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DOI
10.1007/978-3-030-55874-1_70

Publication date
2021

Document Version
Final published version

Published in
Numerical Mathematics and Advanced Applications, ENUMATH 2019 - European Conference

Citation (APA)
Markensteijn, A. S., Romate, J. E., & Vuik, C. (2021). Scaling of the Steady-State Load Flow Equations for Multi-Carrier Energy Systems. In F. J. Vermolen, & C. Vuik (Eds.), *Numerical Mathematics and Advanced Applications, ENUMATH 2019 - European Conference* (pp. 713-721). (Lecture Notes in Computational Science and Engineering; Vol. 139). Springer. https://doi.org/10.1007/978-3-030-55874-1_70

Scaling of the Steady-State Load Flow Equations for Multi-Carrier Energy Systems

A. S. Markensteijn, J. E. Romate, and C. Vuik

Abstract Coupling single-carrier networks (SCNs) into multi-carrier energy systems (MESs) has recently become more important. Steady-state load flow analysis of energy systems leads to a system of nonlinear equations, which is usually solved using the Newton-Raphson method (NR). Due to various physical scales within a SCN, and between different SCNs in a MES, scaling might be needed to solve the nonlinear system. In single-carrier electrical networks, per unit scaling is commonly used. However, in the gas and heat networks, various ways of scaling or no scaling are used. This paper presents a per unit system and matrix scaling for load flow models for a MES consisting of gas, electricity, and heat. The effect of scaling on NR is analyzed. A small example MES is used to demonstrate the two scaling methods. This paper shows that the per unit system and matrix scaling are equivalent, assuming infinite precision. In finite precision, the example shows that the NR iterations are slightly different for the two scaling methods. For this example, both scaling methods show the same convergence behavior of NR in finite precision.

1 Introduction

Multi-carrier energy systems (MESs) have become more important over the years, as the need for efficient, reliable and low carbon energy systems increases. In these energy systems, different energy carriers, such as gas, electricity, and heat, interact

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F. J. Vermolen, C. Vuik (eds.), *Numerical Mathematics and Advanced Applications ENUMATH 2019*, Lecture Notes in Computational Science and Engineering 139,
https://doi.org/10.1007/978-3-030-55874-1_70

with each other leading to one integrated energy network. An important tool for the design and operation of energy systems is steady-state load flow (LF) analysis of the energy networks. LF models for single-carrier networks (SCNs) have been widely studied, but only recently LF models for MESs have been proposed.

Steady-state LF analysis leads to a system of nonlinear equations, which is usually solved using the Newton-Raphson method (NR). The quantities in the LF equations can be several orders of magnitude apart, such that scaling might be needed to solve the nonlinear system.

In single-carrier electricity networks, per unit scaling is generally used (e.g. [1]). In the per unit system, every variable and parameter is scaled to obtain dimensionless equations. In gas and heat networks, a more ad hoc approach to scaling is used. In MESs, the SCN variables, having various scales, are combined. This requires a consistent way to scale the LF equations for MES. In [2], the per unit system is extended to the heat network for consistency throughout an example MES. To the best of the authors knowledge, there is no equivalent of the per unit system for a gas network.

Another option to scale the system of nonlinear LF equations is by scaling the equations and variables using scaling matrices. Even though this method is a well known scaling method, it is not generally used for LF analysis in any of the SCNs.

We introduce a per unit scaling for MESs consisting of gas, electricity, and heat, by extending the per unit scaling of an electricity network to gas and heat. We compare the per unit scaling with matrix scaling for NR, and show that they are equivalent when using the same base values. The advantages and disadvantages of both methods are discussed.

Using a small MES consisting of gas, electricity, and heat, we investigate the effect of the two scaling methods on the convergence of NR. Despite numerical (round-off) errors, both scaling methods show the same convergence behavior.

2 Steady-State Load Flow

An important tool for the design and operation of energy systems is steady-state LF analysis. The inflow and outflow of the energy system are assumed constant, and the network flows and potentials are determined by the LF equations. For instance, in a gas pipeline network, the gas inflow and outflow are assumed constant, and the gas flow in the pipes and the pressures at the start and end of the pipes are determined.

Energy systems are mathematically represented by a network or graph, which is a collection of nodes, connected by (directed) links. Flow enters the network through sources and leaves the network through sinks. This is represented by an open link connected to a single node only, called terminal link and terminal node respectively. For steady-state LF, the variables of interest are associated with the network nodes, links, or terminal links. Conservation of energy holds in all the single-carrier (SC) nodes. All SC (terminal) links representing a physical component have a link equation that relates link and nodal variables. SC nodes and coupling nodes

can have additional node equations that relate the (terminal) link variables of the links connected to that node. There are generally more variables than nodal and link equations. Therefore, some variables are assumed known, called the boundary conditions (BCs) of the network. Collecting all the nodal and link equations, some of which are nonlinear, into one system, and substituting the BCs, gives the system of LF equations:

$$\mathbf{F}(\mathbf{x}) = \mathbf{0} \tag{1}$$

with $\mathbf{F} \in \mathbb{R}^n$ the vector of (nonlinear) LF equations and $\mathbf{x} \in \mathbb{R}^n$ the vector of variables. For specific LF models, see for instance [1] for electricity, [3] for gas, and [4] for MESs.

3 Scaling

The parameters and the dependent and independent variables in the LF equations can be several orders of magnitude apart, even within one SCN. For instance, gas flow $\sim 1\,\mathrm{kg\,s^{-1}}$ whereas pressure $\sim 10^5\,\mathrm{Pa}$. These different scales might result in issues with solving the system of nonlinear equations, see Sect. 4. Normalizing or scaling the variables and parameters for electricity networks is commonly done, and is called the per unit system (e.g. [1]). Another option is to scale the system of equations and the independent variables by scaling matrices, without scaling the equation parameters. To investigate the effect of scaling on the system of equations, we consider dimensional analysis.

The LF equations are a mathematical representation of a physical phenomenon. Physical quantities are not just numerical values, they also have a dimension and a unit measure associated with them. For instance, the diameter D of a gas pipe has dimension 'length', and could have a unit measure of 1 cm and a value of 15. Denoting the unit measure of length by l and the value of D by k, we can write $D = kl$. We can scale D by changing the unit measure with a scaling factor $k_l \in \mathbb{R}$, and generally $k_l > 0$, such that $l \to k_l l$. Using this new unit measure for D will change the unit measure and the value (to k/k_l), but not the dimension.

Based on the logic as laid out for dimensional analysis in for instance [5], quantities can only be combined in limited ways. Quantities can be multiplied, which multiplies the dimension in the same way. To add two quantities, they must have the same dimension and the same unit measure. Other functional relations are only possible if all arguments are dimensionless. For instance, if $f(x) = \sin(x)$, then both $f(x)$ and x must be dimensionless. Using these concepts recursively, a function of multiple arguments can be made. An equation that satisfies these properties is called 'complete' in [5]. A consequence is that the algebraic form of the equation is unit independent. That is, if the unit measure of any dimension is changed, the algebraic form of the equation remains the same. However, the value

of the function might be changed, just like the value of some of the quantities is changed. This can be seen as follows.

Since two (or more) terms can only be added if the terms have the same dimension and unit measure, we can limit ourselves to functions consisting of only one term. Furthermore, for dimensionless quantities, or for a dimensionless group consisting of the power product of some quantities, the changes in unit measures cancel out. Hence, we only need to consider the change in value of functions of the form $f(y_1, \ldots, y_n) = y_1^{a_1} \cdots y_n^{a_n}$. We can assume that all y_i have a single (primary) dimension. Scaling each y_i by changing the unit measures of the primary dimensions by a factor k_i gives

$$\begin{aligned} f(y_1, \ldots, y_n) \to f(k_1 y_1, \ldots, k_n y_n) &= \left(k_1^{a_1} \cdots k_n^{a_n}\right)\left(y_1^{a_1} \cdots y_n^{a_n}\right) \\ &= \left(k_1^{a_1} \cdots k_n^{a_n}\right) f(y_1, \ldots, y_n) \end{aligned} \tag{2}$$

such that f is scaled by a power product of the unit measure scaling factors.

An equation describing a physical model does not need to be complete for the model to be valid. In fact, the commonly used form of the link equation for a transmission line in an electrical network is not a complete equation. It contains terms $\sin \delta_k$ and $\cos \delta_k$, with δ_k the voltage angles difference of link k. Based on the logic provided above, δ_i and δ_j should be dimensionless. However, they have dimension 'plane angle'. The link equation can be turned into a complete equation by using δ_k/δ_0 instead of δ_k, with a δ_0 reference angle.

3.1 Per Unit System

The per unit system is commonly used in electricity networks, and extended in [2] to the heat network. We consider a more general extension of the per unit system to heat and gas networks. In the per unit system, a quantity x is scaled by a base value:

$$x_{\text{p.u.}} = \frac{x_a}{x_b} \tag{3}$$

Here, x_a is the unscaled or actual quantity, usually in S.I. units, x_b is a chosen base value with the same dimension as x_a, and $x_{\text{p.u.}}$ is the scaled quantity. The scaled quantity is dimensionless but is given p.u. as unit. Hence, the scaled quantity is also called the per unit quantity or value.

There are two main differences between the per unit system and changing the unit measures. The first is that the base value has a dimension, unlike the scaling factor of the unit measure. Second, only the unit measure scaling factors of the primary dimensions are chosen, whereas in the per unit system, the base value for derived quantities might be chosen. The first point has no consequence for the argumentation resulting in (2). However, the second point can lead to some difficulties. Since derived quantities are combinations of other quantities, and applying the same

logic that resulted in a complete equation, only a limited set of base values can be specified. The base values for the other quantities then follow from dimensional analysis. The set of base values that can be specified is not unique, neither are the resulting base values of the other quantities. However, it is possible to find a set of base values such that the equation remains a complete equation. For such a set of base values, the argumentation resulting in (2) is still valid. We can now look at the effect of the per unit system of the equation.

Suppose we have a (complete) equation of the form $f\left(\mathbf{x_a},\ \mathbf{p_a}\right)$, with $\mathbf{x_a} \in \mathbb{R}^n$ all the variables, and $\mathbf{p_a} \in \mathbb{R}^m$ all other quantities, dimensionless or not, appearing in the algebraic form of f. We take a set of base values $b_1, \ldots, b_k$, with $k \leq n + m$, and scale each $x \in \mathbf{x_a}$ and $p \in \mathbf{p_a}$ according to (3), with x_b and p_b power products of the base values $b_1, \cdots, b_k$. If the base values are chosen such that the equation f remains a complete equation after scaling, the equation is scaled according to:

$$f\left(\mathbf{x_a},\ \mathbf{p_a}\right) = \left[b_1^{\alpha_1} \cdots b_k^{\alpha_k}\right] f\left(\mathbf{x}_{\text{p.u.}},\ \mathbf{p}_{\text{p.u.}}\right) \tag{4}$$

Usually, only the variables are explicitly denoted as arguments for the function, such that $f\left(\mathbf{x_a},\ \mathbf{p_a}\right)$ is written as $f\left(\mathbf{x_a}\right)$ and $f\left(\mathbf{x}_{\text{p.u.}},\ \mathbf{p}_{\text{p.u.}}\right)$ as $f_{\text{p.u.}}\left(\mathbf{x}_{\text{p.u.}}\right)$. For the scaled equation we then find

$$f\left(\mathbf{x_a}\right) = \left[b_1^{\alpha_1} \cdots b_k^{\alpha_k}\right] f\left(\mathbf{x}_{\text{p.u.}},\ \mathbf{p}_{\text{p.u.}}\right) := f_b\, f_{\text{p.u.}}\left(\mathbf{x}_{\text{p.u.}}\right) \tag{5}$$

where $f_b = \left[b_1^{\alpha_1} \cdots b_k^{\alpha_k}\right]$ is called the base value of the function f. That is, for a suitable set of base values, the same expression of the LF equations can be used for both the unscaled and per unit quantities, and all independent variables and all LF equations can be scaled to similar orders of magnitude.

For an electricity network, the base values of the voltage amplitude and the power are chosen. The base values of the other variable (current amplitude) and of the parameters of the LF equations (admittance) are determined by the requirement that the LF equations remain a complete equation, using dimensional analysis (e.g. [1]).

The per unit system is then easily extended to the gas and heat SCN, and to a MES. We choose the base values for pressure and flow in the gas network, and for pressure, mass flow, temperature, and power in the heat network. The base values of the other variables and parameters are determined based on dimensional analysis. For the couplings in a MES, we choose the base values of the power of every carrier involved in the coupling, and again determine the base values of the other quantities according to dimensional analysis.

The advantage of scaling derived quantities instead of scaling primary dimensions becomes clear when considering transformers in an electrical network, or compressors in a gas network. These components change the voltage or pressure level, and their link equation has the general form $f(x_1,\ x_2,\ r) = x_1 - r x_2 = 0$, with x_1 and x_2 the voltages or pressures, and r some ratio. Since x_1 and x_2 have the same dimension, r must be dimensionless. Hence, changing the unit measures will scale the values of x_1 and x_2 with the same factor, and will leave r unscaled.

In practice, x_1 and x_2 might be orders of magnitude apart when using the same unit measure. In the per unit system, it is possible to use a different base value for x_1 and x_2, such that both $x_1 \sim 1$ p.u. and $x_2 \sim 1$ p.u. Note that the scaled x_1 and x_2 now have different unit measures, despite both of their units being denoted by p.u. Due to the requirement for addition of dimensional quantities, r needs to scaled with $r_b = {}^{(x_1)_b}/_{(x_2)_b}$.

3.2 Matrix Scaling

Another option is to scale the independent variables and the equations only, using scaling matrices [6]. Taking non-singular matrices T_x, $T_F \in \mathbb{R}^{n \times n}$, the scaled variables $\hat{\mathbf{x}}$ and scaled equations $\hat{\mathbf{F}}$ are given by:

$$\hat{\mathbf{x}} = T_x \mathbf{x} \tag{6}$$

$$\hat{\mathbf{F}}\left(\hat{\mathbf{x}}\right) = T_F \mathbf{F}\left(T_x^{-1}\hat{\mathbf{x}}\right) = T_F \mathbf{F}\left(\mathbf{x}\right) \tag{7}$$

Unlike the per unit scaling, scaling with matrices requires us to also choose the scaling for the equations instead of only for the variables. However, per unit scaling requires base values for all parameters in every equation. Furthermore, matrix scaling is generally easier to implement than per unit scaling.

If we take T_x as a diagonal matrix with $(T_x)_{ii} = (x_b)_i$, where $(x_b)_i$ the base value of $x_i \in \mathbf{x}$ used in per unit scaling, it follows from (5) that T_F is a diagonal matrix with $(T_F)_{ii} = (f_b)_i$, where $(f_b)_i$ the base value of $f_i \in \mathbf{F}$ found in per unit scaling. Therefore, in infinite precision, the per unit scaling and matrix scaling will result in the same scaled system of equations $\hat{\mathbf{F}}$ and the same scaled variables $\hat{\mathbf{x}}$. Hence, the per unit system and matrix scaling are said to be equivalent.

4 Newton-Raphson

We use the Newton-Raphson method (NR) to solve the system of non-linear LF equations (1). The iteration scheme in multiple dimensions is given by [6]:

$$J\left(\mathbf{x}^k\right)\mathbf{s}^k = -\mathbf{F}\left(\mathbf{x}^k\right), \text{ with } \mathbf{x}^{k+1} = \mathbf{x}^k + \mathbf{s}^k \tag{8}$$

$J\left(\boldsymbol{x}^k\right)$ is the Jacobian matrix. We take $e^k = ||\mathbf{F}\left(\mathbf{x}^k\right)||_2$ as error of NR at iteration k, with $||\cdot||_2$ the 2-norm. For the stopping criterion we take $e^k \leq \tau$ for some chosen tolerance τ. If the equations in $\mathbf{F}$ are several orders of magnitudes apart, the smaller ones might be ignored during NR, or NR might not convergence to a

solution since the larger ones will never reach the required tolerance. Therefore, we scale all equations in $\mathbf{F}$ to be of same order of magnitude.

Since per unit scaling and matrix scaling are equivalent, we only consider matrix scaling. The iteration scheme of NR is adjusted to:

$$\hat{J}\left(\hat{\mathbf{x}}^k\right)\hat{\mathbf{s}}^k = -\hat{\mathbf{F}}\left(\hat{\mathbf{x}}^k\right), \text{ with } \hat{\mathbf{x}}^{k+1} = \hat{\mathbf{x}}^k + \hat{\mathbf{s}}^k \tag{9}$$

It is straightforward to show that $\hat{J}\left(\hat{\mathbf{x}}\right) = T_F J\left(\mathbf{x}\right) T_x^{-1}$. Then, for the scaled step it holds that $\hat{\mathbf{s}}^k = -T_x J\left(\mathbf{x}\right)^{-1} \mathbf{F}\left(\mathbf{x}^k\right) = T_x \mathbf{s}^k$, meaning that scaling does not affect the NR iterations. We take $\hat{e}^k = ||\hat{\mathbf{F}}^k||_2 = ||T_F \mathbf{F}\left(T_x^{-1}\hat{\mathbf{x}}\right)||_2$ as error of the scaled NR. Since all $\hat{f}_i \in T_F \mathbf{F}$ are roughly of the same order of magnitude, we take $\hat{e}^k \leq \tau$ as stopping criterion.

5 Numerical Results

The previous analysis only holds in infinite precision. In finite precision, an NR step might be affected. In the per unit system, the scaled variables and parameters are plugged into (1) to obtain the scaled system of equations, denoted by $\mathbf{F}_{\text{p.u.}}$. With matrix scaling, the unscaled variables and parameters are used in (1). Then, the scaled system of equations is given by $\hat{\mathbf{F}} = T_F \mathbf{F}\left(T_x^{-1}\hat{\mathbf{x}}\right)$. Due to round-off errors, generally $\mathbf{F}_{\text{p.u.}} \neq \hat{\mathbf{F}}$, even though $\mathbf{F}_{\text{p.u.}}$ and $\hat{\mathbf{F}}$ will be close. Similarly, $J_{\text{p.u.}} \neq \hat{J}$, such that $\hat{\mathbf{s}}^k \neq \mathbf{s}^k_{\text{p.u.}} \neq T_x \mathbf{s}^k$. We model a small MES to investigate the effect of finite precision on NR for the two different scaling options.

We consider the small MES shown in Fig. 1, and use the LF model as described in [4]. The resulting system of nonlinear equations is scaled using the per unit system and using matrix scaling. The resulting scaled systems $\mathbf{F}_{\text{p.u.}}$ and $\hat{\mathbf{F}}$ are solved using NR as described in Sect. 4, with a tolerance of $\tau = 10^{-6}$. For comparison, we also solve the unscaled system using NR. Denoting the unscaled system by $\mathbf{F}$, the (unscaled) error at each NR iteration is given by $e^k = ||\mathbf{F}^k\left(\mathbf{x}\right)||_2$. To compare with the error of NR for the scaled systems, we calculate the scaled error of the unscaled NR iteration by $\tilde{e}^k = ||T_F \mathbf{F}^k\left(\mathbf{x}\right)||_2$. Note that $\tilde{e}^k$ is different from the error $\hat{e}^k = ||\hat{\mathbf{F}}^k\left(\hat{\mathbf{x}}\right)||_2 = ||T_F \mathbf{F}^k\left(T_x^{-1}\hat{\mathbf{x}}\right)||_2$ of scaled NR, since scaled NR uses the scaled update $\hat{\mathbf{s}}^k$ instead of $\mathbf{s}^k$.

Table 1 gives the errors for NR. We can see that the errors for the per unit scaling and the matrix equals are unequal, but close, to each other and to the error of unscaled NR. Hence, scaling affects NR in finite precision. In this example, this effect does not result in a significant difference between the solutions to the LF problem.

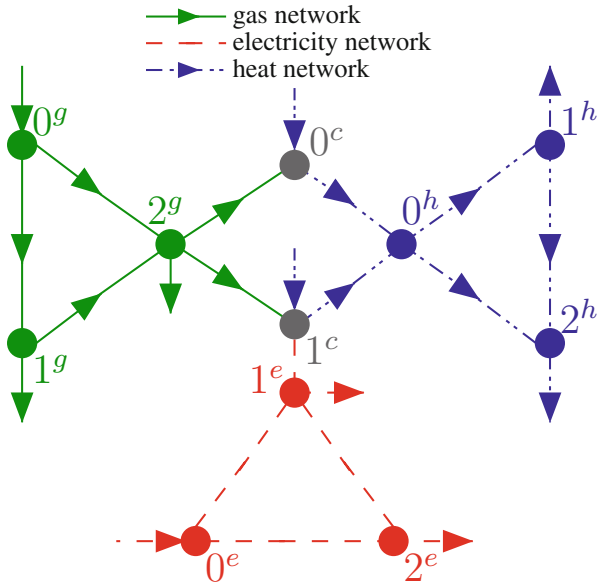

Fig. 1 Network representation of a small MES. Each SCN consists of three nodes. The gas and electricity networks have an external source connected at nodes 0^g and 0^e, the heat network has no external sources. In each SCN, nodes 1 and 2 are sinks. The SCNs are coupled by a gas-boiler, node 0^c, and a combined heat and power plant (CHP), node 1^c. The links show defined direction of flow, the terminal links show actual direction of flow

Table 1 Errors of NR for each iteration k, using a tolerance of $\tau = 10^{-6}$. Here $\tilde{e}^k = ||T_F \mathbf{F}^k||_2$, $\hat{e}^k = ||\hat{\mathbf{F}}^k||_2$ and $e^k_{\text{p.u.}} = ||\mathbf{F}^k_{\text{p.u.}}||_2$. The last column gives the relative difference between the errors of scaled NR and unscaled NR

k	$\tilde{e}^k$	$\hat{e}^k$	$e^k_{\text{p.u.}}$	$\frac{\lvert\tilde{e}^k - \hat{e}^k\rvert}{\lvert\tilde{e}^k\rvert}$	$\frac{\lvert\tilde{e}^k - e^k_{\text{p.u.}}\rvert}{\lvert\tilde{e}^k\rvert}$
0	1.0310×10^{6}	1.0310×10^{6}	1.0310×10^{6}	0.0000	0.0000
1	1.3081×10^{3}	1.3081×10^{3}	1.3081×10^{3}	2.6421×10^{-14}	1.0951×10^{-14}
2	5.7417×10^{-1}	5.7417×10^{-1}	5.7417×10^{-1}	1.5071×10^{-12}	9.6527×10^{-13}
3	7.0379×10^{-4}	7.0379×10^{-4}	7.0379×10^{-4}	6.5244×10^{-10}	7.7472×10^{-10}
4	3.2883×10^{-9}	3.2890×10^{-9}	3.2886×10^{-9}	1.8566×10^{-4}	7.4581×10^{-5}
5	6.6172×10^{-11}	–	–	–	–

6 Conclusion

We extended the per unit system used in electrical networks for scaling the load flow (LF) equations to gas networks, heat networks, and multi-carrier energy networks (MCNs). The per unit system scales the equations by scaling all variables and parameters. The base values are determined by dimensional analysis, such that the scaled system is also dimensionless. Another option is to use scaling matrices, which

explicitly scales the equations. We showed that base values can be chosen such that the per unit system is equivalent to using scaling matrices, in infinite precision.

Newton-Raphson's method (NR) is used to solve the (scaled) system of nonlinear LF equations. In infinite precision NR is unaffected by scaling. Using the LF equations for a small MCN, we showed that both scaling methods lead to slightly different NR steps, meaning that NR is affected by scaling in finite precision. However, the difference in the solution found for the LF problem is small. Hence, for this example, the per unit system and scaling matrices are equivalent in finite precision.

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