\cdot10^{-5}\;\; -1.36\cdot10^{-5}\right]\ m$$

$$error\_min := \Delta H\left(u\_infl\_min, \xi\_in\_fr\_umin\right) - \Delta H\left(u\_infl\_min, \xi\_in\_fr\_avg\right)$$

$$error\_min^{\mathrm{T}} = \left[6.14\cdot10^{-7}\;\; 9.35\cdot10^{-7}\;\; 8.45\cdot10^{-7}\;\; 7.04\cdot10^{-7}\;\; 5.45\cdot10^{-7}\right]\ m$$

Using the average value has only a slight under-estimation in the order of $O\left(10^{-7}\ m\right)$ for the minimum discharge and a slight over-estimation of order $O\left(10^{-5}\ m\right)$ for the maximum discharge. It is therefore deemed acceptable to use the average.

In that case the contribution to the QDC is:

$$C\_3 := \frac{\xi\_in\_fr\_avg}{2\ g\cdot\overrightarrow{A\_infl\cdot A\_infl}}$$

$$C\_3^{\mathrm{T}} = \left[9.06\cdot10^{-8}\;\; 4.01\cdot10^{-7}\;\; 1.78\cdot10^{-7}\;\; 1.4\cdot10^{-7}\;\; 1.01\cdot10^{-7}\right]\ \frac{s^2}{m^5}$$

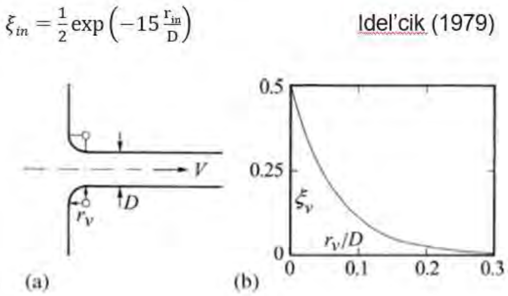
Collecting the contributions:

$C\_dv := \mathrm{augment}\left(C\_dv, C\_3\right)$

Interaction with bulb (section 4)

It is assumed that the rounding of the bulb makes sure that flow seperation doesn't happen and that the contraction losses are therefore negligible. There is however interaction with this shape, so it is assumed that the flow going around the bulb is like an inflow with rounded edges. However the diameter is then the difference between the radius of the inflow pipe and the bulb shape.

```
$\xi\_4\_con :=$
    for $i \in 0..\mathrm{rows}(D\_t\_dv)-1$
        $X_i \leftarrow \frac{1}{2}\cdot\exp\left(-15\cdot\frac{0.5\cdot D\_bulb_i}{\frac{\left(D\_infl_i - D\_bulb_i\right)}{2}}\right)$
    return $X$
```



$$\xi\_4\_con^{\mathrm{T}} = \left[4.85\cdot10^{-7}\;\; 4.85\cdot10^{-7}\;\; 4.85\cdot10^{-7}\;\; 4.85\cdot10^{-7}\;\; 4.85\cdot10^{-7}\right]$$

For such a case the reference flow velocity is after the flow has entered, so that would be past the rounding. The cross-sectional area is then:

$$A\_4 := \frac{\pi}{4}\cdot\left(\overrightarrow{D\_infl\cdot D\_infl} - \overrightarrow{D\_bulb\cdot D\_bulb}\right)$$

$$A\_4^{\mathrm{T}} = \left[60.44\;\; 29.62\;\; 43.67\;\; 48.96\;\; 57.46\right]\ m^2$$

Making the contribution to the QDC:

$$C\_4 := \frac{\xi\_4\_con}{2\ g\cdot\overrightarrow{A\_4\cdot A\_4}}$$

$$C\_4^{\mathrm{T}} = \left[6.77\cdot10^{-12}\;\; 2.82\cdot10^{-11}\;\; 1.3\cdot10^{-11}\;\; 1.03\cdot10^{-11}\;\; 7.49\cdot10^{-12}\right]\ \frac{s^2}{m^5}$$

Collecting the contributions:

$C\_dv := \mathrm{augment}\left(C\_dv, C\_4\right)$

Interaction with bulb (section 4) (continued)
With the bulb the water experiences friction from both the concrete wall and the steel casing of the bulb. Both contributions will be small, but have been taken into account here.

The relevant height for the Reynolds number is the gap between the bulb and the concrete.
$h\_gap\_bulb := 0.5\ (D\_infl - D\_bulb)$

$Re\_4\_umax := Re(u(Q\_t\_max\_g, A\_4), h\_gap\_bulb)$
$Re\_4\_umax^{T} = \begin{bmatrix} 3.57 \cdot 10^{6} & 3.39 \cdot 10^{6} & 3.9 \cdot 10^{6} & 3.56 \cdot 10^{6} & 3.13 \cdot 10^{6} \end{bmatrix}$
$Re\_4\_umin := Re(u(Q\_t\_20pc, A\_4), h\_gap\_bulb)$
$Re\_4\_umin^{T} = \begin{bmatrix} 7.15 \cdot 10^{5} & 6.79 \cdot 10^{5} & 7.8 \cdot 10^{5} & 7.11 \cdot 10^{5} & 6.26 \cdot 10^{5} \end{bmatrix}$

Then for the concrete:
$\lambda\_fr\_4c\_umax := \lambda\_fr(h\_gap\_bulb, k\_c, Re\_4\_umax)$
$\lambda\_fr\_4c\_umax^{T} = \begin{bmatrix} 1.59 \cdot 10^{-2} & 1.72 \cdot 10^{-2} & 1.65 \cdot 10^{-2} & 1.63 \cdot 10^{-2} & 1.6 \cdot 10^{-2} \end{bmatrix}$
$\lambda\_fr\_4c\_umin := \lambda\_fr(h\_gap\_bulb, k\_c, Re\_4\_umin)$
$\lambda\_fr\_4c\_umin^{T} = \begin{bmatrix} 1.66 \cdot 10^{-2} & 1.77 \cdot 10^{-2} & 1.7 \cdot 10^{-2} & 1.69 \cdot 10^{-2} & 1.68 \cdot 10^{-2} \end{bmatrix}$
Again using average:
$\lambda\_fr\_4c := 0.5 \cdot (\lambda\_fr\_4c\_umax + \lambda\_fr\_4c\_umin)$
$\lambda\_fr\_4c^{T} = \begin{bmatrix} 1.63 \cdot 10^{-2} & 1.75 \cdot 10^{-2} & 1.67 \cdot 10^{-2} & 1.66 \cdot 10^{-2} & 1.64 \cdot 10^{-2} \end{bmatrix}$

For steel (new welded steel tube):
$k\_s := 0.2\ mm$

$\lambda\_fr\_4s\_umax := \lambda\_fr(h\_gap\_bulb, k\_s, Re\_4\_umax)$
$\lambda\_fr\_4s\_umax^{T} = \begin{bmatrix} 1.2 \cdot 10^{-2} & 1.27 \cdot 10^{-2} & 1.22 \cdot 10^{-2} & 1.22 \cdot 10^{-2} & 1.21 \cdot 10^{-2} \end{bmatrix}$
$\lambda\_fr\_4s\_umin := \lambda\_fr(h\_gap\_bulb, k\_s, Re\_4\_umin)$
$\lambda\_fr\_4s\_umin^{T} = \begin{bmatrix} 1.35 \cdot 10^{-2} & 1.41 \cdot 10^{-2} & 1.36 \cdot 10^{-2} & 1.37 \cdot 10^{-2} & 1.38 \cdot 10^{-2} \end{bmatrix}$
Again using average:
$\lambda\_fr\_4s := 0.5 \cdot (\lambda\_fr\_4s\_umax + \lambda\_fr\_4s\_umin)$
$\lambda\_fr\_4s^{T} = \begin{bmatrix} 1.28 \cdot 10^{-2} & 1.34 \cdot 10^{-2} & 1.29 \cdot 10^{-2} & 1.29 \cdot 10^{-2} & 1.29 \cdot 10^{-2} \end{bmatrix}$

For the Reynolds number the gap-size is the relevant number, but for the friction the surface area is important. So for the loss coefficient the diameters of the bulb and the concrete tube around it are used:

$$\xi\_4\_fr := \overrightarrow{\left(\frac{\lambda\_fr\_4c}{D\_infl} + \frac{\lambda\_fr\_4s}{D\_bulb}\right) \cdot L\_bulb}$$

$\xi\_4\_fr^{T} = \begin{bmatrix} 4.8 \cdot 10^{-2} & 5.07 \cdot 10^{-2} & 4.89 \cdot 10^{-2} & 4.87 \cdot 10^{-2} & 4.86 \cdot 10^{-2} \end{bmatrix}$

Making the contribution to the QDC:

$$C\_4\_fr := \frac{\xi\_4\_fr}{2\ g \cdot \overrightarrow{A\_4 \cdot A\_4}}$$

$C\_4\_fr^{T} = \begin{bmatrix} 6.7 \cdot 10^{-7} & 2.95 \cdot 10^{-6} & 1.31 \cdot 10^{-6} & 1.04 \cdot 10^{-6} & 7.5 \cdot 10^{-7} \end{bmatrix} \frac{s^{2}}{m^{5}}$

Collecting the contributions:
$C\_dv := \text{augment}(C\_dv, C\_4\_fr)$

Turbine friction and/or turbulence losses

The turbine itself also has some friction or turbulence, but this is out of the scope of this research. For now it is assumed to be zero.

$\xi\_turb := 0.00$

Making the contribution to the QDC:

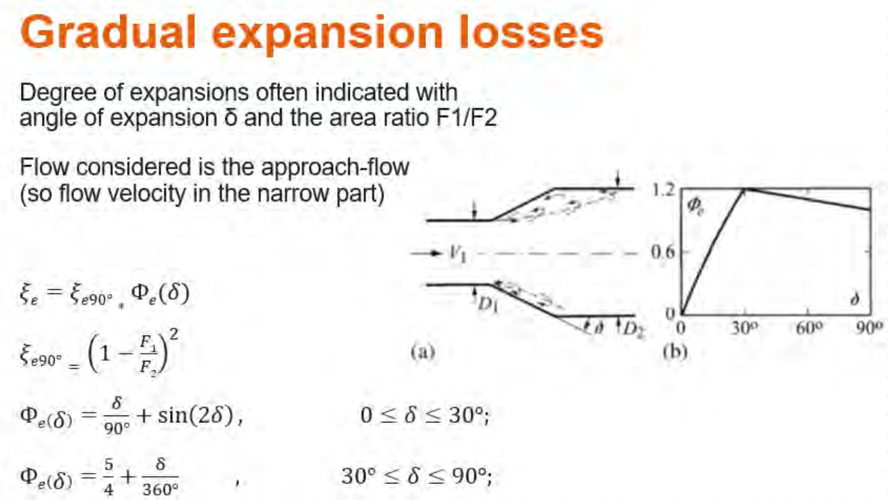
$C\_T := \overrightarrow{\dfrac{\xi\_turb}{2\, g \cdot A\_t \cdot A\_t}} \qquad C\_T^{\mathrm{T}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix} \dfrac{s^2}{m^5}$

Collecting the contributions:

$C\_dv := \mathrm{augment}(C\_dv, C\_T)$

The Draft tube (Section 5 and 6)

From Chapter 2 pg. 36 from [1]:



Gradual expansion losses

Degree of expansions often indicated with angle of expansion δ and the area ratio F1/F2

Flow considered is the approach-flow (so flow velocity in the narrow part)

$\xi_e = \xi_{e90^\circ} \, _{*} \, \Phi_e(\delta)$

$\xi_{e90^\circ} \, _{=} \left(1-\frac{F_1}{F_2}\right)^2$

$\Phi_e(\delta) = \frac{\delta}{90^\circ} + \sin(2\delta), \qquad 0 \le \delta \le 30^\circ;$

$\Phi_e(\delta) = \frac{5}{4} + \frac{\delta}{360^\circ} \quad , \qquad 30^\circ \le \delta \le 90^\circ;$

[1] Source: W.H. Hager, Wastewater Hydraulics - Theory and Practice, 2nd ed., DOI 10.1007/978-3-642-11383-3_2, Springer-Verlag, Berlin Heidelberg 2010

The expansion angle is chosen the same for all design variants:

$\beta\_draft = 5\ deg$ (°)

That means that the phi factor for the expansion is also the same for all variants:

$\Phi(\delta) := \dfrac{\delta}{90^\circ} + \sin(2 \cdot \delta) \qquad \text{--->} \qquad \Phi(\beta\_draft) = 0.23$

Draft tube exit cross-sectional area is:

$A\_exit := \dfrac{\pi}{4} \cdot D\_outfl^2 \qquad A\_exit^{\mathrm{T}} = \begin{bmatrix} 52.8 & 25.87 & 38.15 & 42.77 & 50.2 \end{bmatrix} m^2$

Then the loss coefficients are:

$\xi\_56\_exp := \left(1 - \overrightarrow{\dfrac{A\_t}{A\_exit}}\right) \cdot \Phi(\beta\_draft)$

$\xi\_56\_exp^{\mathrm{T}} = \begin{bmatrix} 1.83 \cdot 10^{-1} & 1.83 \cdot 10^{-1} & 1.83 \cdot 10^{-1} & 1.83 \cdot 10^{-1} & 1.83 \cdot 10^{-1} \end{bmatrix}$

(Note that all the area ratios are the same as well, so thats why all the Xi factors are equal)

Making the contribution to the QDC:

$C\_5 := \overrightarrow{\dfrac{\xi\_56\_exp}{2\, g \cdot A\_4 \cdot A\_4}} \qquad C\_5^{\mathrm{T}} = \begin{bmatrix} 2.56 \cdot 10^{-6} & 1.07 \cdot 10^{-5} & 4.9 \cdot 10^{-6} & 3.9 \cdot 10^{-6} & 2.83 \cdot 10^{-6} \end{bmatrix} \dfrac{s^2}{m^5}$

Collecting the contributions:

$C\_dv := \mathrm{augment}(C\_dv, C\_5)$

Friction in the draft tube (section 5 and 6)
Friction in the draft tube is more challanging due to the fact that the diameter is changing over the length. This causes all kinds of non-linear friction distributions with the current method and that is ignoring the fact that physically other phenomena might be happening (although this is partially solved by calculating the expension losses in the previous page). However, to simplify things the friction in the draft tube is interpolated between the factor just after the turbine and the one just before the outflow.

Reynolds number just after the turbine:

$$Re\_5\_umax := Re\left(u\left(Q\_t\_max\_g , A\_4\right), D\_t\_dv\right)$$
$$Re\_5\_umax^{\mathrm{T}} = \begin{bmatrix} 5.5 \cdot 10^{6} & 5.22 \cdot 10^{6} & 6 \cdot 10^{6} & 5.47 \cdot 10^{6} & 4.82 \cdot 10^{6} \end{bmatrix}$$
$$Re\_5\_umin := Re\left(u\left(Q\_t\_20pc , A\_4\right), D\_t\_dv\right)$$
$$Re\_5\_umin^{\mathrm{T}} = \begin{bmatrix} 1.1 \cdot 10^{6} & 1.04 \cdot 10^{6} & 1.2 \cdot 10^{6} & 1.09 \cdot 10^{6} & 9.64 \cdot 10^{5} \end{bmatrix}$$

Reynolds number just before the outflow:

$$Re\_6\_umax := Re\left(u\left(Q\_t\_max\_g , A\_exit\right), D\_outfl\right)$$
$$Re\_6\_umax^{\mathrm{T}} = \begin{bmatrix} 1.29 \cdot 10^{7} & 1.22 \cdot 10^{7} & 1.41 \cdot 10^{7} & 1.28 \cdot 10^{7} & 1.13 \cdot 10^{7} \end{bmatrix}$$
$$Re\_6\_umin := Re\left(u\left(Q\_t\_20pc , A\_exit\right), D\_outfl\right)$$
$$Re\_6\_umin^{\mathrm{T}} = \begin{bmatrix} 2.58 \cdot 10^{6} & 2.45 \cdot 10^{6} & 2.82 \cdot 10^{6} & 2.57 \cdot 10^{6} & 2.26 \cdot 10^{6} \end{bmatrix}$$

Friction factor right after the turbine:

$$\lambda\_fr\_5\_umax := \lambda\_fr\left(D\_t\_dv , k\_c , Re\_5\_umax\right)$$
$$\lambda\_fr\_5\_umax^{\mathrm{T}} = \begin{bmatrix} 1.45 \cdot 10^{-2} & 1.56 \cdot 10^{-2} & 1.5 \cdot 10^{-2} & 1.48 \cdot 10^{-2} & 1.46 \cdot 10^{-2} \end{bmatrix}$$
$$\lambda\_fr\_5\_umin := \lambda\_fr\left(D\_t\_dv , k\_c , Re\_5\_umin\right)$$
$$\lambda\_fr\_5\_umin^{\mathrm{T}} = \begin{bmatrix} 1.51 \cdot 10^{-2} & 1.61 \cdot 10^{-2} & 1.55 \cdot 10^{-2} & 1.54 \cdot 10^{-2} & 1.53 \cdot 10^{-2} \end{bmatrix}$$

Again using average:

$$\lambda\_fr\_5 := 0.5 \cdot \left(\lambda\_fr\_5\_umax + \lambda\_fr\_5\_umin\right)$$
$$\lambda\_fr\_5^{\mathrm{T}} = \begin{bmatrix} 1.48 \cdot 10^{-2} & 1.59 \cdot 10^{-2} & 1.52 \cdot 10^{-2} & 1.51 \cdot 10^{-2} & 1.5 \cdot 10^{-2} \end{bmatrix}$$

Friction factor right before the outflow:

$$\lambda\_fr\_6\_umax := \lambda\_fr\left(D\_outfl , k\_c , Re\_6\_umax\right)$$
$$\lambda\_fr\_6\_umax^{\mathrm{T}} = \begin{bmatrix} 1.26 \cdot 10^{-2} & 1.34 \cdot 10^{-2} & 1.29 \cdot 10^{-2} & 1.28 \cdot 10^{-2} & 1.26 \cdot 10^{-2} \end{bmatrix}$$
$$\lambda\_fr\_6\_umin := \lambda\_fr\left(D\_outfl , k\_c , Re\_6\_umin\right)$$
$$\lambda\_fr\_6\_umin^{\mathrm{T}} = \begin{bmatrix} 1.3 \cdot 10^{-2} & 1.38 \cdot 10^{-2} & 1.33 \cdot 10^{-2} & 1.32 \cdot 10^{-2} & 1.31 \cdot 10^{-2} \end{bmatrix}$$

Again using average:

$$\lambda\_fr\_6 := 0.5 \cdot \left(\lambda\_fr\_6\_umax + \lambda\_fr\_6\_umin\right)$$
$$\lambda\_fr\_6^{\mathrm{T}} = \begin{bmatrix} 1.28 \cdot 10^{-2} & 1.36 \cdot 10^{-2} & 1.31 \cdot 10^{-2} & 1.3 \cdot 10^{-2} & 1.29 \cdot 10^{-2} \end{bmatrix}$$

The loss-coefficients, interpolating between the start and end of the draft tube are:

$$\xi\_56\_fr := \left\| \begin{array}{l} \text{for } i \in 0 \,..\, \text{rows}\left(n\_t\right) - 1 \\ \quad \left\| \; Y_i \leftarrow \displaystyle\int_{0\,m}^{L\_out_i} \left(\frac{\lambda\_fr\_5_i}{D\_t\_dv_i} \cdot \frac{\left(L\_out_i - x\right)}{L\_out_i} + \frac{\lambda\_fr\_6_i}{D\_outfl_i} \cdot \frac{x}{L\_out_i} \right) \mathrm{d}x \right. \\ \text{return } Y \end{array} \right.$$

$$\xi\_56\_fr^{\mathrm{T}} = \begin{bmatrix} 6.32 \cdot 10^{-2} & 6.76 \cdot 10^{-2} & 6.49 \cdot 10^{-2} & 6.44 \cdot 10^{-2} & 6.37 \cdot 10^{-2} \end{bmatrix}$$

Friction in the draft tube (section 5 and 6) (continued)
To check if the answers are within the expected range first calculating the extreme case as if the draft tube over the whole length has the turbine diameters:

$$\xi\_56\_fr\_check1 := \overrightarrow{\left(\frac{\lambda\_fr\_5}{D\_t\_dv}\right) \cdot L\_out}$$

$$\xi\_56\_fr\_check1^{T} = \begin{bmatrix} 8.9 \cdot 10^{-2} & 9.52 \cdot 10^{-2} & 9.15 \cdot 10^{-2} & 9.07 \cdot 10^{-2} & 8.97 \cdot 10^{-2} \end{bmatrix}$$

Then calculating as if the draft tube has the outflow diameter over the whole length:

$$\xi\_56\_fr\_check2 := \overrightarrow{\left(\frac{\lambda\_fr\_6}{D\_outfl}\right) \cdot L\_out}$$

$$\xi\_56\_fr\_check2^{T} = \begin{bmatrix} 3.74 \cdot 10^{-2} & 3.99 \cdot 10^{-2} & 3.84 \cdot 10^{-2} & 3.81 \cdot 10^{-2} & 3.77 \cdot 10^{-2} \end{bmatrix}$$

To compare with:

$$\xi\_56\_fr^{T} = \begin{bmatrix} 6.32 \cdot 10^{-2} & 6.76 \cdot 10^{-2} & 6.49 \cdot 10^{-2} & 6.44 \cdot 10^{-2} & 6.37 \cdot 10^{-2} \end{bmatrix}$$

The found answer lies within these two extremes and thus is assumed to be reasonable approximation of the actual loss-coefficient.

The Quadratic loss coefficient is also dependent on the discharge area. Using the same integration method as for the Xi-factor the following value is found:

```
C_56 := for $i \in 0 .. \mathrm{rows}(n\_t) - 1$
            $Y_i \leftarrow \int_{0\,m}^{L\_out_i} \left( \frac{1}{(A\_t_i)^2} \left(\frac{\lambda\_fr\_5_i}{D\_t\_dv_i}\right) \cdot \frac{(L\_out_i - x)}{L\_out_i} + \frac{1}{(A\_exit_i)^2} \left(\frac{\lambda\_fr\_6_i}{D\_outfl_i}\right) \cdot \frac{x}{L\_out_i} \right) dx$
        return $\frac{1}{2\,g} \cdot Y$
```

$$C\_56^{T} = \begin{bmatrix} 2.07 \cdot 10^{-5} & 9.23 \cdot 10^{-5} & 4.08 \cdot 10^{-5} & 3.22 \cdot 10^{-5} & 2.31 \cdot 10^{-5} \end{bmatrix} \frac{s^2}{m^5}$$

To check again the same method of finding extremes:

$$C\_56check1 := \frac{\xi\_56\_fr}{\overrightarrow{2\,g \cdot A\_t \cdot A\_t}}$$

$$C\_56check1^{T} = \begin{bmatrix} 2.89 \cdot 10^{-5} & 1.29 \cdot 10^{-4} & 5.69 \cdot 10^{-5} & 4.49 \cdot 10^{-5} & 3.23 \cdot 10^{-5} \end{bmatrix} \frac{s^2}{m^5}$$

$$C\_56check2 := \frac{\xi\_56\_fr}{\overrightarrow{2\,g \cdot A\_exit \cdot A\_exit}}$$

$$C\_56check2^{T} = \begin{bmatrix} 1.16 \cdot 10^{-6} & 5.15 \cdot 10^{-6} & 2.27 \cdot 10^{-6} & 1.8 \cdot 10^{-6} & 1.29 \cdot 10^{-6} \end{bmatrix} \frac{s^2}{m^5}$$

The integration lies withing the expected extreme values and thus accepted as approximation.

Collecting the contributions:

$$C\_dv := \mathrm{augment}(C\_dv, C\_56)$$

Outflow losses (section 7)
The outflow losses are like the expansion losses, only the area downstream is now near infinte. Neglecting the naturally caused flow velocity (which the flow also has before flowing into the turbine), the flow loses all velocity head here. Therefore the loss-coefficient is:

$$\xi\_out := 1$$

Making the contribution to the QDC:

$$C\_7 := \overrightarrow{\frac{\xi\_out}{2\,g \cdot A\_exit \cdot A\_exit}}$$

$$C\_7^{\mathrm{T}} = \begin{bmatrix} 1.83 \cdot 10^{-5} & 7.62 \cdot 10^{-5} & 3.5 \cdot 10^{-5} & 2.79 \cdot 10^{-5} & 2.02 \cdot 10^{-5} \end{bmatrix} \frac{s^2}{m^5}$$

Collecting the contributions:

$$C\_dv := \operatorname{augment}(C\_dv, C\_7)$$

To sumerize the QDC values are:
From left to right:

1. trash rack
2. inflow
3. friction inflow pipe
4. contraction
5. friction bulb
6. turbine
7. expansion draft tube
8. friction draft turbe
9. outflow

$$C\_dv = \begin{array}{c} \begin{array}{ccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{array} \\ \begin{bmatrix} 1.03 \cdot 10^{-6} & 4.59 \cdot 10^{-8} & 9.06 \cdot 10^{-8} & 6.77 \cdot 10^{-12} & 6.7 \cdot 10^{-7} & 0 & 2.56 \cdot 10^{-6} & 2.07 \cdot 10^{-5} & 1.83 \cdot 10^{-5} \\ 4.28 \cdot 10^{-6} & 1.91 \cdot 10^{-7} & 4.01 \cdot 10^{-7} & 2.82 \cdot 10^{-11} & 2.95 \cdot 10^{-6} & 0 & 1.07 \cdot 10^{-5} & 9.23 \cdot 10^{-5} & 7.62 \cdot 10^{-5} \\ 1.94 \cdot 10^{-6} & 8.05 \cdot 10^{-8} & 1.78 \cdot 10^{-7} & 1.3 \cdot 10^{-11} & 1.31 \cdot 10^{-6} & 0 & 4.9 \cdot 10^{-6} & 4.08 \cdot 10^{-5} & 3.5 \cdot 10^{-5} \\ 1.57 \cdot 10^{-6} & 7 \cdot 10^{-8} & 1.4 \cdot 10^{-7} & 1.03 \cdot 10^{-11} & 1.04 \cdot 10^{-6} & 0 & 3.9 \cdot 10^{-6} & 3.22 \cdot 10^{-5} & 2.79 \cdot 10^{-5} \\ 1.14 \cdot 10^{-6} & 5.28 \cdot 10^{-8} & 1.01 \cdot 10^{-7} & 7.49 \cdot 10^{-12} & 7.5 \cdot 10^{-7} & 0 & 2.83 \cdot 10^{-6} & 2.31 \cdot 10^{-5} & 2.02 \cdot 10^{-5} \end{bmatrix} \end{array} \frac{s^2}{m^5}$$

Taking the sum for each design variant:

$$C\_D := \left\| \begin{array}{l} \text{for } i \in 0 .. \operatorname{rows}(C\_dv) - 1 \\ \quad \left\| \; C_i \leftarrow \sum_{j=0}^{\operatorname{cols}(C\_dv)-1} C\_dv_{i,j} \right. \\ \text{return } C \end{array} \right. \qquad C\_D = \begin{bmatrix} 4.34 \cdot 10^{-5} \\ 1.87 \cdot 10^{-4} \\ 8.42 \cdot 10^{-5} \\ 6.67 \cdot 10^{-5} \\ 4.82 \cdot 10^{-5} \end{bmatrix} \frac{s^2}{m^5}$$

Check if ξ_eq lies within expected ranges:

$$\xi\_eq := \overrightarrow{C\_D \cdot A\_t \cdot A\_t} \cdot 2\,g = \begin{bmatrix} 0.095 \\ 0.098 \\ 0.096 \\ 0.096 \\ 0.095 \end{bmatrix}$$

For Maurik this was $\xi\_eq\_M := 0.101$, so these values seem reasonable.

Head-discharge relation

As established in the literature review of the thesis this is accompanying (Chapter 4) the available head is subdivided in the following way:

$$\Delta H\_ava(Q\_t) = \frac{(\eta\_t \cdot Q\_t)^{\frac{2}{3}}}{g} \cdot (r\_s)^{\frac{4}{3}} + Q\_t^{2} \cdot C$$

Solving for Q_t on the next page...

Defining the head over the turbine:

$$\Delta H\_t(Q\_t, r\_s, \eta\_t) := \overrightarrow{\frac{(\eta\_t \cdot Q\_t)^{\frac{2}{3}}}{g} \cdot \left(\frac{r\_s}{s}\right)^{\frac{4}{3}}}$$

Function for turbine discharge:

```
$Q\_t(\Delta H\_ava, C, r\_s, \eta\_t) :=$
  for $i \in 0 ..$ if $\text{rows}(C) = 0$
                        $0$
                     else
                        $\text{rows}(C) - 1$
    if $\text{rows}(C) < 2$
      $C_0 \leftarrow C$
      $r\_s_0 \leftarrow r\_s$
      $\eta\_t_0 \leftarrow \eta\_t$
    $Q_i \leftarrow 1\ \frac{m^3}{s}$
    $D1 \leftarrow \left(\text{UnitsOf}\left(2\ Q_i \cdot \left(C_i\right)\right)\right)^{-1}$
    $D2 \leftarrow \left(\text{UnitsOf}\left(2 \cdot \left(r\_s_i \cdot \frac{1}{s}\right)^{\frac{4}{3}} \cdot \left(3 \cdot Q_i^{\frac{1}{3}} \cdot g\right)^{-1}\right)\right)^{-1}$
    $DH \leftarrow \left(Q_i^{\ 2} \cdot C_i + \frac{\left(\eta\_t_i \cdot Q_i\right)^{\frac{2}{3}}}{g} \cdot \left(r\_s_i \cdot \frac{1}{s}\right)^{\frac{4}{3}} - \Delta H\_ava\right) \cdot \frac{1}{m}$
    while $\frac{|DH|\ m}{\Delta H\_ava} > 10^{-7}$
      $dDH\_dQ \leftarrow 2 \cdot Q_i \cdot C_i \cdot D1 + \frac{2 \cdot \eta\_t_i^{\frac{2}{3}}}{3 \cdot Q_i^{\frac{1}{3}} \cdot g} \cdot D2 \cdot \left(r\_s_i \cdot \frac{1}{s}\right)^{\frac{4}{3}}$
      $Q_i \leftarrow$ if $Q_i - \frac{DH}{dDH\_dQ} \cdot \frac{m^3}{s} < 0$
                  $Q_i + \frac{DH}{dDH\_dQ} \cdot \frac{m^3}{s}$
               else
                  $Q_i - \frac{DH}{dDH\_dQ} \cdot \frac{m^3}{s}$
      $DH \leftarrow \left(Q_i^{\ 2} \cdot C_i + \frac{\left(\eta\_t_i \cdot Q_i\right)^{\frac{2}{3}}}{g} \cdot \left(r\_s_i \cdot \frac{1}{s}\right)^{\frac{4}{3}} - \Delta H\_ava\right) \cdot \frac{1}{m}$
  return $Q$
```

Defining the head over the turbine:

$$\Delta H\_t(Q\_t, r\_s, \eta\_t) := \overrightarrow{\frac{(\eta\_t \cdot Q\_t)^{\frac{2}{3}}}{g} \cdot \left(\frac{r\_s}{s}\right)^{\frac{4}{3}}}$$

Plotting with a available head of 2m, a speed ratio of 1.1 and a constant efficiency:

Define efficiency of the turbines (assumed 90% now):

$$\eta\_t\_test := \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix}^{\mathrm{T}} \cdot 95\%$$

Define speedratios for turbines

$$r\_s\_test := \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix}^{\mathrm{T}} \cdot 1.1$$

Then the discharges with C_D as QD coefficient:

$$Q\_test := Q\_t(2.0\ m, C\_D, r\_s\_test, \eta\_t\_test) = \begin{bmatrix} 65.32 \\ 50.36 \\ 59.38 \\ 61.65 \\ 64.49 \end{bmatrix} \frac{m^3}{s}$$

Related head differences over turbine:

$$\Delta H\_t\_test := \Delta H\_t(Q\_test, r\_s\_test, \eta\_t\_test) = \begin{bmatrix} 1.81 \\ 1.53 \\ 1.7 \\ 1.75 \\ 1.8 \end{bmatrix} m$$

these are all less than the available head, so that seems correct.
Check head over structure (vector over the function making sure matrices are multiplied element for element) :

$$\overrightarrow{\frac{(\eta\_t\_test \cdot Q\_test)^{\frac{2}{3}}}{g} \cdot \left(\frac{r\_s\_test}{s}\right)^{\frac{4}{3}} + Q\_test^2 \cdot C\_D} = \begin{bmatrix} 2.00 \\ 2.00 \\ 2.00 \\ 2.00 \\ 2.00 \end{bmatrix} m$$

All the head-differences are equal to the starting value of 2m. check.

Now calculate the head-differences per system part (except over the turbine):

```
$\Delta H\_test :=$
    $H_{(\mathrm{rows}(C\_dv)-1),(\mathrm{cols}(C\_dv)-1)} \leftarrow 0$
    for $i \in 0..\mathrm{cols}(C\_dv)-1$
        for $j \in 0..\mathrm{rows}(C\_dv)-1$
            $H_{j,i} \leftarrow {Q\_test_j}^2 \cdot C\_dv_{j,i}$
    $H^{\langle 5 \rangle} \leftarrow \Delta H\_t\_test$
    return $H$
```

$$\Delta H\_test = \begin{array}{c} \begin{array}{ccccccccc} \text{TR} & \text{infl} & \text{frict} & \text{bulb} & \text{frict} & \text{turb} & \text{expan} & \text{frict} & \text{outflow} \end{array} \\ \begin{bmatrix} 4.38 & 0.20 & 0.39 & 0.00 & 2.86 & 1814.93 & 10.92 & 88.30 & 78.03 \\ 10.85 & 0.48 & 1.02 & 0.00 & 7.48 & 1525.98 & 27.03 & 234.00 & 193.16 \\ 6.84 & 0.28 & 0.63 & 0.00 & 4.61 & 1703.12 & 17.29 & 143.72 & 123.52 \\ 5.95 & 0.27 & 0.53 & 0.00 & 3.94 & 1746.31 & 14.83 & 122.23 & 105.94 \\ 4.76 & 0.22 & 0.42 & 0.00 & 3.12 & 1799.50 & 11.78 & 96.05 & 84.16 \end{bmatrix} \end{array} mm$$

Plotting the losses for the test case:
Energy head levels:

```
$Hx(dH, dHa) :=$
    $H_0 \leftarrow dHa$
    for $i \in 1 .. \text{rows}(dH) - 4$
        $H_i \leftarrow H_{i-1} - dH_{i-1}$
    $H_6 \leftarrow H_5 - dH_5 - dH_6$
    $H_7 \leftarrow H_6 - dH_7$
    $H \leftarrow \text{stack}(H, 0\ m)$
    return $H$
```

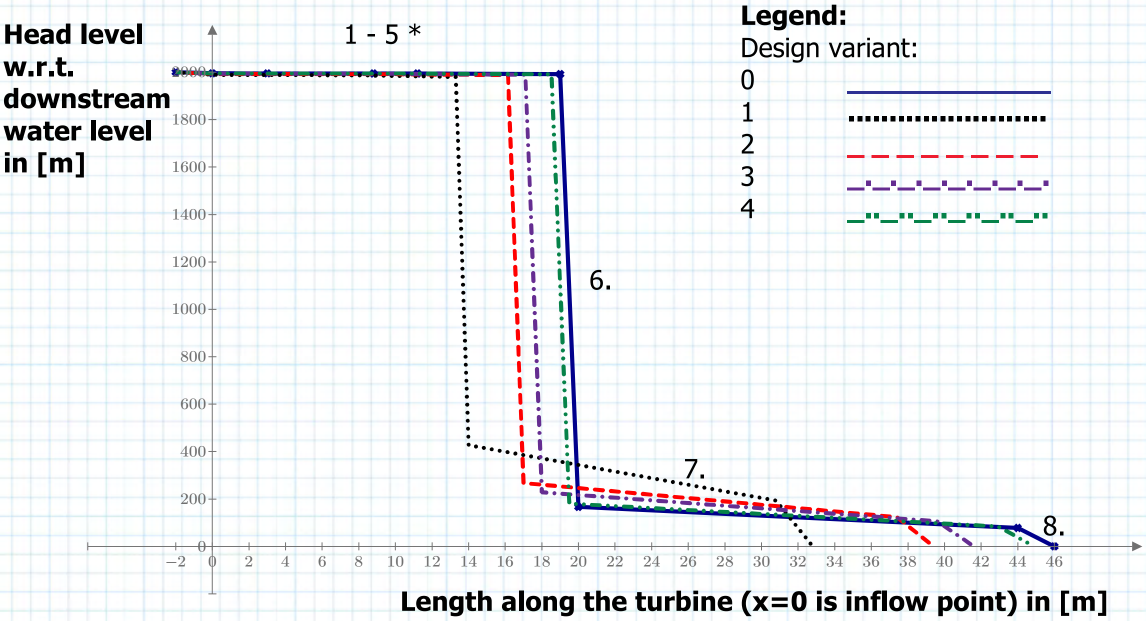
For design variant 0:

$$Hx\left(\left(\Delta H\_test^{T}\right)^{\langle 0\rangle}, 2\ m\right) = \begin{bmatrix} 2000 \\ 1995.62 \\ 1995.42 \\ 1995.04 \\ 1995.04 \\ 1992.18 \\ 166.33 \\ 78.03 \\ 0 \end{bmatrix} mm$$

$$\left(\Delta H\_test^{T}\right)^{\langle 0\rangle} = \begin{bmatrix} 4.38 \\ 0.2 \\ 0.39 \\ 0 \\ 2.86 \\ 1814.93 \\ 10.92 \\ 88.3 \\ 78.03 \end{bmatrix} mm$$

1. trash rack
2. inflow
3. friction inflow pipe
4. Bulb contraction
5. friction bulb
6. turbine
7.1 expansion draft tube
7.2 friction draft turbe
8. outflow

Plot
Keep in mind this is all with the same speed ratio, which is not necessarily the optimal one for each turbine. This plot is just to show where the losses and head-drops occur in the system:



$Hx0 := Hx\left(\left(\Delta H\_test^{T}\right)^{\langle 0\rangle}, 2\ m\right)$

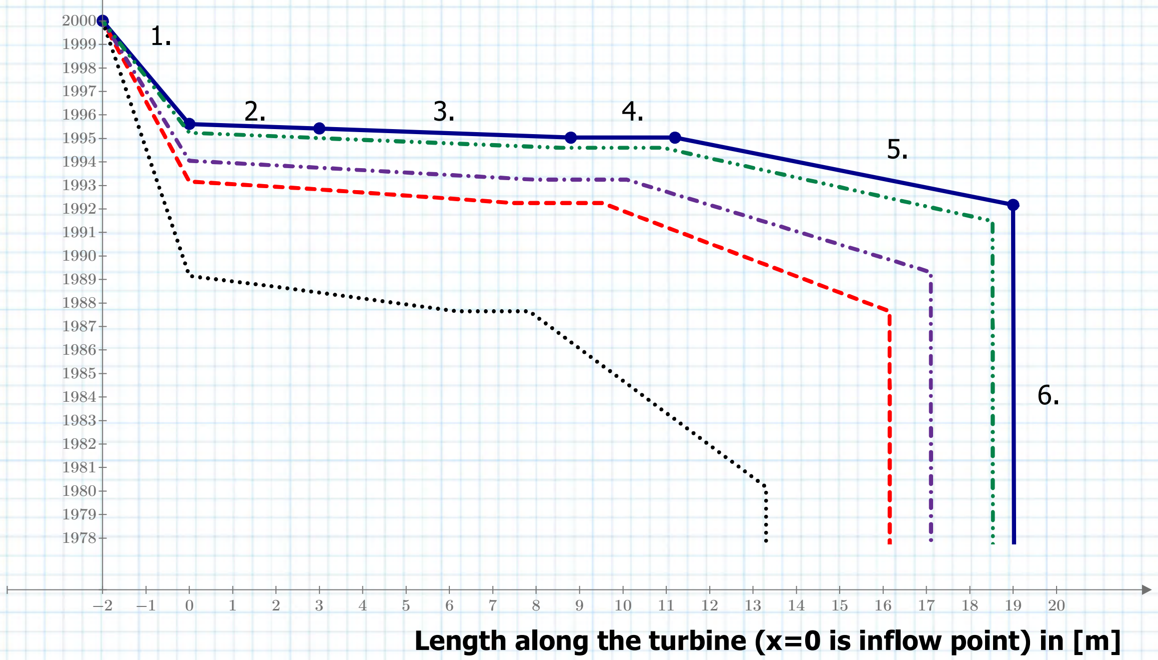
$\widehat{x^{1}} = [-2.00\ \ 0.00\ \ 3.00\ \ 8.80\ \ 11.20\ \ 19.00\ \ 20.00\ \ 44.00\ \ 46.00]\ m$

$Hx0^{T} = [2000\ \ 1995.62\ \ 1995.42\ \ 1995.04\ \ 1995.04\ \ 1992.18\ \ 166.33\ \ 78.03\ \ 0]\ mm$

*For energy head changes in detail for inflow till turbine see next page.

Head levels before the turbine in detail:

Head level w.r.t. downstream water level in [m]



Legend:
Design variant:
0 ————————
1 ·······················
2 – – – – – – – –
3 -·-·-·-·-·-·-·-
4 -··-··-··-··-··-

Markers:
1. trash rack
2. inflow
3. friction inflow pipe
4. Bulb contraction
5. friction bulb
6. turbine

Powerplant operation:
Minimum head level for the design variants 1 to 4 are assumed to be:
$\Delta H\_t\_thres := 0.3\ m$
(as this was claimed by Pentair to be the minimum value)

And for design variant 0, the copy of Maurik, the minimum is:
$\Delta H\_t\_thres0 := 0.6\ m$
(As this was the lowest value on the Hill-chart of Maurik)

Minimum discharge is assumed to be 20% of the maximum, so for the entire power plant:

$$Q\_p\_max\_g := \begin{bmatrix} 333.571 & 110.84 & 232.132 & 298.767 & 356.264 \end{bmatrix}^{\mathrm{T}} \cdot m^3 \cdot s^{-1}$$

$$Q\_p\_20pc^{\mathrm{T}} = \begin{bmatrix} 66.71 & 22.17 & 46.43 & 59.75 & 71.25 \end{bmatrix} \frac{m^3}{s}$$

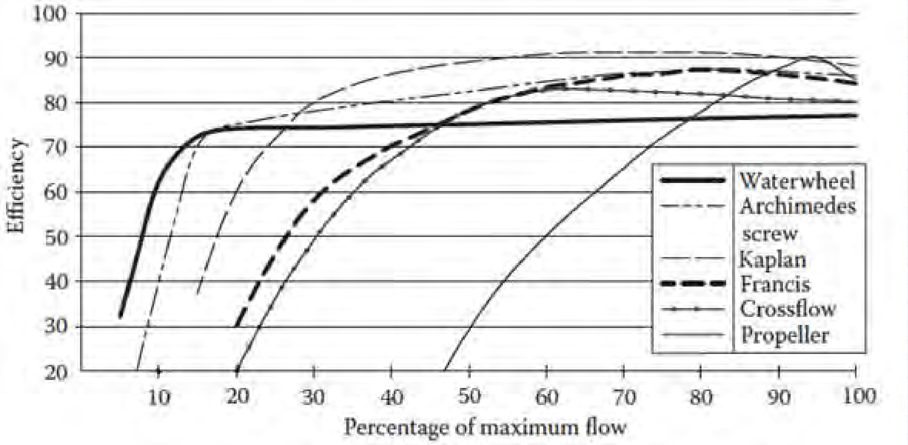
(this last discharge is a bit academic, because in practise no all turbines will run at 20% then, less turbines will run at a higher discharge per turbine...)

Per turbine:
$Q\_t\_max\_g := p2t(Q\_p\_max\_g)$
$Q\_t\_max\_g^{\mathrm{T}} = \begin{bmatrix} 83.39 & 55.42 & 77.38 & 74.69 & 71.25 \end{bmatrix} m^3 \cdot s^{-1}$
$Q\_t\_20pc := p2t(Q\_p\_20pc)$
$Q\_t\_20pc^{\mathrm{T}} = \begin{bmatrix} 16.68 & 11.08 & 15.48 & 14.94 & 14.25 \end{bmatrix} m^3 \cdot s^{-1}$

Powerplant operation (continued)

Efficiency per discharge and per head difference is now also taken into account.
However, the exact curves are not available.

For the discharge the curve from Kardi and Pandey 2016 is used:



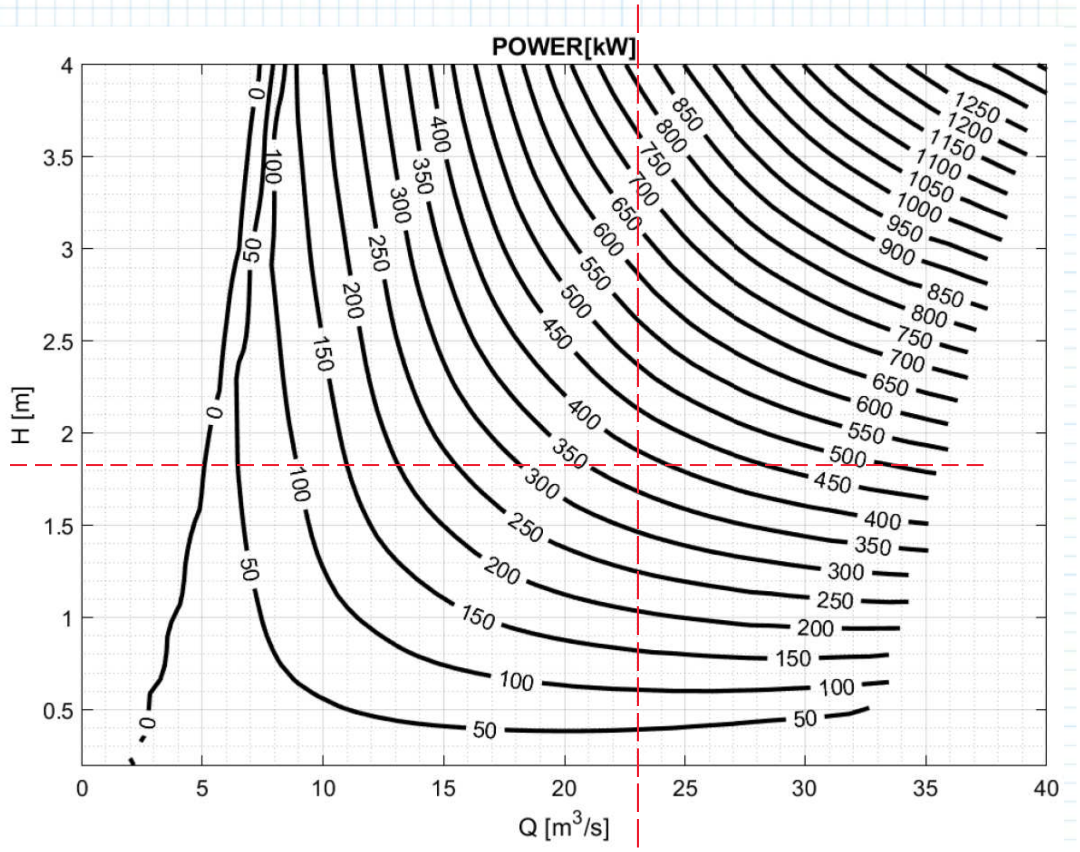
Aproximated for Kaplan with:

$$\eta\_K(Q) := \begin{bmatrix} 0\ Q & 0.1\ Q & 0.2\ Q & 0.3\ Q & 0.4\ Q & 0.5\ Q & 0.6\ Q & 0.7\ Q & 0.8\ Q & 0.9\ Q & 1.0\ Q & 1.1\ Q \\ 0 & 30\% & 70\% & 84\% & 88\% & 90\% & 92\% & 92\% & 92\% & 90\% & 88\% & 80\% \end{bmatrix}$$

"Hill-chart" for D=2.5m fishfriendly Pentair Fairbanks Nijhuijs (PFN) turbine
Efficiency isn't constant, but is dependant on Q and H:

IMPORTANT NOTE:
H shown in the graph below is Pressure-head Δh, **NOT** the Energy head ΔH that has been used in the rest of the sheet!

$\Delta H.t = m$



$Q.t = \frac{m^3}{s}$

Estimating efficiency curves Pentair turbine:
Reading from hill-chart at: At Q_t=30m3/s
Define efficiency estimation function:

$$\eta\_estH(probe, Q) := \text{augment}\left(probe^{\langle 0 \rangle} \cdot m \cdot \rho \cdot g \cdot Q\ m^3 \cdot s^{-1} \cdot kW^{-1}, \overrightarrow{\frac{probe^{\langle 1 \rangle}\ kW}{probe^{\langle 0 \rangle} \cdot m \cdot \rho \cdot g \cdot Q\ m^3 \cdot s^{-1}}}\right)$$

$[\Delta H\_t \quad P\_t]$ $[\rho g Q H \quad \eta]$

$$probe\_H1 := \begin{bmatrix} 0.30 & 35 \\ 0.40 & 50 \\ 0.57 & 75 \\ 0.75 & 100 \\ 1.11 & 150 \\ 1.50 & 200 \\ 1.90 & 250 \\ 2.40 & 300 \\ 2.96 & 350 \end{bmatrix} \quad \eta\_estH(probe\_H1, 15) = \begin{bmatrix} 44.05 & 0.79 \\ 58.73 & 0.85 \\ 83.70 & 0.90 \\ 110.13 & 0.91 \\ 162.99 & 0.92 \\ 220.25 & 0.91 \\ 278.99 & 0.90 \\ 352.40 & 0.85 \\ 434.63 & 0.81 \end{bmatrix}$$

Estimating efficiency curves Pentair turbine (continued):

Reading from hill-chart at: At Q_t=15m3/s

$$\begin{bmatrix} \Delta H\_t & P\_t \end{bmatrix} \qquad \begin{bmatrix} \rho g Q H & \eta \end{bmatrix}$$

$$probe\_H2 := \begin{bmatrix} 0.3 & 0.001 \\ 0.45 & 50 \\ 0.62 & 100 \\ 0.78 & 150 \\ 0.94 & 200 \\ 1.10 & 250 \\ 1.26 & 300 \\ 1.42 & 350 \\ 1.57 & 400 \\ 1.74 & 450 \\ 1.91 & 500 \\ 2.08 & 550 \\ 2.24 & 600 \\ 2.40 & 650 \\ 2.57 & 700 \\ 2.74 & 750 \\ 2.91 & 800 \\ 3.08 & 850 \\ 3.25 & 900 \\ 3.42 & 950 \\ 3.60 & 1000 \\ 3.80 & 1050 \\ 4.00 & 1100 \end{bmatrix} \quad \eta\_estH\left(probe\_H2, 30\right) = \begin{bmatrix} 88.1 & 0 \\ 132.15 & 0.38 \\ 182.08 & 0.55 \\ 229.06 & 0.65 \\ 276.05 & 0.72 \\ 323.04 & 0.77 \\ 370.02 & 0.81 \\ 417.01 & 0.84 \\ 461.06 & 0.87 \\ 510.99 & 0.88 \\ 560.91 & 0.89 \\ 610.83 & 0.9 \\ 657.82 & 0.91 \\ 704.81 & 0.92 \\ 754.73 & 0.93 \\ 804.66 & 0.93 \\ 854.58 & 0.94 \\ 904.5 & 0.94 \\ 954.43 & 0.94 \\ 1004.35 & 0.95 \\ 1057.21 & 0.95 \\ 1115.95 & 0.94 \\ 1174.68 & 0.94 \end{bmatrix}$$

Estimating efficiency curves Pentair (continued)

Reading from hill-chart at: At Q_t=23m3/s

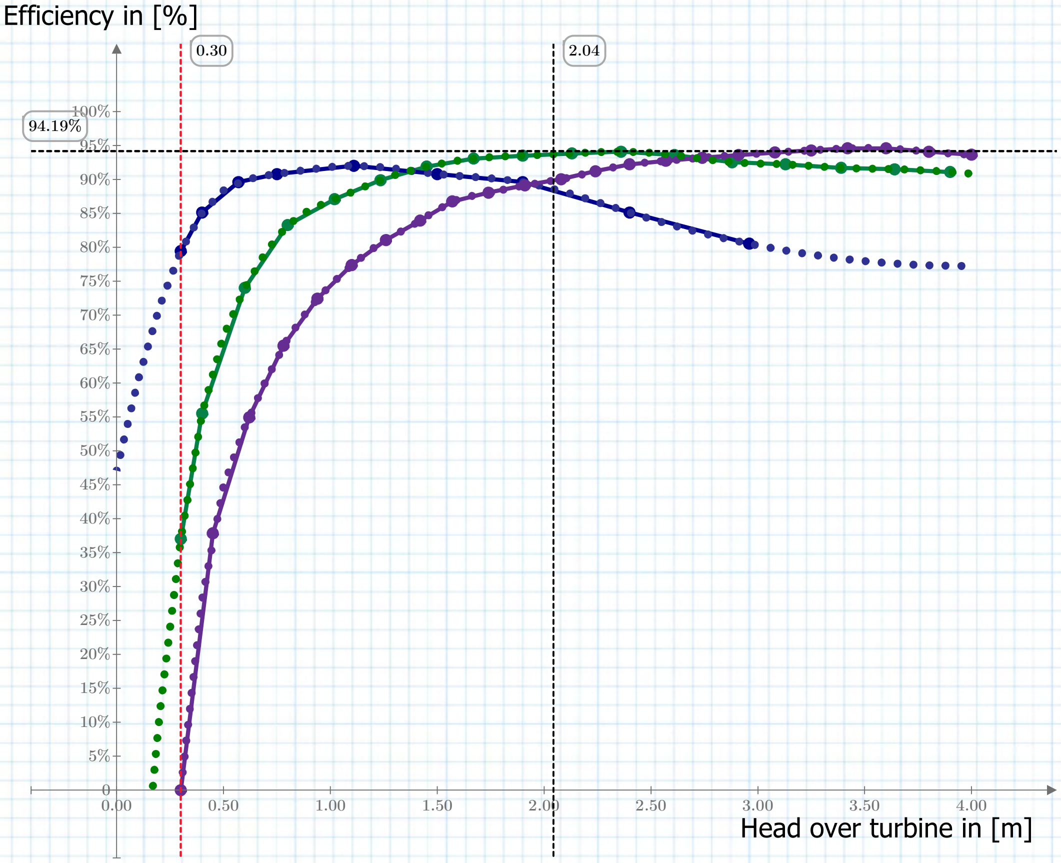
$$\begin{bmatrix} \Delta H\_t & P\_t \end{bmatrix} \qquad \begin{bmatrix} \rho g Q H & \eta \end{bmatrix}$$

$$probe\_H3 := \begin{bmatrix} 0.3 & 25 \\ 0.40 & 50 \\ 0.60 & 100 \\ 0.80 & 150 \\ 1.02 & 200 \\ 1.235 & 250 \\ 1.45 & 300 \\ 1.67 & 350 \\ 1.90 & 400 \\ 2.13 & 450 \\ 2.36 & 500 \\ 2.61 & 550 \\ 2.88 & 600 \\ 3.13 & 650 \\ 3.39 & 700 \\ 3.64 & 750 \\ 3.90 & 800 \end{bmatrix} \quad \eta\_estH\left(probe\_H3, 23\right) = \begin{bmatrix} 67.544 & 0.37 \\ 90.059 & 0.555 \\ 135.088 & 0.74 \\ 180.118 & 0.833 \\ 229.65 & 0.871 \\ 278.056 & 0.899 \\ 326.463 & 0.919 \\ 375.995 & 0.931 \\ 427.779 & 0.935 \\ 479.563 & 0.938 \\ 531.347 & 0.941 \\ 587.634 & 0.936 \\ 648.423 & 0.925 \\ 704.71 & 0.922 \\ 763.248 & 0.917 \\ 819.535 & 0.915 \\ 878.073 & 0.911 \end{bmatrix}$$

Define x-axis and approximate function:

$$H\_spl := 0\ m, 0.1\ m\,..\,4\ m = \begin{bmatrix} 0 \\ 0.1 \\ 0.2 \\ \vdots \end{bmatrix} m$$

$$\eta\_aprH(H, P, Q) := \text{interp}\left(\text{pspline}\left(P^{\langle 0\rangle}, \eta\_estH(P,Q)^{\langle 1\rangle}\right), P^{\langle 0\rangle}, \eta\_estH(P,Q)^{\langle 1\rangle}, H \cdot m^{-1}\right)$$



Legend:

Measured on hill-chart at Q= $15\ m^3 \cdot s^{-1}$	-o---o---o---o-
Cubic spline with parabolic endpoints fit of measurement at Q= $15\ m^3 \cdot s^{-1}$	....
Measured on hill-chart at Q= $23\ m^3 \cdot s^{-1}$	-o---o---o---o-
Cubic spline with parabolic endpoints fit of measurement at Q= $23\ m^3 \cdot s^{-1}$	....
Measured on hill-chart at Q= $30\ m^3 \cdot s^{-1}$	-o---o---o---o-
Cubic spline with parabolic endpoints fit of measurement at Q= $30\ m^3 \cdot s^{-1}$	....

Notes:

jkl

Estimating efficiency-discharge relation:
Reading from hill-chart at: At H_t=1.9m
Define efficiency estimation function:

$$\eta\_estQ\left(probe, H\right) := \text{augment}\left(H \cdot m \cdot \rho \cdot g \cdot probe^{\langle 0 \rangle}\ m^3 \cdot s^{-1} \cdot kW^{-1}, \overrightarrow{\frac{probe^{\langle 1 \rangle}\ kW}{H \cdot m \cdot \rho \cdot g \cdot probe^{\langle 0 \rangle}\ m^3 \cdot s^{-1}}}\right)$$

$$probe\_Q1 := \begin{array}{c} \left[Q\_t\ \ P\_t\right] \\ \begin{bmatrix} 5.1 & 25 \\ 6.5 & 50 \\ 8.9 & 100 \\ 10.9 & 150 \\ 13 & 200 \\ 15 & 250 \\ 17.6 & 300 \\ 20.2 & 350 \\ 23 & 400 \\ 26.5 & 450 \\ 30.7 & 500 \\ 36 & 550 \end{bmatrix} \end{array} \quad \eta\_estQ\left(probe\_Q1, 1.9\right) = \begin{array}{c} \left[\rho g Q H\ \ \eta\right] \\ \begin{bmatrix} 94.86 & 0.26 \\ 120.89 & 0.41 \\ 165.53 & 0.60 \\ 202.73 & 0.74 \\ 241.79 & 0.83 \\ 278.99 & 0.90 \\ 327.34 & 0.92 \\ 375.70 & 0.93 \\ 427.78 & 0.94 \\ 492.88 & 0.91 \\ 570.99 & 0.88 \\ 669.57 & 0.82 \end{bmatrix} \end{array}$$

probe

Define x-axis and approximate function:

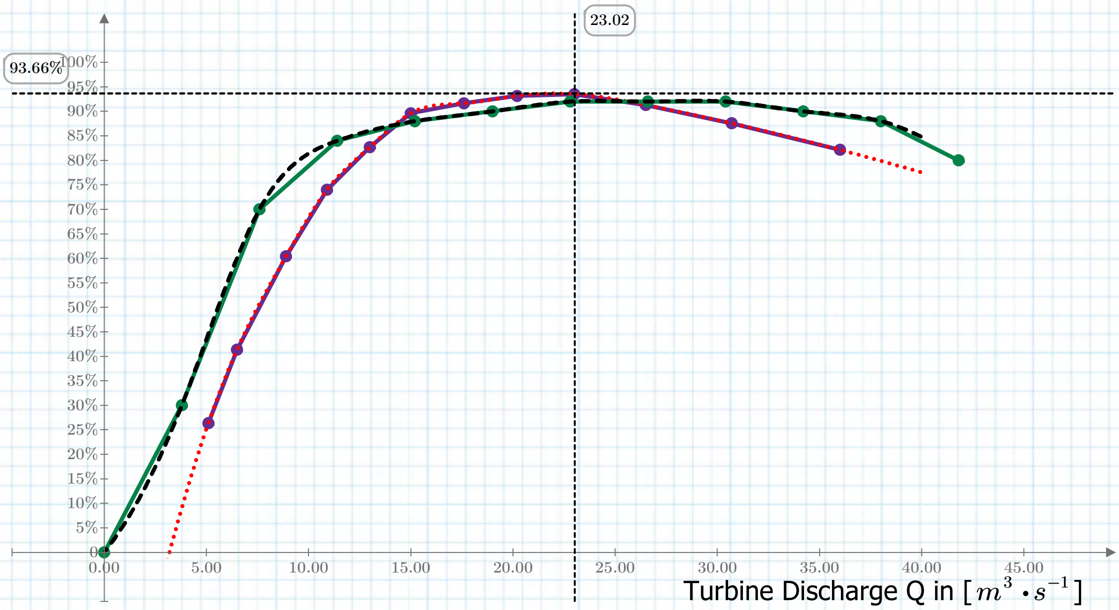
$$Q\_spl := 0\ \left(m^3 \cdot s^{-1}\right), 0.1\ \left(m^3 \cdot s^{-1}\right)..40\ m^3 \cdot s^{-1}$$

$$\eta\_aprQ\left(H, prb, Q\right) := \text{interp}\left(\text{pspline}\left(prb^{\langle 0 \rangle}, \eta\_estQ\left(prb, H\right)^{\langle 1 \rangle}\right), prb^{\langle 0 \rangle}, \eta\_estQ\left(prb, H\right)^{\langle 1 \rangle}, Q \cdot m^{-3} \cdot s^{1}\right)$$

Function from theory (Kardi and Padney (2016)):

$$\eta.Q\left(Q, Qm\right) := \text{interp}\left(\text{pspline}\left(\left(\eta\_K\left(Qm\right)^{T}\right)^{\langle 0 \rangle}, \left(\eta\_K\left(Qm\right)^{T}\right)^{\langle 1 \rangle}\right), \left(\eta\_K\left(Qm\right)^{T}\right)^{\langle 0 \rangle}, \left(\eta\_K\left(Qm\right)^{T}\right)^{\langle 1 \rangle}, Q\right)$$

Efficiency in [%]



Legend:

Efficiency curve from Kardi and Pandey (2016) with Qmax= $38\ m^3 \cdot s^{-1}$	-o---o---o---o-
Cubic spline with parabolic endpoints fit of theory curve with Qmax= $38\ m^3 \cdot s^{-1}$	---------------
Measured on hill-chart at H= $1.90\ m$	-o---o---o---o-
Cubic spline with parabolic endpoints fit of measurement at H= $1.90\ m$	···················

Conclusion efficiency:
For the discharge efficiency curve the one found at 1.90m seems to correspond quite well with the one from theory, so the one from theory is accepted as being representative for all turbine heads and maximum discharges.

For the Head efficiency curve the one found for $23\ m^3 \cdot s^{-1}$ seems a good average and is assumed to be representative for all discharges.

So the following function will be used to determine energy production:
$\eta\_Q(Q, Qm) := \max\left(10^{-7}, \eta.Q(Q, Qm)\right)$
$\eta\_H(H) := \eta\_aprH(H, probe\_H3, 23)$

Combining turbine efficiency:
$$\eta\_t(Q, Qm, H) := \max\left(0.05, \eta\_Q\left(Q \cdot \text{UnitsOf}(Q)^{-1}, Qm \cdot \text{UnitsOf}(Qm)^{-1}\right) \cdot \frac{\eta\_H(H)}{\eta\_H(1.90\ m)}\right)$$

Ecological minimum discharge:
$Q\_eco := 25\ m^3 \cdot s^{-1}$

Maximum turbine discharge:
$Q\_t\_max\_g := p2t(Q\_p\_max\_g)$
$Q\_t\_max\_g^{\mathrm{T}} = \begin{bmatrix} 83.39 & 55.42 & 77.38 & 74.69 & 71.25 \end{bmatrix} m^3 \cdot s^{-1}$

Minimum turbine discharge:
$Q\_t\_20pc := p2t(Q\_p\_20pc)$
$Q\_t\_20pc^{\mathrm{T}} = \begin{bmatrix} 16.68 & 11.08 & 15.48 & 14.94 & 14.25 \end{bmatrix} m^3 \cdot s^{-1}$

Threshold value for head over turbine:
$\Delta Hthres := \begin{bmatrix} 2 & 1 & 1 & 1 & 1 \end{bmatrix}^{\mathrm{T}}\ 0.3\ m$
(Maurik has a minimum head of 0.6m)
Determining the power will go as follows:

Stepst in order of occurance:
1) Looping through all data values of available head.
(index "i").
2) Reducing available discharge with ecological minimum. (this flow is not available for the turbine)
3) looping through all design variants (index "j")
4) Determining number of working turbines n_on for a given available discharge, with a maximum of the number of turbines determined in the generic turbine chapter n_t
5) Determining available discharge per working turbine
$Qavt$
6) making first estimate of turbine discharge Qt with efficiency of 90%. If available discharge is less than would go through the turbine with the available head, then the discharge is obviously reduced to the available discharge.
7) Determine head over turbine ΔHt for this discharge and efficiency
8) Determine efficiency ηt from curves
9) Next iteration of turbine discharge Qt now with found efficiency ηt
10) head over turbine ΔHt with new efficiency ηt and new discharge Qt
11) redetermine efficiency and if necessary reloop discharge and head till the value stabilises.
(More iterations could be made, but choice was made to make just 1 iteration)
12) determine wether minimum head and discharge per turbine are exceeded and if so calculate total power output of the plant
$P = n\_on \cdot \eta t2 \cdot \rho \cdot g \cdot Qt2 \cdot \Delta Ht2$
Otherwise $P = 0\ kW$

(See next page for algorythm)

Define Power function for fixed speed ratio:
Required variables/parameters:

Quadratic Discharge Coeffients C_D ; speed ratios r_s;
Available head data ΔH_ava ; Available discharge data Q_ava;
Threshold values: Q_eco, $Q\_t\_max\_g$, Q_t_20pc, $\Delta Hthres$
Max number of turbines n_t ;

Required functions:

Turbine discharge function: $Q\_t(\Delta H\_ava, C\_D, r\_s, \eta t)$
Turbine head function: $\Delta H\_t(Qt2, r\_s, \eta t)$
Efficiency function (based on curves): $\eta\_t(Qt, Q\_t\_max\_g, \Delta Ht)$

```
$P\_t(Q\_ava, \Delta H\_ava, r\_s) :=$
    for $i \in 0..\text{rows}(Q\_ava)-1$
        (1.) & (2.)  $Qa \leftarrow Q\_ava_i - Q\_eco$
        (3.)  for $j \in 0..(\text{rows}(n\_t)-1)$
            if $Qa \geq Q\_t\_20pc_j \wedge \Delta H\_ava_i \geq 0.3\ m$
                (4.)  $n\_on \leftarrow min\left(n\_t_j, \text{ceil}\left(\frac{Qa}{Q\_t\_max\_g_j}\right)\right)$
                (5.)  $Qavt \leftarrow \frac{Qa}{n\_on}$
                (6.)  $Qt1 \leftarrow \min\left(Qavt, Q\_t\left(\Delta H\_ava_{i,0}, C\_D_j, r\_s_j, 1\right)_{0,0}\right)$
                (7.)  $\Delta Ht \leftarrow \Delta H\_t\left(Qt1, r\_s_j, 1\right)$
                (8.)  $\eta t \leftarrow \eta\_t\left(Qt1, Q\_t\_max\_g_j, \Delta Ht\right)$
                (9.)  $Qt \leftarrow \min\left(Qavt, Q\_t\left(\Delta H\_ava_{i,0}, C\_D_{j,0}, r\_s_{j,0}, \eta t\right)\right)$
                (10.)  $\Delta Ht \leftarrow \Delta H\_t\left(Qt, r\_s_j, \eta t\right)$
                (11.)  $\eta t \leftarrow \eta\_t\left(Qt, Q\_t\_max\_g_j, \Delta Ht\right)$
                while $\left|\frac{Qt}{Qt1}-1\right| > 10^{-3}$
                    $Qt1 \leftarrow Qt$
                    $\eta t \leftarrow \eta\_t\left(Qt, Q\_t\_max\_g_j, \Delta Ht\right)$
                    $\Delta Ht \leftarrow \Delta H\_t\left(Qt, r\_s_j, \eta t\right)$
                    $Qt \leftarrow \min\left(Qavt, Q\_t\left(\Delta H\_ava_{i,0}, C\_D_{j,0}, r\_s_{j,0}, \eta t\right)\right)$
                (12.)  if $\Delta Ht \geq \Delta Hthres_j \wedge Qt \geq Q\_t\_20pc_j$
                    $P_{i,j} \leftarrow n\_on \cdot \eta t \cdot \rho \cdot g \cdot Qt \cdot \Delta Ht$
                else
                    $P_{i,j} \leftarrow 0\ kW$
    return $P$
```

Test the power function with fixed speed ratio:

$$P\_t\left(\begin{bmatrix}200\\38\\400\end{bmatrix}\frac{m^3}{s},\begin{bmatrix}2\\2\\.37\end{bmatrix}m,\begin{bmatrix}1.1\\1.1\\1.1\\1.1\\1.1\end{bmatrix}\right)=\begin{bmatrix}2578 & 1342.02 & 2579.82 & 2584.52 & 2565.13\\0 & 32.2 & 0 & 0 & 0\\0 & 32.9 & 0 & 0 & 96.4\end{bmatrix}kW$$

Notes:
1. For an available discharge of 100m3/s and 2m head all turbines work and give a power output.
2. When the discharge is reduced to 38m3/s only design variant nr. 1 has enough discharge to work (minimum discharge threshold).
3. When the discharge is 100m3/s, but the head is reduced to 0.39m only design variant nr. 2 works. This means that this one has the least losses by a vafourable combination of quadratic discharge coefficient and for this speed ratio and

Define energy function:
Since the discharge data is taken of daily averages it makes sense to assume each power value to be valid for each day. Hence the energy function is defined as follows:

```
$E\_plant(P) :=$
    $E_{cols(P)-1} \leftarrow 0\ kW \cdot hr$
    for $i \in 0 ..$ if $rows(P) = 0$
                         0
                     else
                         $rows(P) - 1$
        for $j \in 0 ..$ if $cols(P) = 0$
                             0
                         else
                             $cols(P) - 1$
            $E_j \leftarrow E_j + P_{i,j} \cdot 24\ hr$
    return $E$
```

Loading flow data from wet, dry and average year:

Looking at reference years

File path:

$FilePath := \text{“C:\textbackslash Users\textbackslash vanerps6413\textbackslash OneDrive – ARCADIS\textbackslash 061 Flow and waterlevel data\textbackslash 01 Di...}$

File name:

$FileName := \text{“002 – OUTPUT–MATHCAD – Datalink QH–t–series for E–calc – v01.xlsx”}$

Load data:

$DatasetWtDrAv := \text{READEXCEL}\left(\text{concat}\left(FilePath, FileName\right), \text{“Reference years!A2:F366”}, 0\right)$

Define data for each year:

$Qwet := DatasetWtDrAv^{\langle 0 \rangle} \cdot m^3 \cdot s^{-1}$

$Qdry := DatasetWtDrAv^{\langle 2 \rangle} \cdot m^3 \cdot s^{-1}$

$Qavg := DatasetWtDrAv^{\langle 4 \rangle} \cdot m^3 \cdot s^{-1}$

$Hwet := DatasetWtDrAv^{\langle 1 \rangle} \cdot m$

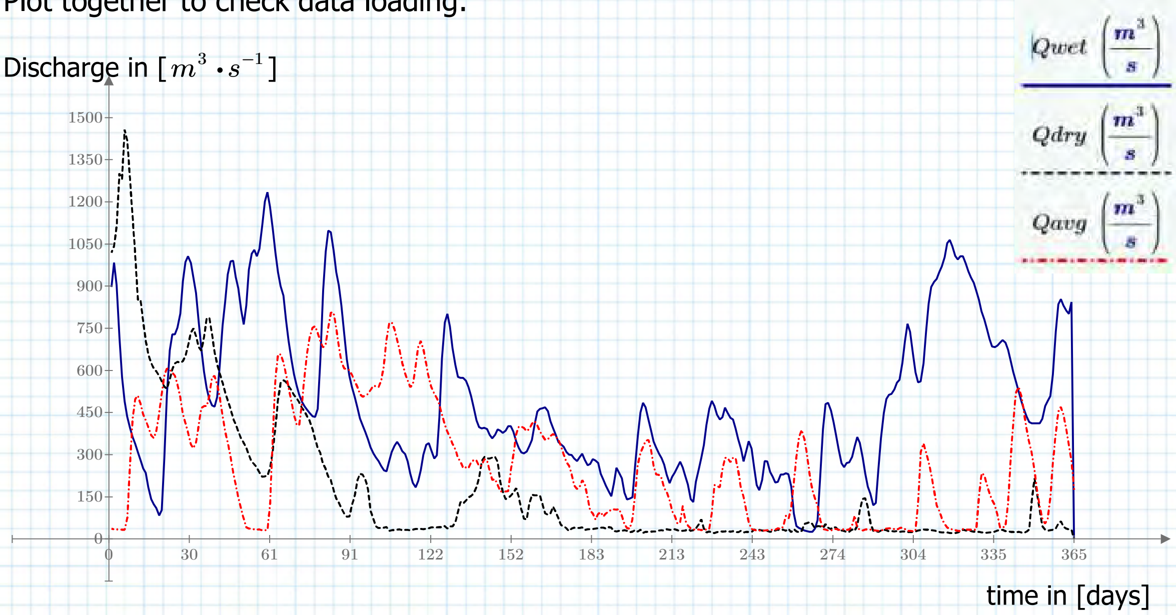
$Hdry := DatasetWtDrAv^{\langle 3 \rangle} \cdot m$

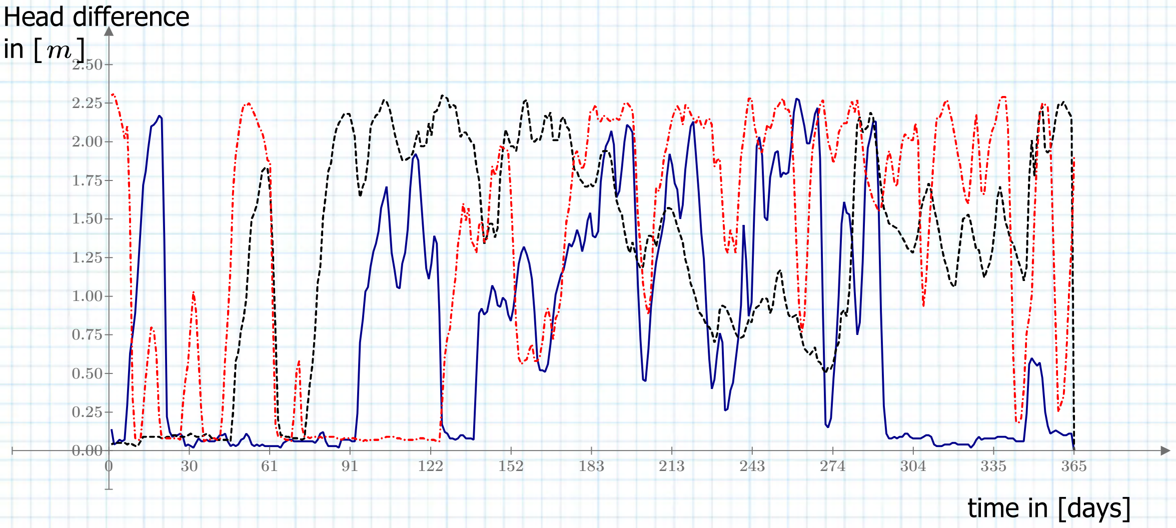
$Havg := DatasetWtDrAv^{\langle 5 \rangle} \cdot m$

Define time axis and discharge area to be a vector:

$$tx := \left\| \begin{array}{l} a_{\text{round}\left(\frac{yr}{day}\right)-1} \leftarrow \text{round}\left(\frac{yr}{day}\right) \\ \text{for } k \in 0,1..\text{round}\left(\frac{yr}{day}\right)-1 \\ \quad \left\| \, a^{\widehat{k}} \leftarrow (k+1) \cdot day \right. \\ \text{return } a \end{array} \right. = \begin{bmatrix} 1 \\ \vdots \end{bmatrix} day$$

Plot together to check data loading:





Plotting power output for fixed speed ratio for wet, dry and average year:

$$r\_s\_guess := \begin{bmatrix} 1.1 & 1.1 & 1.1 & 1.1 & 1.1 \end{bmatrix}^{\mathrm{T}}$$

Average year:

$$E\_avg := E\_plant\left(P\_t\left(Qavg, Havg, r\_s\_guess\right)\right)^{\mathrm{T}}$$
$$E\_avg = \begin{bmatrix} 5411.58 & 3034.28 & 4732.86 & 5431.18 & 5828.65 \end{bmatrix} \ MW \cdot hr$$

$$P\_t\_dv0\_avg := P\_t\left(Qavg, Havg, r\_s\_guess\right)^{\langle 0 \rangle} \qquad \max\left(P\_t\_dv0\_avg\right) = 2733.44 \ kW$$
$$P\_t\_dv1\_avg := P\_t\left(Qavg, Havg, r\_s\_guess\right)^{\langle 1 \rangle} \qquad \max\left(P\_t\_dv1\_avg\right) = 1414.33 \ kW$$
$$P\_t\_dv2\_avg := P\_t\left(Qavg, Havg, r\_s\_guess\right)^{\langle 2 \rangle} \qquad \max\left(P\_t\_dv2\_avg\right) = 2396.41 \ kW$$
$$P\_t\_dv3\_avg := P\_t\left(Qavg, Havg, r\_s\_guess\right)^{\langle 3 \rangle} \qquad \max\left(P\_t\_dv3\_avg\right) = 2495.36 \ kW$$
$$P\_t\_dv4\_avg := P\_t\left(Qavg, Havg, r\_s\_guess\right)^{\langle 4 \rangle} \qquad \max\left(P\_t\_dv4\_avg\right) = 2899.73 \ kW$$

Wet year:

$$E\_wet := E\_plant\left(P\_t\left(Qwet, Hwet, r\_s\_guess\right)\right)^{\mathrm{T}}$$
$$E\_wet = \begin{bmatrix} 6515.84 & 3054.5 & 5348.46 & 6487.05 & 7202.57 \end{bmatrix} \ MW \cdot hr$$

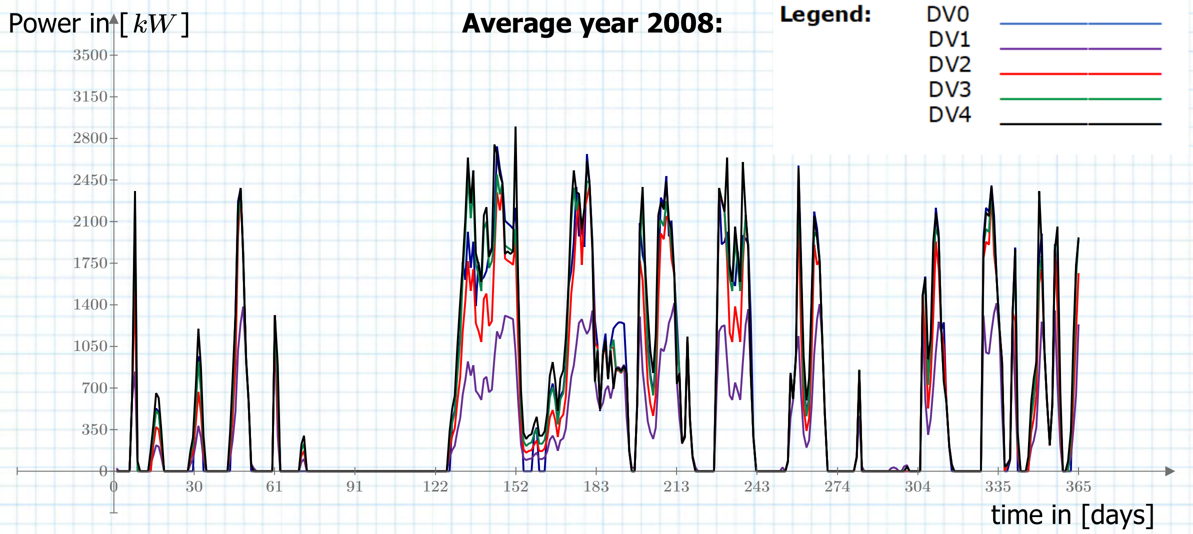
$$P\_t\_dv0\_wet := P\_t\left(Qwet, Hwet, r\_s\_guess\right)^{\langle 0 \rangle} \qquad \max\left(P\_t\_dv0\_wet\right) = 2898.28 \ kW$$
$$P\_t\_dv1\_wet := P\_t\left(Qwet, Hwet, r\_s\_guess\right)^{\langle 1 \rangle} \qquad \max\left(P\_t\_dv1\_wet\right) = 1449.37 \ kW$$
$$P\_t\_dv2\_wet := P\_t\left(Qwet, Hwet, r\_s\_guess\right)^{\langle 2 \rangle} \qquad \max\left(P\_t\_dv2\_wet\right) = 2607.04 \ kW$$
$$P\_t\_dv3\_wet := P\_t\left(Qwet, Hwet, r\_s\_guess\right)^{\langle 3 \rangle} \qquad \max\left(P\_t\_dv3\_wet\right) = 2890.65 \ kW$$
$$P\_t\_dv4\_wet := P\_t\left(Qwet, Hwet, r\_s\_guess\right)^{\langle 4 \rangle} \qquad \max\left(P\_t\_dv4\_wet\right) = 3092.75 \ kW$$

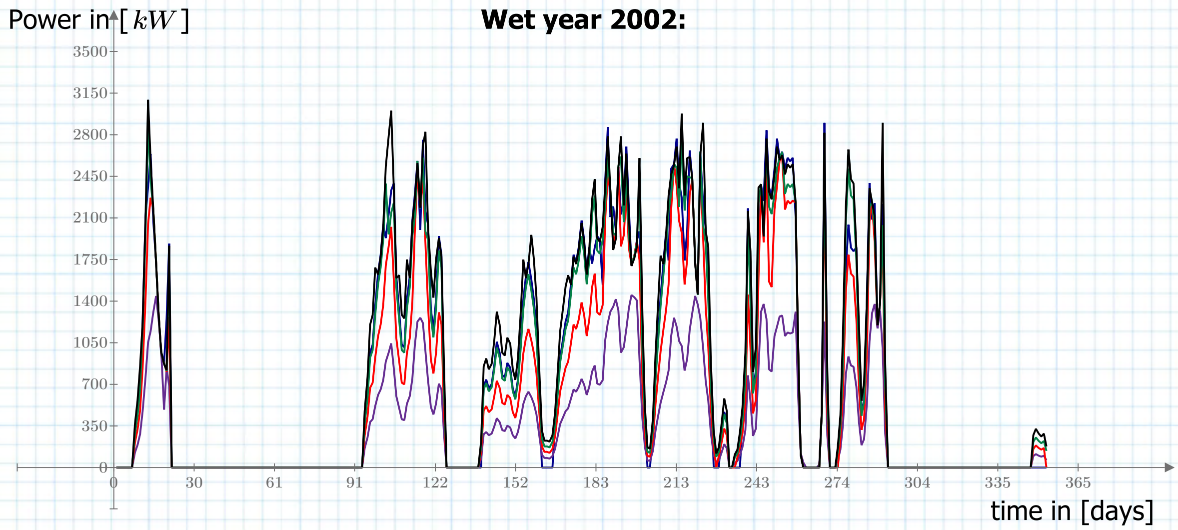
Dry year:

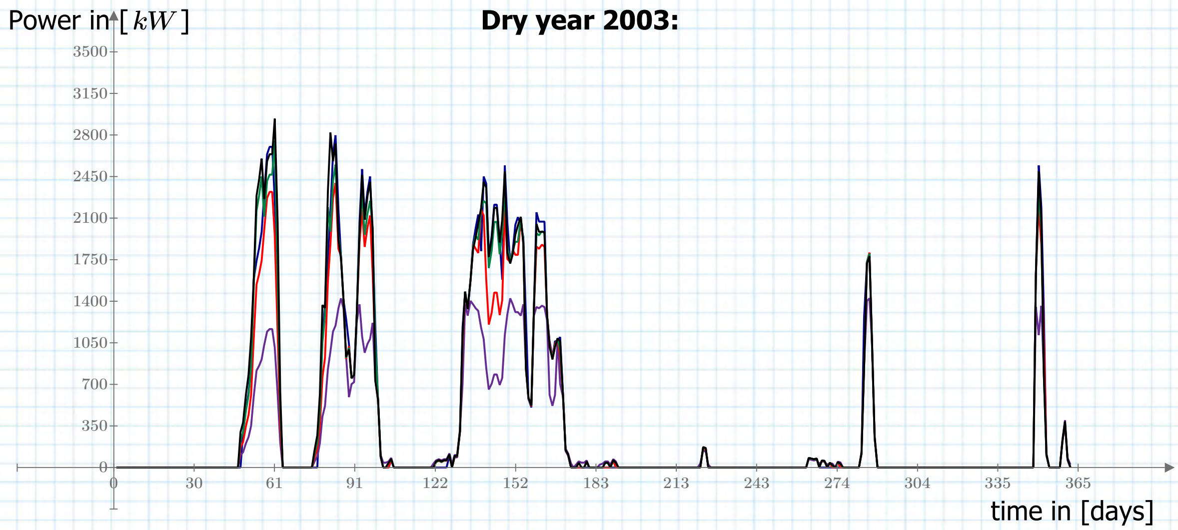
$$E\_dry := E\_plant\left(P\_t\left(Qdry, Hdry, r\_s\_guess\right)\right)^{\mathrm{T}}$$
$$E\_dry = \begin{bmatrix} 3449.94 & 2085.19 & 3042.04 & 3367.58 & 3525.6 \end{bmatrix} \ MW \cdot hr$$

$$P\_t\_dv0\_dry := P\_t\left(Qdry, Hdry, r\_s\_guess\right)^{\langle 0 \rangle} \qquad \max\left(P\_t\_dv0\_dry\right) = 2799.06 \ kW$$
$$P\_t\_dv1\_dry := P\_t\left(Qdry, Hdry, r\_s\_guess\right)^{\langle 1 \rangle} \qquad \max\left(P\_t\_dv1\_dry\right) = 1423.67 \ kW$$
$$P\_t\_dv2\_dry := P\_t\left(Qdry, Hdry, r\_s\_guess\right)^{\langle 2 \rangle} \qquad \max\left(P\_t\_dv2\_dry\right) = 2396.41 \ kW$$
$$P\_t\_dv3\_dry := P\_t\left(Qdry, Hdry, r\_s\_guess\right)^{\langle 3 \rangle} \qquad \max\left(P\_t\_dv3\_dry\right) = 2751.28 \ kW$$
$$P\_t\_dv4\_dry := P\_t\left(Qdry, Hdry, r\_s\_guess\right)^{\langle 4 \rangle} \qquad \max\left(P\_t\_dv4\_dry\right) = 2938.29 \ kW$$

Plots power with fixed speed ratio ($r\_s\_guess_{0,0} = 1.1$):


Average year 2008:


Wet year 2002:


Dry year 2003:

Head ratio and speed ratio

Aimed is to achieve the optimal head ratio as determined in the generic turbine chapter. However, to do this, a relation is needed for the speed ratio as function for the head ratio.

Head ratio:

$$r\_h = \frac{\Delta H\_t}{\Delta H\_ava} = 1 - \frac{\Delta H\_loss}{\Delta H\_ava} = 1 - \frac{Q\_t^2 \cdot C}{\Delta H\_ava}$$

Ideally, the entirety of the available discharge should be used, but if that leads to a lower head ratio than 2/3 power production is no longer optimal. In other words, it is either 2/3 or 1 minus the losses over the available head, making the function for:

the optimal head ratio:

$$r\_h\_opt(Q\_ava, \Delta H\_ava, C) := \max\left(\frac{2}{3}, 1 - \overrightarrow{\frac{Q\_ava^2 \cdot C}{\Delta H\_ava}}\right)$$

That also means that when the available discharge is larger than the discharge the turbine can take at a head ratio equal to 2/3 the optimal discharge can be determined, because then:

$$r\_h(Q) = 1 - \frac{Q^2 \cdot C}{\Delta H\_ava} = \frac{2}{3} \quad \text{----->} \quad Q = \sqrt{\left(1 - \frac{2}{3}\right) \cdot \frac{\Delta H\_ava}{C}}$$

So for $Q\_ava \geq \sqrt{\left(1 - \frac{2}{3}\right) \cdot \frac{\Delta H\_ava}{C}}$ then $r\_h = \frac{2}{3}$ is optimal

That means that the optimal speed ratio can be found with the following equation:

$$\Delta H\_t = r\_h \cdot \Delta H\_ava$$

Substituting relevant relations:

$$\frac{(\eta\_t \cdot Q\_t)^{\frac{2}{3}}}{g} \cdot (r\_s)^{\frac{4}{3}} = r\_h \cdot \Delta H\_ava$$

rewriting:

$$(r\_s)^{\frac{4}{3}} = g \cdot \frac{(r\_h \cdot \Delta H\_ava)}{(\eta\_t \cdot Q\_t)^{\frac{2}{3}}} \quad \text{---->} \quad r\_s = \left(g \cdot \frac{r\_h \cdot \Delta H\_ava}{(\eta\_t \cdot Q\_t)^{\frac{2}{3}}}\right)^{\frac{3}{4}}$$

Define function for:

optimal speed ratio:

$$r\_s\_opt(r\_h, \Delta H\_ava, Q\_t, \eta\_t) := \left(g \cdot \frac{r\_h \cdot \Delta H\_ava}{(\eta\_t \cdot Q\_t)^{\frac{2}{3}}}\right)^{\frac{3}{4}} \; s$$

Power with variable (optimal) head and speed ratio:
Bascially the power function is altered to include the optimal head ratio and related speed ratio.

```
$P\_opt(Q\_ava, \Delta H\_ava) :=$
  for $i \in 0..\text{rows}(Q\_ava) - 1$
    $Qa \leftarrow Q\_ava_i - Q\_eco$
    for $j \in 0..(\text{rows}(n\_t) - 1)$
      if $Qa \geq Q\_t\_20pc_j \wedge \Delta H\_ava_i \geq 0.3\ m$
        $n\_on \leftarrow min\left(n\_t_j, \text{ceil}\left(\frac{Qa}{Q\_t\_max\_g_j}\right)\right)$
        $Qavt \leftarrow \frac{Qa}{n\_on}$
        $r\_h \leftarrow r\_h\_opt\left(Qavt, \Delta H\_ava_i, C\_D_j\right)$
        $r\_s \leftarrow r\_s\_opt\left(r\_h, \Delta H\_ava_i, Qavt, 1.0\right)$
        $Qt1 \leftarrow \min\left(Qavt, Q\_t\left(\Delta H\_ava_i, C\_D_j, r\_s, 1.0\right)_{0,0}\right)$
        $\Delta Ht \leftarrow \Delta H\_t(Qt1, r\_s, 1)$
        $\eta t \leftarrow \eta\_t\left(Qt1, Q\_t\_max\_g_j, \Delta Ht\right)$
        $r\_s \leftarrow r\_s\_opt\left(r\_h, \Delta H\_ava_i, Qt1, \eta t\right)$
        $Qt \leftarrow \min\left(Qavt, Q\_t\left(\Delta H\_ava_{i,0}, C\_D_{j,0}, r\_s, \eta t\right)\right)$
        $\Delta Ht \leftarrow \Delta H\_t(Qt, r\_s, \eta t)$
        $\eta t \leftarrow \eta\_t\left(Qt, Q\_t\_max\_g_j, \Delta Ht\right)$
        while $\left|\frac{Qt}{Qt1} - 1\right| > 10^{-3}$
          $Qt1 \leftarrow Qt$
          $\eta t \leftarrow \eta\_t\left(Qt, Q\_t\_max\_g_j, \Delta Ht\right)$
          $r\_s \leftarrow r\_s\_opt\left(r\_h, \Delta H\_ava_i, Qt, \eta t\right)$
          $\Delta Ht \leftarrow \Delta H\_t(Qt, r\_s, \eta t)$
          $Qt \leftarrow \min\left(Qavt, Q\_t\left(\Delta H\_ava_{i,0}, C\_D_{j,0}, r\_s, \eta t\right)\right)$
        if $\Delta Ht \geq \Delta Hthres_j \wedge Qt \geq Q\_t\_20pc_j$
          $P_{i,j} \leftarrow n\_on \cdot \eta t \cdot \rho \cdot g \cdot Qt \cdot \Delta Ht$
        else
          $P_{i,j} \leftarrow 0\ kW$
  return $P$
```

Compare the methods: variable speed ratio versus fixed:

Fixed:

$$P\_t\left(\begin{bmatrix}200\\48\\400\\37\\100\end{bmatrix}\frac{m^3}{s},\begin{bmatrix}2\\2\\.37\\2\\.37\end{bmatrix}m,\begin{bmatrix}1.1\\1.1\\1.1\\1.1\\1.1\end{bmatrix}\right)=\begin{bmatrix}2578 & 1342.02 & 2579.82 & 2584.52 & 2565.13\\127.75 & 145.34 & 131.85 & 133.58 & 135.82\\0 & 32.9 & 0 & 0 & 96.4\\0 & 23.62 & 0 & 0 & 0\\0 & 32.9 & 0 & 0 & 38.56\end{bmatrix}kW$$

Variable:

$$P\_opt\left(\begin{bmatrix}200\\48\\400\\37\\100\end{bmatrix}\frac{m^3}{s},\begin{bmatrix}2\\2\\.37\\2\\.37\end{bmatrix}m\right)=\begin{bmatrix}2917.71 & 1248.93 & 2696.41 & 2793.24 & 2882.74\\367.38 & 378.22 & 369.41 & 373.81 & 379.08\\0 & 0 & 0 & 0 & 0\\0 & 172.23 & 0 & 0 & 0\\0 & 0 & 0 & 0 & 80.62\end{bmatrix}kW$$

Plotting power output for fixed speed ratio for wet, dry and average year:

Average year:

$E\_opt\_avg:=E\_plant(P\_opt(Qavg,Havg))^{\mathrm{T}}$

$E\_opt\_avg=\begin{bmatrix}6669.1 & 3559.24 & 5842.83 & 7002.76 & 7878.7\end{bmatrix}MW\cdot hr$ (New value)

$E\_avg=\begin{bmatrix}5411.58 & 3034.28 & 4732.86 & 5431.18 & 5828.65\end{bmatrix}MW\cdot hr$ (previous value)

$P\_opt\_dv0\_avg:=P\_opt(Qavg,Havg)^{\langle 0\rangle}$ $\max(P\_opt\_dv0\_avg)=2781.97\ kW$

$P\_opt\_dv1\_avg:=P\_opt(Qavg,Havg)^{\langle 1\rangle}$ $\max(P\_opt\_dv1\_avg)=1457.2\ kW$

$P\_opt\_dv2\_avg:=P\_opt(Qavg,Havg)^{\langle 2\rangle}$ $\max(P\_opt\_dv2\_avg)=2501.97\ kW$

$P\_opt\_dv3\_avg:=P\_opt(Qavg,Havg)^{\langle 3\rangle}$ $\max(P\_opt\_dv3\_avg)=2772.36\ kW$

$P\_opt\_dv4\_avg:=P\_opt(Qavg,Havg)^{\langle 4\rangle}$ $\max(P\_opt\_dv4\_avg)=2967.42\ kW$

Wet year:

$E\_opt\_wet:=E\_plant(P\_opt(Qwet,Hwet))^{\mathrm{T}}$

$E\_opt\_wet=\begin{bmatrix}8368.44 & 3220.73 & 6537.78 & 8250.74 & 9561.2\end{bmatrix}MW\cdot hr$ (New value)

$E\_wet=\begin{bmatrix}6515.84 & 3054.5 & 5348.46 & 6487.05 & 7202.57\end{bmatrix}MW\cdot hr$ (Previous value)

$P\_opt\_dv0\_wet:=P\_opt(Qwet,Hwet)^{\langle 0\rangle}$ $\max(P\_opt\_dv0\_wet)=2975.74\ kW$

$P\_opt\_dv1\_wet:=P\_opt(Qwet,Hwet)^{\langle 1\rangle}$ $\max(P\_opt\_dv1\_wet)=1474.63\ kW$

$P\_opt\_dv2\_wet:=P\_opt(Qwet,Hwet)^{\langle 2\rangle}$ $\max(P\_opt\_dv2\_wet)=2642.94\ kW$

$P\_opt\_dv3\_wet:=P\_opt(Qwet,Hwet)^{\langle 3\rangle}$ $\max(P\_opt\_dv3\_wet)=3031.95\ kW$

$P\_opt\_dv4\_wet:=P\_opt(Qwet,Hwet)^{\langle 4\rangle}$ $\max(P\_opt\_dv4\_wet)=3159.88\ kW$

Dry year:

$E\_opt\_dry:=E\_plant(P\_opt(Qdry,Hdry))^{\mathrm{T}}$

$E\_opt\_dry=\begin{bmatrix}4155.96 & 2439.76 & 3634.1 & 4124.14 & 4421.94\end{bmatrix}MW\cdot hr$ (New value)

$E\_dry=\begin{bmatrix}3449.94 & 2085.19 & 3042.04 & 3367.58 & 3525.6\end{bmatrix}MW\cdot hr$ (previous value)

$P\_opt\_dv0\_dry:=P\_opt(Qdry,Hdry)^{\langle 0\rangle}$ $\max(P\_opt\_dv0\_dry)=2975.69\ kW$

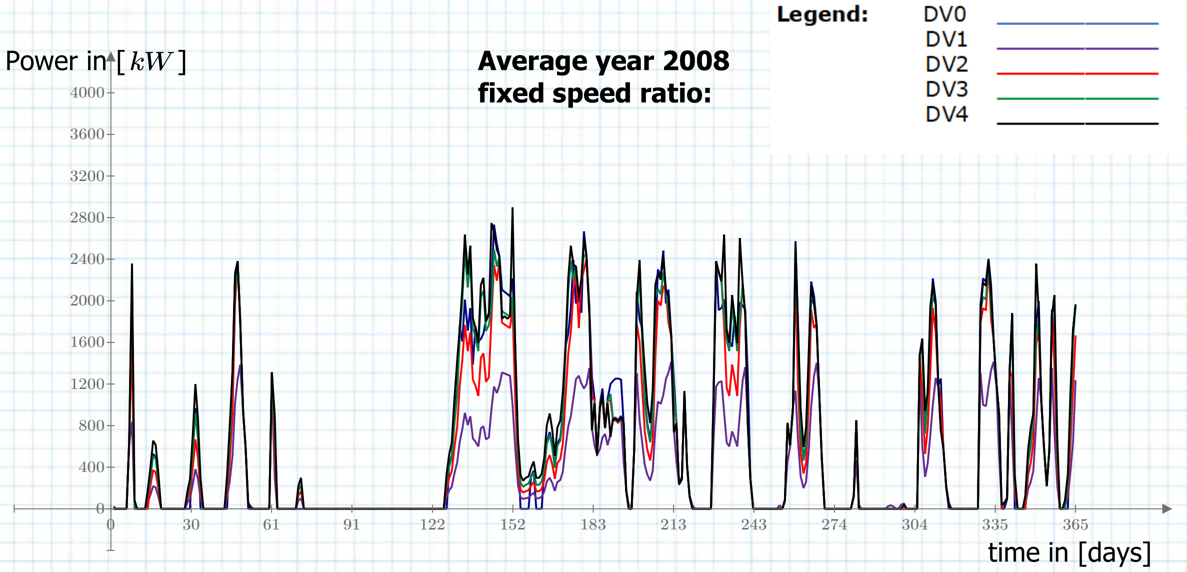
$P\_opt\_dv1\_dry:=P\_opt(Qdry,Hdry)^{\langle 1\rangle}$ $\max(P\_opt\_dv1\_dry)=1470.22\ kW$

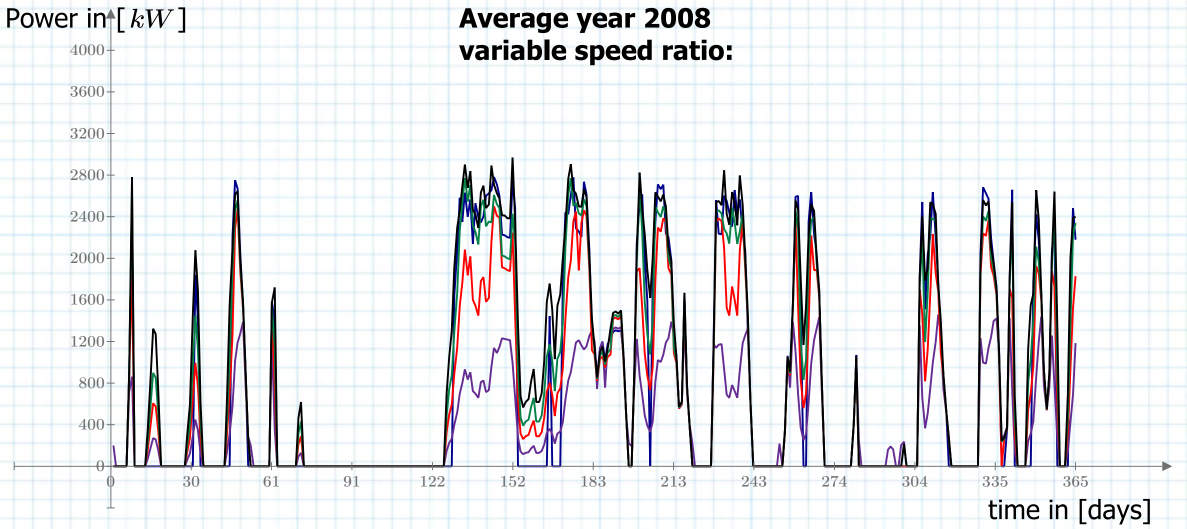
$P\_opt\_dv2\_dry:=P\_opt(Qdry,Hdry)^{\langle 2\rangle}$ $\max(P\_opt\_dv2\_dry)=2544.71\ kW$

$P\_opt\_dv3\_dry:=P\_opt(Qdry,Hdry)^{\langle 3\rangle}$ $\max(P\_opt\_dv3\_dry)=2951.9\ kW$

$P\_opt\_dv4\_dry:=P\_opt(Qdry,Hdry)^{\langle 4\rangle}$ $\max(P\_opt\_dv4\_dry)=3081.09\ kW$

Plots power with variable speed ratio:


Average year 2008 fixed speed ratio:


Average year 2008 variable speed ratio:

$$P\_rated\_dv0 := \max\left(P\_opt\_dv0\_avg, P\_opt\_dv0\_wet, P\_opt\_dv0\_dry\right) = 2975.74\ kW$$

$$P\_rated\_dv1 := \max\left(P\_opt\_dv1\_avg, P\_opt\_dv1\_wet, P\_opt\_dv1\_dry\right) = 1474.63\ kW$$

$$P\_rated\_dv2 := \max\left(P\_opt\_dv2\_avg, P\_opt\_dv2\_wet, P\_opt\_dv2\_dry\right) = 2642.94\ kW$$

$$P\_rated\_dv3 := \max\left(P\_opt\_dv3\_avg, P\_opt\_dv3\_wet, P\_opt\_dv3\_dry\right) = 3031.95\ kW$$

$$P\_rated\_dv4 := \max\left(P\_opt\_dv4\_avg, P\_opt\_dv4\_wet, P\_opt\_dv4\_dry\right) = 3159.88\ kW$$

$$CF(P, E) := \frac{E}{P \cdot yr}$$

$$CF\_dv0 := CF\left(P\_rated\_dv0, E\_opt\_avg^{\langle 0 \rangle}\right)_{0,0} = 25.57\%$$

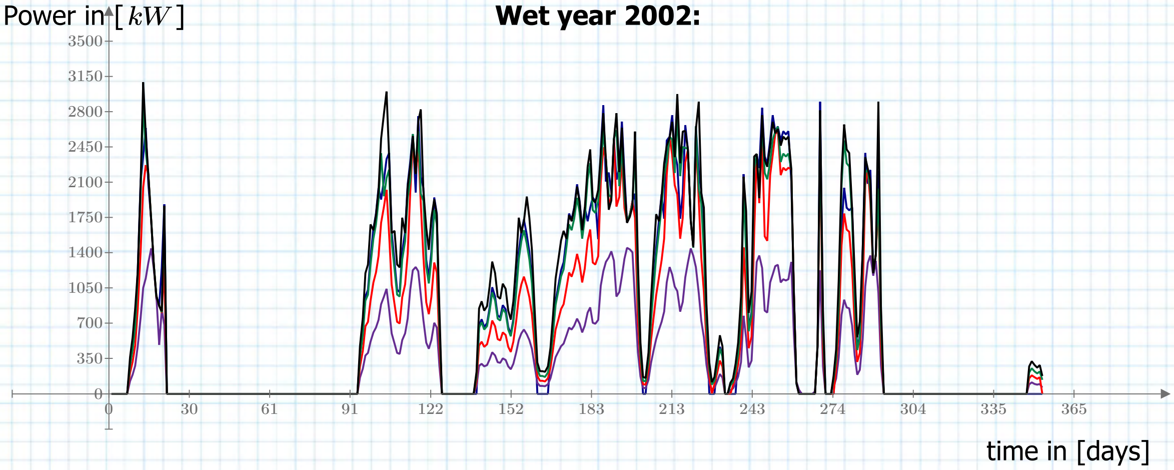
$$CF\_dv1 := CF\left(P\_rated\_dv1, E\_opt\_avg^{\langle 1 \rangle}\right)_{0,0} = 27.53\%$$

$$CF\_dv2 := CF\left(P\_rated\_dv2, E\_opt\_avg^{\langle 2 \rangle}\right)_{0,0} = 25.22\%$$

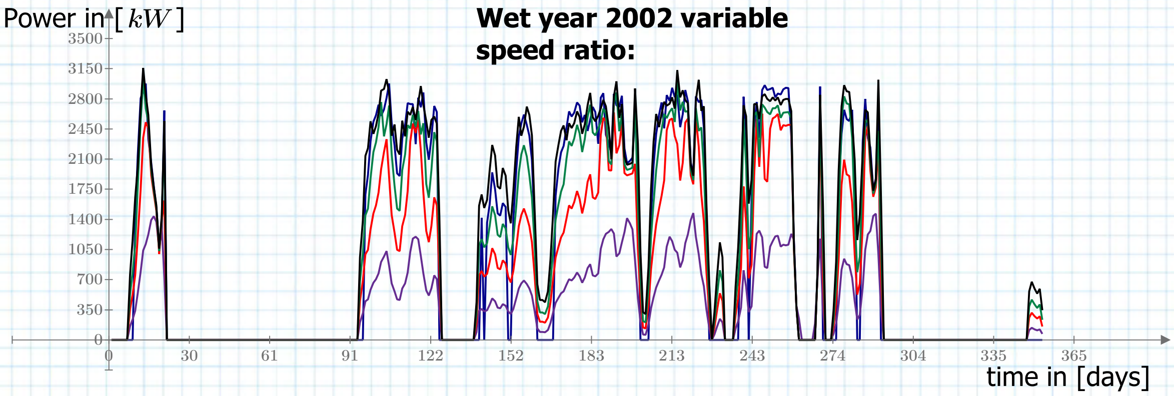
$$CF\_dv3 := CF\left(P\_rated\_dv3, E\_opt\_avg^{\langle 3 \rangle}\right)_{0,0} = 26.35\%$$

$$CF\_dv4 := CF\left(P\_rated\_dv4, E\_opt\_avg^{\langle 4 \rangle}\right)_{0,0} = 28.44\%$$

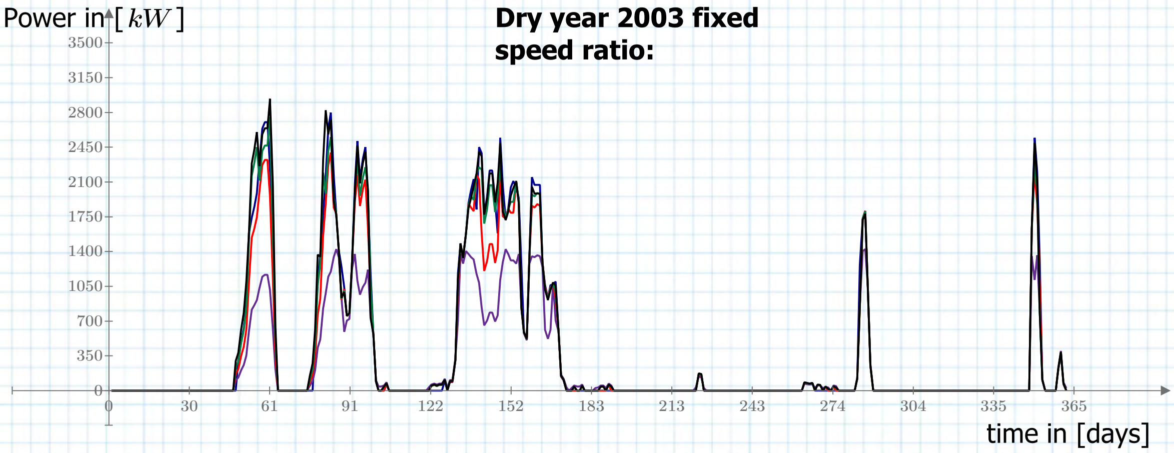
Wet year 2002:



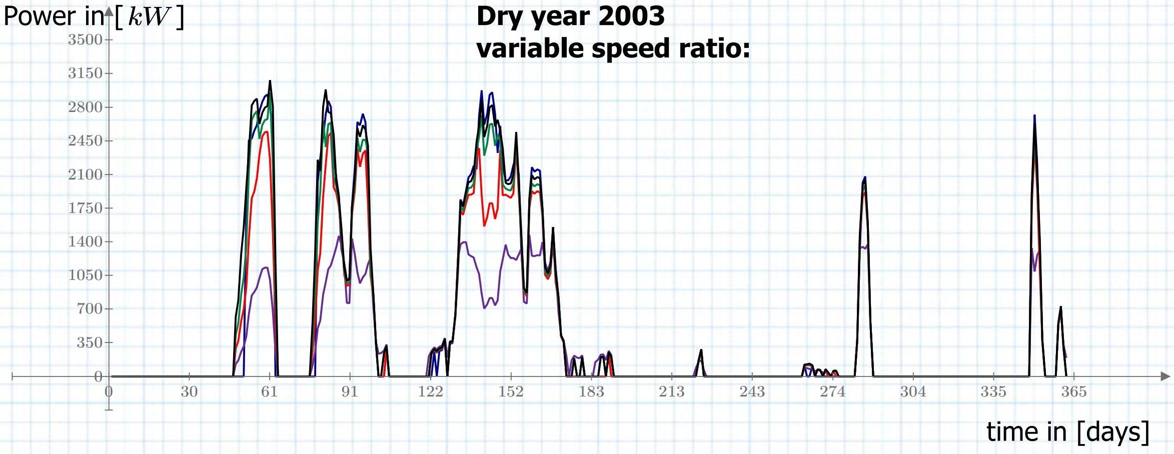
Wet year 2002 variable speed ratio:



Dry year 2003 fixed speed ratio:



Dry year 2003 variable speed ratio:



Load 10 and 30 year data:

$$MH10y := \text{READEXCEL}\left(\text{concat}\left(FilePath, FileName\right), \text{“MH10y!A1:NB11”}, 0\right)$$
$$MQ10y := \text{READEXCEL}\left(\text{concat}\left(FilePath, FileName\right), \text{“MQ10y!A1:NB11”}, 0\right)$$

$$H10y := \left\| \begin{array}{l} H \leftarrow \left(MH10y^{\mathrm{T}}\right)^{\langle 0\rangle} \\ \text{for } i \in 1\,..\,\text{rows}\left(MH10y\right)-1 \\ \quad \left\| \, H \leftarrow \text{stack}\left(H, \left(MH10y^{\mathrm{T}}\right)^{\langle i\rangle}\right) \right. \\ H \end{array} \right. \qquad Q10y := \left\| \begin{array}{l} Q \leftarrow \left(MQ10y^{\mathrm{T}}\right)^{\langle 0\rangle} \\ \text{for } i \in 1\,..\,\text{rows}\left(MQ10y\right)-1 \\ \quad \left\| \, Q \leftarrow \text{stack}\left(Q, \left(MQ10y^{\mathrm{T}}\right)^{\langle i\rangle}\right) \right. \\ Q \end{array} \right.$$

$$E\_opt\_10y := E\_plant\left(P\_opt\left(Q10y \cdot m^{3} \cdot s^{-1}, H10y \cdot m\right)\right)^{\mathrm{T}}$$
$$E\_opt\_10y = \left[71800.17 \;\; 44992.4 \;\; 64532.71 \;\; 73560.44 \;\; 81179.27\right] MW \cdot hr$$
$$E\_opt\_10y\_avg := \frac{E\_opt\_10y}{10}$$

$$MH30y := \text{READEXCEL}\left(\text{concat}\left(FilePath, FileName\right), \text{“MH30y!A1:NB31”}, 0\right)$$
$$MQ30y := \text{READEXCEL}\left(\text{concat}\left(FilePath, FileName\right), \text{“MQ30y!A1:NB31”}, 0\right)$$

$$H30y := \left\| \begin{array}{l} H \leftarrow \left(MH30y^{\mathrm{T}}\right)^{\langle 0\rangle} \\ \text{for } i \in 1\,..\,\text{rows}\left(MH30y\right)-1 \\ \quad \left\| \, H \leftarrow \text{stack}\left(H, \left(MH30y^{\mathrm{T}}\right)^{\langle i\rangle}\right) \right. \\ H \end{array} \right. \qquad Q30y := \left\| \begin{array}{l} Q \leftarrow \left(MQ30y^{\mathrm{T}}\right)^{\langle 0\rangle} \\ \text{for } i \in 1\,..\,\text{rows}\left(MQ30y\right)-1 \\ \quad \left\| \, Q \leftarrow \text{stack}\left(Q, \left(MQ30y^{\mathrm{T}}\right)^{\langle i\rangle}\right) \right. \\ Q \end{array} \right.$$

$$E\_opt\_30y := E\_plant\left(P\_opt\left(Q30y \cdot m^{3} \cdot s^{-1}, H30y \cdot m\right)\right)^{\mathrm{T}}$$
$$E\_opt\_30y = \left[171393.75 \;\; 89892.36 \;\; 148320.68 \;\; 175747.98 \;\; 198349.86\right] MW \cdot hr$$
$$E\_opt\_30y\_avg := \frac{E\_opt\_30y}{30}$$

End results energy production:

$$E\_opt\_avg = \left[6669.1 \;\; 3559.24 \;\; 5842.83 \;\; 7002.76 \;\; 7878.7\right] MW \cdot hr$$
$$E\_avg = \left[5411.58 \;\; 3034.28 \;\; 4732.86 \;\; 5431.18 \;\; 5828.65\right] MW \cdot hr$$
$$E\_opt\_10y\_avg = \left[7180.02 \;\; 4499.24 \;\; 6453.27 \;\; 7356.04 \;\; 8117.93\right] MW \cdot hr$$
$$E\_opt\_30y\_avg = \left[5713.13 \;\; 2996.41 \;\; 4944.02 \;\; 5858.27 \;\; 6611.66\right] MW \cdot hr$$

APPENDIX 17 – HYDRAULIC MODEL VETT

– see inserted page(s) behind this page –

Hydraulic model - Venture-enhanced Kaplan turbine

The idea with the Venturi enhanced turbine is that the flow velocity of the bypass will lower the pressure at the end of the turbine tube and consequently increase its head-difference over the turbine (because upstream head does not change by opening the bypass).

So instead of just 1 pipe there are now 2 pipes, 1 bypass and 1 turbine pipe that houses the turbine rotor. The generator is assumed to be connected with a shaft to the rotor and work like a kaplan bulb turbine.

Method:
1) Determine or assume the dimensions and known parameters of the turbine and the tube-system
2) Calculate the loss-coefficients for all components in the system (inflow, friction, ... , outflow, etc.)
3) Determine dependencies of the loss-coefficients (what determines the magnitude of the loss-coefficients, geometry?, discharge?, etc.).
4) When dependencies are clear define functions for discharge through the system and check if sum of losses equals present water-level difference (an assumption for the water-level difference can be made to check the loss coefficients).
5) Calculate turbine performance in terms of power-output
6) Check power-output with Hill-chart from manufacturer FPN
7) Load discharge and waterlevel data from Driel and determine energy production with various configurations and regimes.

Step 1) Determine and assume dimensions parameters of the turbine and the tube-system:

Physical constants:

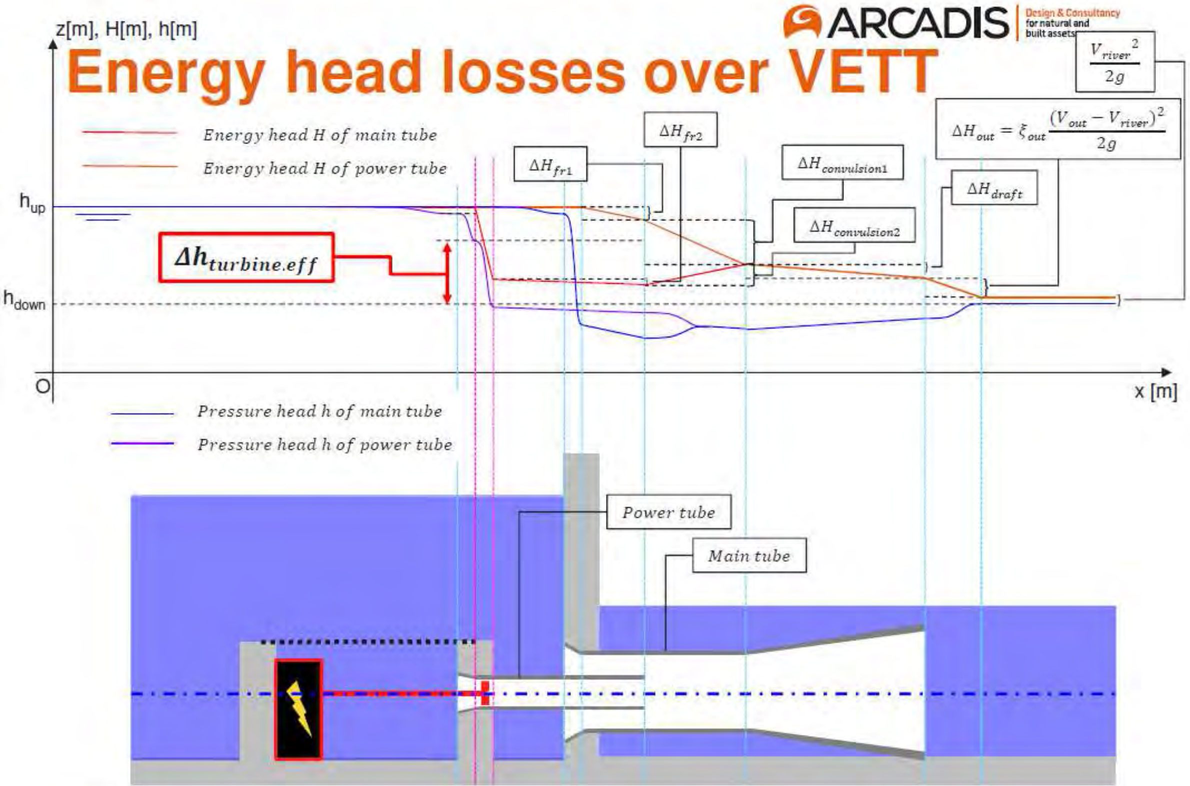
$\rho := 998.2\ kg \cdot m^{-3}$ Density of water at 20°C (assumed constant)

$g = 9.81\ m \cdot s^{-2}$ Gravitational acceleration (assumed constant)

$\mu := 1.002 \cdot 10^{-3} \cdot \frac{kg}{m \cdot s} = \left(1 \cdot 10^{-3}\right) \frac{kg}{m \cdot s}$ Dynamic viscosity at 20°C is 1,002 mPa = 1,002*10-3 Pa

$\nu := \frac{\mu}{\rho} = \left(1 \cdot 10^{-6}\right) \frac{m^2}{s}$ (Kinematic viscosity also assumed constant, value for temperature of 20°C)

Note: this is an initial sketch drawing not to scale!



Sections:

Path A - turbine: upstream 1) 2) 3) 4) 5) 6) down-stream
Path B - Bypass: upstream 1) 2) 4) 5) 6) down-stream

in one pipe the sum of discharge 'Q' is equal everywhere along its length, so cross-sectional areas 'A' determine the flow-velocities 'v' and the sum of ΔH cannot be larger than the present head. Also, after convulsion the flows have merged and the two routes must reach the same values for energy head and discharge at the end of section 4.

Geometry of the turbines

Number of turbines (based on Regular Kaplan Turbine design variants):

$$n\_t := \begin{bmatrix} 4 & 2 & 3 & 4 & 5 \end{bmatrix}^{T}$$

Based on the regular kaplan the design variants (5 variants, where variant 0 is a copy of Maurik) have the following discharge area per turbine:

$$A\_t\_RK := \begin{bmatrix} 10.80 & 5.00 & 7.80 & 8.75 & 10.00 \end{bmatrix}^{T} \ m^2$$

Assuming the bypass is additional the same diameter and area can be used for the VETT:

$$A\_t\_VET := 100\% \cdot A\_t\_RK \qquad A\_t\_VET^{T} = \begin{bmatrix} 10.8 & 5 & 7.8 & 8.75 & 10 \end{bmatrix} \ m^2$$

$$D\_t\_dv := \sqrt{\frac{A\_t\_VET}{\frac{\pi}{4} \cdot \left(1^2 - \left(\frac{1.6}{4.0} \cdot 1\right)^2\right)}} \qquad D\_t\_dv^{T} = \begin{bmatrix} 4.05 & 2.75 & 3.44 & 3.64 & 3.89 \end{bmatrix} \ m$$

Rounding these diameters to nearest 5cm:

$$D\_t\_dv := \text{Round}\left(D\_t\_dv, 5\ cm\right) \qquad D\_t\_dv^{T} = \begin{bmatrix} 4.05 & 2.75 & 3.45 & 3.65 & 3.9 \end{bmatrix} \ m$$

Diameter of rotor shaft/attachement of rotor blades

$$D\_in := \frac{1.6}{4.0} \cdot D\_t\_dv \qquad D\_in^{T} = \begin{bmatrix} 1.62 & 1.1 & 1.38 & 1.46 & 1.56 \end{bmatrix} \ m$$

... continue on next page ...

Geometry of the turbines (continued)

To redefine the discharge area VET:

$$A\_t\_VET := \frac{\pi}{4} \cdot \left(\overrightarrow{D\_t\_dv \cdot D\_t\_dv} - \left(\overrightarrow{D\_in \cdot D\_in} \right) \right)$$

$$A\_t\_VET^{\mathrm{T}} = \begin{bmatrix} 10.82 & 4.99 & 7.85 & 8.79 & 10.03 \end{bmatrix} \, m^2$$

Discharge areas:

After some experimentation with the completed model the area ratio that seemed ideal was:

$r\_A = 0.94$

The area ratio is defined as:

$$r\_A = \frac{A\_tt}{A\_ct} = \frac{A\_tt}{A\_tt + A\_bpt}$$

Where it is assumed that after the turbine the tube has the same diameter as the outerdiameter of the turbine (i.e. the discharge area then doesn't need to be reduced by the rotor inner rotor diameter):

$A\_tt := \frac{\pi}{4} \cdot \overrightarrow{D\_t\_dv \cdot D\_t\_dv}$ The turbine tube discharge area in $[m^2]$ $(A\_tt \neq A\_t\_VET\,!)$

$$A\_tt^{\mathrm{T}} = \begin{bmatrix} 12.88 & 5.94 & 9.35 & 10.46 & 11.95 \end{bmatrix} \, m^2$$

$A\_ct := \frac{A\_tt}{r\_A}$ The common tube discharge area in $[m^2]$

$$A\_ct^{\mathrm{T}} = \begin{bmatrix} 13.7 & 6.32 & 9.94 & 11.13 & 12.71 \end{bmatrix} \, m^2$$

This means that the discharge area of the bypass tube at the conflux zone is:

$$A\_bpt := A\_ct - A\_tt$$

$$A\_bpt^{\mathrm{T}} = \begin{bmatrix} 0.82 & 0.38 & 0.6 & 0.67 & 0.76 \end{bmatrix} \, m^2$$

Note: the diameter of the bypass tube only needs to be the correct size where it connects to the common tube, namely equal to the commont tube diameter:

$$D\_bpt := \sqrt{\frac{4}{\pi} \cdot A\_ct} \qquad D\_bpt^{\mathrm{T}} = \begin{bmatrix} 4.18 & 2.84 & 3.56 & 3.76 & 4.02 \end{bmatrix} \, m$$

Geometry turbine tube

The inflow of the turbine tube is assumed to have the same ratios as the regular Kaplan for now. Again using the size parameter and taking 100% size to reduce losses as much as possible:

$Size := 100$

Diamter of the inflow opening:

$$D\_infl\_tt := \left(1.5 + \frac{Size}{100} \cdot (1) \right) \cdot D\_t\_dv$$

Diameter of the bulb:

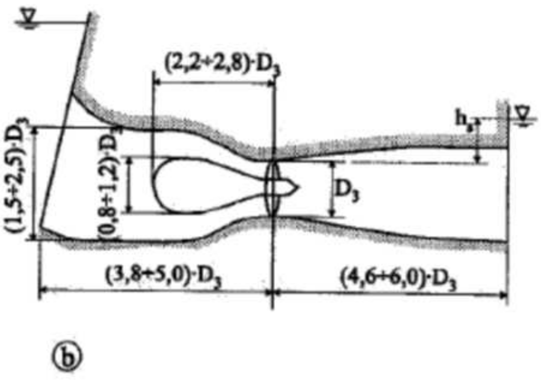
$$D\_bulb := \left(0.8 + \frac{Size}{100} \cdot (0.4) \right) \cdot D\_t\_dv$$

Length of the inflow till rotor blades

$$L\_in\_tt := \left(3.8 + \frac{Size}{100} \cdot (5 - 3.8) \right) \cdot D\_t\_dv$$

Length of the bulb till the rotor blades

$$L\_bulb := \left(2.2 + 0.6 \cdot \frac{Size}{100} \right) \cdot D\_t\_dv$$



1b) Bulb turbine dimension ranges

Radius of the inflow rounding:

$$r\_in\_tt := \mathrm{Round}\left(0.4\ D\_infl\_tt \cdot m^{-1}, 0.1\right) \cdot m$$

To reduce inflow losses to a minimum the radius is 0.4 times the diameter of the inflow pipe and rounded to a multiple of 10cm to have practical values.

Length of the turbine tube till conflux is dependent on the length of the bypass tube, which is based on the rules of thumb for a Kaplan bulb turbine (the bulb length will be used for the contraction towards the desired outflow opening at the conflux zone, but size is set to 0 to keep it short: $L\_out\_tt := \left(3.8 + \frac{0}{100} \cdot (5 - 3.8) \right) \cdot D\_bpt$

Geometry bypass tube

As noted at the turbine tube geometry the dimensions are based on the rules of thumb for a regular Kaplan bulb turbine, but the inflow length is doesn't include the bulb, as it is not there. Instead the

Diamter of the inflow opening:

$$D\_infl\_bpt := \left(1.5 + \frac{Size}{100} \cdot (1)\right) \cdot D\_bpt$$

Length of the inflow till rotor blades

$$L\_in\_bpt := L\_out\_tt$$

Length of the contraction till the conflux zone

$$L\_cont\_bpt := \left(2.2 + 0.6 \cdot \frac{Size}{100}\right) \cdot D\_bpt$$

Radius of the inflow rounding:

$$r\_in\_bpt := \mathrm{Round}\left(0.3\ D\_infl\_bpt \cdot m^{-1}, 0.1\right) \cdot m$$

Geometry common tube

Diameter of the mixing area:

$$D\_conflux := \sqrt{\frac{4}{\pi} \cdot A\_ct}$$

Length of the conflux zone (estimated to be 6 times the conflux zone diameter):

$$L\_conflux := 6 \cdot D\_conflux$$

Draft-tube length:

$$L\_out\_ct := \left(4.6 + \frac{Size}{100} \cdot (6 - 4.6)\right) \cdot D\_conflux$$

Expansion angle of the draft tube:

$$\beta\_draft := 5°$$

Note: Having the steepest possible angle give the lowest outflow losses, but makes the expansion losses larger.

Diameter at outflow:

$$D\_outfl := D\_conflux + 2 \cdot \tan\left(\beta\_draft\right) \cdot L\_out\_ct$$

To summerize the geometry of the inlfow tube:

$$L\_total\_tt := L\_in\_tt + L\_out\_tt$$

$$GEO\_tt := \mathrm{augment}\left(r\_in\_tt, D\_infl\_tt, D\_bulb, D\_t\_dv, L\_in\_tt, L\_bulb, L\_out\_tt, L\_total\_tt\right)$$

Collumns show:

$$GEO\_tt = \begin{array}{cccccccccc} \text{r\_in} & \text{D\_infl} & \text{D\_blb} & \text{D\_t} & \text{L\_infl} & \text{L\_blb} & \text{L\_out} & \text{L\_total} & & \\ 4.10 & 10.13 & 4.86 & 4.05 & 20.25 & 11.34 & 15.87 & 36.12 & & \text{DV0} \\ 2.80 & 6.88 & 3.30 & 2.75 & 13.75 & 7.70 & 10.78 & 24.53 & & \text{DV1} \\ 3.50 & 8.63 & 4.14 & 3.45 & 17.25 & 9.66 & 13.52 & 30.77 & m & \text{DV2} \\ 3.70 & 9.13 & 4.38 & 3.65 & 18.25 & 10.22 & 14.31 & 32.56 & & \text{DV3} \\ 3.90 & 9.75 & 4.68 & 3.90 & 19.50 & 10.92 & 15.29 & 34.79 & & \text{DV4} \end{array}$$

To summerize the geometry of the bypass tube:

$$L\_total\_bpt := L\_in\_bpt$$

$$GEO\_bpt := \mathrm{augment}\left(r\_in\_bpt, D\_infl\_bpt, D\_bpt, L\_in\_bpt, L\_cont\_bpt, L\_total\_bpt\right)$$

Collumns show:

$$GEO\_bpt = \begin{array}{cccccccc} \text{r\_in} & \text{D\_infl} & \text{D\_out} & \text{L\_infl} & \text{L\_con} & \text{L\_total} & & \\ 3.10 & 10.44 & 4.18 & 15.87 & 11.70 & 15.87 & & \text{DV0} \\ 2.10 & 7.09 & 2.84 & 10.78 & 7.94 & 10.78 & & \text{DV1} \\ 2.70 & 8.90 & 3.56 & 13.52 & 9.96 & 13.52 & m & \text{DV2} \\ 2.80 & 9.41 & 3.76 & 14.31 & 10.54 & 14.31 & & \text{DV3} \\ 3.00 & 10.06 & 4.02 & 15.29 & 11.26 & 15.29 & & \text{DV4} \end{array}$$

To summerize the geometry of the common tube:

$L\_total\_ct := L\_conflux + L\_out\_ct$

$GEO\_ct := \text{augment}(D\_conflux, L\_conflux, L\_out\_ct, L\_total\_ct)$

Collumns show:

$$GEO\_ct = \begin{array}{ccccccc} & \text{D\_cnfl} & \text{L\_cnfl} & \text{L\_drft} & \text{L\_total} & & \\ & 4.18 & 25.06 & 25.06 & 50.13 & & \text{DV0} \\ & 2.84 & 17.02 & 17.02 & 34.04 & & \text{DV1} \\ & 3.56 & 21.35 & 21.35 & 42.70 & m & \text{DV2} \\ & 3.76 & 22.59 & 22.59 & 45.18 & & \text{DV3} \\ & 4.02 & 24.14 & 24.14 & 48.27 & & \text{DV4} \end{array}$$

Total length system:
taking turbine path (largest dimension, so this one determines size of power plant):

$L\_sys\_tp := L\_total\_tt + L\_total\_ct$

$L\_sys\_tp^{T} = \begin{bmatrix} 86.25 & 58.57 & 73.47 & 77.73 & 83.06 \end{bmatrix} m$

For information: bypass path:

$L\_sys\_bp := L\_total\_bpt + L\_total\_ct$

$L\_sys\_bp^{T} = \begin{bmatrix} 66 & 44.82 & 56.22 & 59.48 & 63.56 \end{bmatrix} m$

Largest diameter is D_infl_bpt, which determines the width of the power plant.

$D\_infl\_bpt^{T} = \begin{bmatrix} 10.44 & 7.09 & 8.9 & 9.41 & 10.06 \end{bmatrix} m$

(minimum) Land use: $A\_land\_use := \overrightarrow{n\_t \cdot L\_sys\_tp \cdot D\_infl\_bpt}$

$A\_land\_use^{T} = \begin{bmatrix} 3603 & 831 & 1961 & 2926 & 4176 \end{bmatrix} m^{2}$

(minimum) width powerhouse: $B\_land\_use := 1\ m + \overrightarrow{n\_t \cdot (D\_infl\_bpt + 1\ m)}$

$B\_land\_use^{T} = \begin{bmatrix} 46.77 & 17.18 & 30.69 & 42.65 & 56.28 \end{bmatrix} m$

Resistance to flow

To determine the resistance to flow the quadratic discharge coefficient is determined for each part of the system. Assuming the bypass can also be closed off, the resistance of the system with a closed bypass also needs to be determined.

The quadratic discharge coefficient (QDC) is defined as follows:

$$C = \frac{1}{2\ g} \cdot \sum_{i=0}^{N} \frac{\xi_i}{{A_i}^{2}}$$

And is determined for the following sections:

Turbine tube:

C_tt

Losses taken into account:

1. trash rack
2. inflow
3. friction inflow pipe
4. contraction
5. friction bulb
6. turbine
7. friction outflow pipe
8. conflux

Bypass tube:

C_bpt

Losses taken into account:

1. inflow
2. friction inflow pipe

3.1 contraction
3.2 friction in contraction

4. conflux

(Bypass tube is assumed to have no trash rack)

Common tube:

C_ct_bp1 (bypass open)

Losses taken into account:

1. Friction in conflux zone
2. expansion draft tube
3. friction draft turbe
4. outflow

C_ct_bp0 (bypass closed)

1. expansion into conflux zone
2. friction in conflux zone
3. expansion draft tube
4. friction draft turbe
5. outflow

Resistance to flow (continued)
Locations where the losses occur:
x=0 is at inflow of the turbine tube.
For the turbine path:

```
$x\_tp :=$ for $j \in 0..10$
    $x_{0,j} \leftarrow j$
    for $i \in 0..\text{rows}(n\_t)-1$
        $x_{i+1,0} \leftarrow -2\ m$
        $x_{i+1,1} \leftarrow 0\ m$
        $x_{i+1,2} \leftarrow r\_in\_tt_i$
        $x_{i+1,3} \leftarrow L\_in\_tt_i - L\_bulb_i$
        $x_{i+1,4} \leftarrow L\_in\_tt_i - L\_bulb_i + \frac{D\_bulb_i}{2}$
        $x_{i+1,5} \leftarrow L\_in\_tt_i - \frac{D\_t\_dv_i}{4}$
        $x_{i+1,6} \leftarrow L\_in\_tt_i$
        $x_{i+1,7} \leftarrow L\_in\_tt_i + L\_out\_tt_i$
        $x_{i+1,8} \leftarrow L\_total\_tt_i + L\_conflux_i$
        $x_{i+1,9} \leftarrow L\_total\_tt_i + L\_conflux_i + L\_out\_ct_i$
        $x_{i+1,10} \leftarrow L\_sys\_tp_i + 2\ m$
return $x$
```

$$x\_tp = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ -2\ m & 0\ m & 4.1\ m & 8.91\ m & 11.34\ m & 19.24\ m & 20.25\ m & 36.12\ m & 61.19\ m & 86.25\ m & 88.25\ m \\ -2\ m & 0\ m & 2.8\ m & 6.05\ m & 7.7\ m & 13.06\ m & 13.75\ m & 24.53\ m & 41.55\ m & 58.57\ m & 60.57\ m \\ -2\ m & 0\ m & 3.5\ m & 7.59\ m & 9.66\ m & 16.39\ m & 17.25\ m & 30.77\ m & 52.12\ m & 73.47\ m & 75.47\ m \\ -2\ m & 0\ m & 3.7\ m & 8.03\ m & 10.22\ m & 17.34\ m & 18.25\ m & 32.56\ m & 55.14\ m & 77.73\ m & 79.73\ m \\ -2\ m & 0\ m & 3.9\ m & 8.58\ m & 10.92\ m & 18.53\ m & 19.5\ m & 34.79\ m & 58.92\ m & 83.06\ m & 85.06\ m \end{bmatrix}$$

Resistance to flow (continued)
Locations where the losses occur:
x=0 is at inflow of the turbine tube.
For the bypass path:

```
$x\_bpp :=$ for $j \in 0..7$
    $x_{0,j} \leftarrow j$
    for $i \in 0..\mathrm{rows}(n\_t)-1$
        $xbp0_i \leftarrow x\_tp_{i+1,7} - L\_total\_bpt_i$
        $x_{i+1,0} \leftarrow xbp0_i - 2\ m$
        $x_{i+1,1} \leftarrow xbp0_i$
        $x_{i+1,2} \leftarrow xbp0_i + r\_in\_tt_i$
        $x_{i+1,3} \leftarrow xbp0_i + L\_in\_bpt_i - L\_cont\_bpt_i$
        $x_{i+1,4} \leftarrow xbp0_i + L\_in\_bpt_i$
        $x_{i+1,5} \leftarrow xbp0_i + L\_in\_bpt_i + L\_conflux_i$
        $x_{i+1,6} \leftarrow xbp0_i + L\_in\_bpt_i + L\_conflux_i + L\_out\_ct_i$
        $x_{i+1,7} \leftarrow xbp0_i + L\_sys\_bp_i + 2\ m$
    return $x$
```

$$x\_bpp = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 18.25\ m & 20.25\ m & 24.35\ m & 24.43\ m & 36.12\ m & 61.19\ m & 86.25\ m & 88.25\ m \\ 11.75\ m & 13.75\ m & 16.55\ m & 16.59\ m & 24.53\ m & 41.55\ m & 58.57\ m & 60.57\ m \\ 15.25\ m & 17.25\ m & 20.75\ m & 20.81\ m & 30.77\ m & 52.12\ m & 73.47\ m & 75.47\ m \\ 16.25\ m & 18.25\ m & 21.95\ m & 22.01\ m & 32.56\ m & 55.14\ m & 77.73\ m & 79.73\ m \\ 17.5\ m & 19.5\ m & 23.4\ m & 23.52\ m & 34.79\ m & 58.92\ m & 83.06\ m & 85.06\ m \end{bmatrix}$$

Calculation of QDC for turbine tube

Trash rack (Section 1 of turbine tube)

As general formula the trash rack loss coefficient is defined as:

$\xi\_tr = \beta\_tr \cdot \zeta\_tr \cdot c\_tr \cdot sin(\delta\_tr)$

Where:

$\beta\_tr =$ the "rack" coefficient, which is actually more a shape coefficient

$\zeta\_tr =$ the gap geometry coefficient

$c\_tr =$ the "cleaning method" coefficient, determined by how the rack will be cleaned

$\delta\_tr =$ the inclination angle of the rack

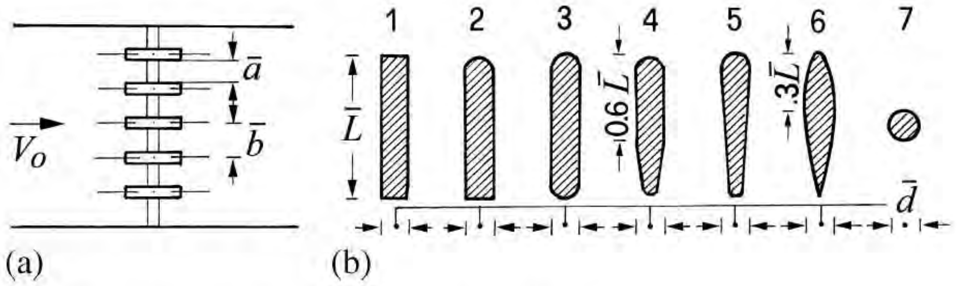


Fig. 2.18 (a) Plan of the rack bars, (b) Types of rack bars

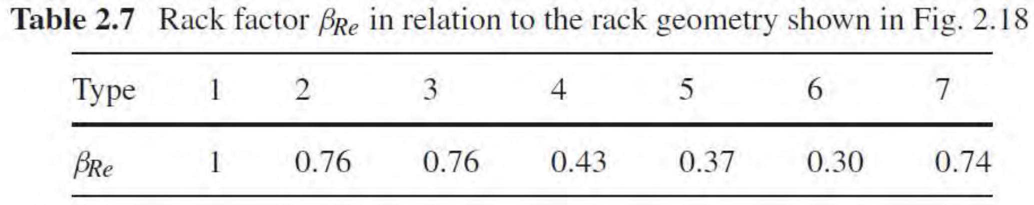


Table 2.7 Rack factor β_{Re} in relation to the rack geometry shown in Fig. 2.18

Type	1	2	3	4	5	6	7
β_{Re}	1	0.76	0.76	0.43	0.37	0.30	0.74

Figure and table above are from pages 44 and 45 (chapter 2) respectively.

[1] Source: W.H. Hager, Wastewater Hydraulics - Theory and Practice, 2nd ed., DOI 10.1007/978-3-642-11383-3_2, Springer-Verlag, Berlin Heidelberg 2010

The following assumptions are made regarding the trash-rack:

1. Rounded bars are used (type 3 from the previous page) with a L/d ratio of about 5.

$\beta\_tr := 0.76$

2. The gap ratio a/b is taken to be larger than 0.5.

3. If for example a/b=0.8 and the gaps are 1cm then the centre to centre distance is 1.25cm and the bar thickness needs to be 0.25cm and L=1.25cm. Comparing with images from existing trash-racks, these seem realistic values. thus:

$r\_ab := 0.8$

3. Assumptions 1 and 2 mean that Idel'cik's simplified formula can be used:

$$\xi\_tr = \frac{7}{3} \beta\_tr \cdot \left(\frac{b}{a} - 1\right)^{\frac{4}{3}} \cdot c\_tr \cdot sin(\delta\_tr)$$

4. Cleaning will be done mechanically, which means c_tr lies between 1.1 and 1.3, of which the average is assumed to be a reasonable value to estimate the losses with.

$c\_tr := 1.2$

5. The inclination will be taken as 10° from vertical.

$\delta\_tr := 90° - 10°$

Therefore, the loss-coefficient is:

$$\xi\_tr := \frac{7}{3} \beta\_tr \cdot \left(\frac{1}{r\_ab} - 1\right)^{\frac{4}{3}} \cdot c\_tr \cdot \sin(\delta\_tr) = 0.33$$

Trash rack (section 1 of turbine tube) (continued):
The flow velocity through the trash rack determines the actual head losses. So for a variable discharge the cross sectional area is required. However, the discharge area is dependent on the waterlevel as there is free surface flow when the water goes through the trash rack.

For the moment the assumption is made that a area with a width of the inflow diameter plus twice the inflow rounding radius and a height of the inflow diameter is the flow area for the trash rack. Due to the bars the area is reduced by factor r_ab.

Related cross-sectional area:

$$A\_tr := r\_ab\ \overrightarrow{(D\_infl\_tt) \cdot (2\ r\_in\_tt + D\_infl\_tt)}$$

$$A\_tr^{\mathrm{T}} = [148.43\ \ 68.61\ \ 107.81\ \ 120.63\ \ 136.89]\ m^2$$

Contribution to quadratic discharge coefficient:

$$C\_tt\_1 := \frac{\xi\_tr}{2\ g \cdot \overrightarrow{A\_tr \cdot A\_tr}}$$

$$C\_tt\_1^{\mathrm{T}} = [7.64 \cdot 10^{-7}\ \ 3.57 \cdot 10^{-6}\ \ 1.45 \cdot 10^{-6}\ \ 1.16 \cdot 10^{-6}\ \ 8.98 \cdot 10^{-7}]\ \frac{s^2}{m^5}$$

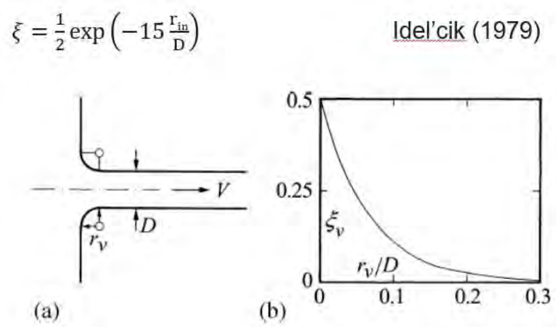
Collecting the contributions:
$C\_tt := C\_tt\_1$

Inflow (section 2 of turbine tube):

Shape of inflow:
"Rounded with large radius"

Xi-loss coefficient for inflow general formula:

```
$\xi\_in(r\_in, D\_in) :=$
    for $i \in 0 .. \mathrm{rows}(D\_in) - 1$
        $X_i \leftarrow \frac{1}{2} \cdot \exp\left(-15 \cdot \frac{r\_in_i}{D\_in_i}\right)$
    return $X$
```



$$\xi\_in\_tt := \xi\_in(r\_in\_tt, D\_infl\_tt)$$

$$\xi\_in\_tt^{\mathrm{T}} = [1.15 \cdot 10^{-3}\ \ 1.11 \cdot 10^{-3}\ \ 1.14 \cdot 10^{-3}\ \ 1.14 \cdot 10^{-3}\ \ 1.24 \cdot 10^{-3}]$$

Related cross-sectional area:

$$A\_infl\_tt := \frac{\pi}{4} \cdot \overrightarrow{D\_infl\_tt \cdot D\_infl\_tt}$$

$$A\_infl\_tt^{\mathrm{T}} = [80.52\ \ 37.12\ \ 58.43\ \ 65.4\ \ 74.66]\ m^2$$

Contribution to quadratic discharge coefficient:

$$C\_tt\_2 := \overrightarrow{\frac{\xi\_in\_tt}{2\ g \cdot A\_infl\_tt \cdot A\_infl\_tt}}$$

$$C\_tt\_2^{\mathrm{T}} = [9.05 \cdot 10^{-9}\ \ 4.11 \cdot 10^{-8}\ \ 1.7 \cdot 10^{-8}\ \ 1.36 \cdot 10^{-8}\ \ 1.13 \cdot 10^{-8}]\ \frac{s^2}{m^5}$$

Collecting the contributions:
$C\_tt := \mathrm{augment}(C\_tt, C\_tt\_2)$

Inlettube wall friction (section 3 of turbine tube)

Wall friction is dependent on the Reynolds number, which is dependent on the flow velocity. This differs for each available discharge and head. However, in the generic turbine model the maximum discharge has been determined. These can be used to estimate the wall friction loss coefficient.

From the model of the generic turbine (total discharge of the plant):

$Q\_p\_max\_g := \begin{bmatrix} 336.92 & 68.1 & 131.7 & 236.56 & 308.85 \end{bmatrix}^{T} \cdot m^{3} \cdot s^{-1}$

After design iterations it is shown that about 80% of the flow will go through the turbine:

$Q\_max\_tt := 80\% \cdot Q\_p\_max\_g$

The lowest discharge that will go through the turbine is about 20% of that:

$Q\_20pc\_tt := 20\% \cdot Q\_max\_tt$

$Q\_20pc\_tt^{T} = \begin{bmatrix} 53.91 & 10.9 & 21.07 & 37.85 & 49.42 \end{bmatrix} m^{3} \cdot s^{-1}$

So per turbine this is:

```
p2t(X) :=  for $i \in 0..\mathrm{rows}(X)-1$
               $Y_i \leftarrow \frac{X_i}{n\_t_i}$
           return $Y$
```

$Q\_t\_max\_g := p2t(Q\_max\_tt)$

$Q\_t\_max\_g^{T} = \begin{bmatrix} 67.38 & 27.24 & 35.12 & 47.31 & 49.42 \end{bmatrix} m^{3} \cdot s^{-1}$

$Q\_t\_20pc := p2t(Q\_20pc\_tt)$

$Q\_t\_20pc^{T} = \begin{bmatrix} 13.48 & 5.45 & 7.02 & 9.46 & 9.88 \end{bmatrix} m^{3} \cdot s^{-1}$

The relevant cross-sectional areas for this part of the system are equal to that of the inflow, so flow velocities are:

```
u(Q, A) :=  for $i \in 0..\mathrm{rows}(Q)-1$
                $Y_i \leftarrow \frac{Q_i}{A_i}$
            return $Y$
```

$u\_infl\_tt\_max := u(Q\_t\_max\_g, A\_infl\_tt)$

$u\_infl\_tt\_max^{T} = \begin{bmatrix} 0.84 & 0.73 & 0.6 & 0.72 & 0.66 \end{bmatrix} m \cdot s^{-1}$

$u\_infl\_tt\_min := u(Q\_t\_20pc, A\_infl\_tt)$

$u\_infl\_tt\_min^{T} = \begin{bmatrix} 0.17 & 0.15 & 0.12 & 0.14 & 0.13 \end{bmatrix} m \cdot s^{-1}$

Making the Reynolds numbers:

$Re(u, D) := \frac{\overrightarrow{u \cdot D}}{\nu}$ $Re\_infl\_tt\_max := Re(u\_infl\_tt\_max, D\_infl\_tt)$

$Re\_infl\_tt\_max^{T} = \begin{bmatrix} 8.44 \cdot 10^{6} & 5.03 \cdot 10^{6} & 5.16 \cdot 10^{6} & 6.58 \cdot 10^{6} & 6.43 \cdot 10^{6} \end{bmatrix}$

$Re\_infl\_tt\_min := Re(u\_infl\_tt\_min, D\_infl\_tt)$

$Re\_infl\_tt\_min^{T} = \begin{bmatrix} 1.69 \cdot 10^{6} & 1.01 \cdot 10^{6} & 1.03 \cdot 10^{6} & 1.32 \cdot 10^{6} & 1.29 \cdot 10^{6} \end{bmatrix}$

Relative roughness (new welded steel tube):

$k\_s := 0.2\ mm$

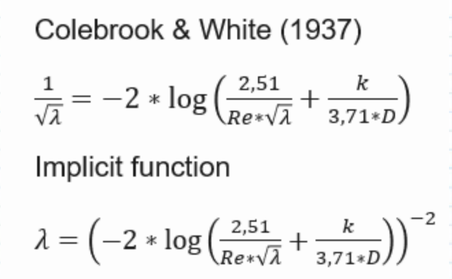
Length over which friction acts:

$L\_fr\_infl\_tt := L\_in\_tt - L\_bulb$ $L\_fr\_infl\_tt^{T} = \begin{bmatrix} 8.91 & 6.05 & 7.59 & 8.03 & 8.58 \end{bmatrix} m$

... continue on next page

Inlettube wall friction (section 3 of turbine tube) (continued)

Colebrook & White (1937) friction factor:


Colebrook & White (1937)

$$\frac{1}{\sqrt{\lambda}} = -2 * \log\left(\frac{2{,}51}{Re*\sqrt{\lambda}} + \frac{k}{3{,}71*D}\right)$$

Implicit function

$$\lambda = \left(-2 * \log\left(\frac{2{,}51}{Re*\sqrt{\lambda}} + \frac{k}{3{,}71*D}\right)\right)^{-2}$$

```
$\lambda\_fr(D,k,Re) :=$ for $i \in 0..\mathrm{rows}(D)-1$
        $a_i \leftarrow 0.010$
        $count \leftarrow 0$
        $b_i \leftarrow \left(-2\cdot\log\left(\left(\frac{2.51}{Re_i\cdot\sqrt[2]{a_i}}+\frac{k}{3.71\cdot D_i}\right),10\right)\right)^{-2}$
        while $\left\|a_i-b_i\right\| > 0.1\%$
            $count \leftarrow count+1$
            $a_i \leftarrow b_i$
            $b_i \leftarrow \left(-2\cdot\log\left(\left(\frac{2.51}{Re_i\cdot\sqrt[2]{a_i}}+\frac{k}{3.71\cdot D_i}\right),10\right)\right)^{-2}$
    return $b$
```

Colebrook & White (1937) friction factors:

$\lambda\_fr\_umax := \lambda\_fr(D\_infl\_tt, k\_s, Re\_infl\_tt\_max)$

$\lambda\_fr\_umax^{\mathrm{T}} = \begin{bmatrix} 9.68\cdot10^{-3} & 1.04\cdot10^{-2} & 1.02\cdot10^{-2} & 9.95\cdot10^{-3} & 9.9\cdot10^{-3} \end{bmatrix}$

$\lambda\_fr\_umin := \lambda\_fr(D\_infl\_tt, k\_s, Re\_infl\_tt\_min)$

$\lambda\_fr\_umin^{\mathrm{T}} = \begin{bmatrix} 1.12\cdot10^{-2} & 1.22\cdot10^{-2} & 1.21\cdot10^{-2} & 1.17\cdot10^{-2} & 1.17\cdot10^{-2} \end{bmatrix}$

Loss coefficients using Darcy-Weisbach (1845):

$\xi\_in\_fr\_umax := \overrightarrow{\lambda\_fr\_umax\cdot L\_fr\_infl\_tt\cdot\frac{1}{D\_infl\_tt}}$

$\xi\_in\_fr\_umax^{\mathrm{T}} = \begin{bmatrix} 8.52\cdot10^{-3} & 9.19\cdot10^{-3} & 8.96\cdot10^{-3} & 8.76\cdot10^{-3} & 8.71\cdot10^{-3} \end{bmatrix}$

$\xi\_in\_fr\_umin := \overrightarrow{\lambda\_fr\_umin\cdot L\_fr\_infl\_tt\cdot\frac{1}{D\_infl\_tt}}$

$\xi\_in\_fr\_umin^{\mathrm{T}} = \begin{bmatrix} 9.9\cdot10^{-3} & 1.08\cdot10^{-2} & 1.06\cdot10^{-2} & 1.03\cdot10^{-2} & 1.03\cdot10^{-2} \end{bmatrix}$

```
$\xi\_in\_fr\_avg :=$ for $i \in 0..\mathrm{rows}(n\_t)-1$
        $a_i \leftarrow \xi\_in\_fr\_umax_i + \xi\_in\_fr\_umin_i$
    return $0.5\cdot a$
```

$\xi\_in\_fr\_avg^{\mathrm{T}} = \begin{bmatrix} 9.21\cdot10^{-3} & 9.98\cdot10^{-3} & 9.8\cdot10^{-3} & 9.51\cdot10^{-3} & 9.49\cdot10^{-3} \end{bmatrix}$

Check error of head-losses for average value:

$$\Delta H(u,\xi) := \frac{\overrightarrow{\xi\cdot u\cdot u}}{2\ g}$$

$\Delta H(u\_infl\_tt\_max, \xi\_in\_fr\_umax)^{\mathrm{T}} = \begin{bmatrix} 3.04\cdot10^{-4} & 2.52\cdot10^{-4} & 1.65\cdot10^{-4} & 2.34\cdot10^{-4} & 1.95\cdot10^{-4} \end{bmatrix}\ \mathrm{m}$

$\Delta H(u\_infl\_tt\_max, \xi\_in\_fr\_avg)^{\mathrm{T}} = \begin{bmatrix} 3.29\cdot10^{-4} & 2.74\cdot10^{-4} & 1.8\cdot10^{-4} & 2.54\cdot10^{-4} & 2.12\cdot10^{-4} \end{bmatrix}\ \mathrm{m}$

$\Delta H(u\_infl\_tt\_min, \xi\_in\_fr\_umin)^{\mathrm{T}} = \begin{bmatrix} 1.41\cdot10^{-5} & 1.18\cdot10^{-5} & 7.83\cdot10^{-6} & 1.1\cdot10^{-5} & 9.17\cdot10^{-6} \end{bmatrix}\ \mathrm{m}$

$\Delta H(u\_infl\_tt\_min, \xi\_in\_fr\_avg)^{\mathrm{T}} = \begin{bmatrix} 1.32\cdot10^{-5} & 1.1\cdot10^{-5} & 7.22\cdot10^{-6} & 1.02\cdot10^{-5} & 8.48\cdot10^{-6} \end{bmatrix}\ \mathrm{m}$

... continue on next page ...

Inlettube wall friction (section 3 of turbine tube) (continued)
Error in using the average instead of the actual loss-coefficient:

$$error\_max := \Delta H\left(u\_infl\_tt\_max, \xi\_in\_fr\_umax\right) - \Delta H\left(u\_infl\_tt\_max, \xi\_in\_fr\_avg\right)$$

$$error\_max^{T} = \left[-2.46 \cdot 10^{-5} \;\; -2.16 \cdot 10^{-5} \;\; -1.53 \cdot 10^{-5} \;\; -2.01 \cdot 10^{-5} \;\; -1.74 \cdot 10^{-5}\right] \, m$$

$$error\_min := \Delta H\left(u\_infl\_tt\_min, \xi\_in\_fr\_umin\right) - \Delta H\left(u\_infl\_tt\_min, \xi\_in\_fr\_avg\right)$$

$$error\_min^{T} = \left[9.82 \cdot 10^{-7} \;\; 8.66 \cdot 10^{-7} \;\; 6.13 \cdot 10^{-7} \;\; 8.05 \cdot 10^{-7} \;\; 6.95 \cdot 10^{-7}\right] \, m$$

Using the average value has only a slight under-estimation in the order of $O\left(10^{-7}\ m\right)$ for the minimum discharge and a slight over-estimation of order $O\left(10^{-5}\ m\right)$ for the maximum discharge. It is therefore deemed acceptable to use the average.

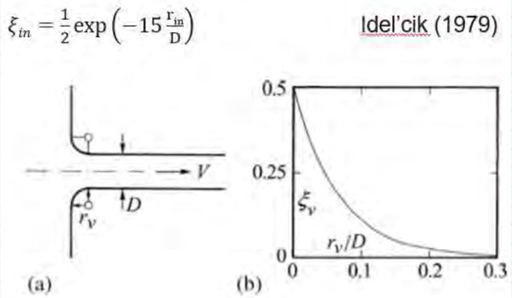
In that case the contribution to the QDC is:

$$C\_tt\_3 := \frac{\xi\_in\_fr\_avg}{2\ g \cdot \overrightarrow{A\_infl\_tt \cdot A\_infl\_tt}}$$

$$C\_tt\_3^{T} = \left[7.24 \cdot 10^{-8} \;\; 3.69 \cdot 10^{-7} \;\; 1.46 \cdot 10^{-7} \;\; 1.13 \cdot 10^{-7} \;\; 8.68 \cdot 10^{-8}\right] \frac{s^{2}}{m^{5}}$$

Collecting the contributions:
$C\_tt := \text{augment}\left(C\_tt, C\_tt\_3\right)$

Interaction with bulb (section 4 of turbine tube)
It is assumed that the rounding of the bulb makes sure that flow seperation doesn't happen and that the contraction losses are therefore negligible. There is however interaction with this shape, so it is assumed that the flow going around the bulb is like an inflow with rounded edges. However the diameter is then the difference between the radius of the inflow pipe and the bulb shape.



$$\xi\_bulb\_tt := \xi\_in\left(0.5 \cdot D\_bulb, \left(D\_infl\_tt - D\_bulb\right)\right)$$

$$\xi\_bulb\_tt^{T} = \left[4.92 \cdot 10^{-4} \;\; 4.92 \cdot 10^{-4} \;\; 4.92 \cdot 10^{-4} \;\; 4.92 \cdot 10^{-4} \;\; 4.92 \cdot 10^{-4}\right]$$

For such a case the reference flow velocity is after the flow has entered, so that would be past the rounding. The cross-sectional area is then:

$$A\_tt\_4 := \frac{\pi}{4} \cdot \left(\overrightarrow{D\_infl\_tt \cdot D\_infl\_tt} - \overrightarrow{D\_bulb \cdot D\_bulb}\right)$$

$$A\_tt\_4^{T} = \left[61.96 \;\; 28.57 \;\; 44.96 \;\; 50.33 \;\; 57.46\right] m^{2}$$

Making the contribution to the QDC:

$$C\_tt\_4 := \frac{\xi\_bulb\_tt}{2\ g \cdot \overrightarrow{A\_tt\_4 \cdot A\_tt\_4}}$$

$$C\_tt\_4^{T} = \left[6.54 \cdot 10^{-9} \;\; 3.08 \cdot 10^{-8} \;\; 1.24 \cdot 10^{-8} \;\; 9.91 \cdot 10^{-9} \;\; 7.6 \cdot 10^{-9}\right] \frac{s^{2}}{m^{5}}$$

Collecting the contributions:
$C\_tt := \text{augment}\left(C\_tt, C\_tt\_4\right)$

Interaction with bulb (section 4) (continued)
With the bulb the water experiences friction from both the concrete wall and the steel casing of the bulb. Both contributions will be small, but have been taken into account here.

The relevant height for the Reynolds number is the gap between the bulb and the concrete.
$h\_gap\_bulb := 0.5\ (D\_infl\_tt - D\_bulb)$

$Re\_4\_umax := Re(u(Q\_t\_max\_g, A\_tt\_4), h\_gap\_bulb)$
$Re\_4\_umax^{T} = [2.85 \cdot 10^{6}\ \ 1.7 \cdot 10^{6}\ \ 1.74 \cdot 10^{6}\ \ 2.22 \cdot 10^{6}\ \ 2.17 \cdot 10^{6}]$
$Re\_4\_umin := Re(u(Q\_t\_20pc, A\_tt\_4), h\_gap\_bulb)$
$Re\_4\_umin^{T} = [5.7 \cdot 10^{5}\ \ 3.4 \cdot 10^{5}\ \ 3.49 \cdot 10^{5}\ \ 4.44 \cdot 10^{5}\ \ 4.34 \cdot 10^{5}]$

Then for outer wall
$\lambda\_fr\_tt\_4out\_umax := \lambda\_fr(h\_gap\_bulb, k\_s, Re\_4\_umax)$
$\lambda\_fr\_tt\_4out\_umax^{T} = [1.21 \cdot 10^{-2}\ \ 1.31 \cdot 10^{-2}\ \ 1.27 \cdot 10^{-2}\ \ 1.24 \cdot 10^{-2}\ \ 1.23 \cdot 10^{-2}]$
$\lambda\_fr\_tt\_4out\_umin := \lambda\_fr(h\_gap\_bulb, k\_s, Re\_4\_umin)$
$\lambda\_fr\_tt\_4out\_umin^{T} = [1.39 \cdot 10^{-2}\ \ 1.52 \cdot 10^{-2}\ \ 1.5 \cdot 10^{-2}\ \ 1.44 \cdot 10^{-2}\ \ 1.44 \cdot 10^{-2}]$
Again using average:
$\lambda\_fr\_tt\_4out := 0.5 \cdot (\lambda\_fr\_tt\_4out\_umax + \lambda\_fr\_tt\_4out\_umin)$
$\lambda\_fr\_tt\_4out^{T} = [1.3 \cdot 10^{-2}\ \ 1.42 \cdot 10^{-2}\ \ 1.38 \cdot 10^{-2}\ \ 1.34 \cdot 10^{-2}\ \ 1.34 \cdot 10^{-2}]$

For the inner (bulb) wall
$\lambda\_fr\_tt\_4in\_umax := \lambda\_fr(h\_gap\_bulb, k\_s, Re\_4\_umax)$
$\lambda\_fr\_tt\_4in\_umax^{T} = [1.21 \cdot 10^{-2}\ \ 1.31 \cdot 10^{-2}\ \ 1.27 \cdot 10^{-2}\ \ 1.24 \cdot 10^{-2}\ \ 1.23 \cdot 10^{-2}]$
$\lambda\_fr\_tt\_4in\_umin := \lambda\_fr(h\_gap\_bulb, k\_s, Re\_4\_umin)$
$\lambda\_fr\_tt\_4in\_umin^{T} = [1.39 \cdot 10^{-2}\ \ 1.52 \cdot 10^{-2}\ \ 1.5 \cdot 10^{-2}\ \ 1.44 \cdot 10^{-2}\ \ 1.44 \cdot 10^{-2}]$
Again using average:
$\lambda\_fr\_tt\_4in := 0.5 \cdot (\lambda\_fr\_tt\_4in\_umax + \lambda\_fr\_tt\_4in\_umin)$
$\lambda\_fr\_tt\_4in^{T} = [1.3 \cdot 10^{-2}\ \ 1.42 \cdot 10^{-2}\ \ 1.38 \cdot 10^{-2}\ \ 1.34 \cdot 10^{-2}\ \ 1.34 \cdot 10^{-2}]$

For the Reynolds number the gap-size is the relevant number, but for the friction the surface area is important. So for the loss coefficient the diameters of the bulb and the concrete tube around it are used:

$$\xi\_tt\_4\_fr := \overrightarrow{\left(\frac{\lambda\_fr\_tt\_4out}{D\_infl\_tt} + \frac{\lambda\_fr\_tt\_4in}{D\_bulb}\right) \cdot L\_bulb}$$

$\xi\_tt\_4\_fr^{T} = [4.49 \cdot 10^{-2}\ \ 4.9 \cdot 10^{-2}\ \ 4.78 \cdot 10^{-2}\ \ 4.64 \cdot 10^{-2}\ \ 4.62 \cdot 10^{-2}]$

Making the contribution to the QDC:

$$C\_tt\_4\_fr := \frac{\xi\_tt\_4\_fr}{2\ g \cdot \overrightarrow{A\_tt\_4 \cdot A\_tt\_4}}$$

$$C\_tt\_4\_fr^{T} = [5.96 \cdot 10^{-7}\ \ 3.06 \cdot 10^{-6}\ \ 1.21 \cdot 10^{-6}\ \ 9.34 \cdot 10^{-7}\ \ 7.14 \cdot 10^{-7}]\ \frac{s^{2}}{m^{5}}$$

Collecting the contributions:
$C\_tt := \text{augment}(C\_tt, C\_tt\_4\_fr)$

Turbine friction and/or turbulence losses
The turbine itself also has some friction or turbulence, but this is out of the scope of this research. For now it is assumed to be zero.
$\xi\_turb := 0.00$

Making the contribution to the QDC:

$$C\_T := \frac{\xi\_turb}{2\ g \cdot \overrightarrow{A\_t\_VET \cdot A\_t\_VET}} \qquad C\_T^{T} = [0\ \ 0\ \ 0\ \ 0\ \ 0]\ \frac{s^{2}}{m^{5}}$$

Collecting the contributions: $C\_tt := \text{augment}(C\_tt, C\_T)$

Wall friciton connection tube (section 5 of turbine tube)

The Reynolds number is:

$Re\_5\_tt\_umax := Re\left(u\left(Q\_t\_max\_g, A\_tt\right), D\_t\_dv\right)$

$Re\_5\_tt\_umax^{T} = \left[2.11\cdot 10^{7} \quad 1.26\cdot 10^{7} \quad 1.29\cdot 10^{7} \quad 1.64\cdot 10^{7} \quad 1.61\cdot 10^{7}\right]$

$Re\_5\_tt\_umin := Re\left(u\left(Q\_t\_20pc, A\_tt\right), D\_t\_dv\right)$

$Re\_5\_tt\_umin^{T} = \left[4.22\cdot 10^{6} \quad 2.51\cdot 10^{6} \quad 2.58\cdot 10^{6} \quad 3.29\cdot 10^{6} \quad 3.21\cdot 10^{6}\right]$

Friciton factor:

$\lambda\_fr\_tt\_5\_umax := \lambda\_fr\left(D\_t\_dv, k\_s, Re\_5\_tt\_umax\right)$

$\lambda\_fr\_tt\_5\_umax^{T} = \left[1.07\cdot 10^{-2} \quad 1.15\cdot 10^{-2} \quad 1.1\cdot 10^{-2} \quad 1.09\cdot 10^{-2} \quad 1.08\cdot 10^{-2}\right]$

$\lambda\_fr\_tt\_5\_umin := \lambda\_fr\left(D\_t\_dv, k\_s, Re\_5\_tt\_umin\right)$

$\lambda\_fr\_tt\_5\_umin^{T} = \left[1.12\cdot 10^{-2} \quad 1.21\cdot 10^{-2} \quad 1.18\cdot 10^{-2} \quad 1.15\cdot 10^{-2} \quad 1.14\cdot 10^{-2}\right]$

Again using average:

$\lambda\_fr\_tt\_5 := 0.5\cdot\left(\lambda\_fr\_tt\_5\_umax + \lambda\_fr\_tt\_5\_umin\right)$

$\lambda\_fr\_tt\_5^{T} = \left[1.09\cdot 10^{-2} \quad 1.18\cdot 10^{-2} \quad 1.14\cdot 10^{-2} \quad 1.12\cdot 10^{-2} \quad 1.11\cdot 10^{-2}\right]$

Loss coefficient:

$$\xi\_tt\_5\_fr := \overrightarrow{\left(\frac{\lambda\_fr\_tt\_5}{D\_t\_dv}\right)\cdot L\_out\_tt}$$

$\xi\_tt\_5\_fr^{T} = \left[4.29\cdot 10^{-2} \quad 4.62\cdot 10^{-2} \quad 4.47\cdot 10^{-2} \quad 4.39\cdot 10^{-2} \quad 4.36\cdot 10^{-2}\right]$

Making the contribution to the QDC:

$$C\_tt\_5\_fr := \frac{\xi\_tt\_5\_fr}{2\ g\cdot\overrightarrow{A\_tt\cdot A\_tt}}$$

$$C\_tt\_5\_fr^{T} = \left[1.32\cdot 10^{-5} \quad 6.68\cdot 10^{-5} \quad 2.61\cdot 10^{-5} \quad 2.05\cdot 10^{-5} \quad 1.56\cdot 10^{-5}\right]\frac{s^{2}}{m^{5}}$$

Collecting the contributions:

$C\_tt := \mathrm{augment}\left(C\_tt, C\_tt\_5\_fr\right)$

Energy gain from conflux zone (section 6 of turbine tube)

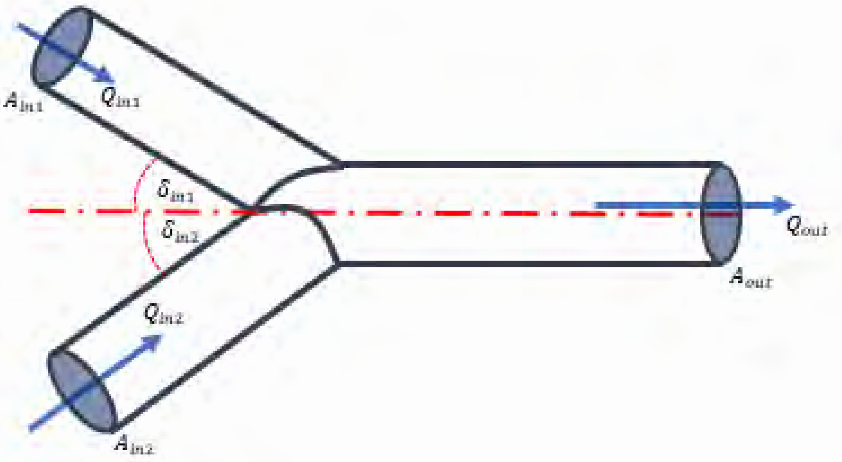
For a conduit junction with sharp edges Vischer (1958):

$$\xi\_in1 = 1 - 2\cdot r\_A1^{-1}\cdot r\_qd\cdot cos\left(\delta\_1\right) - 2\cdot r\_A1^{-1}\cdot\left(1 - r\_qd\right)^{2}\cdot cos\left(\delta\_2\right) + \left(r\_A1^{-1}\cdot r\_qd\right)^{2}$$

$$\xi\_in2 = 1 - 2\cdot r\_A1^{-1}\cdot r\_qd\cdot cos\left(\delta\_1\right) - 2\cdot r\_A1^{-1}\cdot\left(1 - r\_qd\right)^{2}\cdot cos\left(\delta\_2\right) + \left(r\_A1^{-1}\cdot\left(1 - r\_qd\right)\right)^{2}$$

Where:

$r\_A1 = r\_A = \frac{A\_tt}{A\_ct}$ and $r\_A1 = 1 - r\_A = 1 - \frac{A\_tt}{A\_ct}$ ... continue on next page ...



Energy gain from conflux zone (section 6 of turbine tube) (continued)

Where if the turbine tube is defined as the second pipe then:

$r\_A2 = r\_A = \frac{A\_tt}{A\_ct}$ ----> $r\_A2 := r\_A$

and

$r\_A1 = 1 - r\_A = 1 - \frac{A\_tt}{A\_ct} = \frac{A\_bp}{A\_ct}$ ----> $r\_A1 := 1 - r\_A$

Since for the design of the VETT the angle of the tubes are actually 0 both angles δ_1 and δ_2 should be zero degrees with respect to the commont tube axis. Since there is some sideways movement due to the fact that the bypass flow has a higher discharge and flow velocity, the assumption is made that the bypass tube has a connecting angle of:

$$\delta\_2 := \operatorname{atan}\left(\frac{D\_t\_dv - 0.5\left(D\_bpt - D\_t\_dv\right)}{2 \cdot 0.5 \cdot L\_conflux}\right)$$

Which is the angle a water particle the middle of the flow in the bypass tube has to make to get to the middle of the common tube (half length at half height). This seems a fair angle.

The flow from the turbine tube doesn't really change direction, so this angle is assumed to be zero for all design variants:

$\delta\_1 := 0°$

then the "loss" coefficient or energy gain coefficient is:

$$\xi\_tt\_cfx = 1 - 2 \cdot r\_A1^{-1} \cdot r\_qd^2 \cdot cos\left(\delta\_1\right) - 2 \cdot r\_A1^{-1} \cdot \left(1 - r\_qd\right)^2 \cdot cos\left(\delta\_2\right) + \left(r\_A1^{-1} \cdot \left(1 - r\_qd\right)\right)^2$$

$$\xi\_cfx2\left(r\_qd\right) := \overrightarrow{1 - 2 \cdot \frac{r\_qd^2}{r\_A1} \cdot \cos\left(\delta\_1\right) - 2 \cdot \frac{\left(1 - r\_qd\right)^2}{r\_A2} \cdot \cos\left(\delta\_2\right) + \left(\frac{\left(1 - r\_qd\right)}{r\_A2}\right)^2}$$

Also defining the head loss from the bypass tube towards the conflux zone:

$$\xi\_cfx1\left(r\_qd\right) := \overrightarrow{1 - 2 \cdot \frac{r\_qd^2}{r\_A1} \cdot \cos\left(\delta\_1\right) - 2 \cdot \frac{\left(1 - r\_qd\right)^2}{r\_A2} \cdot \cos\left(\delta\_2\right) + \left(\frac{r\_qd}{r\_A1}\right)^2}$$

Discharge ratio is the turbine discharge over the common discharge. The company VerdErg claimed to be 20%, but it isn't clear if this is a constant ratio (actually very likely it is not).

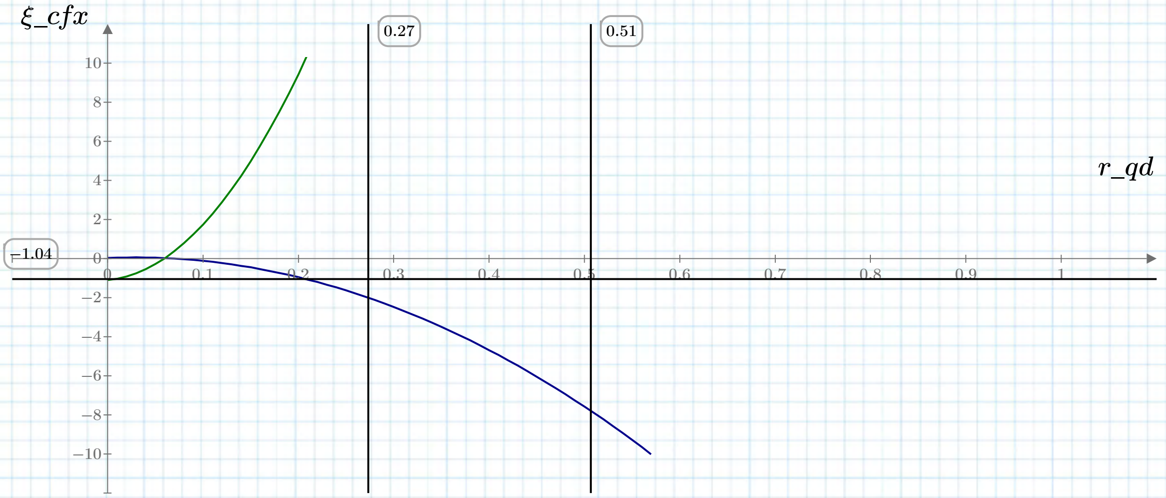
So for now it will remain a variable, as later this ratio is encountered again and turns out to be one of the important variables to solve the system.

$r\_qd = \frac{Q\_1}{Q\_3} = \frac{Q\_pbt}{Q\_ct}$ so the bypass tube discharge over the common tube discharge

With the chosen area ratios, the xi loss coefficient has the following curve shape:

define a r_qd axis:

$r\_qdx := 0, 0.01 .. 1 = \begin{bmatrix} 0 \\ \vdots \end{bmatrix}$ making it a matrix: $r\_qdxM := \left[r\_qdx \;\; r\_qdx \;\; r\_qdx \;\; r\_qdx \;\; r\_qdx\right]^T$



Energy gain from conflux zone (section 6 of turbine tube) (continued)

The discharge ratio needs to be at least larger than 0.55 to give an increase in head for the turbine tube. At the suggested 80% discharge ratio the ξ-coefficient is about -0.82, so the gain would be 82% of the velocity head in the common tube. However, because it isn't certain the discharge ratio is constant the conflux zone energy head gain will remain a function of the discharge ratio.

Making the contribution to the QDC:

$$C\_tt\_confl(r\_qd) := \frac{\xi\_cfx2(r\_qd)}{\overrightarrow{2\ g \cdot A\_ct \cdot A\_ct}}$$

$$\xi\_cfx2(80\%)^{\mathrm{T}} = \begin{bmatrix} -20.37 & -20.37 & -20.37 & -20.37 & -20.37 \end{bmatrix}$$

$$C\_tt\_confl(80\%)^{\mathrm{T}} = \begin{bmatrix} -5.53 \cdot 10^{-3} & -2.6 \cdot 10^{-2} & -1.05 \cdot 10^{-2} & -8.38 \cdot 10^{-3} & -6.43 \cdot 10^{-3} \end{bmatrix} \frac{s^2}{m^5}$$

Defining the constant part of the QDC:

$$C\_tt\_c := C\_tt$$

Collecting the contributions (as can be seen below, from this point on the QDC becomes a function of the discharge ratio):

$$C\_tt(r\_qd) := \mathrm{augment}(C\_tt, C\_tt\_confl(r\_qd))$$

Summation of Quadratic discharge coefficients turbine tube:

After this point the flow enters the conflux zone, rest of the losses are determined by the common tube. Summing the QDC contributions the total value for the turbine tube is found:

With discharge ratio of 80%:

$$C\_tt(80\%) = \begin{array}{cccccccc} \text{TR} & \text{infl} & \text{friction} & \text{bulb} & \text{bulb.fr} & \text{Turb} & \text{friction} & \text{head gain} \\ \hline 7.64 \cdot 10^{-7} & 9.05 \cdot 10^{-9} & 7.24 \cdot 10^{-8} & 6.54 \cdot 10^{-9} & 5.96 \cdot 10^{-7} & 0 & 1.32 \cdot 10^{-5} & -5.53 \cdot 10^{-3} \\ 3.57 \cdot 10^{-6} & 4.11 \cdot 10^{-8} & 3.69 \cdot 10^{-7} & 3.08 \cdot 10^{-8} & 3.06 \cdot 10^{-6} & 0 & 6.68 \cdot 10^{-5} & -0.03 \\ 1.45 \cdot 10^{-6} & 1.7 \cdot 10^{-8} & 1.46 \cdot 10^{-7} & 1.24 \cdot 10^{-8} & 1.21 \cdot 10^{-6} & 0 & 2.61 \cdot 10^{-5} & -0.01 \\ 1.16 \cdot 10^{-6} & 1.36 \cdot 10^{-8} & 1.13 \cdot 10^{-7} & 9.91 \cdot 10^{-9} & 9.34 \cdot 10^{-7} & 0 & 2.05 \cdot 10^{-5} & -8.38 \cdot 10^{-3} \\ 8.98 \cdot 10^{-7} & 1.13 \cdot 10^{-8} & 8.68 \cdot 10^{-8} & 7.6 \cdot 10^{-9} & 7.14 \cdot 10^{-7} & 0 & 1.56 \cdot 10^{-5} & -6.43 \cdot 10^{-3} \end{array} \frac{s^2}{m^5}$$

With discharge ratio that has negligible head gain ($rqd0 := 6.93\%$):

$$C\_tt(rqd0) = \begin{array}{cccccccc} \text{TR} & \text{infl} & \text{friction} & \text{bulb} & \text{bulb.fr} & \text{Turb} & \text{friction} & \text{head gain} \\ \hline 7.64 \cdot 10^{-7} & 9.05 \cdot 10^{-9} & 7.24 \cdot 10^{-8} & 6.54 \cdot 10^{-9} & 5.96 \cdot 10^{-7} & 0 & 1.32 \cdot 10^{-5} & 3.3 \cdot 10^{-8} \\ 3.57 \cdot 10^{-6} & 4.11 \cdot 10^{-8} & 3.69 \cdot 10^{-7} & 3.08 \cdot 10^{-8} & 3.06 \cdot 10^{-6} & 0 & 6.68 \cdot 10^{-5} & 1.55 \cdot 10^{-7} \\ 1.45 \cdot 10^{-6} & 1.7 \cdot 10^{-8} & 1.46 \cdot 10^{-7} & 1.24 \cdot 10^{-8} & 1.21 \cdot 10^{-6} & 0 & 2.61 \cdot 10^{-5} & 6.27 \cdot 10^{-8} \\ 1.16 \cdot 10^{-6} & 1.36 \cdot 10^{-8} & 1.13 \cdot 10^{-7} & 9.91 \cdot 10^{-9} & 9.34 \cdot 10^{-7} & 0 & 2.05 \cdot 10^{-5} & 5 \cdot 10^{-8} \\ 8.98 \cdot 10^{-7} & 1.13 \cdot 10^{-8} & 8.68 \cdot 10^{-8} & 7.6 \cdot 10^{-9} & 7.14 \cdot 10^{-7} & 0 & 1.56 \cdot 10^{-5} & 3.84 \cdot 10^{-8} \end{array} \frac{s^2}{m^5}$$

Define summation function:

```
C_sum(C_c) := for $i \in 0..\mathrm{rows}(C\_c)-1$
                  $C_i \leftarrow \sum_{j=0}^{\mathrm{cols}(C\_c)-1} C\_c_{i,j}$
              return $C$
```

$$C\_TT(r\_qd) := C\_sum(C\_tt(r\_qd))$$

With discharge ratio of 80%:

$$C\_TT(80\%)^{\mathrm{T}} = \begin{bmatrix} -5.52 \cdot 10^{-3} & -0.03 & -0.01 & -8.36 \cdot 10^{-3} & -6.41 \cdot 10^{-3} \end{bmatrix} \frac{s^2}{m^5}$$

With discharge ratio that has negligible head gain ($rqd0 = 6.93\%$):

$$C\_TT(rqd0)^{\mathrm{T}} = \begin{bmatrix} 1.47 \cdot 10^{-5} & 7.4 \cdot 10^{-5} & 2.9 \cdot 10^{-5} & 2.27 \cdot 10^{-5} & 1.73 \cdot 10^{-5} \end{bmatrix} \frac{s^2}{m^5}$$

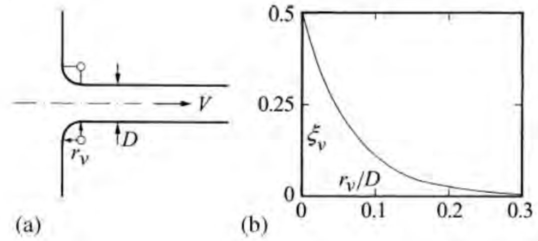
Calculation of QDC for bypass tube

Inflow (section 1 of bypass tube):

$\xi = \frac{1}{2} \exp\left(-15 \frac{r_{in}}{D}\right)$ Idel'cik (1979)

Shape of inflow:
"Rounded with large radius"

Xi-loss coefficient for inflow general formula:



$\xi\_in\_bpt := \xi\_in(r\_in\_bpt, D\_infl\_bpt)$
$\xi\_in\_bpt^{T} = [5.82 \cdot 10^{-3} \;\; 5.88 \cdot 10^{-3} \;\; 5.27 \cdot 10^{-3} \;\; 5.77 \cdot 10^{-3} \;\; 5.7 \cdot 10^{-3}]$

Related cross-sectional area:
(Note: The bulb of the turbine tube ends at the inflow of the bypass, so the tube installed within the bypass tube has the diameter of the connecting tube from the turbine path.

$A\_infl\_bpt := \frac{\pi}{4} \cdot \left(\overrightarrow{D\_infl\_bpt \cdot D\_infl\_bpt} - \overrightarrow{D\_t\_dv \cdot D\_t\_dv}\right)$

$A\_infl\_bpt^{T} = [72.77 \;\; 33.55 \;\; 52.81 \;\; 59.11 \;\; 67.48] \; m^2$

Contribution to quadratic discharge coefficient:

$C\_bpt\_1 := \overrightarrow{\frac{\xi\_in\_bpt}{2 \; g \cdot A\_infl\_bpt \cdot A\_infl\_bpt}}$

$C\_bpt\_1^{T} = [5.61 \cdot 10^{-8} \;\; 2.67 \cdot 10^{-7} \;\; 9.64 \cdot 10^{-8} \;\; 8.42 \cdot 10^{-8} \;\; 6.38 \cdot 10^{-8}] \; \frac{s^2}{m^5}$

Collecting the contributions:
$C\_bpt := C\_bpt\_1$

Wall friciton connection tube (section 2 of bypass tube)

In the bypass tube friction is experienced by both the wall from the bypass tube and the turbine tube (tube within a tube). That makes the gap the following size:

$h\_gap\_bpt\_infl := 0.5 \; (D\_infl\_bpt - D\_t\_dv)$

Discharge in bypass tube is determined to be about 20% of total discharge and $Q\_t\_max\_g$ is assumed to be 80% of this total. Therefore:

$Q\_bp\_max := Q\_t\_max\_g \cdot \frac{0.20}{0.80}$ and $Q\_bp\_20pc := Q\_t\_20pc \cdot \frac{0.20}{0.80}$

Bypass max discharge: $Q\_bp\_max^{T} = [16.85 \;\; 6.81 \;\; 8.78 \;\; 11.83 \;\; 12.35] \; \frac{m^3}{s}$

Turbine max discharge: $Q\_t\_max\_g^{T} = [67.38 \;\; 27.24 \;\; 35.12 \;\; 47.31 \;\; 49.42] \; \frac{m^3}{s}$

Bypass min discharge: $Q\_bp\_20pc^{T} = [3.37 \;\; 1.36 \;\; 1.76 \;\; 2.37 \;\; 2.47] \; \frac{m^3}{s}$

Turbine min discharge: $Q\_t\_20pc^{T} = [13.48 \;\; 5.45 \;\; 7.02 \;\; 9.46 \;\; 9.88] \; \frac{m^3}{s}$

This makes the Reynolds number is:
$Re\_2\_bpt\_umax := Re(u(Q\_bp\_max, A\_infl\_bpt), h\_gap\_bpt\_infl)$
$Re\_2\_bpt\_umax^{T} = [7.37 \cdot 10^{5} \;\; 4.39 \cdot 10^{5} \;\; 4.51 \cdot 10^{5} \;\; 5.74 \cdot 10^{5} \;\; 5.61 \cdot 10^{5}]$
$Re\_2\_bpt\_umin := Re(u(Q\_bp\_20pc, A\_infl\_bpt), h\_gap\_bpt\_infl)$
$Re\_2\_bpt\_umin^{T} = [1.47 \cdot 10^{5} \;\; 8.78 \cdot 10^{4} \;\; 9.02 \cdot 10^{4} \;\; 1.15 \cdot 10^{5} \;\; 1.12 \cdot 10^{5}]$

... Continue on next page ...

Wall friciton inflow (section 2 of bypass tube) (continued)

Friction factor:

$$\lambda\_fr\_bpt\_2\_umax := \lambda\_fr\left(h\_gap\_bpt\_infl, k\_s, Re\_2\_bpt\_umax\right)$$

$$\lambda\_fr\_bpt\_2\_umax^{T} = \begin{bmatrix} 1.33 \cdot 10^{-2} & 1.45 \cdot 10^{-2} & 1.43 \cdot 10^{-2} & 1.38 \cdot 10^{-2} & 1.38 \cdot 10^{-2} \end{bmatrix}$$

$$\lambda\_fr\_bpt\_2\_umin := \lambda\_fr\left(h\_gap\_bpt\_infl, k\_s, Re\_2\_bpt\_umin\right)$$

$$\lambda\_fr\_bpt\_2\_umin^{T} = \begin{bmatrix} 1.7 \cdot 10^{-2} & 1.9 \cdot 10^{-2} & 1.88 \cdot 10^{-2} & 1.79 \cdot 10^{-2} & 1.79 \cdot 10^{-2} \end{bmatrix}$$

Again using average:

$$\lambda\_fr\_bpt\_2 := 0.5 \cdot \left(\lambda\_fr\_bpt\_2\_umax + \lambda\_fr\_bpt\_2\_umin\right)$$

$$\lambda\_fr\_bpt\_2^{T} = \begin{bmatrix} 1.52 \cdot 10^{-2} & 1.68 \cdot 10^{-2} & 1.65 \cdot 10^{-2} & 1.58 \cdot 10^{-2} & 1.59 \cdot 10^{-2} \end{bmatrix}$$

Loss coefficients for both inner and outer wall:

$$\xi\_bpt\_2\_fr\_out := \overrightarrow{\left(\frac{\lambda\_fr\_bpt\_2}{D\_infl\_bpt}\right) \cdot \left(L\_in\_bpt - L\_cont\_bpt\right)}$$

$$\xi\_bpt\_2\_fr\_out^{T} = \begin{bmatrix} 6.07 \cdot 10^{-3} & 6.7 \cdot 10^{-3} & 6.61 \cdot 10^{-3} & 6.34 \cdot 10^{-3} & 6.35 \cdot 10^{-3} \end{bmatrix}$$

$$\xi\_bpt\_2\_fr\_in := \overrightarrow{\left(\frac{\lambda\_fr\_bpt\_2}{D\_t\_dv}\right) \cdot \left(L\_in\_bpt - L\_cont\_bpt\right)}$$

$$\xi\_bpt\_2\_fr\_in^{T} = \begin{bmatrix} 1.56 \cdot 10^{-2} & 1.73 \cdot 10^{-2} & 1.7 \cdot 10^{-2} & 1.63 \cdot 10^{-2} & 1.64 \cdot 10^{-2} \end{bmatrix}$$

Making the contribution to the QDC:

$$C\_bt\_2\_fr := \frac{\xi\_bpt\_2\_fr\_out + \xi\_bpt\_2\_fr\_in}{2\ g \cdot \overrightarrow{A\_infl\_bpt \cdot A\_infl\_bpt}}$$

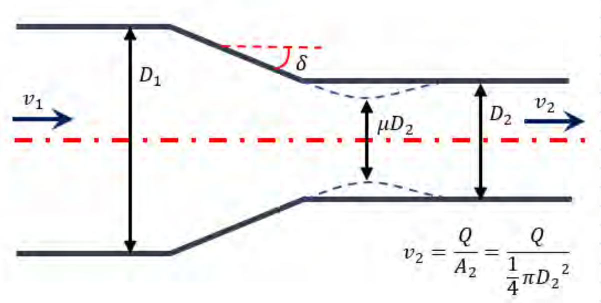
$$C\_bt\_2\_fr^{T} = \begin{bmatrix} 2.09 \cdot 10^{-7} & 1.09 \cdot 10^{-6} & 4.33 \cdot 10^{-7} & 3.31 \cdot 10^{-7} & 2.54 \cdot 10^{-7} \end{bmatrix} \frac{s^{2}}{m^{5}}$$

Collecting the contributions:

$$C\_bpt := \text{augment}\left(C\_bpt, C\_bt\_2\_fr\right)$$

Contraction towards conflux (section 3 of bypass tube)

The contraction losses are determined by the area ratio and the angle of contraction.



Area ratio:

$$r\_A\_bpt := \overrightarrow{\frac{A\_bpt}{A\_infl\_bpt}}$$

Angle of contraction:

$$\delta\_bpt\_cont := \text{atan}\left(\frac{0.5\ \left(D\_infl\_bpt - D\_bpt\right)}{L\_cont\_bpt}\right)$$

Loss coefficient:

$$\xi\_contraction = \frac{1}{2} \cdot (1 - \varphi) \cdot \left(\frac{\delta}{90\ °}\right)^{1.83 \cdot (1-\varphi)^{0.4}}$$

$$\xi\_bpt\_3\_cont := \overrightarrow{\frac{1}{2} \cdot \left(1 - r\_A\_bpt\right) \cdot \left(\frac{\delta\_bpt\_cont}{90\ °}\right)^{1.83 \cdot (1 - r\_A\_bpt)^{0.4}}}$$

Making the contribution to the QDC:

$$C\_bt\_3\_contr := \frac{\xi\_bpt\_3\_cont}{2\ g \cdot \overrightarrow{A\_bpt \cdot A\_bpt}}$$

$$C\_bt\_3\_contr^{T} = \begin{bmatrix} 1.42 \cdot 10^{-3} & 6.7 \cdot 10^{-3} & 2.7 \cdot 10^{-3} & 2.16 \cdot 10^{-3} & 1.66 \cdot 10^{-3} \end{bmatrix} \frac{s^{2}}{m^{5}}$$

Collecting the contributions:

$$C\_bpt := \text{augment}\left(C\_bpt, C\_bt\_3\_contr\right)$$

Friction in the contracting bypass tube (section 3 of bypass tube)
Friction in the contracting part of the bypass tube is more challanging due to the fact that the diameter and thus the gap between the inner and out tube is changing over the length. This causes all kinds of non-linear friction distributions with the current method and that is ignoring the fact that physically other phenomena might be happening (although this is partially solved by calculating the expansion losses in the previous page).

However, to simplify things the friction in the draft tube is interpolated between the factor at the start of the contraction and and the one at the end of the contraction near the outflow into the conflux zone.

Reynolds number at the start of the contraction:

$Re\_bpt\_3\_1\_umax := Re\left(u\left(Q\_bp\_max, A\_infl\_bpt\right), h\_gap\_bpt\_infl\right)$

$Re\_bpt\_3\_1\_umax^{T} = \left[7.37 \cdot 10^{5} \quad 4.39 \cdot 10^{5} \quad 4.51 \cdot 10^{5} \quad 5.74 \cdot 10^{5} \quad 5.61 \cdot 10^{5}\right]$

$Re\_bpt\_3\_1\_umin := Re\left(u\left(Q\_bp\_20pc, A\_infl\_bpt\right), h\_gap\_bpt\_infl\right)$

$Re\_bpt\_3\_1\_umin^{T} = \left[1.47 \cdot 10^{5} \quad 8.78 \cdot 10^{4} \quad 9.02 \cdot 10^{4} \quad 1.15 \cdot 10^{5} \quad 1.12 \cdot 10^{5}\right]$

The gap at the end of the contraction is:

$h\_gap\_bpt\_out := 0.5 \left(D\_bpt - D\_t\_dv\right)$

Reynolds number at the end of the contraction :

$Re\_bpt\_3\_2\_umax := Re\left(u\left(Q\_t\_max\_g, A\_bpt\right), h\_gap\_bpt\_out\right)$

$Re\_bpt\_3\_2\_umax^{T} = \left[5.19 \cdot 10^{6} \quad 3.09 \cdot 10^{6} \quad 3.18 \cdot 10^{6} \quad 4.05 \cdot 10^{6} \quad 3.96 \cdot 10^{6}\right]$

$Re\_bpt\_3\_2\_umin := Re\left(u\left(Q\_t\_20pc, A\_bpt\right), h\_gap\_bpt\_out\right)$

$Re\_bpt\_3\_2\_umin^{T} = \left[1.04 \cdot 10^{6} \quad 6.18 \cdot 10^{5} \quad 6.36 \cdot 10^{5} \quad 8.09 \cdot 10^{5} \quad 7.91 \cdot 10^{5}\right]$

Friction factor at the start of the contraction:

$\lambda\_fr\_3\_1\_umax := \lambda\_fr\left(h\_gap\_bpt\_infl, k\_s, Re\_bpt\_3\_1\_umax\right)$

$\lambda\_fr\_3\_1\_umax^{T} = \left[1.33 \cdot 10^{-2} \quad 1.45 \cdot 10^{-2} \quad 1.43 \cdot 10^{-2} \quad 1.38 \cdot 10^{-2} \quad 1.38 \cdot 10^{-2}\right]$

$\lambda\_fr\_3\_1\_umin := \lambda\_fr\left(h\_gap\_bpt\_infl, k\_s, Re\_bpt\_3\_1\_umin\right)$

$\lambda\_fr\_3\_1\_umin^{T} = \left[1.7 \cdot 10^{-2} \quad 1.9 \cdot 10^{-2} \quad 1.88 \cdot 10^{-2} \quad 1.79 \cdot 10^{-2} \quad 1.79 \cdot 10^{-2}\right]$

Again using average:

$\lambda\_fr\_3\_1 := 0.5 \cdot \left(\lambda\_fr\_3\_1\_umax + \lambda\_fr\_3\_1\_umin\right)$

$\lambda\_fr\_3\_1^{T} = \left[1.52 \cdot 10^{-2} \quad 1.68 \cdot 10^{-2} \quad 1.65 \cdot 10^{-2} \quad 1.58 \cdot 10^{-2} \quad 1.59 \cdot 10^{-2}\right]$

Friction factor at the end of the contraction :

$\lambda\_fr\_3\_2\_umax := \lambda\_fr\left(h\_gap\_bpt\_out, k\_s, Re\_bpt\_3\_2\_umax\right)$

$\lambda\_fr\_3\_2\_umax^{T} = \left[2.65 \cdot 10^{-2} \quad 2.97 \cdot 10^{-2} \quad 2.78 \cdot 10^{-2} \quad 2.73 \cdot 10^{-2} \quad 2.68 \cdot 10^{-2}\right]$

$\lambda\_fr\_3\_2\_umin := \lambda\_fr\left(h\_gap\_bpt\_out, k\_s, Re\_bpt\_3\_2\_umin\right)$

$\lambda\_fr\_3\_2\_umin^{T} = \left[2.66 \cdot 10^{-2} \quad 2.98 \cdot 10^{-2} \quad 2.79 \cdot 10^{-2} \quad 2.74 \cdot 10^{-2} \quad 2.69 \cdot 10^{-2}\right]$

Again using average:

$\lambda\_fr\_3\_2 := 0.5 \cdot \left(\lambda\_fr\_3\_2\_umax + \lambda\_fr\_3\_2\_umin\right)$

$\lambda\_fr\_3\_2^{T} = \left[2.66 \cdot 10^{-2} \quad 2.97 \cdot 10^{-2} \quad 2.78 \cdot 10^{-2} \quad 2.74 \cdot 10^{-2} \quad 2.69 \cdot 10^{-2}\right]$

The loss-coefficients, interpolating between the start and end of the draft tube are:

$$\xi\_bpt\_3\_fr := \left\| \begin{array}{l} \text{for } i \in 0 \,..\, \text{rows}\left(n\_t\right) - 1 \\ \quad \left\| \; Y_{i} \leftarrow \displaystyle\int_{0\,m}^{L\_cont\_bpt_{i}} \left(\frac{\lambda\_fr\_3\_1_{i}}{D\_infl\_bpt_{i}} \cdot \frac{\left(L\_cont\_bpt_{i} - x\right)}{L\_cont\_bpt_{i}} + \frac{\lambda\_fr\_3\_2_{i}}{D\_bpt_{i}} \cdot \frac{x}{L\_cont\_bpt_{i}} \right) \mathrm{d}x \right. \\ \text{return } Y \end{array} \right.$$

$\xi\_bpt\_3\_fr^{T} = \left[4.57 \cdot 10^{-2} \quad 5.1 \cdot 10^{-2} \quad 4.82 \cdot 10^{-2} \quad 4.72 \cdot 10^{-2} \quad 4.65 \cdot 10^{-2}\right]$

... Continued on nex page ...

Friction in the draft tube (section 3) (continued)
To check if the answers are within the expected range first calculating the extreme case as if the draft tube over the whole length has the turbine diameters:

$$\xi\_bpt\_3\_fr\_check1 := \overrightarrow{\left(\frac{\lambda\_fr\_3\_1}{D\_infl\_bpt}\right) \cdot L\_cont\_bpt}$$

$$\xi\_bpt\_3\_fr\_check1^{\mathrm{T}} = \left[1.7 \cdot 10^{-2} \;\; 1.88 \cdot 10^{-2} \;\; 1.85 \cdot 10^{-2} \;\; 1.77 \cdot 10^{-2} \;\; 1.78 \cdot 10^{-2}\right]$$

Then calculating as if the draft tube has the outflow diameter over the whole length:

$$\xi\_bpt\_3\_fr\_check2 := \overrightarrow{\left(\frac{\lambda\_fr\_3\_2}{D\_bpt}\right) \cdot L\_cont\_bpt}$$

$$\xi\_bpt\_3\_fr\_check2^{\mathrm{T}} = \left[7.44 \cdot 10^{-2} \;\; 8.33 \cdot 10^{-2} \;\; 7.8 \cdot 10^{-2} \;\; 7.67 \cdot 10^{-2} \;\; 7.53 \cdot 10^{-2}\right]$$

To compare with:

$$\xi\_bpt\_3\_fr^{\mathrm{T}} = \left[4.57 \cdot 10^{-2} \;\; 5.1 \cdot 10^{-2} \;\; 4.82 \cdot 10^{-2} \;\; 4.72 \cdot 10^{-2} \;\; 4.65 \cdot 10^{-2}\right]$$

The found answer lies within these two extremes and thus is assumed to be reasonable approximation of the actual loss-coefficient.

The Quadratic loss coefficient is also dependent on the discharge area. Using the same integration method as for the Xi-factor the following value is found:

```
$C\_bpt\_3 :=$
    for $i \in 0 .. \mathrm{rows}(n\_t) - 1$
        $A1 \leftarrow A\_infl\_bpt_i$
        $A2 \leftarrow A\_bpt_i$
        $L \leftarrow L\_cont\_bpt_i$
        $Y_i \leftarrow \int_{0\,m}^{L} \left(\frac{1}{(A1)^2}\left(\frac{\lambda\_fr\_3\_1_i}{D\_infl\_bpt_i} \cdot \frac{(L-x)}{L}\right) + \frac{1}{(A2)^2}\left(\frac{\lambda\_fr\_3\_2_i}{D\_bpt_i} \cdot \frac{x}{L}\right)\right) \mathrm{d}x$
    return $\frac{1}{2\,g} \cdot Y$
```

$$C\_bpt\_3^{\mathrm{T}} = \left[2.8 \cdot 10^{-3} \;\; 1.48 \cdot 10^{-2} \;\; 5.58 \cdot 10^{-3} \;\; 4.38 \cdot 10^{-3} \;\; 3.3 \cdot 10^{-3}\right] \frac{s^2}{m^5}$$

To check again the same method of finding extremes:

$$C\_bpt\_3\_check1 := \frac{\xi\_bpt\_3\_fr\_check1}{2\,g \cdot \overrightarrow{A\_infl\_bpt \cdot A\_infl\_bpt}}$$

$$C\_bpt\_3\_check1^{\mathrm{T}} = \left[1.64 \cdot 10^{-7} \;\; 8.5 \cdot 10^{-7} \;\; 3.38 \cdot 10^{-7} \;\; 2.59 \cdot 10^{-7} \;\; 1.99 \cdot 10^{-7}\right] \frac{s^2}{m^5}$$

$$C\_bpt\_3\_check2 := \frac{\xi\_bpt\_3\_fr\_check2}{2\,g \cdot \overrightarrow{A\_bpt \cdot A\_bpt}}$$

$$C\_bpt\_3\_check2^{\mathrm{T}} = \left[5.61 \cdot 10^{-3} \;\; 2.95 \cdot 10^{-2} \;\; 1.12 \cdot 10^{-2} \;\; 8.76 \cdot 10^{-3} \;\; 6.6 \cdot 10^{-3}\right] \frac{s^2}{m^5}$$

The integration lies withing the expected extreme values and thus accepted as approximation.

Collecting the contributions:

$$C\_bpt := \mathrm{augment}(C\_bpt, C\_bpt\_3)$$

Energy loss from conflux zone (section 4 of bypass tube)

Earlier defined at the turbine tube losses head loss for flow going from bypass to the conflux zone:

$$\xi\_cfx1\left(r\_qd\right) = 1 - 2 \cdot \frac{r\_qd^{2}}{r\_A1} \cdot \cos\left(\delta\_1\right) - 2 \cdot \frac{\left(1 - r\_qd\right)^{2}}{r\_A1} \cdot \cos\left(\delta\_2\right) + \left(\frac{r\_qd}{r\_A1}\right)^{2}$$

Reference flow velocity is the flow in the common tube so that makes the QDC contribution:

$$C\_bt\_4\_confl\left(r\_qd\right) := \frac{\xi\_cfx1\left(r\_qd\right)}{2\ g \cdot \overrightarrow{A\_ct \cdot A\_ct}}$$

For a discharge ratio of 80%

$$C\_bt\_4\_confl\left(80\%\right)^{T} = \left[4.27 \cdot 10^{-2}\ \ 2.01 \cdot 10^{-1}\ \ 8.11 \cdot 10^{-2}\ \ 6.48 \cdot 10^{-2}\ \ 4.97 \cdot 10^{-2}\right] \frac{s^{2}}{m^{5}}$$

Define constant part of QDC:

$$C\_bpt\_c := C\_bpt$$

Collecting the contributions:

$$C\_bpt\left(r\_qd\right) := \text{augment}\left(C\_bpt, C\_bt\_4\_confl\left(r\_qd\right)\right)$$

Summation of Quadratic discharge coefficients bypass tube:

After this point the flow enters the conflux zone, rest of the losses are determined by the common tube. Summing the QDC contributions the total value for the bypass tube is found:

With discharge ratio of 80%:

$$C\_bpt\left(80\%\right) = \begin{array}{ccccc} \text{infl} & \text{friction} & \text{contract} & \text{friction} & \text{conflux} \\ \end{array}$$
$$\begin{bmatrix} 5.61 \cdot 10^{-8} & 2.09 \cdot 10^{-7} & 1.42 \cdot 10^{-3} & 2.8 \cdot 10^{-3} & 0.04 \\ 2.67 \cdot 10^{-7} & 1.09 \cdot 10^{-6} & 6.7 \cdot 10^{-3} & 0.01 & 0.2 \\ 9.64 \cdot 10^{-8} & 4.33 \cdot 10^{-7} & 2.7 \cdot 10^{-3} & 5.58 \cdot 10^{-3} & 0.08 \\ 8.42 \cdot 10^{-8} & 3.31 \cdot 10^{-7} & 2.16 \cdot 10^{-3} & 4.38 \cdot 10^{-3} & 0.06 \\ 6.38 \cdot 10^{-8} & 2.54 \cdot 10^{-7} & 1.66 \cdot 10^{-3} & 3.3 \cdot 10^{-3} & 0.05 \end{bmatrix} \frac{s^{2}}{m^{5}}$$

With discharge ratio that has negligible head gain ($rqd0 = 0.07$):

$$C\_bpt\left(rqd0\right) = \begin{array}{ccccc} \text{infl} & \text{friction} & \text{contract} & \text{friction} & \text{conflux} \\ \end{array}$$
$$\begin{bmatrix} 5.61 \cdot 10^{-8} & 2.09 \cdot 10^{-7} & 1.42 \cdot 10^{-3} & 2.8 \cdot 10^{-3} & 9.61 \cdot 10^{-5} \\ 2.67 \cdot 10^{-7} & 1.09 \cdot 10^{-6} & 6.7 \cdot 10^{-3} & 0.01 & 4.52 \cdot 10^{-4} \\ 9.64 \cdot 10^{-8} & 4.33 \cdot 10^{-7} & 2.7 \cdot 10^{-3} & 5.58 \cdot 10^{-3} & 1.82 \cdot 10^{-4} \\ 8.42 \cdot 10^{-8} & 3.31 \cdot 10^{-7} & 2.16 \cdot 10^{-3} & 4.38 \cdot 10^{-3} & 1.46 \cdot 10^{-4} \\ 6.38 \cdot 10^{-8} & 2.54 \cdot 10^{-7} & 1.66 \cdot 10^{-3} & 3.3 \cdot 10^{-3} & 1.12 \cdot 10^{-4} \end{bmatrix} \frac{s^{2}}{m^{5}}$$

Summation:

$$C\_BPT\left(r\_qd\right) := C\_sum\left(C\_bpt\left(r\_qd\right)\right)$$

With discharge ratio of 80%:

$$C\_BPT\left(80\%\right)^{T} = \left[0.05\ \ 0.22\ \ 0.09\ \ 0.07\ \ 0.05\right] \frac{s^{2}}{m^{5}}$$

With discharge ratio that has negligible head gain ($rqd0 = 6.93\%$):

$$C\_BPT\left(rqd0\right)^{T} = \left[4.33 \cdot 10^{-3}\ \ 0.02\ \ 8.47 \cdot 10^{-3}\ \ 6.69 \cdot 10^{-3}\ \ 5.07 \cdot 10^{-3}\right] \frac{s^{2}}{m^{5}}$$

Calculation of QDC for common tube

Expansion losses into conlfux zone (section 1 of common tube closed bypass):
The expansion angle is 90° when the bypass is closed (assuming the valve is perpendicular to the tube axes.

That means that the phi factor for the expansion is also the same for all variants:

$\Phi 3090(\delta) := \frac{5}{4} + \frac{\delta}{360°}$ ---> $\Phi 3090(90°) = 1.5$

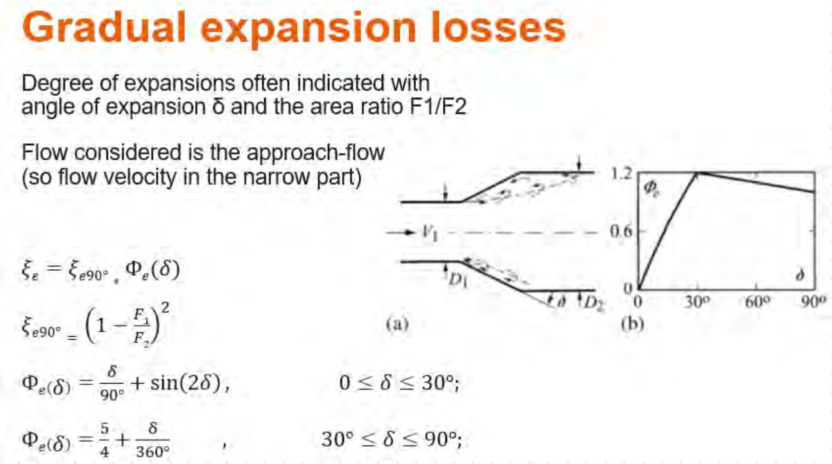
$\xi\_ct\_infl := \overrightarrow{\left(1 - \frac{A\_tt}{A\_ct}\right)} \cdot \Phi 3090(90°)$ $\xi\_ct\_infl^{T} = [0.09 \;\; 0.09 \;\; 0.09 \;\; 0.09 \;\; 0.09]$

Making the contribution to the QDC:

$$C\_ct\_1 := \frac{\xi\_ct\_infl}{2\, g \cdot \overrightarrow{A\_ct \cdot A\_ct}}$$

$$C\_ct\_1^{T} = [2.44 \cdot 10^{-5} \;\; 1.15 \cdot 10^{-4} \;\; 4.64 \cdot 10^{-5} \;\; 3.7 \cdot 10^{-5} \;\; 2.84 \cdot 10^{-5}] \, \frac{s^2}{m^5}$$

Collecting the contributions:
$C\_ct\_bp0 := C\_ct\_1$



Gradual expansion losses

Degree of expansions often indicated with angle of expansion δ and the area ratio F1/F2

Flow considered is the approach-flow (so flow velocity in the narrow part)

$\xi_e = \xi_{e90°} * \Phi_e(\delta)$

$\xi_{e90°} = \left(1 - \frac{F_1}{F_2}\right)^2$

$\Phi_{e(\delta)} = \frac{\delta}{90°} + \sin(2\delta),$ $0 \le \delta \le 30°;$

$\Phi_{e(\delta)} = \frac{5}{4} + \frac{\delta}{360°}$, $30° \le \delta \le 90°;$

[1] Source: W.H. Hager, Wastewater Hydraulics - Theory and Practice, 2nd ed., DOI 10.1007/978-3-642-11383-3_2, Springer-Verlag, Berlin Heidelberg 2010

Wall friction in the conflux zone (section 2 of common tube):
Discharge in common tube is equal to turbine and bypass discharge combined and $Q\_t\_max\_g$ is 20% of this total discharge. Therefore:

$Q\_ct\_max := \frac{Q\_t\_max\_g}{0.2}$ and $Q\_ct\_20pc := \frac{Q\_t\_20pc}{0.2}$

Common tube max discharge: $Q\_ct\_max^{T} = [336.92 \;\; 136.2 \;\; 175.6 \;\; 236.56 \;\; 247.08] \, \frac{m^3}{s}$

Common tube min discharge: $Q\_ct\_20pc^{T} = [67.38 \;\; 27.24 \;\; 35.12 \;\; 47.31 \;\; 49.42] \, \frac{m^3}{s}$

Discharge area at this point is:
$A\_ct^{T} = [13.7 \;\; 6.32 \;\; 9.94 \;\; 11.13 \;\; 12.71] \, m^2$

This makes the Reynolds number is:

$Re\_2\_ct\_umax := Re(u(Q\_ct\_max, A\_ct), D\_conflux)$

$Re\_2\_ct\_umax^{T} = [1.02 \cdot 10^{8} \;\; 6.09 \cdot 10^{7} \;\; 6.26 \cdot 10^{7} \;\; 7.97 \cdot 10^{7} \;\; 7.79 \cdot 10^{7}]$

$Re\_2\_ct\_umin := Re(u(Q\_ct\_20pc, A\_ct), D\_conflux)$

$Re\_2\_ct\_umin^{T} = [2.05 \cdot 10^{7} \;\; 1.22 \cdot 10^{7} \;\; 1.25 \cdot 10^{7} \;\; 1.59 \cdot 10^{7} \;\; 1.56 \cdot 10^{7}]$

... continue on next page

Friction factor:
$\lambda\_fr\_ct\_2\_umax := \lambda\_fr\left(D\_conflux, k\_s, Re\_2\_ct\_umax\right)$
$\lambda\_fr\_ct\_2\_umax^{\mathrm{T}} = \left[1.05\cdot10^{-2}\ \ 1.13\cdot10^{-2}\ \ 1.08\cdot10^{-2}\ \ 1.07\cdot10^{-2}\ \ 1.06\cdot10^{-2}\right]$
$\lambda\_fr\_ct\_2\_umin := \lambda\_fr\left(D\_conflux, k\_s, Re\_2\_ct\_umin\right)$
$\lambda\_fr\_ct\_2\_umin^{\mathrm{T}} = \left[1.06\cdot10^{-2}\ \ 1.14\cdot10^{-2}\ \ 1.1\cdot10^{-2}\ \ 1.09\cdot10^{-2}\ \ 1.07\cdot10^{-2}\right]$
Again using average:
$\lambda\_fr\_ct\_2 := 0.5\cdot\left(\lambda\_fr\_ct\_2\_umax + \lambda\_fr\_ct\_2\_umin\right)$
$\lambda\_fr\_ct\_2^{\mathrm{T}} = \left[1.06\cdot10^{-2}\ \ 1.13\cdot10^{-2}\ \ 1.09\cdot10^{-2}\ \ 1.08\cdot10^{-2}\ \ 1.07\cdot10^{-2}\right]$

Loss coefficient:

$$\xi\_ct\_2\_fr := \overrightarrow{\left(\frac{\lambda\_fr\_ct\_2}{D\_conflux}\right)\cdot\left(L\_conflux\right)}$$

$\xi\_ct\_2\_fr^{\mathrm{T}} = \left[6.34\cdot10^{-2}\ \ 6.8\cdot10^{-2}\ \ 6.54\cdot10^{-2}\ \ 6.47\cdot10^{-2}\ \ 6.4\cdot10^{-2}\right]$

Making the contribution to the QDC:

$$C\_ct\_2\_fr := \frac{\xi\_ct\_2\_fr}{2\ g\cdot\overrightarrow{A\_ct\cdot A\_ct}}$$

$$C\_ct\_2\_fr^{\mathrm{T}} = \left[1.72\cdot10^{-5}\ \ 8.69\cdot10^{-5}\ \ 3.37\cdot10^{-5}\ \ 2.66\cdot10^{-5}\ \ 2.02\cdot10^{-5}\right]\ \frac{s^2}{m^5}$$

Collecting the contributions, friction happens for both open and closed bypass tube:
$C\_ct\_bp1 := C\_ct\_2\_fr$ (BP open)
$C\_ct\_bp0 := \mathrm{augment}\left(C\_ct\_bp0, C\_ct\_2\_fr\right)$ (BP closed)

Expansion losses draft tube (section 3 of common tube)
The expansion angle is chosen the same for all design variants:
$\beta\_draft = 5\ deg$ (°)

That means that the phi factor for the expansion is also the same for all variants:

$$\Phi(\delta) := \frac{\delta}{90^\circ} + \sin(2\cdot\delta) \quad \text{--->} \quad \Phi(\beta\_draft) = 0.23$$

Draft tube exit cross-sectional area is:

$$A\_exit := \frac{\pi}{4}\cdot D\_outfl^2 \qquad A\_exit^{\mathrm{T}} = \left[57.59\ \ 26.55\ \ 41.79\ \ 46.77\ \ 53.4\right]\ m^2$$

Then the loss coefficients are:

$$\xi\_ct\_3\_exp := \left(1 - \overrightarrow{\frac{A\_ct}{A\_exit}}\right)\cdot\Phi(\beta\_draft)$$

$\xi\_ct\_3\_exp^{\mathrm{T}} = \left[1.75\cdot10^{-1}\ \ 1.75\cdot10^{-1}\ \ 1.75\cdot10^{-1}\ \ 1.75\cdot10^{-1}\ \ 1.75\cdot10^{-1}\right]$

(Note that all the area ratios are the same as well, so thats why all the Xi factors are equal)

Making the contribution to the QDC:

$$C\_ct\_3\_exp := \frac{\xi\_ct\_3\_exp}{2\ g\cdot\overrightarrow{A\_ct\cdot A\_ct}}$$

$$C\_ct\_3\_exp^{\mathrm{T}} = \left[4.74\cdot10^{-5}\ \ 2.23\cdot10^{-4}\ \ 9\cdot10^{-5}\ \ 7.19\cdot10^{-5}\ \ 5.51\cdot10^{-5}\right]\ \frac{s^2}{m^5}$$

Collecting the contributions:
$C\_ct\_bp1 := \mathrm{augment}\left(C\_ct\_bp1, C\_ct\_3\_exp\right)$ (BP open)
$C\_ct\_bp0 := \mathrm{augment}\left(C\_ct\_bp0, C\_ct\_3\_exp\right)$ (BP closed)

Friction in the draft tube (section 3 of common tube)
Friction in the draft tube is more challanging due to the fact that the diameter is changing over the length. This causes all kinds of non-linear friction distributions with the current method and that is ignoring the fact that physically other phenomena might be happening (although this is partially solved by calculating the expension losses in the previous page).

However, to simplify things the friction in the draft tube is interpolated between the factor just after the turbine and the one just before the outflow.

Reynolds number just after the turbine:

$Re\_ct\_3\_1\_umax := Re\left(u\left(Q\_ct\_max, A\_ct\right), D\_conflux\right)$

$Re\_ct\_3\_1\_umax^{\mathrm{T}} = \left[1.02 \cdot 10^{8} \;\; 6.09 \cdot 10^{7} \;\; 6.26 \cdot 10^{7} \;\; 7.97 \cdot 10^{7} \;\; 7.79 \cdot 10^{7}\right]$

$Re\_ct\_3\_1\_umin := Re\left(u\left(Q\_ct\_20pc, A\_ct\right), D\_conflux\right)$

$Re\_ct\_3\_1\_umin^{\mathrm{T}} = \left[2.05 \cdot 10^{7} \;\; 1.22 \cdot 10^{7} \;\; 1.25 \cdot 10^{7} \;\; 1.59 \cdot 10^{7} \;\; 1.56 \cdot 10^{7}\right]$

Reynolds number just before the outflow:

$Re\_ct\_3\_2\_umax := Re\left(u\left(Q\_ct\_max, A\_exit\right), D\_outfl\right)$

$Re\_ct\_3\_2\_umax^{\mathrm{T}} = \left[4.99 \cdot 10^{7} \;\; 2.97 \cdot 10^{7} \;\; 3.05 \cdot 10^{7} \;\; 3.89 \cdot 10^{7} \;\; 3.8 \cdot 10^{7}\right]$

$Re\_ct\_3\_2\_umin := Re\left(u\left(Q\_ct\_20pc, A\_exit\right), D\_outfl\right)$

$Re\_ct\_3\_2\_umin^{\mathrm{T}} = \left[9.98 \cdot 10^{6} \;\; 5.94 \cdot 10^{6} \;\; 6.11 \cdot 10^{6} \;\; 7.78 \cdot 10^{6} \;\; 7.6 \cdot 10^{6}\right]$

Friction factor right after the turbine:

$\lambda\_fr\_ct\_3\_1\_umax := \lambda\_fr\left(D\_conflux, k\_s, Re\_ct\_3\_1\_umax\right)$

$\lambda\_fr\_ct\_3\_1\_umax^{\mathrm{T}} = \left[1.05 \cdot 10^{-2} \;\; 1.13 \cdot 10^{-2} \;\; 1.08 \cdot 10^{-2} \;\; 1.07 \cdot 10^{-2} \;\; 1.06 \cdot 10^{-2}\right]$

$\lambda\_fr\_ct\_3\_1\_umin := \lambda\_fr\left(D\_conflux, k\_s, Re\_ct\_3\_1\_umin\right)$

$\lambda\_fr\_ct\_3\_1\_umin^{\mathrm{T}} = \left[1.06 \cdot 10^{-2} \;\; 1.14 \cdot 10^{-2} \;\; 1.1 \cdot 10^{-2} \;\; 1.09 \cdot 10^{-2} \;\; 1.07 \cdot 10^{-2}\right]$

Again using average:

$\lambda\_fr\_ct\_3\_1 := 0.5 \cdot \left(\lambda\_fr\_ct\_3\_1\_umax + \lambda\_fr\_ct\_3\_1\_umin\right)$

$\lambda\_fr\_ct\_3\_1^{\mathrm{T}} = \left[1.06 \cdot 10^{-2} \;\; 1.13 \cdot 10^{-2} \;\; 1.09 \cdot 10^{-2} \;\; 1.08 \cdot 10^{-2} \;\; 1.07 \cdot 10^{-2}\right]$

Friction factor right before the outflow:

$\lambda\_fr\_ct\_3\_2\_umax := \lambda\_fr\left(D\_outfl, k\_s, Re\_ct\_3\_2\_umax\right)$

$\lambda\_fr\_ct\_3\_2\_umax^{\mathrm{T}} = \left[9.36 \cdot 10^{-3} \;\; 1 \cdot 10^{-2} \;\; 9.67 \cdot 10^{-3} \;\; 9.55 \cdot 10^{-3} \;\; 9.45 \cdot 10^{-3}\right]$

$\lambda\_fr\_ct\_3\_2\_umin := \lambda\_fr\left(D\_outfl, k\_s, Re\_ct\_3\_2\_umin\right)$

$\lambda\_fr\_ct\_3\_2\_umin^{\mathrm{T}} = \left[9.78 \cdot 10^{-3} \;\; 1.05 \cdot 10^{-2} \;\; 1.02 \cdot 10^{-2} \;\; 1 \cdot 10^{-2} \;\; 9.97 \cdot 10^{-3}\right]$

Again using average:

$\lambda\_fr\_ct\_3\_2 := 0.5 \cdot \left(\lambda\_fr\_ct\_3\_2\_umax + \lambda\_fr\_ct\_3\_2\_umin\right)$

$\lambda\_fr\_ct\_3\_2^{\mathrm{T}} = \left[9.57 \cdot 10^{-3} \;\; 1.03 \cdot 10^{-2} \;\; 9.96 \cdot 10^{-3} \;\; 9.79 \cdot 10^{-3} \;\; 9.71 \cdot 10^{-3}\right]$

The loss-coefficients, interpolating between the start and end of the draft tube are:

```
$\xi\_fr\_ct\_3 :=$
    for $i \in 0 .. \mathrm{rows}(n\_t) - 1$
        $L \leftarrow L\_out\_ct_i$
        $Y_i \leftarrow \int_{0\,m}^{L} \left( \frac{\lambda\_fr\_ct\_3\_1_i}{D\_conflux_i} \cdot \frac{(L-x)}{L} + \frac{\lambda\_fr\_ct\_3\_2_i}{D\_outfl_i} \cdot \frac{x}{L} \right) \mathrm{d}x$
    return $Y$
```

$\xi\_fr\_ct\_3^{\mathrm{T}} = \left[4.57 \cdot 10^{-2} \;\; 4.91 \cdot 10^{-2} \;\; 4.73 \cdot 10^{-2} \;\; 4.67 \cdot 10^{-2} \;\; 4.62 \cdot 10^{-2}\right]$

Friction in the draft tube (section 3 of common tube)(continued)

To check if the answers are within the expected range first calculating the extreme case as if the draft tube over the whole length has the turbine diameters:

$$\xi\_fr\_ct\_3\_check1 := \overrightarrow{\left(\frac{\lambda\_fr\_ct\_3\_1}{D\_conflux}\right)\cdot L\_out\_ct}$$

$$\xi\_fr\_ct\_3\_check1^{T} = \left[6.34\cdot10^{-2}\ \ 6.8\cdot10^{-2}\ \ 6.54\cdot10^{-2}\ \ 6.47\cdot10^{-2}\ \ 6.4\cdot10^{-2}\right]$$

Then calculating as if the draft tube has the outflow diameter over the whole length:

$$\xi\_fr\_ct\_3\_check2 := \overrightarrow{\left(\frac{\lambda\_fr\_ct\_3\_2}{D\_outfl}\right)\cdot L\_out\_ct}$$

$$\xi\_fr\_ct\_3\_check2^{T} = \left[2.8\cdot10^{-2}\ \ 3.01\cdot10^{-2}\ \ 2.91\cdot10^{-2}\ \ 2.87\cdot10^{-2}\ \ 2.84\cdot10^{-2}\right]$$

To compare with:

$$\xi\_fr\_ct\_3^{T} = \left[4.57\cdot10^{-2}\ \ 4.91\cdot10^{-2}\ \ 4.73\cdot10^{-2}\ \ 4.67\cdot10^{-2}\ \ 4.62\cdot10^{-2}\right]$$

The found answer lies within these two extremes and thus is assumed to be reasonable approximation of the actual loss-coefficient.

The Quadratic loss coefficient is also dependent on the discharge area. Using the same integration method as for the Xi-factor the following value is found:

```
$C\_ct\_3\_fr :=$
    for $i \in 0..\text{rows}(n\_t)-1$
        $L \leftarrow L\_out\_ct_i$
        $A1 \leftarrow A\_ct_i$
        $A2 \leftarrow A\_exit_i$
        $Y_i \leftarrow \int_{0\,m}^{L}\left(\frac{1}{(A1)^2}\left(\frac{\lambda\_fr\_ct\_3\_1_i}{D\_conflux_i}\cdot\frac{(L-x)}{L}\right)+\frac{1}{(A2)^2}\left(\frac{\lambda\_fr\_ct\_3\_2_i}{D\_outfl_i}\cdot\frac{x}{L}\right)\right)dx$
    return $\frac{1}{2\,g}\cdot Y$
```

$$C\_ct\_3\_fr^{T} = \left[8.82\cdot10^{-6}\ \ 4.45\cdot10^{-5}\ \ 1.73\cdot10^{-5}\ \ 1.36\cdot10^{-5}\ \ 1.03\cdot10^{-5}\right]\frac{s^2}{m^5}$$

To check again the same method of finding extremes:

$$C\_ct\_3\_fr\_check1 := \frac{\xi\_fr\_ct\_3\_check1}{2\,g\cdot\overrightarrow{A\_ct\cdot A\_ct}}$$

$$C\_ct\_3\_fr\_check1^{T} = \left[1.72\cdot10^{-5}\ \ 8.69\cdot10^{-5}\ \ 3.37\cdot10^{-5}\ \ 2.66\cdot10^{-5}\ \ 2.02\cdot10^{-5}\right]\frac{s^2}{m^5}$$

$$C\_ct\_3\_fr\_check2 := \frac{\xi\_fr\_ct\_3\_check2}{2\,g\cdot\overrightarrow{A\_exit\cdot A\_exit}}$$

$$C\_ct\_3\_fr\_check2^{T} = \left[4.31\cdot10^{-7}\ \ 2.18\cdot10^{-6}\ \ 8.51\cdot10^{-7}\ \ 6.68\cdot10^{-7}\ \ 5.08\cdot10^{-7}\right]\frac{s^2}{m^5}$$

The integration lies withing the expected extreme values and thus accepted as approximation.

Collecting the contributions:

$C\_ct\_bp1 := \text{augment}(C\_ct\_bp1, C\_ct\_3\_fr)$ (BP open)

$C\_ct\_bp0 := \text{augment}(C\_ct\_bp0, C\_ct\_3\_fr)$ (BP closed)

Outflow losses (section 4 of common tube)

The outflow losses are like the expansion losses, only the area downstream is now near infinte. Neglecting the naturally caused flow velocity (which the flow also has before flowing into the turbine), the flow loses all velocity head here. Therefore the loss-coefficient is:

$\xi\_out := 1$

Making the contribution to the QDC:

$$C\_ct\_4 := \frac{\xi\_out}{\overrightarrow{2\,g \cdot A\_exit \cdot A\_exit}}$$

$$C\_ct\_4^{\mathrm{T}} = \begin{bmatrix} 1.54 \cdot 10^{-5} & 7.23 \cdot 10^{-5} & 2.92 \cdot 10^{-5} & 2.33 \cdot 10^{-5} & 1.79 \cdot 10^{-5} \end{bmatrix} \frac{s^2}{m^5}$$

Collecting the contributions:

$C\_ct\_bp1 := \text{augment}\left(C\_ct\_bp1, C\_ct\_4\right)$ (BP open)

$C\_ct\_bp0 := \text{augment}\left(C\_ct\_bp0, C\_ct\_4\right)$ (BP closed)

To sumerize the QDC values are:

In case the bypass is open:

$$C\_ct\_bp1 = \begin{bmatrix} 1.72 \cdot 10^{-5} & 4.74 \cdot 10^{-5} & 8.82 \cdot 10^{-6} & 1.54 \cdot 10^{-5} \\ 8.69 \cdot 10^{-5} & 2.23 \cdot 10^{-4} & 4.45 \cdot 10^{-5} & 7.23 \cdot 10^{-5} \\ 3.37 \cdot 10^{-5} & 9 \cdot 10^{-5} & 1.73 \cdot 10^{-5} & 2.92 \cdot 10^{-5} \\ 2.66 \cdot 10^{-5} & 7.19 \cdot 10^{-5} & 1.36 \cdot 10^{-5} & 2.33 \cdot 10^{-5} \\ 2.02 \cdot 10^{-5} & 5.51 \cdot 10^{-5} & 1.03 \cdot 10^{-5} & 1.79 \cdot 10^{-5} \end{bmatrix} \frac{s^2}{m^5}$$

In case the bypass is closed (note: in such a case the discharge ratio $q\_rd = \frac{Q\_bp}{Q\_ct} = 0$):

$$C\_ct\_bp0 = \begin{bmatrix} 2.44 \cdot 10^{-5} & 1.72 \cdot 10^{-5} & 4.74 \cdot 10^{-5} & 8.82 \cdot 10^{-6} & 1.54 \cdot 10^{-5} \\ 1.15 \cdot 10^{-4} & 8.69 \cdot 10^{-5} & 2.23 \cdot 10^{-4} & 4.45 \cdot 10^{-5} & 7.23 \cdot 10^{-5} \\ 4.64 \cdot 10^{-5} & 3.37 \cdot 10^{-5} & 9 \cdot 10^{-5} & 1.73 \cdot 10^{-5} & 2.92 \cdot 10^{-5} \\ 3.7 \cdot 10^{-5} & 2.66 \cdot 10^{-5} & 7.19 \cdot 10^{-5} & 1.36 \cdot 10^{-5} & 2.33 \cdot 10^{-5} \\ 2.84 \cdot 10^{-5} & 2.02 \cdot 10^{-5} & 5.51 \cdot 10^{-5} & 1.03 \cdot 10^{-5} & 1.79 \cdot 10^{-5} \end{bmatrix} \frac{s^2}{m^5}$$

Summation:

$C\_CT\_BP1 := C\_sum\left(C\_ct\_bp1\right)$

$$C\_CT\_BP1^{\mathrm{T}} = \begin{bmatrix} 8.88 \cdot 10^{-5} & 4.27 \cdot 10^{-4} & 1.7 \cdot 10^{-4} & 1.35 \cdot 10^{-4} & 1.04 \cdot 10^{-4} \end{bmatrix} \frac{s^2}{m^5}$$

$C\_CT\_BP0 := C\_sum\left(C\_ct\_bp0\right)$

$$C\_CT\_BP0^{\mathrm{T}} = \begin{bmatrix} 1.13 \cdot 10^{-4} & 5.42 \cdot 10^{-4} & 2.17 \cdot 10^{-4} & 1.72 \cdot 10^{-4} & 1.32 \cdot 10^{-4} \end{bmatrix} \frac{s^2}{m^5}$$

Head discharge relation

Two flow conditions are considered:
1. Bypass open and turbine active
2. Bypass closed and turbine active.

Situation 1 is the most complex. In the thesis report was derived that a system of 2 equations needs to be solved. First defining the speed ratio to reduce further the amount of variables:

$$r\_s = \frac{N}{N\_s}$$

System of equations:

1) $\Delta H\_ava = \Delta H\_par + \Delta H\_ct$

2) $\Delta H\_par = \Delta H\_bpt = \Delta H\_tt$

Expanding:

1)

$$\Delta H\_ava = Q\_sys^2 \cdot C\_PAR + Q\_sys^2 \cdot C\_CT\_BP1$$

Where:

$$C\_PAR = \left(\frac{1}{C\_TT\left(\frac{Q\_bp}{Q\_sys}\right)} - \frac{\left(\frac{\left(\eta \cdot Q\_sys \cdot \left(1 - \frac{Q\_bp}{Q\_sys}\right)\right)^{\frac{2}{3}}}{g} \cdot \left(r\_s\right)^{\frac{4}{3}} \right)}{\left(\left(Q\_bp\right)^2 \cdot C\_BPT\left(\frac{Q\_bp}{Q\_sys}\right)\right) C\_TT\left(\frac{Q\_bp}{Q\_sys}\right)} + \frac{1}{C\_BPT\left(\frac{Q\_bp}{Q\_sys}\right)} \right)^{-1}$$

2)

$$C\_BPT\left(\frac{Q\_bp}{Q\_sys}\right) \cdot \left(Q\_bp\right)^2 = \left(\frac{\left(\eta \cdot Q\_sys \cdot \left(1 - \frac{Q\_bp}{Q\_sys}\right)\right)^{\frac{2}{3}}}{g} \cdot \left(r\_s\right)^{\frac{4}{3}} \right) + C\_TT\left(\frac{Q\_bp}{Q\_sys}\right) \cdot \left(Q\_sys \cdot \left(1 - \frac{Q\_bp}{Q\_sys}\right)\right)^2$$

Both need to be solved for system discharge Q_sys and discharge ratio Q_bp.

(Note: solving for r_qd didn't work, but entering the ratio as bypass over system discharge does lead to a sensible solution for most combinations of the variables)

Analytically solving is clearly not an option. Numerically a system is more challenging, but the quasi Newtonian method can be used.

Let **F** be a vector of equations for which the root is being sought and **X** a vector of (estimate) roots values for the variables of that system. Then using the Jacobian matrix **J**:

$$\overrightarrow{x}_{n} = \overrightarrow{x}_{n-1} - J^{-1} \cdot \overrightarrow{F\left(x_{n-1}\right)}$$

For the situation to be solved:

$$\begin{bmatrix} x\_1 \\ x\_2 \end{bmatrix} = \begin{bmatrix} Q\_sys \\ Q\_bp \end{bmatrix}$$

Head discharge relation (continued)

Unfortunately the formulas used, don't fit on width of the sheet anymore...
so to shorten notation further and show what happens in the algorythm in $Q\_s1\left(\Delta H\_ava, r\_s, \eta\_t\right)$
on the next page:

$$f1\left(\frac{x\_2}{x\_1}\right) = C\_TT\left(\frac{x\_2}{x\_1}\right)$$

is:

$$f1\left(\frac{x\_2}{x\_1}\right) = \sum_{j=0}^{cols\left(C\_tt\_c\right)-1} C\_tt\_c_{i,j} + \frac{\left(1 - 2 \cdot \frac{\left(\frac{x\_2}{x\_1}\right)^2}{r\_A2} \cdot cos\left(\delta\_1\right) - 2 \cdot \frac{\left(1 - \frac{x\_2}{x\_1}\right)^2}{r\_A1} \cdot cos\left(\delta\_2_i\right) + \left(\frac{1 - \frac{x\_2}{x\_1}}{r\_A1}\right)^2\right)}{2\ g \cdot A\_ct_i \cdot A\_ct_i}$$

and

$$f2\left(\frac{x\_2}{x\_1}\right) = C\_BPT\left(\frac{x\_2}{x\_1}\right)$$

is:

$$f2\left(\frac{x\_2}{x\_1}\right) = \sum_{j=0}^{cols\left(C\_bpt\_c\right)-1} C\_bpt\_c_{i,j} + \frac{\left(1 - 2 \cdot \frac{\left(\frac{x\_2}{x\_1}\right)^2}{r\_A2} \cdot cos\left(\delta\_1\right) - 2 \cdot \frac{\left(1 - \frac{x\_2}{x\_1}\right)^2}{r\_A1} \cdot cos\left(\delta\_2_i\right) + \left(\frac{\frac{x\_2}{x\_1}}{r\_A2}\right)^2\right)}{2\ g \cdot A\_ct_i \cdot A\_ct_i}$$

Define turbine parameter:

$$K\_t = \frac{\eta^{\frac{2}{3}} \cdot \left(r\_s\right)^{\frac{4}{3}}}{g}$$

Define common tube parameter:
$K\_ct = C\_CT\_BP1$
Define available head parameter:
$K\_h = \Delta H\_ava$

System of equations in short:

$$F\_1 \leftarrow x\_1^2 \cdot \left(\frac{1}{f1\left(\frac{x\_2}{x\_1}\right)} - \frac{\left(\left(x\_1 \cdot \left(1 - \frac{x\_2}{x\_1}\right)\right)^{\frac{2}{3}} \cdot K\_t\right)}{\left(\left(x\_1 \cdot \frac{x\_2}{x\_1}\right)^2 \cdot f2\left(\frac{x\_2}{x\_1}\right)\right) f1\left(\frac{x\_2}{x\_1}\right)} + \frac{1}{f2\left(\frac{x\_2}{x\_1}\right)}\right)^{-1} + x\_1^2 \cdot K\_ct - K\_h$$

$$F\_2 \leftarrow \left(\left(x\_1 \cdot \left(1 - \frac{x\_2}{x\_1}\right)\right)^{\frac{2}{3}} \cdot K\_t\right) + f1\left(\frac{x\_2}{x\_1}\right) \cdot \left(x\_1 \cdot \left(1 - \frac{x\_2}{x\_1}\right)\right)^2 - f2\left(\frac{x\_2}{x\_1}\right) \cdot \left(x\_2\right)^2$$

Note: $C\_TT\left(r\_qd\right)$ and $C\_BPT\left(r\_qd\right)$ both only has 1 term that is a function of r_qd, the rest are constant with respect to r_qd. However, the parts that are dependent on r_qd are quadratic.

```
$Q\_s1(\Delta H\_ava, r\_s, \eta\_t) :=$
  for $k \in 0 ..$ if $\mathrm{rows}(\Delta H\_ava) = 0$
                        $\Delta H\_ava \leftarrow [\Delta H\_ava]$
                        $0$
                     else
                        $\mathrm{rows}(\Delta H\_ava) - 1$
    for $i \in 0 ..$ if $\mathrm{rows}(r\_s) = 0$
                          $r\_s \leftarrow [r\_s]$
                          $\eta\_t \leftarrow [\eta\_t]$
                          $0$
                       else
                          $\mathrm{rows}(r\_s) - 1$
      $n \leftarrow 0$
      $x\_ini \leftarrow \begin{bmatrix} \sqrt{\dfrac{\Delta H\_ava_k}{\sum_{j=0}^{\mathrm{cols}(C\_bpt\_c)-1} C\_bpt\_c_{i,j}}} \\ 80\% \cdot \sqrt{\dfrac{\Delta H\_ava_k}{\sum_{j=0}^{\mathrm{cols}(C\_bpt\_c)-1} C\_bpt\_c_{i,j}}} \end{bmatrix}$
      $K\_t \leftarrow \dfrac{\eta\_t_i^{\frac{2}{3}} \cdot \left(\dfrac{r\_s_i}{s}\right)^{\frac{4}{3}}}{g}$
      $K\_h \leftarrow \Delta H\_ava_k$
      $K\_ct \leftarrow C\_CT\_BP1_i$
      $f1(x\_1, x\_2) \leftarrow \sum_{j=0}^{\mathrm{cols}(C\_tt\_c)-1} C\_tt\_c_{i,j} + \dfrac{(1 - 2 \cdot \dfrac{\left(\dfrac{x\_2}{x\_1}\right)^2}{r\_A1} \cdot \cos(\delta\_1) - 2 \cdot \dfrac{\left(1 - \dfrac{x\_2}{x\_1}\right)^2}{r\_A2}}{2\ g \cdot A\_ct_i \cdot A\_ct}$
      $f2(x\_1, x\_2) \leftarrow \sum_{j=0}^{\mathrm{cols}(C\_bpt\_c)-1} C\_bpt\_c_{i,j} + \dfrac{(1 - 2 \cdot \dfrac{\left(\dfrac{x\_2}{x\_1}\right)^2}{r\_A1} \cdot \cos(\delta\_1) - 2 \cdot \dfrac{(1 - \dfrac{x\_2}{x\_1}}{r\_A2}}{2\ g \cdot A\_ct_i \cdot A\_c}$
      $F\_1(x\_1, x\_2) \leftarrow (x\_1 - x\_2)^2 \cdot f1(x\_1, x\_2) + \left((x\_1 - x\_2)^{\frac{2}{3}} \cdot K\_t\right) + x\_1^2 \cdot K\_ct$
      $F\_2(x\_1, x\_2) \leftarrow \left(\left(x\_1 \cdot \left(1 - \dfrac{x\_2}{x\_1}\right)\right)^{\frac{2}{3}} \cdot K\_t\right) + f1(x\_1, x\_2) \cdot \left(x\_1 \cdot \left(1 - \dfrac{x\_2}{x\_1}\right)\right)^2 - f$
      $dF11(x\_1, x\_2) \leftarrow \dfrac{\mathrm{d}}{\mathrm{d}x\_1} F\_1(x\_1, x\_2)$
      $dF12(x\_1, x\_2) \leftarrow \dfrac{\mathrm{d}}{\mathrm{d}x\_2} F\_1(x\_1, x\_2)$
      $dF21(x\_1, x\_2) \leftarrow \dfrac{\mathrm{d}}{} F\_2(x\_1, x\_2)$
```

```
            $\mathrm{d}x\_1$
            $dF22(x\_1,x\_2) \leftarrow \frac{\mathrm{d}}{\mathrm{d}x\_2} F\_2(x\_1,x\_2)$
            $J(x\_1,x\_2) \leftarrow \begin{bmatrix} dF11(x\_1,x\_2) & dF12(x\_1,x\_2) \\ dF21(x\_1,x\_2) & dF22(x\_1,x\_2) \end{bmatrix}$
            $Jinv(x\_1,x\_2) \leftarrow (J(x\_1,x\_2))^{-1}$
            $\begin{bmatrix} x\_1 \\ x\_2 \end{bmatrix} \leftarrow x\_ini$
            while $|F\_1(x\_1,x\_2)| > 10^{-7}\ m \lor |F\_2(x\_1,x\_2)| > 10^{-7}\ m$
                $x \leftarrow \begin{bmatrix} x\_1 \\ x\_2 \end{bmatrix} - Jinv(x\_1,x\_2) \cdot \begin{bmatrix} F\_1(x\_1,x\_2) \\ F\_2(x\_1,x\_2) \end{bmatrix}$
                $\begin{bmatrix} x\_1 \\ x\_2 \end{bmatrix} \leftarrow \begin{bmatrix} |x_0| \\ |x_1| \end{bmatrix}$
                $n \leftarrow n+1$
                if $n > 1000$
                    break
            $Q\_sys_{k,i} \leftarrow |x\_1|$
            $Q\_bpt_{k,i} \leftarrow |x\_2|$
            $F_{k,i} \leftarrow F\_1(x\_1,x\_2)$
    return $[Q\_sys\ \ Q\_bpt\ \ F]$
```

Head discharge relation (continued)

From the solution the formula for system discharge:
$Q\_sys(\Delta H\_ava, r\_s, \eta\_t) := Q\_s1(\Delta H\_ava, r\_s, \eta\_t)_{0,0}$

From the solution the formula for bypass discharge:
$Q\_bp(\Delta H\_ava, r\_s, \eta\_t) := Q\_s1(\Delta H\_ava, r\_s, \eta\_t)_{0,1}$

Discharge through the turbine tube:
$Q\_t(\Delta H\_ava, r\_s, \eta\_t) := Q\_sys(\Delta H\_ava, r\_s, \eta\_t) - Q\_bp(\Delta H\_ava, r\_s, \eta\_t)$

Set a test value for the speed ratio for all design variants: $r\_s\_test := [1.6\ \ 2.35\ \ 1.9\ \ 1.8\ \ 1.7]^{\mathrm{T}}$

And a value for the efficiency: $\eta\_ttest := [90\%\ \ 90\%\ \ 90\%\ \ 90\%\ \ 90\%]^{\mathrm{T}}$

Define area ratio: $r\_A \equiv 0.94$

$[1.6\ \ 2.4\ \ 1.9\ \ 1.8\ \ 1.7]$	Optimal speed ratios for 2.5m available head
$[1.6\ \ 2.35\ \ 1.9\ \ 1.8\ \ 1.7]$	Optimal speed ratios for 1.4m available head
$[0.9\ \ 1.3\ \ 1.0\ \ 0.9\ \ 0.9]$	Optimal speed ratios for 0.5m available head

Head discharge relation (continued)

System discharge:

$$Qtest1 := Q\_sys\left(\begin{bmatrix} 2.5\ m \\ 2\ m \\ 1.4\ m \\ 0.5\ m \end{bmatrix}, r\_s\_test, \eta\_ttest\right) = \begin{bmatrix} 84.31 & 38.22 & 59.93 & 67.12 & 76.11 \\ 66.15 & 30.05 & 46.91 & 52.5 & 59.38 \\ 42.33 & 19.32 & 29.94 & 33.46 & 37.69 \\ 10.34 & 4.77 & 7.36 & 8.22 & 9.26 \end{bmatrix} \frac{m^3}{s}$$

Bypass discharge:

$$Qtest2 := Q\_bp\left(\begin{bmatrix} 2.5\ m \\ 2\ m \\ 1.4\ m \\ 0.5\ m \end{bmatrix}, r\_s\_test, \eta\_ttest\right) = \begin{bmatrix} 16.67 & 7.53 & 11.98 & 13.45 & 15.4 \\ 14.72 & 6.66 & 10.57 & 11.86 & 13.55 \\ 11.62 & 5.27 & 8.33 & 9.33 & 10.64 \\ 4.98 & 2.29 & 3.58 & 4 & 4.55 \end{bmatrix} \frac{m^3}{s}$$

Turbine discharge:

$$Qtest3 := Q\_t\left(\begin{bmatrix} 2.5\ m \\ 2\ m \\ 1.4\ m \\ 0.5\ m \end{bmatrix}, r\_s\_test, \eta\_ttest\right) = \begin{bmatrix} 67.64 & 30.69 & 47.94 & 53.67 & 60.7 \\ 51.44 & 23.39 & 36.35 & 40.65 & 45.83 \\ 30.71 & 14.05 & 21.61 & 24.12 & 27.05 \\ 5.36 & 2.48 & 3.78 & 4.21 & 4.71 \end{bmatrix} \frac{m^3}{s}$$

Check with:

$$Q\_t\_max\_g^{\mathrm{T}} = \begin{bmatrix} 67.38 & 27.24 & 35.12 & 47.31 & 49.42 \end{bmatrix} \frac{m^3}{s}$$

Discharge ratios:

$$\overrightarrow{\frac{Qtest2}{Qtest1}} = \begin{bmatrix} 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0.22 & 0.22 & 0.23 & 0.23 & 0.23 \\ 0.27 & 0.27 & 0.28 & 0.28 & 0.28 \\ 0.48 & 0.48 & 0.49 & 0.49 & 0.49 \end{bmatrix}$$

Sum of head "losses" in turbine tube:

```
$\Delta H\_tt1\_loss(\Delta H\_ava, r\_s, \eta\_t) :=$
    $Q1 \leftarrow Q\_t(\Delta H\_ava, r\_s, \eta\_t)$
    $Q2 \leftarrow Q\_bp(\Delta H\_ava, r\_s, \eta\_t)$
    $Q3 \leftarrow Q\_sys(\Delta H\_ava, r\_s, \eta\_t)$
    for $i \in 0 ..$ if $\mathrm{rows}(\Delta H\_ava) = 0$
                        $\Delta H\_ava \leftarrow [\Delta H\_ava]$
                        $0$
                     else
                        $\mathrm{rows}(\Delta H\_ava) - 1$
        for $j \in 0 ..$ if $\mathrm{rows}(r\_s) = 0$
                            $r\_s \leftarrow [r\_s]$
                            $\eta\_t \leftarrow [\eta\_t]$
                            $0$
                         else
                            $\mathrm{rows}(r\_s) - 1$
            $H_{i,j} \leftarrow {Q1_{i,j}}^{2} \cdot C\_TT\left(\frac{Q2_{i,j}}{Q3_{i,j}}\right)_j$
    return $H$
```

$$Htest1 := \Delta H\_tt1\_loss\left(\begin{bmatrix} 2.5\ m \\ 2\ m \\ 1.4\ m \\ 0.5\ m \end{bmatrix}, r\_s\_test, \eta\_ttest\right)$$

$$Htest1 = \begin{bmatrix} -1.08 & -1.03 & -1.06 & -1.07 & -1.08 \\ -0.85 & -0.81 & -0.83 & -0.83 & -0.84 \\ -0.5 & -0.49 & -0.49 & -0.49 & -0.49 \\ -0.05 & -0.05 & -0.05 & -0.05 & -0.05 \end{bmatrix} m$$

Head discharge relation (continued)

Head over the turbine:

```
$\Delta H\_t1(\Delta H\_ava, r\_s, \eta\_t) :=$
    $Q1 \leftarrow Q\_t(\Delta H\_ava, r\_s, \eta\_t)$
    for $i \in 0 ..$ if rows$(\Delta H\_ava) = 0$
                         $\Delta H\_ava \leftarrow [\Delta H\_ava]$
                         $0$
                     else
                         rows$(\Delta H\_ava) - 1$
        for $j \in 0 ..$ if rows$(r\_s) = 0$
                             $r\_s \leftarrow [r\_s]$
                             $\eta\_t \leftarrow [\eta\_t]$
                             $0$
                         else
                             rows$(r\_s) - 1$
            $H_{i,j} \leftarrow \dfrac{\left(Q1_{i,j} \cdot \eta\_t_j\right)^{\frac{2}{3}} \cdot \left(\dfrac{r\_s_j}{s}\right)^{\frac{4}{3}}}{g}$
    return $H$
```

$$Htest2 := \Delta H\_t1\left(\begin{bmatrix} 2.5\ m \\ 2\ m \\ 1.4\ m \\ 0.5\ m \end{bmatrix}, r\_s\_test, \eta\_ttest\right) = \begin{bmatrix} 2.95 & 2.91 & 2.95 & 2.96 & 2.98 \\ 2.46 & 2.43 & 2.45 & 2.46 & 2.47 \\ 1.74 & 1.73 & 1.74 & 1.74 & 1.74 \\ 0.54 & 0.54 & 0.54 & 0.54 & 0.54 \end{bmatrix} m$$

Head difference over the turbine tube:

```
$\Delta H\_tt(\Delta H\_ava, r\_s, \eta\_t) :=$
    for $i \in 0 ..$ rows$(n\_t) - 1$
        $H^{\langle i \rangle} \leftarrow \Delta H\_ava - \Delta H\_tt1\_loss(\Delta H\_ava, r\_s, \eta\_t)^{\langle i \rangle}$
    return $H$
```

$$Htest11 := \Delta H\_tt\left(\begin{bmatrix} 2.5\ m \\ 2\ m \\ 1.4\ m \\ 0.5\ m \end{bmatrix}, r\_s\_test, \eta\_ttest\right) = \begin{bmatrix} 3.58 & 3.53 & 3.56 & 3.57 & 3.58 \\ 2.85 & 2.81 & 2.83 & 2.83 & 2.84 \\ 1.9 & 1.89 & 1.89 & 1.89 & 1.89 \\ 0.55 & 0.55 & 0.55 & 0.55 & 0.55 \end{bmatrix} m$$

Head ratio:

$$r\_h\_test := \overrightarrow{\frac{Htest2}{Htest11}} = \begin{bmatrix} 0.82 & 0.82 & 0.83 & 0.83 & 0.83 \\ 0.86 & 0.86 & 0.87 & 0.87 & 0.87 \\ 0.92 & 0.92 & 0.92 & 0.92 & 0.92 \\ 0.98 & 0.98 & 0.98 & 0.98 & 0.98 \end{bmatrix}$$

Head discharge relation (continued)

Head difference over the bypass tube:

```
$\Delta H\_bpt(\Delta H\_ava, r\_s, \eta\_t) :=$
    $Q1 \leftarrow Q\_t(\Delta H\_ava, r\_s, \eta\_t)$
    $Q2 \leftarrow Q\_bp(\Delta H\_ava, r\_s, \eta\_t)$
    $Q3 \leftarrow Q\_sys(\Delta H\_ava, r\_s, \eta\_t)$
    for $i \in 0 ..$ if $\text{rows}(\Delta H\_ava) = 0$
                         $0$
                     else
                         $\text{rows}(\Delta H\_ava) - 1$
        for $j \in 0 ..$ if $\text{rows}(r\_s) = 0$
                             $0$
                         else
                             $\text{rows}(r\_s) - 1$
            $H_{i,j} \leftarrow {Q2_{i,j}}^2 \cdot C\_BPT\left(\frac{Q2_{i,j}}{Q3_{i,j}}\right)_j$
    return $H$
```

$$Htest3 := \Delta H\_bpt\left(\begin{bmatrix} 2.5\ m \\ 2\ m \\ 1.4\ m \\ 0.5\ m \end{bmatrix}, r\_s\_test, \eta\_ttest\right) = \begin{bmatrix} 1.87 & 1.88 & 1.89 & 1.89 & 1.9 \\ 1.61 & 1.61 & 1.63 & 1.63 & 1.63 \\ 1.24 & 1.24 & 1.25 & 1.25 & 1.25 \\ 0.49 & 0.49 & 0.49 & 0.49 & 0.49 \end{bmatrix} m$$

Head difference over the common tube:

```
$\Delta H\_ct(\Delta H\_ava, r\_s, \eta\_t) :=$
    $Q1 \leftarrow Q\_t(\Delta H\_ava, r\_s, \eta\_t)$
    $Q2 \leftarrow Q\_bp(\Delta H\_ava, r\_s, \eta\_t)$
    $Q3 \leftarrow Q\_sys(\Delta H\_ava, r\_s, \eta\_t)$
    for $i \in 0 ..$ if $\text{rows}(\Delta H\_ava) = 0$
                         $0$
                     else
                         $\text{rows}(\Delta H\_ava) - 1$
        for $j \in 0 ..$ if $\text{rows}(r\_s) = 0$
                             $0$
                         else
                             $\text{rows}(r\_s) - 1$
            $H_{i,j} \leftarrow {Q3_{i,j}}^2 \cdot C\_CT\_BP1_j$
    return $H$
```

$$Htest4 := \Delta H\_ct\left(\begin{bmatrix} 2.5\ m \\ 2\ m \\ 1.4\ m \\ 0.5\ m \end{bmatrix}, r\_s\_test, \eta\_ttest\right) = \begin{bmatrix} 0.63 & 0.62 & 0.61 & 0.61 & 0.6 \\ 0.39 & 0.39 & 0.37 & 0.37 & 0.37 \\ 0.16 & 0.16 & 0.15 & 0.15 & 0.15 \\ 0.01 & 0.01 & 0.01 & 0.01 & 0.01 \end{bmatrix} m$$

Head discharge relation (continued)

Conclusions method of calculation:
Head losses of the turbine and the turbine tube equal the head losses of the bypass, so that equation holds:

$$Htest1+Htest2=\begin{bmatrix}1.87 & 1.88 & 1.89 & 1.89 & 1.9\\ 1.61 & 1.61 & 1.63 & 1.63 & 1.63\\ 1.24 & 1.24 & 1.25 & 1.25 & 1.25\\ 0.49 & 0.49 & 0.49 & 0.49 & 0.49\end{bmatrix} m$$

$$Htest3=\begin{bmatrix}1.87 & 1.88 & 1.89 & 1.89 & 1.9\\ 1.61 & 1.61 & 1.63 & 1.63 & 1.63\\ 1.24 & 1.24 & 1.25 & 1.25 & 1.25\\ 0.49 & 0.49 & 0.49 & 0.49 & 0.49\end{bmatrix} m$$

The head difference over the entire system is equal to the available head difference

$$Hcheck:=Htest3+Htest4=\begin{bmatrix}2.5 & 2.5 & 2.5 & 2.5 & 2.5\\ 2 & 2 & 2 & 2 & 2\\ 1.4 & 1.4 & 1.4 & 1.4 & 1.4\\ 0.5 & 0.5 & 0.5 & 0.5 & 0.5\end{bmatrix} m \qquad \begin{bmatrix}2.5\ m\\ 2\ m\\ 1.4\ m\\ 0.5\ m\end{bmatrix}$$

Head gain over the turbine tube:

$$\frac{Htest11}{Hcheck}=\begin{bmatrix}1.43 & 1.41 & 1.43 & 1.43 & 1.43\\ 1.42 & 1.41 & 1.41 & 1.42 & 1.42\\ 1.36 & 1.35 & 1.35 & 1.35 & 1.35\\ 1.11 & 1.11 & 1.1 & 1.1 & 1.1\end{bmatrix}$$

Head gain over turbine itself:

$$\frac{Htest2}{Hcheck}=\begin{bmatrix}1.18 & 1.16 & 1.18 & 1.18 & 1.19\\ 1.23 & 1.21 & 1.23 & 1.23 & 1.23\\ 1.25 & 1.24 & 1.24 & 1.24 & 1.24\\ 1.09 & 1.09 & 1.09 & 1.09 & 1.08\end{bmatrix}$$

Discharge head relation when the bypass is closed
One QDC can be calculated for this situation:

$$C\_bp0:=\sum_{j=0}^{\mathrm{cols}\left(C\_tt\_c\right)-1} C\_tt\_c^{\langle j\rangle}+C\_CT\_BP0$$

The regular Kaplan discharge function can be used shown on the next page, because it didn't fit on the remainder of this one.

Discharge head relation when the bypass is closed (continued)

```
$Q\_s2(\Delta H\_ava, C, r\_s, \eta\_t) :=$
  for $k \in 0 ..$ if rows$(\Delta H\_ava) = 0$
                      $\Delta H\_ava \leftarrow [\Delta H\_ava]$
                      $0$
                    else
                      rows$(\Delta H\_ava) - 1$
    for $i \in 0 ..$ if rows$(C) = 0$
                       $0$
                     else
                       rows$(C) - 1$
      if rows$(C) < 2$
        $C_0 \leftarrow C$
        $r\_s_0 \leftarrow r\_s$
        $\eta\_t_0 \leftarrow \eta\_t$
      $Q_{k,i} \leftarrow 1 \frac{m^3}{s}$
      $D1 \leftarrow \left(\text{UnitsOf}\left(2\ Q_{k,i} \cdot \left(C_i\right)\right)\right)^{-1}$
      $D2 \leftarrow \left(\text{UnitsOf}\left(2 \cdot \left(r\_s_i \cdot \frac{1}{s}\right)^{\frac{4}{3}} \cdot \left(3 \cdot Q_{k,i}^{\frac{1}{3}} \cdot g\right)^{-1}\right)\right)^{-1}$
      $DH \leftarrow \left(Q_{k,i}^{2} \cdot C_i + \frac{\left(\eta\_t_i \cdot Q_{k,i}\right)^{\frac{2}{3}}}{g} \cdot \left(r\_s_i \cdot \frac{1}{s}\right)^{\frac{4}{3}} - \Delta H\_ava_k\right) \cdot \frac{1}{m}$
      if $\Delta H\_ava_k = 0$
        $Q_{k,i} \leftarrow 0\ m^3 \cdot s^{-1}$
      else
        while $\frac{|DH \cdot m|}{\Delta H\_ava_k} > 10^{-7}$
          $dDH\_dQ \leftarrow 2 \cdot Q_{k,i} \cdot C_i \cdot D1 + \frac{2 \cdot \eta\_t_i^{\frac{2}{3}}}{3 \cdot Q_{k,i}^{\frac{1}{3}} \cdot g} \cdot D2 \cdot \left(r\_s_i \cdot \frac{1}{s}\right)^{\frac{4}{3}}$
          $Q_{k,i} \leftarrow$ if $Q_{k,i} - \frac{DH}{dDH\_dQ} \cdot \frac{m^3}{s} < 0$
                                 $Q_{k,i} + \frac{DH}{dDH\_dQ} \cdot \frac{m^3}{s}$
                               else
                                 $Q_{k,i} - \frac{DH}{dDH\_dQ} \cdot \frac{m^3}{s}$
          $\left( \frac{\left(\eta\_t_i \cdot Q_{k,i}\right)^{\frac{2}{3}}}{} \right)^{\frac{4}{3}}$
```

```
                $DH \leftarrow \left(Q_{k,i}^{2} \cdot C_i + \frac{(\quad)}{g} \cdot \left(r\_s_i \cdot \frac{1}{s}\right) - \Delta H\_ava_k\right) \cdot \frac{1}{m}$
    return $Q$
```

Discharge head relation when the bypass is closed

$$Qtest4 := Q\_s2\left(\begin{bmatrix} 2.5\ m \\ 2\ m \\ 1.4\ m \\ 0.5\ m \end{bmatrix}, C\_bp0, r\_s\_test, \eta\_ttest\right) = \begin{bmatrix} 44.79 & 20.67 & 31.9 & 35.61 & 40.11 \\ 33.68 & 15.56 & 23.96 & 26.73 & 30.07 \\ 20.79 & 9.62 & 14.77 & 16.47 & 18.5 \\ 4.67 & 2.17 & 3.31 & 3.69 & 4.14 \end{bmatrix} \frac{m^3}{s}$$

Head over turbine in the case the bypass is closed:

```
$\Delta H\_t0(\Delta H\_ava, C, r\_s, \eta\_t) :=$
    $Q1 \leftarrow Q\_s2(\Delta H\_ava, C, r\_s, \eta\_t)$
    for $i \in 0 ..$ if rows$(\Delta H\_ava) = 0$
                        $\Delta H\_ava \leftarrow [\Delta H\_ava]$
                        $0$
                     else
                        rows$(\Delta H\_ava) - 1$
        for $j \in 0 ..$ if rows$(r\_s) = 0$
                            $r\_s \leftarrow [r\_s]$
                            $\eta\_t \leftarrow [\eta\_t]$
                            $0$
                         else
                            rows$(r\_s) - 1$
            $H_{i,j} \leftarrow \dfrac{\left(Q1_{i,j} \cdot \eta\_t_j\right)^{\frac{2}{3}} \cdot \left(\dfrac{r\_s_j}{s}\right)^{\frac{4}{3}}}{g}$
    return $H$
```

$$\Delta Htest4 := \Delta H\_t0\left(\begin{bmatrix} 2.5\ m \\ 2\ m \\ 1.4\ m \\ 0.5\ m \end{bmatrix}, C\_bp0, r\_s\_test, \eta\_ttest\right) = \begin{bmatrix} 2.24 & 2.24 & 2.25 & 2.25 & 2.26 \\ 1.86 & 1.85 & 1.86 & 1.86 & 1.87 \\ 1.34 & 1.34 & 1.35 & 1.35 & 1.35 \\ 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \end{bmatrix} m$$

Compared to when the bypass is opened: $Htest2 = \begin{bmatrix} 2.95 & 2.91 & 2.95 & 2.96 & 2.98 \\ 2.46 & 2.43 & 2.45 & 2.46 & 2.47 \\ 1.74 & 1.73 & 1.74 & 1.74 & 1.74 \\ 0.54 & 0.54 & 0.54 & 0.54 & 0.54 \end{bmatrix} m$

The gain by opening the bypass is then:

$$\overrightarrow{\frac{Htest2 - \Delta Htest4}{\Delta Htest4}} = \begin{bmatrix} 0.32 & 0.3 & 0.31 & 0.31 & 0.32 \\ 0.33 & 0.31 & 0.32 & 0.32 & 0.32 \\ 0.3 & 0.29 & 0.29 & 0.29 & 0.29 \\ 0.1 & 0.09 & 0.09 & 0.09 & 0.09 \end{bmatrix}$$

at the cost of a discharge of:

$$Qtest2 = \begin{bmatrix} 16.67 & 7.53 & 11.98 & 13.45 & 15.4 \\ 14.72 & 6.66 & 10.57 & 11.86 & 13.55 \\ 11.62 & 5.27 & 8.33 & 9.33 & 10.64 \\ 4.98 & 2.29 & 3.58 & 4 & 4.55 \end{bmatrix} \frac{m^3}{s}$$

Energy per second gained:

$$P\_gain := \rho \cdot g \cdot \left(\overrightarrow{Htest2 \cdot Qtest3 - \Delta Htest4 \cdot Qtest4}\right)$$

$$P\_gain = \begin{bmatrix} 971.61 & 422.06 & 682.7 & 770.49 & 882.79 \\ 627.23 & 274.26 & 437.25 & 492.15 & 559.03 \\ 250.81 & 111.38 & 172.59 & 193.2 & 215.96 \\ 5.82 & 2.68 & 3.96 & 4.4 & 4.81 \end{bmatrix} kW$$

Energy per second lost through bypass:

$$P\_lost := \rho \cdot g \cdot \overrightarrow{Htest2 \cdot Qtest2} = \begin{bmatrix} 481.8 & 214.5 & 346.3 & 389.99 & 449.11 \\ 354.42 & 158.27 & 253.94 & 285.63 & 327.72 \\ 198.35 & 89.28 & 141.49 & 158.76 & 180.94 \\ 26.56 & 12.18 & 19.02 & 21.28 & 24.13 \end{bmatrix} kW$$

Ratio gained/lost:

$$r\_bypass := \overrightarrow{\frac{P\_gain}{P\_lost}} = \begin{bmatrix} 2.02 & 1.97 & 1.97 & 1.98 & 1.97 \\ 1.77 & 1.73 & 1.72 & 1.72 & 1.71 \\ 1.26 & 1.25 & 1.22 & 1.22 & 1.19 \\ 0.22 & 0.22 & 0.21 & 0.21 & 0.2 \end{bmatrix}$$

Optimal speed ratios for this specific case (iteratively determined):

$r\_s\_max := \begin{bmatrix} 1.6 & 2.4 & 1.9 & 1.8 & 1.7 \end{bmatrix}^{\mathrm{T}}$ Optimal speed ratios for 2.5m available head

$r\_s\_min := \begin{bmatrix} 0.9 & 1.3 & 1.0 & 0.9 & 0.9 \end{bmatrix}^{\mathrm{T}}$ Optimal speed ratios for 0.5m available head

An interpolation function can be made for the speed ratio. Optimizing this speed ratio is complex and out of the scope of this research.

$$r\_s\_f(H\_ava, dv) := \mathrm{interp}\left(\mathrm{lspline}\left(\left(\begin{bmatrix} 0.5 & 1.4 & 1.5 & 2.0 & 2.5 \end{bmatrix} m\right)^{\mathrm{T}}, \left(\mathrm{augment}\left(r\_s\_min, r\_s\_max, r\_s\_max, r\_s\_ma\right.\right.\right.\right.$$

e.g. available head 2.13m design variant 1 the approximate optimal speed ratio is then:

Power output
The efficiency curves from the regular Kaplan are used.

For the discharge efficiency the following information is required:

$$\eta\_K(Q) := \begin{bmatrix} 0\ Q & 0.1\ Q & 0.2\ Q & 0.3\ Q & 0.4\ Q & 0.5\ Q & 0.6\ Q & 0.7\ Q & 0.8\ Q & 0.9\ Q & 1.0\ Q & 1.1\ Q \\ 0 & 30\% & 70\% & 84\% & 88\% & 90\% & 92\% & 92\% & 92\% & 90\% & 88\% & 80\% \end{bmatrix}$$

$$\eta.Q(Q,Qm) := \text{interp}\left(\text{pspline}\left(\left(\eta\_K(Qm)^{T}\right)^{\langle 0\rangle}, \left(\eta\_K(Qm)^{T}\right)^{\langle 1\rangle}\right), \left(\eta\_K(Qm)^{T}\right)^{\langle 0\rangle}, \left(\eta\_K(Qm)^{T}\right)^{\langle 1\rangle}, Q\right)$$

So the following function will be used to determine energy production:
$\eta\_Q(Q,Qm) := \max\left(10^{-7}, \eta.Q(Q,Qm)\right)$

Where:
Qm is the maximum discharge
Q is the instantaious discharge

For the head efficiency curve the following information is required:

$$\eta\_estH(probe,Q) := \text{augment}\left(probe^{\langle 0\rangle}\cdot m\cdot\rho\cdot g\cdot Q\ m^{3}\cdot s^{-1}\cdot kW^{-1}, \overrightarrow{\frac{probe^{\langle 1\rangle}\ kW}{probe^{\langle 0\rangle}\cdot m\cdot\rho\cdot g\cdot Q\ m^{3}\cdot s^{-1}}}\right)$$

$$[\Delta H\_t\ \ P\_t] \qquad\qquad [\rho gQH\ \ \eta]$$

$$probe\_H3 := \begin{bmatrix} 0.3 & 25 \\ 0.40 & 50 \\ 0.60 & 100 \\ 0.80 & 150 \\ 1.02 & 200 \\ 1.235 & 250 \\ 1.45 & 300 \\ 1.67 & 350 \\ 1.90 & 400 \\ 2.13 & 450 \\ 2.36 & 500 \\ 2.61 & 550 \\ 2.88 & 600 \\ 3.13 & 650 \\ 3.39 & 700 \\ 3.64 & 750 \\ 3.90 & 800 \end{bmatrix} \qquad \eta\_estH(probe\_H3, 23) = \begin{bmatrix} 67.54 & 0.37 \\ 90.06 & 0.56 \\ 135.09 & 0.74 \\ 180.12 & 0.83 \\ 229.65 & 0.87 \\ 278.06 & 0.9 \\ 326.46 & 0.92 \\ 376 & 0.93 \\ 427.78 & 0.94 \\ 479.56 & 0.94 \\ 531.35 & 0.94 \\ 587.63 & 0.94 \\ 648.42 & 0.93 \\ 704.71 & 0.92 \\ 763.25 & 0.92 \\ 819.53 & 0.92 \\ 878.07 & 0.91 \end{bmatrix}$$

$$\eta\_aprH(H,P,Q) := \text{interp}\left(\text{pspline}\left(P^{\langle 0\rangle}, \eta\_estH(P,Q)^{\langle 1\rangle}\right), P^{\langle 0\rangle}, \eta\_estH(P,Q)^{\langle 1\rangle}, H\cdot m^{-1}\right)$$

$\eta\_H(H) := \eta\_aprH(H, probe\_H3, 23)$

Where H is the in instantaious head difference.

Combining turbine efficiency:

$$\eta\_t(Q,Qm,H) := \max\left(0.05, \eta\_Q\left(Q\cdot\text{UnitsOf}(Q)^{-1}, Qm\cdot\text{UnitsOf}(Qm)^{-1}\right)\cdot\frac{\eta\_H(H)}{\eta\_H(1.90\ m)}\right)$$

Ecological minimum discharge:
$Q\_eco := 25\ m^{3}\cdot s^{-1}$

Power output (continued)

Maximum turbine discharge:
$Q\_t\_max\_g^{T} = \left[67.38\ \ 27.24\ \ 35.12\ \ 47.31\ \ 49.42\right]\ m^{3} \cdot s^{-1}$

Minimum turbine discharge:
$Q\_t\_20pc := 20\% \cdot Q\_t\_max\_g$
$Q\_t\_20pc^{T} = \left[13.48\ \ 5.45\ \ 7.02\ \ 9.46\ \ 9.88\right]\ m^{3} \cdot s^{-1}$

Threshold value for head over turbine:
$\Delta Hthres := \left[1\ \ 1\ \ 1\ \ 1\ \ 1\right]^{T}\ 0.3\ m$
(all assumed to have pentair turbines that start at 0.3m head difference).

The available head-discharge relation is a bit computaitonally demanding, so a interpolation function is made to calculate the discharge for a given head difference more quickly:

$$H\_ava\_set := 0.1\ m, 0.2\ m..4\ m = \begin{bmatrix}0.1\\0.2\\0.3\\0.4\\0.5\\0.6\\\vdots\end{bmatrix} m$$

```
$r\_s\_f\_set :=$
    for $j \in 0..\text{rows}(H\_ava\_set) - 1$
        for $i \in 0..\text{rows}(n\_t) - 1$
            $r\_s_{j,i} \leftarrow r\_s\_f\left(H\_ava\_set_{j}, i\right)$
    return $r\_s^{T}$
```

$= \left[0.48\ \ 0.57\ \ 0.68\ \ 0.79\ \ 0.9\ \ 1.01\ \ 1.12\ \ 1.\right.$ ...

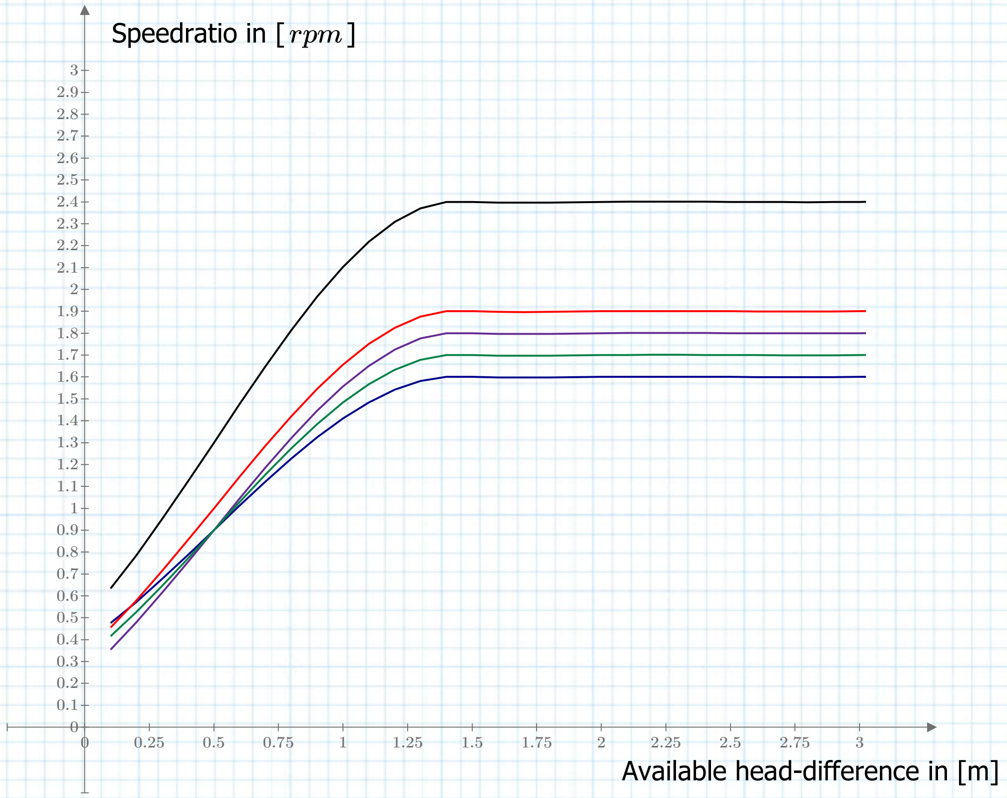
```
$Q\_sys\_set :=$
    $Q \leftarrow Q\_s1\left(H\_ava\_set_{0}, r\_s\_f\_set^{\langle 0\rangle}, \eta\_ttest\right)_{0,0}$
    for $i \in 1..\text{rows}(H\_ava\_set) - 1$
        $Q \leftarrow \text{stack}\left(Q, Q\_s1\left(H\_ava\_set_{i}, r\_s\_f\_set^{\langle i\rangle}, \eta\_ttest\right)_{0,0}\right)$
    return $Q$
```

$= \left[9.36\ \ 5.09\ \ 9.5\right.$ ...

```
$Q\_bp\_set :=$
    $Q \leftarrow Q\_s1\left(H\_ava\_set_{0}, r\_s\_f\_set^{\langle 0\rangle}, \eta\_ttest\right)_{0,1}$
    for $i \in 1..\text{rows}(H\_ava\_set) - 1$
        $Q \leftarrow \text{stack}\left(Q, Q\_s1\left(H\_ava\_set_{i}, r\_s\_f\_set^{\langle i\rangle}, \eta\_ttest\right)_{0,1}\right)$
    return $Q$
```

$= \left[2.93\ \ 1.4\ \ 2.32\right.$ ...

Approximated optimal speed ratio (with spline function with linear ends)



$$Q\_cubs\_sys(H\_ava, dv) := \text{interp}\left(\text{pspline}\left(H\_ava\_set, (Q\_sys\_set)^{\langle dv\rangle}\right), H\_ava\_set, (Q\_sys\_set)^{\langle dv\rangle}, H\_ava\right)$$

$$Q\_cubs\_bp(H\_ava, dv) := \text{interp}\left(\text{pspline}\left(H\_ava\_set, (Q\_bp\_set)^{\langle dv\rangle}\right), H\_ava\_set, (Q\_bp\_set)^{\langle dv\rangle}, H\_ava\right)$$

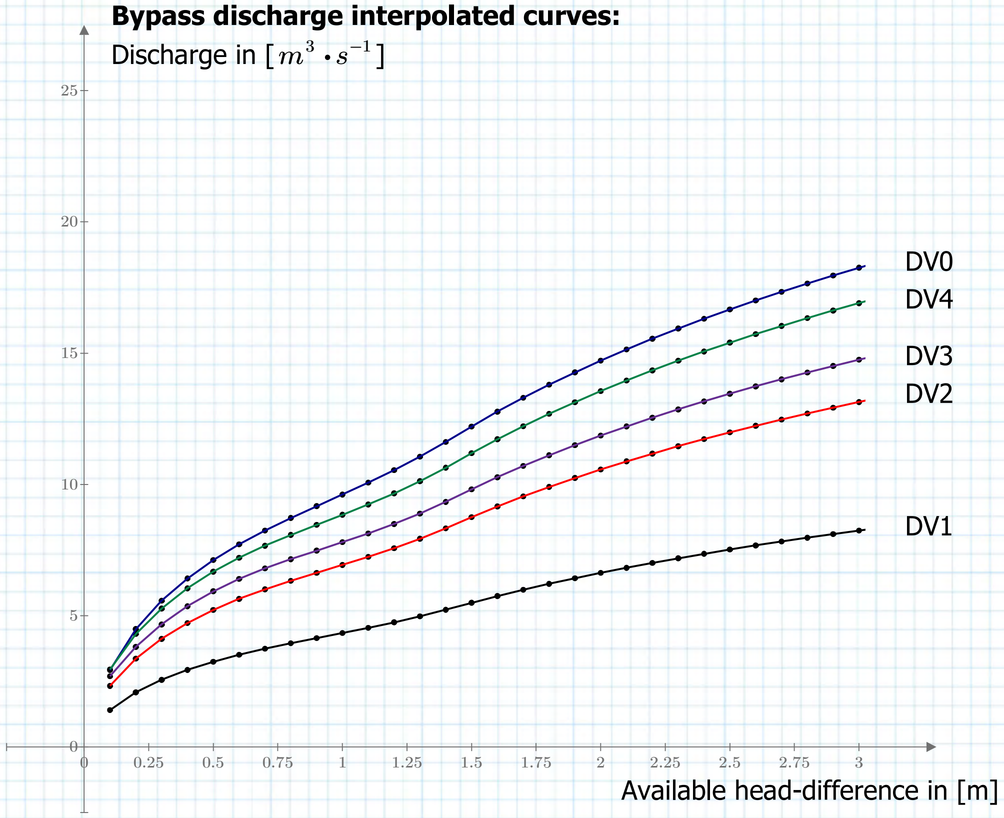
$$H\_t1\_set := \left\| \begin{array}{l} H \leftarrow \Delta H\_t1\left(H\_ava\_set_i, r\_s\_f\_set^{\langle i\rangle}, \eta\_ttest\right) \\ \text{for } i \in 1..\text{rows}(H\_ava\_set)-1 \\ \quad \left\| H \leftarrow \text{stack}\left(H, \Delta H\_t1\left(H\_ava\_set_i, r\_s\_f\_set^{\langle i\rangle}, \eta\_ttest\right)\right) \right. \\ \text{return } H \end{array} \right. = \begin{bmatrix} 0.25 & 0.25 & 0.24 & 0.23 & 0.25 \\ 0.25 & 0.25 & 0.24 & 0.23 & 0.25 \\ 0.37 & 0.37 & 0.37 & 0.35 & 0.37 \\ 0.5 & 0.49 & 0.49 & 0.48 & 0.5 \\ 0.62 & 0.62 & 0.62 & 0.62 & 0.62 \\ 0.75 & 0.74 & 0.75 & 0.75 & 0.75 \\ & & & & \vdots \end{bmatrix} m$$

$$H\_cubs\_t1(H\_ava, dv) := \text{interp}\left(\text{pspline}\left(H\_ava\_set, H\_t1\_set^{\langle dv\rangle}\right), H\_ava\_set, H\_t1\_set^{\langle dv\rangle}, H\_ava\right)$$

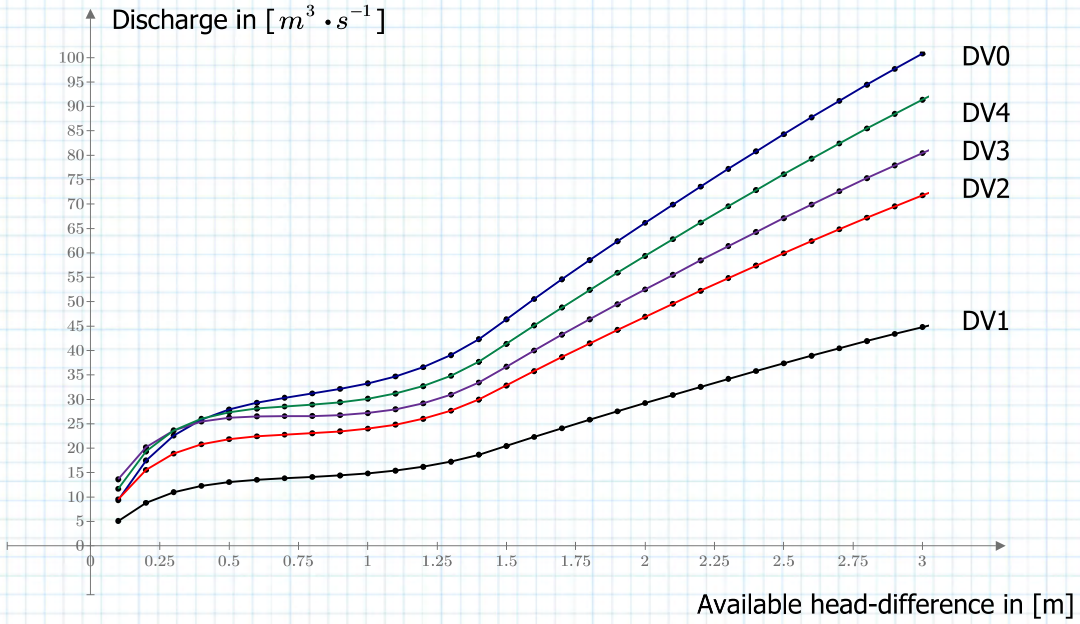
$$H\_t0\_set := \left\| \begin{array}{l} H \leftarrow \Delta H\_t0\left(H\_ava\_set_i, C\_bp0, r\_s\_f\_set^{\langle i\rangle}, \eta\_ttest\right) \\ \text{for } i \in 1..\text{rows}(H\_ava\_set)-1 \\ \quad \left\| H \leftarrow \text{stack}\left(H, \Delta H\_t0\left(H\_ava\_set_i, C\_bp0, r\_s\_f\_set^{\langle i\rangle}, \eta\_ttest\right)\right) \right. \\ \text{return } H \end{array} \right. = \left[\begin{array}{ccccc} 0.19 & 0.19 & 0.18 & 0.18 & 0.19 \\ 0.19 & 0.19 & 0.18 & 0.18 & 0.19 \\ & & & & \vdots \end{array} \right.$$

$$H\_cubs\_t0(H\_ava, dv) := \text{interp}\left(\text{pspline}\left(H\_ava\_set, H\_t0\_set^{\langle dv\rangle}\right), H\_ava\_set, H\_t0\_set^{\langle dv\rangle}, H\_ava\right)$$

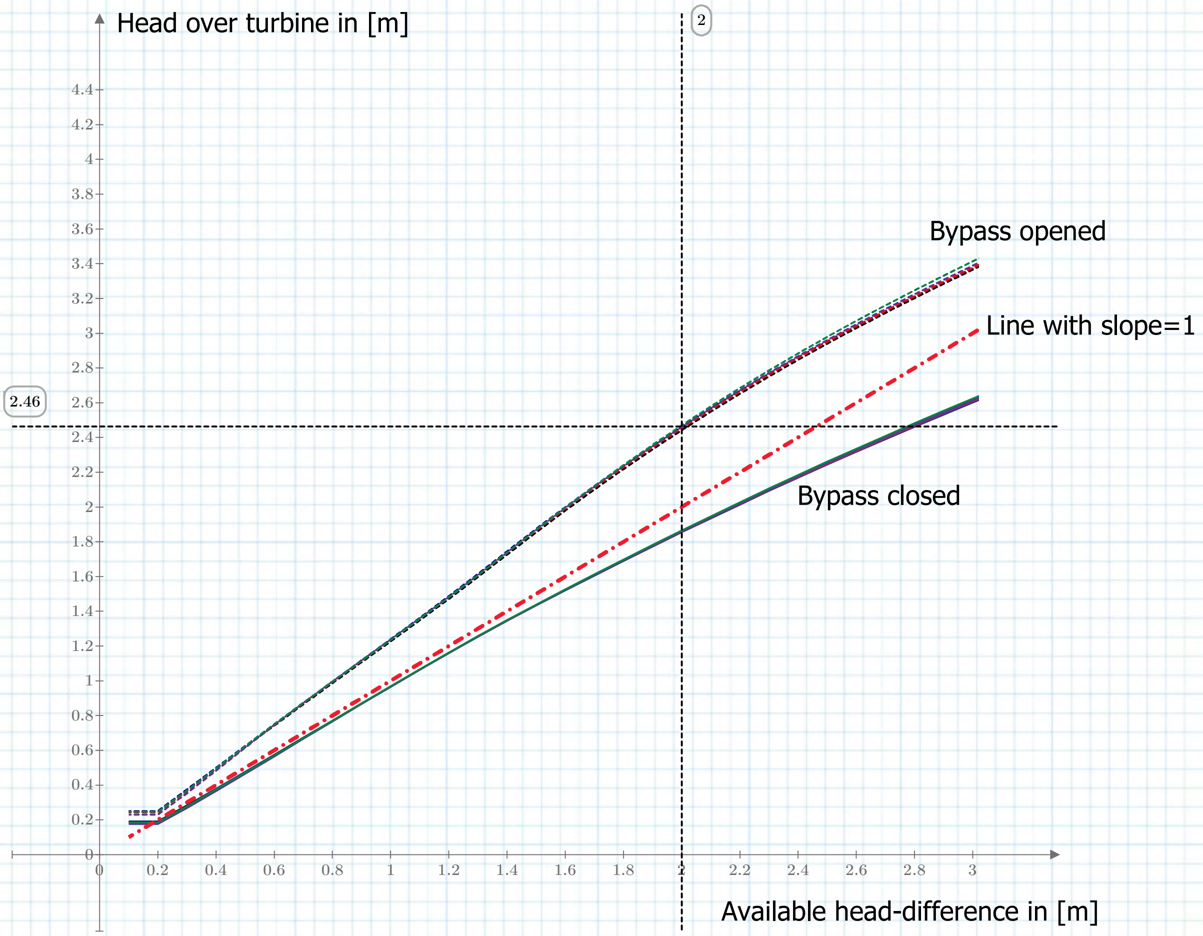
Bypass discharge interpolated curves:



System discharge interpolated curves:



Head over turbine, both with and without open bypass



Stepst in order of occurance:

1) Looping through all data values of available head.
(index "i").
2) Reducing available discharge with ecological minimum. (this flow is not available for the turbine)
3) looping through all design variants (index "j")
4) Determining number of working turbines n_on for a given available discharge, with a maximum of the number of turbines determined in the generic turbine chapter n_t
5) Determining available discharge per working turbine
$Qavt$
6) making first estimate of turbine discharge Qt with efficiency of 90%. If available discharge is less than would go through the turbine with the available head, then the discharge is obviously reduced to the available discharge.
7) Determine head over turbine ΔHt for this discharge and efficiency
8) Determine efficiency ηt from curves
9) Next iteration of turbine discharge Qt now with found efficiency ηt
10) head over turbine ΔHt with new efficiency ηt and new discharge Qt
11) redetermine efficiency and if necessary reloop discharge and head till the value stabilises.
(More iterations could be made, but choice was made to make just 1 iteration)
12) determine wether minimum head and discharge per turbine are exceeded and if so calculate total power output of the plant
$P = n\_on \cdot \eta t2 \cdot \rho \cdot g \cdot Qt2 \cdot \Delta Ht2$
Otherwise $P = 0\ kW$

(See next page for algorythm)

Define Power function
The approximate optimal speed ratios determined earlier have been used here as well (they were iteratively fine-tuned for these design variants):

```
$P\_t(Q\_ava, \Delta H\_ava) :=$
  $\eta t \leftarrow [0.9\ \ 0.9\ \ 0.9\ \ 0.9\ \ 0.9]^{T}$
  for $i \in 0..\text{rows}(Q\_ava) - 1$
    (1.) & (2.)  $Qa \leftarrow Q\_ava_{i} - Q\_eco$
    (3.)  for $j \in 0..\text{rows}(n\_t) - 1$
      if $Qa \geq Q\_t\_20pc_{j} \wedge \Delta H\_ava_{i} > 0.15\ m$
        (4.)  $r\_s_{i,j} \leftarrow r\_s\_f(\Delta H\_ava_{i}, j)$
        $Q\_s\_bp1 \leftarrow \max(Q\_t\_20pc_{j}, (Q\_cubs\_sys(\Delta H\_ava_{i}, j)))$
        (5.)  $Q\_bp\_bp1 \leftarrow \max(Q\_t\_20pc_{j}, Q\_cubs\_bp(\Delta H\_ava_{i}, j))$
        (6.)  $Q\_t\_bp1 \leftarrow Q\_s\_bp1 - Q\_bp\_bp1$
        $Qt1\_bp0 \leftarrow Q\_s2(\Delta H\_ava_{i}, C\_bp0_{j}, r\_s_{i,j}, \eta t_{j})^{T}$
        $n\_on\_bp1 \leftarrow \min\left(n\_t_{j}, \text{ceil}\left(\frac{Qa}{\max(Q\_t\_max\_g_{j}, Q\_s\_bp1)}\right)\right)$
        $n\_on\_bp0 \leftarrow \min\left(n\_on\_bp1, \text{ceil}\left(\frac{Qa - n\_on\_bp1 \cdot Q\_s\_bp1}{\max(Q\_t\_max\_g_{j}, Qt1\_bp0)}\right)\right)$
        $Qavt\_bp1 \leftarrow \frac{Qa - n\_on\_bp0 \cdot Qt1\_bp0}{n\_on\_bp1}$
        if $n\_on\_bp0 > 0$
          $Qavt\_bp0 \leftarrow \frac{Qa - n\_on\_bp1 \cdot Q\_s\_bp1}{n\_on\_bp0}$
        else
          $Qavt\_bp0 \leftarrow 0\ m^{3} \cdot s^{-1}$
        $Qt1\_bp1 \leftarrow \min(Qavt\_bp1, Q\_s\_bp1)$
        $Qt1\_bp0 \leftarrow \min(Qavt\_bp0, Qt1\_bp0)$
        $\Delta Ht\_bp1 \leftarrow H\_cubs\_t1(\Delta H\_ava_{i}, j)$
        $\Delta Ht\_bp0 \leftarrow H\_cubs\_t0(\Delta H\_ava_{i}, j)$
        $\eta t\_b1 \leftarrow \eta\_t(Qt1\_bp1, Q\_t\_max\_g_{j}, \Delta Ht\_bp1)$
        $\eta t\_b0 \leftarrow \eta\_t(Qt1\_bp0, Q\_t\_max\_g_{j}, \Delta Ht\_bp0)$
        $Qt\_bp1 \leftarrow min(Qavt\_bp1, Q\_s\_bp1 - Q\_bp\_bp1)$
        $Qt\_bp0 \leftarrow min\left(Qavt\_bp0, Q\_s2(\Delta H\_ava_{i}, C\_bp0_{j}, r\_s_{i,j}, \eta t\_b0)_{0,0}\right)$
        $\eta t\_b1 \leftarrow \eta\_t(Qt\_bp1, Q\_t\_max\_g_{j}, \Delta Ht\_bp1)$
        $\eta t\_b0 \leftarrow \eta\_t(Qt\_bp0, Q\_t\_max\_g_{j}, \Delta Ht\_bp0)$
        if $\Delta Ht\_bp1 \geq \Delta Hthres_{j}$
          $P1_{i,j} \leftarrow n\_on\_bp1 \cdot \eta t\_b1 \cdot \rho \cdot g \cdot Qt\_bp1 \cdot \Delta Ht\_bp1$
        else
          $P1_{i,j} \leftarrow 0\ kW$
        if $\Delta Ht\_bp0 \geq \Delta Hthres_{j}$
```

```
                $P2_{i,j} \leftarrow n\_on\_bp0 \cdot \eta t\_b0 \cdot \rho \cdot g \cdot Qt\_bp0 \cdot \Delta Ht\_bp0$
            else
                $P2_{i,j} \leftarrow 0\ kW$
        $P \leftarrow P1 + P2$
return $P$
```

Define energy function:
Since the discharge data is taken of daily averages it makes sense to assume each power value to be valid for each day. Hence the energy function is defined as follows:

```
$E\_plant(P) :=$
    $E_{\text{cols}(P)-1} \leftarrow 0\ kW \cdot hr$
    for $i \in 0 ..$ if $\text{rows}(P) = 0$
                        0
                     else
                        $\text{rows}(P) - 1$
        for $j \in 0 ..$ if $\text{cols}(P) = 0$
                            0
                         else
                            $\text{cols}(P) - 1$
            $E_j \leftarrow E_j + P_{i,j} \cdot 24\ hr$
    return $E$
```

Loading flow data from wet, dry and average year:

Looking at reference years

File path:

$FilePath := \text{“C:\textbackslash Users\textbackslash vanerps6413\textbackslash OneDrive – ARCADIS\textbackslash 061 Flow and waterlevel data\textbackslash 01 Di...}$

File name:

$FileName := \text{“002 – OUTPUT–MATHCAD – Datalink QH–t–series for E–calc – v01.xlsx”}$

Load data:

$DatasetWtDrAv := \text{READEXCEL}\left(\text{concat}\left(FilePath, FileName\right), \text{“Reference years!A2:F366”}, 0\right)$

Define data for each year:

$Qwet := DatasetWtDrAv^{\langle 0 \rangle} \cdot m^3 \cdot s^{-1}$

$Qdry := DatasetWtDrAv^{\langle 2 \rangle} \cdot m^3 \cdot s^{-1}$

$Qavg := DatasetWtDrAv^{\langle 4 \rangle} \cdot m^3 \cdot s^{-1}$

$Hwet := DatasetWtDrAv^{\langle 1 \rangle} \cdot m$

$Hdry := DatasetWtDrAv^{\langle 3 \rangle} \cdot m$

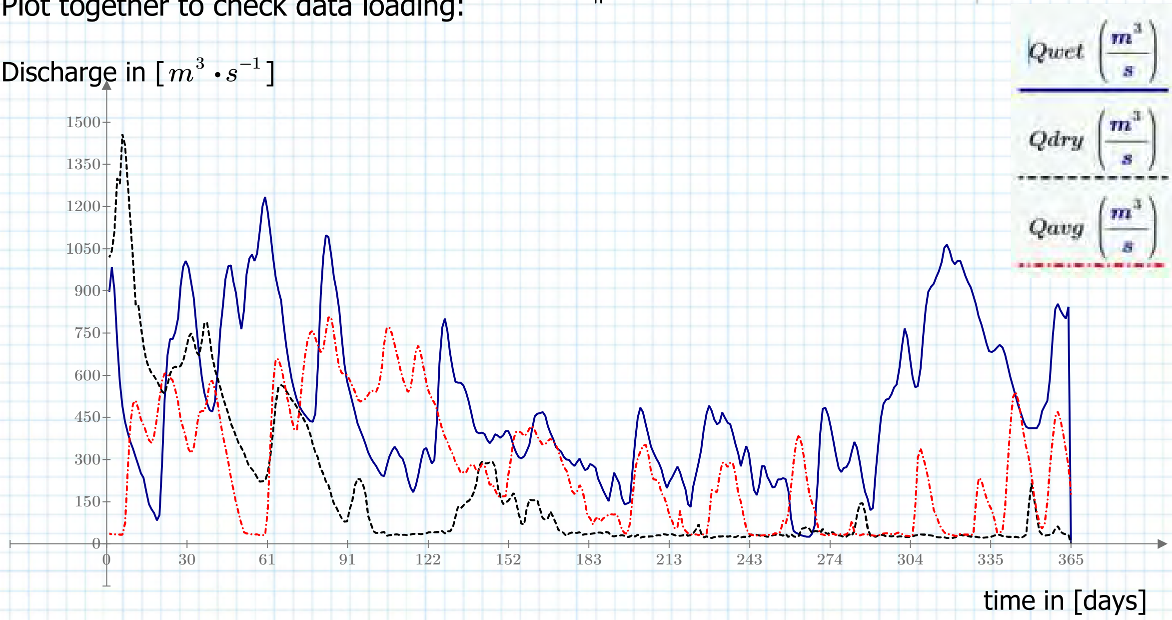
$Havg := DatasetWtDrAv^{\langle 5 \rangle} \cdot m$

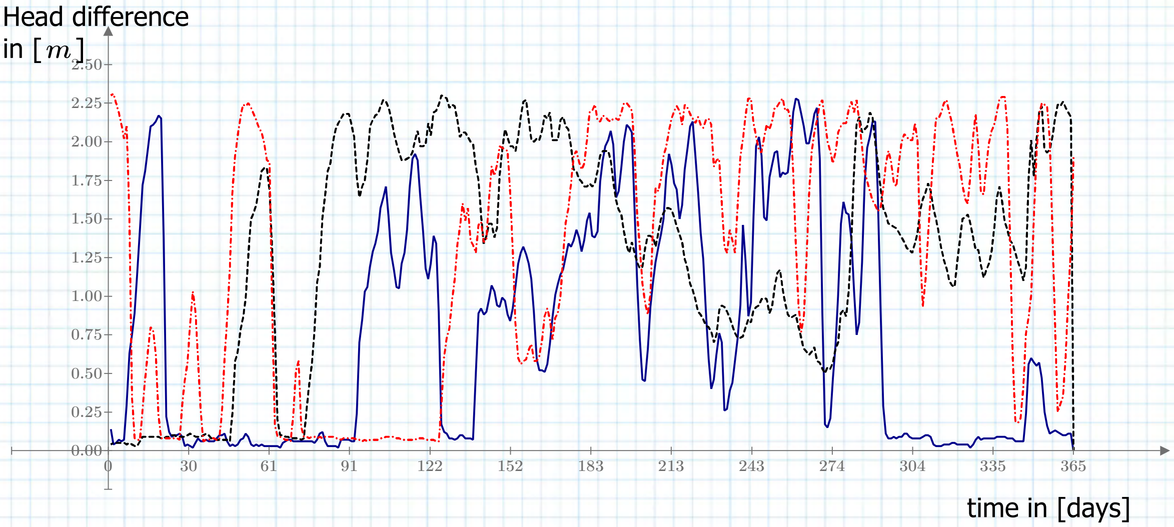
Define time axis and discharge area to be a vector:

```
tx := | $a_{\text{round}\left(\frac{yr}{day}\right)-1} \leftarrow \text{round}\left(\frac{yr}{day}\right)$
      | for $k \in 0, 1 .. \text{round}\left(\frac{yr}{day}\right) - 1$
      |     $a^{\widehat{k}} \leftarrow (k+1) \cdot day$
      | return $a$
```

$$= \begin{bmatrix} 1 \\ \vdots \end{bmatrix} day$$

Plot together to check data loading:





Plotting power output for fixed speed ratio for wet, dry and average year:

Capacity factor:

$$CF(E,P) := \frac{E}{P \cdot yr}$$

Average year:

$E\_avg := E\_plant(P\_t(Qavg, Havg))$

$E\_avg^{T} = [8087.31 \;\; 3993.58 \;\; 7104.73 \;\; 8097.06 \;\; 9255.97]\; MW \cdot hr$

$P\_t\_dv0\_avg := P\_t(Qavg, Havg)^{\langle 0 \rangle}$ $\max(P\_t\_dv0\_avg) = 3302.84\; kW$

$P\_t\_dv1\_avg := P\_t(Qavg, Havg)^{\langle 1 \rangle}$ $\max(P\_t\_dv1\_avg) = 1698.17\; kW$

$P\_t\_dv2\_avg := P\_t(Qavg, Havg)^{\langle 2 \rangle}$ $\max(P\_t\_dv2\_avg) = 2622.48\; kW$

$P\_t\_dv3\_avg := P\_t(Qavg, Havg)^{\langle 3 \rangle}$ $\max(P\_t\_dv3\_avg) = 3374.53\; kW$

$P\_t\_dv4\_avg := P\_t(Qavg, Havg)^{\langle 4 \rangle}$ $\max(P\_t\_dv4\_avg) = 3667.15\; kW$

Wet year:

$E\_wet := E\_plant(P\_t(Qwet, Hwet))$

$E\_wet^{T} = [8909.68 \;\; 3163.22 \;\; 6747.72 \;\; 8636.39 \;\; 9984.66]\; MW \cdot hr$

$P\_t\_dv0\_wet := P\_t(Qwet, Hwet)^{\langle 0 \rangle}$ $\max(P\_t\_dv0\_wet) = 3673.22\; kW$

$P\_t\_dv1\_wet := P\_t(Qwet, Hwet)^{\langle 1 \rangle}$ $\max(P\_t\_dv1\_wet) = 1684.39\; kW$

$P\_t\_dv2\_wet := P\_t(Qwet, Hwet)^{\langle 2 \rangle}$ $\max(P\_t\_dv2\_wet) = 2874.48\; kW$

$P\_t\_dv3\_wet := P\_t(Qwet, Hwet)^{\langle 3 \rangle}$ $\max(P\_t\_dv3\_wet) = 3510.43\; kW$

$P\_t\_dv4\_wet := P\_t(Qwet, Hwet)^{\langle 4 \rangle}$ $\max(P\_t\_dv4\_wet) = 3977.1\; kW$

Dry year:

$E\_dry := E\_plant(P\_t(Qdry, Hdry))$

$E\_dry^{T} = [4914.15 \;\; 2922.11 \;\; 4657.97 \;\; 5075.82 \;\; 5688.57]\; MW \cdot hr$

$P\_t\_dv0\_dry := P\_t(Qdry, Hdry)^{\langle 0 \rangle}$ $\max(P\_t\_dv0\_dry) = 3299.74\; kW$

$P\_t\_dv1\_dry := P\_t(Qdry, Hdry)^{\langle 1 \rangle}$ $\max(P\_t\_dv1\_dry) = 1699.46\; kW$

$P\_t\_dv2\_dry := P\_t(Qdry, Hdry)^{\langle 2 \rangle}$ $\max(P\_t\_dv2\_dry) = 2908.94\; kW$

$P\_t\_dv3\_dry := P\_t(Qdry, Hdry)^{\langle 3 \rangle}$ $\max(P\_t\_dv3\_dry) = 3370.94\; kW$

$P\_t\_dv4\_dry := P\_t(Qdry, Hdry)^{\langle 4 \rangle}$ $\max(P\_t\_dv4\_dry) = 3562.69\; kW$

Rated power estimation:

$P\_rated\_dv0 := \max(\max(P\_t\_dv0\_avg), \max(P\_t\_dv0\_dry), \max(P\_t\_dv0\_wet)) = 3673\ \mathbf{kW}$

$P\_rated\_dv1 := \max(\max(P\_t\_dv1\_avg), \max(P\_t\_dv1\_dry), \max(P\_t\_dv1\_wet)) = 1699\ \mathbf{kW}$

$P\_rated\_dv2 := \max(\max(P\_t\_dv2\_avg), \max(P\_t\_dv2\_dry), \max(P\_t\_dv2\_wet)) = 2909\ \mathbf{kW}$

$P\_rated\_dv3 := \max(\max(P\_t\_dv3\_avg), \max(P\_t\_dv3\_dry), \max(P\_t\_dv3\_wet)) = 3510\ \mathbf{kW}$

$P\_rated\_dv4 := \max(\max(P\_t\_dv4\_avg), \max(P\_t\_dv4\_dry), \max(P\_t\_dv4\_wet)) = 3977\ \mathbf{kW}$

$CF\_avg\_dv0 := CF\left(E\_avg_0, P\_rated\_dv0\right) = 25.12\%$

$CF\_avg\_dv1 := CF\left(E\_avg_1, P\_rated\_dv1\right) = 26.81\%$

$CF\_avg\_dv2 := CF\left(E\_avg_2, P\_rated\_dv2\right) = 27.86\%$

$CF\_avg\_dv3 := CF\left(E\_avg_3, P\_rated\_dv3\right) = 26.31\%$

$CF\_avg\_dv4 := CF\left(E\_avg_4, P\_rated\_dv4\right) = 26.55\%$

$CF\_wet\_dv0 := CF\left(E\_wet_0, P\_rated\_dv0\right) = 27.67\%$

$CF\_wet\_dv1 := CF\left(E\_wet_1, P\_rated\_dv1\right) = 21.23\%$

$CF\_wet\_dv2 := CF\left(E\_wet_2, P\_rated\_dv2\right) = 26.46\%$

$CF\_wet\_dv3 := CF\left(E\_wet_3, P\_rated\_dv3\right) = 28.07\%$

$CF\_wet\_dv4 := CF\left(E\_wet_4, P\_rated\_dv4\right) = 28.64\%$

$CF\_dry\_dv0 := CF\left(E\_dry_0, P\_rated\_dv0\right) = 15.26\%$

$CF\_dry\_dv1 := CF\left(E\_dry_1, P\_rated\_dv1\right) = 19.62\%$

$CF\_dry\_dv2 := CF\left(E\_dry_2, P\_rated\_dv2\right) = 18.27\%$

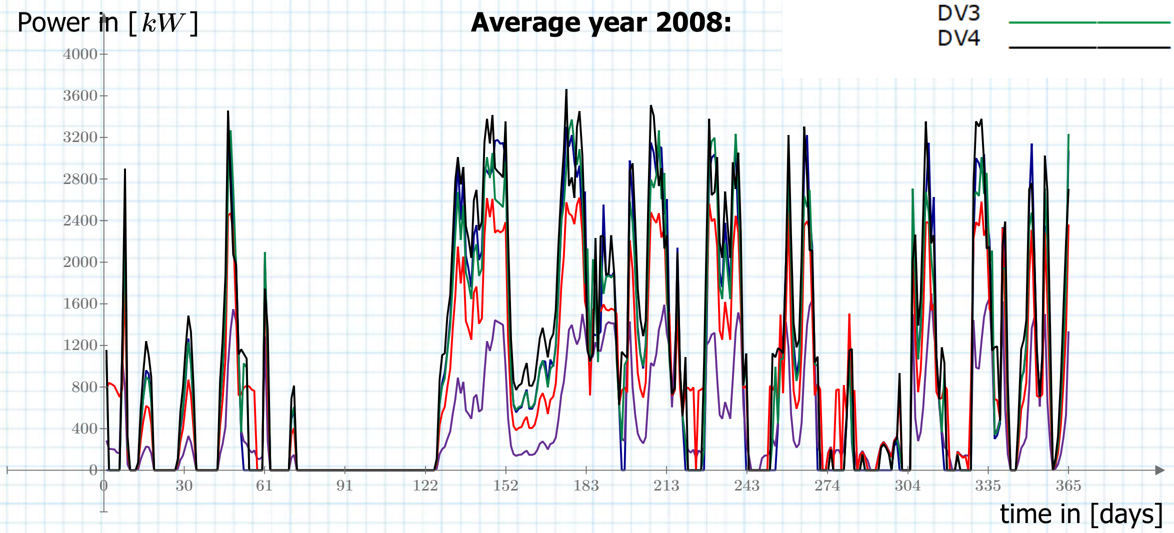
$CF\_dry\_dv3 := CF\left(E\_dry_3, P\_rated\_dv3\right) = 16.5\%$

$CF\_dry\_dv4 := CF\left(E\_dry_4, P\_rated\_dv4\right) = 16.32\%$

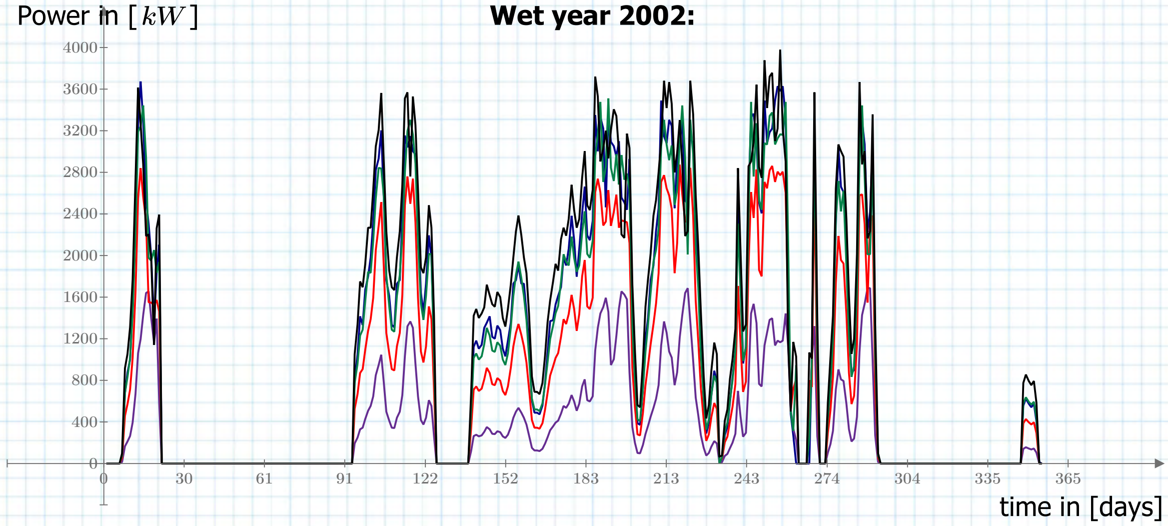
Plots power with fixed speed ratio:

Legend:
DV0 ——
DV1 ——
DV2 ——
DV3 ——
DV4 ——

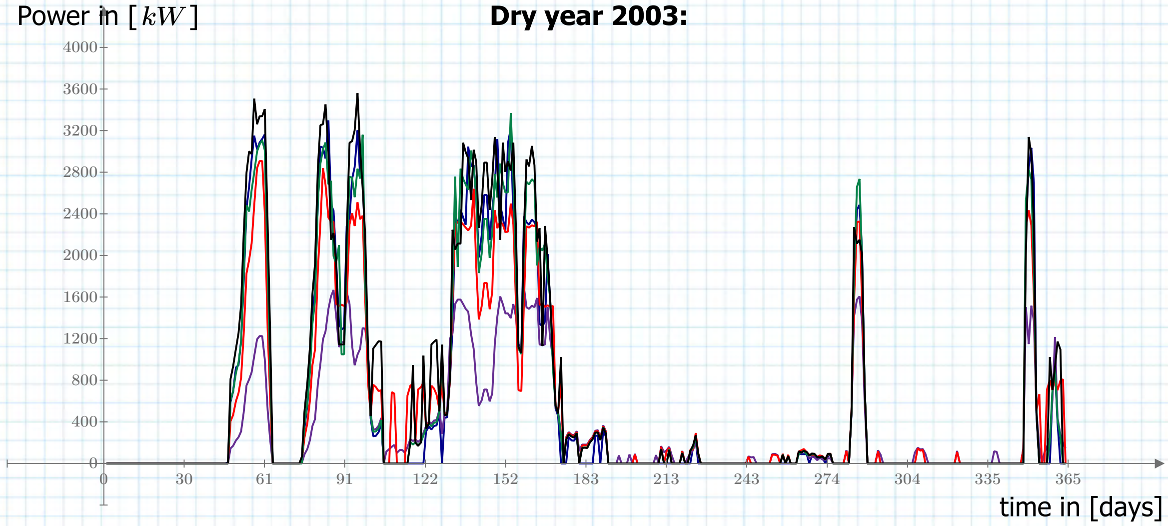
Average year 2008:



Wet year 2002:



Dry year 2003:



Energy calculation (continued)

Load 10 and 30 year data:

$$MH10y := \text{READEXCEL}\left(\text{concat}\left(FilePath, FileName\right), \text{“MH10y!A1:NB10”}, 0\right)$$
$$MQ10y := \text{READEXCEL}\left(\text{concat}\left(FilePath, FileName\right), \text{“MQ10y!A1:NB10”}, 0\right)$$

$$H10y := \left\| \begin{array}{l} H \leftarrow \left(MH10y^{\mathrm{T}}\right)^{\langle 0 \rangle} \\ \text{for } i \in 1 .. \text{rows}\left(MH10y\right) - 1 \\ \quad \left\| \; H \leftarrow \text{stack}\left(H, \left(MH10y^{\mathrm{T}}\right)^{\langle i \rangle}\right) \right. \\ H \end{array} \right. \qquad Q10y := \left\| \begin{array}{l} Q \leftarrow \left(MQ10y^{\mathrm{T}}\right)^{\langle 0 \rangle} \\ \text{for } i \in 1 .. \text{rows}\left(MQ10y\right) - 1 \\ \quad \left\| \; Q \leftarrow \text{stack}\left(Q, \left(MQ10y^{\mathrm{T}}\right)^{\langle i \rangle}\right) \right. \\ Q \end{array} \right.$$

$$E\_opt\_10y := E\_plant\left(P\_t\left(Q10y \cdot \frac{m^3}{s}, H10y \cdot m\right)\right)^{\mathrm{T}}$$
$$E\_opt\_10y = \begin{bmatrix} 63071.8 & 31838.12 & 55076.98 & 63715.68 & 73336.47 \end{bmatrix} MW \cdot hr$$
$$E\_opt\_10y\_avg := \frac{E\_opt\_10y}{10}$$

$$MH30y := \text{READEXCEL}\left(\text{concat}\left(FilePath, FileName\right), \text{“MH30y!A1:NB30”}, 0\right)$$
$$MQ30y := \text{READEXCEL}\left(\text{concat}\left(FilePath, FileName\right), \text{“MQ30y!A1:NB30”}, 0\right)$$

$$H30y := \left\| \begin{array}{l} H \leftarrow \left(MH30y^{\mathrm{T}}\right)^{\langle 0 \rangle} \\ \text{for } i \in 1 .. \text{rows}\left(MH30y\right) - 1 \\ \quad \left\| \; H \leftarrow \text{stack}\left(H, \left(MH30y^{\mathrm{T}}\right)^{\langle i \rangle}\right) \right. \\ H \end{array} \right. \qquad Q30y := \left\| \begin{array}{l} Q \leftarrow \left(MQ30y^{\mathrm{T}}\right)^{\langle 0 \rangle} \\ \text{for } i \in 1 .. \text{rows}\left(MQ30y\right) - 1 \\ \quad \left\| \; Q \leftarrow \text{stack}\left(Q, \left(MQ30y^{\mathrm{T}}\right)^{\langle i \rangle}\right) \right. \\ Q \end{array} \right.$$

$$E\_opt\_30y := E\_plant\left(P\_t\left(Q30y \cdot \frac{m^3}{s}, H30y \cdot m\right)\right)^{\mathrm{T}}$$
$$E\_opt\_30y = \begin{bmatrix} 196946.13 & 99364.67 & 172237.8 & 199616.77 & 228082.1 \end{bmatrix} MW \cdot hr$$
$$E\_opt\_30y\_avg := \frac{E\_opt\_30y}{30}$$

End results energy production:

$$E\_avg^{\mathrm{T}} = \begin{bmatrix} 8087.31 & 3993.58 & 7104.73 & 8097.06 & 9255.97 \end{bmatrix} MW \cdot hr$$
$$E\_opt\_10y\_avg = \begin{bmatrix} 6307.18 & 3183.81 & 5507.7 & 6371.57 & 7333.65 \end{bmatrix} MW \cdot hr$$
$$E\_opt\_30y\_avg = \begin{bmatrix} 6564.87 & 3312.16 & 5741.26 & 6653.89 & 7602.74 \end{bmatrix} MW \cdot hr$$

APPENDIX 18 – OUTPUT ENERGY CALCULATION AST

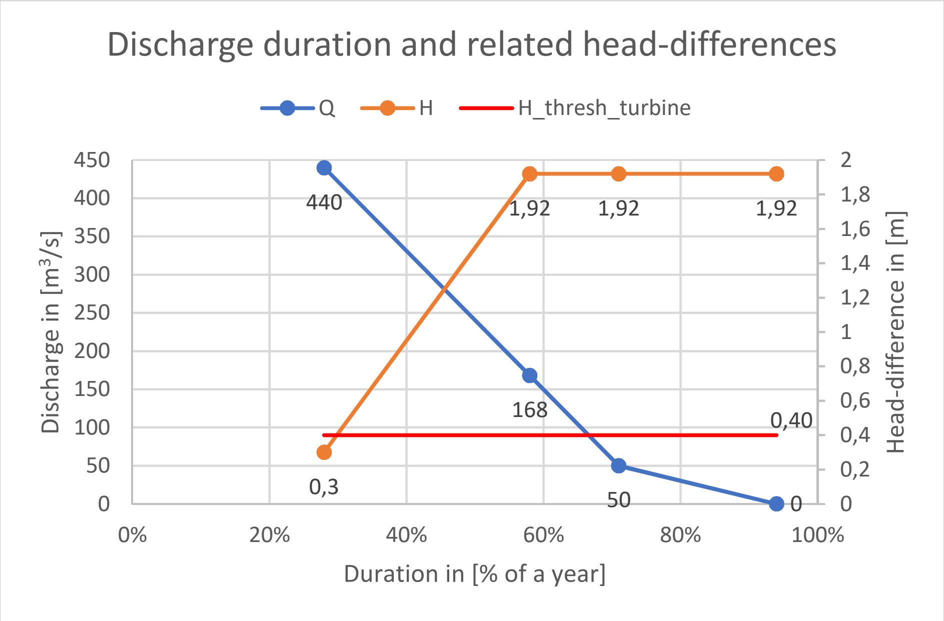
– see inserted page(s) behind this page –

Energy production estimates for AST			
Description	**Quantity**	**Unit**	**Value**
Max available design discharge	Q_avail_d	m^3/s	440,0
Maximum nr. of turbines	N_turb	-	36
Max discharge	Q_100%	m^3/s	10,0
Min discharge	Q_min	m^3/s	1,0
Rated power (dH=1,92 Q=Q_100%)	P_t_rated	kW	156
Results:			
Maximum available energy	E_available_max	GWh	11,05
Energy production N=1	E_annual n= 1	GWh	0,63
Energy production N=5	E_annual n= 5	GWh	2,55
Energy production N=11	E_annual n= 11	GWh	4,46
Energy production N=23	E_annual n= 23	GWh	6,77
Energy production N=30	E_annual n= 30	GWh	7,32
Energy production N=33	E_annual n= 33	GWh	7,44
Energy production N=36	E_annual n= 36	GWh	7,50
Capacity factor:			
Capacity factor for N=1	CF_N= 1	%	45,9%
Capacity factor for N=5	CF_N= 5	%	37,4%
Capacity factor for N=11	CF_N= 11	%	29,7%
Capacity factor for N=23	CF_N= 23	%	21,6%
Capacity factor for N=30	CF_N= 30	%	17,9%
Capacity factor for N=33	CF_N= 33	%	16,5%
Capacity factor for N=36	CF_N= 36	%	15,3%

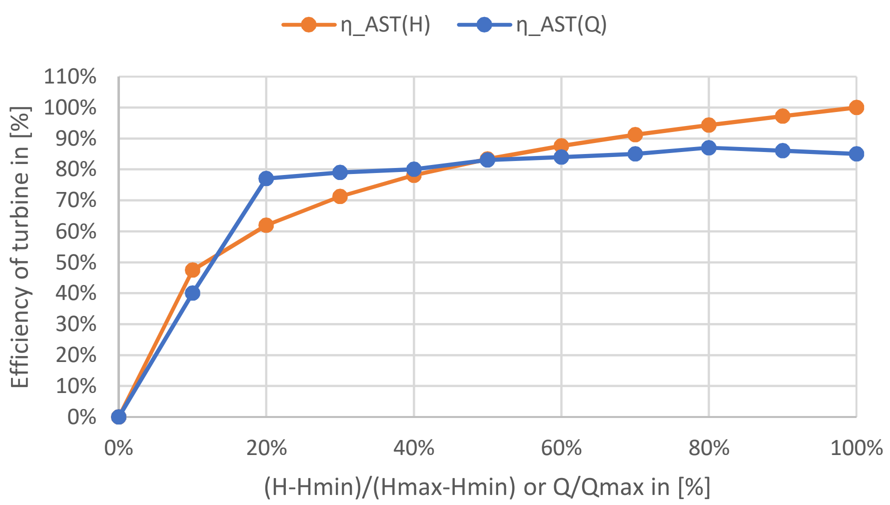
Used formulae:

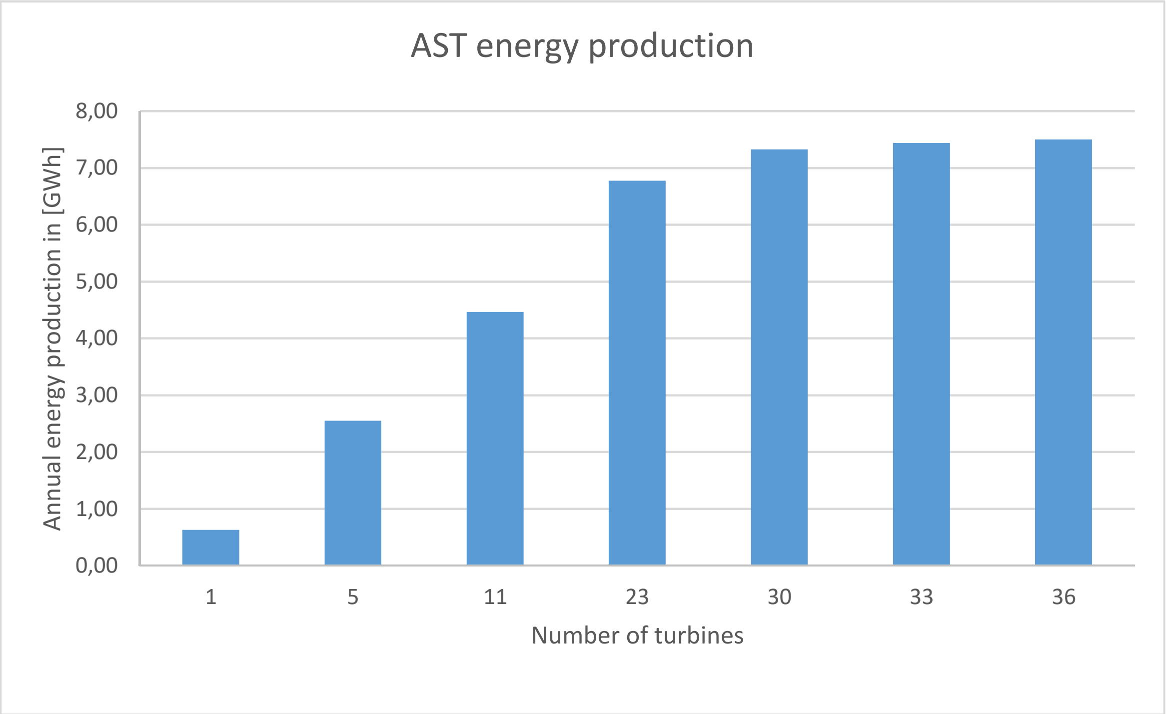
$$P_t = \eta * \rho * g * \Delta H * Q$$

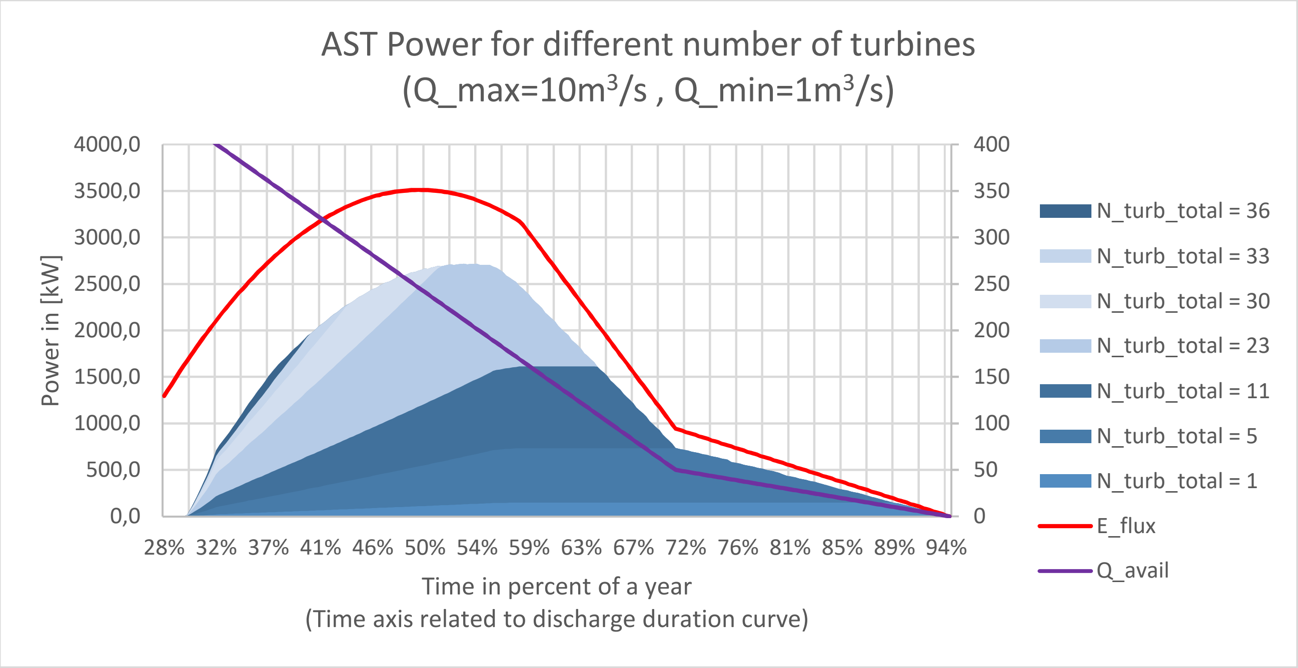
$$CF = \frac{E_{annual}}{t_{year} * P_{rated}} = \frac{t_{full-load}}{t_{year}}$$



P_total (kW)
156
778
1.712
3.580
4.670
5.137
5.604







- END OF APPENDICES –