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Cost-effective design of long spatially variable soil slopes using conditional simulation

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ABSTRACT

The three dimensional nature of soil spatial variability implies the need for 3D analysis of geotechnical structures. This paper presents the probabilistic analysis of long slopes such as levees and highway embankments, which are usually analysed unrealistically in plane strain, thereby ignoring the discrete failure mechanisms often encountered in practice. Conditional random fields of soil heterogeneity have been generated based on 3D Kriging, so that they match measurement data at borehole locations and honour the spatial correlations of the soil properties. A simple example involving the cost-effective design of an excavation in a 3D clay deposit has been investigated. It has been demonstrated that, by using conditional random fields within the random finite element method, more cost-effective geotechnical designs can be achieved while maintaining the same calculated reliability.

INTRODUCTION

In recent years, three dimensional reliability assessments concerning the stability of long 'linear' soil structures have been gaining increasing attention (Spencer and Hicks, 2006; 2007; Spencer, 2007; Hicks et al., 2008; Griffiths et al., 2009; Hicks and Spencer, 2010; Vanmarcke, 2011; Li et al., 2013; 2015a; 2015c; Vanmarcke and Otsubo, 2013; Hicks et al., 2014; Li and Hicks, 2014; Ji and Chan, 2014; Ji, 2014; Li et al., 2016; Xiao et al., 2016; Varkey et al., 2016). The reasons for this are three-fold: (1) the three dimensional nature of soil spatial variability necessitates 3D analysis of geotechnical structures, as this is more realistic than a plane strain analysis which ignores the discrete 3D failure mechanisms generally encountered in practice; (2) the increasing computational power makes 3D analyses possible (Li et al., 2015b); (3) the increasing attention paid by regulatory bodies in asking for rational risk assessments and cost-effective design of important infrastructures, e.g. as in levee/embankment design and maintenance in the Netherlands.

However, the spatial distribution of related measurement data were not utilised to constrain the random fields of soil properties in the 3D geotechnical applications mentioned above. Only partial use was made of available data, with field data at the measurement locations only being used to derive statistical properties for input into random field generation (i.e. without being directly used in the simulation). As this results in an exaggerated range of responses in the analysis of geotechnical performance, it is desirable to make more effective use of field data.

Unconditional random fields can easily be conditioned to known measurements by Kriging (Journel, 1974; Delfiner and Chiles, 1977), and conditional simulations using Kriging have long been available in geostatistics in the field of reservoir engineering and hydrogeology (Delhomme, 1979; Clifton and Neuman, 1982). However, the application of conditional simulation in geotechnical studies has been limited, although some 2D exceptions include, e.g., Vanmarcke and Fenton (1991), Van den Eijnden and Hicks (2011), Lloret-Cabot et al. (2012; 2014), Van den Eijnden et al. (2017). This is mainly due to the smaller amount of data generally available in geotechnical projects (e.g. compared to hydrogeology). However, it is desirable to use conditional simulation to make more realistic geotechnical performance predictions. Hence, following the previous 2D work of Van den Eijnden and Hicks (2011) and Lloret-Cabot et al. (2012), this paper applies conditional simulation in three dimensional space, in order to reduce uncertainty in the field when cone penetration tests (CPTs) (De Gast et al., 2017) are carried out.

In this paper, a simple illustrative example compares different candidate slope designs, in order to choose the best (most cost-effective) design satisfying the reliability requirements.

CONDITIONAL SIMULATION OF RANDOM FIELDS

The generation of a conditional random field involves two steps (Journel and Huijbregts, 1978; Frimpong and Achireko, 1998; Fenton and Griffiths, 2008):

(i) Generation of an unconditional random field, $Z_{ru}(\mathbf{x})$, of the spatial variability of soil properties (where $\mathbf{x}$ denotes a location in space);

(ii) Conditioning the random field; e.g. Kriging estimates, $Z_{km}(\mathbf{x})$, based on measured values at $\mathbf{x}_i$ ($i = 1, 2, \ldots, N$) and Kriging estimates, $Z_{ks}(\mathbf{x})$, based on unconditionally (or randomly) simulated values at the same positions $\mathbf{x}_i$ ($i = 1, 2, \ldots, N$), where N is the number of measurement locations, are combined with $Z_{ru}(\mathbf{x})$ from step (*i*) to give the conditional random field, $Z_{rc}(\mathbf{x})$, where

$$Z_{rc}(\mathbf{x}) = Z_{km}(\mathbf{x}) + \left(Z_{ru}(\mathbf{x}) - Z_{ks}(\mathbf{x})\right) \tag{1}$$

The readers are referred to Li et al. (2016) for a detailed implementation of the conditional simulator.

The unconditional random field in step (i) can be simulated using any one of several methods (Fenton, 1994); for example, interpolated autocorrelation (e.g. Ji et al., 2012), covariance matrix decomposition (e.g. Zhu and Zhang, 2013), discrete Fourier transform or Fast Fourier transform (e.g. Fenton, 1994), turning bands method (e.g. Matheron, 1973; Delhomme, 1979), local average subdivision (LAS) (Fenton and Vanmarcke, 1990), and Karhunen–Loeve expansion (Phoon et al., 2002), among others. The LAS method is used in this paper.

Figure 1 shows an example realisation of a 3D conditional random field, together with known CPT profiles taken from the cross-section. It is seen that the conditional field honours the measurement data at the measurement locations. CPT data, i.e. cone resistance and sleeve friction, cannot be used directly in the analyses reported here. A conversion or transformation model is needed to relate the test measurement to an appropriate design property (e.g. the undrained shear strength). As this paper theoretically shows how such converted data can be used, artificial data of undrained shear strength have been used in the following example.

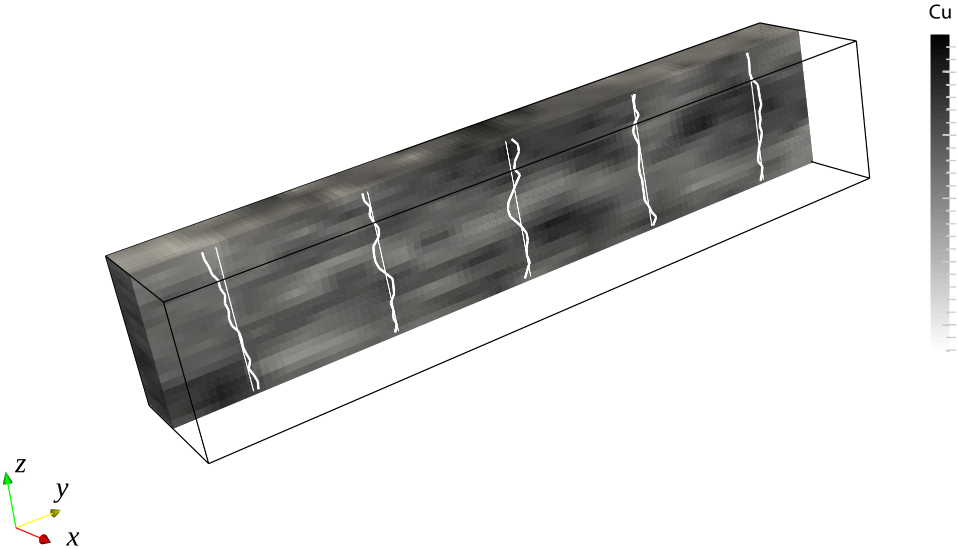

Figure 1. Illustration of conditional simulation of random field of undrained shear strength c_u (black and white indicate high and low values, respectively). Curves indicate CPT locations and measurements.

COST-EFFECTIVE DESIGN OF A LONG SLOPE

The conditional simulator introduced above has previously been used to investigate cost-effective plans for site investigations using the random finite element method (RFEM), i.e. in terms of sampling locations and sampling intensity (Li et al., 2016). For the 3D problem considered, it was shown that the optimum spacing for CPTs was around half the horizontal scale of fluctuation. Moreover, it was demonstrated that the design of geotechnical structures such as slopes and embankments also became more cost-effective as the uncertainty is reduced. In this section, an example involving cost-effective design with regard to slope angle is presented.

Figure 2 shows the *x*-*z* cross-sections (see Fig. 1) of three possible angles of a slope to be constructed (vertical:horizontal ratios of 1:2, 1:1 and 2:1). Also shown are the corresponding finite element mesh discretisations (involving 20-node, 3D elements), the boundary conditions (a fixed base, rollers on the back face preventing *x*-displacements, and rollers on the two ends allowing only settlements), and the location where CPT data were taken prior to the slope construction. The slope is H = 5 m high and L = 50 m long in the third dimension, and the left-hand boundary is taken to be W = 15 m from the slope toe. Five equally spaced CPTs were taken along the length of the slope, at intervals of 10 m (see Fig. 1). The clay soil was modelled by a linear elastic, perfectly plastic Tresca soil model and by a spatially varying undrained shear strength represented by a truncated normal distribution and an exponential covariance function, with a mean μ = 21 kPa, coefficient of variation V = 0.2, and vertical and horizontal scales of fluctuation of θ_v = 1 m and θ_h = 12 m (Spencer, 2007), respectively. The other parameter values were assumed to be deterninistic: elastic modulus E = 100 MPa, Poisson's ratio ν = 0.3, and soil unit weight γ= 20 kN/m^3.

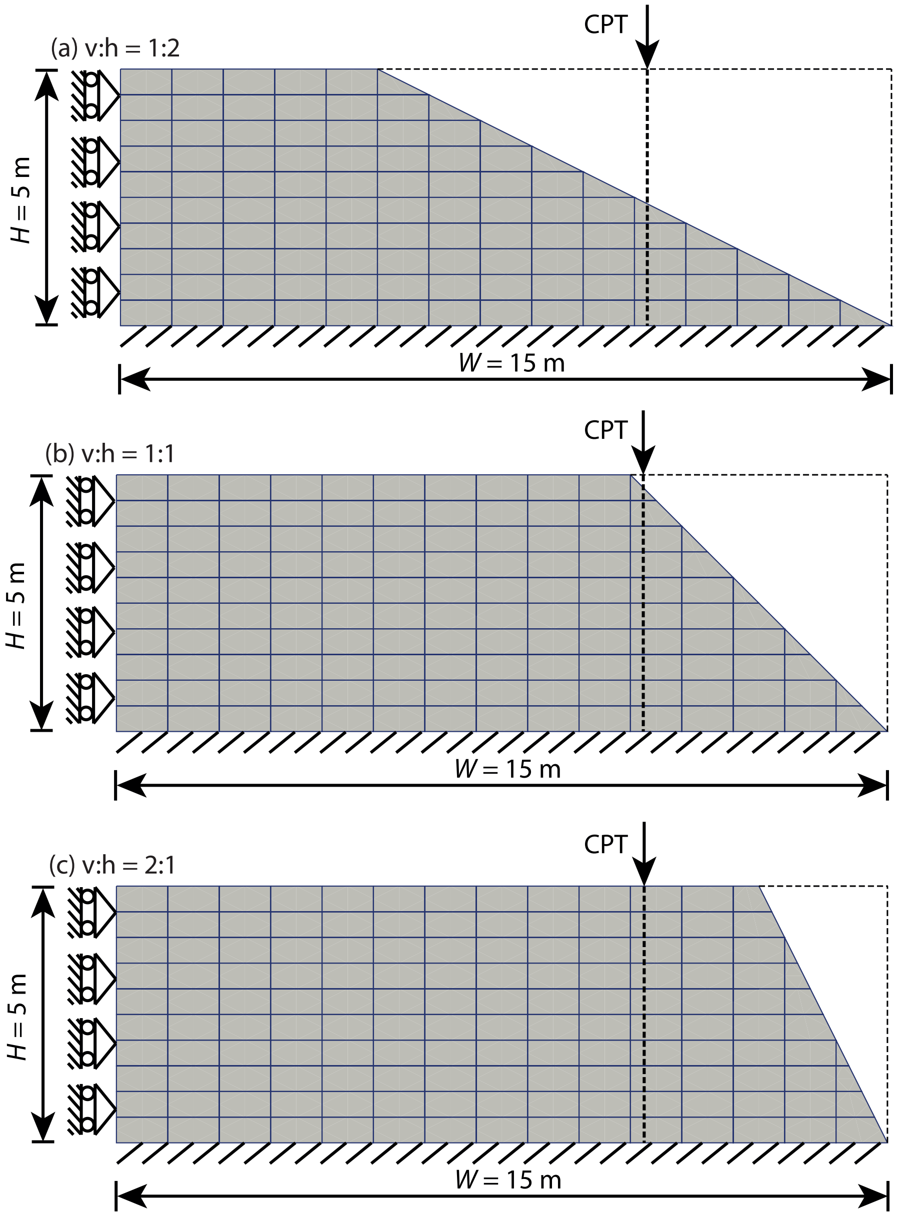

Figure 2. Finite element mesh discretization, problem geometry, CPT measurement locations and boundary conditions for different slope angles.

The slope factors of safety were calculated using the strength reduction method, by applying gravitational loading to generate the in situ stresses (Smith et al., 2013). The analyses have been undertaken within a probabilistic (RFEM) framework; specifically, for each candidate slope angle shown in Fig. 2, two RFEM simulations comprising 500 realisations have been carried out, i.e. a conditional simulation and, for comparative purposes, an unconditional simulation. Note that the CPT measurements used in the conditional random fields in the RFEM analyses are taken from one reference random field that was generated and assumed to represent the 'real' field situation.

Figure 3 shows the probability density functions (fitted normal) of the realised factor of safety (F_R) for the three slopes, for both conditional (cond) and unconditional (uncond) simulations. The deterministic (traditional) factors of safety F_T, i.e. the factors of safety based on the mean property values, are also shown as vertical lines. It is seen that the probability of the 2:1 slope failing down is significant (23%) when unconditional simulation is performed (the probability of failure is the area under the pdf for the realised factor of safety that is smaller than 1.0). It is not surprising that the gentlest (i.e. 1:2) slope has the lowest probability of failure. However, the narrower probability distributions (shown as dashed curves in the figure) demonstrate that conditional simulation significantly reduces the uncertainty in the structural response (i.e. F_R). In particular, the reliability (i.e. 1 – probability of failure) of the steepest slope increases from 77% to greater than 99% when the CPT measurement data are directly taken into account.

A target reliability level of 95% is suggested in Eurocode 7 (2004). The results show that the 1:1 and 1:2 slopes, but not the 2:1 slope, satisfy this criterion if unconditional simulations are used. However, when additional (i.e. actual spatial distribution) information from the CPT profiles is used, even the steepest slope meets the target reliability. Hence, the embankment may be designed to a slope angle of 2:1 if the CPT measurements are directly used in the simulation. This has implications for the soil volume to be excavated (i.e. the volume above the slope faces in Fig. 2) and thereby the cost, although the cost can be dependent on the site-specific situation (e.g. whether there are nearby structures). The 'best design' can be defined as a design that minimises the cost while meeting the requirements set by standards. In this case, the steepest slope which is 95% reliable is the best design.

CONCLUSION

Unconditional and conditional simulations involving differing levels of uncertainty have been carried out to demonstrate the cost effective design of a long slope characterised by a spatially varying undrained shear strength. A 3D (conditional) random field generator, coupled with a 3D finite element model, has been used for this purpose. The model output is described by the variance of the structure response (in this case, the factor of safety), which is a measure of the prediction error resulting from the uncertainty involved. It is shown that the prediction error/variance can be reduced and thereby the confidence in a project's success or failure increased, by making use of the spatial arrangement of measurements (e.g. Cone Penetration Test (CPT), Vane Shear Test (VST)) via conditional simulation. For the problem analysed, a steeper slope was found to be sufficiently reliable (i.e. in line with Eurocode 7). This was in contrast to the finding if unconditional simulation was carried out, where only partial use was made of available measurement data. The potential benefit of a 3D conditional simulation in geotechnical cost-effective designs has therefore been highlighted.

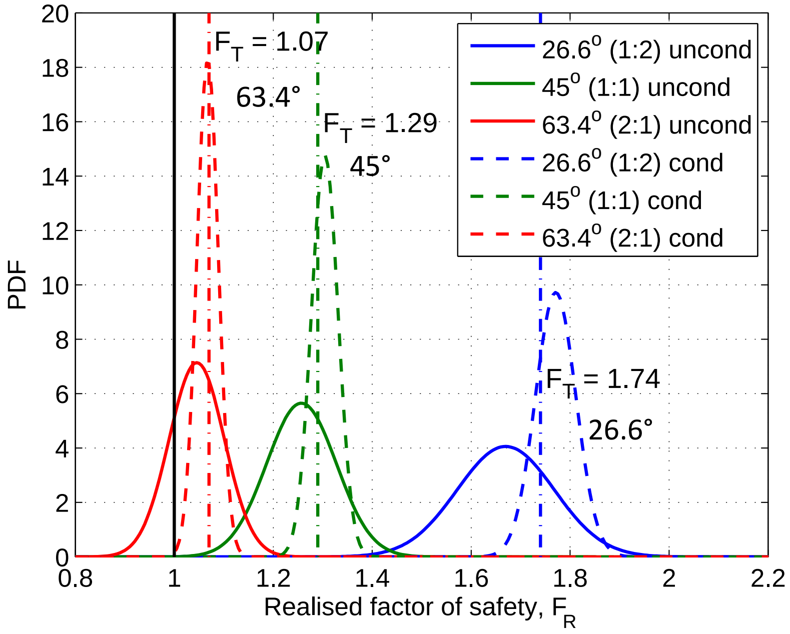

Figure 3. Probability density functions of realised factor of safety for three slopes, based on conditional and unconditional simulations.

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