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## A Flexible Strategy for Efficient Merging Maneuvers of Connected Automated Vehicles

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### ABSTRACT

Merging is a challenging task for automated vehicles. This paper proposes a strategy for connected automated vehicles (CAVs) to guide merging on-ramp vehicles efficiently while ensuring safe interactions with the mainline vehicles. Point-mass kinematic models are used to describe 2-D vehicle motion and receding horizon control is used to generate optimal trajectories of interacting vehicles. The strategy determines the optimal merging time instant for merging vehicles and acceleration of all involved vehicles to minimize deviation from the preceding vehicles' speed, deviation from preferred inter-vehicle gaps, accelerations, and the time spent merging. The strategy builds on a pre-determined order of vehicles passing the conflict zone but is not restricted to fixed merging points as previous research assumes. It resembles human-like behavior in the sense that on-ramp vehicles will accept smaller gaps when approaching the end of the acceleration lane. The performance of the strategy is demonstrated in simulations.

### INTRODUCTION

Connected Automated Vehicles (CAVs), enabled by Vehicle-to-Vehicle (V2V) or Vehicle-to-Infrastructure (V2I) communication, have the ability to bring driving comfort and increase traffic performance (van Arem et al. 2006). The performance of automated vehicles depends on the design of their decision-making systems. For automated vehicles to fully replace human drivers, their controllers should handle different foreseen scenarios in real traffic. On-ramp merging is a common but challenging task for drivers, during which inappropriate maneuvers can lead to traffic perturbation and even crashes. On-ramps are also typical bottlenecks of road networks, where considerable vehicle hours are lost when activated. With the possibility of cooperation among CAVs and coordination between CAVs and infrastructure, the merging process can be improved, leading to safer and more efficient traffic.

Efforts to design cooperative merging strategies have been reported in literature. Wang et al. (2013) proposed to convert merging into a virtual platooning control problem. The conversion is based on mapping an on-ramp vehicle or platoon onto the mainstream lane to form a virtual platoon. The motions of all vehicles in the virtual platoon are controlled using a geometric method to make such that inter-vehicle distances reach a constant value at a fixed merging point (Wang et al. 2013). However, this method seriously affects the traffic stream of the main lane, even though it can give a smooth speed trajectory to the merging vehicle. Another merging strategy is given by designing an upper-level controller and a lower-level fuzzy controller. The upper level controller gives desired inter-vehicles' distances laterally and longitudinally using

the geometric method, and the lower level fuzzy controller functions by managing the vehicles' actuators to follow the given distances (Milanes et al. 2011). Some researchers tried to control the merging process laterally and longitudinally by learning from insects. A vehicle merging strategy based on dragonflies' prey-seeking strategies is proposed (Ito et al. 2016). Formulating merging as a constrained nonlinear optimization problem or using a Model Predictive Control (MPC) method is another stream of approach. A cooperative path generation using an optimal merging point for one on-ramp and one mainstream vehicles is given using MPC (Cao et al. 2015). A longitudinal trajectory planning methodology to facilitate merging process while minimizing the engine effort and passenger discomfort is proposed (Ntousakis et al. 2016). The target of the trajectory planning is, at the final merging point, the speed of the controlled vehicle equals to its putative leader and the desired time-headway is reached. The objectives of minimizing the engine effort and passenger discomfort are reached through minimizing the acceleration and its first and second derivatives of the controlled vehicle. Using Hamiltonian analysis, an analytical optimal solution format is available offline. A similar method is minimizing the square of accelerations to reduce fuel consumption (Rios-Torres e Malikopoulos 2017). Analytic solution can also be achieved. To avoid collision, the potential collision zone comprised of the acceleration lane and the mainstream lane is treated as a merging zone, and only one vehicle is allowed there before its driver takes over to complete the final lane changing maneuver (Rios-Torres e Malikopoulos 2017). This concept of using the merging zone to avoid conflicts is also used when the designed merging strategy aims to reduce travel time and increase average speed by minimizing the standard deviation of accelerations and the opposite values of vehicles' speeds (Xie et al. 2016). A longitudinal and lateral control strategy is proposed by evaluating discrete lane change decisions and continuous accelerations jointly based on receding horizon control and dynamic game theory (Wang et al. 2015). The discrete lane change time instants and directions are given in the prediction horizon according to the predicted desired lane sequence.

The role of infrastructure is also considered in the existing merging strategies. A slot-based method is proposed, in which the CAVs travel within virtual slots generated by traffic management system (Marinescu et al. 2012). For on-ramps with ramp metering, CAVs on the mainstream lane collect gaps based on fundamental diagram theory, and then the ramp metering regulates on-ramp vehicles to use the gaps for merging (Scarinci et al. 2015). A similar strategy is that the infrastructure sends signals to the upstream CAVs to make gaps for the upcoming merging vehicles (Jin et al. 2016).

Summarizing the relevant literature, we find that the majority of existing methods ignore the lateral motion of merging CAVs and only longitudinal motion is controlled. The merging starts when the longitudinal inter-vehicle gaps for merging vehicles reaches a constant or a speed-dependent value at a fixed merging point. The lateral motion is completed by drivers or controlled using a simple geometric method. The merging condition is rigid and has no flexible acceptable gap depending on the urgency of merge, and the merging can fail in dense traffic without sufficiently large gaps. However, human-drivers behave differently. When approaching the terminal of the acceleration lane, drivers will accept smaller gaps (Daamen et al. 2010). This increases the traffic efficiency to a certain degree. Therefore, it is better for CAVs to have human-like behavior to increase the feasibility of finding an acceptable gap and yield gaps for the merger to reach the equilibrium state after merging.

This paper proposes a strategy for CAVs to optimally guide the on-ramp vehicles to the mainline efficiently while ensuring safe interactions with the mainline vehicles. Point-mass

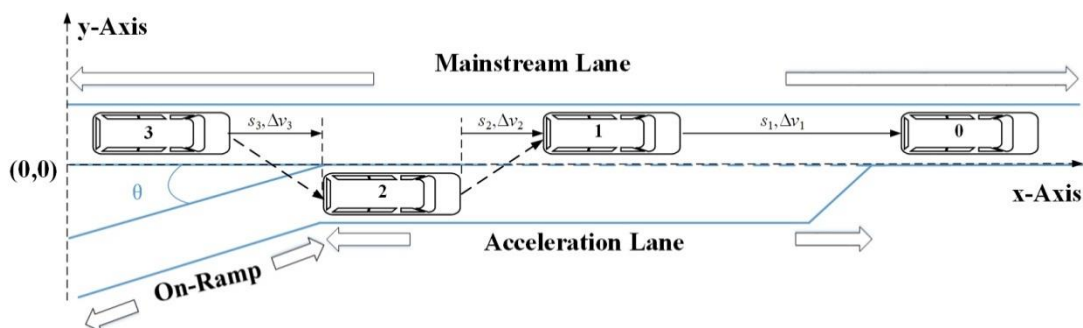
kinematic models are used to describe 2-D vehicle motion and receding horizon control is used to generate optimal trajectories of interacting vehicles. The strategy determines optimal merging time for merging vehicles and accelerations of all involved vehicles to minimize the deviation from the preceding vehicles' speed, deviation from preferred inter-vehicle gaps, accelerations and the time spent on the acceleration lane. The strategy builds on a pre-determined order of passing the conflict zone but is not restricted to fixed merging points. It resembles some human-like behavior in the sense that on-ramp vehicles will accept smaller gaps when approaching the end of the acceleration lane. Performance of the strategy is demonstrated in simulation.

The paper is organized as follows. First, cooperative merging concepts and operational preliminaries are stated. After that, vehicle dynamics models are presented. The cooperative merging strategy design process is given. Finally, the design of simulations and simulation results are illustrated.

### COOPERATIVE MERGING CONCEPTS AND OPERATIONAL PRELIMINARIES

We consider a merging scenario with one mainstream lane and an on-ramp connected with an acceleration lane (as shown in Figure 1). We postulate a future passing order of the vehicles at the conflict zone (Ntousakis et al. 2016). For merging vehicle 2 in Figure 1, the putative leader and putative follower on the target lane are numbered as vehicle 1 and 3 respectively.

The three vehicles are CAVs whose longitudinal and lateral accelerations are controlled. We do not consider communication imperfection, nor any measurement noise in this paper. A receding horizon control method is used to design the merging strategy. At the current time instant, the interacting vehicles communicate their current states represented by the inter-vehicle gap and relative speeds. With that, the controller predicts the future motions of the three vehicles using a model (described in the next section) and decides the optimal acceleration trajectories of all three vehicles and the initiation time of lane change for vehicle 2 over a time horizon  $T^p$ . The decisions are generated to optimize a performance index or cost function of the whole CAV group, reflecting safety, efficiency, and control effort. The lane change of vehicle 2 can be triggered when the predicted gaps to vehicle 1 and 3 are larger than a minimum gap in the predicted future. Only the first sample of the acceleration trajectories is implemented and at the next time instant, the whole procedure is repeated.



**Figure 1. The selected merging scenario with one mainstream lane and an on-ramp connected with an acceleration lane**

### VEHICLE DYNAMICS MODELS

The system we considered consists of the three vehicles shown in Figure 1. The passing order of the three vehicles is as shown by a dotted arrow line in Figure 1. This paper denotes  $x_i$ ,

$y_i, v_{ix}, v_{iy}, a_{ix}$  and  $a_{iy}$  as the longitudinal location, lateral location, longitudinal speed, lateral speed, longitudinal acceleration, and lateral acceleration of vehicle  $i$  ( $i=1,2,3$ ). The merging process is that vehicle 2 moves laterally and longitudinally from the acceleration lane to the mainstream lane while vehicle 1 and 3 move longitudinally. For simplicity, the time argument is dropped where no misunderstanding exists.

The state variable and command variable are defined  $\mathbf{X}=(s_1, \Delta v_1, s_2, \Delta v_2, s_3, \Delta v_3, y_2)^T$  and  $\mathbf{U}=(a_{1x}, a_{2x}, a_{3x}, t_2^l)^T$  separately, where  $s_i = x_{i-1} - x_i - l_i$  and  $\Delta v_i = v_{i-1} - v_i$  ( $i=1,2,3$ ) denote gap (or net spacing) and relative speed of vehicle  $i$  with respect to vehicle  $i-1$  respectively,  $t_2^l$  denotes the merging time instant for vehicle 2 and  $l_i$  denotes the length of vehicle  $i$ .  $y_2$  is a continuous function of  $t_2^l$  as shown in Eq. (1), where  $h$  denotes the lane width. Point mass models are chosen to describe the motion of a vehicle. A second order dynamics is used to represent both the longitudinal and lateral motion of vehicles, as shown in Eq. (2) and Eq. (3).

$$y_2 = \begin{cases} -h/2 & t \leq t_2^l \\ f(t_2^l) & t_2^l < t < t_2^l + t_m \\ h/2 & t \geq t_2^l + t_m \end{cases} \quad (1)$$

$$\dot{x}_i = v_{ix}, \dot{v}_{ix} = a_{ix} \quad (2)$$

$$\dot{y}_i = v_{iy}, \dot{v}_{iy} = a_{iy} \quad (3)$$

The admissible values of lateral and longitudinal accelerations are subject to the physical characteristics of vehicles (Mehar et al. 2013).

To this end, the system state dynamics is as shown in Eq.(4).

$$\frac{d}{dt} \mathbf{X} = (\Delta v_1, a_0 - a_1, \Delta v_2, a_1 - a_2, \Delta v_3, a_2 - a_3, \dot{d}_2^m)^T \quad (4)$$

## COOPERATIVE MERGING STRATEGY DESIGN

This section gives a detailed description of the design of cooperative merging controller for CAVs.

### Cooperative merging control formulation

The objectives of the cooperative merging controller are to efficiently and safely facilitate the on-ramp merging vehicle to merge into the mainstream. Model Predictive Control (MPC) method is applied to design the controller. At each time instant  $t_0$ , the controller solves an optimal control problem as shown in Eqs. (5) and (6).

$$\min_{\mathbf{U}[t_0, t_0+T_p]} J(\mathbf{X}, \mathbf{U}) = \min_{\mathbf{U}[t_0, t_0+T_p]} \int_{t_0}^{t_0+T_p} L(\mathbf{X}, \mathbf{U}) dt \quad (5)$$

$$L = c_1 \cdot \underbrace{\sum_{i=1}^3 \Delta s_i^2}_{safety} + c_2 \cdot \underbrace{\sum_{i=1}^3 \Delta v_i^2}_{efficiency} + c_3 \cdot \underbrace{\sum_{i=1}^3 a_{ix}^2}_{control} + c_4 \cdot \underbrace{t_2^l}_{lane\ switch} \quad (6)$$

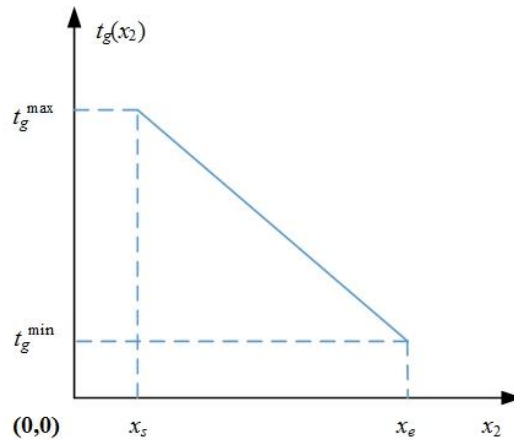
where  $\Delta s_i = s_i - s_i^d$  denotes the gap error, i.e. the deviation of the real gap to the desired gap. The desired gap  $s_i^d$  is determined using constant time gap, i.e.  $s_i^d = v_i \cdot t_d + s_0$ , where  $s_0$  is the minimum gap at standstill.  $T_p$  is the prediction horizon.

The cost function is comprised of safety, efficiency, control, and lane switch costs. With the safety cost, the CAVs have a tendency to reach the desired gap. The efficiency cost implies that

CAVs follows the speeds of their preceding CAVs according to the future passing order. The control cost penalizes large values of desired longitudinal accelerations. The lane switch cost represents the cost for late merging and thus the time spent on the acceleration lane is penalized.

The optimal control problem is subject to the following constraints on state and control variables:

- 1) the system dynamic model of Eq. (2) and (3).
- 2) the initial condition:  $\mathbf{X}(t_0) = \tilde{\mathbf{X}}(t_0)$ , where  $\tilde{\mathbf{X}}(t_0)$  represents the initial state for the controller at  $t_0$ .
- 3) state constraints of speed bound:  $v_i \in [0, v^{\text{limit}}]$ , where  $v^{\text{limit}}$  represent the speed limit.
- 4) admissible acceleration bound:  $a_{ix} \in [a_{\min}^i, a_{\max}^i]$ , where  $-a_{\min}^i$  and  $a_{\max}^i$  denote the largest deceleration and acceleration separately.
- 5) Minimum gap to initiate lane change of vehicle 2: the predicted time gap between vehicle 1 and 3 should be no less than some threshold  $t_g(x_2)$ , which is dependent on vehicle 2's position on the acceleration lane. The chosen simple relationships between  $t_g(x_2)$  and  $x_2$  is linear, as shown in Figure 2, where  $x_s$  and  $x_e$  denotes the start and end locations of the acceleration lane separately, and  $t_g^{\max}$  and  $t_g^{\min}$  denotes the maximum and minimum time gap for the acceptance of the merging CAV and the putative follower during the merging process.



**Figure 2. The linear relationship of desired net gap headway and the location of CAV 2 on the acceleration lane**

The solution to the constrained optimization problem is based on Pontryagin's Minimum Principle. The optimal longitudinal accelerations are first obtained. Then the controller compares the predicted gaps between vehicle 1 and 2 and between vehicle 2 and 3 with the acceptable gap to have the differences in the prediction horizon. The first time instant that satisfies all the differences are non-negative is the optimal merging time instant.

To have the optimal longitudinal accelerations, we use  $X_1 = (s_1, s_2, s_3)^T$ ,  $X_2 = (\Delta v_1, \Delta v_2, \Delta v_3)^T$  and  $S^d = (s_1^d, s_2^d, s_3^d)^T$  and construct its Hamiltonian function as shown in Eq. (7) considering Eqs. (4) and (6).

$$H = c_1 \cdot X_1 - S^d + c_2 \cdot X_2^2 + c_3 \cdot \sum_{i=1}^3 a_{ix}^2 + \lambda_1 \cdot X_2 + \lambda_2 \cdot \sum_{i=1}^3 a_{i-1,x} - a_{ix} \quad (7)$$

where  $\lambda_1$  and  $\lambda_2$  are co-state variables. The necessary conditions for optimality are as shown in Eq. (8).

$$\dot{\lambda}_1 = -\frac{\partial H}{\partial X_1}; \dot{\lambda}_2 = -\frac{\partial H}{\partial X_2} \tag{8}$$

subject to initial state conditions and terminal conditions:  $\lambda_1(t_0+ T_p) = 0$  and  $\lambda_2(t_0+ T_p) = 0$ .

The process then turns to two-point boundary value problem. It is then solved with gradient method (Wang et al. 2015).

**Human-like lane change**

The lateral motion of CAV 2 resembles a human-like behavior. When the lane changing manoeuvre starts, it will follow an empirical human-like lane change path model shown in Eq. (9), where  $t_m$  and  $h$  represent the lane change execution time and the lane width separately (Samiee et al. 2016). The positive value of  $h$  indicates a lane change to the right side of the road.

$$f(t'_2) = \left(\frac{-6h}{t_m^5}\right)(t-t'_2)^5 + \left(\frac{15h}{t_m^4}\right)(t-t'_2)^4 + \left(\frac{-10h}{t_m^3}\right)(t-t'_2)^3 - \frac{h}{2} \tag{9}$$

**SIMULATIONS AND RESULTS**

This section designs two experiments to test the performance of the designed merging strategy to achieve cooperative on-ramp merging with short gap acceptance characteristics.

**Experiment Design**

The merging controller parameters are set as follows:  $c_1 = 0.1, c_2 = 0.5, c_3 = 0.5, c_4 = 0.5, T_p = 6$  s,  $t_d = 1$  s,  $v^{\text{limit}} = 30$  m/s,  $a^i_{\text{max}} = -2$  m/s<sup>2</sup> ( $i=1,2,3$ ),  $a^i_{\text{max}} = 2$  m/s<sup>2</sup> ( $i=1,2,3$ ),  $t_m = 2$  s,  $s_0 = 2$  m,  $t_g^{\text{min}} = 0.25$  s and  $t_g^{\text{max}} = 1$  s. It is the observed minimum acceptable time gap to the putative leader and follower of human-driven vehicles, i.e. the time gap between the putative leader and follower is larger than  $0.5 + l_2/v_2$  (Daamen et al. 2010). The simulation time step is 0.1 s. A feedback delay  $\tau^s = 0.2$  s due to discrete sampling process is introduced in simulation, i.e.  $\mathbf{X}(t) = \mathbf{X}(t - \tau^s)$ . Thus the performance of the strategy is tested against model mismatch. The length of the acceleration lane is 300 m, setting  $x_s = 0$  m and  $x_e = 300$  m.

**Table 1. Initial conditions for experiment 1**

<i>i</i>	$x_i$ (m)	$y_i$ (m)	$v_{ix}$ (m/s)	$l_i$ (m)
1	18	3.5/2	30	4
2	0	-3.5/2	30	4
3	-18	3.5/2	30	4

**Table 2. Initial conditions for experiment 2**

<i>i</i>	$x_i$ (m)	$y_i$ (m)	$v_{ix}$ (m/s)	$l_i$ (m)
1	32	3.5/2	30	4
2	0	-3.5/2	30	4
3	-4	3.5/2	30	4

To prove the efficiency of the controller, the two experiments are designed, and the settings are as shown in Table 1 and

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Table 2. In experiment 1, the initial inter-vehicle distances between CAV 2 and CAV 1 and between CAV 2 and CAV 3 are the same; in experiment 2, the inter-vehicle distance of CAV 2 and CAV 3 is 0, and the merging situation is much more difficult than the first experiment.

The speed of CAV 1 is set constant for two experiments and the simulation time is 30 s.

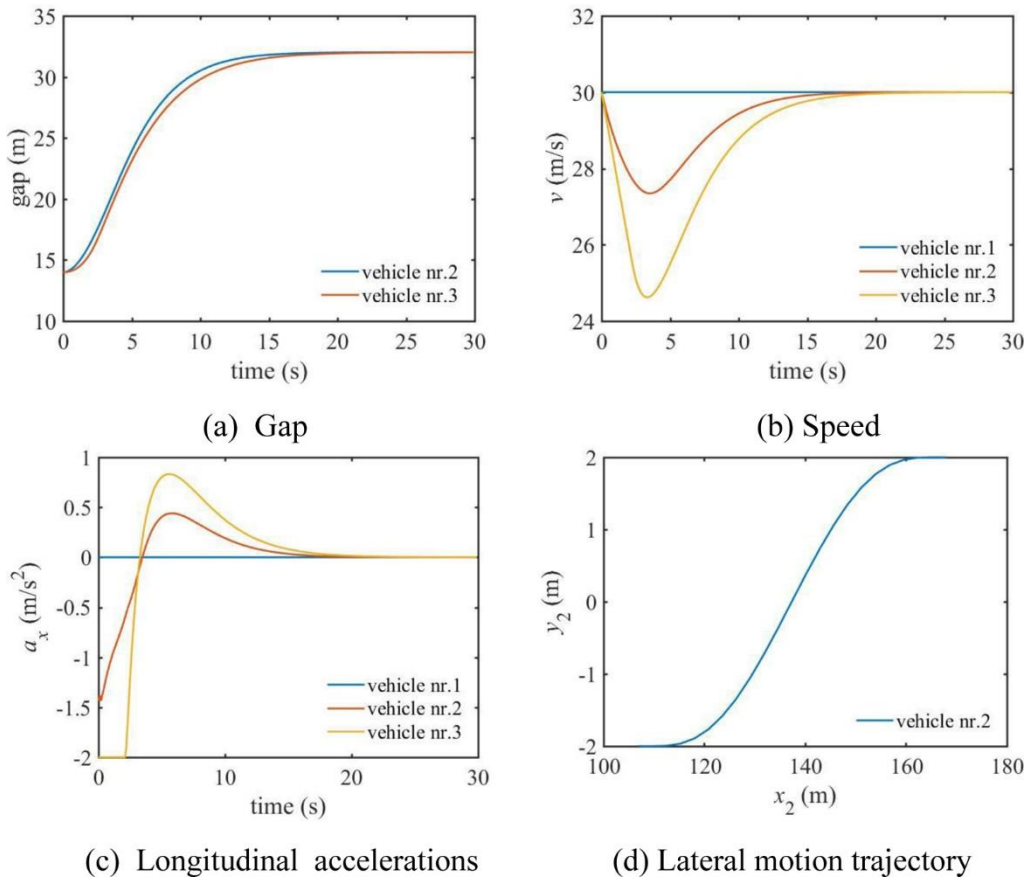
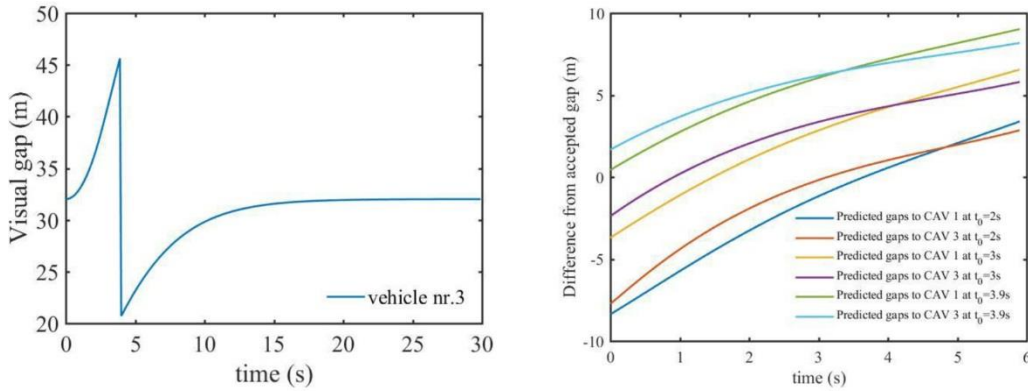


Figure 3. Simulation results of experiment 1

### Results and analysis

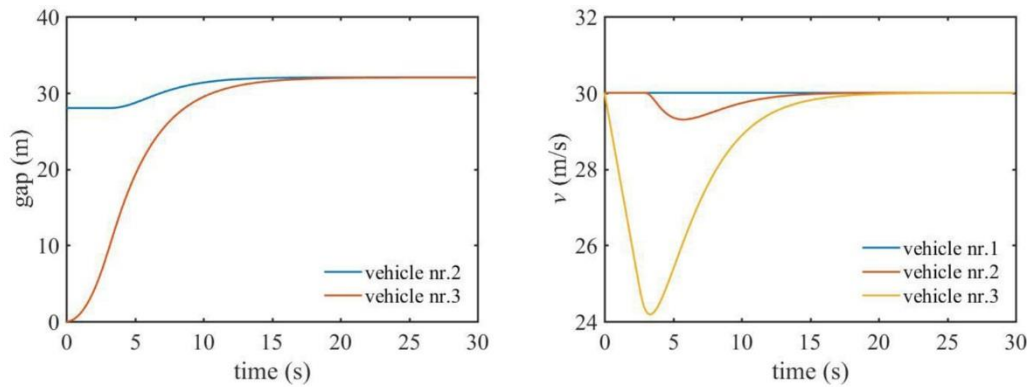
The simulated vehicle trajectories of experiment 1 are shown in Figure 3 and Figure 4. The merging strategy generates reasonable behavior. At the initial condition, the actual gaps of CAV 2 and 3 are smaller than their desired gaps, and the gap errors are negative. To reduce the gap errors, CAV 2 and 3 decelerate to reduce their speeds and then the desired gaps decrease, and the gap errors are reduced. However, after the decelerations of CAV 2 and 3, the relative speeds become positive and the values of the longitudinal accelerations are non-zero. Then there exists a tradeoff for these cost terms, and finally the gaps settle down to the desired gap of 32 m at the speeds to 30 m/s with zero longitudinal accelerations. In the lateral dimension, according to differences of the predicted gaps of merging CAV 2 to CAV 1 and CAV 3 to the accepted gap as shown in Figure 4 (b), the merging time instant is  $t_2^l = 3.9$  s. The merging trajectory is as shown in Figure 3 (d). Before merging, the predecessor of CAV 3 is CAV 1; however, after merging it becomes CAV 2. Accordingly, the visual gap of CAV 3 to its preceding CAV on the mainstream lane reduces sharply and then gradually increases to reach the equilibrium state, as shown in Figure 4 (a).



(a) Distance gaps of CAV 3

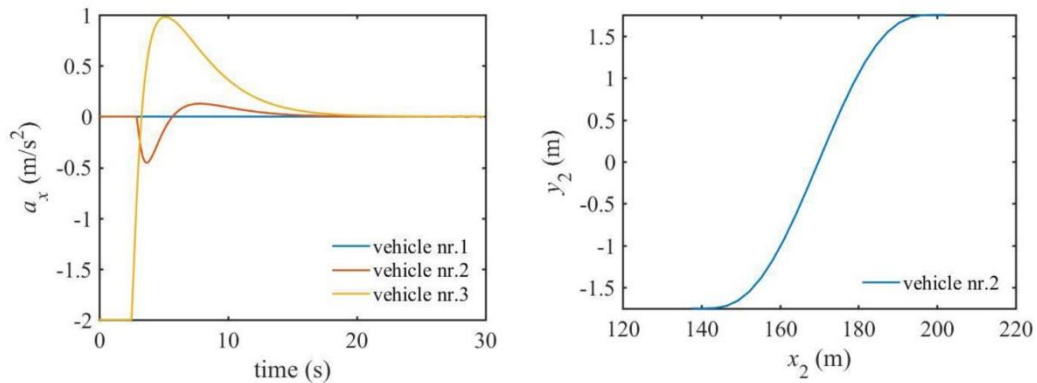
(b) Predicted gap differences

**Figure 4. Experiment 1: (a) Distance gap of CAV 3 to its preceding CAV on the mainstream; (b) the difference of the predicted gaps to the accepted gaps over the prediction horizon at several different simulation time**



(a) Gap

(b) Speed



(c) Longitudinal accelerations

(d) Lateral motion trajectory

**Figure 5. Simulation results of experiment 2**

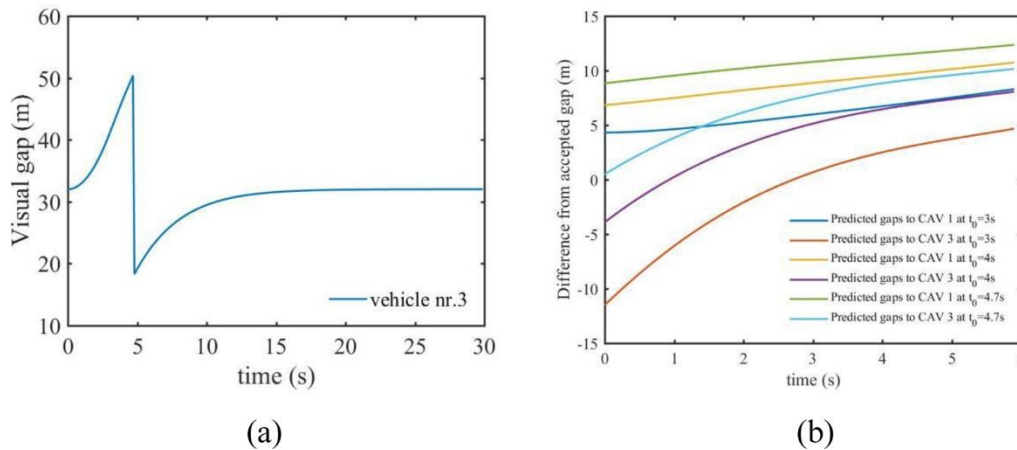
The simulation results of experiment 2 are shown in Figure 5 and Figure 6. The controller still generates reasonable behavior in this challenging scenario. Longitudinally, the initial gap for CAV 3 is 0 and the gap error is large and negative. The controller gives commands for CAV 3 to decelerate. The behavior of CAV 2 is affected by the trade-off of different cost terms and it starts

with constant speeds. With deceleration, the gap error of CAV 3 starts to be small; however, the relative speed becomes positive. By trading off different cost terms, finally CAV 2 and 3 reach the equilibrium state as in experiment 1. Laterally the merging time instant is  $t_2^l = 4.7$  s as shown in Figure 6(b).

The simulation results indicate the feasibility of the designed controller to complete merging automatically and safely, and the capability to accept small gaps for merging. Even though only two settings of initial states of CAVs are given in this paper, we have tested many other possible settings.

### CONCLUSION

This paper proposes a cooperative merging control strategy to complete the challenging merging task for connected automated vehicles. The controller is based on model predictive control. It regulates the longitudinal motions of CAVs and gives the initiation time of lane change for the merging vehicle. The effectiveness of the proposed strategy is tested through simulation experiments in merging scenarios with model mismatch. Results show the proposed approach generates feasible and smooth trajectories at challenging initial conditions even when feedback delay is not modeled. The strategy does not require a predefined merging point and resembles human-like behavior in accepting smaller gaps with increasing lane change desires.



**Figure 6. Experiment 2: (a) Distance gap of CAV 3 to its preceding CAV on the mainstream; (b) the difference of the predicted gaps to the accepted gaps over the prediction horizon at several different simulation time**

The current design used simple vehicle dynamics models and the robustness of the approach against uncertainties has not been addressed systematically. This will be addressed in future research. Furthermore, this study is done with 100% CAVs with one lane in the mainline. Future research will focus on heterogeneous traffic conditions with multiple mainstream lanes.

### ACKNOWLEDGEMENTS

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