Reliability and Degradation of Power Electronic Materials

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Chapter 14
Reliability and Degradation of Power Electronic Materials

R. Ross and G. Koopmans

1 Introduction to Reliability and Degradation

Power electronics (PE) obtain their properties from a constellation of materials. The performance of such devices depends on the properties of the applied materials and the quality of the assembly. This chapter describes reliability matters related to performance of materials and systems in R&D experiments and production. The focus is on the statistical description, failure data analytics, and inferences.

The chapter starts out with a description of the basic parts in PE and how their degradation can jeopardize the integrity of PE. Next the basics of relevant statistical analysis are discussed. This is followed by describing the acceleration due to enhanced stresses. The effect on the system lifetime by size, redundancy, and reparability is discussed next. Finally, it is discussed that a product batch may consist of various subpopulations (e.g., defected and sound products) and that various processes may occur simultaneously (e.g., random and wear-out failure) which leads to the phenomenon of mixed failure distributions. Some examples illustrate how to deal with such complicated situations that nevertheless occur frequently in testing.

A full review of the extensive literature that exists on reliability analysis is beyond the scope of this chapter. For a more extensive introduction the reader is referred to textbooks such as on lifetime data [7], accelerated testing [10], and reliability data [8]. The present approach also transfers practice on asset management in electric power supply [13] to power electronics. As a special topic the present

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contribution also discusses the fact that multiple processes can lead to the degradation of devices, which leads to the concept of combined distributions. With accelerated aging the original distributions can all change, but probably at different rates. This is a noteworthy phenomenon which may help or hinder R&D testing.

2 Power Electronic Materials

The basic functionalities in PE devices are conduction, semi-conduction, insulation, and stress control. The performance and degradation of electrical materials in PE devices is described in industry handbooks and textbooks, e.g., [3], and relevant information can also be found in electrical materials handbooks, e.g., [4] and, e.g., IEEE transactions. The background is that phenomena that happen in high voltage engineering also appear to happen in miniaturized low voltage PE devices, because the electric fields are in the same range.

Most applied materials in increasing order of conductivity are aluminum, gold, copper, and silver. Other important properties of conductors are mechanical strength, thermal expansion, and chemical stability. Lack of conductivity can stem from design (insufficient cross section) or high Ohmic contact (e.g., fracture in soldered connection). Overheating may be caused by forced current (e.g., by inductances) through high impedances.

Semi-conducting materials owe their conduction to available charges and electric fields that advance or block current. This is the essential functionality of PE devices. Particularly during switching, electric losses occur that cause heating and may lead to thermal degradation of the device. Lack of cooling may lead to a thermal runaway that damages the insulating layers.

Insulating materials serve to keep charges separated. Such materials are often referred to as dielectric materials because many are polarized in an electric field. Characteristic dielectric properties are electric susceptibility χ_e and permittivity ε, of which the real part ε' reflects the capacitive properties and the imaginary part ε" the conductive part (i.e., the leakage current through the insulator). The ratio ε"/ε' = tan δ is the dielectric loss factor that quantifies the ratio of the leaked and the stored energy. Tan δ may increase with temperature which may contribute to a thermal runaway. The electric breakdown strength is the electric field at which an insulating material breaks down. A local transgression of the breakdown strength can lead to electrical treeing with high fields at the branch tips which may quickly lead to a full breakdown. The electric field may also exceed the strength in cracks and cavities (e.g., from delamination) which may first lead to partial discharging in such spaces and next to eroding the walls and consequently cause treeing and breakdown. Such phenomena are also quite common in high voltage (i.e., high electric field) engineering.

Finally, stress control materials are used to control the electric field in and at the surface of insulators in order to prevent exceeding the breakdown strength.

All the above-mentioned functionalities of the materials are essential to the performance of PE devices and malfunction can jeopardize the PE device. Not only the quality of the materials, but also the configuration and assembly are essential. As mentioned above, multiple processes can be expected to occur simultaneously.

3 Reliability

When multiple objects fail as a result of a given failure process, the individual failures will be distributed in time (or any other relevant variable X that reflects the object performance, but for clarity time is used as the representative variable). This distribution of failure events in time is often characteristic for the process. Even being one specific distribution in time, such a distribution is still represented by various basic functions: the cumulative failure distribution $F(x)$, the survival function $R(x)$, the distribution density $f(x)$, the hazard rate $h(x)$, and the cumulative hazard rate $H(x)$. A noteworthy distinction is made in statistics between the variable X and the values x it can adopt. A statement that the (variable) time X is smaller than time value x is written as: $X < x$.

3.1 Basic Functions

The description of the basic functions follows that of [13] maintaining the different meanings of distribution and probability function. A distribution describes how failure times are spread in time. Probability of failure describes the perceived likelihood of failure which depends on the available knowledge of individual objects. E.g., we may know how failure times overall tend to spread in time (the distribution), but if we also know that a specific test object, e.g., runs hot before it fails, we may be able to assign a higher probability of failure to those objects that feature a significant rise in temperature. The probablity of failure may differ per object, while the distribution remains the same for the group.

The cumulative distribution $F(x)$ describes the population part that failed at or before x (so $X \leq x$). The survival function $R(x)$ describes the population part that survived at least up to x (so $X > x$). The values of the function F range from 0 to 1 and those of R range from 1 to 0. $F(x)$ and $R(x)$ relate as:

$$R(x) = 1 - F(x) \tag{14.1}$$

The distribution density $f(x)$ describes the rate of failures:

$$f(x) = \frac{\mathrm{d}F(x)}{\mathrm{d}x} \tag{14.2}$$

The density is normalized, which means that its integral over the complete range of x equals 1. The mass function $f(x_\mathrm{i})$ has values at discrete values x_i. It is the discrete analog of the continuous density $f(x)$. The mass function is also normalized meaning that the sum of its values over all x_i equals 1.

The hazard rate $h(x)$ looks much like the density but only applies to the surviving population part $R(x)$.

$$h(x) = \frac{f(x)}{R(x)} = \frac{-d\ln(R(x))}{dx} \tag{14.3}$$

Since R ≤ 1 the hazard rate $h(x) \geq f(x)$. A discrete hazard rate is based on the mass function. The cumulative hazard rate $H(x)$ is the integral or sum of the hazard rates. A noteworthy relationship between R and H is:

$$R(x) = \exp(-H(x)) \quad \Leftrightarrow \quad H(x) = -\ln(R(x)) \tag{14.4}$$

If one of the fundamental functions F, R, f, h, or H is known, the other functions can be derived from that in principle.

Distributions are often reflected in measures like the mean time to failure and the variance (i.e., squared standard deviation σ) of failure times, which follow from the fundamental functions. The *mean* $\langle x\rangle$ follows from integrating x with $f(x)$:

$$x = \frac{\int_{x_{\min}}^{x_{\max}} x \cdot f(x)\mathrm{d}x}{\int_{x_{\min}}^{x_{\max}} f(x)\mathrm{d}x} \tag{14.5}$$

A non-trivial equation for the mean is also:

$$x = \int_0^1 x(F)\mathrm{d}F = \int_{x_{\min}}^{x_{\max}} R(x)\mathrm{d}x \tag{14.6}$$

The *variance* var(x) is found as (using the above equation for $\langle x\rangle$):

$$\mathrm{var}(x) = \sigma^2 = x^2 - x^2 = \frac{\int_{x_{\min}}^{x_{\max}} x^2 \cdot f(x)dx}{\int_{x_{\min}}^{x_{\max}} f(x)dx} - x^2 \tag{14.7}$$

The mean (and similarly variance) may also follow from the <u>complete</u> set of N values x_i (i = 1,..,N) as:

$$x = \frac{1}{N} \cdot \sum_{i=1}^{N} x_i \tag{14.8}$$

If only a *subset* of n observations is available with $n < N$, then this is a sample of which the *average* $\overline{x}$ and *sample variance* s^2 can be determined. This average and sample variance often serve as estimator of the mean and variance of the complete population.

3.2 Some Important Distribution Families

The previous sections discussed distributions as a concept. Various types of distribution families exist for various purposes. Here we will address four of those distributions: the Normal, Lognormal, Weibull, and Exponential distribution.

The Normal or Gaussian distribution is the best-known and possibly also the most misused distribution. The distribution has parameters mean μ and standard deviation σ. Because of the central limit theorem, the Normal distribution is particularly adequate (the asymptotic distribution) for describing the sums or averages of large data sets in the range (−∞, ∞). Many physical quantities are positive and do not fulfill this property. If the standard deviation σ is smaller than 3 or 4 times μ), the fraction $F(x)$ with $x < 0$ is small, but that remains to be checked. There is no analytical expression for the cumulative distribution F, but for the Normal distribution density f there is:

$$f\left(x;,\mu;,\sigma\right)=\frac{1}{\sigma\sqrt{2\pi}}\cdot\exp\left(-\frac{1}{2}\cdot\left(\frac{x-\mu}{\sigma}\right)^2\right) \tag{14.9}$$

Various approximations exist that enable numerical calculation of $F(x)$ to any desired precision ([18]). The mean x is equal to μ.

As mentioned, −∞ < x < ∞ and therefore x is not time in this case. The Lognormal distribution offers a way to fix the range-issue. If x follows a Normal distribution, then $t = \exp(x)$ follows the so-called Lognormal distribution. It also has parameters μ and σ but they are the mean and standard deviation of $\ln(t)$, not of t. The Lognormal distribution is particularly useful for describing the sum or mean of large sets of log-positive t-values (if $t > 0$ then $-\infty < \log(t) = x < \infty$ for which the Normal distribution applies). It can also be applicable in its own right. Again, there is no analytical expression for the cumulative distribution, but the distribution density is:

$$f\left(t;,\mu;,\sigma\right)=\frac{1}{\sigma\sqrt{2\pi}}\cdot\exp\left(-\frac{1}{2}\cdot\left(\frac{\ln\left(t\right)-\mu}{\sigma}\right)^2\right) \tag{14.10}$$

The mean of the Lognormally distributed t is not equal to μ (which is the mean of the Normally distributed x). The mean θ of a Lognormal distribution is:

$$\theta_{\text{Lognormal}}=\exp\left(\mu+\frac{\sigma^2}{2}\right) \tag{14.11}$$

The Weibull distribution is the asymptotic distribution for the smallest of an increasingly large set of positive values t. While the Normal and Lognormal distribution focus on the center (i.e., the mean), the Weibull distribution focuses on the lower extreme such as the lowest breakdown time that characterizes the electric strength of a device. The Weibull distribution usually is applied with two parameters: namely a scale parameter α and a shape parameter β. Analytical expressions exist for all basic functions. The cumulative distribution function reads:

$$F(t;,\alpha;,\beta) = 1 - \exp\left(-\left(\frac{t}{\alpha}\right)^{\beta}\right) \tag{14.12}$$

The other functions follow from this as discussed before. A three-parameter version employs a threshold parameter δ as lower limit for t, in which case t in the equation above is replaced by t–δ. A one-parameter version employs only the scale parameter α, and sets β to a fixed value by definition. The Weibull mean θ is:

$$\theta_{\mathrm{Weibull}} = \alpha \cdot \Gamma\left(1 + \frac{1}{\beta}\right) \tag{14.13}$$

Here Γ is the Gamma distribution. If β ≥ 1 then θ closely resembles α with a maximum deviation of −11.4% at β = 2. The ratio θ/α runs up fast with decreasing β < 1.

The exponential distribution can be regarded as such a one-parameter Weibull distribution with β = 1 per definition. In that case the scale parameter is often denoted as θ. This is the mean failure time, also called the characteristic time. Again, analytical expressions exist for all basic functions and the cumulative function reads:

$$F(t;\theta) = 1 - \exp\left(-\frac{t}{\theta}\right) \tag{14.14}$$

An important property of the Exponential distribution is its constant hazard rate h(t) = h = 1/θ. Working devices having a constant hazard rate do not suffer from child disease, nor from wear-out. Failure is random in time ('bad luck') and often due to external impact like cosmic radiation or mechanical impact, if unrelated to the use and age of the device, etc. Another situation where the Exponential distribution applies is where maintenance is carried out to keep the hazard rate at a more or less constant level. This constant hazard rate implies an Exponential distribution.

3.3 Competition of Processes and Mixed Subpopulations

A batch of devices under test will often suffer from multiple processes. For instance, devices may be simultaneously exposed to child mortality processes (e.g., an ongoing cracking or chemical reaction originating from the production process, random failure (e.g., cosmic radiation), and wear (e.g., electrochemical degradation). Additionally, the batch of devices may be inhomogeneous if there are multiple subpopulations with distinct failure behavior. For instance, a certain subpopulation may contain defects from production that leads to failure behavior that is not found for the rest of the population. Competing processes and mixed populations lead to more complicated failure distributions.

As for competing processes, the functions F_{total}, etc., representing the total distribution are illustrated with the case of two competing processes A and B. A device only survives if it survives both A and B. From this principle follow:

$$R_{\text{total}} = R_A \cdot R_B \tag{14.15}$$

$$F_{\text{total}} = 1 - R_{\text{total}} = F_A + F_B - F_A \cdot F_B \tag{14.16}$$

$$f_{\text{total}} = \frac{\mathrm{d}F_{\text{total}}}{\mathrm{d}x} = f_A \cdot R_B + f_B \cdot R_A \tag{14.17}$$

$$h_{\text{total}} = h_A + h_B \tag{14.18}$$

$$H_{\text{total}} = H_A + H_B \tag{14.19}$$

It is noteworthy that items that fail by process A still bear information about process B. The reason is that the moment they fail at time t_A by process A, then it is also clear that they survived process B up to t_A. This finding should be taken into account with the data analytics for process B (see also the section on censored data).

In case of mixed subpopulations a similar set of equations can be derived. Let us assume each subpopulation is indexed with 'i' and forms a fraction p_i of the combined population. The sum of the fractions p_i amounts to 1. This leads to the following:

$$F_{\text{total}} = \sum_i p_i \cdot F_i \tag{14.20}$$

$$R_{\text{total}} = \sum_i p_i \cdot R_i \tag{14.21}$$

$$f_{\text{total}} = \sum_i p_i \cdot f_i \tag{14.22}$$

$$h_{\text{total}} = \frac{f_{\text{total}}}{R_{\text{total}}} = \frac{1}{R_{\text{total}}} \sum_i p_i \cdot f_i \tag{14.23}$$

$$H_{\text{total}} = \int \sum_i \left(p_i \cdot \frac{f_i}{R_{\text{total}}} \right) \mathrm{d}t \tag{14.24}$$

The hazard rates are less straightforward than in the case of competing processes.

As discussed, the total distribution can be deduced conveniently if the distributions of the competing processes and fractions of subpopulations are known. Unfortunately, starting from observed failure times without further knowledge, it may be much harder to disentangle distributions and reconstruct the competing processes and mixed subpopulations. Moreover, observation data generally scatter which can give the false impression of entangled distributions. Therefore, objective measures are needed to evaluate whether a study into competing processes and mixed subpopulations is justified.

However, plots may nevertheless reveal entangled distributions if they are sufficiently distinctive. As an illustration consider the case of two mixed subpopulations. Subpopulation 1 has a Weibull distribution with parameters $\alpha = 1$ and $\beta = 0.6$ (which has a character of a child mortality process as $\beta < 1$). Subpopulation 2 follows a Weibull distribution with parameters $\alpha = 20$ and $\beta = 3$ (which has a character of a wear-out process as $\beta > 1$). If these are mixed the resulting distribution functions can be found with Eqs. 14.18, 14.19, 14.20, 14.21, and 14.22. Figures 14.1 and 14.2 show respectively the hazard rate plot and distribution density plot. The figures show the original functions and the total function. The figures also show the observations that can be expected on average if the total population consists of $n_{total} = 13$ test objects ($n_1 = 4$ and $n_2 = 9$). Particularly the distribution density plot gives a clear indication of a mixed case.

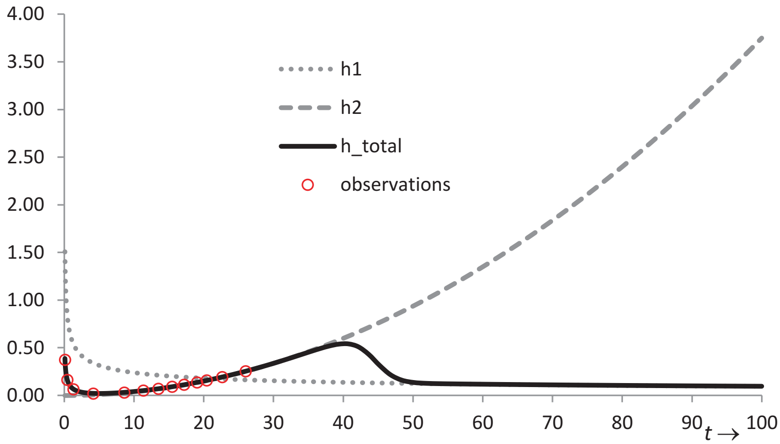

Fig. 14.1 Hazard rate plot of two entangled distributions due to a mixed population

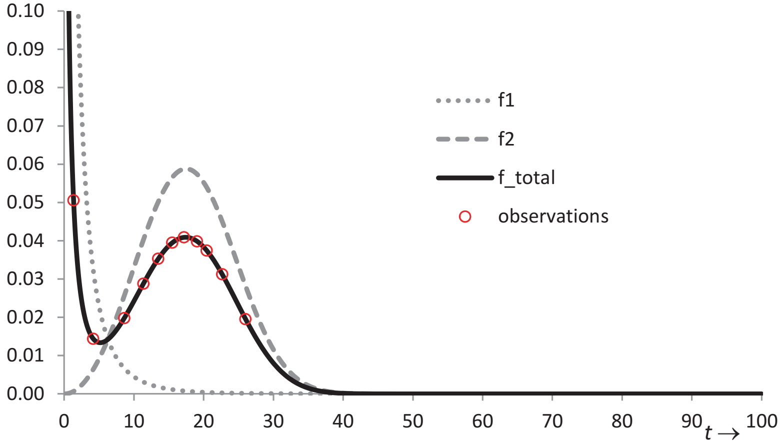

Fig. 14.2 Distribution density plot of two entangled distributions due to a mixed population

Table 14.1 Example composition of batch of devices. The total population consist of two subpopulations ('Normal' and 'Weak subpopulation') with multiple competing processes

Subpopulation	Normal	Weak subpopulation	Weibull parameters
Percentage	90%	10%	
Child mortality or early failures	Present	Present	$\alpha = 10{,}000$, $\beta = 0.005$
Random failure	Present	Present	$\alpha = 200$, $\beta = 1$
Wear-out	Present	Present	$\alpha = 25$, $\beta = 4$
Additional wear out	N.A.	Present	$\alpha = 0.3$, $\beta = 6$

3.4 *Bath Tub Curves and Screening*

Well-known examples of competing processes and mixed subpopulations are often found in so-called bath-tub curves. These are hazard rate graphs plotted as a function of time.

Table 14.1 shows an illustrative example of entangled distributions. The example is based on a case where typically three processes compete in each device, namely: child mortality or also called early failures; random failure; and wear-out failure. The early failures can be due to production errors or curing materials. With time this process loses importance. Wear-out can typically be due to degradation of insulating layers or other changing material properties in the long run. In between is an interval where, e.g., cosmic radiation or other often external influences deteriorate the devices. In this example this would be a set of competing processes that are typical for a 'normal' batch.

In this particular example 10% of the devices have been assigned an additional failure mechanism that causes fast wear-out and competes with the other three processes. This is a subpopulation in the total batch and turns the total batch into a mixed population.

Table 14.1 details the composition of the total. The rules in the previous section can be used to obtain the overall hazard rate for each subpopulation (cf. Eq. 14.18) and subsequently the total hazard rate (cf. Eq. 14.23).

Figure 14.3 shows this example of entangled distributions in a hazard rate plot. The entanglement is due to both competing processes and a mixed population. In various stages the underlying distributions dominate here. In a first stage a child mortality or also called ‘early failure’ process dominates. If such a phase is normal to produced batches of devices, a screening or burn-in process may be applied to prevent the delivery of early failing devices to customers. This process is normally done by applying enhanced stresses beyond rating which causes accelerating aging. This can be conceived as speeding up the consumption of operational life until a point that the hazard rate is reduced to acceptable levels.

A disadvantage of burn-in is that operational life of all devices is reduced. Therefore, the enhanced stress preferably targets the early failure mechanism specifically. Another approach is investigating whether the early failures can be prevented by improved production, after which the burn-in process may be shortened or even skipped, which translates in longer operational life of all devices.

Secondly, the plot shows a peak where the fast wear-out process of the weak subpopulation is dominant. This applies only to a part of the total batch and once this part is eradicated, the hazard rate drops. If batches normally contain such imperfection, a burn-in process may cause too much damage to the healthy part in case the mechanism cannot be targeted specifically. Other strategies may be developed to detect imperfect devices, e.g., by condition indicators such as the occurrence of hot spots, unusual leakage currents, partial discharge, etc.

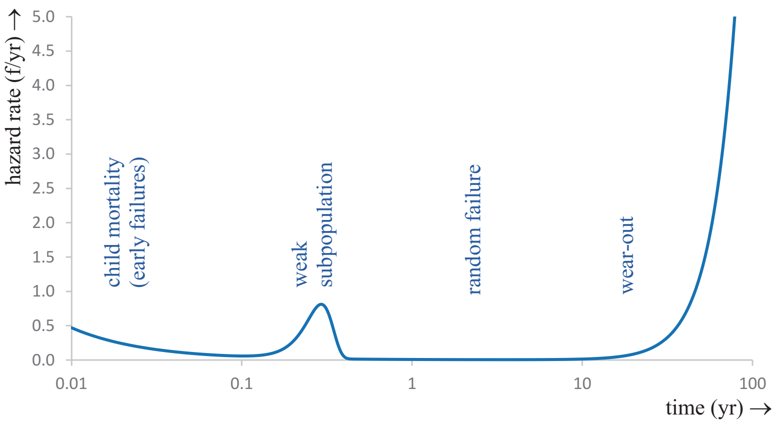

Fig. 14.3 A bath tub curve combined with subpopulation of weak devices based on Table 14.1. This is an example of entangled hazard rates based on competing processes (the bath tub) and a mixed population. The data here are illustrative and can be very different in other cases

The third stage is the desired operational mode where typically only random failures occur, i.e., the ‘bad luck’ region. Such failures may be due to effects that are unrelated to the age of the device, such as solar flames, lightning nearby, etc.

The final stage is the wear-out phase. Material and composition may deteriorate over time due to TEMA (thermal, electrical, mechanical, and ambient) processes. Knowledge of the timing of this wear-out is necessary for predicting product life, warranty, replacement planning, etc.

The same combination of distributions is shown in a reliability plot in Fig. 14.4. In this plot three types of screening are illustrated. The first is the full batch without screening which exhibits a considerable number of failures due to the child mortality process. The second graph shows the effect of screening for these early failures. The third graph shows the effect of extra screening until also the weak population is taken out. The resulting cumulative failure distribution $F_{\text{tot},s}(t)$ after screening until $t = t_s$ is obtained from:

$$F_{\text{tot},s}(t) = 1 - \frac{R_{\text{tot}}(t)}{R_{\text{tot}}(t_s)} \tag{14.25}$$

The screening or burn-in process has a great influence on the quality as perceived by customers. Whether or not the weak subpopulation is timely known will determine whether screening is applied and/or other measures are taken to prevent these low-quality devices to be delivered.

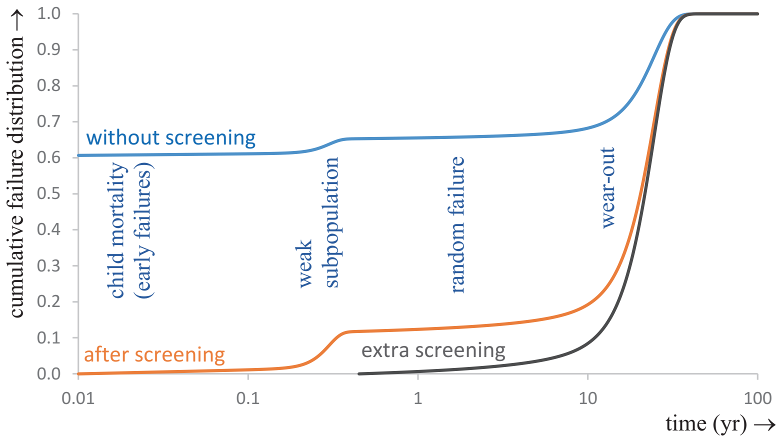

Fig. 14.4 Failure probability graph of the same population as in Fig. 14.3. In addition to the population without screening, also plots are added for screening (up to $t = 0.01$ year) to mitigate child mortality and extra screening (until $t = 0.45$ year) to also mitigate the weak population

4 Data Analytics

Failure data analysis is an important part of R&D and quality assurance. Quite straightforward is the analysis of results if all objects failed in the test and only one mechanism is active that is characterized by a single distribution. In general, many experiments and tests will not reveal all failure data to each failure mechanism. Such hidden data are called censored or suspended data and will be discussed first.

The two main categories of data analytics are graphical analysis and parameter estimation. Both have their distinct value. In the following these two are reviewed after discussing censored data.

4.1 Censored Data

As mentioned above, at the time of evaluation not all test results may be available, e.g., objects may survive a test for a certain time without a failure. This period of survival bears information about the failure distribution even if no failure is observed. Data that remain hidden are called censored or suspended data. Some causes of censored data are: withdrawal of test objects, disrupted tests, unequal starting times, and data from competing processes.

Test objects may be withdrawn from a test for various reasons like noticeably deviating and irrelevant aging conditions, or the data may be regarded unreliable and be discarded for that matter. Tests may also be terminated before all objects fail.

If the lifetime of devices in service is monitored, the tracked devices may not have been taken into operation at the same time. Some may have replaced failed devices from the same batch. This usually also leads to censored data.

If testing or aging in operation involves competing failure processes, then failure of an object by one process means it does not fail by other processes. For the process in question, it is a valid observation, but with respect to the other processes it is censored. Other objects will fail by other competing processes for which the observation is valid. Each failure principally yields a valid observation for one of the processes and is censored for the other competing processes. This implies that the data sets for each process can be significantly censored.

In summary, censoring is a very common phenomenon. It occurs not only in incomplete tests, but also in experiments with competing processes. Proper evaluation of test results requires analysis methods that can deal with censored data and are addressed in the following sections.

4.2 Graphical Analysis

Graphs are visuals that are informative about evolution and scatter of data. They are used to identify outliers, check the family to which the failure distribution might belong, check for entangled distributions, etc. In a graphical representation of observed failure times, it is customary to plot the observations along the horizontal axis. Along the vertical axis ranked theoretical values based on the assumed distribution are plotted. These are often means (or medians) of ranked values from transformed distribution functions like F, R, f, h, or H. These theoretical values are the so-called 'plotting positions' Z and are paired with the individual observations to form the plot. The purpose of the graph should determine the choice of the plotting positions.

Two major analysis families are non-parametric and parametric plots. Non-parametric plots show the distribution as a function of time without assumptions about the distribution. There are various methods. A rather straightforward technique is to plot the cumulative distribution, i.e., the failed fraction, $F(t)$ against time t. Assume a complete set of n failure times t_i ($i = 1,..,n$) is observed. The values for the corresponding fractions F_i are to be defined. Often the data set is treated as an n-sized sample randomly drawn from a (very) large N-sized population of devices. When repeatedly done, the mean fraction F_i of the i^{th} test object can be proven to be:

$$F_i = \frac{i}{n+1} \tag{14.26}$$

A popular alternative for the mean is the median plotting position $F_{M,i}$. If this choice is made, the median plotting position is usually approximated with:

$$F_{M,i} = \frac{i-0.3}{n+0.4} \tag{14.27}$$

A typical example of a non-parametric plot of only the wear distribution in Table 14.1 is shown in Fig. 14.5. No assumptions have been made with respect to the type of the distribution like Lognormal or Weibull.

The parametric plots assume a type of distribution. The plotting position commonly depends on this type in such a way that the graph of the assumed distribution type becomes a straight line. This is very helpful for estimating parameters, for evaluating outliers, for extrapolating the graph to do predictions, etc. The challenge is to find a linear relationship between (functions of) the distribution function and the observed failure times.

For instance, for the Weibull distribution equation Eq. 14.11 can be rewritten as a linear relation between log(−ln(1−F)) and log(t). Other sources give the relation in terms of ln(−ln(1−F)) and ln(t). The choice in the present paper is to use 'ln' as the natural logarithm when taking the inverse of 'exp'. The choice for log or ln can be regarded a matter of taste. The expression for Z is:

$$Z = \log\left(-\ln\left(1-F\right)\right) = \beta \cdot \log\left(t\right) - \beta \cdot \log\left(\alpha\right) \tag{14.28}$$

The slope of such a graph is β and the intercept or initial value is −β•log(α). A graph is prepared by plotting the logarithm of observation t_i along the horizontal axis and

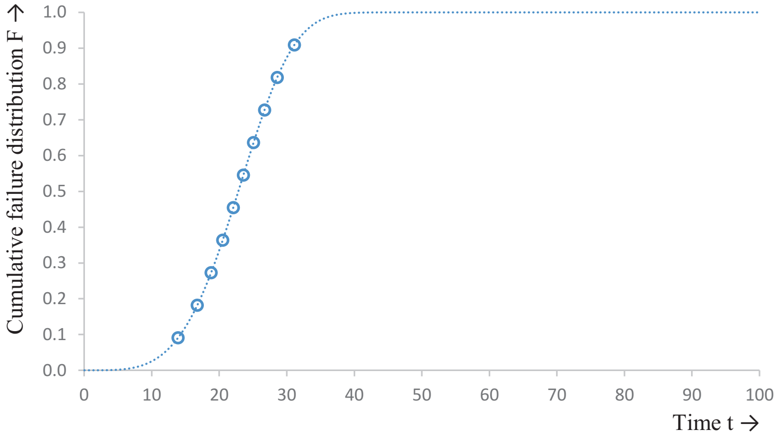

Fig. 14.5 Non-parametric plot with expected F_i plotting position of the wear distribution. The points have coordinates (t_i,‹F_i›). The dotted line would be the best fit

the plotting position can be chosen as the expected values ‹$Z_{i,n}$› and be plotted along the vertical axis. The exact expression for can be found with the Beta function, Sect. 6.2 in [1], as:

$$Z_{i,n} = \left[-\gamma + i \binom{n}{i} \sum_{j=0}^{i-1} \binom{i-1}{j} \frac{(-1)^{i-j} \cdot \ln(n-j)}{n-j} \right] \cdot \log e \tag{14.29}$$

A good approximation for finding ‹$Z_{i,n}$› is [5]:

$$Z_{i,n} \approx \log\left(-\ln\left(1 - \frac{i-0.44}{n+0.25} \right) \right) \tag{14.30}$$

Although Z-values are plotted along the vertical axis, normally the scale is displayed such that it shows F-values. As a result the scale is not linear anymore. An example is shown in Fig. 14.6. Here the same wear data as for Fig. 14.5 were used. From the graph the data can be fit quite well with a straight line, which supports the assumption that the wear-out process follows a Weibull distribution.

As mentioned previously, for various reasons data may be incomplete. If the first r of n observations are available, but the remaining n-r are missing, then this is called ‘right censoring’. The failure times t_i with $i \leq r$ are known and $t_i \leq t_r$. Furthermore, for $i > r$ the times t_i are unknown as yet, but $t_i > t_r$. This typically happens if all objects entered the test simultaneously and the data are evaluated before the test is completed and/or the test is terminated while some objects survive as yet.

For plotting, the consequences are little. The plotting positions for n observations (censored or not) can be determined as above, but the last n-r observations are left open. Particularly parametric plots like Fig. 14.6 can still be conveniently produced.

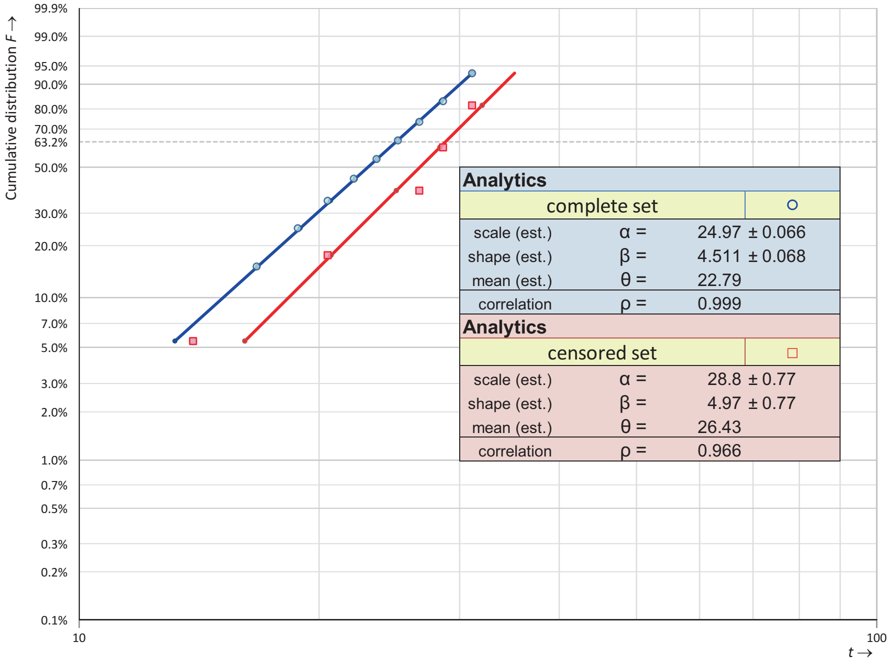

Fig. 14.6 Weibull plot employing a mean plotting position. The observations for the complete set are the same as in Table 14.2 and Fig. 14.5. The censored set is given in Table 14.3

However, in many other cases the censoring can be random: one of more failure times t_i are known, but the censored observations can be larger but also smaller than existing observations. This typically happens when a fleet of devices is taken into operation and failed devices are replaced by new devices. The new ones start their operational life later than other as yet surviving devices and they may fail at a younger or older age than the already failed devices. As a consequence, the ranking of the first r observations of n is no longer fixed. E.g., assume the items in Table 14.2 were each replaced after failure and the test was continued. At some moment, there may be as yet surviving objects with a shorter lifetime than, say, t_4. These surviving objects may fail before or after t_4, which implies that the presently fourth failure ultimately may be ranked higher like the sixth, tenth, or higher. The exact ultimate ranking remains hidden until the data are complete. However, there are methods that estimated the ultimate ranking based on actual observations and the lifetimes of the as yet surviving objects. A widely used method is ‘adjusted ranking’ that adjust the ranking indices i of observed data to adjusted ranking indices I, which no longer need to be integers. The adjusted ranks are calculated as [5]:

$$I(i) = I(i-1) + \frac{n+1-I(i-1)}{n+2-C_i} \tag{14.31}$$

Table 14.2 Observed breakdown times for Fig. 14.5

Index i	1	2	3	4	5	6	7	8	9	10
Time t_i (y)	13.9	16.7	18.8	20.5	22.1	23.6	25.1	26.7	28.6	31.1

C_i is the sum of the number of censored and uncensored observations including the failure i. $I(0)$ is 0 by definition. As an example, assume not all observations of Table 14.2 would be available, but at some time Table 14.3 would apply instead. Failure times are in the second row and surviving times without failure (i.e., the censored failure times) are in the third row. The adjusted rank follows from Eq. 14.31.

The data are included in Fig. 14.6. With data lacking, the fit is not necessarily the same as for the complete set. With increasing number of observations it approaches the fit for the complete data set. In some cases the set will never become complete. For instance if failures occur due to a competing process earlier, the failure time according to the studied process will never be determined. With entangled distributions, this can be very relevant to the data analysis.

There are alternative methods to analyze censored data sets, e.g., [15] and to use adjusted plotting positions (cf. Sect. 5.2.2 in [13]).

4.3 Parameter Estimation

Distributions have parameters and if these are known, various characteristics can be estimated such as the expected lifetime, confidence limits (e.g., a probability that the first out on n devices will fail before a certain time), etc. The parameters are usually not known, but are estimated based on a data set of n failure observations. Various methods exist to estimate parameters.

At least three aspects of parameter estimation are important: bias, efficiency, and consistency. Bias is the systematic error of an estimator. Preferably, the estimator should be unbiased, but if it is known at least, it may be corrected for with an unbiasing operation. The efficiency is related to the scatter or random error. It should be as small as possible and cannot be corrected for, it can be reduced by averaging the results from multiple experiments. A consistent estimator fulfills the requirement of becoming more accurate with increasing sample size such that bias and scatter approach zero.

Two classes of parameter estimators are maximum likelihood (ML) and linear regression using least squares (LS). The latter can be improved by applying weights (WLS). For large sample sizes n these classes are consistent and perform quite comparably. For small sample sizes n the bias and scatter can be significant. This effect should be considered when parameters are estimated with data sets that differ in size n. A comparison may be flawed if the bias depends on sample size.

Both classes optimize an expression to estimate parameters. For the ML method the so-called likelihood function L is maximized, which is a function of the

Table 14.3 A set of observed and censored data. Adjusted ranks are shown in the fourth row. The set is consistent with Table 14.2, i.e., after completion, the test could yield the same results

Rank i	1	2	3	4	5	6	7	8	9	10
Failure t_i (y)	13.9			20.5		22.1			28.6	31.1
Censored (y)		14.0	14.3		21.3		22.3	23.1		
Adj.rank I	1			2.25		3.71			6.14	8.57

parameters u_j ($j = 1,..,m$) to be estimated. For instance, u_1 could be the Weibull scale parameter estimator a of α and u_2 could be the shape parameter estimator b of β. The observations t_i ($i = 1,..,n$) are parameters. More convenient is to maximize ln(L):

$$\ln\left[L\left(u_1,\ldots,u_m,t_1,\ldots,t_n\right)\right] = \sum_{i=1}^{n} \ln f\left(t_i|u_1|\ldots|u_m\right) \tag{14.32}$$

In case of $n-r$ censored data, ln(L) includes a term R^{n-r} that describes the surviving fraction at given $t = \tau$:

$$L\left(u_1,\ldots,u_m,n,r,\tau,t_1,\ldots,t_r\right) = \left(R\left(\tau\right)\right)^{n-r} \cdot \prod_{r}^{i=1} f\left(t_i|u_1|\ldots|u_m\right) \tag{14.33}$$

The parameters can now be found by solving:

$$d\ln\left[L\right] = \sum_{i=1}^{n} \frac{\partial \ln\left[L\right]}{\partial u_i} du_i = 0 \tag{14.34}$$

In case of the Weibull scale parameter this yields (with τ_j the survival time of object j):

$$a^b = \frac{1}{r} \cdot \left(\sum_{i=1}^{r} t_i^{\,b} + \sum_{j=1}^{n-r} \tau_j^{\,b} \right) \tag{14.35}$$

And for the Weibull shape parameter:

$$\frac{\sum_{i=1}^{r} t_i^b \ln\, t_i + \sum_{j=1}^{n-r} \tau_j^b \ln\, \tau_j}{\sum_{i=1}^{r} t_i^b + \sum_{j=1}^{n-r} \tau_j^b} - \frac{1}{r} \cdot \sum_{i=1}^{r} \ln\, t_i = \frac{1}{b} \tag{14.36}$$

These expressions cannot be solved analytically but require a numerical method such as the Newton-Raphson method. The ML method can be extended to handle more complicated situations, such as accelerated aging testing or mixed distributions. Formulas for the bias and scatter exist [13]. There is also extensive literature on the subject and in various studies Eq. 14.36 is modified in order to even more decrease the bias and/or increase the efficiency of b, such as [16]. Bias formulas also conveniently enable to define unbiased estimators.

For (W)LS the (weighted) sum of residues is minimized to obtain estimators u_j. In the following weights w_i are included in the equations. The method is elaborated for the Weibull distribution. For ordinary LS (OLS) all weights w_i can be set to unity. The index 'i' can be replaced by the adjusted rank 'I' in case of censored data. The WLS weights w_i are defined as:

$$w_i = \frac{1}{Z_i^2 - Z_i^2} \tag{14.37}$$

The weights also for non-integer adjusted ranking can be calculated conveniently with an approximation [19]. First two averages are assessed:

$$\overline{Z}_w = \frac{\sum_{i=1}^{n} w_i Z_i}{\sum_{i=1}^{n} w_i} \quad \text{and} \quad \overline{\log t}_w = \frac{\sum_{i=1}^{n} w_i \log t_i}{\sum_{i=1}^{n} w_i} \tag{14.38}$$

Next the Weibull shape parameter b is found as:

$$b = \frac{\sum_{i=1}^{n} w_i \left(\langle Z_i\rangle - \overline{\langle Z\rangle}_w\right)^2}{\sum_{i=1}^{n} w_i \left(\langle Z_i\rangle - \overline{\langle Z\rangle}_w\right)\cdot\left(\log\ t_i - \overline{\log\ t}_w\right)} \tag{14.39}$$

And the scale parameter a as:

$$a = 10^{\overline{\log\ t}_w - \frac{\overline{\langle Z\rangle}_w}{b_{WLS}}} \tag{14.40}$$

The WLS is a special case of the generalized LS (GLS) [2].

The estimators of both are biased except the (W)LS-estimator of $1/b$. As an advantage, (W)LS is consistent with plotting if the expected plotting position $\langle Z_i\rangle$ is used rather than the median plotting position.

The ML and (W)LS methods are widely used. The ML estimators are found by a numerical procedure, whereas the (W)LS method employs analytical expressions. As a consequence, error calculations can be performed for (W)LS in case the observations vary in accuracy. The ML and (W)LS estimators of the Weibull shape parameter β are biased. The same applies to the ML estimator $1/b$. Table 14.4 gives expressions for unbiasing factors and for the standard deviations of these estimators for complete data sets as a function of their sample size n. For example, the factor to unbias the ML estimator b is found to be 4/5.32 = 0.75, 6/7.32 = 0.82, and

Table 14.4 Unbiasing factors and standard deviations (st. dev.) of the ML, WLS, and LS estimators for the Weibull shape parameter, valid for complete data sets. The original estimators are b and $1/b$. The unbiased estimators are b_u and $(1/b)_u$. The true or theoretical parameters are β and 1/β. The formulas are discussed in Chap. 6 of [13]

Estimator b of Weibull shape parameter β			
Method	ML	WLS	LS
Bias	$b_U \approx b \cdot \frac{n-2}{n-0.68}$	$b_U \approx b \cdot \frac{n-2}{n-1.34}$	$b_U \approx b \cdot \frac{(n-2)^{0.9}}{(n-2)^{0.9}+0.68}$
St. dev.	$\sigma_{b_U} \approx \beta \cdot \frac{0.8}{\sqrt{n-3}}$	$\sigma_{b_U} \approx \beta \cdot \frac{0.8}{\sqrt{n-1.2}}$	$\sigma_{b_U} \approx \beta \cdot \frac{0.8}{(n-2.9)^{0.45}}$
Estimator 1/b of the inverse Weibull shape parameter 1/β			
Method	ML	WLS	LS
Bias	$\left(\frac{1}{b}\right)_U \approx \frac{1}{b} \cdot \frac{n-0.18}{n-0.94}$	$\left(\frac{1}{b}\right)_U \approx \frac{1}{b}$	$\left(\frac{1}{b}\right)_U \approx \frac{1}{b}$
St. dev.	$\sigma_{\left(\frac{1}{b}\right)_U} \approx \frac{1}{\beta} \cdot \frac{0.8}{\sqrt{n-1.2}}$	$\sigma_{\left(\frac{1}{b}\right)_U} \approx \frac{1}{\beta} \cdot \frac{0.8}{\sqrt{n-1.2}}$	$\sigma_{\left(\frac{1}{b}\right)_U} \approx \frac{1}{\beta} \cdot \frac{0.9}{(n-0.9)^{0.47}}$

8/9.32 = 0.86 for n = 6, 8 and 10 observations respectively. This signals that estimators with different sample sizes n require unbiasing to make a fair comparison possible. For censored data sets the bias and scatter are worse generally and often unbiasing expressions are as yet to be developed.

5 Stress-Related Lifetimes

Power electronics are designed to operate in a range of stresses like voltage, current, temperature, humidity, exposure to contamination, etc. Usage within a rated range should result in a satisfactory lifetime of the device. However, in the long run devices will age. If enhanced stresses are applied, then this will generally accelerate the deterioration of devices. In normal use the rated stresses should not be exceeded, but in testing this provides a means to assess the quality of devices in a relatively convenient short time. Accelerated aging in tests is very useful, but care must be taken to not enhance the stresses to such an extent that the deterioration becomes unrepresentative of normal operation. At that point a test loses its value.

The Acceleration Factor (AF) is defined as the ratio of the lifetime t_{B0} at a rated level B_0 (i.e., the assumed usage stress level) and the lifetime t_B at an enhanced stress level B. For instance, if a testing period of 2 h at an enhanced stress level is the equivalent of a service lifetime of 50 h, then the AF is 50/2 = 25. Put in other words, 1 h at the enhanced stress level is the equivalent of 25 h at rated stress level. In short:

$$AF = \frac{t_{B0}}{t_B} \tag{14.41}$$

The dependency of *AF* on the stresses varies with the underlying aging mechanism(s), as discussed in the following.

5.1 Power Law

Some stresses are known to influence the lifetime through a so-called power law. Again, if B is a given test stress, B_0 the rated stress, t_B the (expected, median) lifetime under stress B, and t_{B0} the (expected, median) lifetime under stress B_0, then the power law with power parameter p reads:

$$B^p \cdot t_B = B_0^p \cdot t_{B0} \tag{14.42}$$

The acceleration factor of the power law, AF_P, follows as:

$$AF_P = \frac{t_{B0}}{t_B} = \left(\frac{B}{B_0}\right)^p \tag{14.43}$$

Degradation types where the power law usually applies:

- Electric breakdown with electric field E versus rated field E_0.
- Humidity impact in e.g. breakdown with relative humidity RH.

For failure mechanisms that follow a Weibull distribution it can be shown that a power law acceleration only influences the scale parameter α and not the shape parameter β. This can be conveniently demonstrated with the cumulative hazard rate. The distributions at rated stress and test are the same except for the acceleration of time or put in other words the compression of the time scale. The cumulative hazard $H_{B0}(t_0)$ at stress B_0 and $H_B(t_B)$ at stress B relate as:

$$H_{B0}\left(t_{B0}\right) = H_B\left(t_B\right) \Leftrightarrow \left(\frac{t_{B0}}{\alpha_{B0}}\right)^{\beta_{B0}} = \left(\frac{t_B}{\alpha_B}\right)^{\beta_B} \tag{14.44}$$

$$\Rightarrow \ \beta_B = \beta_{B0} \ \wedge \ \alpha_B = \alpha_{B0} \cdot \left(\frac{B_0}{B}\right)^p \tag{14.45}$$

The fact that the shape parameter remains constant is apparent in a Weibull plot where data from different stresses have practically the same slope 1/β, but are shifted along the time axis. Figure 14.7 shows a typical Weibull plot of failure data

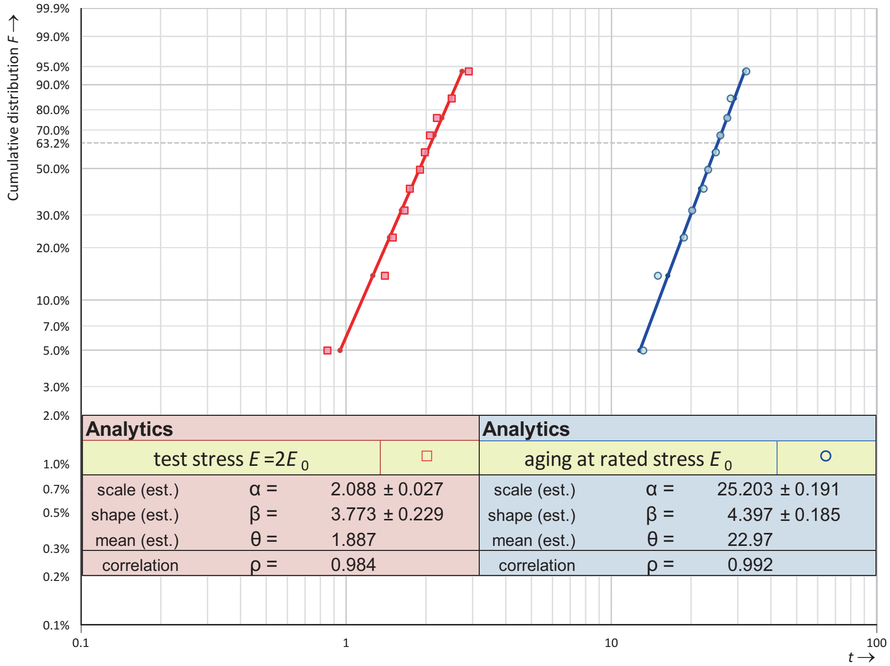

Fig. 14.7 Comparison of failure data at rated stress E_0 and enhanced test stress $E = 2E_0$. The power in the power law is about 3.6 in this case. The shape parameters of the series are similar

under rated stress E_0 and an accelerated aging test with stress $E = 2{\bullet}E_0$. AF is 2^p in this example (Eq. 14.43).

If graphs of distributions at various stresses do not run more or less parallel in a Weibull plot, this may indicate that, e.g., the stress enhancement leads to unrepresentative degradation, multiple mechanisms play a role, the test objects are different, or that the sample sizes n of the test series significantly differ. The last situation may be corrected for with the equations in Table 14.4. Comparing the shape parameters from tests at different stresses is a good check whether the application of a power law is justified.

The purpose of accelerated testing is often to estimate the expected life t_{B0} under rated stress B_0 with Eq. 14.43 and given t_B and AF. This requires knowledge of p.

The parameter p may be known from experience. If not, it can be determined experimentally from tests at various fixed stresses B_i or from ramped stress tests. The latter involves a series of tests in which the stress B_i is linearly increased at a rate c_i with time. On a logarithmic scale the ramped stresses $B(t) = c{\bullet}t$ at various rates c show as parallel lines (cf. Fig. 14.8). The stresses $B(\tau_i)$ at mean breakdown times τ_i form the power law line for ramped stresses that runs parallel to the power law line for fixed stresses. The slope of these lines is $-1/p$ which provides an estimator for p.

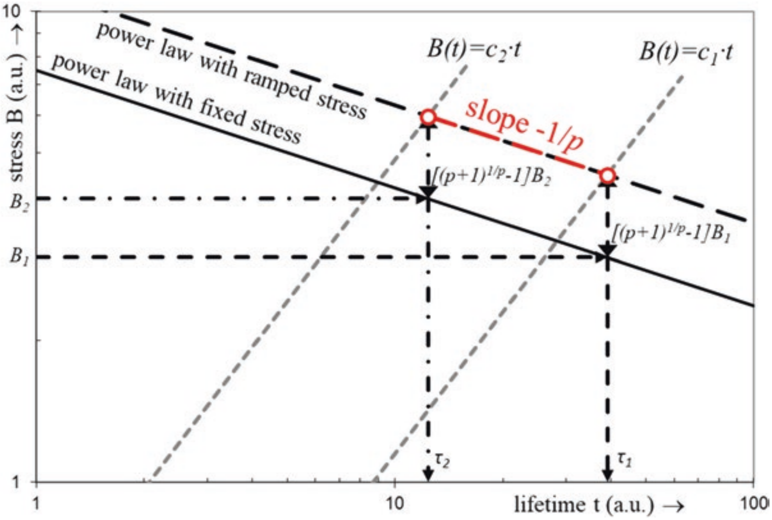

Fig. 14.8 Experiment to estimate the parameter p of the power law. In two or more tests the stress B is ramped up linearly in time. The combined mean or median breakdown times at the various ramped stress tests yield the power law line for ramped stress. The figure also shows the power law line for fixed stresses

5.2 Arrhenius Law

Some *AF*s have an activation energy relationship with the applied stress. Particularly temperature follows such an exponential law, that is called the Arrhenius law and is also referred to as the Eyring model. This type of *AF* is often encountered in thermal and chemical processes. According to the Arrhenius law the rate constant k of a chemical reaction depends on the absolute temperature T as:

$$k = A \cdot \exp\left(\frac{-E_a}{k_B T}\right) \tag{14.46}$$

Here A is the pre-exponential factor, k_B is the Boltzmann constant ($8.62 \bullet 10^{-5}$ eV/K) and E_a is a so-called activation energy (eV) that is related to an energy barrier to be overcome for the process to take place. With respect to reliability, such reactions can lead to degradation and failure. If applied to degradation of devices, the reaction rate is directly related to the failure rate in this model. When comparing aging at different temperatures, a reference or rated temperature T_0 (K) and a test temperature T (K) may be defined. The acceleration factor AF_T at T is then given by:

$$AF_T = \exp\left[\frac{E_a}{k_B} \cdot \left(\frac{1}{T_0} - \frac{1}{T}\right)\right] \tag{14.47}$$

For voltage stress a similar law is found to describe the physical damage on oxide layers. This is probably because of heating the layers by leakage currents. If the reference voltage is V_0 and the test voltage V, then the acceleration factor AF_V is found to be:

$$AF_V = \exp\left[\gamma \cdot \left(V - V_0\right)\right] \tag{14.48}$$

Similar to the power law, an enhanced temperature reduces the scale parameter and leaves the shape parameter untouched. Again, this may be employed to check whether an accelerated aging test remains representative. Transitions in material structure may have a large impact and limit the applicability of the Arrhenius law. For instance, relatively abrupt changes like the glass transition affect reaction rates (much) more than the Arrhenius model predicts [11].

The parameters of the Arrhenius law A and E_a are usually found by plotting the logarithmic reaction or hazard rate against the applicable aging parameter like temperature or voltage. The case of temperature follows from Eq. 14.46:

$$\ln k = \ln A - \frac{E_a}{k_B} \cdot \frac{1}{T} \tag{14.49}$$

Therefore, the slope of this plot is $-E_a/k_B$, from which E_a follows.

5.3 Combined Accelerated Aging Factors

Some processes are influenced in various ways. For instance aging of dielectric layers may be accelerated not only by increasing the electric field, but also by an increase in temperature and humidity. If these factors act independently, the combined acceleration factor AF_{combi} is the product of the separate factors AF_i:

$$AF_{\text{combi}} = \prod AF_i \tag{14.50}$$

As an example, an AF due to combined independent influences of an electric field, relative humidity, and temperature would yield:

$$AF_{E,RH,T} = \left(\frac{E}{E_0}\right)^{p_E} \cdot \left(\frac{RH}{RH_0}\right)^{p_{RH}} \cdot \exp\left[\frac{E_a}{k_B} \cdot \left(\frac{1}{T_0} - \frac{1}{T}\right)\right] = \frac{t_{E_0,RH_0,T_0}}{t_{E,RH,T}} \tag{14.51}$$

This can be extended with other factors. If the influences are not independent, then a tempering, but also often a synergistic effect may occur. E.g. a higher humidity may cause ions to be dissolved. The higher mobility or greater availability of ions may increase the rate of a reaction. As a second example, a higher electric field may help ions to overcome electrostatic barriers. The increased presence of ions in an insulating layer increases the hydrophilicity and changes the dielectric properties ε' and ε''. That can have a synergistic effect where leakage currents play a role.

It should be noted that the action of combined stresses as discussed above applies to a single process. In case of competing processes and mixed subpopulations, the effects are evidently more complex.

6 System Lifetime

Many applications of power electronics concern a system or assembly of components. For instance, an HVDC converter may consist of various modules, one or more power supplies, filters, etc. Not only the component reliabilities, but also the exact configuration determine the system reliability R_S.

The basic building blocks for a system configuration are series and parallel connections. A block diagram visualizes the dependency of the system reliability on the component reliabilities. The diagram consists of a starting point (usually not indicated explicitly) on the left of the diagram. From this point a network of blocks and interconnecting lines or arrows develops towards an exit. If all elements between start and exit are functioning, then the system works, otherwise it does not. The former state is the working or up state, the latter the failed or down state.

Figure 14.9 shows the diagram of a series system. The system failure distribution follows the same set of rules as for the case of competing processes. In a series system the processes of all components compete. Some rules to calculate the distribution functions of a series system are:

$$R_S = \prod_i R_i \tag{14.52}$$

$$h_S = \sum_i h_i \tag{14.53}$$

$$H_S = \sum_i H_i \tag{14.54}$$

It can be seen that the hazard rate of a series system is always larger than that of each of its components. A general lesson is that longer series configurations tend to increase failure rates. This is why keeping systems as simple as possible may be a good strategy to make them more reliable. A chain is not as weak as its weakest link; it is weaker. This is because it may also fail due to another link.

Fig. 14.9 Block diagram of a series system

A special case of a series system is the situation where the system length is a variable. What if a connection would have M times the length of another connection with length L and otherwise the same properties? From Eq. 14.53 it follows that the hazard rate is increased with a factor M. It depends on the character of the distribution how this translates into lifetime. With an exponential distribution the mean lifetime is the inverse of the hazard rate and it would thus be reduced with a factor M. For a Weibull distribution the shape parameter would remain the same, but the scale parameter would be reduced:

$$\beta_{M \cdot L} = \beta_L \quad \text{and} \quad \alpha_{M \cdot L} = \frac{\alpha_L}{M^{1/\beta}} \tag{14.55}$$

As β remains constant with varying M, the ratio of θ and α also remains constant (cf. Eq. 14.13). Therefore, θ changes with the same factor as α with varying M.

Figure 14.10 shows the diagram of a parallel system. It is down only if all parts fail. Some rules to calculate the distribution functions for a parallel system are:

$$F_S = \prod_i F_i \tag{14.56}$$

$$R_S = 1 - F_S = 1 - \prod_i (1 - R_i) \tag{14.57}$$

$$h_S = \frac{1}{R_S} \cdot \frac{\partial F_S}{\partial t} \tag{14.58}$$

The reliability and hazard rate are more complicated than in the series system case. If it comes to a parallel system of n identical components following an Exponential failure distribution with mean lifetime θ, then the mean system lifetime θ_S can be shown to be (Sect. 7.6 in [13]):

$$\theta_S = \theta \cdot \left(1 + \frac{1}{2} + .. + \frac{1}{n} \right) \tag{14.59}$$

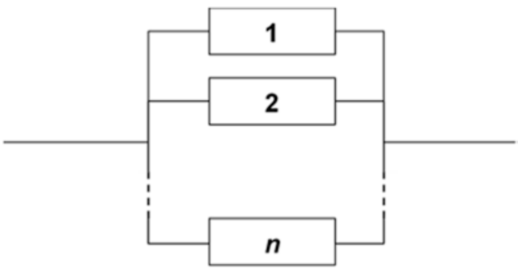

Fig. 14.10 Block diagram of a parallel system

This relates redundancy to mean lifetime. For instance, the electrical connection of TO-220 packaged devices to the top-side of the chip consists of two parallel Al wire-bonds [11]. Provided each wire is able to fulfill the function, this is a redundant connection with $n = 2$. There can be various processes that would disrupt this connection. However, if these would fail at random and independently then the mean lifetime of the wire pair is 150% of the mean lifetime of a single wire connection. In practice there are also other aspects to consider like thermal impact on degradation. In those cases, the Exponential distribution may not be applicable.

Systems may be more complicated. It may not be possible to analyze it in terms of pure series and parallel systems. Methods like the so-called conditional method, truth tables, and minimal path method can be applied to analyze the reliabilities (e.g. [12], Chap. 7 in [13]).

6.1 Reparability Considerations

Many power electronic modules are used with a run-to-failure maintenance strategy. This means the devices are operated until failure at which moment the system is down unless the system was equipped with redundancy. After failure the module is replaced to lift the state of failure. This type of maintenance is called corrective maintenance.

So far, corrective maintenance is quite standard on electronic devices and circuits because there is often little preventive maintenance that can be done, except for auxiliary systems such as the cooling. However, with Reliability 4.0 the ambition grows to monitor devices, and rather than to run until failure, the condition of circuits may be monitored as well. This would open the door to health index methodologies and timely replacement of modules to prevent system outages (Ross & Koopmans, Timely Detection of Non-compliance, [14]). This is particularly possible if redundancy is available and problematic modules can be exchanged without interrupting services. Such maintenance strategies are standard in infrastructures for energy and water, and may increasingly be applied in the domain of power electronics.

The reliability of components that cannot be repaired is usually described in terms of the survival function $R(t)$ and/or expected lifetime θ. Reparable systems are subject to an equilibrium between up and down states. For that purpose the availability is defined as the ratio between up time and total time (i.e., up time plus down time). Apart from the failure rate(s) also repair rate(s) are considered. Where reliability focuses on mean-time-until-failure, with availability mean-time-between-failures becomes important. The analysis of availability is often carried out with concepts as system states, Markov chains and Laplace transforms to solve the differential equations that describe the balance between failure and repair and the effect of absorbing states (i.e., unrepairable states).

With the upcoming smart systems, asset management strategies like condition-based and risk-based maintenance with concepts like health index and risk index may also become more important.

7 Disentangling Combined Distributions

So far, various aspects of reliability degradation of power electronics materials were discussed. If only a single process is active, the failure data analysis can be quite straightforward. However, the situation is more complicated if:

- Various processes can compete within each test object (cf. Figs. 14.3 and 14.4). The rules to calculate the resulting distributions are given in Eq. 14.15 through Eq. 14.19 for two processes. These can conveniently be extended to multiple processes starting from Eq. 14.15. An important example of competing processes is the conventional bath tub curve.
- Populations can be a mix of subpopulations with different processes (cf. Figs. 14.1 and 14.2). The rules to calculate the resulting distribution of a mixed population are given in Eq. 14.20 through Eq. 14.24. An important example is a subpopulation of defect items mixed into a batch of good products.
- When tests are terminated before all items fail and/or when multiple processes compete, there will be censored data (cf. Fig. 14.6). This means that the data sets for at least one (if not all) processes are incomplete. Times until which items were observed to survive, are relevant data and should be involved in the statistical analysis.
- Enhanced stresses speed up degradation (cf. Fig. 14.7). This acceleration depends on various circumstances such as the applied materials, stresses, types of degradation, ambient aspects, etc. In various cases the acceleration factors can be described by laws and characteristic parameters. Discussed are the power law and the Arrhenius law. The parameters associated with these laws differ from process to process.

Single processes are associated with single distributions. The discussed aspects can result in more complicated distributions. These can shift in time by enhanced stresses. However, since various processes can have a different acceleration factor, the appearance of the total distribution can change as well. We used the expression 'entangled distributions' for this combination.

If data are collected in such a complicated situation, the challenge is to recognize that situation and to disentangle the distributions. A general strategy is to firstly assume a model and secondly evaluate whether the observations match the model. For instance, one might assume that a bath tub model of competing processes applies. A basic bath tub model would consist of a child mortality process, a random process, and a wear-out process (cf. Fig. 14.3 without the weak subpopulation). This might be described with two Weibull-2 distributions and an Exponential distribution in the middle of the operational life. This would involve five parameters (α_1, β_1, θ,

α_2, β_2). Based on the observations, the parameter values must be estimated that best fit the assumed model.

Thirdly, another model might be proposed for which also the parameter values are estimated that fit that model. The more complicated the model, the more parameters will be involved. These are not only distribution parameters, but also the fractions p_i in case of subpopulations and parameters in acceleration factors if applicable. These parameter sets can grow rapidly. Therefore, models must be chosen wisely.

Instead of crunching the numbers to produce best-estimated parameters, a graphical approach might be used to explore models. Figs. 14.1 and 14.2 are examples of two mixed subpopulations. The Weibull plot of the total cumulative distribution with the original distributions is shown in Fig. 14.11. The observations are ideal in this illustrative example in the sense that the expected observations are used as data. Naturally these data are not obscured by scatter as practical data would. All figures indicate a mix of a child mortality type and a wear-out type of distribution. If confidence intervals were drawn, a part would probably exceed the boundaries. Such patterns can be recognized by eye or pattern recognition methods and would suggest to use a two-distribution model.

As a rule of thumb, an upward bend indicates a competition where a process at some moment in time starts to take over. In contrast, an zigzag shape indicates a mixed population of which a weak subpopulation is depleted (see also the example in Figs. 14.3 and 14.4). It is also possible that upper levelling off part of the zigzag shape is not yet visible in the observations (cf. Fig. 14.11). So, care must be taken when interpreting these shapes.

There are two optimizations required in this approach. Firstly, the best-estimated parameters require an objective method. With a single distribution ML and (W)LS methods are commonly used. For entangled distributions, it must be considered what is the most effective way to estimate parameters. Number crunching on the

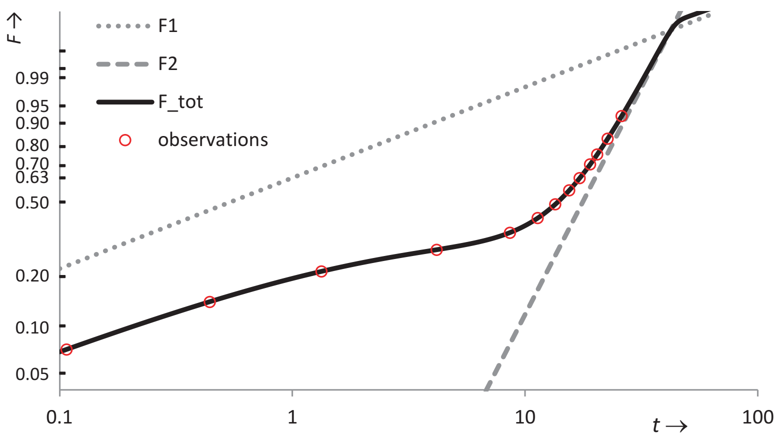

Fig. 14.11 Weibull plot of two entangled Weibull distributions and their combination due to a mixed population. See Figs. 14.1 and 14.2 for the corresponding hazard rate and distribution density. Note, the vertical scale is actually the Z-scale (cf. Eq. 14.30)

complete set is one way. Another way is to divide the data into subsets and subsequently estimate parameters for each data subset (acknowledging censored data). Examples employing the ML method are described in (Nelson, Applied Life Data Analysis, [9]).

Secondly, alternative models may be compared and the choice of model that matches the data best should also be based on objective grounds. Various measures may be used to quantitatively evaluate the best model and its parameters. Such measures include a range of goodness of fit tests [6] and measures for the difference between observed and expected data such as L2 (sum of squared residues) and the similarity index [17].

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