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DOI
10.1364/OFC.2019.W2A.50

Publication date
2019

Document Version
Accepted author manuscript

Published in
Proceedings of the 2019 Optical Fiber Communications Conference and Exhibition (OFC 2019)

Wiener-Hopf Method for b-Modulation

Sander Wahls, Shrinivas Chimmalgi and Peter J. Prins
Delft Center for Systems and Control, Delft University of Technology, Mekelweg 2, 2628 CD Delft, The Netherlands
{*s.wahls, s.chimmalgi, p.j.prins*}*@tudelft.nl*

Abstract: A numerical method for the generation of fiber inputs in nonlinear frequency division multiplexing (NFDM) systems based on b-modulation is provided. The method is parallelizable, does not suffer from error propagation, and converges exponentially.
OCIS codes: 060.2330, 000.4430.

1. Introduction

A few years ago, Yousefi and Kschischang [1] proposed to utilize nonlinear Fourier transforms (NFTs) for data transmission in optical fiber. Their concept, called *nonlinear frequency division multiplexing (NFDM)*, allows to account for the Kerr nonlinearity in a simple, analytical way. Since then, the field has grown significantly [2]. Early NFDM systems suffered from slowly decaying tails in the time domain, which was one of several key factors that limits spectral efficiency. Recently, it has been shown that this problem can be resolved using a concept called b-modulation [3], which provides precise control over the temporal support of the generated signals. (An alternative, but mathematically more involved approach based on periodic NFTs has just been proposed in [4].) Several experiments [5, 6] and simulations [7] furthermore show that b-modulation is less sensitive to noise than earlier NFDM systems.

While b-modulation now allows to generate signals without detrimental tails, it has been observed that the performance of b-modulation decreases as the signal duration increases in relation to the memory of the channel [8]. In [5, 8], this effect has been attributed to growing signal-noise interactions; as a theoretical justification, a lower bound on capacity [9] that decreases with signal energy is cited in [5]. This bound, however, is not for b-modulation, but for conventional NFT-based systems that modulate the reflection coefficient. It is also a *lower* bound that might be very loose [9, p. 5]. The signal-noise interaction hypothesis also does not explain why b-modulation systems currently achieve peak performance for signal durations that are much shorter than those in conventional systems, which suffer from signal-noise interactions as well. Hence, other factors might currently be limiting signal duration in b-modulation.

We conjecture that numerical problems during the computation of the time-domain representation of the fiber input from the b-coefficient are currently a major limiting factor when signal durations are increased within b-modulation systems. (Similar effects are known in the literature on fiber Bragg gratings [10].) Numerically efficient algorithms are, with the exception of the Toeplitz inner bordering (TIB) method, all based on the so-called layer peeling principle. Layer peeling has the drawback that already reconstructed samples are reused in the reconstruction of later samples. Numerical errors therefore start to accumulate – in the worst case, exponentially fast. We remark that the TIB method is implicitly reusing already reconstructed samples as well and therefore also suffers from error propagation. *At this point, it is not known how strong the impact of numerical error propagation on the performance of b-modulation employing current algorithms actually is. The key missing ingredient to answer this question is a reliable baseline method that does not suffer from numerical error propagation. Such a method is derived in this paper.*

2. Wiener-Hopf Method for b-modulation

The most popular approaches to compute inverse NFTs are solving Gelfand-Levithan-Marcenko (GLM) integral equations, Riemann-Hilbert problems, and layer-peeling techniques [1, 2]. In this paper, we instead utilize a formulation of the b-modulation problem as a standard (matrix) Wiener-Hopf problem that is given in the book [11].

Theory Let $q(t)$, where $t \in \mathbb{R}$ denotes time, denote the complex envelope of a fiber input signal. The NFT of this signal is defined in terms of the scattering coefficients $a(\xi) := a(\infty, \xi)$ and $b(\xi) := b(\infty, \xi)$, $\xi \in \mathbb{R}$, defined by (e.g., [1])

$$\frac{\partial}{\partial t}\begin{bmatrix}\phi_1(t,\xi)\\ \phi_2(t,\xi)\end{bmatrix} = \begin{bmatrix}-j\xi & q(t)\\ -q^*(\xi) & j\xi\end{bmatrix}\begin{bmatrix}\phi_1(t,\xi)\\ \phi_2(t,\xi)\end{bmatrix}, \quad \begin{bmatrix}a(t,\xi)\\ b(t,\xi)\end{bmatrix} := \begin{bmatrix}e^{-j\xi t}\phi_1(t,\xi)\\ e^{j\xi t}\phi_2(t,\xi)\end{bmatrix} \longrightarrow \begin{bmatrix}1\\ 0\end{bmatrix} \text{ for } t \to -\infty.$$

This project has received funding from the European Research Council (ERC) under the European Unions Horizon 2020 research and innovation programme (grant agreement No 716669).

The NFT contains two parts: a continuous spectrum and a discrete spectrum. Typically, the reflection coefficient $b(\xi)/a(\xi)$ is used to describe the continuous spectrum, and conventional NFDM systems embed data in it [2]. Recently, it was proposed to embed data in $b(\xi)$ instead [3]. The advantages of this approach are explicit control over the temporal support of the generated $q(t)$ and less sensitivity w.r.t. noise [5–8]. The b-modulation problem is to generate the correct fiber input $q(t)$ from a given $b(\xi)$ in which data has been embedded. Let $B(\tau) := \int_{-\infty}^{\infty} b(\xi) e^{j\xi\tau} \frac{d\xi}{2\pi}$ denote the inverse Fourier transform of $b(\xi)$.[1] We consider, with t fixed, the matrix Wiener-Hopf problem [11, Eq. II-2.53]

$$\Omega_+(t,\tau) + \Phi(t,\tau) + \int_0^\infty \Omega_+(t,\tau')\Phi(t,\tau-\tau')d\tau' = 0,\ \tau \geq 0, \quad \Phi(t,\tau) := \begin{bmatrix} 0 & -B^*(-\tau+2t) \\ -B(\tau+2t) & 0 \end{bmatrix}. \tag{1}$$

The fiber input $q(t)$ that corresponds to the given $b(\xi)$ is then found from the relation [11, Eqs. II-2.51+II-2.58]

$$\Omega_+(t,0) \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \Omega_+(t,0) = \begin{bmatrix} 0 & q(t) \\ -q^*(t) & 0 \end{bmatrix}. \tag{2}$$

We emphasize that the time t is constant in this problem. Hence, the Wiener-Hopf problem can be solved for different times in parallel. Moreover, there will be no error propagation between different times as in layer peeling methods.

Numerical Method We first truncate the integral in (1) at $L > 0$ and discretize the result using Clenshaw-Curtis quadrature. The quadrate nodes τ_m and weights w_m, where $m = 1,2,\ldots,N$, have been found with the chebpts function in Chebfun [12]. The resulting equation is then evaluated at $\tau = \tau_n$, which leads us to the linear system

$$\tilde{\Omega}_+(t,\tau_n) + \Phi(t,\tau_n) + \sum_{m=0}^{N-1} w_m \tilde{\Omega}_+(t,\tau_m)\Phi(t,\tau_n-\tau_m) = 0, \quad n,m = 1,2,\ldots,N \quad \Longleftrightarrow \quad X(I+A) = B, \tag{3}$$

where I is an identity matrix, $A := [w_m \Phi(t,\tau_n-\tau_m)]_{m,n=1}^N$ is a block matrix, and $X := [\tilde{\Omega}_+(t,\tau_n)]_{n=1}^N$ and $B := [\Phi(t,\tau_n)]_{n=1}^N$ are block row vectors. The linear system in the right hand side of (3) is solved for $X = B(I+A)^{-1}$, which provides us numerical approximations $\tilde{\Omega}_+(t,\tau_n) \approx \Omega_+(t,\tau_n)$. In order to recover $q(t)$ via (2), we require an estimate of $\Omega_+(t,0)$. Note that $\tau_1 \neq 0$. We hence use linear extrapolation to obtain this estimate, i.e., $\tilde{\Omega}_+(t,0) := \frac{-\tau_2}{\tau_1-\tau_2}\tilde{\Omega}_+(t,\tau_1) + \frac{-\tau_1}{\tau_2-\tau_1}\tilde{\Omega}_+(t,\tau_2)$. We substitute $\tilde{\Omega}_+(t,0)$ for $\Omega_+(t,0)$ in (2), and solve the resulting equation for $q(t)$.

The error of our numerical method has two components: a truncation error, and a discretization error. The discretization error decays exponentially with N if $B(\tau)$ is smooth. The overall error follows this trend until the truncation error, which is controlled through the truncation parameter L, becomes dominant and creates an error floor.

Comparison With Other Methods The only other method for the inverting continuous spectrum that is immune to error propagation and converges exponentially seems to be the Riemann-Hilbert approach in [14]. From a technical point of view, it is similar to our method; the Wiener-Hopf problem used here arises as a reformulation of a Riemann-Hilbert problem [11]. Similar to [14], we (implicitly) use Chebychev polynomials, which results in rapid convergence. The method in [14] however works on the reflection coefficient $b(\xi)/a(\xi)$ instead of $B(\tau)$; in b-modulation, however the latter is of prime interest. (It is possible to recover $a(\xi)$ from $b(\xi)$, but this introduces another source of error.)

3. Numerical Example

We consider the example $B(\tau) = \frac{j}{2\pi}\sin(0.49\pi)\text{sech}(\tau/2-1)$. The exact solution is known to be $q(t) = 0.49j\text{sech}(t-1)$. Fig. 1 (left) compares the performance of the proposed method with that of the software library FNFT [13] (version 0.2.1, with N samples and temporal support $[-\frac{T}{2},\frac{T}{2}]$) for various configurations. The plots confirm that the proposed method indeed converges exponentially until an error floor due to truncation and/or finite machine precision is hit. The algorithm in FNFT is a second-order method and thus converges much slower, but note that its numerical complexity is much lower as well: $O(N\log^2 N)$ floating point operators instead of $O(N^3)$ for the proposed method. FNFT also hits error floors. At this point, it is interesting to look at the signal generated by FNFT in Fig. 1 (right). The algorithm in FNFT is a layer peeling procedure that suffers from propagation of numerical errors. It starts generating $q(t)$ at a large t and then proceeds to other values. The error floor indeed occurs on the left side of the signal, which is reconstructed later. However, by extending the temporal support, we are able to suppress the error floor. Thus, *at least in this very specific example*, error propagation does not seem to be a limiting factor for the layer peeling method in FNFT. However, this does not mean that numerical error propagation does not play a role in more realistic scenarios.

[1]The precise definition of the NFT varies slightly throughout the literature. Some formulas used here differ slightly from [11] for this reason.

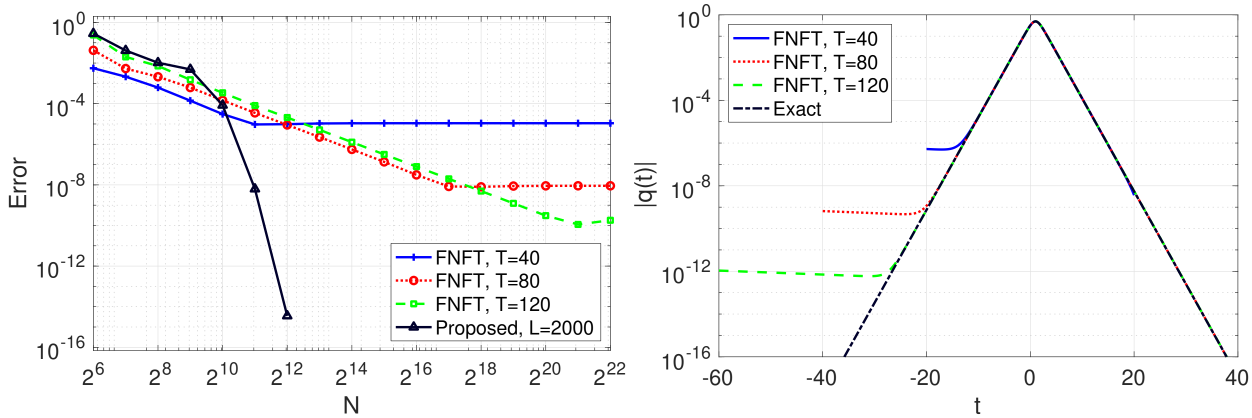

Fig. 1. *Left:* Sum of errors in $q(t)$ at the grid points closest to $t \in \{-2, 0.5, 2.5\}$. *Right:* Time-domain.

4. Conclusion and Outlook

The proposed numerical Wiener-Hopf method for performing b-modulation is simple but can achieve very high precision. Since it does not suffer from error propagation, we plan to use it to analyze the impact of error propagation on other numerical methods in scenarios that are more realistic than the analytical example considered in this paper. Since Wiener-Hopf methods parallelize naturally, we also want to investigate fast implementations in the future.

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