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RESEARCH ARTICLE

# On the tails of the wind ramp distributions

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## Abstract

We analyzed several multiyear wind speed datasets from 4 different geographical locations. The probability density functions of wind ramps from all these sites revealed remarkably similar shapes. The tails of the probability density functions are much heavier than a Gaussian distribution, and they also systematically depend on time increments. Quite interestingly, from a purely statistical standpoint, the characteristics of the extreme ramp-up and ramp-down events are found to be almost identical. With the aid of extreme value theory, we describe several other inherent features of extreme wind ramps in this paper.

## KEYWORDS

extreme value theory, Hill plot, Pareto distribution, tail-index

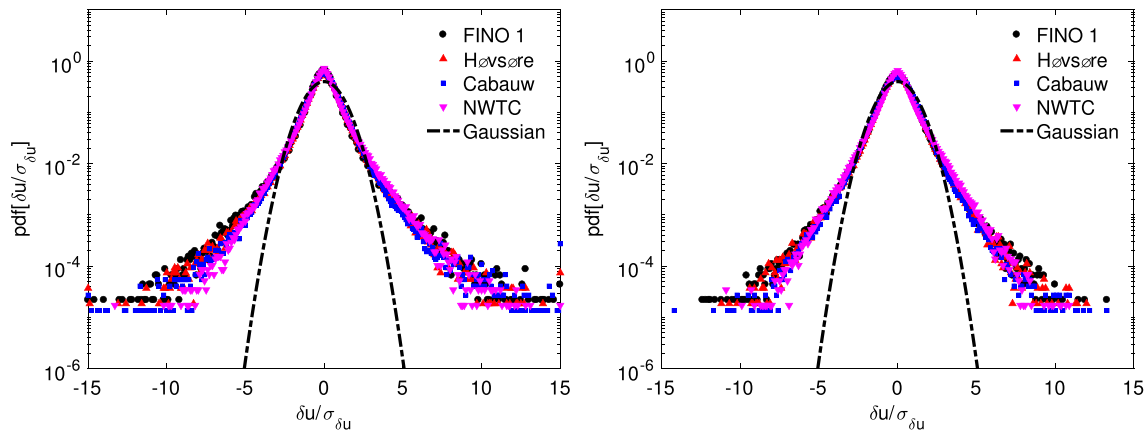
## 1 | INTRODUCTION

One of the major challenges facing the wind energy industry is the accurate prediction of sudden and sharp fluctuations in the wind field (aka wind ramps) near the lower part of the atmospheric boundary layer.<sup>1-4</sup> These not-so-rare and inauspicious events can drastically modulate deficiencies (ramp-down) and/or surpluses (ramp-up) in wind power production causing disruptions in operations and energy supply balance. Therefore, as the demand for more reliable wind power increases there is an ever-present need for further advancement in the understanding of how to properly characterize and quantify the ramp events.<sup>5</sup> It is well-known in the literature that various meteorological factors can contribute to ramp events including (but not limited to): thunderstorm outflows, low-level jets, dry lines, and cold fronts.<sup>6</sup> Since most of these phenomena are location-specific (for example, dry lines occur predominantly over the southern Great Plains of the United States), one would naïvely expect the statistical properties of the ramp events to also be site-dependent. In this paper, we confront this expectation with reality. By making use of long-term observational data from several field sites, we demonstrate that a key trait of wind ramp statistics behaves in a quasi-universal manner.

Before delving into the detailed quantitative aspects, we provide qualitative support for our claim via Figure 1. Here, we have plotted the probability density functions (pdfs) of wind speed increments ( $\delta u$ ) normalized by standard deviation ( $\sigma_{\delta u}$ ). For a specific time-increment ( $\tau$ ), the wind speed increments (or ramps) are defined as follows:

$$\delta u(t) = u(t + \tau) - u(t), \quad (1)$$

where  $u(t)$  is the wind speed at time  $t$ . Positive (negative) values of  $\delta u$  signify ramp-up (down) events. From a wind energy perspective,  $\tau$  values on the order of a few minutes to a few hours (the so-called mesoscale regime) are of utmost importance.<sup>5,7-9</sup> As illustrative examples, in Figure 1, we show the observed wind ramp pdfs corresponding to  $\tau = 10$  min (left panel) and  $\tau = 60$  min (right panel). A number of inferences can be drawn by visual inspection of this figure. First and foremost, all the pdfs, representing 4 diverse geographical and meteorological conditions (ranging from coastal environment to complex terrain), reveal remarkably similar shapes. They portray strong peakedness near the mode of the distribution, and more importantly, they all possess tails that are much heavier than a Gaussian pdf. These tails seem to depend on  $\tau$  in a subtle yet systematic manner. These unexpected findings inspired us to probe further into this problem by addressing a suite of science questions in this paper:



**FIGURE 1** Probability density functions of wind ramps ( $\delta u$ ) from 4 tall-tower sites (FINO1, Høvsøre, Cabauw, and NWTC). Multiyear, 10-min averaged wind data measured by the topmost sensors on these towers are used here. Further details are provided in Section 2. The wind increment values are normalized by the corresponding standard deviations ( $\sigma_{\delta u}$ ). The left and right panels represent time-increments ( $\tau$ ) of 10 and 60 min, respectively. A Gaussian pdf is overlaid (dashed line) as a reference. NWTC, National Wind Technology Center; pdf, probability density function [Colour figure can be viewed at [wileyonlinelibrary.com](http://wileyonlinelibrary.com)]

- Do the pdf tails corresponding to the ramp-up and ramp-down events behave differently?
- How do the tails depend on the height (above ground level)?
- What is the impact of aggregation (filtering) on the tails?
- Can the dependence of tail properties on the sample size be quantified?

To the best of our knowledge, none of these questions have been answered in the literature in a comprehensive manner. We do point out that a handful of studies<sup>10–16</sup> provided important building blocks for our research. Unfortunately, several of these papers focused on wind gusts, and thus, their findings cannot be very relevant for mesoscale wind ramps. More critically, most of these studies used very limited amount of observational data (often with durations of a few hours to merely a few days) and came up with conflicting results. For example, by analyzing only a few days worth of data, Liu et al<sup>13</sup> concluded that wind ramps follow *truncated stable distributions*. In a follow-up study, however, Liu and Hu<sup>12</sup> arrived at an opposite conclusion when they made use of a slightly larger dataset. We argue that, in lieu of converged statistics, the results from these past studies cannot be faithfully generalized.

The present study differs from the others in 2 areas. First, it primarily relies on rigorous statistical analyses for pdf characterization instead of qualitative visual inspection. Second, it uses multiyear wind datasets from 4 tall-tower sites: FINO 1 (North Sea), Høvsøre (Denmark), Cabauw (the Netherlands), and National Wind Technology Center (NWTC; USA). Since these sites are quite diverse in nature, we have more confidence in generalizing the outcomes. In the following section, we briefly describe these datasets.

## 2 | DESCRIPTION OF WIND DATASETS

Over the past decades, the wind energy and boundary layer meteorology communities have invested significant resources in installing and operating a few tall-towers around the world. A multitude of research-grade sensors (eg, high-fidelity cup anemometers) are mounted on these towers at various heights. Owing to their periodic calibration and regular maintenance, the meteorological datasets (including wind speed time series) collected by these sensors are deemed to be of the highest quality. Thus, it is not surprising that these datasets have been heavily used to advance our understanding of the lower part of the atmospheric boundary layer. For example, very recently, Kiliyanpilakkil et al<sup>17,18</sup> conducted rigorous scaling analyses of wind datasets from 3 of these prominent tall-towers: FINO 1, Cabauw, and NWTC. We also leverage on the same datasets, supplemented by measurements from Høvsøre, to demonstrate the statistical characterization of wind ramps. Table 1, along with the following subsections, provides more details into these locations.

### 2.1 | FINO 1

It is an offshore platform in the North Sea.<sup>19–21</sup> It consists of a 100-m tall meteorological tower equipped with wind speed measurement sensors (cup anemometers) at heights of 33, 40, 50, 60, 70, 80, 90, and 100 m. A total of 91 months of wind speed data collected over a period of 9 years (2004–2012) are used in the present study. Each time series (output rate: 10 min) contains approximately 478 000 samples.

**TABLE 1** Description of measurement sites

| Site    | Elevation (m; Mean sea level) | Location            | No. of Months |
|---------|-------------------------------|---------------------|---------------|
| FINO 1  | 0                             | 54.01° N, 6.59° E   | 91            |
| Høvsøre | 0                             | 58.44° N, 8.15° E   | 132           |
| Cabauw  | −0.7                          | 51.97° N, 4.93° E   | 170           |
| NWTC    | 1855                          | 39.91° N, 105.23° W | 132           |

Abbreviation: NWTC, National Wind Technology Center.

## 2.2 | Høvsøre

This meteorological tower is situated in a rural area close to the west coast of Jutland, Denmark and played a pivotal role in numerous wind energy studies.<sup>22,23</sup> We analyze 10-min average wind data from 6 levels: 10, 40, 60, 80, 100, and 116 m collected during the years 2005-2015. In this case, each time series consists of approximately 567 000 samples.

## 2.3 | Cabauw

The Cabauw Experimental Site for Atmospheric Research tower is located in the western part of the Netherlands.<sup>24-26</sup> We use 170 months of 10-min average wind speed data from the years 2001-2015 (approximately 736 000 samples) measured by propeller wind vanes at heights of 10, 20, 40, 80, 140, and 200 m.

## 2.4 | NWTC

We analyze multiyear (2004-2014) wind data from a 80-m tall tower (called M2) located at the foothills of the Colorado Rocky near Boulder, Colorado and maintained by the National Renewable Energy Laboratory NWTC. This location represents complex terrain and is prone to various wind flows and disturbances.<sup>27</sup> The NWTC dataset includes 1-min averaged, cup anemometer-based, wind speed time series from 4 heights: 10, 20, 50, and 80 m. Each time series is made up of approximately 5.78 million points with virtually no data gaps.

## 3 | METHODOLOGY

To investigate the tail features of the wind ramp events, we have borrowed a well-established methodology, called the Hill plot,<sup>37</sup> from the extreme value (EV) theory.<sup>28-30</sup> In this section, we explain this approach in detail by using synthetically generated random variates from 2 heavy-tailed distributions.

By definition, a heavy-tailed distribution ( $F$ ) satisfies<sup>31</sup>:

$$\bar{F}(x) = 1 - F(x) \sim \frac{\lambda}{x^\gamma}; \quad x \rightarrow \infty, \gamma > 0, \tag{2}$$

where  $\bar{F}$  is the so-called complementary cumulative distribution function (ccdf),  $\lambda$  is a positive constant, and  $\gamma$  is known as the tail-index (or shape parameter). In principle,  $\gamma$  can be estimated from the slope,  $-\frac{d \log \bar{F}(x)}{d \log x}$ .<sup>31</sup> However, in practice, the most common approach is to invoke the concept of order statistics.<sup>32</sup>

The rank-ordered values (in decreasing order) of  $x$  can be written as follows:  $\Phi_k = x_k$ , where  $k = 1, \dots, N$ . If the variates  $x$  follow Equation 2, it is expected to exhibit the following power-law behavior (aka Zipf law<sup>33</sup>):

$$\Phi_k \propto \left(\frac{k}{N}\right)^{-\frac{1}{\gamma}}. \tag{3}$$

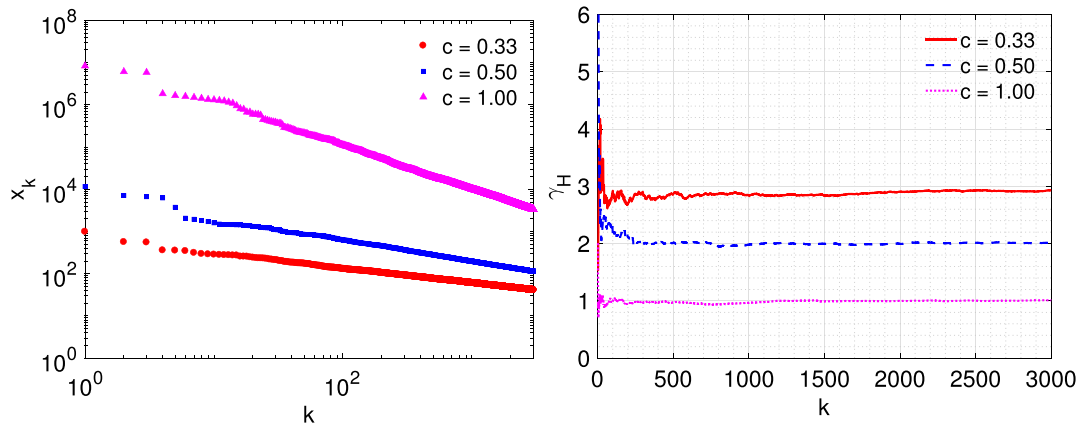
Over the years, several estimators for  $\gamma$  have been proposed in the literature, including (but not limited to) Pickand's estimator,<sup>34</sup> Hill estimator,<sup>35</sup> and the Dekkers-Einmahl-de Haan estimator.<sup>36</sup> In this work, we use the popular Hill estimator ( $\gamma_H$ ):

$$\gamma_H = \left[ \frac{1}{k} \sum_{i=1}^k \log \left( \frac{\Phi_i}{\Phi_{k+1}} \right) \right]^{-1}, \tag{4}$$

where  $k = 1, \dots, N - 1$ . When  $\gamma_H$  is plotted against  $k$ , it is known as the Hill plot.<sup>37</sup> For EV distributions (eg, Pareto), estimated  $\gamma_H$  is supposed to stabilize with increasing values of  $k$ . In Figure 2, we show an illustrative example using generalized Pareto (GP) distributed variates.

The pdf of the GP distribution can be written as follows:

$$f(x) = \left(\frac{1}{a}\right) \left(1 + \frac{c(x-b)}{a}\right)^{-1-1/c}; \quad x > b, c > 0, \tag{5}$$



**FIGURE 2** Left panel: rank-order (aka Zipf) plots for generalized Pareto distributed variates with 3 different  $c$  values. The parameters  $a$  and  $b$  are assumed to be equal to 1 and 0, respectively. Right panel: the estimated  $\gamma_H$  values for these cases. It is clear that  $\gamma_H \approx 1/c$  for  $k > 1000$  [Colour figure can be viewed at [wileyonlinelibrary.com](http://wileyonlinelibrary.com)]

where  $a, b, c$  are the parameters of GP. By integrating this equation, one can derive the ccdf of GP as follows:

$$\bar{F}(x) = \left(1 + \frac{c(x-b)}{a}\right)^{-1/c}. \quad (6)$$

Thus, the ccdf of GP is expected to decline as a power-law with tail-index  $\gamma = 1/c$ .

In Figure 2 (left panel), the rank-order plots for the GP distribution are shown for 3 values of  $c$ . For each case, the sample size is  $10^7$  and the parameters  $a$  and  $b$  are assumed to be equal to 1 and 0, respectively. By construction, only positive random variates are generated in this case. The tail indices are determined via the Hill plot in the right panel of Figure 2. Clearly, the  $\gamma_H$  values rapidly stabilize towards  $\frac{1}{c}$  for all the 3 cases, as would be desired. This example attests to the prowess of the Hill plot in estimating the tail indices from a rather simple EV distribution. Next, we investigate the usefulness of the Hill plot using a far more complicated distribution with 2 distinct tail behaviors.

The generalized hyperbolic skew student's  $t$  (GHSST) distribution is often used in financial modeling and risk management.<sup>38,39</sup> It has the innate ability to fit pdfs with heavy tails and significant asymmetry. A brief overview of this distribution is provided in Appendix A. A realization of the GHSST variates is shown in the top panel of Figure 3. Large positive values, signifying a heavy right tail, can be readily observed in this plot.

The pdf of the generated GHSST variates is shown in the middle-left panel of Figure 3. For comparison, a Gaussian pdf is overlaid on this plot. Clearly, both the left and right tails of the GHSST variates are much heavier than the Gaussian pdf. They also show different decaying behaviors as evident in the middle-right panel. For large values of  $x$ , the right tail portrays a quasi-linear appearance in this log-log representation. In other words, the right tail is characterized by a power-law distribution, which is in-line with the asymptotic limit discussed in Appendix A. In contrast, the left tail strongly departs from linearity highlighting its mixed-exponential-power-law behavior. The rank-order plot, shown in the bottom-left panel of Figure 3, provides further supporting evidence.

At this point, we would like to point out that Equation 2 has limited applicability in real-world scenarios. For such cases, this equation should be generalized as follows:

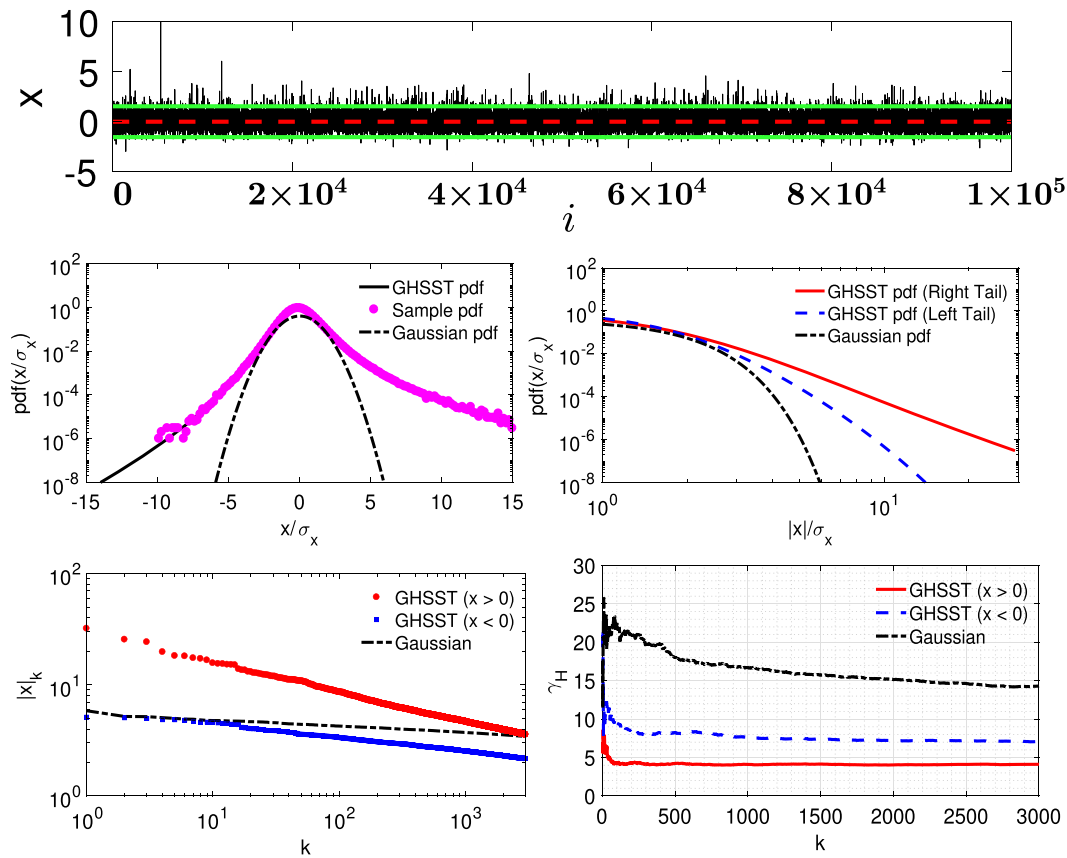
$$\bar{F}(x) = 1 - F(x) \sim \frac{L(x)}{x^\gamma}; \quad x \rightarrow \infty, \gamma > 0, \quad (7)$$

where,  $L$  is a slowly varying function (eg, exponential). For large  $x$ ,  $L(x)$  may be approximated as a constant  $\lambda$ .

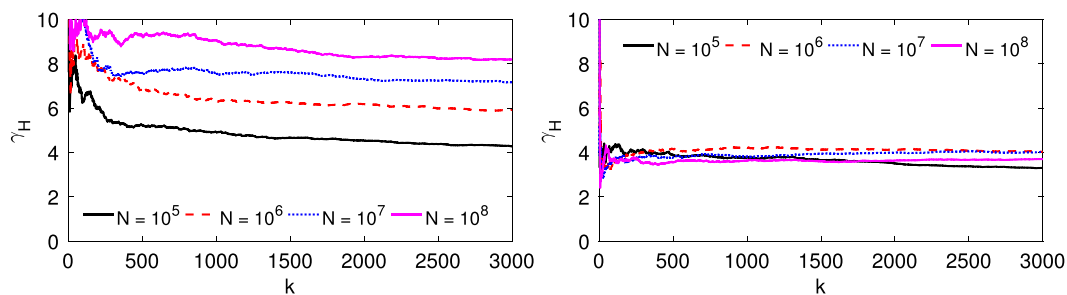
The tail indices from the GHSST variates are estimated via the Hill plot and shown in the bottom-right panel of Figure 3. These results should be interpreted carefully by taking into consideration Equation 7. In the case of right tail, the  $\gamma_H$  values stabilize rapidly as in the case of GP distributed variates. For the left tail, the  $\gamma_H$  values are significantly higher; also, the stabilization is slightly slower (difficult to detect in this figure without zooming in). In this case, the exponential term modulates the power-law tail. Please note that the estimated  $\gamma_H$  values are very high for the random Gaussian variates and they never stabilize as the tails simply follow exponential behavior. In Figure 4, we document the influence of sample size on  $\gamma_H$  estimation using the GHSST variates. In the case of the right tail (exhibiting power-law behavior),  $\gamma_H$  values are more or less insensitive to sample size ( $N$ ). In contrast, for the left tail,  $\gamma_H$  keeps increasing as  $N$  increases. This nonconvergence essentially corroborates the fact that the left tail of the GHSST pdf does not exhibit a purely power-law behavior; rather, it follows a mixed-exponential-power-law.\*

In summary, with the aid of randomly generated variates, we have demonstrated that the Hill plots can be very effective in characterizing different types of tail behaviors. It is also computationally very efficient. For these reasons, in the following section, we will invoke this methodology to address the science questions posed in Section 1. The idealized examples shown in Figures 2, 3, and 4 will provide guidance for interpreting the wind ramp characteristics observed within various observational datasets.

\* In a recent paper,<sup>40</sup> we reported a similar trend for the normal inverse Gaussian distribution, which also possesses mixed-exponential-power-law tails.



**FIGURE 3** A realization of the GHSST variates (sample size =  $10^7$ ) is generated using the following parameters:  $\nu = 6$ ,  $\beta = 0.5$ ,  $\mu = -0.125$ , and  $\delta = 1$ . A subset of these variates is shown in the top panel as an illustrative example. The mean of the variates is depicted by the dashed red line. The green lines denote 3 times the standard deviation around the mean. The analytical<sup>38</sup> (black line) and sample (magenta circles) pdfs are shown in the middle-left panel. For comparison, a Gaussian pdf (dot-dashed line) with zero mean and unit variance is overlaid on this panel. The tails of the GHSST and Gaussian pdfs are shown the middle-right panel. Since the right and the left tails of the GHSST pdf behave differently, they are shown separately. Clearly, the right tail exhibits a linear behavior in this log-log representation. Rank-order (aka Zipf) plots for the GHSST and Gaussian distributed variates are shown in the bottom-left panel. Estimated tail indices ( $\gamma_H$ ) using the Hill plot are documented in the bottom-right panel. GHSST, generalized hyperbolic skew student's t; pdf, probability density function [Colour figure can be viewed at [wileyonlinelibrary.com](http://wileyonlinelibrary.com)]

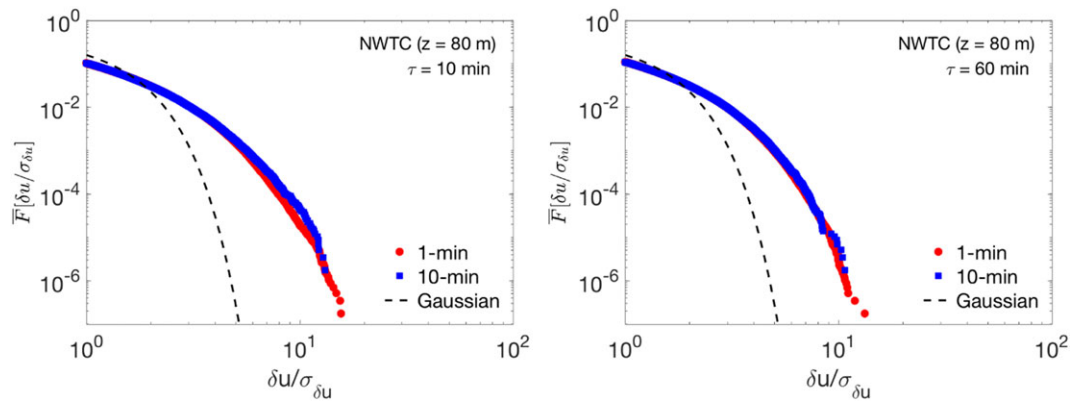


**FIGURE 4** Sensitivity of estimated  $\gamma_H$  values with respect to sample size. All the generalized hyperbolic skew student's t variates are generated using the same parameters as in Figure 3. The left and right panels correspond to the left and right tails, respectively [Colour figure can be viewed at [wileyonlinelibrary.com](http://wileyonlinelibrary.com)]

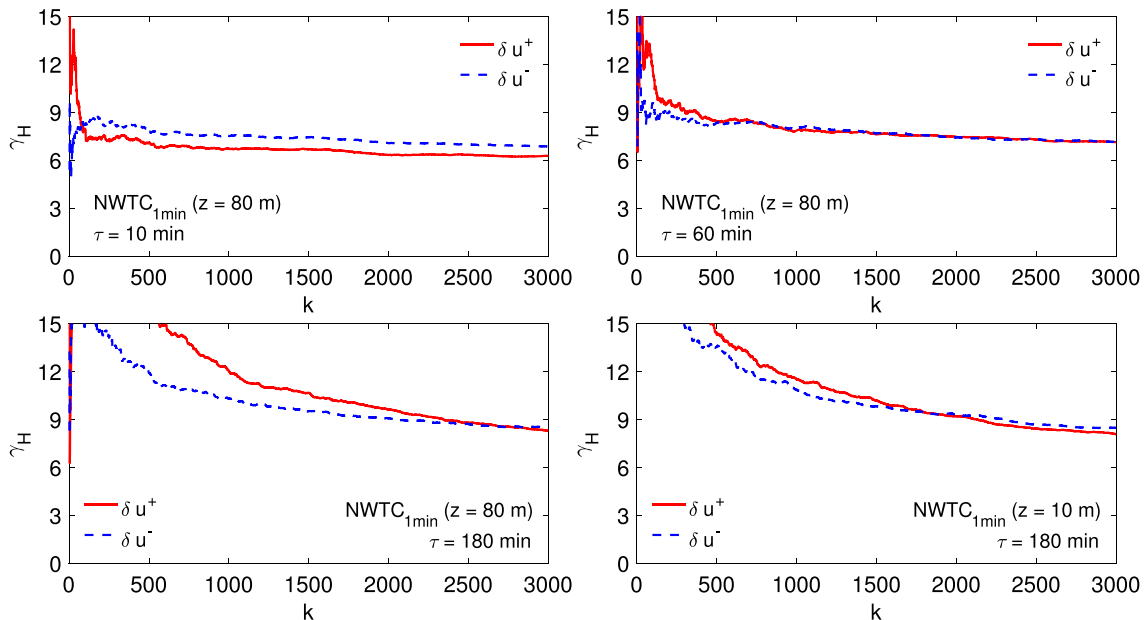
### 4 | RESULTS

Given that the NWTC dataset offers the largest sample size, we select it first for comprehensive analysis. The original granularity of the wind time series is 1 min. Henceforth, we refer to this series as  $NWTC_{1min}$ . To study the effects of aggregation on the tail characteristics, we created a 10-min average series (sample size: approximately 578 000) by simple moving averaging (followed by downsampling) of the  $NWTC_{1min}$  series. This new series will be identified as  $NWTC_{10min}$ . In addition to the results reported in the current section, these series are also used in Appendices B and C to investigate the issues of nonstationarity and correlation.

The ccdf ( $\bar{F}$ ) for both the  $NWTC_{1min}$  and  $NWTC_{10min}$  time series are shown in Figure 5. The left and right panels represent  $\tau = 10$  min and  $\tau = 60$  min, respectively. On these plots,  $\bar{F}$  for a Gaussian distribution is also shown for comparison. In this log-log representation, we only focus on the right tail



**FIGURE 5** Complementary cumulative distribution function ( $\bar{F}$ ) from the  $NWTC_{1min}$  and  $NWTC_{10min}$  datasets at  $z = 80$  m are shown. The left and right panels represent time increments ( $\tau$ ) of 10 min and 60 min, respectively. The wind increment values are normalized by the corresponding standard deviations ( $\sigma_{\delta u}$ ). A Gaussian pdf is overlaid (dashed line) as a reference. NWTC, National Wind Technology Center [Colour figure can be viewed at [wileyonlinelibrary.com](http://wileyonlinelibrary.com)]



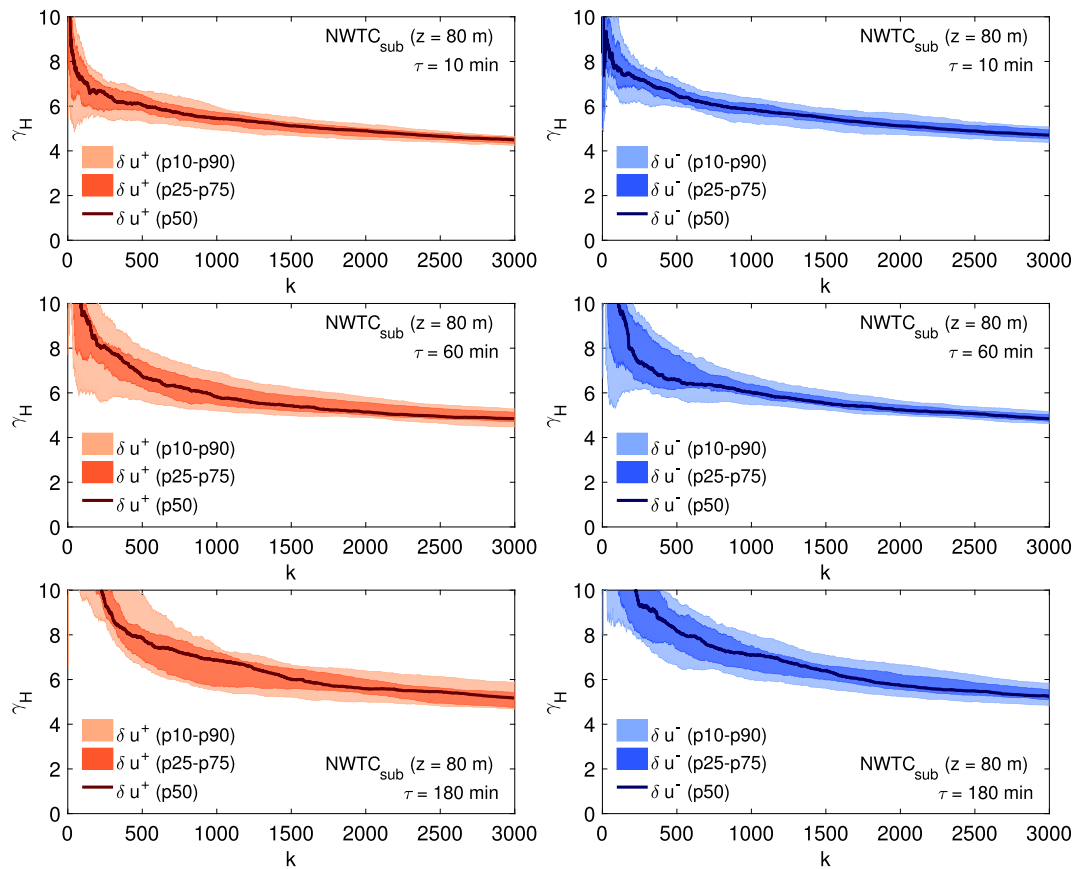
**FIGURE 6** The Hill plots for wind ramp distributions based on the  $NWTC_{1min}$  time series. The red (solid) and blue (dashed) lines represent ramp-up ( $\delta u^+$ ) and ramp-down ( $\delta u^-$ ) cases, respectively. The top-left, top-right, bottom-left, and bottom-right panels correspond to the following scenarios, respectively: (1)  $\tau = 10$  min,  $z = 80$  m; (2)  $\tau = 60$  min,  $z = 80$  m; (3)  $\tau = 180$  min,  $z = 80$  m; and (4)  $\tau = 180$  min,  $z = 10$  m. NWTC, National Wind Technology Center [Colour figure can be viewed at [wileyonlinelibrary.com](http://wileyonlinelibrary.com)]

(ramp-up) of the pdf. Several remarks can be made from this figure. First of all, both the  $NWTC_{1min}$  and  $NWTC_{10min}$  cases clearly portray non-Gaussian tails. The implication of this heavy-tail behavior is rather crucial for the wind energy community. For example, the exceedance probability of a strong ramp-up event of magnitude  $5\sigma_{\delta u}$  is very small (much less than  $10^{-6}$ ) if one assumes Gaussianity. According to observations, however, the exceedance probability is almost  $10^{-2}$ . In other words, the assumption of Gaussianity leads to severe underestimation of extreme wind ramp events.

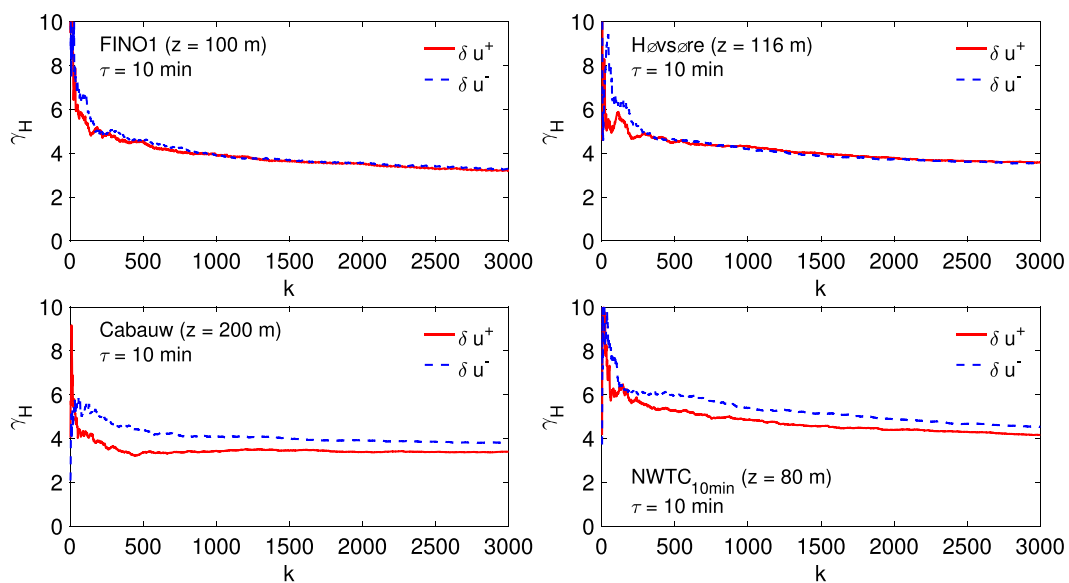
In Figure 1, we reported that the tails of the wind ramp pdfs systematically depend on  $\tau$ . Thus, it is not surprising that the same dependence is also evident from the cdfs. From Figure 5, one can discern that, in comparison with  $\tau = 10$  min, the right tail decays faster in the case of  $\tau = 60$  min. Later on, we will establish that this trend is actually monotonic in the range of  $\tau = 10$  to 360 min.

According to Figure 5, the agreement between  $NWTC_{1min}$ - and  $NWTC_{10min}$ -based  $\bar{F}$  curves are excellent up to  $\delta u \approx 5\sigma_{\delta u}$ . Beyond that point, the  $NWTC_{1min}$ -based  $\bar{F}$  curve starts to decline faster. With the aid of the Hill plots, we will further investigate if this discrepancy is because of the disparity in sample sizes, or it is an artifact of aggregation.

The Hill plots for  $NWTC_{1min}$  are shown in Figure 6. The values of  $\gamma_H$  are found to be noticeably higher for larger  $\tau$  values. In other words, the wind ramp pdf tails decay faster for larger  $\tau$  values, which is in-line with our earlier finding. Both the ramp-up (noted as  $\delta u^+$ ) and ramp-down ( $\delta u^-$ ) cases follow similar trends. However, the  $\gamma_H$  curves never fully stabilize for either case in the considered range ( $1 < k \leq 3000$ ). Thus, we can deduce that the wind ramp pdf tails do not obey power-laws. Later on, we will explore further if these NWTC data-based results also hold for other datasets.



**FIGURE 7** The Hill plots for wind ramp distributions based on the subsets of 1-min average NWTC wind speed time series. Each subset contains contiguous 578 000 samples. A total of one hundred randomly selected subsets are used for these plots. The top, middle, and bottom panels represent  $\tau = 10$  min, 60 min, and 180 min, respectively. Ramp-up and ramp-down results are shown in left and right panels, respectively. The solid lines, dark shaded areas, and the light shaded areas correspond to the medians, 25th-75th percentile ranges, and 10th-90th percentile ranges, respectively. NWTC, National Wind Technology Center [Colour figure can be viewed at wileyonlinelibrary.com]



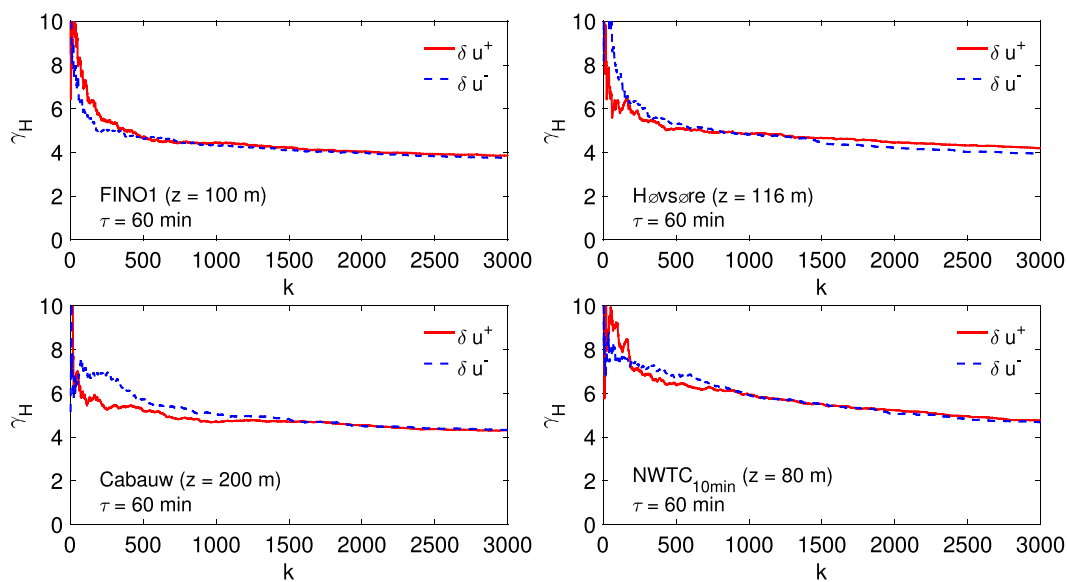
**FIGURE 8** The Hill plots of wind ramp distributions from 4 field sites: FINO1 (top-left panel; sensor height: 100 m), Høvsøre (top-right panel; sensor height: 116 m), Cabauw (bottom-left panel; sensor height: 200 m), NWTC (bottom-right panel; sensor height: 80 m). For all the cases, 10-min averaged wind speed are used. The time increment ( $\tau$ ) is 10 min. The ramp-up ( $\delta u^+$ ) and ramp-down ( $\delta u^-$ ) statistics are denoted by red (solid) and blue (dashed) lines, respectively. NWTC, National Wind Technology Center [Colour figure can be viewed at wileyonlinelibrary.com]



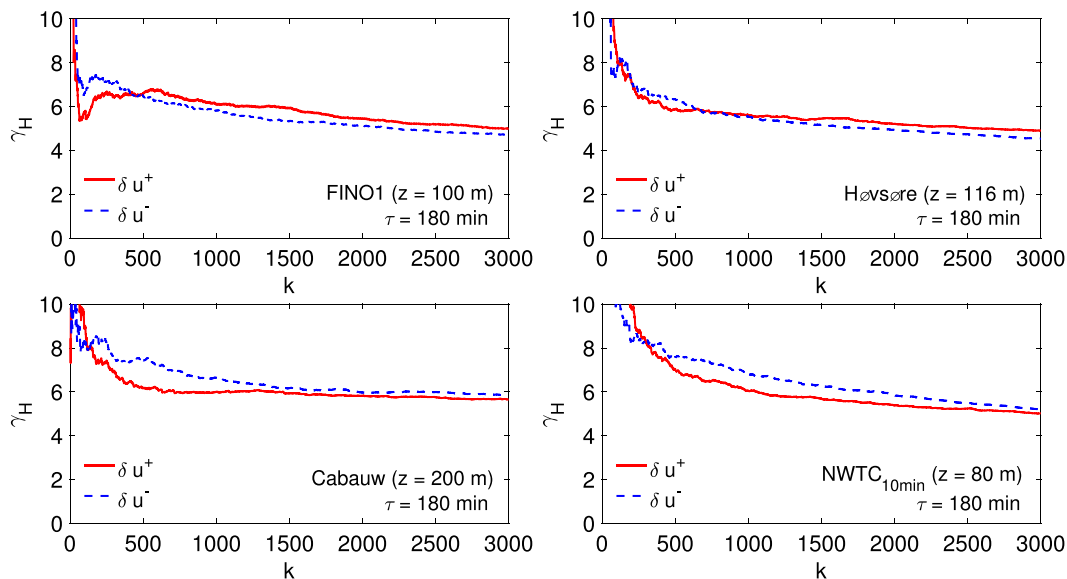
To quantify the effect of sample size on the  $\gamma_H$  values, we adopted a Monte-Carlo-type strategy. From the  $NWTC_{1min}$  time series (sample size 5.78 million), we extract 100 contiguous subsets from random locations. Each subset is called  $NWTC_{sub}^i$  and contains 578 000 samples. The index  $i$  varies from 1 to 100 to demarcate each subset. We then perform Hill plot analysis on each subset separately and compute ensemble statistics. These ensemble Hill plots are shown in Figure 7. The ramp-up and ramp-down cases are shown separately in the left and right panels, respectively.

The overall trends of the  $\gamma_H$  values reported in Figures 6 and 7 are qualitatively very similar. However, the magnitudes of  $\gamma_H$  for the  $NWTC_{sub}^i$  cases are significantly lower than the  $NWTC_{1min}$  series. We would like to remind the readers that a similar sample size dependency was reported earlier in the case of the left tail (depicting a mixed-exponential-power-law behavior) of the GHSST pdf (refer to bottom-left panel of Figure 4). Thus, on the basis of the Hill plot analyses and ccdf plots, we can confidently claim that the wind ramp distributions do not exhibit power-law tails.

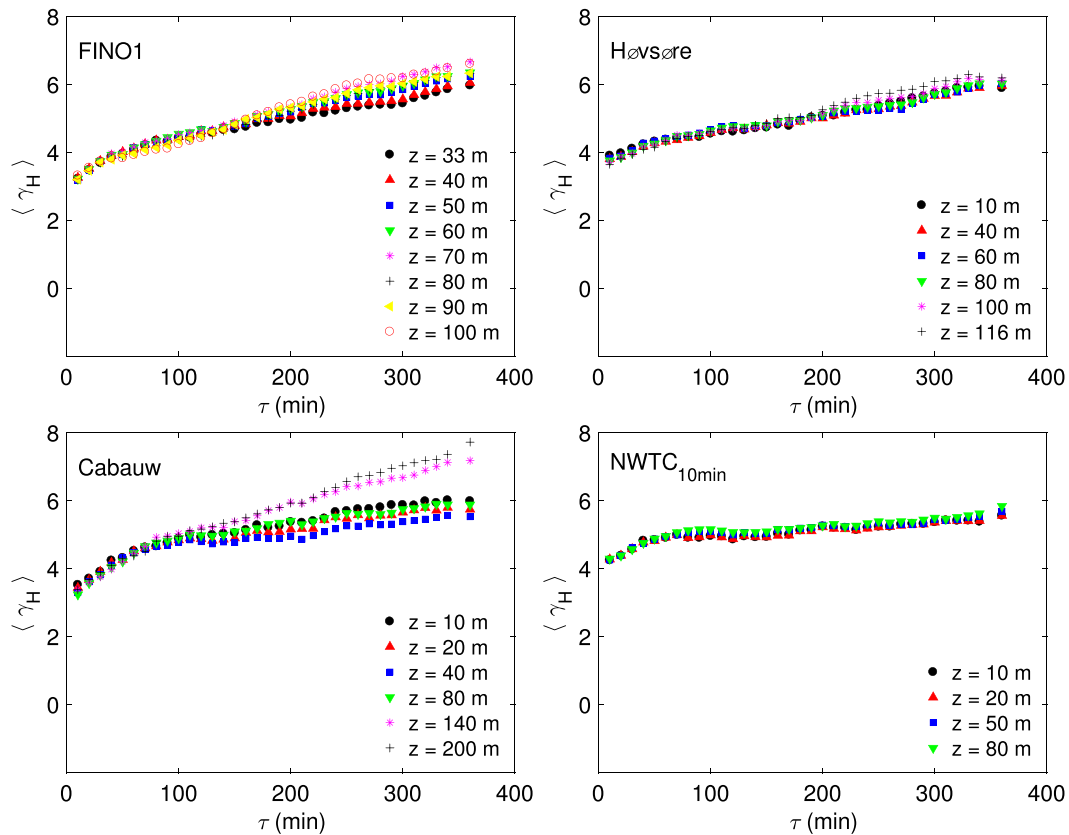
To further bolster our claim, we have computed  $\gamma_H$  for 3 other locations: FINO 1, Høvsøre, and Cabauw. Wind data from the topmost sensor levels are used. Figures 8, 9, and 10 show the Hill plots for 3 different values of  $\tau$ , 10, 60, and 180 min, respectively. In each of these figures, we also included the Hill plot for the  $NWTC_{10min}$  series. Based on these figures, several assertions can be made. First of all, the Hill plots from all the locations look remarkably similar. For (almost) all the cases, the differences between the ramp-up and ramp-down events are marginal. At the same time, for



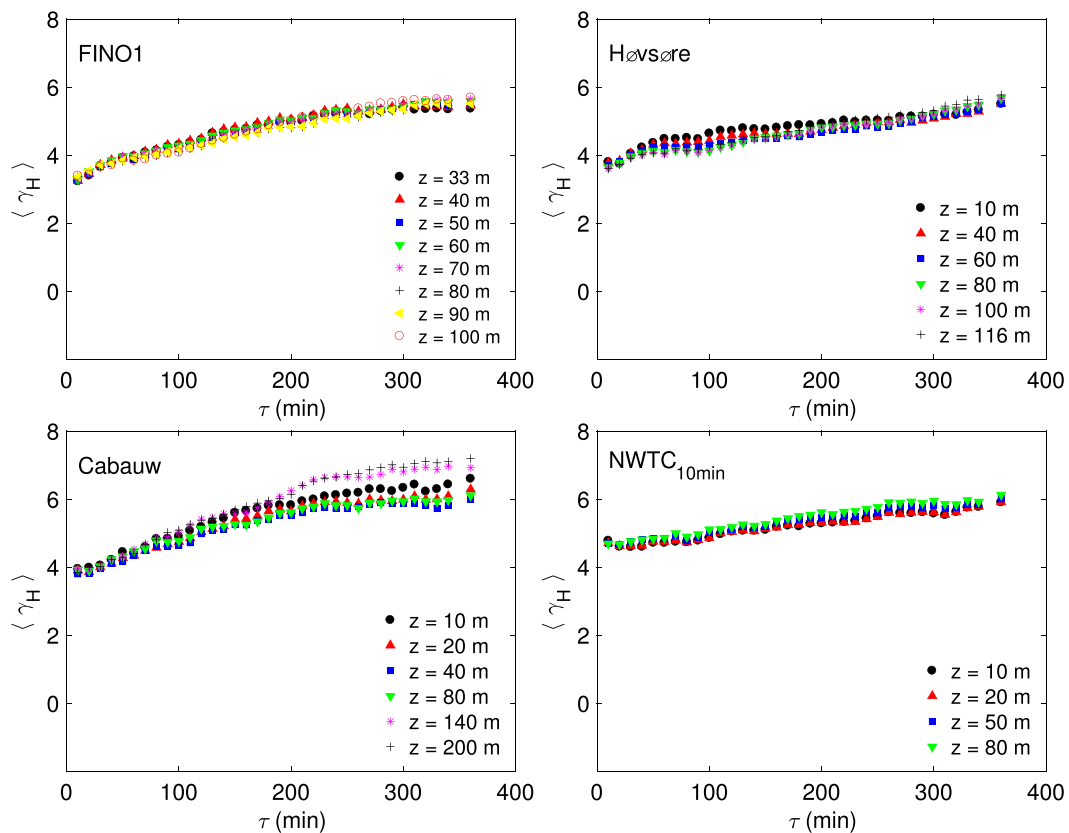
**FIGURE 9** Same as Figure 8, except for time increment ( $\tau$ ) of 60 min. NWTC, National Wind Technology Center [Colour figure can be viewed at [wileyonlinelibrary.com](http://wileyonlinelibrary.com)]



**FIGURE 10** Same as Figure 8, except for time increment ( $\tau$ ) of 180 min. NWTC, National Wind Technology Center [Colour figure can be viewed at [wileyonlinelibrary.com](http://wileyonlinelibrary.com)]



**FIGURE 11** The averaged values of tail indices ( $\langle \gamma_H \rangle$ ) as a function time-increment ( $\tau$ ). Wind ramp-up data from all the available sensors from all the 4 sites are used here. NWTC, National Wind Technology Center [Colour figure can be viewed at wileyonlinelibrary.com]



**FIGURE 12** Same as Figure 11, except for ramp-down events. NWTC, National Wind Technology Center [Colour figure can be viewed at wileyonlinelibrary.com]

all plots, the values of  $\gamma_H$  do not stabilize and continue to decrease with increasing  $k$ . Thus, we can safely rule out power-law being a viable candidate for wind ramp distributions. In-line with our earlier finding, the  $\gamma_H$  values exhibit dependence on  $\tau$ . Thus, stable distributions<sup>32</sup> should not be used to characterize wind ramp distributions.

Earlier, we have concluded that the  $\gamma_H$  values strongly depend on sample size. Now, in order to probe the impact of aggregation, we compare the  $\gamma_H$  values for the  $NWTC_{10\text{min}}$  series (bottom-right panels of Figures 8, 9, and 10) against the corresponding values from the  $NWTC_{\text{sub}}^i$  series (reported in Figure 7). Please note that these series have identical sample size; albeit, they have different granularity. From the plots, it is quite evident that the  $\gamma_H$  values from the  $NWTC_{10\text{min}}$  and  $NWTC_{\text{sub}}^i$  series are very much comparable. Thus, within the limited filtering range of 1 to 10 min, the aggregation effect is negligible. However, one should not downplay the effects of sample size.

Thus far, we have only focused on wind data from the topmost sensors of all the 4 meteorological towers. It would be interesting to find out if/how the  $\gamma_H$  values depend on sensor height. Instead of plotting several individual Hill plots, we opt for plotting averaged  $\gamma_H$  values so as to report all the results succinctly in Figures 11 and 12. The values of  $\langle \gamma_H \rangle$  are computed for  $2000 \leq k \leq 3000$ . We intentionally (and incorrectly) assume that over this range the values of  $\gamma_H$  have fully stabilized. Despite this ad hoc assumption, the results are quite revealing. For both the ramp-up and ramp-down cases,  $\langle \gamma_H \rangle$  increase monotonically from approximately 4 (at  $\tau = 10$  min) to approximately 6–7 (at  $\tau = 360$  min). The diversity in  $\langle \gamma_H \rangle$  values across various heights is rather small (especially for  $\tau < 180$  min). Among all the locations, the spread of  $\langle \gamma_H \rangle$  is the most significant at Cabauw; the  $\langle \gamma_H \rangle$  curves from the top two sensors, located at heights of 140 and 200 m, seem to branch out from others for  $\tau > 180$  min. We speculate that certain meteorological processes (eg, low-level jets) influence the wind ramp statistics at higher altitudes. However, we need more observational datasets from higher altitudes (possibly collected by lidars and/or sodars) to shed further light on this intriguing finding.

## 5 | CONCLUSIONS

In this study, we analyzed several long-term wind speed datasets composed of 4 different geographical locations, from offshore to complex terrain. We showed that the wind ramp pdfs from all the sites reveal amazingly similar shape characteristics. Most interestingly, the tails of the wind ramp pdfs are much heavier than Gaussian and decay faster as time increments increase. With the aid of the Hill plots, we showed that the extreme ramp-up and ramp-down events behave similarly from a statistical point of view. Moreover, the tail-index statistics exhibited minimal dependence with respect to height above the ground.

Another important aspect of these results showed that the tails of the wind ramp distributions do not follow a power-law distribution, rather modulated by a slowly varying function. We speculate this function to be an exponential. Therefore, in future work, we will use several types of pdfs from the generalized hyperbolic distribution family (eg, GHSST, normal inverse Gaussian) to determine the ideal candidate for capturing the tail characteristics of wind ramp distributions.

As the wind energy industry continues to flourish, and wind ramp prediction becomes increasingly important, the results from this study should be used for model validations and improvement. It would be critical to find out if the state-of-the-art numerical weather prediction models and time-series forecasting tools are able to capture the extreme ramp behaviors accurately. It is also envisaged that the contemporary synthetic wind speed generators (eg, D'Amico et al<sup>41</sup> and Negra et al<sup>42</sup>), heavily relying on statistical information, will tremendously benefit from our findings.

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## APPENDIX A: GENERALIZED HYPERBOLIC SKEW STUDENT'S T (GHSST) DISTRIBUTION

The GHSST distribution is a subclass of the generalized hyperbolic (GH) family, and its probability density function (pdf) is defined as follows<sup>38,43,44</sup>:

$$f(x; \nu, \beta, \mu, \delta) = \frac{2^{\frac{1-\nu}{2}} \delta^\nu}{\sqrt{\pi} \Gamma(\nu/2)} \left( \frac{y_x}{|\beta|} \right)^{-\frac{\nu+1}{2}} K_{\frac{\nu+1}{2}}(|\beta| y_x) e^{\beta(x-\mu)}, \quad \beta \neq 0, \quad (\text{A1})$$

where  $\nu$ ,  $\beta$ ,  $\mu$ , and  $\delta$  are the 4 parameters of the GHSST distribution.  $y_x = \sqrt{\delta^2 + (x - \mu)^2}$ , and  $\Gamma$  is the gamma function. The parameters  $\nu$  and  $\beta$  together control the degree of heavy-tailedness and skewness of the tails.  $\mu$  is the location parameter and is slightly different from the mean of the distribution.  $\delta$  is a scale or peakedness parameter and it controls the shape of the pdf near its mode.  $K_\nu(x)$  is the so-called modified Bessel function.<sup>45</sup>

The tails of the distribution exhibit the following traits:

$$f_x(x) \sim C|x|^{-\nu/2-1} \exp(-|\beta x| + \beta x) \quad \text{as } x \rightarrow \pm\infty. \quad (\text{A2a})$$

Thus, the heavier tail decays as follows:

$$f_x(x) \sim C|x|^{-\nu/2-1} \quad \text{when } \begin{cases} \beta < 0 & \text{and } x \rightarrow -\infty, \\ \beta > 0 & \text{and } x \rightarrow +\infty, \end{cases} \quad (\text{A2b})$$

and the lighter tail behaves as follows:

$$f_x(x) \sim C|x|^{-\nu/2-1} \exp(-2|\beta x|) \quad \text{when } \begin{cases} \beta < 0 & \text{and } x \rightarrow +\infty, \\ \beta > 0 & \text{and } x \rightarrow -\infty. \end{cases} \quad (\text{A2c})$$

In this work, we generated GHSST distributed random variates following an algorithm described by Aas and Hobæk Haff<sup>43</sup>:

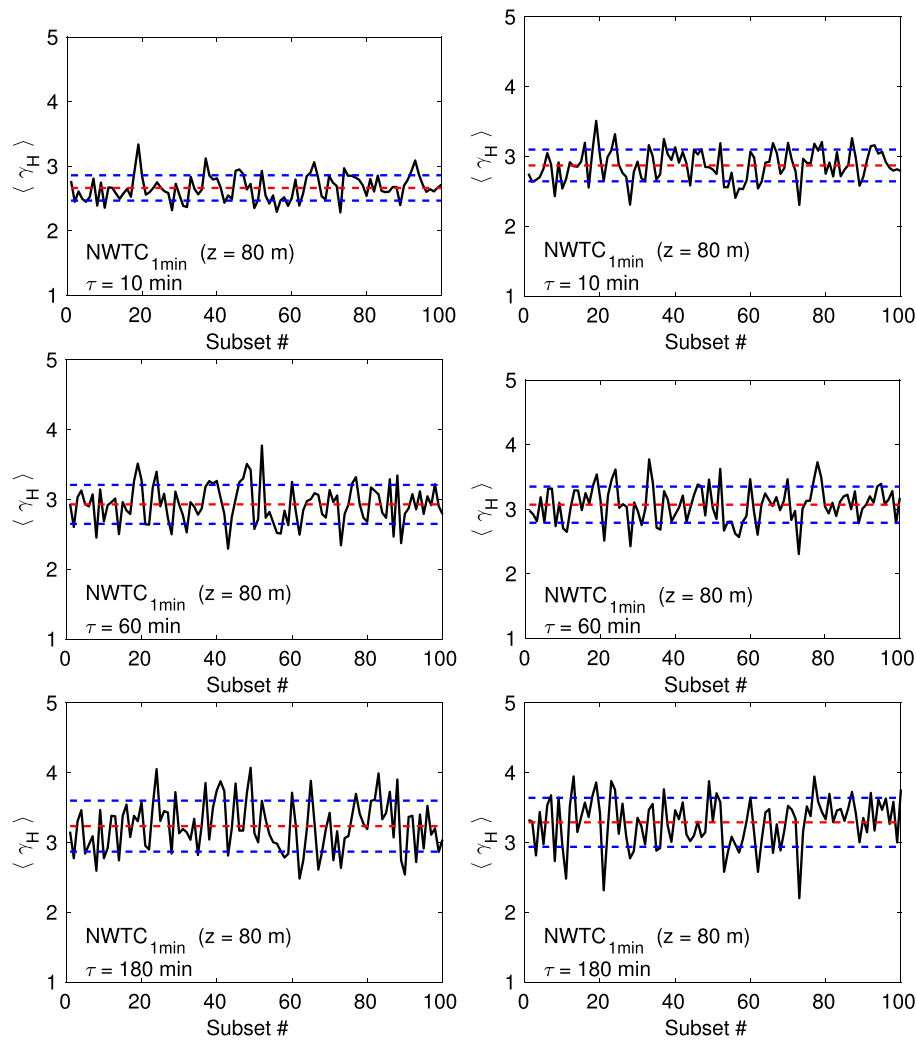
- Generate Gamma distributed variates (G) with parameters  $\nu/2$  and  $\delta^2/2$ .
- Obtain the Inverse Gamma variates by simply using  $Z = G^{-1}$ .
- Generate Gaussian random variates Y with zero mean and unit variance.
- Compute the GHSST random variates,  $X = \mu + \beta Z + \sqrt{Z}Y$ .

## APPENDIX B: EFFECTS OF NONSTATIONARITY

For observational time series, quantifying the effects of nonstationarity on the computed statistics is a challenging task. In meteorology literature, quite often time-varying mean and variance of a series are used to establish the (non)existence of nonstationarity (for example, see Mahrt<sup>46</sup> and the references therein). A far more stringent test of nonstationarity is usually employed in the dynamical systems literature. Quite often a specific higher-order statistic (eg, correlation dimension, Lyapunov exponent, and Shannon entropy) is computed for different segments of a series.<sup>47</sup> In the case of systematic trends and/or significant variations of this specific statistic, the series is flagged as nonstationary.

In this appendix, we use the  $\text{NWTC}_{1\text{min}}$  time series to probe into the issue of nonstationarity. Given the large sample size, we are able to divide it into 100 nonoverlapping and contiguous subsets. Each subset has a sample size of approximately 57 800. We perform Hill plot analysis on each subset in sequence. For each subset,  $\langle \gamma_H \rangle$  is computed for  $2000 \leq k \leq 3000$ . The resultant  $\langle \gamma_H \rangle$  series for ramp-up and ramp-down cases with various time-increment ( $\tau$ ) values are reported in Figure B1.

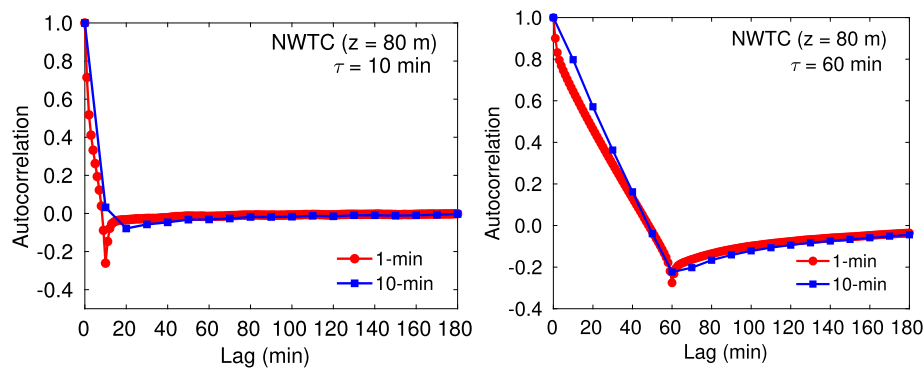
A few observations can be readily made based on Figure B1. First of all, the estimated  $\langle \gamma_H \rangle$  values are much smaller than those reported in Figures 6 to 7. This behavior is completely in-line with our earlier findings related to the sample size dependency of  $\gamma_H$  (thus,  $\langle \gamma_H \rangle$ ) values. Second, the average values of  $\langle \gamma_H \rangle$  marginally increases with  $\tau$ . This trend was also reported earlier in Section 4. Furthermore, similar to Figure 7, we see that the variability of  $\langle \gamma_H \rangle$  increases slightly with larger  $\tau$  values. Most importantly, we do not see any trend or significant variations of  $\langle \gamma_H \rangle$  with time (proxied by subset #). In other words, there is no tell-tale sign of nonstationarity that we can detect in the National Wind Technology Center (NWTC) dataset.



**FIGURE B1** The averaged values of tail indices ( $\langle \gamma_H \rangle$ ) based on the nonoverlapping and contiguous subsets of  $NWTC_{1min}$  time series. The left and right panels correspond to ramp-up and ramp-down events, respectively. Time increments ( $\tau$ ) of 10, 60, and 180 min are used in top, middle, and bottom panels, respectively. For visual aid, the temporal mean values of  $\langle \gamma_H \rangle$  and one standard deviation around the mean are depicted with the red and blue dashed lines, respectively. NWTC, National Wind Technology Center [Colour figure can be viewed at [wileyonlinelibrary.com](http://wileyonlinelibrary.com)]

**APPENDIX C: EFFECTS OF CORRELATION**

Rigorously speaking, the tail-index analysis should be performed on independent and identically distributed (iid) random variates. However, observational data often possess intrinsic correlation. In this appendix, we report the autocorrelation functions of wind ramp time series. We make use



**FIGURE C1** Autocorrelation functions based on  $NWTC_{1min}$  (red lines with circles) and  $NWTC_{10min}$  (blue lines with squares) time series. The left and right panels correspond to time increments ( $\tau$ ) of 10 and 60 min, respectively. NWTC, National Wind Technology Center [Colour figure can be viewed at [wileyonlinelibrary.com](http://wileyonlinelibrary.com)]

of both the  $NWTC_{1min}$  and  $NWTC_{10min}$  time series from  $z = 80$ -m level. In Figure C1, the results are shown for  $\tau = 10$  min (left panel) and  $\tau = 60$  min (right panel). Clearly, the autocorrelation function decreases rapidly for all the cases. In the case of  $\tau = 10$  min, the minimum autocorrelation occurs around lag = 10 min; in contrast, the minimum occurs at lag = 60 min for  $\tau = 60$  min. As a matter of fact, for all the other cases that we analyzed (not shown), the autocorrelation function always passed through a local minimum close to their respective  $\tau$  values. Several years ago,<sup>48</sup> reported identical results in the context of fully developed turbulence.

Last, we would like to point out that most of our analyses reported in Section 4 were based on 10-min averaged data from several sites. Like the  $NWTC$  time series, the autocorrelation function in other cases were also close to zero for  $\tau = 10$  min (not shown); for these cases, the samples could be considered independent. For higher values of  $\tau$ , the samples were correlated.