The Ripple Effect in the Housing Market

An Economic Engineering Model to Control Spatial Price Dynamics

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Master of Science Thesis

Delft University of Technology

Delft Center for Systems and Control

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For the degree of Master of Science in Systems and Control at Delft University of Technology

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April 14, 2022

Faculty of Mechanical, Maritime and Materials Engineering (3mE) · Delft University of Technology

The work in this thesis was supported by Brainbay. Their cooperation is hereby gratefully acknowledged. With special thanks to F. Harleman and S. Hommes

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The undersigned hereby certify that they have read and recommend to the Faculty of Mechanical, Maritime and Materials Engineering (3mE) for acceptance a thesis entitled

The Ripple Effect in the Housing Market

by

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in partial fulfillment of the requirements for the degree of

Master of Science Systems and Control

Dated: April 14, 2022

Supervisor(s): dr.ir. M.B. Mendel

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Abstract

In this thesis, an economic-engineering model of the ripple effect in the housing market is put forward. The ripple effect in the housing market is modeled by integrating into one model the local price dynamics of the rental market, the spatial price dynamics between local rental markets, and the valuation of real estate. Such a model is lacking in current housing-market literature.

The economic-engineering model design has two main aspects. First, economic-engineering model design uses analogs between mechanical systems and the housing market, ensuring that the model contains only economically and mechanically interpretable parameters. In this way input optimization and parameter optimization have a direct implementation in the real world. Second, economic-engineering model design relies on classical-mechanical modeling techniques that make the model suitable for optimally and robustly determining governmental policy using control formalism.

This thesis takes the perspective of the housing market as a heterogeneous economic space, where the dimensions are geographic proximity and economic influence. The model put forward in this thesis spatially discretizes this heterogeneous economic space into homogeneous local markets, such that local price dynamics govern the local markets and inter-local differences add spatial price dynamics.

The model put forward in this thesis is designed to predict where shortages will arise due to the introduction of rent control. Such a model guides policymakers in where to build for effectively relieving shortages in the housing market. Finally, such a model informs investors about the value change to expect due to policy changes and migration trends.

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Preface

During the research for this thesis, I learned a lot, both academically and about life itself. One of the most important lessons learned is that the story is told not for the storyteller but for the listener. The storyteller should thus contain the story to the part that is of utmost importance to the listener to bring the main point across. Then, an interested listener can ask for elaboration once the main point is already made. Digressing in details and underlying considerations before making the main point can leave the listener lost and disengaged.

To demonstrate this lesson, I tried to keep the central part of this thesis as condensed as possible and added a few appendices containing further details. After two years of research, I could talk about my research for days. Therefore, interested readers are encouraged to ask about considerations, and side quests endeavored on along the way.

“In theory there is no difference between theory and practice, while in practice there is”

— *Benjamin Brewster*

Chapter 1

Introduction

During my internship at Brainbay, I discovered that the housing market is a complex system of interacting subsystems, legislation, and time horizons. In the housing market, local policies can cause price perturbations and housing shortages on the other side of the country. Such perturbations causing side effects in different parts of the system is called the ripple effect. In the housing market, local housing markets located at different sides of the country as well as the rental market and the real estate market are connected in this ripple effect. The housing market ripple effect consist out of three aspects, the local pricing mechanism, the spatial interaction, and the interaction between rental market and real estate market. Chapter 2 will explain in more detail what the ripple effect in the housing market is and what the side effects are.

Accurate housing market models are essential for the government to formulate efficient policies and foresee the implications of those policies. The current academic standard for housing market models ignores the ripple effect by focusing on a single aspect of the housing market without considering spatial interference of the rest of the market [1]. (Appendix A gives as background reading an overview of the current academic standard for housing market models. The full report can be found in the literature review preceding this thesis [2].) Because current housing market models focus on the aspects of the housing market system independently, current governmental housing-market policies also focus on a single aspect of the housing market system at a time [3, 4]. This leads to the observation that there is a need for a model integrating the whole housing market ripple effect.

Can economic-engineering model design integrate the three aspects of the housing-market ripple effect into one model? Economic engineering theory is specifically developed to handle complex dynamic economic systems. Economic engineering uses classical mechanical modeling techniques to break down complex economic systems into simpler subsystems by drawing analogies between economic systems and mechanical systems. Economic-engineering model design first lays out the structure with first-principal elements before fitting with empirical data, with the direct consequence that interpretability is built into the design method itself. Interpretability allows for insight into the way a parameter tuned in the model or input tuned with the model would correspond to a real-world change [5]. (As background reading, Appendix B explains the importance of different kinds of models for different kinds of situations and explains why an interpretable model is preferred as a basis for governmental policy.) From an economic-engineering perspective, the rental market and the real estate market are not separate markets, nor are the local markets detached from other local markets. Each housing market can be considered a subsystem of a complete multi-market system.

Chapter 2 reviews the economic variables of the ripple effect in the housing market and splits the ripple effect up in three aspects. The economic variables are interpreted and given units to give them an exact definition. Based on these definitions, state variables, prerequisites for the model presented in Chapter 3, are determined.

Chapter 3 puts forward an economic-engineering model that integrates the three aspects of the ripple effect in the housing market in one model. This thesis takes the perspective of the housing market as a heterogeneous economic space, and discretizes that into spatially-homogeneous local markets. Each local market has local price dynamics and spatial price dynamics between local markets. The rent price dynamics happen in the time domain; valuation of the real estate market is modeled using a frequency-domain analysis of the rental market.

Chapter 4 summarizes the conclusions of Chapter 2 and Chapter 3 and concludes by answering the research question "Can economic-engineering model design integrate the three aspects of the housing-market ripple effect into one model?" affirmatively.

Lastly, Chapter 5 proposes some opportunities for further research. This thesis is written in the brother ambition of incorporating control theory into the housing-market policymaking process. The main contribution of this thesis lies in modeling the ripple effect in the housing market. This thesis opens the door for control theory to housing market policymaking by developing a model suitable for control theory and integrally modeling the complete housing market ripple effect. Chapter 5 suggests further research into the development of a controller as the basis for governmental policy, the extension of the model to capture the asymmetric effects that locations could have on each other, and the effect of the rest of the capital market on the real estate market. These three proposals will further the goal of incorporating control theory into the housing-market policymaking process.

Chapter 2

Preliminary Analysis: The Ripple Effect in the Housing Market

2-1 Introduction

For the housing market, a change in local rent prices affects the rent price at neighboring locations, which in turn affects the rent price at their neighboring locations. The rental and real estate markets are closely related; an unexpected change in rent price affects real estate valuation. In this manner, a local change in rent price ripples out from one location across the land and into the capital market [6, 7, 8].

In economics, an economic factor that, when increased or changed, causes increases or changes in many other related economic variables is called a ripple effect. This effect is also called the spillover effect, or an economic multiplier [9, 10]. The *ripple* effect is an analogy to ripples expanding across a fluid surface when an object is dropped into it. This thesis envisions the ripple effect in the housing market as a literal ripple in a mechanical aspect perturbing through the housing market space. This thesis envisions the rental market and the real estate market as two sides of the same coin, where the price dynamics of the rental markets happen in the time domain and the valuation of real estate happens in the frequency domain. Chapter 3 will elaborate on the division between time domain and frequency domain.

This chapter will introduce the prerequisite concepts of the housing market ripple effect, and define state variables using these concepts in preparation for the models put forward in Chapter 3. Current housing market literature is vague about the used variables' units. This chapter gives an exact definition of the state variables by determining the corresponding units.

Section 2-2 discusses the first and second aspect of the ripple effect, the price dynamics within the rental market and the necessary variables. Subsection 2-2-1 discusses the first aspect, this subsection assumes the housing market is spatially homogeneous and isolated from the rest of the world. Subsection 2-2-2 discusses the second aspect of the housing market ripple effect by letting go of these assumptions and introduces inter-local effects such as migration and arbitrage. Finally, Section 2-3 discusses the third and final aspect of the housing market ripple effect. Section 2-3 discusses how the rental market is the basis for the value of the real estate market, and introduces the supplementary state variables for the real estate market.

2-2 Price Dynamics in the Rental Market

2-2-1 Spatially Homogeneous Case

A housing service is a consumer good. It is the comfort and utility of occupying a residence. Like every other consumer good, the laws of supply and demand determine the quantities traded and the market price for the traded goods or services. The house itself is seen as a capital asset, which produces a flow of housing services. Like a cookie machine produces cookies, chickens produce eggs, and laborers produce labor. The price of housing services is called the rent price. The amount of housing services times the rent price equals the rent. The demander of housing services is the resident. The supplier of housing services is the landlord. The resident pays the landlord rent [1, 11].

Housing Service

The main features differentiating the amount of housing services a residence supplies are size and duration. A big house does not supply the same housing services as a small apartment, and a house where one can live for the rest of one's life does not supply the same service as a house where one could only stay the night [12, 13]. Other features such as location, architectonics, and interior design are harder to quantify and less critical to the rent price. Further sub-classification falls outside the scope of this thesis. The hedonic pricing method was developed for further sub-classification. Further reading about the hedonic pricing method can be found in Appendix A.

Housing services, indicated with the variable q, are measured in area times duration, $\mathrm{m}^2{\cdot}\mathrm{yr}$. Staying at a residence for one year equals q housing services; staying there for two years or one year at a residence twice the size equals $2q$ housing services. Often it will be necessary to normalize housing services to a per citizen basis. Normalizing housing services allows comparing densely populated and sparsely populated areas and brings the focus to a demand-sided analysis, compared to the non-normalized case which is used for a supplier or capital-focused analysis. Living in a dwelling alone equals q housing services per citizen. Living in that same dwelling together equals $0.5q$ housing services per citizen. Normalized housing services are are measured in area times duration per citizen, $\frac{\mathrm{m}^2\cdot\mathrm{yr}}{\mathrm{ctz}}$.

Flow of Housing Services

The flow of housing services produced, indicated by the variable $\dot{q}$, is ideally equal to the size of the housing stock; both are measured in area, m^2. One housing service is the benefits one square meter of residence produces in one year. However, capital does not always produce its maximum amount of services. Capital often has a downtime; human capital produces labor, one man·hour per hour. However, this laborer typically produces only eight man·hour per day and only 40 man·hour per week. The same can be said about real estate. During construction and renovation, it does not produce the same flow of housing services as when somebody is living there. It becomes a gray area if a landlord leaves a residence vacant in hopes of better renter prospects. This thesis argues that the landlord has access to these housing services, thus utilizing those housing services. This argument follows the same line of reasoning like the following case. When one is standing in the living room, and nobody is in the bedroom, is one still utilizing the benefits of having access to the bedroom? This thesis assumes one does, and missed rental income is a form of opportunity cost. Housing services can not be stored. This means that the amount of housing services produced equals the amount used.

Rent and Rent Price

The price of a housing service is the rent price, indicated by p and measured in dollars per housing service, $\frac{\$}{\mathrm{m}^2\cdot\mathrm{yr}}$. Rent is the rent price times the flow of housing services. Rent is indicated by R and measured in dollars per year, $\frac{\$}{\mathrm{yr}}$, or dollar per year per citizen, $\frac{\$}{\mathrm{yr}\cdot\mathrm{ctz}}$.

$$R(t) = p(t)\dot{q}(t) \tag{2-1}$$

There is an assumed indifference as to whether the occupant of a residence is the owner or a renter. In the case of a renter, the rent is simply the rent paid. In the owner-occupied case, rent is an imputed payment to oneself. One could have received this imputed rent by renting out the property to a third person. The imputed rent is thus a form of opportunity cost [14].

Economic Forces

The laws of supply and demand state that the market price will adjust in a free market until the quantity supplied meets the quantity demanded. The market price adjustment happens via economic forces or wants, indicated by F. The sum of all wants result in a price change, indicated by $\dot{p}$.

The mechanism behind market price adaptation in a free market has three components: Brokers do not have any inventory but match suppliers with demanders and gain a fee for this service. Dealers have an inventory and want to buy low and sell high. Speculators buy when prices are going up and sell when prices are going down. These three functions describe the basics of a market price mechanism [15, 16]. However, these are roles somebody can fulfill. Most people fulfill multiple roles at the same time. The rental market knows only brokers because housing services can not be stored, and subletting is often a neglectable part of the market.

Besides the laws of supply and demand, the price of housing services can also be determined or adjusted by governmental policy. In this case, the market is not entirely free. Government can impose rent control. Rent control is often put in place to keep prices affordable in places where high demand would otherwise drive rent prices up to levels only affordable for the rich. By lowering the rent price, people with lower incomes are capable of demanding a socially acceptable amount of housing services. Thus the demand for housing services increases [17, 4].

The laws of supply and demand state that the quantity supplied and the quantity demanded will change by changing the price. How much the quantity supplied and the quantity demanded will change, depends on the price elasticity. The more price elastic the quantity supplied and the quantity demanded are, the more they will react to a price change. Housing service supply is commonly assumed to be price inelastic. To change the housing service supply per citizen, either the housing stock has to expand or contract, which is a slow process compared to how quickly the demand can adapt, or the population size has to expand or contract, which is also slow and mostly a non-price-driven process. Thus for the rental market, the quantity supplied is price inelastic relative to the price elasticity of demand [11, 18, 8].

Variable	Description	Units	Units Normalized
$q(t)$	Housing Services	$\mathrm{m}^2{\cdot}\mathrm{yr}$	$\frac{\mathrm{m}^2\cdot\mathrm{yr}}{\mathrm{ctz}}$
$\dot{q}(t)$	Housing-Service Flow	m^2	$\frac{\mathrm{m}^2}{\mathrm{ctz}}$
$p(t)$	Rent Price	$\frac{\$}{\mathrm{m}^2\cdot\mathrm{yr}}$	$\frac{\$}{\mathrm{m}^2\cdot\mathrm{yr}}$
$\dot{p}(t)$	Rent-Price Change	$\frac{\$}{\mathrm{m}^2\cdot\mathrm{yr}^2}$	$\frac{\$}{\mathrm{m}^2\cdot\mathrm{yr}^2}$
$F(t)$	Economic Force or Want	$\frac{\$}{\mathrm{m}^2\cdot\mathrm{yr}^2}$	$\frac{\$}{\mathrm{m}^2\cdot\mathrm{yr}^2}$

Table 2-1: Variables and units for the rental market

2-2-2 Spatially Heterogeneous Case

The heterogeneous housing market is spatially discretized into many small and approximately homogeneous housing markets. Every small homogeneous market is called a local market. Every local market functions like a spatially homogeneous market. These locations are linked to their spatially or economically proximate neighbors (e.g., adherent locations and the capital). The interaction between locations can be both migration and arbitrage. The interaction between locations causes spatial price dynamics. Local price changes spread out over the region, first affecting the regions nearby then further away [19, 20, 21, 11, 22].

Migration

Migration is a person who leaves one location, decreasing the housing services demanded there and increasing the housing services demanded at the location of arrival. Migration is thus an expansion or contraction of the housing services demanded from one location to another location. Migration can roughly be split into two forms: active and passive migration. In the case of passive migration, people move from one location to another with similar facilities but with a higher excess supply of housing services. Passive migration is a statistical result of a better chance of finding residence in a location with excess supply than in a location with excess demand. Active migration is driven by people searching for better facilities, better job opportunities, or more nearby friends. Active migration is often a factor of urbanization [21]. To quantify how interconnected two locations are, one can look at migration patterns between locations [23, 11].

Arbitrage

The common definition of arbitrage is buying a product in one market and directly selling it in another market, profiting from the price difference [24]. This thesis uses a more abstract interpretation of arbitrage. Arbitrage is an economic force bringing together the price of similar products sold in different markets. Arbitrage occurs when there is a prices difference between markets. One expects a similar price for a similar product. When housing services are only distinguished by location, and there is no apparent reason why one location is better than the other, one expects a similar rent price. Every location has its particular price due to fluctuating demand per location caused by migration, uneven construction, and uneven population growth. However, the intrinsic housing service stays the same.

Summary

- Housing services are consumers goods
- Housing services are measured in $\mathrm{m^2 \cdot yr}$ or in $\frac{\mathrm{m^2 \cdot yr}}{\mathrm{ctz}}$
- Demand for and supply of housing services is measured as a flow of housing services
- Only in the ideal case, one square meter of real estate produces a flow of one housing service per year
- The price of a housing service is the rent price and measured in $\frac{\$}{\mathrm{m^2 \cdot yr}}$
- Rent is the rent price times the flow of housing services and is measured in $\frac{\$}{\mathrm{yr}}$ or $\frac{\$}{\mathrm{yr \cdot ctz}}$
- The rental market knows only brokers, named real estate agents
- Supply of housing services is price inelastic
- Supply of housing services is changed by construction and demolition and in the normalized case also by population growth
- Besides the laws of supply and demand, the rental market knows other economic forces such as rent control
- A heterogeneous housing market can be spatially discretized into small homogeneous housing markets
- These small homogeneous housing markets are interconnected to spatially or economically proximate neighbors
- Migration is the expansion or contraction of demand for housing services from one location to another
- Migration can be divided into two sorts: passive migration and active migration
- Arbitrage is an economic force bringing tougher the price of similar products in different markets

2-3 Valuation in the Real Estate Market

Real estate is a capital asset. The value is determined by housing services still to be produced. All housing services produced in the past are no longer important for the present value of the capital asset. Rent is the cash flow basis for calculating the present value of a real estate portfolio. The discounted cash flow method calculates the present value as the integrated discounted cash flow [25]. In this manner, the discounted cash flow method provides the connection between the price of housing services and the value of real estate capital [3, 11, 26, 27, 28, 29, 30, 31, 32].

Real Estate Portfolio

A square meter is the base unit of real estate. A residence is considered a real estate portfolio because it consists of multiple square meters. A portfolio can contain one residence; however, a portfolio can also contain multiple residences or the complete housing stock. A portfolio can fluctuate in size over time, and thus the flow of housing services supplied by a portfolio can fluctuate.

The housing stock is a portfolio containing all residences in a market. As discussed in Section 2-2, one housing service is the benefit one square meter of residence produces in one yr. Only in the ideal case, one m^2 of housing stock produces one housing service per year. During downtime, when the real estate is under construction or renovation, the real estate does not produce the maximum amount of housing services. During downtime, real estate will be indicated by inactive real estate. The housing stock is active and inactive housing stock together.

Discounting

Discounting, indicated by d, says that a dollar invested today could have earned something by tomorrow, making it worth more than a dollar received tomorrow. Discounting imputes a return on the capital assets. Imputed return is income one could have had if the capital is well invested. When the capital is well invested, and all the earnings are physically paid out, the imputed return should match the physical payout. Rent is the cash flow generated in return for the flow of housing services. When no renter is profiting from these housing services, it is seen as an opportunity cost because the landlord could have earned the imputed return.

When the earnings are reinvested, they generate earnings on earnings, resulting in exponentially increasing returns. The simplest form of discounting is exponential discounting. Exponential discounting compensates for earnings on earnings. Exponential discounting is the form most commonly discussed in economic textbooks [16]; however, more elaborate discount rates are more realistic. These do not only compensate for the imputed returns but also for the changing opportunities to use the returns. The discount rate is case and person-specific. Yong people saving for their pension might have a low discount rate at the beginning and a high discount rate when they plan to retire. People living paycheck to paycheck might have a steep discount rate for the current month and might be indifferent to what comes next because they can not plan past the current survival mode. Older people without children might have a high discount rate because they can be dead tomorrow and have nobody to pass the remaining wealth to, whereas grandparents might want to leave something behind for their children and grandchildren [16, 33, 34].

Effective Amount of Housing Services

Q are the effective housing services. Effective housing services are a measure of all the future housing services. Just like a dollar, a housing service is worth more today than tomorrow. Effective housing services are the integrated discounted future flow of housing services. These are portfolio dependent. Effective housing services based on the flow of housing services from the whole housing stock are important for a government. Depending on the need for housing, the government can announce big plans far in the future or a direct, quick relief. An investor might want to schedule renovation or acquire a new building. At this point, $\dot{q}$ will not stay constant. The investor can choose up to some degree the moment of renovation. The investor can thus optimize this depending on the expected future rental price, the frequency of renovation, and the increasing cost of prolonged renovation.

Ideal Present Value

The ideal present value per square meter, V, is the integrated discounted rent price. In the ideal case that a portfolio constantly contains one square meter of real estate and produces a constant flow of one housing service per year, then the value of that portfolio would be the discounted rent price. However, real estate has some downtime, and portfolios expand and contract. In addition to this, there is the discount profile; thus, determining the value of a portfolio is more convoluted than just the ideal present value.

Variable	Description	Unit
V	Ideal Present Value	$\frac{\$}{\mathrm{m}^2}$
Q	Effective Housing Services	m^2
d	Discount Factor	%
PV	Present Value	$

Table 2-2: Variables and units for real estate valuation

Summary

- Real estate is a capital asset
- A real estate portfolio consists of some number of square meters
- Housing services are the product real estate portfolios produce
- The housing stock consists of all the residences in a market
- Real estate can be split into active and inactive
- The discounted cash flow method is used to evaluate the present value of capital assets
- Rent is the cash flow basis for the present value of real estate
- The discount rate is case-specific and does not have to be simply exponential
- Q represents the effective amount of housing services in a real estate portfolio
- V represents the ideal present value per square meter of real estate

2-4 Conclusions

Local rental market policy does not stop at the border of local rental markets. The housing market ripple effect involves local rental market policy that affect other local rental markets and real estate markets. An unexpected rent price change in a local rental market on one side of the country can cause a present value change in a local real estate market on the other side of the country. Thus, to determine efficient governmental policy for the housing market, the full scope of this ripple effect should be considered.

Chapter 3

Economic Engineering Model of the Housing Market

3-1 Introduction

This chapter puts forward an economic engineering model that integrates the ripple effect in the housing market in one model. This is done in four steps. In the first step, analogs are drawn between the housing-market variables, presented in Chapter 2, and mechanical variables. The second step, the pricing mechanism of the rental market is assumed to be spatially homogeneous and modeled as a mechanical system. The third step, the rental market is assumed to be spatially heterogeneous and is spatially discretized into local rental markets. Each local rental market is assumed spatially homogeneous. The spatial pricing mechanism governs the interaction of the locations such that the price distribution spans a scalar price field and migration spans a vector field. Forth and final step, the rent-price dynamics happen in the time domain; valuation of the real estate market is modeled using a frequency-domain analysis of the rental market. In this manner step two to four step through the three aspects of the housing market ripple effect by expanding on the previous step.

Section 3-2 takes the first step. Economic engineering is introduced, and analogies are drawn between the housing market and mechanics following the economic engineering convention. The rest of this chapter will use these analogs by approaching the housing market similarly as a mechanical system. Section 3-3 takes the second and third steps. The second step is put forward in Subsection 3-3-1. This subsection models the pricing mechanism of a single spatially-homogeneous rental market. The third step is put forward in Subsection 3-3-2, where the spatial pricing mechanism of a heterogeneous rental market is introduced. In Subsection 3-3-2, multiple spatially-homogeneous rental market models are interconnected to form a heterogeneous rental market model. The interconnections expand on the pricing mechanism of a single spatially-homogeneous rental market to form the spatial pricing mechanism. Finally, Section 3-4 makes the fourth and final step of integrating the housing-market ripple effect in one model. In Section 3-4 a frequency-domain analysis of the rental market model is used to determine the present value of real estate.

3-2 Economic Engineering Analogies

Economic engineering theory is specifically developed to handle complex dynamic economic systems. Economic engineering uses classical mechanical modeling techniques to break down complex economic systems into simpler subsystems by drawing analogies between economic systems and mechanical systems. Economic-engineering model design first lays out the structure with first-principal elements before fitting with empirical data. From an economic engineering perspective, the rental and the real estate markets are not separate markets, nor are the local markets detached from other local markets. Each local market is considered a subsystem of one complete multi-market system, and the rental market and the real estate market are two sides of the same coin, a time and frequency domain approach of the same process.

This section interprets and implements for the housing market the modeling techniques and analogies developed within the economic engineering group of the TU Delft, put forward in the following research [15, 34, 35, 33, 36, 37].

All variables in the housing market domain interact with each other in the same way as their mechanical domain analogs. The main analogies used in this chapter are those presented in Figure 3-1. These are the analogs between the rental market variables presented in Figure 2-1 and mechanical variables. The analogs are chosen such that the laws of supply and demand for the housing market variables correspond to Newtons' laws of motion for the mechanical variables.

Economic Variable	Housing Market Variable	Mechanical Variable	Variable symbol
Price	Rent Price	Momentum	$p(t)$
Price Change	Rent-Price Change	Momentum Change	$\dot{p}(t)$
Want	Want	Force	$F(t)$
Stock	Housing Service	Position	$q(t)$
Supply or Demand	Housing-Services Flow	Speed	$\dot{q}(t)$

Table 3-1: Yet developed economic engineering analogues with the added interpretation for the housing market

Newtons' first law, a body remains at rest or in motion at a constant speed in a straight line, unless acted upon by a force. This corresponds for the rental market to a resident will demand a constant flow of housing services, unless acted upon by a want.

Newtons' second law, when a body is acted upon by a force, the time rate of change of its momentum equals the force. This corresponds for the housing market to the time rate of change of rent price is equals the want. With a net-zero want, the price stays constant.

Newtons' third law, if two bodies exert forces on each other, these forces have the same magnitude but opposite directions. This corresponds to arbitrage between two local rental markets. If the rent price drops in one local market due to arbitrage, the same arbitrage will raise the rent price in the other local rental market.

3-3 Rental Market Model

The rental market is a competition between residents and landlords for market-clearing of the housing services. Real estate agents work as brokers to match the flow supplied to the flow demanded. Construction and population growth expand or contract the flow of housing services available per citizen. Government can put in place rent control policies to regulate the rent price. Migration increases and decreases the demand for housing services at particular locations. Arbitrage smooths out price differences between locations if there is no apparent reason for the difference.

Many landlords, residents and real estate agents make up a housing market, all with their particular wishes and needs. However, it is assumed that the supplier group, representing all the landlords, and the demand group, representing all the occupants, and the broker group, representing all real estate agents, are locally homogeneous groups. A person can full fill multiple functions. Being a landlord, an occupant, or a broker is a function. A person owning and occupying a house while working as a real estate agent thus populates three groups.

3-3-1 Spatially Homogeneous Model

This subsection models the first aspect of the housing market ripple effect. For the spatially-homogeneous rental-market model, a spatially-isolated market is assumed, disconnected from the rest of the world. The only inputs into the system are rent regulation, construction, and natural population growth.

Figure 3-1 is a bond graph representation from a spatially-homogeneous rental-market model. Equation (3-9) and (3-10) are the state space representation of the spatially-homogeneous rental-market model. The code for the spatially-homogeneous rental-market model can be found in Appendix C-1

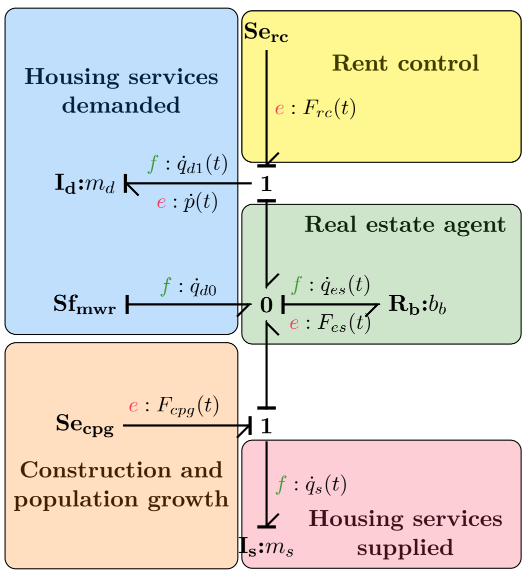


Figure 3-1: Bond graph representation of the homogeneous rental market model

Model Design

Figure 3-1 shows a bond graph representation of the homogeneous rental market model. The model consists of five main parts. The housing services demanded, the housing services supplied, pricing mechanism in the form of real estate agents, construction and population growth, and governmental input in the form of rent control. The following section will explain how this bond graph model is designed. Table 3-2 gives a recap of all the variables used in the design.

Flow of housing services demanded per citizen
The flow of housing services demanded, $\dot{q}_d(t)$, exists out of two parts: The maximum willingness to receive, $\dot{q}_{d0}$, together with a linear price-dependent component, $\dot{q}_{d1}(t)$.

$$\dot{q}_d(t) = \dot{q}_{d0} + \dot{q}_{d1}(t) \tag{3-1}$$

$\dot{q}_{d1}(t)$ is imposed by $\mathbf{I}_\mathrm{d}$ and $\dot{q}_{d0}$ is imposed by $\mathbf{Sf}_\mathrm{mwr}$, they sum together on the 0-junction. $\mathbf{I}_\mathrm{d}$ integrates all price changes, $\dot{p}(t)$, and, depending on the current price level, imposes the price-dependent component of the flow demanded.

$$\dot{q}_{d1}(t) = \int_{-\infty}^{t} \frac{\dot{p}(\tau)}{m_d} d\tau \tag{3-2}$$

m_d is the price inelasticity of demand, linking the price level to the flow demanded. The maximum willingness to receive is the flow of housing services a demander would want if the housing services were free. $\dot{q}_{d0}$ is a constant flow of housing services offsetting the flow demanded. This model uses the normalized per citizen basis, which means that m_d and $\dot{q}_{d0}$ are constants, independent of the population size.

Flow of housing services supplied per citizen
The flow of housing services supplied per citizen, $\dot{q}_s(t)$, is produced by the active housing stock divided by the number of citizens. $\dot{q}_s(t)$ is imposed by $\mathbf{I}_\mathrm{s}$. $\mathbf{I}_\mathrm{s}$ integrates all changes in active housing stock and population size and, based on that, imposes the normalized flow of housing services supplied as the current ratio of active housing stock per citizen.

Real estate agents and excess supply
There is constant competition between residents and landlords for market-clearing of housing services. This happens in the bond graph on the 0-junction. On the 0-junction the flow of housing services supplied adds to the flow of housing services demanded. Unlike in most conventional economic literature, in this thesis, flow demanded is a negative flow supplied.

When the flow demanded plus the flow supplied does not equal zero, there is an excess supply, $\dot{q}_{es}(t)$. When the excess supply is negative, it is called excess demand.

$$\dot{q}_{es}(t) = \dot{q}_{d0} + \dot{q}_{d1}(t) + \dot{q}_s(t) \tag{3-3}$$

When the housing market is in equilibrium, and there are no inputs into the system, the flow demanded plus the flow supplied is zero. For a market equilibrium rent price, the amount of housing services people are willing to pay for equals exactly the amount of housing services supplied.

When the housing market is out of equilibrium, and there are no inputs into system, the real estate agents, indicated by $\mathbf{R}_\mathrm{b}$ in the bond graph, will try to bring the system back into equilibrium by steering the excess supply to zero. $\mathbf{R}_\mathrm{b}$ takes in the excess supply and brokers, steering the market to equilibrium. Brokering is indicated by $F_{es}(t)$.

$$F_{es}(t) = b_b \dot{q}_{es}(t) \tag{3-4}$$

b_b represents the effectiveness of the real estate agent to couple supply and demand. The more effective real estate agents are, the faster they can steer the market to market equilibrium. When the market is most out of equilibrium, the need for a real estate agent is greatest. Depending on an excess demand or excess supply of housing services, the real estate agents will have to work harder to find a suitable residence or renter respectively.

Construction and population growth
The supply of housing services is price inelastic, which means that only construction and population growth, indicated by $\ddot{q}_{cpg}(t)$, change the housing services available per citizen as an expansion or contraction of the flow of housing services per citizen, $\ddot{q}_s(t)$.

$$\ddot{q}_{cpg}(t) = \ddot{q}_s(t) \tag{3-5}$$

The bond graph method can not handle second-order differentials. For this drawback, a workaround is put forward. $F_{cpg}(t)$ and m_s are figments of the bond graph method and are back-engineered such that.

$$F_{cpg}(t) = m_s \ddot{q}_{cpg}(t) \tag{3-6}$$

The state space method, in Equation (3-15), does not need this work-around and sets $\ddot{q}_{cpg}(t)$ directly equal to $\ddot{q}_s(t)$.

The bottom 1-junction, in Figure 3-1 directly above $\mathbf{I}_\mathrm{s}$, sums all efforts affecting the flow of housing services supplied and imposes a net effort on $\mathbf{I}_\mathrm{s}$. $\mathbf{I}_\mathrm{s}$ integrates the imposed net effort and, based on that, imposes the normalized flow of housing services supplied as the current ratio of active housing stock per citizen.

Pure price inelasticity also does not fit neatly in the bond graph method. The effect of the brokering real estate agent on the flow of available housing services should be zero. By choosing m_s much bigger than m_d, supply becomes relatively price inelastic compared with demand.

$$m_d << m_s \quad \Longrightarrow \quad F_{es}(t)/m_s \approx 0 \quad \text{and} \quad F_{es}(t)/m_d \approx \dot{p}(t) \tag{3-7}$$

As m_s goes in the limit to infinity, housing services supplied become completely price inelastic, and expansion or contraction of the flow of housing services supplied per citizen becomes only affected by construction and population growth.

$$\lim_{m_s \to \infty} \underbrace{\frac{F_{cpg}(t)}{m_s}}_{\ddot{q}_{cpg}(t)} = \ddot{q}_s(t) \tag{3-8}$$

Rent control
Rent control, indicated by $F_{rc}(t)$, is an effort on the market price directly. In the bond graph model, rent control is imposed by the effort source $\mathbf{Se}_{\mathrm{rc}}$. The rent control does not change the amount of housing services available, only the rent price. The rent price, in turn, affects the demand for housing services. Rent control is an input to the system. The upper 1-junction, in Figure 3-1, sums the effort from the real estate agents and the rent control and imposes a net price change on $\mathbf{I}_{\mathrm{d}}$. $\mathbf{I}_{\mathrm{d}}$ integrates this net price change and imposes correspondingly the price-dependent component of the flow of housing services demanded, as indicated in (3-2).

State space representation Equation (3-9) and Equation (3-10) form the state space representation of the homogeneous rental market model. Here, $\ddot{q}_{cpg}$ is used directly and m_s is completely inelastic, such that the lower row of Equation (3-9) contains almost only zero coefficients, except for $\ddot{q}_{cpg}$.

$$\underbrace{\begin{bmatrix} \dot{p}(t) \\ a_s(t) \end{bmatrix}}_{\dot{x}(t)} = \underbrace{\begin{bmatrix} -\frac{b_b}{m_d} & -b_b \\ 0 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} p(t) \\ \dot{q}_s(t) \end{bmatrix}}_{x(t)} + \underbrace{\begin{bmatrix} 0 & -b_b & 1 \\ 1 & 0 & 0 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} \ddot{q}_{cpg}(t) \\ \dot{q}_{d0} \\ F_{rc}(t) \end{bmatrix}}_{u(t)} \tag{3-9}$$

$$\underbrace{\begin{bmatrix} p(t) \\ \dot{q}_s(t) \\ \dot{q}_d(t) \\ \dot{q}_{es}(t) \end{bmatrix}}_{y(t)} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \frac{1}{m_d} & 0 \\ \frac{1}{m_d} & 1 \end{bmatrix}}_{C} \begin{bmatrix} p(t) \\ \dot{q}_s(t) \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}}_{D} \begin{bmatrix} \ddot{q}_{cpg}(t) \\ \dot{q}_{d0} \\ F_{rc}(t) \end{bmatrix} \tag{3-10}$$

Variable	Discribtion	Unit
$p(t)$	rent price	$\frac{\$}{\mathrm{m}^2\cdot\mathrm{yr}}$
$\dot{p}(t)$	change rent price	$\frac{\$}{\mathrm{m}^2\cdot\mathrm{yr}^2}$
$\dot{q}_{d1}(t)$	price depended flow of housing services demanded per citizen	$\frac{\mathrm{m}^2}{\mathrm{ctz}}$
$\dot{q}_{d0}$	demanded offset / max willingness to receive per citizen	$\frac{\mathrm{m}^2}{\mathrm{ctz}}$
$\dot{q}_d(t)$	flow of housing services demanded per citizen	$\frac{\mathrm{m}^2}{\mathrm{ctz}}$
$\dot{q}_s(t)$	housing services supplied per citizen	$\frac{\mathrm{m}^2}{\mathrm{ctz}}$
$\ddot{q}_s(t)$	change housing services supplied per citizen	$\frac{\mathrm{m}^2}{\mathrm{yr}\cdot\mathrm{ctz}}$
$\dot{q}_{es}(t)$	excess housing services supplied	$\frac{\mathrm{m}^2}{\mathrm{ctz}}$
$\ddot{q}_{cpg}(t)$	construction and population growth	$\frac{\mathrm{m}^2}{\mathrm{yr}\cdot\mathrm{ctz}}$
$F_{cpg}(t)$	construction and population growth (bond graph figment)	$\frac{\$}{\mathrm{m}^2\cdot\mathrm{yr}^2}$
$F_{rc}(t)$	rent control	$\frac{\$}{\mathrm{m}^2\cdot\mathrm{yr}^2}$
$F_{es}(t)$	brokering	$\frac{\$}{\mathrm{m}^2\cdot\mathrm{yr}^2}$

Table 3-2: Recap variable description for the spatially-homogeneous rental-market model

Simulation Results

In Figure 3-2 the homogeneous rental market model is perturbed by an input from construction and population growth. Red represents the flow of housing services supplied, $\dot{q}_s(t)$. Blue represents the flow demanded, $\dot{q}_d$. Green represents the excess supply, and when it is negative, the excess demand, $\dot{q}_{es}$. Magenta represents the rent price, $p(t)$. Finally, orange represents the input, construction and population growth, $\ddot{q}_{cpg}(t)$.

The homogeneous rental market model is initiated in equilibrium, after that perturbed by a population growth followed by a pause, then some construction, and finally again a pause. First, the effect is a decline in housing services per citizen followed by an increase in housing services per citizen. The decline in living space per person causes a price increase. As the flow of housing services supplied per citizen swells, the price drops again.

In Figure 3-3 the homogeneous housing market model is perturbed by rent control as input. Orange represents the input signal, now rent control, $F_{rc}(t)$. Red represents the flow of housing services supplied again, $\dot{q}_s(t)$. Blue represents the flow demanded again, $\dot{q}_d(t)$. Green represents the excess supply or excess demand again, $\dot{q}_{es}(t)$. Magenta represents the price per housing service again, $p(t)$.

The homogeneous housing market model is initiated in equilibrium, thereafter perturbed by rent control regulations, lowering the rent price. This causes the demand for housing services to increase. However, the supply for housing services is price inelastic and thus does not react to the change in price. An excess supply is created.

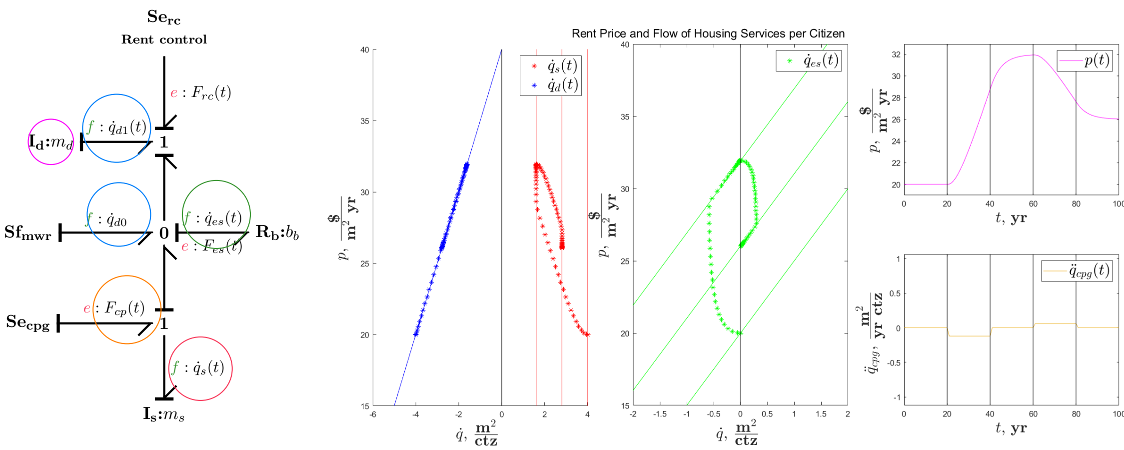


Figure 3-2: Simulation results with construction and population growth as input to the homogeneous rental market model

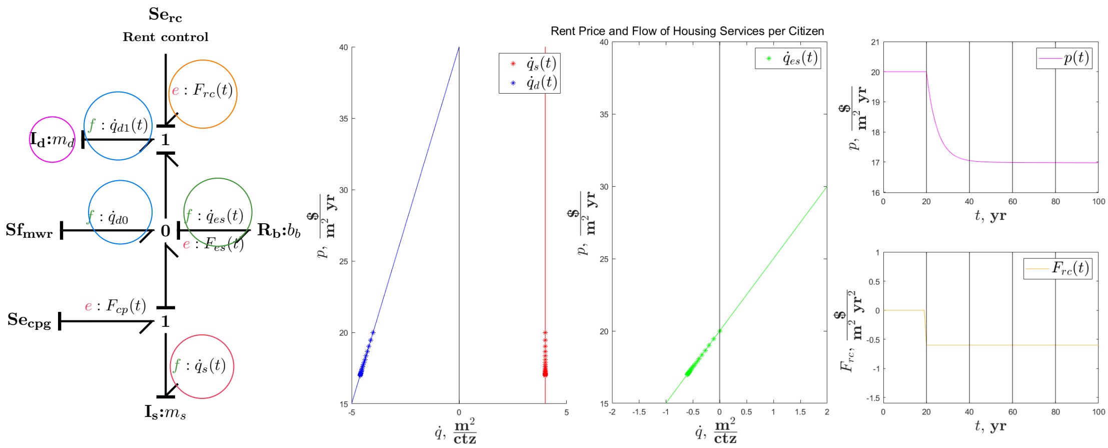


Figure 3-3: Simulation results with rent control as input to homogeneous rental market model

3-3-2 Spatially Heterogeneous Model

This subsection builds on the previous subsection and models the second aspect of the housing market ripple effect. The heterogeneous rental market model is a spatial discretization of the spatially heterogeneous rental market. All spatial discretizations are called locations and are assumed to be internally spatially homogeneous. All locations are modeled using the spatially-homogeneous rental-market model presented in Subsection 3-3-1. The coming subsection expands on this model by introducing interconnections between locations. The interconnections consist of two parts, migration and arbitrage.

Figure 3-5 represents a bond graph model of a heterogeneous rental market consisting of two locations and one interconnection. For the sake of clarity, only two interconnected locations are shown. Adding more locations in the bond graph representation would clutter the figure and would not contribute much more insight. Equations (3-15) and (3-16) are a state-space representation of a heterogeneous rental market consisting of N locations and K interconnections.

The rental market occupies a heterogeneous economic space, where the spatial dimensions are geographic proximity and economic influence. This means that in economic space, two locations can be proximate and directly connected, while in geographic space, they are not. I.e., a capital city can influence all locations in the region while it is not directly geographically connected. This is shown in Figure 3-4, where a,b,d,e,f are rural areas and C is the capital city.

The heterogeneous rental-market model is built up from multiple building blocks: the homogeneous rental market, the interconnection to the capital city, the interconnection between neighboring locations, and the interconnection to regional urban centers. The code for the building blocks of the heterogeneous rental market model is shown in Appendix C-2, and the code for the entire interconnected rental market is shown in C-3

Model Design

In Figure 3-5 two interconnected locations are shown. Both locations 1 and 2 are modeled as a homogeneous rental market, as in Figure 3-1. The allowed difference between locations and the interconnection are the main contributions of the heterogeneous market model over the homogeneous market model.

The interconnection between locations consists of two parts. Migration (bottom) and arbitrage (top). Mind, this thesis uses an abstract definition of arbitrage as discussed in Section 2-2-2.

Migration
Migrated people increase the available housing services per citizen at the location of departure and decrease the available housing services per citizen at the location of arrival. $\dot{q}_{m_{\{1/2\}}}(t)$ is imposed by $\mathbf{I}_{\mathrm{m}}$ and represents the flow of housing services demanded by the people who migrated from location 1 to location 2.

In this thesis, migration between locations is modeled separately from locally-born population growth because migration is seen as an inter-local effect, whereas locally-born population growth can happen independently from all other locations, thus a local effect.

Migration, $\ddot{q}_{m_{\{1/2\}}}(t)$, is the rate at which the flow of housing services demanded by migrated people increases or decreases. $m_{m_{\{1/2\}}}$ represents the reluctance to migrate from location 1 to location 2. A larger $m_{m_{\{1/2\}}}$ implies that people tend to stay longer in their current location and need more motivation to move.

Migration can be divided into two parts passive and active migration. Passive migration is motivated by a difference in excess supply. When two locations are similar, people will, on average, move towards a location where excess supply is highest, simply because finding an available home there is more likely.

Active migration is caused by a migratory push not captured by the difference in excess supply. This migratory push is represented by $F_{m_{\{1/2\}}}(t)$ and is imposed by $\mathbf{Se}_{m_{\{1/2\}}}$. For example, there often is a migratory direction towards the capital city and urban centers. This kind of migration is often motivated by better job opportunities, better facilities, friends living there, or studies, and not because finding housing there is more likely [11].

In the bond graph representation, $\mathbf{I}_{m_{\{1/2\}}}$ integrates all migratory wants, both active and passive, and stores it as the price for which people are willing to move. $p_{m_{\{1/2\}}}(t)$ is the price

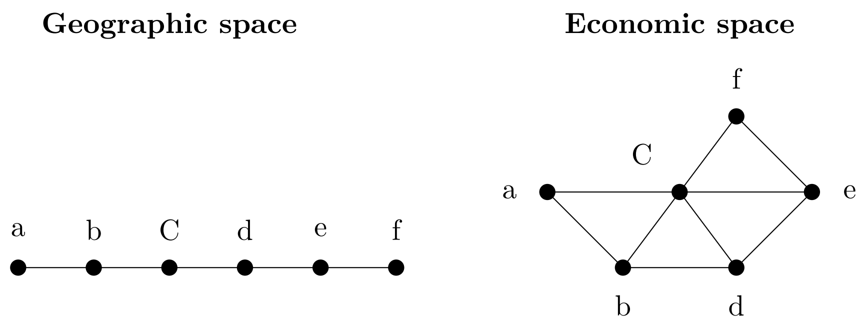


Figure 3-4: Proximity and connections in both geographic space and economic space

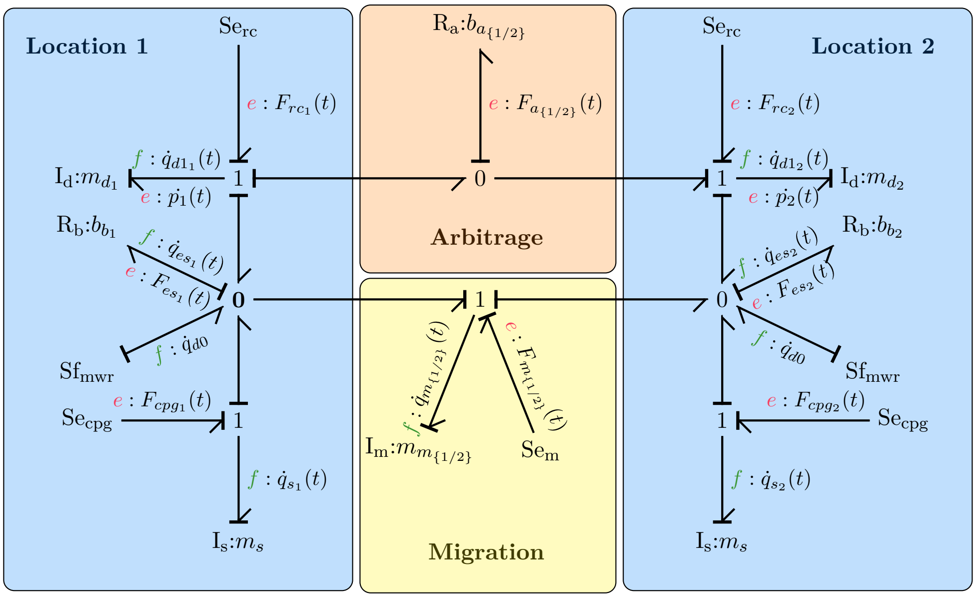


Figure 3-5: Bond graph representation of a heterogeneous rental market model

for which people are willing to move on average from location 1 to location 2.

Arbitrage
The element $\mathbf{R}_\mathrm{a}$ represents the inter-local real estate agents, informing local markets participants of prices at other locations. $b_{a_{\{1/2\}}}$ is the effectiveness of the inter-local real estate agents informing local markets participants at location 1 of the prices in location 2 and informing local markets participants at location 2 of the prices in location 1. $\mathbf{R}_\mathrm{a}$ imposes an economic force called arbitrage, $F_{a_{\{1/2\}}}(t)$, based on the price difference between location 1 and location 2.

$$F_{a_{\{1/2\}}}(t) = b_b(p_1 - p_2) \tag{3-11}$$

This drives the prices of locations 1 and 2 towards each other. This kind of arbitrage changes the perspective of market participants on what a normal and fair price is, independent of the actual supply and demand at that local market.

Because arbitrage moves the price independent from supply and demand, it can move the price out of equilibrium, causing excess supply at location 1 and excess demand at location 2. This difference in excess supply, in turn, causes migration from location 2 to location 1, bringing both markets back to equilibrium.

State space representation
In the state space representation, Equation (3-15) and (3-15), the heterogeneous rental market model is discretized in N local rental markets with K interconnections between them. All locations have the subscript n, all interconnections are indicated by k.

$$n = 1, ..., N \tag{3-12}$$

$$k = \{1/2\}, ..., K \tag{3-13}$$

The interconnections between locations can be divided into connections to neighboring locations, a capital city, and regional urban centers.

$$k = \{1/2\}, ..., K1, ..., K2, ...K \tag{3-14}$$

$k1$ starts at $\{1/2\}$ up to and including $K1$, representing all the connections to adherent locations. $k2$ starts after $K1$ up to and including $K2$, representing all the connections with the capital city. $k2$ starts after $K2$ up to and including K, representing all the connections to urban centers.

H is the incidence matrix of a directed graph, where all locations are nodes, and all connections between locations are edges. H holds for every interconnection a -1 for the location it is leaving and a 1 for the location it arrives at. B_b is a diagonal matrix that holds all location-specific real estate agents' effectiveness. M_d is a diagonal matrix holding all location-specific price elasticities. B_a is a diagonal matrix holding the inter-local specific arbitrage effectiveness between locations. M_m is a diagonal matrix holding all inter-local specific reluctance to move. I and 0 are identity and zero matrices respectively.

$$B_b = \begin{bmatrix} b_1 & & \\ & \ddots & \\ & & b_n \end{bmatrix}, \quad B_a = \begin{bmatrix} b_1 & & \\ & \ddots & \\ & & b_k \end{bmatrix}, \quad M_d = \begin{bmatrix} m_1 & & \\ & \ddots & \\ & & m_n \end{bmatrix}, \quad M_m = \begin{bmatrix} m_1 & & \\ & \ddots & \\ & & m_k \end{bmatrix}$$

HH^T is the node-based Laplacian matrix, whereas the alternative product $H^T H$ defines the edge-based Laplacian. The graph Laplacian matrix can be viewed as a matrix form of the negative discrete Laplace operator on a graph approximating the negative continuous Laplacian obtained by the finite difference method [38, 39].

$$\underbrace{\begin{bmatrix} \dot{p}_n(t) \\ \ddot{q}_{s_n}(t) \\ \ddot{q}_{m_k}(t) \end{bmatrix}}_{\dot{x}(t)} = \underbrace{\begin{bmatrix} -(HB_aH^T + B_b)M_d^{-1} & -B_b & -B_bH \\ 0 & 0 & 0 \\ -B_bM_m^{-1}H^TM_d^{-1} & -B_bM_m^{-1}H^T & -B_bM_m^{-1}H^TH \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} p_n(t) \\ \dot{q}_{s_n}(t) \\ \dot{q}_{m_k}(t) \end{bmatrix}}_{x(t)} + \dots$$

$$\underbrace{\begin{bmatrix} 0 & -B_b & I & 0 \\ I & 0 & 0 & 0 \\ 0 & 0 & 0 & M_m^{-1} \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} \ddot{q}_{cpg_n}(t) \\ \dot{q}_{d0_n} \\ F_{rc_n}(t) \\ F_{m_k}(t) \end{bmatrix}}_{u(t)} \tag{3-15}$$

$$\underbrace{\begin{bmatrix} p_n(t) \\ \dot{q}_{s_n}(t) \\ \dot{q}_{d_n}(t) \\ \dot{q}_{m_k}(t) \\ \dot{q}_{es_n}(t) \\ \dot{q}_{mnet_n}(t) \end{bmatrix}}_{y(t)} = \underbrace{\begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ M_d^{-1} & 0 & 0 \\ 0 & 0 & I \\ M_d^{-1} & I & I \\ 0 & 0 & H \end{bmatrix}}_{C} \begin{bmatrix} p_n(t) \\ \dot{q}_{s_n}(t) \\ \dot{q}_{m_k}(t) \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{D} \begin{bmatrix} \ddot{q}_{cpg_n}(t) \\ \dot{q}_{d0_n} \\ F_{rc_n}(t) \\ F_{m_k}(t) \end{bmatrix} \tag{3-16}$$

Simulation Results

Figure 3-6 shows the results of the heterogeneous rental market model for a grid of 21x21 interconnected locations with at grid point (16,4) capital city **A** and at (4,16) the regional urban core **B** of this interconnected rental market. **A** functions both as capital for the whole country and as a regional urban core for the region. The active migration is simulated such that there is an overall migration from the whole country towards the capital city **A** and there is a regional migration from the region surrounding the urban core towards the regional urban cores **A** and **B**.

The price subfigure (left Figure 3-6) shows that rent prices are highest at **A** and **B**. The capital city, **A**, and the direct surroundings are more expensive than the second city, **B**, and its surroundings.

The excess supply subfigure (mid Figure 3-6) shows that both urban cores **A** and **B** have significant excess demand. However, for the surroundings of the capital **A** the excess demand tapers off as the locations are further away from the capital center, whereas for the regional urban core **B** the surrounding locations have an excess supply. This is caused by migration from the region to the urban center, followed by high prices caused by arbitrage with the urban center.

The migration flow direction subfigure (right Figure 3-6) shows the normalized net migration minus the direct migration towards the urban cores, indicating the migration direction. The migration direction is away from the urban cores indicating that once the people are not migrating particularly towards the urban core, they are, on average, more likely to move further away from the urban core.

Variable	Description	Unit
$p_n(t)$	local price housing service	$\frac{\$}{\mathrm{m}^2 \cdot \mathrm{yr}}$
$\dot{p}_n(t)$	change local price housing service	$\frac{\$}{\mathrm{m}^2 \cdot \mathrm{yr}^2}$
$\dot{q}_{d1_n}(t)$	local price depended flow of housing services demanded per citizen	$\frac{\mathrm{m}^2}{\mathrm{ctz}}$
$\dot{q}_{d0_n}$	local demanded offset / max willingness to receive per citizen	$\frac{\mathrm{m}^2}{\mathrm{ctz}}$
$\dot{q}_{s_n}(t)$	local flow of housing services available per citizen	$\frac{\mathrm{m}^2}{\mathrm{ctz}}$
$\ddot{q}_{s_n}(t)$	local change flow of housing services available per citizen	$\frac{\mathrm{m}^2}{\mathrm{yr} \cdot \mathrm{ctz}}$
$\dot{q}_{es_n}(t)$	local excess flow of housing services supplied per citizen	$\frac{\mathrm{m}^2}{\mathrm{ctz}}$
$\ddot{q}_{cpg_n}(t)$	local construction and population growth	$\frac{\mathrm{m}^2}{\mathrm{yr} \cdot \mathrm{ctz}}$
$F_{cpg_n}(t)$	(bond graph figment)	$\frac{\$}{\mathrm{m}^2 \cdot \mathrm{yr}^2}$
$F_{es_n}(t)$	brokering	$\frac{\$}{\mathrm{m}^2 \cdot \mathrm{yr}^2}$
$F_{a_k}(t)$	arbitrage	$\frac{\$}{\mathrm{m}^2 \cdot \mathrm{yr}^2}$
$F_{rc_n}(t)$	local rent control	$\frac{\$}{\mathrm{m}^2 \cdot \mathrm{yr}^2}$
$\ddot{q}_{m_{k1}}(t)$	migration to adherent location	$\frac{\mathrm{m}^2}{\mathrm{ctz}}$
$\ddot{q}_{m_{k2}}(t)$	migration to capital	$\frac{\mathrm{m}^2}{\mathrm{ctz}}$
$\ddot{q}_{m_{k3}}(t)$	migration to urban center	$\frac{\mathrm{m}^2}{\mathrm{ctz}}$
$\ddot{q}_{mnet_n}(t)$	net migration	$\frac{\mathrm{m}^2}{\mathrm{ctz}}$

Table 3-3: Recap variable description for the heterogeneous rental market model

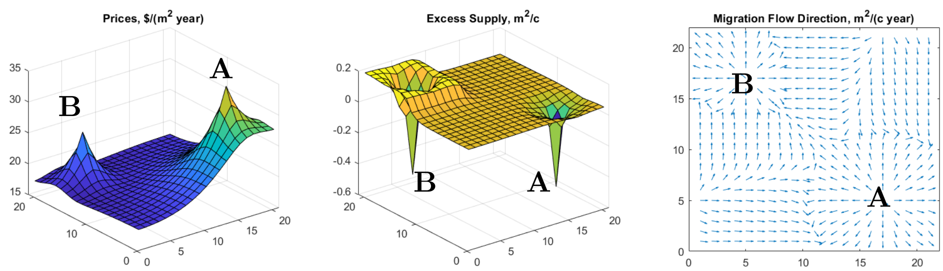


Figure 3-6: Simulation results with active migration as input of the heterogeneous rental market model

3-3-3 Conclusions

The spatially-homogeneous rental-market model successfully modeled the laws of supply and demand. A decrease in available housing services per citizen drives rent prices up, and demand for housing services reacts to this increased rent prices by reducing the demand for housing services. Rent control drives rent prices down. Demand for housing services reacts to the decreased rent price by increasing demand. Supply is price inelastic and does not react to this price change, thus resulting in excess demand. The spatially-homogeneous rental-market model thus captures the theoretically expected price response in a spatially-homogeneous rental market in complete concurrence with the theory of supply and demand for the housing market [16].

The heterogeneous rental-market model successfully modeled migratory patterns and arbitrage following from differences between locations. The heterogeneous rental-market model is a finite difference approximation of the continuous heterogeneous economic space of the rental market and spans a scalar price field and a migratory vector field. The net migration is either directly to the urban centers or dispersing away from the urban centers. Migration towards the urban cores causes excess demand in these urban cores, driving prices up. High prices in the urban centers drive prices up in surrounding locations through arbitrage. When prices are driven up because of arbitrage, but no people migrate out to the surrounding location, an excess supply is created in the surrounding locations. Thus, the heterogeneous rental-market model captures the spatial pricing mechanism in concurrence with the theory [16, 13, 40].

3-4 Real Estate Valuation Model

The discounted cash flow method calculates the present value of real estate as the integrated discounted future rent. In this manner, the discounted cash flow method provides the connection between the rent price and real estate value. The connection between the price of the product and the value of the asset.

Within economic engineering, the discounted cash flow method is calculated by applying the Laplace transform on LTI models [33, 34, 41]. With this goal in mind, the homogeneous and the heterogeneous rental market models are designed as LTI models in Section 3-3. This section will discuss how the Laplace transform, in conjunction with the rental market model, can be used to calculate real estate's present value. First in Subsection 3-4-1 the theory is explained. Second in Subsection 3-4-2 the mathematical argument is given for the real estate valuation variables discussed in Section 2-3.

3-4-1 Frequency Analysis of the Rental Market Model

This section expands on the previous section and considers the third and final aspect of the housing market ripple effect. As described in 2-3, the present value of a portfolio containing real estate is calculated using the discounted cash flow method over the rent. Rent consists of the rent price, $p(t)$, times the flow of housing services, $\dot{q}(t)$. The discount rate, $d(t)$, specifies how much future cash is discounted.

$$\text{PV} = \int_0^\infty p(t)\dot{q}(t)d(t)dt \tag{3-17}$$

The discount rate can be split into two parts. The exponential part, $d_e(t)$, and a cyclical wavefront, $d_c(t)$. $d_e(t)$ compensates for earnings on earnings. $d_c(t)$ is determined by a person's profile as discussed in 2-3 (e.g., the profile of older people, young people, immortal pension funds, or people living from paycheck to paycheck).

$$d(t) = \underbrace{e^{-\sigma t}}_{d_e(t)} d_c(t) \tag{3-18}$$

The present value formula is rewritten as a function of rent price, discount profile, and active portfolio size. The present value is a function in the complex plane. Where the bode plot a cross-section is of the complex plain over the imaginary axis, is the present value function a cross-section over the real axis. A positive discount rate means a dollar tomorrow is worth less than a dollar today, a negative discount rate thus means a dollar today is worth less than a dollar tomorrow. Positive discount rates lay in the right-half plane and negative discount rates lay in the left-half plane.

$$\text{PV}(\sigma) = \int_0^\infty p(t)\dot{q}(t)d_c(t)\underbrace{e^{-\sigma t}}_{d_e(t)}dt \tag{3-19}$$

$$= \int_{\sigma-\infty i}^{\sigma+\infty i} (V(s) * Q(s) * D_c(s))\delta(\omega)d\omega \tag{3-20}$$

$$V(s) = \mathscr{L}\{p(t)\} \qquad Q(s) = \mathscr{L}\{\dot{q}(t)\} \qquad D_c(s) = \mathscr{L}\{d_c(t)\}$$

Variable	Description	Unit
$p(t)$	rent price	$\frac{\$}{\mathrm{m}^2 \cdot \mathrm{yr}}$
$\dot{q}(t)$	flow housing services	m^2
$d(t)$	discount factor	%
$d_c(t)$	cyclical discount factor or profile dependent discount factor	%
$d_e(t)$	exponential discount factor	%

Table 3-4: Recap description of the time domain variables for real estate valuation

Variable	Description	Unit
$V(s)$	Ideal Present Value	$\frac{\$}{\mathrm{m}^2}$
$Q(s)$	Effective Housing Services	m^2
$D_c(s)$	Discount Profile	year
$PV(\sigma)$	Present Value	\$

Table 3-5: Recap description of the frequency domain variables for real estate valuation

3-4-2 Examples and an Application

Ideal Present Value Example

$V(s)$ is the ideal present value per square meter. For the particular case of a portfolio constantly producing the same amount of housing service per year, and the discount profile has a constant discount factor, the present value is equal to the constant number of square meters in the portfolio times the ideal present value per square meter. This is because the flow of housing services and the profile discount factor are now constants and can be taken out of the Laplace transform.

$$\mathrm{PV}(\sigma) = \int_{\sigma-\infty i}^{\sigma+\infty i} \mathscr{L}\{d_c\dot{q}p(t)\}\delta(\omega)d\omega = d_c\dot{q}V(\sigma) \tag{3-21}$$

Effective Housing Services Example

The same can be done for the effective housing services, $Q(s)$. For the particular case where the rent price stays constant and the discount profile has a constant discount factor, the present value equals the constant rent price times the profile discount factor times the effective housing services in a portfolio. Thus implying that housing services received in the future are worth less than housing services received now.

$$\mathrm{PV}(\sigma) = \int_{\sigma-\infty i}^{\sigma+\infty i} \mathscr{L}\{d_c\dot{q}(t)p\}\delta(\omega)d\omega = d_c pQ(\sigma) \tag{3-22}$$

Discount Profile Example

Finally, the same can be done for the discount profile $D_c(s)$, in the particular case that the future rent is fixed. Thus the rent price times the flow of housing services stays constant. The present value is equal to the constant rent times the discount profile. Thus implying that depending on a person's discount profile the same real estate can be worth more or less.

$$\mathrm{PV}(\sigma) = \int_{\sigma-\infty i}^{\sigma+\infty i} \mathscr{L}\{d_c(t)\dot{q}p\}\delta(\omega)d\omega = p\dot{q}D_c(\sigma) \tag{3-23}$$

Determining the Value of a Policies Change Application

let $x(0)$ be the rental market's present state. $U(s)$ is the effective expected inputs to the system, e.g., the present effect of where, when, and how much rent control is applied. $F_{rc_k}(t)$ is the original rent-control plan. Following some policy change, the original plan is reevaluated, and a new plan is put forward $F'_{rc_k}(t)$. Using (3-24) the exact effect of that policy change can be calculated for all locations in the heterogeneous market. $\Delta V(s)$ is a vector with all the ideal present value changes for every location due to the policy change. Assuming all other inputs to the rental market stay constant.

$$V(s) = C(Is - A)^{-1}x(0) + (C(Is - A)^{-1}B + D)U(s) \tag{3-24}$$

$$\Delta V(s) = (C(Is - A)^{-1}B + D)\mathscr{L}\{F_{rc_k}(t) - F'_{rc_k}(t)\} \tag{3-25}$$

3-4-3 Conclusions

The frequency-domain analysis of the rental market model successfully determines the present value of real estate. The frequency-domain analysis successfully enables profile-specific cyclical discounting. The frequency-domain analysis gives clear insights into policy changes' location-specific value changes. The frequency-domain analysis is able to determine the present value of yet to be constructed real estate, which does not have any rental income at present, by using the future active square meters.

The costs and benefits of owning a house consist of much more than just the rent (e.g., maintenance costs, mortgage interest, opportunity costs over the equity invested, and taxes [42]). These cash flows should all be considered for determining the net present value. Determining cash flows other than rent is outside the scope of this thesis. The frequency-domain analysis can be applied for finding the net present value when the other cash flows are known; however, the homogeneous and heterogeneous rental market models do not suffice in modeling other cash flows than the rent.

Chapter 4

Conclusions

This chapter will answer the research question, discuss the implications and applications of the model put forward in this thesis, and concludes by proposing further research to build on this thesis and work towards the incorporation of control theory in housing-market policymaking.

The research question 'Can economic-engineering model design integrate the three aspects of the housing-market ripple effect into one model?' can be answered affirmatively. This thesis put forward a generic economic-engineering housing-market model that can be fitted and adjusted to suit real-world housing markets. This model combines the supply and demand for housing services, the spacial effects of migration and arbitrage, and the present value of real estate into one model. Thus this leads to the conclusion that economic-engineering model design can indeed integrate the three aspects of the housing-market ripple effect into one model.

The first aspect of the ripple effect is modeled in the spatially-homogeneous rental-market model. The spatially-homogeneous rental-market model successfully models the laws of supply and demand in a dynamic manner. The simulations show that a decrease in available housing services per citizen increases the rent price. The simulations further show that demand for housing services reacts to this increased rent price by reducing the demand for housing services. When rent control drives down rent prices, demand for housing services reacts to the decreased rent price by increasing demand. Supply is price inelastic and does not react to this price change, which results in excess demand. These simulations following from the spatially-homogeneous rental-market model thus show the theoretically expected price response in a spatially-homogeneous rental market in concurrence with the theory of supply and demand for the housing market.

The second aspect of the ripple effect is modeled by expanding the spatially-homogeneous rental-market model to the heterogeneous rental-market model. The heterogeneous rental-market model successfully models migratory patterns and arbitrage following from differences between locations. The heterogeneous rental-market model is a finite difference approximation of the continuous heterogeneous economic space of the rental market and spans a scalar price field and a migratory vector field. The simulations show that the net migration is either directly to the urban centers or dispersing away from the urban centers. The simulations show that migration towards the urban cores causes excess demand in these urban cores, driving prices up. High prices in the urban centers drive prices up in surrounding locations through arbitrage. The simulations show that when prices are driven up because of arbitrage, but no people migrate towards the surrounding location, an excess supply is created in the surrounding locations. Thus, the heterogeneous rental-market model simulations show that the modeled spatial pricing mechanism concurs with the theory.

The third aspect of the ripple effect is modeled by expanding the time-domain analysis of the heterogeneous rental market model by adding a frequency-domain analysis. The frequency-domain analysis links the rental market to the real estate market. The rental and real estate markets are two sides of the same coin; The rent-price dynamics take place in the time domain, and the valuation of real estate is modeled as a frequency-domain analysis of the rental market. The frequency-domain analysis of the rental market model successfully determines the present value of real estate. The frequency-domain analysis successfully enables profile-specific cyclical discounting. The frequency-domain analysis gives clear insights into policy changes' location-specific value effects. The frequency-domain analysis is able to determine the present value of yet to be constructed real estate, which does not have any rental income at present, by using the expected rent, dependent on future active housing stock and rent price forecasts.

The model put forward in this thesis differs from existing housing market models in the following ways:

- The model put forward in this thesis integrates all three aspects of the housing-market ripple effect. Current housing market models focus on one aspect at a time.
- The economic-engineering model design used in this thesis allows for forecasts of prices and values for the housing market, where data-driven models struggle due to insufficient data.
- The economic-engineering model design also builds in the interpretability of the parameters where data-driven models also fall short. The value of the parameters still need to be determined, but at least the parameters have an actual economic interpretation.
- The economic-engineering model design makes the model put forward in this thesis suitable for control. This opens up the door for the development of a controller based on this model.

The application of the model put forward in this thesis includes predicting where shortages will arise due to rent control, building stops, migration, and population booms. The model provides both policymakers and investors with insights into the workings of the pricing mechanisms in the housing market. Such a model guides policymakers in determining where to build in order to relieve shortages in the housing market effectively. Such a model also informs investors about the value change to expect due to policy changes and migration trends. Such a model can be designed even for a housing market with little data, due to lack of administration or a rapidly changing circumstances. The interpretability of such a model helps policymakers explain how parameter tuned in the model and input tuned with the model correspond to a real-world change. The development of this economic engineering model ultimately makes a positive contribution to the housing market because it opens the door for introducing control theory into housing market policymaking.

This thesis is written in the broader ambition of incorporating control theory into the housing-market policymaking process. This thesis opens the door for control theory to housing market policymaking by developing a model suitable for control theory and integrally modeling the complete housing market ripple effect. Chapter 5 will discuss some recommendations for future research to further this goal of introducing control theory to housing market policymaking. These recommendations for future research include the asymmetric influence two locations have on each other, such as between a capital city and a small village. They further include the inclusion of the rest of the capital market into the model. Modeling the rest of the capital market will incorporate discount rate dynamics. These two recommendations will make the model more accurate and increase the predictive power. Finally, Chapter 5 recommends the development of the actual controller to make the housing market policy.

Chapter 5

Recommendations

5-1 Control Law Based Housing Policy

I propose expanding the housing market model with a case study and developing a controller for future research. This thesis aimed to develop a model to guide housing market policy. Currently, the housing market model put forward in this thesis can guide policymakers by forecasting the broader effects of proposed policies. However, the end goal is to make housing-market policy using control theory. This thesis developed a theoretical framework for a new kind of housing market model. This generic model can be fitted and adjusted to suit real-world housing markets, and then a controller can be tailored toward that particular situation. The development of a controller based on this model could further guide housing market policymaking and maybe someday essentially replace the policymaking process completely, at least up to political preferences.

Management as a controller is an idea already proposed multiple times within the economic engineering group [33, 34, 36, 43]. Following this idea, the government should be seen as the management of an economy.

5-2 Asymmetric Inter-Local Effects

I propose expanding the housing market model with asymmetric dependencies between locations for future research. Currently, the housing market model put forward in this thesis is designed such that the effect location A has on B is the same as the effect location B has on A. Reality is more nuanced. Firstly, large and densely populated locations, like a capital city, have a more significant impact on smaller villages than the other way around. An x amount of excess supply in the capital could distort the housing market of an adherent village completely, while that same amount of excess supply in the village would not even be noticeable in the housing market of the capital city. Secondly, arbitrage happens via inter-local real estate agents; however, the national media also influences. When the national news speaks of a significant housing shortage, it mainly covers the big cities. Villages, where there is no shortage, do not make the news. This asymmetric reporting can distort the perception of a country as a whole. Thus, I propose further research to consider asymmetric dependencies between locations for a more accurate model.

5-3 Interconnected Capital Markets

For future research, I propose expanding the housing market model by including the rest of the capital market. In essence, all forms of capital are basically just that, capital (e.g., bonds, stocks, land, real estate, machinery). As a rule, one wants to receive a benefit flow from invested capital [16, 42]. Short-term benefits are traded for long-term benefits. The proportion of benefits over the capital invested is the yield. Depending on how the capital is invested, the yield differs. Often the yield is dependent on the perceived risk; different risk perceptions per investment cause a strongly heterogeneous capital market. Capital investments will be rearranged when the risk perception changes, causing capital sub-markets to fluctuate with respect to each other. However, when there is more capital for the same flow of benefits, the yield goes down in the capital market as a whole. Figure 5-1 shows how the yield evolution on the Dutch government bond market, the real estate market of the 31 largest Dutch municipalities, and the Dutch stock market over the past 15 years. Although all capital markets move predominantly independently, it is clear that there is an interconnection between the markets: First, the yield of the bond market peaks; then, the yield of the stock market peaks, followed by the yield of the real estate market. After that, all markets follow the same downward trend. Figure Figure 5-1 summarizes the data analysis done with the help of and under the supervision of Brainbay.

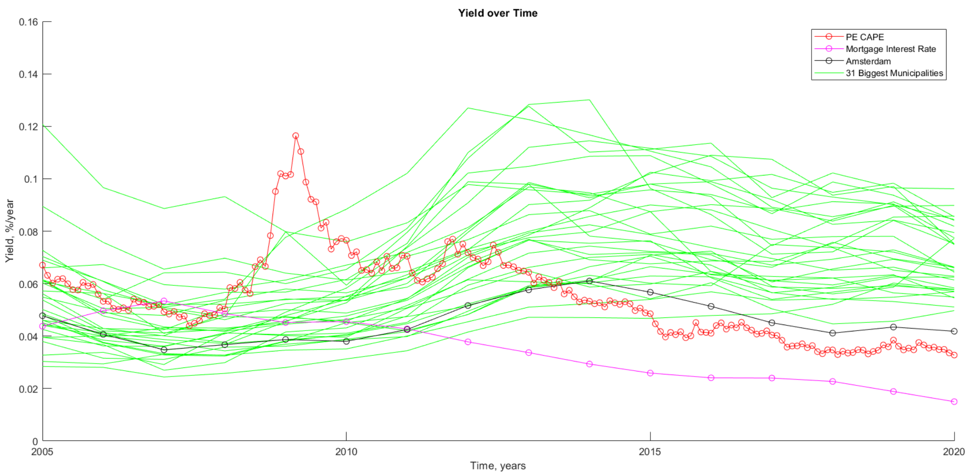


Figure 5-1: Data analyses of the yield over time of different capital markets

Appendix A

Machine Learning Price Models of the Housing Market

A-1 Introduction

Econometric models are the current academic modeling standard for the housing market. Econometrics is the application of machine learning methods to economic data in order to give content to empirical economic data [44]. This appendix will first discuss current machine learning approaches to estimate a current real estate value. Subsequently, dynamic value prediction methods are discussed. Finally, this chapter concludes by discussing whether machine learning methods are suited for applying control theory to guide housing market policymaking.

A-2 Machine Learning Appraisal Models

To give an accurate appraisal is to classify real estate value correctly within the existing data set. Machine learning appraisal models are not concerned with price dynamics. These models acknowledge that house prices change throughout time and aim to be as accurate as possible within a time step by constantly updating the training data and, therefore, the model itself. Machine learning appraisal models focus on the transaction price of a similar transaction shortly before. The to be classified prices are considered an interpolation within the data set. These appraisal models are not concerned with predicting what will happen in the next time step. Often a time dummy variable is given to different periods, not to track the evolution from one to the next but to classify the periods to separate groups.

A-2-1 Appraisal Methods

The comparative pricing method is the oldest appraisal method. A real estate agent selects a couple of comparable houses then estimates the value of the yet to be appraised house based on the prices of that sample set. This method is labor-intensive and sensitive to the subjectivity of the appraiser.

The hedonic price method expresses value as a function of distinctive quality characteristics of a house. Quality characteristics are all features a consumer is prepared to pay for (e.g., location, type of accommodation, floor space)[11]. Hedonic models are developed to separate price changes from quality changes in capital assets [45]. The coefficients of distinctive hedonic attributes can be seen as shadow prices, reflecting the value of each feature independently [46, 47, 48].

Automated valuation models based on hedonic pricing become more and more complex. The abundance of data allows for nonlinear data enrichment and opens up classifying methods such as neuron networks and random forest. Their linear regressed counterparts were highly interpretable with their shadow prices for each feature. However, the individual feature prices are not of interest, but the most accurate final appraisal is desired. Automated valuation models are becoming the industry standard as there is plenty of data to train them, replacing the expert opinion of the comparative pricing method [49].

A-3 Machine Learning index Prediction Models

Whereas appraisal models focus on giving the best estimate of the current value of a particular house, index prediction models focus on predicting the future index movements of a market. By averaging all market transactions, an index reflecting the price level of a market can be composed. This index reflects how the average house price develops over time.

The current standard in literature for index prediction models for the housing market is the ARIMA method. ARIMA models do not require as much knowledge about the mechanism influencing a variable as structural models. The only prior knowledge required is a list of variables that can be hypothesized to drive the dependent variable over time. From economic theory the candidate variables are selected; by regression the weights are set [22, 50, 18, 32, 51, 52, 53, 54, 7, 55, 56].

ARIMA models aim to find a weighted set of variables that explains the index movements Granger-causally; thus, hypothetically predicting future prices. The dependent variables are then tested for statistical significance to support that the results produced by these empirical models are not merely the result of chance. R-squared, t-tests, p-values, and null-hypothesis testing are all methods used by econometricians to evaluate the validity of their model results [57].

The use of ARIMA for house price predictions is often criticized for depending too heavily on statistical significance without linking it to established economic theory and looking for the causal mechanism [58]. Even the most rigorous statistical tests cannot exclude spurious effects when there is only one and limited data set, as is the case for the housing market. There is often no more than 20 years of data for the housing market, resulting in at most 80 strongly autocorrelated and noisy data points. Therefore Vastmans [1] argues that ARIMA models should only serve an exploratory role for the dynamics of house prices. Tu [52] acknowledges that ARIMA models are almost certainly wrong on a fundamental level because they do not match the economic theory.

A-3-1 Example Flaw Current Index Prediction Models

This example comes from Tu [51]

This index prediction model is based on the assumption that a (semi)fixed part of income is spent on mortgage interest [59]. This assumption matches the Cobb-Douglas utility function and the resulting Marshallian demand function. Such a theoretical model has also been proposed by Boelhouwer [55] where a (semi)fixed part of income is spent on mortgage interest 0.3.

$$\gamma P_h i = \theta * Y \tag{A-1}$$

Table A-1: Variables description of the exemplary econometric function

Variable	Description
P_h	House Price
γ	Percentage Borrowed
i	Average Interest Rate
θ	Percentage of Income Spent on Interest
Y	Income

This model evidently omits many effects. For example, it does not consider the rest of the user costs, regulations, or the asset pricing theory. However, it gives a crude explanation of how the interest rate links the part of income spent on housing services to the price of buying a house. Tu [51] sees the low variance accounted for of this theoretical model. Therefore Tu [51] suggests an adaptation to the model proposed by Boelhouwer [55], with the introduction of extra degrees of freedom.

$$log(P) = \alpha_1 * \log(\phi) + \alpha_2 * \log(Y) + \alpha_3 * \log(HS) + \pi \tag{A-2}$$

For comparison, this model can be rewritten as follows:

$$P = \phi^{\alpha_1} * Y^{\alpha_2} * HS^{\alpha_3} * i^{\alpha_4} \tag{A-3}$$

This model increased the variance accounted for the one period ahead prediction from $R^2 = 0.62$ to $R^2 = 0.78$ over 1982-2008 for the Netherlands. The regressed weights are $\alpha_1 = -65$, $\alpha_2 = 1.6$, $\alpha_3 = 5.0$ and $\alpha_4 = -0.008$. These coefficients do not come close to the theoretical coefficients of a fixed part of income spent on interest, (A-1), which both Boelhouwer [55] and Tu [51] use as an argument for their proposed models. However, almost all coefficients fall within the 1% significance interval. Using a vector error correction model method [60] and an Engle-Granger method [61] cointegration is established. Using an augmented Dickey-Fuller test and an Ng-Perron test, stationary for the first-order difference is established to fall within the 5% significance level. This model is thus statistically completely sound but misses any theory of why this holds and, most importantly, why this will hold in the future. It turns out it does not; this model quickly became inaccurate between 2008-2014.

$\alpha_1, \alpha_2, \alpha_3, \alpha_4$ have no physical interpretation. Nor can these regressed coefficients be used outside this specific model for this specific period. Other studies that used the same period and slightly changed the composition of explanatory variables got wildly different results [52, 59]. The influence of individual variables is thus completely dependent on the composition in which they accrue.

> "One can hardly fault the econometric rigor, however despite the fact that their model passes the tests for random residuals, parameter stability, and model consistency, theoretical arguments backed by our empirical results suggest that the 'true' model of house prices has not yet been found."
>
> — *Buckley*[62]

Conclusions

In general, machine learning models become reliable when there is enough data to rule out spurious effects. There is often enough cross-sectional data for static pricing models to make accurate appraisals. If not, one can start at that moment by collecting more data to increase the accuracy of the model. The final valuation is of most importance for the appraisals, and all the internal shadow prices are less so.

Machine-learning index-prediction models need, per definition, a significantly spread out period over which historical data is collected. Significantly spread out historical data is often scarce for the housing market, making current dynamic price models for the housing market often spurious and unreliable. When the data is not collected in the past, it takes a long time to supplement it with present-day measurements to form a long enough measurement period.

The question remains if machine-learning index-prediction models should be used to determine efficient long-term policies. Policymaking is often feedforward control over a long period, and feedback is only collected after this long period. Feedforward control needs reliable models to be efficient. Otherwise, the planned control action does not establish the desired effect. Current machine-learning index-prediction models for the housing market are based on only a few data points, have barely any theoretical backing, and are ad hoc at best.

Moreover, the point of implementing policy is to steer the system's state away from its natural state towards a more desired state. It seems illogical to base policy on correlations valid for the deserted state. Reforms alter the situation the data is gathered on and thus result immediately in empirical models which are out of date.

In conclusion, current machine learning models of the housing market are suited to appraise the current house prices; however, they are unsuited to predict future index movements and form the basis for policy.

Appendix B

Model Characteristics: Machine Learning Models vs. First Principles Models

One of the most important applications for mathematical or scientific models is to predict the future accurately. The second most important aspect is understanding the cause of the predicted. The kind of model to design depends on what is needed to predict and what is already known. In particular, when the mathematical equations governing the system's dynamics are known, a fundamental model might be preferred. However, an empirical model might be a more suitable option when there is no understanding of the system dynamics, but there is plenty of observable data. When accuracy is essential, the choice may fall on a more complex model, whereas a linearized and simplified model might be preferred when the interpretability is critical.

B-1 Complexity vs. Simplicity

Accuracy concerns the ability of a model to make correct predictions, while interpretability is the degree the model allows for human understanding. Complex models have more degrees of freedom to capture more detailed and subtle effects, whereas simple models are more straightforward to understand. Especially when the underlying system itself is complex, the trade-off between accuracy and interpretability becomes apparent. This trade-off occurs when in favor of interpretability, the model is simplified, and details are left out.

Although complexity decreases the interpretability of a model, complexity is not the only component for interpretability. Abstract parameters in the model also decrease the interpretability of a model. Highly complex models, but composed of parameters that can be linked directly to the physical world, can be more easily interpretable than a simple model with only abstract parameters [5].

B-2 Theoretic vs. Empirical

Theoretic models, sometimes called fundamental or first principal models, have multiple advantages. Theoretic models are able to extrapolate and generalize, whereas empirical models are always case-specific. Only a few data points are needed to fit a theoretic model to a particular situation, whereas the number of observations required for empirical models tends to grow exponentially with the number of variables included. Theoretic models can therefore make highly reliable predictions outside the range of previous observations, whereas empirical models are only reliable for interpolation.

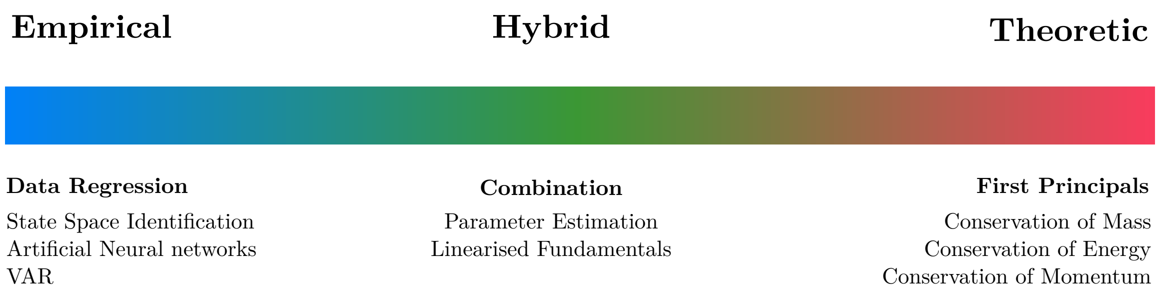
Often, complications arise when theoretical models are applied in the real world, primarily when the predictions concern more complex, less understood phenomena. Often the first principals are not available or require lengthy computation. When the only concern is a primarily reliable prediction, a causative explanation of the system's dynamics is nice to have but no necessity.

Machine learning models, sometimes called statistical, empirical, or data-driven models, relies on observation and correlation rather than first principles. A pattern is determined solely on observations; this pattern is then used to predict the future. Armed with correlations between observations and outcomes, no knowledge about the laws governing the system or about the equations describing these governing laws is needed. Correlation does not imply causation. However, it does not have to imply causation as long as it is a useful predictor.

Outside the STEM fields, the question of "true causality" can be deeply philosophical. The "post hoc ergo propter hoc" fallacy states that one thing preceding another can be used as proof of "predictive causality" or Granger-causality [63]. Using the term "causality" alone is a misnomer, as Granger-causality is better described as "precedence" [64], as Granger himself claimed, "temporally related" [65]. Rather than stating that X causes Y, it states X forecasts Y [66]. Because of this, correlations do not represent causal relationships unless all spurious relationships can be ruled out. In experiments, first principles can be identified by controlling for all spurious explanations. However, these experiments can be costly, time-consuming, and simply herculean to differentiate between all individual factors of highly complex systems [67].

Under the hood of an empirical model, an algorithm sifts and re-sifts all the available data, comparing and weighing the combined input signals with the outcome, and eventually returns the best-weighed set of predictive signals. The user then ought to validate the empirical model on a new independent set of data to check whether the empirical model is more generally applicable than solely within the training set.

Almost all models are a combination of fundamental and empirical. Fundamental models must allow for some element of empiricism to switch from a general model to a specific model. In addition, reality almost always suffers some disturbance and uncertainty, for which a purely fundamental model can not account. The reverse is equally true. Empirical models include fundamental elements, if only in the selection of candidate predictor variables to investigate [68].



B-3 Conclusions

When the outcome of a model is presumed to be a given on which there is no or negligible influence to be exerted (e.g., the weather forecast), the accuracy of the outcome is most important. However, an interpretable model is essential when people make, adjust, and explain policy based on model predictions.

A direct link between the model parameters and reality exists within fundamental models, whereas the parameters in empirical models are often an abstract aggregation of multiple physical features. Abstract parameters do not have to decrease the interpretability and accuracy of the model prediction. However, it does decrease the interpretability of the model itself. With decreasing interpretability of the model comes a decrease in understanding the cause of the predicted outcome. This understanding of the cause of the predicted outcome allows control theorists and policymakers to change something in the real world to create a more desirable reality. Thus, when humans are involved in making policies, interpretability is essential.

Appendix C

Matlab Code

C-1 MATLAB Code: Spatially Homogeneous Rental Market Model

```
close all
md=5;      %price inelasticity of demand
ms=10000; %price inelasticity of supply
ba=1;      %real estate broker
p10=20;    %inital price housing services
vs10=4;    %inital supply housing services
% exces supply is ves=p/m+mwr+vs
% broker effect is Fes=b(p/m+mwr+vs)
mwr=-8; %max willingness to recive, ofset demand

A=[-ba/md -ba;
    -ba/md/ms -ba/ms];


B=[0 -ba 1;1 -ba/ms 0]; %u=[a_cpg;v_mwr;F_rc]
C=[eye(2);1/md 0;1/md 1];    %y=[p;vs;vd;ves]
D=[0 0 0;0 0 0;0 1 0;0 1 0];
sys=ss(A,B,C,D);

x10=[p10; vs10];  %[price; supply housing services]


t=0:100;
t1=floor(length(t)/5);     % input time change points
t2=floor(length(t)/5*2);
t3=floor(length(t)/5*3);
t4=floor(length(t)/5*4);
t5=length(t);

%u=-6*[zeros(t1,1); 0.1*ones(t2-t1,1); zeros(t3-t2,1); -0.05*ones(t4-t3
    ,1); zeros(t5-t4,1)];
```

```
%u=-6*[zeros(t1,1); 0.1*ones(t5-t1,1)];
u=-1.2*[zeros(ceil(length(t)/5),1); 0.1*ones(floor(length(t)/5),1); zeros(floor(length(t)/5),1); -0.05*ones(floor(length(t)/5),1); zeros(floor(length(t)/5),1)];


[y,t]=lsim(sys,[u u*0+mwr u*0],t,x10);
%%

fig=figure('Color',[1 1 1],'InnerPosition',[-100 30 1700 750]);

subplot(2,6,[1 2 7 8])
hold on
plot(y(:,2),y(:,1),'r*')
plot(y(:,3),y(:,1),'b*')

plot([3 8]-8,[3 8]*md,'b')
plot(y(floor(length(t)/5),2)*[1 1],[3; 8]*md,'r')
plot(y(floor(length(t)/5*3),2)*[1 1],[3; 8]*md,'r')
plot(y(end,2)*[1 1],[3; 8]*md,'r')
plot([0 0],[15 40],'k')
%axis([-2 2 15 40])
xlabel('Living Space per Citizen v, m^2/c')
ylabel('Price p, $/(m^2 year)')
legend('Living Space Supplied','Living Space Demanded','v_d(p)','v_s(t)')

subplot(2,6,[3 4 9 10])
plot(y(:,4),y(:,1),'g*')
hold on
plot([-2:2],[-2:2]*md+y(floor(length(t)/5),1),'g')
plot([-2:2],[-2:2]*md+y(floor(length(t)/5*3),1),'g')
plot([-2:2],[-2:2]*md+y(end,1),'g')
%plot([-2:2],[-2:2]*md+y(1,1),'g')
plot([0 0],[15 40],'k')
axis([-2 2 15 40])
xlabel('Living Space per Citizen v, m^2/c')
ylabel('Price p, $/(m^2 year)')
legend('Excess Living Space Supplied per Citizen ','v_{es}(p,t)')

subplot(2,6,[5 6])
plot(t,y(:,1),'m',[t1 t1;t2 t2;t3 t3;t4 t4]',[min(y(:,1))-1 max(y(:,1))+1],'k')
axis([0 t(end) min(y(:,1))-1 max(y(:,1))+1])
ylabel('Price, $/m^2 year')
xlabel('Time, year')

subplot(2,6,[11 12])
plot(t,u,'color',[0.9290 0.6940 0.1250])
hold on
plot([t1 t1;t2 t2;t3 t3;t4 t4]',[min(u)-1 max(u)+1],'k')
axis([0 t(end) min(u)-1 max(u)+1])
%ylabel('Rent Control, $/(m^2 year^2)')
ylabel('Construction and Population Growth, m^2/(c year)')
```

```
xlabel('Time, year')

sgtitle('Rental Price and Living Space per Citizen')
```

C-2 MATLAB Code: Spatially Heterogeneous Rental Market Building Blocks

```
%% alleen de migratie

b1=1;
b2=1*b1;
m=pi;

A=-(b1+0.1*b2)/m;
B=[b1/m 1/m -b2/m/10];
C=[1;-1;.1];
D=[0 0 0; 1 0 0; 0 0 1];

sys=ss(A,B,C,D);

t=0:100;
u=[1 0 -1]'*(t*0+1);
x0=0;
lsim(sys,u,t,x0)


%% 2 prijzen en migratie

b=1;
%b=10*b1;
m=5;
J=10;
A=[-b/m 0 b;
    0 -b/m -b;
    b/(m*J) -b/(m*J) -2*b/J];


B=[-b 0;
    0 -b;
    b/J -b/J];
C=[eye(3);
    1/m 0 -1;
    0 1/m 1];
D=[zeros(3,2);
    1 0;
    0 1];

sys=ss(A,B,C,D);

t=0:10;
u=[-3 -4]'*(t*0+1);
x0=[20 15 0];
lsim(sys,u,t,x0)

%% 2 prijzen met migratie en makelaars

```

```
b1=1;
ba=1;
m=5;
J=1;

A=[-(b1+ba)/m ba/m b1;
    ba/m -(b1+ba)/m -b1;
    b1/(m*J) -b1/(m*J) -2*b1/J];


B=[-b1      0     0;
    0     -b1     0;
    b/J -b/J    1];
C=[eye(3);
    1/m 0 -1;
    0 1/m 1];
D=[zeros(3,3);
    1 0 0;
    0 1 0];

sys=ss(A,B,C,D);

t=0:10;
u=[-4 -4 5]'*(t*0+1);
x0=[20 20 0];
lsim(sys,u,t,x0)

%%  2 prijzen met migratie transformer en makelaars
b1=1;
ba=1;
m=5;
J=1;
TF=10;



A=[-(b1+ba)/m ba/m b1;
    ba/m -(b2+ba)/m -b2*TF;
    b1/(m*J) -b2*TF/(m*J) -(b1+TF^2*b2)/J];


B=[-b1      0     0;
    0     -b2     0;
    b1/J -b2*TF/J  1];
C=[eye(3);
    1/m 0 -1;
    0 1/m 1*TF];
D=[zeros(3,3);
    1 0 0;
    0 1 0];

sys=ss(A,B,C,D);

```

```
t=0:30;
u=[-4 -4 5]'*(t*0+1);
x0=[20 20 0];
lsim(sys,u,t,x0)


%% 3 prijzen, migratie en makelaars op 0->-0->-0

b1=1;
ba=1;
m=5;
J=1;

A=[-(b1+ba)/m    ba/m          0          b1           0;
   ba/m       -(b1+2*ba)/m   ba/m       -b1           b1;
   0              ba/m     -(b1+ba)/m     0           -b1;
   b1/(m*J)   -b1/(m*J)       0        -2*b1/J      b1/J;
   0            b1/(m*J)    -b1/(m*J)  b1/J      -2*b1/J];


B=[-b1 0 0 0 0;
    0 -b1 0 0 0;
    0 0 -b1 0 0;
    b1/J -b1/J 0 1 0;
    0 b1/J -b1/J 0 1];


C=[eye(5);
    1/m 0 0 1 0;
    0 1/m 0 -1 -1;
    0  0  1/m 0 1];


D=[zeros(5,5);
    eye(3,5)];

sys=ss(A,B,C,D);

t=0:10;
u=[-4 -4 -4 -5 -5]'*(t*0+1);
x0=[20 20 20 0 0];
lsim(sys,u,t,x0)


%%  4 prijzen, migratie en makelaars circulair
%  0->-0
%  |   |
%  v   v
%  |   |
%  0->-0

b1=1;
ba=1;
```

```
m=5;
J=1;

%syms b1 ba m J

A=[-(b1+2*ba)/m     ba/m       ba/m              0                     b1   b1   0   0;
   ba/m       -(b1+2*ba)/m      0              ba/m                   -b1   0    b1   0;
   ba/m                  0      -(b1+2*ba)/m   ba/m                    0   -b1    0   b1;
   0                ba/m          ba/m       -(b1+2*ba)/m              0    0    -b1  -b1;

   b1/(m*J)   -b1/(m*J)        0                0                    -2*b1/J      -b1/J     b1/J         0  ;
   b1/(m*J)        0        -b1/(m*J)           0                      -b1/J    -2*b1/J        0        b1/J;
      0          b1/(m*J)        0          -b1/(m*J)                   b1/J          0     -2*b1/J     -b1/J;
      0              0         b1/(m*J)     -b1/(m*J)                    0         b1/J    -b1/J   -2*b1/J];

% [L,V]= eig(A)

Ab=[b1   b1   0       0;
   -b1   0    b1      0;
    0   -b1    0     b1;
    0    0    -b1   -b1];
%
% Ac=[b1/(m*J)  -b1/(m*J)       0              0 ;
%      b1/(m*J)       0       -b1/(m*J)         0 ;
%        0        b1/(m*J)       0          -b1/(m*J);
%        0            0        b1/(m*J)     -b1/(m*J)];
% Ad=-Ac*Ab*m;
%T=[eye(5) zeros(5,3);zeros(3,5) [0 1 0; 0 0 1;1 0 0]']
%AAA=T*A*T';




B=[-b1*eye(4) zeros(4);
  [ b1/J    -b1/J       0        0;
    b1/J       0     -b1/J       0;
    0        b1/J       0    -b1/J
    0          0      b1/J  -b1/J] eye(4)];


C=[eye(8);
   1/m*eye(4) [-1  -1  0  0;
                1  0  -1  0;
                0  1   0  -1;
                0  0   1  1]];
```

```
C=[eye(8);
    1/m*eye(4) -Ab/b1];


D=[zeros(8);
   [eye(4) zeros(4)]];

sys=ss(A,B,C,D);

t=0:100;
u=[-4 -4 -4 -4 -5 5 0 0]'*(t*0+1);
x0=[20 20 20 20 0 0 0 0];
lsim(sys,u,t,x0)


  %% 4 prijzen, migratie en makelaars centraal

%  0-->--0
%  |\
%  v _|
%  |    \
%  0     0


b1=1;
ba=1;
m=5;
J=1;

%syms b1 ba m J

A=[-(b1+3*ba)/m    ba/m       ba/m          ba/m                  b1   b1   b1;
    ba/m       -(b1+ba)/m       0              0                 -b1    0     0;
    ba/m                 0     -(b1+ba)/m      0                  0   -b1     0;
    ba/m                 0        0        -(b1+ba)/m             0    0    -b1;

    b1/(m*J)   -b1/(m*J)        0              0              -2*b1/J      -b1/J
          -b1/J;
    b1/(m*J)         0       -b1/(m*J)         0              -b1/J      -2*b1/J
          -b1/J;
    b1/(m*J)         0          0          -b1/(m*J)          -b1/J        -b1/J
        -2*b1/J];



%[L,V]=eig(A)

Aa=[-(b1+3*ba)/m    ba/m       ba/m          ba/m;
    ba/m       -(b1+ba)/m       0              0   ;
    ba/m                 0     -(b1+ba)/m      0   ;
    ba/m                 0        0        -(b1+ba)/m];

```

```
Ab=[b1  b1  b1;
   -b1  0    0;
    0  -b1   0;
    0   0   -b1];

Ac=[b1/(m*J)  -b1/(m*J)      0            0;
    b1/(m*J)      0       -b1/(m*J)       0;
    b1/(m*J)      0            0       -b1/(m*J)];


%Ac-Ab'/(m*J)

Ad=-Ac*Ab*m;

AA=[Aa Ab;Ac Ad];

A=AA

B=[-b1*eye(4) zeros(4,3);
  [ b1/J   -b1/J     0       0;
    b1/J     0     -b1/J     0;
    b1/J     0       0     -b1/J] eye(3)];


C=[eye(7);
   1/m*eye(4) -Ab/b1];

D=[zeros(7);
  [eye(4) zeros(4,3)]];

sys=ss(A,B,C,D);

t=0:100;
u=[-4 -4 -4 -4 0 5 5]'*(t*0+1);
x0=[20 20 20 20 0 0 0];
lsim(sys,u,t,x0)




%% 4 prijzen, migratie en makelaars centraal + circulair


%  O-->--O       O->-O
%  |\            |   |
%  v _|      +   v   v
%  |   \         |   |
%  O     O       O->-O



b1=1;
```

```
ba=1;
bb=1;
m=5;
J=1;

%syms b1 ba m J

A=[-(b1+2*ba+3*bb)/m (ba+bb)/m (ba+bb)/m bb/m b1 0 b1 0 b1 b1 b1;
   (ba+bb)/m -(b1+2*ba+bb)/m 0 ba/m -b1 0 0 b1 -b1 0 0;
   (ba+bb)/m 0 -(b1+2*ba+bb)/m ba/m 0 b1 -b1 0 0 -b1 0;
   bb/m ba/m ba/m -(b1+2*ba+bb)/m 0 -b1 0 -b1 0 0 -b1;



   b1/(m*J) -b1/(m*J) 0 0 -2*b1/J 0 -b1/J b1/J -2*b1/J -b1/J -b1/J;
   0 0 b1/(m*J) -b1/(m*J) 0 -2*b1/J b1/J -b1/J 0 b1/J -b1/J;
   b1/(m*J) 0 -b1/(m*J) 0 -b1/J b1/J -2*b1/J 0 -b1/J -2*b1/J -b1/J;
   0 b1/(m*J) 0 -b1/(m*J) b1/J -b1/J 0 -2*b1/J b1/J 0 -b1/J;
   b1/(m*J) -b1/(m*J) 0 0 -2*b1/J 0 -b1/J b1/J -2*b1/J -b1/J -b1/J;
   b1/(m*J) 0 -b1/(m*J) 0 -b1/J b1/J -2*b1/J 0 -b1/J -2*b1/J -b1/J;
   b1/(m*J) 0 0 -b1/(m*J) -b1/J -b1/J -b1/J -b1/J -b1/J -b1/J -2*b1/J];


% [L,V]=eig(A);
% real(V)

Ab=[b1 0 b1 0 b1 b1 b1;
   -b1 0 0 b1 -b1 0 0;
    0 b1 -b1 0 0 -b1 0;
    0 -b1 0 -b1 0 0 -b1];
%
Ac=[b1/(m*J) -b1/(m*J) 0 0 ;
    0 0 b1/(m*J) -b1/(m*J);
   b1/(m*J) 0 -b1/(m*J) 0 ;
    0 b1/(m*J) 0 -b1/(m*J);
   b1/(m*J) -b1/(m*J) 0 0 ;
   b1/(m*J) 0 -b1/(m*J) 0 ;
   b1/(m*J) 0 0 -b1/(m*J)];

% %Ac-Ab'/(m*J)
%
% Ad=-Ac*Ab*m;
```

```
%
% AA=[Aa Ab;Ac Ad];
%
% A-AA
%

B=[-b1*eye(4) zeros(4,3);
  Ac*m [zeros(4,3);eye(3)]];


C=[eye(4,11);
   1/m*eye(4) -Ab/b1];

D=[zeros(4,7);
  [eye(4) zeros(4,3)]];

sys=ss(A,B,C,D);

t=0:100;
u=[-4 -4 -4 -4 50 50 50]'*(t*0+1);
x0=[20 20 20 20 0 0 0 0 0 0 0];
lsim(sys,u,t,x0)
```

C-3 MATLAB Code: Spatially Heterogeneous Rental Market Building Full Size

```
close all
n=21; % amount of grid points width and hight
dx1=2; % influence region second city
b1=1;  % reaction force internal market
ba=2;  % reaction force price difference naboring markets
bb=0.001; % reaction force price difference migration capital
bc=0.01; %
m=5;  %inverce price elasticity
J=1;  % reluctance to migrate
u=[-4;5;5];  %input [Sf;Se_mc;Se_msc]


%%%%%%%% position captial and second city
Size=10;   % lot size

mux1=0;    % x location capital
muy1=0;    % y location capital
mux2=8;    % x location second city
muy2=2;    % y location second city

mun1=mux1*(n-1)/Size+muy1*(n-1)*n/Size+1; % place grid vector notation capital
munx1=mux1*(n-1)/Size+1;                  % row grid matrix notation capital
muny1=muy1*(n-1)/Size+1;                  % collom grid matrix notiation capital
mun2=mux2*(n-1)/Size+muy2*(n-1)*n/Size+1;   % place grid vector notation capital
munx2=mux2*(n-1)/Size+1;                    % row grid matrix notation capital
muny2=muy2*(n-1)/Size+1;                    % collom grid matrix notiation capital



%%%%%%% Aa
%%% Aa  diagonal
AD0a=[(-b1-2*ba)/m; ones(n-2,1)*(-b1-3*ba)/m;(-b1-2*ba)/m];
AD0b=[(-b1-3*ba)/m; ones(n-2,1)*(-b1-4*ba)/m;(-b1-3*ba)/m];
AD0=[AD0a;kron(ones(n-2,1),AD0b);AD0a];

%%% Aa one above and below diagonal
AD1a=kron(ones(n-1,1),ba/m);
AD1=[kron(ones(n-1,1),[AD1a; 0]);AD1a];

%%% Aa one grid size offset diagonal
ADn=kron(ones(n^2-n,1),ba/m);

%%% inclution reaction force price difference migration capital
```

```
Aabb=spdiags(-bb/m*ones(n^2,1),0,n^2,n^2);
Aabb(mun1,:)=Aabb(mun1,:)+bb/m*ones(1,n^2);
Aabb(:,mun1)=Aabb(:,mun1)+bb/m*ones(n^2,1);
Aabb(mun1,mun1)=-(n^2-1)*bb/m;

%%% construction Aa
Aa=spdiags(AD0,0,n^2,n^2)+spdiags([0;AD1],1,n^2,n^2)+spdiags([AD1],-1,n^2,n^2)+...
  spdiags([zeros(n,1); ADn],n,n^2,n^2)+spdiags( ADn,-n,n^2,n^2)...
  +Aabb;


%%%%%%% Ab
%%%%%%%% Ab1 vertical flow
Ab1=spdiags((b1)*ones(n^2,1),0,n^2,n^2)+spdiags((-b1)*ones(n^2,1),-1,n^2,n^2);
Ab1(:,n*[1:n])=[];

%%%%%%%  Ab2 horizontal flow
Ab2=spdiags((b1)*ones(n^2-n,1),0,n^2,n^2-n)+spdiags((-b1)*ones(n^2-n,1),-n,n^2,n^2-n);


%%%%%%% Ab3 migration flow capital
Ab3=spdiags(-b1*ones(n^2,1),0,n^2,n^2);
Ab3(mun1,:)=Ab3(mun1,:)+b1;
Ab3(:,mun1)=[];


%%%%%%% Ab4  migration flow secon city
dummi2=zeros(n);
dx=dx1*((n-1)/Size);
dummi2([munx2-dx:munx2+dx],[(muny2-dx):(muny2+dx)])=1;
dummi3=reshape(dummi2,n^2,1);
Ab4=spdiags(-b1*dummi3,0,n^2,n^2);
Ab4(mun2,:)=Ab4(mun2,:)+b1;
dummi4=-dummi3+1;
dummi4(mun2)=1;
Ab4(:,find(dummi4))=[];

%%% construction Ab
Ab=[Ab1 Ab2 Ab3 Ab4];


%%%%%%% Ac
Ac=Ab'/(m*J);


%%%%%%% Ad
Ad=(-Ac*Ab*m);


%%%%%%%% A, x=[P;V_v;V_h;V_mc;V_msc]  v=vertical h=horizontal
```

```
%%%%                mc= magration capital msc=migration secon city

A=[Aa Ab;Ac Ad];
%plot(10.^real(eig(full(A)))) % verify stability. all zero eigen values are redundend pathways

%%%%%%%%  B,  u=[Sf;Se_mc;Se_msc]
B=[[-b1*ones(n^2,1); zeros(size(Ab,2),1)]...
    [zeros(size(Aa,2)+size([Ab1 Ab2],2),1); ones(size(Ab3,2),1); zeros(size(Ab4,2),1)]...
    [zeros(size([Aa Ab1 Ab2 Ab3],2),1); ones(size(Ab4,2),1)] ];

%%%%%%%  C    y=[P;V_v+V_h+V_mc+V_msc;V_v+V_h+V_mc+V_msc] +[0;Sf;0]   v= vertical h=horizontal
%%%%                mc= magration capital msc=migration secon city
C1=speye(size(Aa,2),size(A,2));  % prices
C2=[speye(size(Aa,2)).*(1/m) -Ab/b1]; % exces supply
C3=[zeros(size(Aa,2)) -Ab/b1];        % net flow in


%%%%%% D  y=[P ; V_v+V_h+V_mc+V_msc ; V_v+V_h+V_mc+V_msc] +[0;Sf;0]   v= vertical h=horizontal
%%%%                mc= magration capital msc=migration secon city

D1=zeros(n^2,3); % prices
D2=[ones(n^2,1) zeros(n^2,2)];  % exces supply
D3=[zeros(n^2,3)];              % net flow in

clearvars -except A B C1 C2 C3  D1 D2 D3 n mun1 u

%%
dt=0.001;   % time step, a lagrger grid need smaller steps
t=0:dt:20;  % Simulation time

p0=20;      % initial price p0 = 20, m=5 and Sf=-4 is a stable inital conditon
            % p0+m*Sf=0

x0=[10000;kron(ones(n^2-1,1),p0);zeros(size(A,2)-n^2,1)];
                            %x=[P;V_v;V_h;V_mc;V_msc]  v=vertical h=horizontal
                            %    mc= magration capital msc=migration secon city
xo=x0;                      % initiate initial state
Uo=zeros(n);                % initiate initial horizontal naboring flow
Vo=zeros(n);                % initiate initial vertical naboring flow
Wo=zeros(n);                % initiate initial vertical naboring flow

[X,Y]=meshgrid(1:n,1:n);


%%%%% initiate figure
```

```
fig=figure('Color',[1 1 1],'Position',[10 50 1500 700],'InnerPosition',[0 30 1500 750]);
dn=200;  %%% time steps between snap shots
j=0;     %%% initiate snap shot counter

for i=1:length(t)
xn=(A*xo+B*u)*dt+xo;  % update state


if rem(i,dn)==0        % selecte snapshot moments
y1n=C1*xn+D1*u;        % update prices

y2n=C2*xn+D2*u;        % update exces demand
y2o=C2*xo+D2*u;        % previous exces demand
dy2=y2n-y2o;           % change excces demand

y3n=C3*xn+D3*u;        % net total in flow
y3o=C3*xo+D3*u;
dy3=y3n-y3o;           % instant net in flow

Vd= xn(2*n^2-n+1:3*n^2-2*n);         %selection horizontal flow from state vector
V1=[zeros(n,1) reshape(Vd ,n,n-1)]; %reshape horizontal flow to matrix
V2=[reshape(Vd ,n,n-1) zeros(n,1)]; % and add zeros for stagard grid
Vn=(V1+V2)/2;                         % take mean of stagard grid

Ud=xn(n^2+1:2*n^2-n);                % same vertecaly
U1=[zeros(1,n);reshape(Ud,n-1,n)];
U2=[reshape(Ud,n-1,n);zeros(1,n)];
Un=(U1+U2)/2;

Wd=xn(3*n^2-2*n+1:4*n^2-2*n-1);                % same vertecaly
W1=[Wd(1:mun1-1); 0 ;Wd(mun1:end)];
W2=reshape(W1,n,n);
Wn=W2;


% Us=Un./sqrt(Un.^2+Vn.^2);          %scaled total migration from one
                                     % location to the naboring location
% Vs=Vn./sqrt(Un.^2+Vn.^2);

dV=Vn-Vo;                            % migration on that instant
dU=Un-Uo;
dW=Wn-Wo;

dUs=dU./sqrt(dU.^2+dV.^2);           % scaled migration on that instant
dVs=dV./sqrt(dU.^2+dV.^2);



subplot(2,2,1)
surf(reshape(y1n,n,n))               % 3d representation
%  pcolor(1:n,1:n,reshape(y1n,n,n)) %top view
```

```
   set(gca,'xtick',[])
   set(gca,'ytick',[])
   title('Prices, $/(m^2 year)')


subplot(2,2,2)
  surf(reshape(y2n,n,n))
 % pcolor(1:n,1:n,reshape(y2n,n,n))
  set(gca,'xtick',[])
  set(gca,'ytick',[])
  title('Excess Supply, m^2/c')
hold on
quiver(X,Y,-dVs,-dUs,0.5)      % migration from exces demand to exces supply
hold off


subplot(2,2,3)
surf(reshape(dy2,n,n))
%pcolor(1:n,1:n,reshape(y2n-y2o,n,n))
  set(gca,'xtick',[])
  set(gca,'ytick',[])
  title('change Excess Supply, m^2/c')
hold on
%quiver(X,Y,-V,-U,0.5)
  hold off

subplot(2,2,4)
surf(-dW)
% pcolor(1:n,1:n,y3)
  set(gca,'xtick',[])
  set(gca,'ytick',[])
 title('Migration to Capital, m^2/c')
% hold on
% quiver(X,Y,-V,-U,0.5)
%   hold off



  drawnow
%j=j+1
%FlipBook(j)=getframe(fig);
%print(['TestTest/test-' num2str(j)],'-dpng')
Uo=Un;
Vo=Vn;
Wo=Wn;
else
end


xo=xn;
end
```

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