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Early-stage design of novel vessels: How can we take a step forward?

N.D Charisi¹, A. Kana¹ and J.J. Hopman¹

ABSTRACT

The aim of this paper is to discuss the challenges associated with the early-stage design of novel and reliable vessels, and discuss some of the expected benefits of the application of multi-fidelity models in addressing some of their early-stage design problems. Traditionally, early-stage design tools are computationally cheap, but lack in accuracy. However, for the design of novel vessels, these tools are not sufficient. The first part of the paper discusses the challenges associated with the design of novel vessels. The second part of the paper focuses on a literature review on the application of the multi-fidelity models to the design of complex engineering systems. Finally, the most promising methods are identified and discussed.

KEY WORDS

Early-stage design; Novel vessels; Reliable vessels; Multi-fidelity models; Design Framework

INTRODUCTION

Early-stage design is considered by many to be one of the most critical design phases because many of the major design decisions are being made, and as a consequence, most of the costs (approximately 70% of the total lifecycle cost (Dierolf & Richter 1989)) are committed (Andrews 2018). One of the major challenges is that the engineers need to make these big design decisions with limited knowledge of the design (Mavris, et al. 1998). Knowledge regarding the performance of the vessel is important to making informed design decisions and in turn, make better designs. Therefore, it is critical to introduce the “right” knowledge earlier on in the design process (Willcox 2018).

Knowledge relevant for early-stage design can come from many sources, for example: data from reference vessels, empirical and semi-empirical methods (Papanikolaou 2014), experimental data from hull series, simplified physics models or experts’ opinion (DeNucci 2012). Traditionally, early-stage design knowledge comes from low fidelity models meaning computationally cheap and less accurate models. This approach is dictated by the constraint of limited time and (computational) budget. Higher fidelity methods are adopted gradually throughout the design process. Higher fidelity analysis becomes possible because we reduce the design space to include only the most promising candidate solutions. This practice is successful when the performance analysis of a vessel based on low fidelity models is reliable enough. For example, Nikolopoulos and Boulougouris (2019) adapted the Holtrop and Mennen method to better fit modern hulls (based on the KVLCC2 database), the method was developed for future use in power estimation during early-stage design. However, this approach has real limitations when considering the early-stage design of novel vessels.

For novel vessels, there is very little, if any previous knowledge by the sheer nature of them being “novel”. Therefore, the low fidelity models and tools that designers have developed based on experience may not be fully suitable for such vessels. Instead, designers need to perform accurate analysis to generate insights into the performance of these vessels. For example, the naval architect needs to perform accurate CFD analysis or model testing to assess the hydro-structural performance of novel hull shapes. This creates a scenario where the design space may not be fully explored before down selecting candidate design concepts. A promising compromise to such engineering problems can be given by adopting multi-fidelity models (MFMs).

Multi-fidelity models (MFMs) have the potential to facilitate the introduction of high-fidelity models earlier on the design process. MFMs are a current state-of-the-art in many engineering fields for solving outer-loop applications such as

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optimization. A MFM is a model combining both a high-fidelity model and low-fidelity models (Peherstorfer, et al. 2018). The high-fidelity model ensures accuracy, whereas the low fidelity models lead to computational speedups. MFMs have been already developed for design optimization applications such as the design optimization of a supersonic air vehicle presented by (Allison, et al. 2015). To identify their full potential in the design of novel vessels, further investigation is required.

The aim of the paper is to investigate the challenges associated with the early-stage design of novel vessels and identify the associated design drivers (DDs) and key performance indicators (KPIs). The findings are used to sketch the backbone of an early-stage design framework for novel vessels. The second goal of the paper is to provide a literature study on the applicability of MFMs in design optimization problems, and identify the research gaps connected to their applicability in early-stage ship design problems.

EARLY-STAGE DESIGN OF NOVEL VESSELS

Overview

There are multiple commonly used definitions of complex vessels (e.g., Andrews 1998; Gaspar, et al. 2012; Kana, et al. 2016). Some interesting aspects given by these definitions for this research are the following: (1) early-stage design of complex vessels is a wicked problem meaning that such a design problem is novel and unique, and (2) the overall performance of the vessel is hard to predict even if all the design components are well understood. The present research focuses on novel vessels. Complexity and novelty are interrelated. Complexity as mentioned refers also to the interaction of the various components of the vessel, whereas in the context of the present research, novelty links to a specific design feature of the vessel.



Figure 1(a): Ramform Titan. Retrieved from (PGS n.d)



Figure 1(b): Baltika. Retrieved from (Aker Arctic n.d)



Figure 1(c): Pioneering spirit. Retrieved from (Allseas n.d)



Figure 1(d): Bottsand class oil recovery ship. Retrieved from (AO 'Bottsand' Klasse n.d)

Currently, naval architects face several technological challenges. These connect to the deployment of unmanned or autonomous marine systems, the design of zero-emission ships, the application of new technologies onboard, the design of large and stationary marine systems, and the design of complex sea-going structures. These technological challenges drive

the need to design vessels that go beyond the capabilities of traditional vessels. Innovation is another important business driver that leads the maritime industry to maintain its competitiveness (Hopman 2008). Some examples of novel vessels are the following: (1) the *Ramform Titan* aims to collect seismic data from the ocean, its sinusoidal waterline increases the vessel's stability, (2) the *Baltika* icebreaker which has an asymmetrical hull to be able to break the ice sideways, (3) the *Pioneering Spirit* which is the largest construction vessel designed for the single-lift installation and removal of oil and gas platforms (lifting capability of platform topsides up to 48,000 t and jackets up to 20,000t (Allseas n.d)), and (4) the *Bottsand* vessel is able to split in two parts along its length to effectively support its mission of oil recovery in oceans. These unique designs are examples of revolutionary ideas aimed to design vessels with improved capabilities.

The paper aims to sketch an early-stage design framework for novel vessels. The first step is to identify the DDs and KPIs connected to the design of such vessels. The DDs connect to the important design features, which are investigated during early design stages, and the KPIs associate with the quantities of interest, which determine each design's performance. DDs can be defined as the design aspects that directly influence the technical feasibility, performance and cost, for decision making (Duchateau 2016). According to System's Based Design (SBD) proposed by Levander (1991), the KPIs are focus on the performance of the vessel, such as the structural and seakeeping performance, as well as to economics, such as the building and operational cost.

While there are various types of novel concepts in ship design, this paper focuses specifically on novel hull form design. This research decision was made because the research is part of the Dutch Research Council (NWO) funded project "Multi-fidelity probabilistic design framework for complex marine structures", which aims to introduce extreme wave loading analysis earlier on in the design process. For the DDs and KPIs a bottom-up approach is followed to extract insight from two design cases: the USS Zumwalt (DDG 1000) and the Littoral Combat Ship (LCS - Independence variant). It is important to note that these cases were analyzed based on publicly available information sources.

USS Zumwalt (DDG 1000)

The Zumwalt-class destroyer (DDG 1000) was developed under the DD(X) destroyer program, and it is the lead ship of next-generation multi-mission surface combatants. The vessel is part of a series of three Zumwalts. Originally, the vessels were designed to replace the large-caliber naval gun capability that the US Navy lost when it retired its Iowa-class battleships (O'Rourke 2021).



Figure 2(a): DDG 1000 (La Grone 2021)

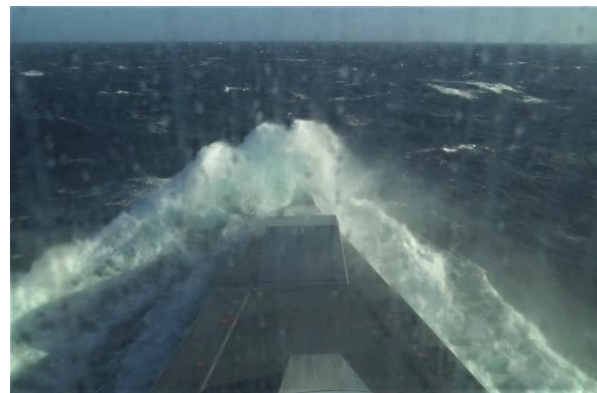


Figure2(b): DDG 1000 during testing in sea state 6 (Hernandez 2021)

According to (O'Rourke 2021), the main mission related design requirements were the following:

- Capability of naval surface fire support (NSFS)
- Operations in littoral waters
- Introduction of several technologies (such as integrated electric-drive propulsion system and automation technologies) which will be available for future naval vessels
- Operation with reduced crew size
- Reduced detectability

By grouping the design requirements, the DDs for the DDG 1000 are the following: (1) concept operations (CONOPS), (2) introduction of new technologies, (3) automation to enable operations with less crew members, and (4) improved stealth

capabilities. The aforementioned DDs connect to the design variables which together make up the design space. The design choice for the wave-piercing tumblehome hull connects to the stealth performance of the vessel, as this hull shape offers smaller radar cross-section compared to conventional hulls (Htet, et al. 2019). The hydrodynamic performance of the tumblehome hull has been researched in literature (parametric roll in head waves by using unsteady Reynolds-averaged Navier-Stokes (URANS) CFD code (Liu, et al. 2021), sway and yaw moment in stern quartering waves (Htet, et al. 2021), extreme event analysis based on the critical wave groups and fully nonlinear CFD code (Silva & Maki 2021). The large body of literature shows that extensive research effort is required to understand the hydrodynamic performance of such a novel hull shape.

On the other hand, from the industry's perspective, this novel hull form leads to concerns regarding the vessel's seakeeping performance. A senior naval officer (Cavas 2015) expressed his concerns by stating that

“In conventional hulls, we have done more with model testing and design work. We have correlation with ships we've built and sent to sea. There's a lot of confidence in designing a conventional hull. We have not had tumblehome wave-piercing hulls at sea. So how would the real ship motions track with the ways we have traditionally modeled ships? How accurate is it?”

The aforementioned quote highlights the challenges associated with the design of novel concepts. These challenges relate to the uncertainty associated with the lack of knowledge for the design itself, as well as the uncertainty introduced by the analysis of novel designs with existing tools.

Littoral Combat Ship (Independence variant)

The LCS vessels were designed to be inexpensive surface combatants with modular mission packages (Kana 2016; O' Rourke 2019). The LCS class consists of two variants, namely the Freedom and Independence variant. The design of the LCS Freedom variant is a steel semi-planing monohull, whereas for the LCS Independence variant the design is an aluminum trimaran hull. For this analysis, the Independence variant is considered due to its novel hull shape.

The main design requirements for the vessel, as presented in (O' Rourke 2019; *Preliminary Design Interim Requirements Document for Littoral Combat Ship (LCS)* 2003) are the following:

- The primary missions of the vessel are antisubmarine warfare (ASW), mine countermeasures (MCM), and surface warfare against small boats (SUW). The targeted area of operation is littoral waters.
- Additional missions include partnership-building operations, intelligence, surveillance, and reconnaissance operations, maritime security and intercept operations, support of Marines and special operations forces, and homeland defense operations.
- An important feature of the design is that it is based on modular mission packages to support the main war fighting capabilities. Modular design supports flexibility for the use of future systems upgrades and mission systems change-out.
- A performance requirement is to be capable to achieve high speed to support operations (the required speed of the vessel is 50 knots in sea state 3).

By examining and grouping the aforementioned design requirements, the DDs for the LCS are: (1) CONOPS, (2) modularity, and (3) high speed. The driver of high speed connects to the design decision of the trimaran hull shape. Recently, researchers put effort on gaining knowledge regarding the behavior of trimaran hull designs (“High-Speed, Multi-Hull Vessel Optimization” project (Weidle, et al. 2019)). The trimaran hull shape offers the following advantages compared to a conventional monohull: (1) improved seakeeping (Yun, et al. 2018) (2) low resistance at high speed (Hamed 2022) (3) increased deck area for operations (Hamed 2022). The selected hull material was aluminum. Aluminum is used as an alternative to high-tensile steel due to its high strength to weight ratio, and ease of manufacture (Benson, et al. 2011). However, aluminum is more susceptible to fatigue, whipping, corrosion and heat than steel and therefore, this material is less suitable for vessels designed for longer service life (Weidle, et al. 2019) and combat situations. Overall, aluminum was a new material for naval applications, and thus there was introduced uncertainty regarding its use for the LCS.

This uncertainty was expressed by the defense analyst, Loren Thompson (Shalal-Esa 2010), who stated that

“It is hard to understand how the Navy could consider selecting a design that it says it does not understand very well.”

“It is surprising that they would say at this point in the evolution of the program that they do not understand how aluminum might operate under certain difficult conditions.”

The aforementioned statements highlight that there is introduced uncertainty due to the selection of the aluminum hull because its performance is not well-understood for the intended application. Regarding the assessment of the LCS vessels, the LCS program has been controversial because of the cost growth, the design and construction problems of the first LCSs, and the remaining concerns regarding the vessel's technical performance and effectiveness of the modular packages (O'Rourke 2019)

Identification of the DDs and KPIs for novel vessels

The findings from these design cases are used to extract conclusions for the DDs and KPIs of novel vessels by following a bottom-up approach. However, to generalize the findings from the previous design cases, some well-established methods were retrieved from literature. Regarding the design case of the DDG1000, the requirement of improved stealth led the "novel" design decision of adopting a tumblehome hull, whereas the driver of high speed led the "novel" design decision of adopting an aluminum trimaran hull for the LCS-2. The S^5 scheme proposed by Brown and Andrews (1980) aimed to identify the important design characteristics. The scheme proposes that the following aspects need to be considered: Speed, Stability, Strength, Seakeeping, and Style. The scheme is valid in the discussed design cases. For the DDG1000, the driver related to improved stealth capabilities may fit under the Style aspect of S^5 .

Regarding the KPIs, the assessment of the technical performance of the designs has been the most important aspect in every design case. In real-world applications, cost assessment is crucial as well. In addition, the design cases of the DDG 1000 and the LCS-2 showed that it is important to account for the safety performance. In the case of the DDG 1000, the challenge was to ensure that the vessel's motions in rough seas would not exceed the acceptable limits. On the other hand, in the case of the LCS-2, the challenge was to ensure that aluminum was a suitable material for the construction of a naval vessel. The assessment of the designs based on the technical performance, cost and safety is in accordance with the risk-based design proposed by (Vassalos 2009).

Regarding the examined design cases, the design drivers lead to novel design decisions. These design decisions introduce uncertainty related to both the design itself, as well as, the analysis methods needed for the performance assessment. This is a major difference between the design of traditional vessels compared to the design of novel vessels. In addition, apart from the technical feasibility and cost assessment of the design, it is important to account for the safety performance early on in the design process. These ideas are summarized in Figure 3.

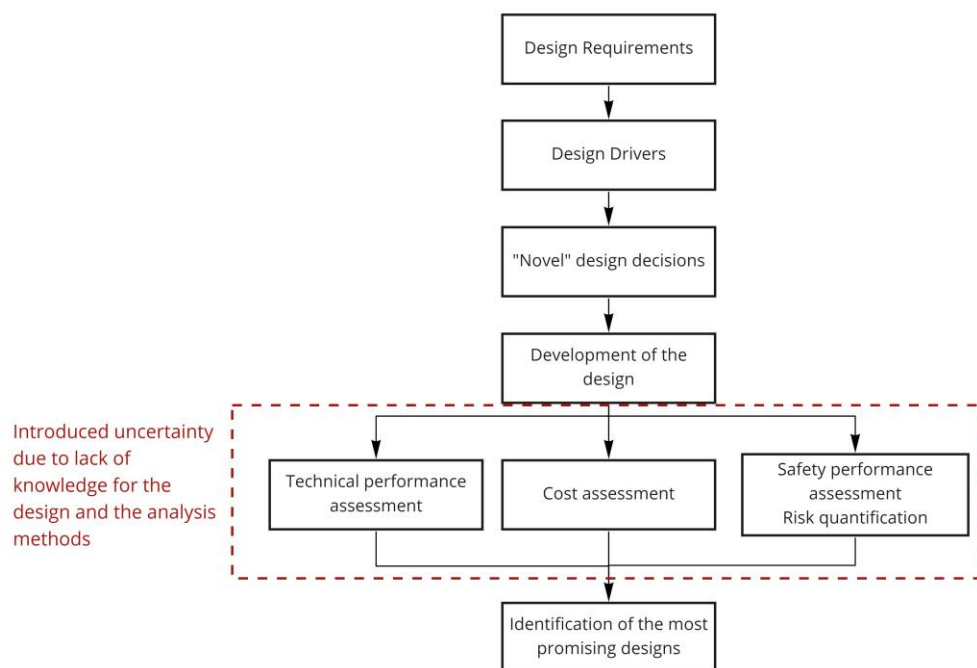


Figure 3: DDs and KPIs for novel vessels

DESIGN ARCHITECTURAL FRAMEWORK FOR NOVEL VESSELS

In this section, a high-level description of an early-stage design framework for novel vessels is given. The definition of a design framework, given by The Open Group Architecture Framework (TOGAF), is the following

a foundational structure, or a set of structures, which can be used for developing a broad range of different architectures. It should describe a method for designing a target state of the enterprise in terms of a set of building blocks, and for showing how the building blocks fit together. It should contain a set of tools and provide a common vocabulary. It should also include a list of recommended standards and compliant products that can be used to implement the building blocks.

Several design frameworks have been proposed in literature such as a framework for analysing distributed systems for naval vessels proposed by (Brefort, et al. 2018), and the design and engineering engine for conceptual aircraft design proposed by (La Rocca, et al. 2012). The DAF in this research aims to address the problem of early-stage design optimization under uncertainty for novel vessels. The DAF consists of three parts: the generative, analysis and optimization engine. The generative parts aims to generate the designs which will be further analysed by the analysis engine. The optimization engine aims to explore the design space to identify the best solutions for a specific design problem. A high level representation of the DAF can be seen in Figure 4.

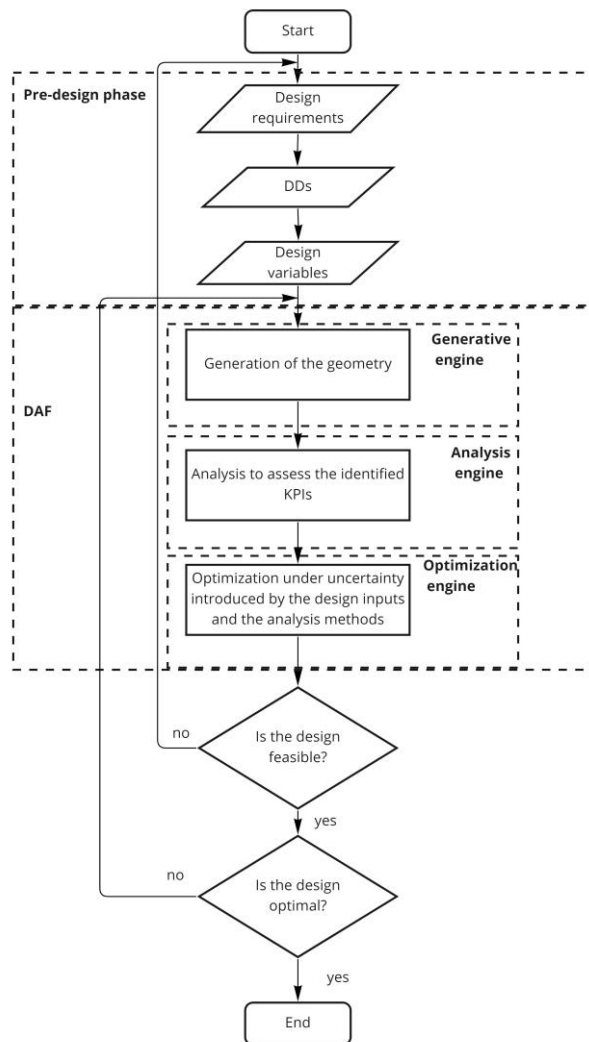


Figure 4: High level sketch of the DAF

Uncertainty in early-stage design

The analysis of the DDs and the KPIs showed that the introduced uncertainty connected to novel concepts should be taken into account for early-stage design exploration. The identification and quantification of uncertainty is a current state-of-the-art for early-stage design. Although, uncertainty is closely related to risk, the major difference between the two is that risk is something that we can put a price on, while uncertainty is a risk that is hard to measure (Silver 2013). Uncertainty has been divided into two categories, the aleatory and epistemic uncertainty (Ghanem, et al. 2017). Aleatory uncertainty connects to random variability and it is irreducible, whereas epistemic uncertainty relates to the lack of knowledge and it is reducible when more knowledge is available. The part of uncertainty related to the lack of knowledge is hard to measure due to its subjected nature, summarized by (Ghanem, et al. 2017) as follows:

“How well can we predict what we do not know yet? The answer lies in the realm of mental processing - in the brain of the predictor, who use their state of knowledge to make the prediction. Uncertainty is a lack of knowledge - in the human brain, and not some sort of objective reality. Probability, as a measure of uncertainty, reflects one's state of mind and not a state of things.”

In engineering design, uncertainty is defined as the difference between models and reality (Kennedy & O'Hagan 2001; Mavris, et al. 1998). Uncertainty is higher during the early-stages of design because knowledge is limited (Figure 5). During early stages of design, uncertainty is introduced by the assumptions, the analysis codes of various fidelities, economic uncertainty or technological risks (Mavris et al. 1998). Specifically for analysis codes, Kennedy & O'Hagan (2001) identified the following uncertainties: (1) parameter uncertainty (unknown parameters of the model), (2) model inadequacy (difference between the value of the real world process and the code output), (3) residual variability (the real world process may not take the same inputs when specific conditions are repeated), (4) parametric variability (some of the conditions associated with the inputs are uncontrolled and unspecified), (5) observation error, and (6) code uncertainty (in practice the output is not known before the code runs). The identification and quantification of uncertainty is relevant to early-stage design because it “creates value only to the extent that it holds the possibility of changing a decision that would otherwise be made differently” (Bickel & Bratvold 2008).

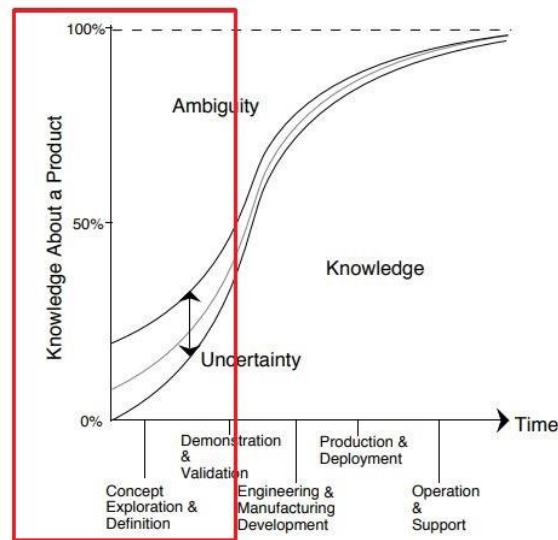


Figure 5: Uncertainty throughout the design process (Mavris, et al. 1998)

From a practical point of view, it is not possible to calculate the model inadequacy because the true mean value of the real world process is not known. Lam et al. (2015) suggest that the fidelity function, which quantifies our confidence in the underlying information source using a standard deviation of the predictions from the value of the high fidelity analysis prediction, can be used to characterize uncertainty of an information source. This approach is suitable to be adopted for the development of an early-stage design framework. A further step is to assess whether taking into account the quantified uncertainty reliably produces meaningful results and leads to improved designs (see Hulse, et al. 2020).

Reliability assessment in early-stage design

During the early design stages, different designs are developed to explore the design space. To assess each design, its performance needs to be analyzed and evaluated. The reliability of different design solutions, given a long exposure to a

potentially harsh excitation environment, is a crucial factor to design decision making (Seyffert, et al. 2019). Reliability should be taken into account early in the design process. Seyffert (2018) states that it is desirable to consider earlier in the design cycle a design's performance over the intended lifetime taking into account the harshening ocean environments and the push to extend the service life of marine systems. From a budget perspective, accounting for reliability early on in the design process ensures the avoidance of costly design iterations during late design stages (Chaudhuri, et al. 2021).

For well-established designs, formulae provided by the classification societies are used to calculate the waves and the vessel's response. The process is explained by Bai & Jin (2015). More specifically, the process consists of the following steps: (1) determine the design load, (2) define the acceptance criteria, and (3) make the strength assessment. The most challenging task is to define the design load as the ship may be exposed to various sea and wave conditions during its lifetime. Classification societies use two methods to determine the design value. The first one is the Design Wave Method, which assumes that the largest wave occurs in the most severe stationary sea state. The second method requires that all possible sea states the ship is likely to encounter in its lifetime are evaluated. A complete analysis of all the sea states is carried out, and the different sea states are weighted according to the likelihood of being encountered by the ship. The former method is less accurate but less computationally expensive, while the latter is computationally expensive but provides more realistic results. The acceptable risk is specified by the approval authority (flag state administration and/or classification society) taking into account the aspects of human life and environmental protection. Over the last 250 years, classification societies have been developing rules which have been backed-up by in-service experience and account for wave effects; however, in design cases where the established rules are not sufficient, direct analysis design assessment is required (Hirdaris, et al. 2014).

However, the challenge arises when considering extreme loading conditions on a novel hull. The classification societies address the problem of defining simultaneous load combination cases by an Equivalent Design Wave (EDW) method for use in a finite element model. Seyffert & Kana (2019) examine whether this approach defines realistic lifetime combined loading scenarios, especially for novel hulls. The research concludes that the simplicity of EDW may not be worth the potential losses in accuracy when considering lifetime combined loading scenarios for a novel hull such as a trimaran. Therefore, this established method may not be sufficient enough for the assessment of complex marine systems. In addition, Vassalos (2009) states that the optimum design solution may lie outside the regulatory envelope. Thus, a broader exploration of the design space in the early stages is more suitable to find 'better' solutions for innovative complex vessels.

To sum up, the paper has so far discussed the challenge of introducing high fidelity analysis tools early on in the design process to assess the performance of novel designs. However, this is a challenging task due to budget and time constraints. By analyzing the design of the DDG1000 and the LCS-Independence, the findings showed that there is introduced uncertainty resulted from the lack of knowledge for the design itself and the design methods and tools. A promising direction in introducing high fidelity tools earlier in the design process is the development of MFM models. The second part of the paper dives into the applicability of these models to early-stage ship design exploration.

MULTI-FIDELITY MODELS IN DESIGN EXPLORATION

MFMs were developed in the field of Machine Learning, and are based on the combination of High-fidelity models (HFMs) and Low-fidelity models (LFMs) to achieve accuracy at a reasonable computational cost (Fernández-Godino, et al. 2016). Peherstorfer et al. (2018) distinguish two key components of the MFMs: (1) the low fidelity models, which are useful approximations of the HFM, and (2) the model management strategy, which distributes the analysis amongst the different models. A detailed overview of the MF techniques is given in (Fernández-Godino, et al. 2016; Peherstorfer, et al. 2018). The goal of this section is to provide a targeted review of MFMs and to investigate their applicability in an early stage design framework for novel vessels.

The main advantage of MFMs is that they enable accurate inference of quantities of interest by synergistically combining realizations of low-cost/ fidelity models (e.g., a simplified physics approximation, a reduced model, a data-fit surrogate) with a small set of high-cost/ fidelity observations (Peherstorfer, et al. 2018; Raissi, et al. 2016). Therefore, MF methods can accelerate the solution of outer-loop applications such as optimization and uncertainty quantification, where several model evaluations are required. Based on the available literature, the ratio of the computational cost of the MFM over the computational cost of the HFM can vary up to 80% in the field of fluid mechanics (Fernández-Godino, et al. 2016). The disadvantage of the method is that MFMs require a substantial amount of time and effort to successfully build and validate (Fernández-Godino, et al. 2016).

In the following example, the potential of applying MFMs in early-stage ship design is illustrated. During early design stages, the naval architect aims to identify the relationship between the DDs, connected with the design variables, and the KPIs to lead the exploration to the identification of a "better design". Let's assume that the design problem is bounded to one design driver, described by the design variable A, related to one KPI A via the Forrester function (Forester, et al. 2008), which is a

one-dimensional test function. In reality, this assumption simplifies the design problem as the designers have to deal with multi-dimensional spaces in real design problems.

For this simplified design example, the performance analysis is shown in Figures 6(a), 6(b). The red line represents the HF model which can be seen as the “true” function representing the involved physics, whereas the blue line represents the LF model which is the computationally cheaper approximation of the physical problem. In practice, the true shape of the HF function cannot be computed due to computational power, time, and budget restrictions. As a result, traditionally, LF models and data are used in the early design stages to identify design trends. Let's assume that the presented KPI A represents a quantity that should be minimized to achieve optimal performance. As it can be seen in Figure 6(a), *design point B* represents the minimization point of the HF model, whereas *design point A* represents the minimization point of the LF model. As a result, the relationship of the design variable A and the KPI A is misinterpreted and we get a wrong conclusion about the range of values of the design variable A that minimizes KPI A. This problem is addressed by introducing more accurate analysis earlier on the design process and construct the MFM. The MFM (Figure 6(b)) was built by using both the LF model to decrease the required computational power and the HF model to ensure the physics of the problem. The result was generated by following the method proposed by Kennedy and O’Hagan (2000) and by using the python package Emukit (Paleyes, et al. 2021). In that case, it can be seen that there is a small deviation between *design points A* and *B*. Thus, the MFM is a good approximation of the physical model and provides meaningful results for the identification of the design trends. This simplified example shows promise that MFMs have the potential to facilitate ship design exploration by introducing high fidelity analysis earlier on the design process.

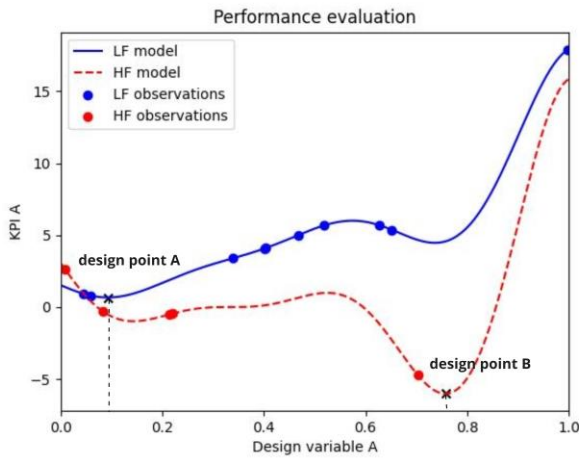


Figure 6(a). Performance assessment based on the LFM and HFM

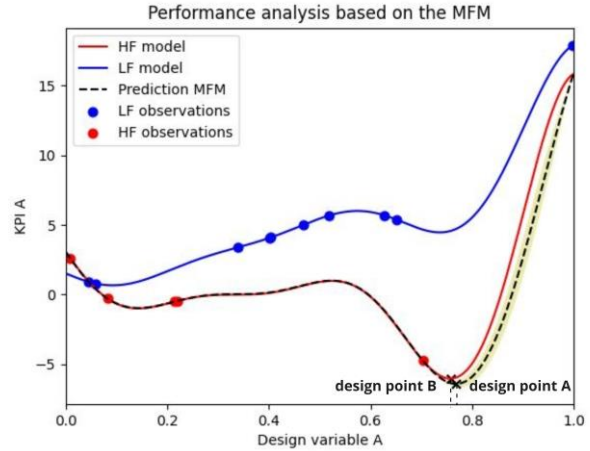


Figure 6(b). Performance assessment based on the MFM

MFMs are a current state-of-the-art in engineering, applied mathematics, and machine learning research groups. MFMs have been built based on various methods such as Gaussian Processes (GPs) (Damianou and Lawrence 2013; Kennedy & O’Hagan 2000; Le Gratiet and Garnier 2014; Perdikaris, et al., 2017;), Monte Carlo approaches (Peherstorfer, et al. 2016), and Neural Networks (Meng & Karniadakis 2020). These methods have been widely applied in different engineering problems such as design analysis and optimization problems in the aerospace (i.e. Chaudhuri et al., 2021; Di Fiore et al., 2021) and maritime field (Bonfiglio, et al. 2018; Gaggero, et al. 2021), solving PDEs (Raissi & Karniadakis 2016), and developing bioengineering applications (Raissi, et al. 2018).

GPs have been widely successfully applied in engineering problems when the analysis is based on computationally costly functions. Information about the GPs can be found in (Rasmussen and Williams). A multi-fidelity framework for GPs was proposed by (Kennedy & O’Hagan 2000). A recursive formulation of the auto-regressive scheme (AR1) was proposed by (Le Gratiet & Garnier 2014). This approach reduces the computational complexity of the original model. An extension of the linear autoregressive GPs, called NARGP, was proposed by Perdikaris, et al. (2017) to account for the complex nonlinear cross-correlations between the various models of the MFM. Damianou & Lawrence (2013) proposed a Deep GP framework structured as a deep belief network based on GP mappings. The aforementioned frameworks have been successfully applied in many applications. A detailed review of the applicability of MF-GPs in aerospace systems can be found in (Brevault, et al. 2020). The benefits of the GPs are that: (1) the model gives accurate predictions for limited data as well as large data sets (Durrande 2021), and (2) the model quantifies the introduced uncertainty. On the other hand, its limitations (Durrande 2021)

are (1) its computational complexity in case of large data sets due to the inversion of the covariance matrix (Durrande 2021) (2) numerical stability issues due to the calculation of the inverse covariance matrix (Durrande 2021).

These methods have been applied to solve ship design problems. Gaggero et al. (2021) proposed a two-fidelity framework for marine propellers design optimization. The integrated methods were a boundary element method as the low-fidelity method, and a RANSE solver as the high fidelity method. These were combined by multi-output Gaussian Processes. In addition, a multi-fidelity framework based on MF-GPs and Bayesian optimization to effectively exploit data from multi-resolution simulations (3D URANS solver) was proposed by (Bonfiglio, et al. 2018a). The framework was applied to the shape optimization of 3D super-cavitating hydrofoils. Bonfiglio, et al. (2018b) developed a MF-GPs and Bayesian optimization to build probabilistic surrogate models and efficiently explore a 35-dimensional design space to optimize SWATH hull shapes that minimize wave-induced motions and accelerations and satisfy specific requirements in terms of displacement and metacentric height. The level of modeling fidelity is a strip theory and a boundary element method based on potential flow. From a practical ship design perspective, Raven and Scholcz (2019) reviews ship hull form optimization at MARIN performed via multi-fidelity surrogate models. One important limitation of the MF-GPs is the reduced effectiveness of the method when there is a small degree of correspondence between the fidelity levels (Gaggero, et al. 2021; Raven and Scholcz 2019).

The Monte Carlo (MC) method can be seen as a methodological way to perform what-if analysis based on repeated sampling and statistical analysis to calculate the results (Raychaudhuri 2008). Multi-fidelity Monte Carlo methods (MF-MC) have been developed and applied in various engineering applications (i.e., Ng & Willcox 2015). The MF-MC method is robust, flexible and simple for implementation (Zhang 2020). In addition, MF-MC methods are suitable for analysis in high dimensional spaces (Peherstorfer, et al. 2018; Zhang 2020). These methods are also able to capture nonlinearities (Peherstorfer, et al. 2018). However, for the crude MC approach, many realizations are required to achieve accurate results. For problems requiring the estimation of the probability of failure, the MF-MC importance sampling method is applied (i.e., Chaudhuri, et al. 2020; Peherstorfer, et al. 2016). In design optimization problems, the MF-MC control variates method has been applied (Ng & Willcox 2014). An application of the method to address conceptual design optimization under uncertainty of aircrafts can be found in (Ng & Willcox 2015). Both the importance sampling and the control variate methods lead to computational savings compared to crude MC.

A neural network (NNs) is an interconnected assembly of processing elements, the neurons, whose processing ability of the network is expressed by the interunit connection strengths, the weights and obtained by the process of learning from a set of training patterns (Gurney 1997). NNs have been successfully employed to solve complex physical problems in different scientific domains. The NNs are capable of identifying nonlinear complex relations (Guenther 2001), however this comes with the requirement of a sufficiently large training set of data (Gurney 1997). Some examples of scientific research related to fluid dynamics are the prediction of the nonlinear motions of vessels in irregular long-crested and oblique seas (Ferrandis, et al. 2019), and the estimation of pressure and velocity fields from images (Raissi, et al. 2020). A multi-fidelity scheme for NNs has been proposed by Meng & Karniadakis (2020). Few applications of the multi-fidelity NN scheme can be found in literature. For example, He et al. (2020) proposed a deep NN to combine low- and high- fidelity aerodynamic data tested in predicting the lift and drag coefficient of a typical airfoil. Regarding design optimization, some design optimization methods based on multi-fidelity NNs can be found in (Zhang et al. 2021) addressing aerodynamic shape optimization, and in (Yoo et al. 2021) targeting design optimization of composite structures. It is important to mention that quantifying uncertainty in NNs is complicated and it is a current field of research and debate. For further information, the reader is referred to (Psaros et al. 2022).

To sum up, MF-GPs are strong and well-understood mathematical methods that provide robust predictions including the underlying uncertainty associated with the predictions. For ship design applications, these methods offer a great advantage since these are suitable for computationally expensive analysis. The limitations of these methods connect to problems characterized by high complexity and dimensionality. MF-MC methods are flexible and suitable for high dimensional spaces. Most of the applications focus on the control variate method for design optimization and importance sampling for failure estimation. MF-NNs seem to be very promising methods for solving highly complex problems. However, the uncertainty quantification in MF-NNs is still an open question.

Identification of the research gaps

The applicability of MFMs in design optimization problems of complex engineering systems is a current state-of-the-art. Most of the research in ship design focuses on the application of MF-GPs. However, the potential of applying the nonlinear schemes (NARGP and MF Deep GPs) is not yet explored. The performance of the different schemes for aerospace problems has been investigated by (Brevault, et al. 2020), and the findings showed that different schemes perform better for different

problems. Thus, there is evidence that the nonlinear schemes may be more suitable for specific analysis and optimization ship design problems. In addition, MF-NNs are very powerful methods to address complex analysis problems and support design optimization. To the best of the authors' knowledge, there is no such framework targeted specifically for early-stage ship design. Regarding design frameworks, (Peherstorfer, et al. 2018) suggests to go beyond multi-fidelity frameworks that focus solely on methods and work on including various information sources, so that decision making is based on a wider range of information sources. Such a goal is in line with the aim of early-stage design as a decision making process. Finally, complex analysis such as the assessment of extreme events can be introduced earlier on the design process in a reliability-based design framework by effectively using MFMs. These developments would help the naval architects to make one step forward towards an early-stage design framework for novel vessels.

CONCLUSIONS

The present research discuss the challenges associated with the early-stage design of novel and reliable vessels. In the first part of the paper, the DDs and KPIs associated with the design of novel vessels were identified. The findings showed that the DDs for novel vessels are not necessarily different from those associated with traditional vessels. However, the novel design features of the vessel introduce uncertainty resulted from the design itself and the analysis methods. The introduced uncertainty needs to be taken into account for decision making during early-stage design. For traditional vessels, the KPIs connect to the technical feasibility and the cost assessment, whereas for novel vessels it is important to account for safety performance as well. The second part of the paper focuses on exploring the potential of introducing MFMs in an early-stage design framework for novel vessels. MFMs are a current state-of-the-art, and have already proven powerful tools in design problems in different fields. The most suitable methods were identified and discussed in the context of early-stage ship design. Finally, some promising research directions were outlined.

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