Optimisation methods for the multi-period petrol station replenishment problem

A case study at AMCS

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TU Delft

AMCS

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Preface

With this thesis project, there comes an end to my time at Delft University of Technology. The thesis has been written to fulfil the graduation requirements of the MSc. program Transport, Infrastructure and Technology. The project concerns optimisation methods for the petrol station replenishment problem. I enjoyed working on the topic, because of my passion for digital technology and optimisation. Some parts of the project, like developing the heuristic with improvement procedure, have been challenging. I am, however, very happy with the end result.

I would like to thank AMCS for making this thesis project possible. I really enjoyed my time with the friendly colleagues at the office in Rotterdam. Most especially, I would like to thank Jelmer Brandt for guiding me during the project and for making time to attend the meetings in Delft. I would also like to thank Kristian Hauge, who came up with the subject for the assignment and who was always available for questions.

The result could not have been achieved without the assessment committee. First of all, I would like to thank Bilge Atasoy, for the help with the mathematical models and always making time for me. Gonçalo Correia, for being critical about the conceptual model and the report. And lastly, Rudy Negenborn, for the critical attitude towards the methodology, structure and presentation of the research.

Studying at the university would of course not have been possible without the support of my mother Lia and the rest of my family, who always respected the choices I made. And of course, not to forget, thanks to my friends and roommates, with whom I have had a great time as student.

Luke Boers
December 20, 2019

Summary

Worldwide, almost 90% of vehicles on the road are powered by internal combustion engines that run on petroleum products. Although vehicles with alternative power sources and more efficient vehicles are becoming available, the oil demand for transportation is not expected to decrease in the next decades, due to the drastic growth of the car fleet worldwide. At the same time, leaders around the world agreed to reduce green gas house emissions. The European Commission has committed itself to a 20% reduction of transport emissions from 2008 levels by 2030.

The aim of this research is to contribute to the sustainability goals by developing efficient planning methods for the distribution of petroleum. In the literature, this problem is well known as the Petrol Station Replenishment Problem (PSRP). The PSRP concerns the distribution planning of petroleum products from a central depot to a set of gas stations by compartmentalised vehicles. The literature gap addressed in this research is twofold. Firstly, new Mixed Integer Linear Programming (MILP) models are developed for a new rich version of the PSRP, to better represent the real-life situation. The main characteristics of the variant of the PSRP are considering inventory management, multiple time periods, allowing multiple trips per day, allowing split loads, considering time constraints and station restrictions. Secondly, a new heuristic is proposed and the *simultaneous dry run* inventory policy is introduced. The planning is optimised over multiple days, since stations do not have to be replenished every day. Therefore, the main research question of this research is formulated as "*What is an efficient planning method for the multi-period petrol station replenishment problem, in terms of computation time and solution quality?*".

The MILP models and the heuristic are presented as planning methods. MILP model variants are presented as well, which are based on allowing the inventory level to drop below the safety stock level by introducing soft constraints, including average levels in the objective and considering a service time that depends on the delivery quantity. The solutions of the planning methods are evaluated with Key Performance Indicators (KPIs), which are the total travel distance, average stock level, number of dry runs, run time, vehicle utilisation and average number of stops per trip. In this research, dry run is the moment when the inventory level of a tank drops below the safety stock level.

To evaluate the performance of the planning methods, a case study based on a real-life petrol distributor is used. The distributor uses software developed by Advanced Manufacturing Control Systems (AMCS) to create the planning for their operations. The case study concerns a distribution network with 59 gas stations, one depot and four vehicles.

The MILP models can be used to find solutions for up to 20 stations and 7 days. The models can, however, not be used to find a solution for the real-life size problem. Therefore, a decomposition heuristic is presented, based on decomposing the decision process of the supplier. Decomposition heuristics have proven to be effective methods to solve the rich Vehicle Routing Problemss (VRPs). Since the decisions of the supplier are not optimised simultaneously, a local improvement procedure based on visiting stations earlier is proposed. To determine delivery quantities, the *simultaneous dry run* inventory policy is applied. This new concept, which has been observed in practice, ensures that a station is visited as few times as possible.

The exact solutions found with the MILP models are used to evaluate the heuristic solution quality. In terms of computation time, the heuristic outperforms the MILP models. For experiments with seven days, the average increase in travel distance for heuristic solutions is 15.6%. At the same time, vehicle utilisation increases and the number of stops per trip decreases, indicating more efficient transport. A heuristic solution could be found within two hours for the full case study data set, consisting of 59 stations, 4 vehicles and one depot. Therefore, the proposed heuristic is an effective planning method for the multi-period petrol station replenishment problem. Presenting the heuristic is the answer to the main research question.

Moreover, MILP model variants are introduced to gain insight into trade-offs between KPIs. If inventory levels and travel distance are optimised simultaneously, it can be advised to check if the solution is Pareto efficient. In general, lower inventory levels require higher travel distance. If dry runs are allowed and penalised in the objective, what means that costs are added when the inventory level drops below the safety stock level, this could result in a decrease of 15.6% in total travel distance.

Future research in this area is encouraged. The value of the *simultaneous dry run* inventory policy could be further explored by considering the exact inventory level in the tank at the time of delivery, and adjusting maximum delivery quantities according to the remaining capacity. This allows to also replenish the quantity of fuel that has already been sold during the day till the moment of delivery. Moreover, the proposed local improvement procedure of the heuristic is based on recreating the planning when evaluating candidate solutions. If this process is sped up, more candidate solutions could be evaluated.

List of abbreviations

AMCS Advanced Manufacturing Control Systems. v, 2, 3, 17, 47, 49, 50, 52, 55–57, 69, 70

CVRP Capacitated Vehicle Routing Problem. 14

ILP Integer Linear Programming. 7

IRP Inventory Routing Problem. 9, 11, 12

KPI Key Performance Indicator. v, vi, 3, 4, 15, 17–19, 25, 48, 63, 64, 70, 71, 89

MCVRP Multi-Compartment Vehicle Routing Problem. 9

MILP Mixed Integer Linear Programming. v, vi, 7–15, 20–22, 30, 33, 44, 46, 48, 56, 58, 59, 62, 64–68, 70, 71

MP-PSRP Multi-Period Petrol Station Replenishment Problem. 8

PPSRP Periodic Petrol Station Replenishment Problem. 8, 11, 12

PSC Petroleum Supply Chain. 3, 11

PSRP Petrol Station Replenishment Problem. v, 2–4, 7–15, 19, 20, 48, 71

RVRP Rich Vehicle Routing Problem. 2, 8, 13

TTLP Tank Truck Loading Problem. 12, 36, 41

VMI Vendor-Managed Inventory. 9, 10, 15

VNS Variable Neighbourhood Search. 8

VRP Vehicle Routing Problems. v, 2, 7, 8, 10, 11, 13, 21, 36, 71

VRPTW Vehicle Routing Problem with Time Windows. 9, 10

Contents

1

Introduction

Worldwide, almost 90% of vehicles on the road are powered by internal combustion engines that run on oil, which makes road transport currently responsible for 44% of the total oil consumption. Oil demand in road transport has grown steadily over the last decades and the demand is not expected to decline soon. Although the global car fleet will grow drastically in the next decades, the growth in oil demand for cars and trucks will only slightly increase (see Figure 1.1), because of the availability of more efficient vehicles, a shift to alternative power sources and more efficient operations (International Energy Agency, 2018).

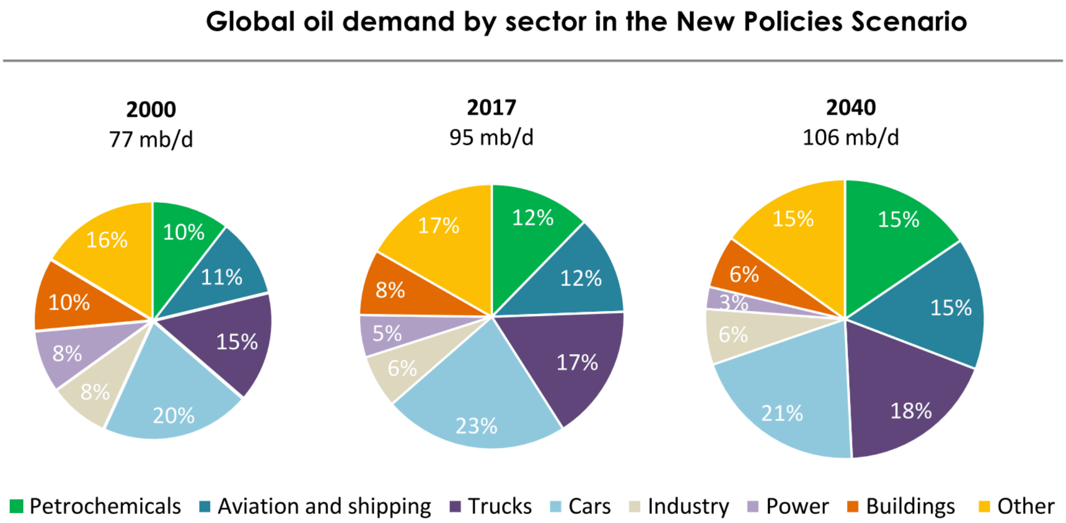

Figure 1.1: Oil demand per sector (International Energy Agency, 2018)

The urgency of reducing emission of greenhouse gasses has been recognised by many countries worldwide by signing the *Paris Agreement* (United Nations, 2015). The European Commission has committed itself to a 20 % reduction of transport emissions from 2008 levels by 2030 (European Commission, 2019). Improving the efficiency of operations is an important way to contribute to these goals.

Petrol distribution companies can make their operations more efficient by using modern optimisation tools, which will result in less kilometres that need to be driven to fulfil their services (AMCS Group, 2019b). This will not only lower their carbon footprint by saving fuel, but it will also improve the competitiveness of the company. Profit margins are low in the oil distribution sector due to fierce competition among distribution companies. Because transport activities are the main cost factors for these companies, savings on operations will directly improve the competitive position (Lima, Relvas, & Barbosa-Póvoa, 2016).

1. INTRODUCTION

The petrol distribution problem is well-known in literature as the PSRP, which concerns the distribution of petroleum products from a depot to a set of petrol stations. These stations have underground tanks in which the fuel is stored and compartmentalised vehicles are used to transport the fuel (see Figure 1.2 for a schematic overview). Since the car fleet is increasing and a reduction of emissions is required, efficient planning methods are needed. Optimising the planning over multiple days leads to better solutions, since stations do not have to replenished every day. Therefore, the objective of this research is to develop an efficient planning method for the multi-period PSRP with real-life characteristics. Considering the most relevant real-life characteristics of the problem leads to models that better represent the real world situation.

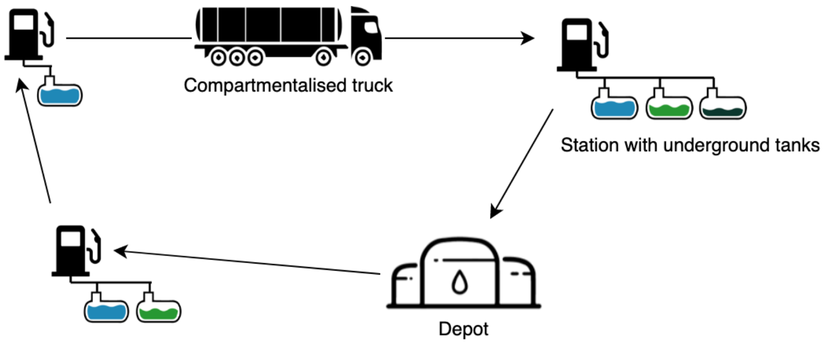

Figure 1.2: Schematic overview of the petrol distribution problem

According to the definition for Rich Vehicle Routing Problems (RVRPs) presented by Lahyani, Khemakhem, and Semet (2015), the problem researched in this report can be considered as ”rich”. RVRPs are extensions of academic variants of VRPs with complicating and challenging features to represent the complexity of the real-life situation.

Since the problem is NP-hard and the exact mathematical models will only be able to find solutions for instances of limited size, a heuristic is proposed to find solutions for instances of real-life size. Braekers, Ramaekers, and Van Nieuwenhuyse (2016) recognised that real-life characteristics are often considered with a limited number of other characteristics. Combining all relevant characteristics for real-life problems in new models and developing efficient solutions for these models are additions to the literature. Therefore, the gap in literature addressed by this research is to develop new planning methods for the PSRP with an unexplored combination of real-life characteristics. These characteristics include an extended time horizon, single depot with unlimited supply, heterogeneous fleet of capacitated vehicles with compartments, flow metres, vehicle restrictions, multiple stops per trip, multiple trips per time period, split loads, time windows, vendor inventory management and a safety stock.

The newly formulated mathematical models and the heuristic are additions to the literature. The heuristic can be classified as decomposition heuristic, because it divides the rich PSRP into smaller sub problems based on the decomposition of the decision process of the supplies. This approach makes it easier to find a solution. This approach, however, leads to lower quality solutions, since the decisions are not optimised simultaneously. Therefore, a local improvement procedure is used to improve the initial solution found by the decomposition heuristic. Candidate solutions are based on visiting a station one day earlier.

The heuristic introduces the *simultaneous dry run* inventory policy. This concept, which has been observed in practice, is used to determine the delivery amounts that ensure that gas stations are visited as few times as necessary to prevent dry run. In this research, dry run is the moment when an underground tank is considered as empty, which is the moment that the inventory level drops below the safety stock level.

Moreover, to evaluate the planning methods, data from a client of AMCS is used. AMCS is a company that specialises in software and vehicle technology for the waste, recycling and material resources industry. One of the company's focus areas is routing solutions for the waste and resources industries. With the product Oil Planner, AMCS automates the petrol distribution operations planning for their customers. The current planning process requires many characteristics to be set manually. This may lead to a sub-optimal solution and setting up the software for a new client requires a thorough analysis of the distribution network. More automation of the planning process could be a way to improve the efficiency and reduce the required customisation of the software product.

1.1. Research questions

Resulting from the above formulated problem definition and research objective, the main research question of this thesis is:

"What is an efficient planning method for the multi-period petrol station replenishment problem, in terms of computation time and solution quality?"

Furthermore, to help answering the main research question, sub-questions have been defined to divide the problem into several relevant parts. The sub-questions are defined as follows:

1. What is the current state of the PSRP in the literature and practice?
2. Which KPIs are relevant for assessing a planning method?
3. Which planning methods can be developed for planning petrol station replenishment?
4. How do the proposed methods perform according to the KPIs?

1.2. Scope

The Petroleum Supply Chain (PSC) can be divided into an upstream, midstream and downstream segment. The upstream segment comprises the exploration of oil wells, oil production and transportation until the refineries. The midstream part comprises petrochemical operations at the refineries. Transportation to wholesale customers and retail stations is considered as the downstream segment (Lima et al., 2016). The optimisation models presented in this research consider the downstream segment of the supply chain, which refers to the distribution from an oil depot to gas stations.

The problem can be optimised at different planning levels. In the review written by Lima et al. (2016), a distinction has been made between on the one hand the strategic and tactical level and on the other hand the tactical and operational level. Both approaches differ in the length of the time span and the consideration of investment costs of vehicles, depots and gas stations. In this research, the aim is to focus on the operational level, which means that a fixed network and vehicle fleet is considered. The goal of the models is to generate a daily planning for the distribution of fuel products.

1.3. Methodology and research structure

The structure of the research is based on the double diamond method (Design Council, 2019). The method consists of four phases;

- **Discover** and understand what the problem is.
- **Define** the problem.
- **Develop** different solutions for the problem.
- **Deliver** the best solutions.

A distinction is made between diverging (discover and develop) and converging thinking (define and deliver). Idea generation is a diverging process, while narrowing ideas down to a concept is a converging process. In the double diamond method, these two processes happen twice, firstly to define the problem and secondly to develop a solution. Both processes are represented by two diamonds.

The first process, defining the problem, is covered by the first two chapters of the research. A review of the literature is, in the discover phase, used to understand the problem. A detailed overview of the problem, its characteristics and the state of practice are presented. This leads in the define phase to a clearly identified literature gap which covers the state-of-the-art situation for the PSRP. The first two chapters conclude with the answer to the first sub-question.

A solution is developed in the second process, elaborated in chapters 3 and 4. The develop phase translates itself to the elaboration of different solution methods, which are the methods used to create the planning for the PSRP in this case. These methods and variations to these methods are first introduced in the conceptual model (section 3.1) and presented in more detail in sections 3.2 - 3.4. These planning methods are the answer to sub-question three. Moreover, objectives are identified in the conceptual model, what leads to KPIs (answer to the second sub-question). The KPIs can be used to compare the solutions from different planning methods. These planning methods are evaluated with a case study. The case study and the corresponding data set are introduced in chapter 4. Experiments are performed with this data sat and results are compared based on the values for the KPIs. The evaluation of the methods is the deliver phase, of which the result is then used to answer sub-question four.

Chapter 5 concludes with an answer to the main research question in the conclusion and recommendations for further research. An overview of the research structure is shown in Figure 1.3.

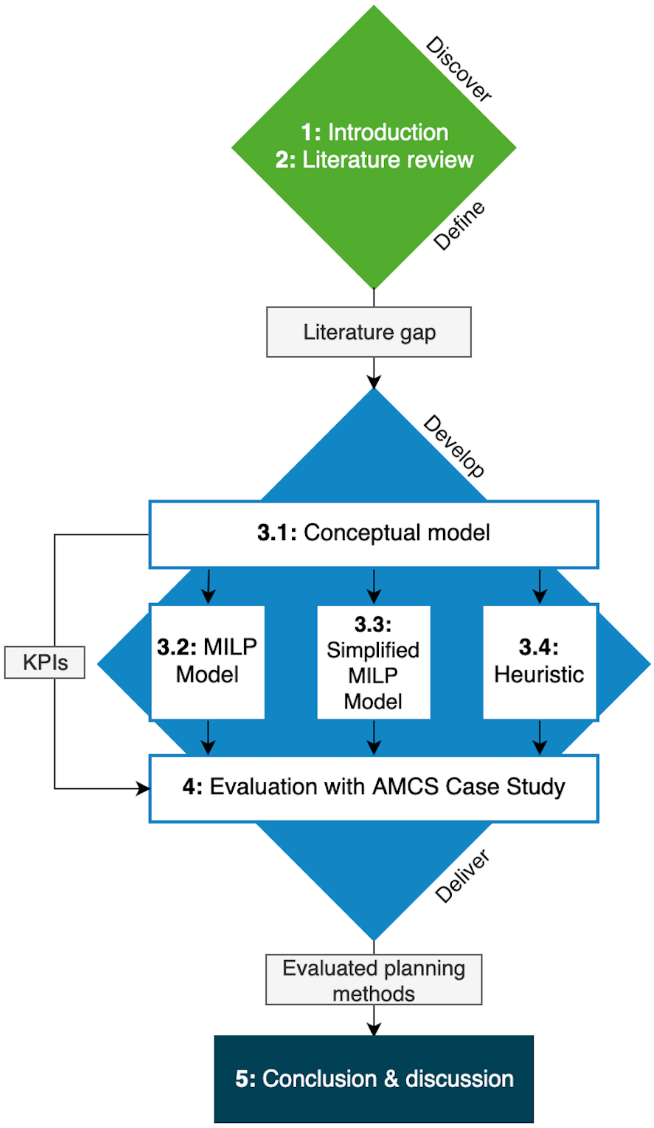

Figure 1.3: Research structure

2

Literature review

The PSRP has received substantial attention in the literature over the last decades. A brief overview of recent literature covering this problem is presented in the next section, followed by a a description of the main characteristics considered in this research. Section 2.3 then gives an overview of the state of practice, after which solution methods for the PSRP with inventory management are discussed in section 2.4. The chapter concludes with a clear definition of the literature gap addressed by this research.

2.1. Petrol station replenishment problem

The PSRP is an extended version of the VRP, of which the latter was formulated for the first time by Dantzig and Ramser (1959). A VRP considers the problem of servicing a set of customers with a fleet of vehicles. The goal is to find the best set of routes, according to an objective function and without violating operational constraints.

The PSRP concerns the optimisation of the distribution of several petroleum products to a set of petrol stations over a given planning horizon. The products, stored in underground tanks, are delivered to the stations using a fleet of vehicles with multiple compartments. Maximum delivery amounts are limited by the available capacity of a storage tank at a station and the capacity of vehicles. The aim is to determine the most optimal route planning for the vehicles, according to the objective of maximising revenue by minimising costs or number of kilometres driven (Benantar, Ouafi, & Boukachour, 2016; Cornillier, Laporte, Boctor, & Renaud, 2009; Cornillier, Boctor, & Renaud, 2012; Triki, Chefi, 2013).

Several versions of the PSRP have been researched over the last three decades. Brown and Graves (1981) were one of the first to publish about this problem. The authors described an Integer Linear Programming (ILP) for a real-time dispatch system for petroleum tank trucks, which were dispatched from multiple depots. A comprehensive overview of the history of the PSRP can be found in the papers published by Cornillier et al. (2012) and Benantar et al. (2016). This review focuses on recent and relevant research related to the version of the PSRP considered in this research.

Cornillier, Boctor, Laporte, and Renaud (2007) developed an exact algorithm for the single day PSRP by decomposing the problem into a routing and a truck loading problem, of which the latter was used to determine delivery quantities and assign quantities to compartments of vehicles. The number of stops per trip was limited to two which made it possible to find an optimal solution. The problem is extended to multiple days by Cornillier, Boctor, Laporte, and Renaud (2008), in which the researchers presented a MILP model with the objective to minimise costs with a penalty for overtime use of vehicles.

Popović, Bjelić, and Radivojević (2011) defined a MILP model for the PSRP which minimises both inventory and routing costs over a time span of multiple days. The number of stations that can be visited in a trip is limited to three and a station cannot be visited by more than one vehicle during a time period. The authors used a fixed consumption rate per tank, which makes the demand deterministic.

In the paper published by Popović, Vidović, and Radivojević (2012), a Variable Neighbourhood Search (VNS) heuristic with a shaking procedure based on shifting deliveries between days on the planning horizon. In this research, homogeneous vehicles are used, which simplifies the situation. Vidović, Popović, and Ratković (2014) also used a homogeneous fleet of compartmentalised vehicles to distribute fuel and presented two MILP models which minimise inventory and routing costs. One of the model also includes vehicle fleet costs.

Li, Chen, Sivakumar, and Wu (2014) presented a version of the PSRP with a fixed fleet of homogeneous vehicles. The stations are visited after a minimum delivery quantity can be delivered, which is set to improve vehicle utilisation.The problem is solved with a tabu search algorithm. The lower bounds of a reasonable sized problem are determined with Langrangian relaxation. The tabu search algorithm proved to be able to find solutions that are near optimal.

An exact model for a variant of the PSRP with multiple compartments and time windows is presented by Benantar et al. (2016). The time windows represent the scheduling horizon for each vehicle and there are two sets of compartments. The problem is solved using an MILP model and a heuristic is proposed.

2.2. Problem characteristics

The importance of combining real-life characteristics in VRPs has been recognised by Braekers et al. (2016), who state that these characteristics are mostly studied individually or with a limited number of other characteristics. The challenge is not only to create these ”richer” models, but also to develop efficient solution methods, which is the aim of this research.

The consideration of real-life characteristics simultaneously puts this research in the category of the RVRPs. In the survey written by Lahyani et al. (2015), a definition was presented for the RVRP. According to this definition, a VRP can be called ”rich”, when at least four strategic and tactical aspects are included in the distribution system and at least six restrictions related to the physical network, which is the case for this research. The main characteristics are discussed in the following sections.

2.2.1. Multi-period time horizon

The PSRP has traditionally been solved over a single day time span, mainly because of the complexity of the problem, what makes it an NP-hard problem (Al-Hinai & Triki, 2018). Because of the fact that stations do not have to be replenished every day, what happens in case when the remaining inventory is enough to fulfil the demand. Solving the problem for a time span of multiple days leads to more efficient solutions (Triki & Al-hinai, 2016). Cornillier et al. (2008) classified this problem as the Multi-Period Petrol Station Replenishment Problem (MP-PSRP).

Triki, Chefi (2013) introduced a variant of the MP-PSRP, called the Periodic Petrol Station Replenishment Problem (PPSRP). When a certain replenishment pattern can be established, frequencies can be determined for how often a station should be visited. In the PPSRP, this periodicity is recognised and the delivery of fuel is optimised to determine the optimal day to visit a station, while minimising total travel distance. A mathematical formulation of the PPSRP is presented by Triki and Al-hinai (2016), which has later been extended with frequency choice by Al-Hinai and Triki (2018).

2.2.2. Inventory routing

Although transportation and inventory costs generate most of the supply chain costs, modelling approaches for supply chain usually consider these two aspects independently. Since inventory and routing decisions are interrelated, solving both problems simultaneously may lead to better solutions, since the customer satisfaction can be improved while the distribution costs are decreased through efficient vehicle utilisation (Vidović et al., 2014; Cordeau, Laganà, Musmanno, & Vocaturo, 2015).

Problems in which inventory management, vehicle routing and scheduling decisions are considered, can be classified as Inventory Routing Problems (IRPs). These problems assume that the Vendor-Managed Inventory (VMI) concept is applied, what means that inventory levels are known by the supplier and that the supplier makes decisions about replenishment. Decisions that need to be taken simultaneously are when to serve a customer, quantities to deliver to this customer and the routes to serve the customers. Both the supplier and vendors benefit from this situation, since the overall distribution costs are minimised and vendors don't have to spend resources on inventory management (Coelho, Cordeau, & Laporte, 2014).

Different policies can be applied to determine the quantities that are delivered. The main inventory policies are maximum level policy and the order-up-to level policy. Under the maximum level policy, the order quantity is flexible, but lower than the maximum capacity. The order-up-to level policy uses a fixed order quantity, which sets the inventory to a certain level (Coelho et al., 2014). The most common policy used for the PSRP is the maximum capacity policy. Cornillier et al. (2007) and Popović et al. (2011) applied this policy by setting bounds on the delivery quantities, based on a the capacity of a tank. Li et al. (2014) applied an order up to policy, which means that the delivery quantity is set to the remaining tank capacity at the moment of delivery. The *simultaneous dry run* inventory policy presented in this research, is a maximum level policy.

Several researchers have considered the PSRP as IRP, with often the objective to minimise both routing and inventory costs (Popović et al., 2011; Popović et al., 2012; Vidović et al., 2014). Li et al. (2014) presented a model with the objective to minimise the maximum route duration. This was done to determine the trade off between number of vehicles and maximum route duration, which is relevant because the use of a vehicle during a day is limited to the driver schedules.

2.2.3. Multi-compartment and split loads

In the PSRP, compartmentalised vehicles are used to distribute fuel. Only one product can be loaded in the same compartment. The problem is then modelled as a variant of the Multi-Compartment Vehicle Routing Problem (MCVRP), which requires that products must be transported in different compartments, because of incompatibility constraints (Kaabi & Jabeur, 2015). Coelho and Laporte (2015) presented different MILP models for this problem, which the authors classified as split or unsplit delivery and tanks. Split delivery means that the content of a compartment can be split over multiple stations, where unsplit delivery means that the full content of the compartment must be delivered to a station. With split tanks was meant that a tank can receive delivery from multiple vehicles.

Moreover, researchers often assumed that vehicles are not equipped with flow meters, what implies that the full content of a compartment must be emptied at a station (Macedo, Alves, De Carvalho, Clautiaux, & Hanafi, 2011; Cornillier et al., 2007; Cornillier et al., 2008; Benantar et al., 2016; Popović et al., 2012; Vidović et al., 2014). The absence of flow meters limits the flexibility in utilisation of vehicles, while it is possible to equip vehicles with flow meters nowadays (Coelho & Laporte, 2015). In this research, vehicles are assumed to be equipped with flow meters, which allows split delivery. Tanks can be resupplied by multiple vehicles as well.

2.2.4. Time constraints and multiple use of vehicles

Several researchers have considered the PSRP with time windows, during which a station needs to be replenished. This problem can be classified as the Vehicle Routing Problem with Time Windows (VRPTW). Benantar et al. (2016) considered this problem with compartmentalised vehicles for a single day. The delivery quantities were determined by the demand for that day. A tabu search algorithm with neighbourhoods defining the visiting order was used to solve the problem.

A variant of the VRPTW is problem with multiple use of vehicles, where each vehicle can perform multiple trips per time period. Azi, Gendreau, and Potvin (2010) studied this problem with a homogeneous fleet of capacitated vehicles. The researchers presented a MILP model and showed how the problem can be solved using a branch-and-price algorithm.

Another MILP model for the single day VRPTW with multiple use of vehicles is presented by Macedo et al. (2011). The fleet of vehicles is homogeneous with fixed capacity. The authors also proposed a network flow model with all feasible routes generated prior to the optimisation. An interval was determined for each route, which defines when the route is feasible according to the time windows for delivery at the stations.

Time windows can also be applied to vehicles, to represent driver schedules. Cornillier et al. (2008) limited the use of vehicles to a number of available hours. In this research, it was also possible to use vehicles in overtime hours, which had higher costs.

2.2.5. Stochastic VRPs

The PSRP is usually solved by deterministic models. However, in the real-life situation, the fuel consumption at stations cannot be known beforehand and is therefore stochastic. Uncertainty in actual fuel consumption can be corrected by safety stock levels or emergency deliveries (Popović et al., 2011). Cornillier et al. (2008) recognised the stochastic nature of demand, as well as the fact that the stock is sold during the day and that the delivery amount should depend on the time of arrival. However, the authors considered demand as deterministic and applied a safety stock level to make it feasible to deliver during the entire day.

Demand, presence of customers, travel times and service times can be modelled as stochastic in the VRPs. In stochastic models, not all information is available before decisions are made, where deterministic models have complete information (Oyola, Arntzen, & Woodruff, 2018). In this research, demand is considered as deterministic and a safety stock is applied to prevent stock out due to uncertainty in demand. Service times are based on realistic values for delivering a certain quantity. Travel times are deterministic as well. To determine if the planning is affected by uncertainty in travel times, for example caused by congestion, a sensitivity analysis is conducted.

Popović et al. (2011) developed a simulation model to determine the effect of uncertainty on the PSRP planning, with different levels of safety stock applied. The authors showed that higher safety stock levels are necessary when there the demand is less predictable. The same approach, increasing the safety stock, can be used in practice if the uncertainty in demand is high for some of the stations.

2.3. State of practice

Brown and Graves (1981) was one of the first researchers to consider a real-life case for the PSRP for a petroleum distributor in the United States. The research was used to automate the dispatching of trucks. Brown, Ellis, Graves, and Ronen (1987) also described a planning optimisation for a oil company in the United States, which was used to improve the productivity of personnel and to save costs. Van der Bruggen, Gruson, and Salomon (1995) published about their consultancy study they performed in the Netherlands, in which the distribution network for a large oil company was considered. Distribution of petroleum products form the depot to gas stations was optimised to assist the management with the restructuring of the network.

The distribution network of petroleum distribution is considered by Cornillier et al. (2007). A heuristic is proposed to solve the problem. The proposed model was used to create awareness by the company about the benefits of optimisation, which resulted in the integration of the algorithm in the planning systems of the distribution company.

Ng, Leung, Lam, and Pan (2008) improved the distribution of petrol for a network in Hong Kong, in which they applied the VMI concept. The proposed approach helped the company to increase the delivery volume and decrease in driver costs.

Triki, Chefi (2013) used real-life cases to evaluate the performance of the solution methods proposed by the authors to solve the PPSRP. Compared to a planning made by a human operator, a saving of 17.7 % was achieved with the most effective presented heuristic. Triki, Al-hinai, Kaabachi, and Krichen (2016) also investigated the PPSRP for a distributor of petroleum products in Oman to evaluate the performance of the solver used to find an exact solution. The solver was able to find a solution for up to 22 stations. The model will eventually be used to prepare a bid for auctions of transportation procurement.

Li et al. (2014) considered a large petroleum company in China, with 40,000 gas stations spread all over the country. The distribution of petroleum products in provinces was modelled as IRP. Regulations applied by the company and the industry was translated to the model. The results of a heuristic are proven to be the near optimal.

A case study of the downstream PSC in the United States is modelled using MILP by Kazemi and Szmerekovsky (2015). The model is used to determine the optimal supply chain design, while considering multi-modal transportation methods when determining locations for the facilities.

The case of distribution for an Algerian petroleum company has been evaluated by Benantar et al. (2016), with a model for the PSRP with compartmentalised vehicles and time windows. The method proposed by the authors outperformed the solution created by the company, in terms of number of vehicles and total travel distance.

The case studies mentioned in this sections show that developing MILP models and heuristic are suitable and effective methods for the planning of the PSRP. These methods can be used to gain insight in tactical and operational choices that need to be made. New trends like the possibility to equip vehicles with flow meters (Coelho & Laporte, 2015) and more interest in ”richer” models promise even more effective methods for the future.

2.4. Matheuristics for solving the rich PSRP

Including real-life characteristics in a MILP model for the PSRP makes the model NP-hard (Li et al., 2014; Cornillier et al., 2008; Vidović et al., 2014; Kaabi & Jabeur, 2015; Al-Hinai & Triki, 2018). Matheuristics have proven to be an efficient category of heuristics that can be used to solve rich VRPs. An introduction to matheuristics is given below, after which is shown that matheuristics can also be used to solve rich variants of the PSRP.

The definition of matheuristics is defined by Boschetti, Maniezzo, Roffilli, and Bolufé Röhler (2009) as ”heuristic algorithms made by the interoperation of metaheuristics and mathematical modelling techniques”. Archetti and Speranza (2014) defines three categories of matheuristics. Firstly, there are decomposition approaches, where the problem is divided into sub problems and MILP models are used to find a solution to at least one of these sub problems. Secondly, there are improvement matheuristics, where MILP models are used to improve solutions from other heuristics. The third category is column generation based approaches.

Decomposition matheuristics is the category that is especially relevant for solving the rich PSRP. Decomposing the complex problem, in which often many routing and inventory decisions need to be made, into smaller sub problems makes it easier to find a solution. A sub class of the decomposition matheuristics are the multi-phase approaches, where sub problems are solved after each other (Archetti & Speranza, 2014).

Several researchers solved the PSRP with decomposition heuristics. Triki, Chefi (2013) has solved the PPSRP, in which inventory was not considered, by decomposing the problem into an assignment, routing and improvement procedure. The authors presented different methods to solve the assignment model; minimising daily demand, minimising daily number of stations, minimising depot distance and minimising the distance of virtual clusters. The last mentioned approach proved to be most effective. To improve the initial solution, the researchers used a local search technique based on switching any two stations between service days and checking if improvement has been made. Another solution method for the PPSRP is presented by Carotenuto, Giordani, Massari, and Vagaggini (2015), where the authors use a hybrid genetic algorithm to solve the routing model.

Cordeau et al. (2015) solved an IRP by decomposing the decision process of the vendor into a three-phase heuristic. The decisions that are made in the phases are the replenishment plan, the delivery sequence and determining which routes to drive. Inventory management has also been included by (Cornillier et al., 2007), who decomposed the problem into a Tank Truck Loading Problem (TTLP) and a routing problem. The objective of the TTLP is to maximise the total delivery quantity, while considering vehicle and underground tank capacity constraints. Similar models are used in the heuristic presented by Cornillier et al. (2008). The authors extended the heuristic by constructing the planning per day and shifting the day of visit to a station. Both heuristic approaches discussed in this paragraph limit the number of stops per trip to a maximum of two, what simplifies the problem.

Vidović et al. (2014) developed a MILP model and a heuristic to solve the PSRP with inventory management, multiple products and multiple periods. The authors decomposed the problem into an inventory and a routing problem. The initial solution found by the heuristic is improved by a local search procedure. This local search procedure is based on shifting the content of the compartment of a vehicle between days in the planning. The same ”compartment transfer” heuristic has been used by Popović et al. (2012), with an intra-period heuristic for route optimisation within one time period and a inter-period route optimisation with compartment shaking for route optimisation over the full time horizon. The assumption that only full compartments can be delivered to stations simplifies the problem. In this research, delivery quantities are determined per litre.

The brief overview in this section shows that decomposing the complex PSRP into smaller sub problems is an efficient solution method. Decisions that need to be taken by the supplier can then be made apart from each other. The same approach will be used in the heuristic presented in this research. To improve the solution found by the decomposition heuristic, a local improvement procedure is applied.

2.5. Literature gap addressed by this research

The literature gap addressed by this research consists of presenting new optimisation models for the considered version of the PSRP and the presentation of a heuristic in which the *simultaneous dry run* inventory policy is applied. The version of the PSRP considered in this research is a combination of real-life characteristics that has not been researched before. Table 2.1 presents an overview of relevant papers in which MILP models are presented for VRPs with some of the characteristics. This research considers all the characteristics mentioned in the table simultaneously. More commonly used characteristics are considered as well. These are one depot with unlimited supply, heterogeneous fleet of vehicles, a fixed number of vehicles and deterministic demand.

Table 2.1: Comparison between MILP models in the literature and in this research

Paper	D	S/T	TH	MP	VU	VF	MC	SL	TC	VR
Azi et al. (2010)	R	unlimited	SP		MT	HO		✓	✓	
Benantar et al. (2016)	R	unlimited	SP	✓	ST	HE	✓		✓	✓
Coelho and Laporte (2015)	R + I	unlimited	MP	✓	ST	HE	✓	✓		
Cornillier et al. (2008)	R + I	max. 2	MP	✓	MT	HE	✓		✓	
Li et al. (2014)	R + I	unlimited	SP		SU	HO.			✓	
Macedo et al. (2011)	R	unlimited	SP		MT	HO		✓	✓	
Popović et al. (2012)	R + I	max. 3	MP	✓	ST	HO	✓			
Al-Hinai and Triki (2018)	R	comp. + 1	MP	✓	ST	HO	✓			
Vidović et al. (2014)	R + I	max. 3	MP	✓	ST	HO	✓			
This research	R + I	unlimited	MP	✓	MT	HE	✓	✓	✓	✓

Description of abbreviations used in the table:

- D = decision: routing (R) or routing and inventory (R + I)
- S/T = allowed stops per trip
- MP = multi-product / multi-commodity
- TH = time horizon: single (SP) or multiple period (MP)
- VU = vehicle use: single (ST) or multiple (MT) trip
- VF = vehicle fleet: homogeneous (HO) or heterogeneous (HE)
- MC = multi-compartment
- SL = split loads
- TC = time constraints
- VR = vehicle restrictions

The combination of considering these characteristics simultaneously has, to the best of my knowledge, not been researched before. The developed MILP models are one part of the addition to the literature. The mathematical formulations show how all the considered characteristics can be combined into a single RVRPs, representing a new version of the PSRP. The mathematical formulation of these models does not only consists of combining constraints that have been presented by other researchers. Constraints are adapted and new constraints are added.

The second part of addition to the literature is the proposed heuristic. The distinctive aspect of the heuristic presented in this paper is the application of the introduced *simultaneous dry run* inventory policy, which is used to determine the delivery quantities to a station. The heuristic procedure consists of a decomposition heuristic and a local improvement procedure. A decomposition heuristic is used to decompose the complex problem into smaller sub problems, for which it is easier to find a solution. Since decisions are not optimised simultaneously, a local improvement procedure is proposed to improve the initial solution.

2.6. Concluding remarks on the literature

The sub-question answered in the first part of this research is formulated as:

What is the current state of the PSRP in the literature and practice?

The PSRP is a variant of the well-known Capacitated Vehicle Routing Problem (CVRP) and has received significant interest from researchers over the last decades. Researchers have been adding more and more problem specific constraints to the PSRP, to create more representative models that better reflect the complex real-life situation. The problem, that concerns the delivery of petroleum products from a depot to a set of gas stations by a fleet of compartmentalised vehicles, has traditionally been solved over a single time span. Recent research considers the problem together with inventory management over a longer time horizon, since optimising inventory and routing simultaneously leads to more efficient distribution. The availability of vehicles equipped with flow meters allow to split the content of a compartment between stations and time constrains make the models more realistic.

Solving the PSRP with MILP models and decomposition heuristics have proven to be effective solution methods. Many researchers have used such models to mathematically formulate and solve the real-life problem. These studies provide insight in tactical and operational decisions. The trend of more interest in ”richer” models promise even more effective evaluation methods for this problem in the future.

The version of the PSRP presented in this research considers a combination of characteristics that has not been modelled simultaneously before (as shown in table 2.1). MILP models are presented for this new rich variant, which are additions to the literature. Furthermore, the distinctive aspect of the decomposition heuristic developed in this research is the newly introduced *simultaneous dry run* inventory policy, which is observed in practice and ensures that stations are visited as few times as necessary. This is done by determining minimal delivery amounts in a way that the next visit to the station is postponed for as long as possible.

3

Planning methods

Planning methods for the PSRP are presented in this chapter. Firstly, the problem is described conceptually, after which two MILP models are shown. The secondly presented model uses predetermined routes, which makes it easier (faster) to find a solution compared to the firstly presented "full" model. Furthermore, a decomposition heuristic is presented, in which the new *simultaneous dry run* inventory policy is applied. The chapter concludes with answers to sub research questions 2 and 3.

3.1. Conceptual model

The variant of the PSRP considered in this research is first described in words in this section. An overview of definitions and model characteristics is presented, after which the main assumptions are discussed. Thereafter, objectives are identified which lead to KPIs. The section concludes with an overview of planning methods, for which requirements are discussed first.

3.1.1. Model characteristics

The goal of the models is to create the planning for the distribution of petrol products from a central depot to a set of gas stations with one or multiple underground tanks over a planning horizon of multiple days. Petrol products are transported by a heterogeneous fleet of compartmentalised vehicles. An example of a vehicle driving a trip to three stations with different numbers of underground tanks is shown by Figure 3.1. The models assume the application of the VMI concept, which means that the supplier knows the inventory levels and that the supplier determines when and with which quantities the stations are replenished.

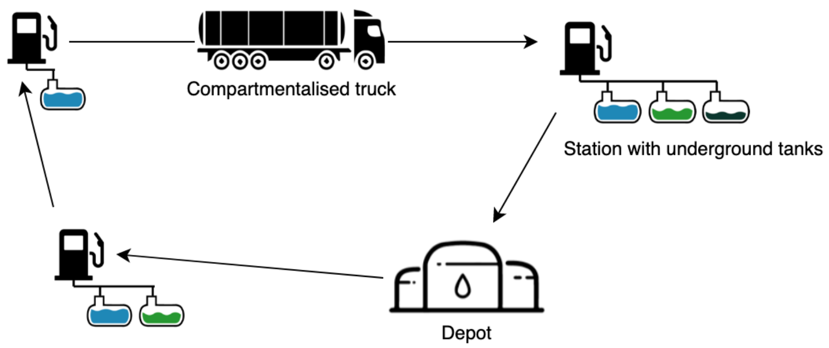

Figure 3.1: Visualisation of a compartmentalised vehicle making a trip to multiple stations with underground tanks

Stations need to be replenished before a minimum level is reached. For each time period, the supplier makes the following decisions:

- Determine when stations are visited and which stations are combined in trips

- Determine which vehicles are used to perform the trips
- Determine the delivery quantities that are transported to each station
- Determine which compartments are used to deliver the fuel
- Determine for each trip the time a vehicle leaves and returns to the depot, while considering service times at the depot and at the stations

An important distinction that has to be made is the difference between a stop, a route and a trip. In this research a stop is when a vehicle visits a station. A route is a sequence of stops, and when a trip is when a vehicle drives a route. A vehicle can perform multiple routes during a time period. Since the vehicles are equipped with flow meters, the load from one compartment can be divided over multiple stations. Another important characteristic is that some of the stations are not accessible by all vehicles, because of spatial constraints. An overview of the model definitions and characteristics are presented by Tables 3.1 and 3.2. A more detailed description of the model characteristics can be found in the literature review in chapter 2.

Table 3.1: Model definitions

Term	Definition
Fuel products	Petrol products with different brands and types
Gas station	Places where fuel products are sold to end customers
Depot	Place from where the fuel products are distributed
Vehicle	Tank trucks are used to distribute fuel from the depot to stations
Underground tank	Fuel is stored at stations in underground tanks which can hold one type of product
Safety stock	Safety stock is the inventory level at which a underground tank is considered empty
Dry run	When the inventory level drops below the safety stock level
Stop	When a vehicle visits a station to deliver fuel, it is called a stop
Route	A route is a sequence of stops/stations to visit, starting and ending at the depot
Trip	A trip is a vehicle that drives a route
Time period	A planning is created per time period, which represents a day
Planning horizon	The time horizon is the set of considered time periods

Table 3.2: Model characteristics

Characteristic	Description
IRP	Decision to take are inventory and routing decisions
Multi-period	The planning is optimised for a time horizon of multiple days
Multi commodity	Multiple types of products are distributed
Compartmentalised vehicles	Vehicles have compartments which can contain one type of product
Heterogeneous fleet	The number of compartments and compartment capacities differ for each vehicle
Vehicles with flow meters	Vehicles are equipped with flow meters
Fixed number of vehicles	The number of available vehicles is fixed
Deterministic demand	Demand is considered as deterministic
Split loads	Vehicles are allowed to split the load from one compartment over multiple stations
Vehicles with time windows	Use of vehicles is limited by time windows representing operating hours
Multiple trip	Vehicles can perform multiple trips during one time period
Station restrictions	Some stations cannot be reached by each vehicle, because of spatial limitations

3.1.2. Main assumptions

The list below gives an overview of the assumptions made:

- There is enough inventory at the depot to fulfil the replenishment plan.
- Inventory levels at stations are known by the supplier.
- Demand is considered as deterministic, based on a forecast. In real-life, demand is created by selling fuel during the day, which is a stochastic process. To prevent stock out, a a safety stock level is determined for each tank at each station and the inventory level should always be higher than the safety stock level.
- A station can have only one underground tank per product. If this is the case in real-life, tank capacities and inventory levels should be combined.
- To transport the petrol products, a heterogeneous fleet of vehicles is used with compartments of known size. The vehicles are assumed to be equipped with flow meters, which enables split loads.
- Vehicle compartments do not have to be cleaned between transporting different types of fuel. In real-life, compartments are not cleaned (based on personal communication with AMCS planning expert K. Hauge) and comparable studies discussed in the literature review do not require cleaning.
- All vehicles are assumed to drive with the same speed, which means that they have the same travel time.
- Some stations can not be visited by all vehicles, because of spatial restrictions.
- Stations can be visited multiple times a day and $24/7$, since it is assumed that a truck can always deliver the petrol products at the station.
- Vehicles can only be used within the operating hours. This represents the time schedules of workers. It is only possible to have one time schedule per vehicle per time period, so if there are multiple work shifts during one day, for each shift a vehicle should be added.

3.1.3. Objectives supplier and KPIs

An objective tree is used to identify KPIs that are relevant for the supplier, who makes the decision about replenishment in the considered problem. The main objective of the supplier is to maximise the company's profit. This can be done by minimising cost and by maximising revenue. The objectives of the supplier are shown in the objective tree in Figure 3.2. Personal communication with petrol station replenishment expert K. Hauge helped to draw the objective tree.

Revenue can be maximised by maximising sales and minimising fines, what can be accomplished by minimising the number of dry runs. In this research, dry run is the moment when the inventory level of a tank drops below the safety stock level. If dry run is prevented, stock out is unlikely to occur and fuel can be sold at the gas stations. In this way, sales are maximised. Furthermore, contractual liabilities may allow the gas stations to impose a fine on the supplier if dry run occurs.

Routing costs and inventory costs are seen as the main factors of petrol distribution costs (Popović et al., 2011; Vidović et al., 2014). Minimising costs can thus be achieved by minimising the travel distance of all vehicles and by minimising the average stock level. Lower travel distance leads to less use of vehicles, resulting in lower vehicle and driver costs. Lower travel distance will also lead to less CO2 emissions, since less fuel is consumed by the vehicles. The travel distance can be decreased by maximising vehicle utilisation while minimising the average number of stops per trip. This limits the visits to a station, because driving routes with many stops multiple times will increase the travel distance.

Average stock levels should be minimised, because excessive stock means that the fuel cannot be sold elsewhere and that the value of excessive stock cannot be used for other purposes.

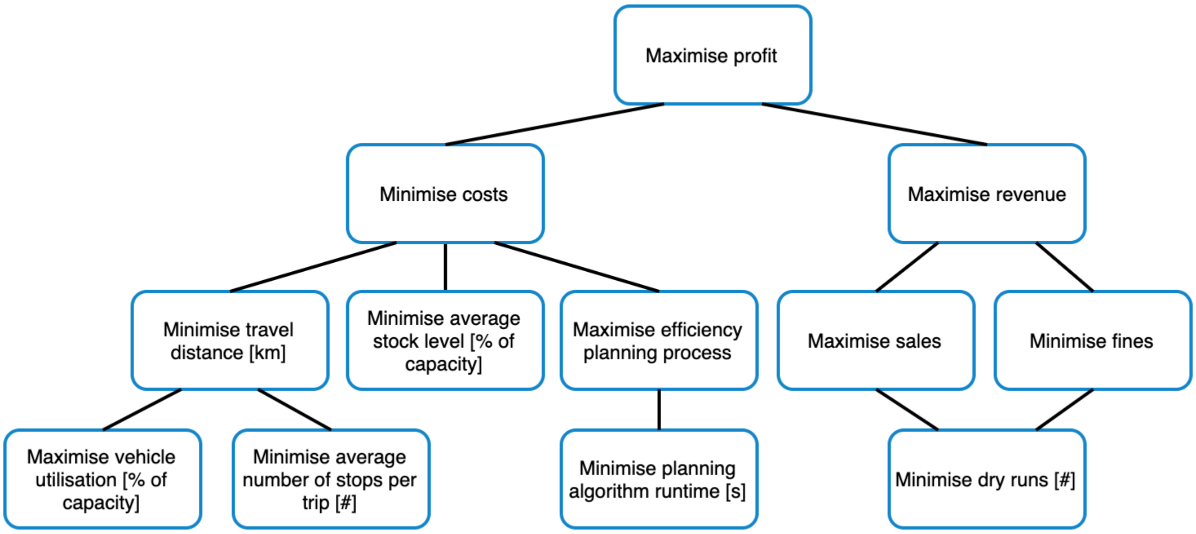

Figure 3.2: Objective tree and identified KPIs

Preventing dry runs, minimising travel distance and minimising the average stock level are conflicting goals. For the total travel distance, it might be more efficient to visit a station earlier than necessary, which will increase the average stock level. Moreover, allowing dry runs might lower the total travel distance and average stock level as well. These trade-offs will be evaluated with the case study in chapter 4.

Lastly, costs can be minimised with efficient planning. If the time the planning algorithm takes is minimised, this means that there is more time for manual adjustments of the planning, before the drivers need to know their schedule. Another benefit is that the algorithm can start later, which improves the information quality since the used information is more representative.

The KPIs which will later be used to evaluate the solutions created by the planning methods follow from the objective tree. These KPIs are calculated as follows:

- **Total travel distance [km]**
  Total travel distance is calculated by enumerating the distances of routes all the vehicles during the full time horizon make.

- **Average stock level [%]**
  Average stock levels are calculated by dividing the sum of the total stock in all underground tanks at the end of the day by the sum of all underground tanks' capacities. An average of all days of the time horizon is taken.

- **Number of dry runs [#]**
  The number of dry runs is the number of tanks with an inventory level below the safety stock level. All days in the time horizon are considered, so if the inventory level of a tank is lower than the safety stock for two days in the time horizon, this is seen as two dry runs.

- **Algorithm run time [s]**
  The run time is the time in seconds an algorithm needs to find a solution. The time it takes to load input data or to write output data to files is excluded form the algorithm run time.

- **Vehicle utilisation [%]**
  Vehicle utilisation is the sum of all delivery quantities divided by the sum of the capacity of vehicles. The total capacity of vehicles is calculated by multiplying the sum of the capacity of the vehicles' compartments with the number of trips performed by the vehicle. The *total delivered quantity*

is a supporting indicator, because it can be used to compare solutions from different planning methods.

- **Average number of stops per trip [#]**
  The ratio between stops and trips is determined by dividing the total number of stops by the number of trips during the full time horizon. The *number of stops* and *number of trips* are seen as supporting indicators, which can be used to compare solutions from different planning methods.

3.1.4. Objective and requirements of the planning methods

The objective of the planning methods considered in this research, is to create the planning for the replenishment of petrol stations with one or multiple underground tanks by compartmentalised vehicles. Each planning method must meet the requirements, which are presented below.

- **A planning must be generated**
  The output of the planning methods should be a replenishment plan, with shows drivers how much and which type of fuel to load in a compartment, and the quantity of a product to deliver at a station. The planning can be seen as the solution found by the planning method.
- **The planning must be generated within seven hours**
  The planning methods must generate a planning within reasonable time. The planning is normally made over night. Starting as late as possible will provide more up-to-date data. On the other hand, there should be some time for manual adjustments before drivers get their schedule. Therefore, seven hours is seen as acceptable for the run time, while lower runtime is preferred.
- **KPIs must be calculated**
  For each solution generated by the planning methods, KPIs must be calculated in exactly the same way. The values for the KPIs show the quality of the found solution, what can be used to evaluate the different methods by comparing the solutions.
- **The solution must be feasible**
  The solution must be feasible according to the applicable inventory, capacity and time constraints. Solutions created by the planning methods must all be feasible for the variant of the PSRP considered in this research. In Chapter 4, where the performance of the planning methods is evaluated with a case study, the created solutions must all be feasible for the considered case study.

3.1.5. Planning methods

From the literature review in Chapter 2 can be concluded that MILP models and heuristic approaches are effective methods to formulate and solve PSRP. Two MILP models are presented in this research. Since the PSRP is NP-hard and a solution can only be found by a solver for limited size instances, a heuristic is proposed to find a solution for real-life size instances. These three models are the main planning methods considered in this research. Variations to these planning methods are proposed as well. An overview of the differences between these planning methods and variations is shown by Table 3.3. Each planning method meets the requirements set in the previous section.

Table 3.3: Overview of planning methods *Implicit objective **Quantitatively rated relative to each other

	Full MILP Model				Simplified MILP model			Heuristic
	Basic	**Var. 1**	**Var. 2**	**Var. 3**	**Basic**	**Var. 1**	**Var. 2**	**Basic**
Objective								
Minimise travel distance	✓			✓	✓			✓*
Min. travel dist. + penalty dry run		✓				✓		
Min. travel distance + inventory			✓				✓	
Routes created								
With binary x_{ij} variables	✓	✓	✓	✓				
Prior to the optimisation					✓	✓	✓	✓
Constrained by								
Inventory level ≥ safety stock	✓		✓	✓	✓		✓	✓
Inventory level ≥ 0		✓				✓		
Inventory level ≤ tank capacity	✓	✓	✓	✓	✓	✓	✓	✓
1 product per compartment/tank	✓	✓	✓	✓	✓	✓	✓	✓
Station restrictions	✓	✓	✓	✓	✓	✓	✓	✓
Compartment capacities vehicles	✓	✓	✓	✓	✓	✓	✓	✓
Trip duration and driver schedule	✓	✓	✓	✓	✓	✓	✓	✓
Begin and start at depot	✓	✓	✓	✓	✓	✓	✓	✓
Deterministic daily demand	✓	✓	✓	✓	✓	✓	✓	✓
Solution method								
Solved to optimality by solver	✓	✓	✓	✓	✓	✓	✓	
Constructed by heuristic								✓
(Dis)advantages								
Computation time**	- -	- -	- -	- - -	-	-	-	+ +
Quality solution**	+ +	+ +	+ +	+ + +	+	+	+	-
Quantity based service time				✓				
Time at station determined	✓	✓	✓	✓				
Stops per trip possible	unl.	unl.	unl.	unl.	limited	limited	limited	limited

Main planning methods

The main planning methods are the full MILP model, the simplified MILP model and the heuristic. The objective of the **basic** version of each planning method is to minimise the total travel distance. This is explicitly implemented in the MILP models by including travel distance in the objective function. Minimising travel distance is implicitly implemented in the heuristic, since the heuristic consists of multiple phases for which it is not possible to set an overall objective. The aim if the heuristic is, however, to minimise total travel distance. Moreover, dry run is not allowed for the basic planning methods, what means that the inventory level is not allowed to drop below the safety stock level. The **basic** version of the main planning methods are now discussed in more detail.

- **Full MILP model** (section 3.2)
  As can be concluded from the literature review in chapter 2, using a MILP model to is a common and effective method to formulate the PSRP. The model presented in this research considered all characteristics as defined in the conceptual model. A solution is found with a commercial solver. In the full MILP model, routes are constructed with a sequence of binary variables x_{ij}, representing a vehicle driving a route from nodes i to nodes j. The advantage of this model is that an unlimited number of stations can be visited during a trip and that the time a station is visited is

considered. The disadvantage is that this makes the model more complex, which makes it harder (slower) for the solver to find a solution.

- **Simplified MILP model** (section 3.3)
  Cordeau, Gendreau, Laporte, Potvin, and Semet (2002) described that solution methods for VRPs are often measured against accuracy and speed, where accuracy is used to describe the quality of the solution and speed is measured in computation time. The authors recognised that there is a trade-off between accuracy and speed. In this planning method, simplifications are made without violating the characteristics as described in the conceptual model. The main simplification is that routes are generated prior to the optimisation, which decreases the computation time for the solver to find a solution. The disadvantage is that the number of stops per trip is limited and that the time a station is visited is not considered.

- **Heuristic** (section 3.4)
  As appears from the analysis in Chapter 4, the solver can only find a solution for the MILP models for small size instances. To find solutions for larger real-life instances, a heuristic is proposed. The objective of the heuristic is to implicitly minimise the total travel distance, since the model consists of several phases with different objective, which means that an explicit overall objective cannot be set. The main advantage of the heuristic is the decrease in time it takes to find a solution, where the main disadvantage is a lower quality solution. The quality of the solution is lower, because the heuristic divides the problem into sub problems, which are not optimised simultaneously.

Planning method variations
Model variations will be proposed to each of the above presented models. These variations are:

1. **Soft constraints on inventory**
   Preventing dry runs and minimising travel distance can be conflicting goals in planning policy. If it would be allowed to visit a station a day later, even if the inventory level of one of the tanks drops below the minimum level, this could lead to lower total travel distance. The trade-off between travel distance and dry runs is investigated by introducing soft constraints on minimum inventory level. This means that the inventory level is allowed to drop below the safety stock level. When this happens, a penalty is applied to the objective function. Varying the penalty costs provides insight in the trade-off between travel distance and dry runs. This model variation is only applied to both MILP models, since the trade-off cannot be determined with the proposed heuristic.

2. **Minimising travel distance and inventory**
   Another trade-off that is investigated, is the trade-off between travel distance and inventory. Visiting a station a day earlier might decrease total travel distance, while it will lead to higher average inventory. Including average inventory levels in the objective together with the travel distance will give useful information about this planning policy trade-off. The inventory levels are multiplied with a certain factor, which is used to set the relative weight of inventory compared to travel distance. This variation is not applied to the heuristic, because of the fact that the heuristics consists of multiple phases and the decisions about routing and inventory are not made simultaneously.

3. **Including service rate**
   The last model variation is calculating the service time at station with a service rate which is multiplied by the delivered quantity. This could lead to a more precise planning, because the service time is more precisely determined. The main planning methods use a fixed service time, based on full truckloads and thus representing a worst-case situation. Considering the service rate might lead to better solutions in some cases. The reason that it is not included in the main models, is that the service rates adds complexity to the models. This complexity increases the computation time necessary to find a solution. This variation is only applied to the full MILP model, since this is the only model that considers the time a vehicle arrives at a station.

3.2. Full MILP method

The basic version of the full MILP model is presented in this section. Firstly, the mathematical formulation is presented with valid inequalities, after which verification experiments are performed and the model is validated. The section concludes with the presentation of the variations of the full MILP model.

3.2.1. Mathematical formulation

The problem can be formulated as follows. Let $G = (V, A)$ be a complete directed graph, where $V = (0, 1, ..., n)$ is a set of nodes and $A = \{(i, j) : i, j \in V\}$ is the set of arcs. Each arc has a travel distance d_{ij} and a travel time t_{ij}. The stations are represented by nodes $N = (1, ..., n)$ and the depot is defined by node 0 or $n + 1$, depending on whether it is the initial or final node in a trip. Each node has a fixed service time St_i. Set $T = (1, ..., n)$ defines the time periods in the planning horizon.

Stations have one or multiple underground tanks which store one type of product p. Set P contains all products, which are stored at the depot. Each underground tank has a maximum capacity Lc_i^p, a safety stock Ls_i^p and an initial inventory level at the start of the planning horizon Li_i^p. Ld_{it}^p gives the demand for each product at each station per day t.

Fuel is transported by a heterogeneous fleet of vehicles. The configuration of each vehicle k is determined by Q^{km}, which denotes the capacity of each compartment m. Station restrictions are included in the model by variable δ_i^k, which equals 1 if station i can be visited by vehicle k. Operating hours of vehicles are represented by a time window $[a_t^k, b_t^k]$. Within the daily schedule, a vehicle can perform multiple trips, which all start and end at the depot. Set R contains the indices for these trips, with $|R|$ chosen large enough to enable the maximum number of trips a vehicle can perform during a daily work schedule. The trip indices are used in increasing order, which means that $s > r$ if a vehicle performs trip s after trip r. L is an arbitrary large constant. An overview of the used parameters and variables is presented below.

Parameters

d_{ij}	driving distance between node i and j
t_{ij}	travel time between node i and j
δ_i^k	1 if station i can be visited by vehicle k, 0 otherwise
Ls_i^p	safety stock level for product p at station i
Lc_i^p	tank capacity for product p at station i
Li_i^p	inventory level at start of first time period
Ld_{it}^p	demand for product p at station i during day t
Q^{km}	capacity of compartment m of vehicle k
a_t^k	start time workday for vehicle k
b_t^k	end time workday b
St	fixed service time in seconds
L	arbitrary large value

Variables

$$x_{ijrt}^k = \begin{cases} 1 & \text{if vehicle } k \text{ drives from } i \text{ to } j \text{ in trip } r \text{ during day } t \\ 0 & \text{otherwise} \end{cases}$$

$$y_{rt}^k = \begin{cases} 1 & \text{if vehicle } k \text{ drives trip } r \text{ during day } t \\ 0 & \text{otherwise} \end{cases}$$

$$z_{irt}^k = \begin{cases} 1 & \text{if vehicle } k \text{ visits station } i \text{ during trip } r \text{ on day } t \\ 0 & \text{otherwise} \end{cases}$$

$$w_{rt}^{kmp} = \begin{cases} 1 & \text{if product } p \text{ is loaded into compartment } m \text{ of vehicle } k \text{ during trip } r \text{ on day } t \\ 0 & \text{otherwise} \end{cases}$$

$$u_{rst}^k = \begin{cases} 1 & \text{if trip } s \text{ is driven after trip } r \text{ by vehicle } k \text{ on day } t \\ 0 & \text{otherwise} \end{cases}$$

q_{irt}^{kmp} = quantity of product p in compartment m of vehicle k delivered to location i during trip r on day t
S_{irt}^{k} = the time vehicle k can start the service at station i during trip r on day t
I_{it}^{p} = inventory level of product p at station i at the end of day t

With the variables and parameters defined above, the problem can be stated as:

$$MIN \sum_{t\in T} \sum_{r\in R} \sum_{k\in K} \sum_{(i,j)\in A} d_{ij}\, x_{ijrt}^{k} \tag{1}$$

Subject to

$$I_{it}^{p} \geq Ls_{i}^{p} \qquad \forall p \in P, \forall i \in N, \forall t \in T \tag{2}$$

$$I_{i,t-1}^{p} + \sum_{r\in R} \sum_{k\in K} \sum_{m\in M} q_{irt}^{kmp} \leq Lc_{i}^{p} \qquad \forall p \in P, \forall i \in N, \forall t \in T \tag{3}$$

$$I_{i0}^{p} = Li_{i}^{p} \qquad \forall p \in P, \forall i \in N \tag{4}$$

$$I_{it}^{p} = I_{i,t-1}^{p} + \sum_{r\in R} \sum_{k\in K} \sum_{m\in M} q_{irt}^{kmp} - Ld_{it}^{p} \qquad \forall p \in P, \forall i \in N, \forall t \in T \tag{5}$$

$$z_{irt}^{k} \leq \delta_{i}^{k} \qquad \forall k \in K, \forall i \in N, \forall r \in R, \forall t \in T \tag{6}$$

$$\sum_{j\in V} x_{0jrt}^{k} = 1 \qquad \forall k \in K, \forall r \in R, \forall t \in T \tag{7}$$

$$\sum_{i\in V} x_{ijrt}^{k} - \sum_{i\in V} x_{jirt}^{k} = 0 \qquad \forall k \in K, \forall j \in V, \forall r \in R, \forall t \in T \tag{8}$$

$$z_{irt}^{k} = \sum_{j\in V} x_{ijrt}^{k} \qquad \forall k \in K, \forall i \in N, \forall r \in R, \forall t \in T \tag{9}$$

$$q_{irt}^{kmp} \leq L\, z_{irt}^{k} \qquad \forall k \in K, \forall p \in P, \forall m \in M, \forall i \in N, \forall r \in R, \forall t \in T \tag{10}$$

$$\sum_{i\in N} q_{irt}^{kmp} \leq w_{rt}^{kmp}\, Q^{km} \qquad \forall k \in K, \forall m \in M, \forall p \in P, \forall r \in R, \forall t \in T \tag{11}$$

$$\sum_{p\in P} w_{rt}^{kmp} \leq 1 \qquad \forall k \in K, \forall m \in M, \forall r \in R, \forall t \in T \tag{12}$$

$$S_{irt}^{k} \geq a_{t}^{k} \qquad \forall k \in K, \forall i \in V, \forall r \in R, \forall t \in T \tag{13}$$

$$S_{irt}^{k} \leq b_{t}^{k} \qquad \forall k \in K, \forall i \in V, \forall r \in R, \forall t \in T \tag{14}$$

$$S^k_{irt} + t_{ij} + St_i - L(1 - x^k_{ijrt}) \leq S^k_{jrt} \qquad \forall k \in K, \forall (i,j) \in A, \forall r \in R, \forall t \in T \qquad (15)$$

$$S^k_{0st} + L(1 - u^k_{rst}) \geq S^k_{n+1,rt} \qquad \forall r,s \in R, r < s, \forall k \in K, \forall t \in T \qquad (16)$$

$$\sum_{r \in R} \sum_{s \in R | s > r} u^k_{rst} \geq \sum_{r \in R} y^k_{rt} - 1 \qquad \forall k \in K, \forall t \in T \qquad (17)$$

$$z^k_{irt} \leq y^k_{rt} \qquad \forall k \in K, \forall i \in N, \forall r \in R, \forall t \in T \qquad (18)$$

$$x^k_{ijrt} \in \{0,1\} \qquad \forall k \in K, \forall i,j \in V, \forall r \in R, \forall t \in T \qquad (19)$$

$$y^k_{rt} \in \{0,1\} \qquad \forall k \in K, \forall r \in R, \forall t \in T \qquad (20)$$

$$z^k_{irt} \in \{0,1\} \qquad \forall k \in K, \forall i \in N, \forall r \in R, \forall t \in T \qquad (21)$$

$$w^{kmp}_{rt} \in \{0,1\} \qquad \forall k \in K, \forall m \in M, \forall p \in P, \forall r \in R, \forall t \in T \qquad (22)$$

$$u^k_{rst} \in \{0,1\} \qquad \forall k \in K, \forall r,s \in R, r < s, s = r+1, \forall t \in T \qquad (23)$$

$$q^{kmp}_{irt} \geq 0 \qquad \forall k \in K, \forall m \in M, \forall p \in P, \forall i \in V, \forall r \in R, \forall t \in T \qquad (24)$$

$$S^k_{irt} \geq 0 \qquad \forall k \in K, \forall i \in V, \forall r \in R, \forall t \in T \qquad (25)$$

$$I^p_{it} \geq 0 \qquad \forall p \in P, \forall i \in N, \forall t \in T \qquad (26)$$

The objective function (1) minimises the total number of kilometres driven by all vehicles during the full time horizon. Constraints (2) and (3) ensure that inventory levels for fuel in underground tanks stay above the safety stock level and below the maximum capacity. Constraints (4) set the initial inventory level for each product at each station and constraints (5) set the daily inventory level, taking into account the demand and deliveries. Station restrictions are imposed by constraints (6), which means that some stations cannot be accessed by all vehicles. Constraints (7) make sure that all vehicles start and end at the depot. Constraints (8) represent the flow conservation constraints. Constraints (9) and (10) are used to link x and q to z, which means that a vehicle can only drive to a station and deliver fuel when it is visited by that vehicle in a certain time period. Constraints (11) ensures that compartment capacities can not be exceeded and constraints (12) makes sure that only one type of product is loaded in a compartment.

Constraints (13-14) forces the arrival time in node i to be within the work schedule of the vehicles. The start and end time of a trip must be within this time window. Constraints (15) take driving and service time into account, to calculate at what time the service can start at a node. A variant to this model is the version with a quantity dependent service time, described in section 3.2.7. Constraints (16-18) make sure that, in case a vehicle performs multiple trips during a time period, these trips are driven consecutively.

Finally, constraints (19-23) are used to define x, y, z, w and u as binary values and constraints (24-26) enforce the integer variables to be non-negative.

3.2.2. Valid inequalities to speed up the model

Valid inequalities are proposed to speed up the process of finding a solution. Constraint (27) ensures that all vehicles return to the depot and constraint (28) strengthens the relationship between the binary variables x and y. Finally, (29) is used to link the constraint q and w.

$$\sum_{i \in V} x^k_{i,n+1,rt} = 1 \qquad \forall k \in K, \forall r \in R, \forall t \in T \tag{27}$$

$$x^k_{ijrt} \leq y^k_{rt} \qquad \forall k \in K, \forall i,j \in V, \forall r \in R, \forall t \in T \tag{28}$$

$$q^{kmp}_{irt} \leq L\, w^{kmp}_{rt} \; \forall k \in K, \forall m \in M, \forall p \in P, \forall i \in N, \forall r \in R, \forall t \in T \tag{29}$$

3.2.3. Verification of the model

To answer the question *"is model is right?"*, experiments are performed with the model to determine if the model behaves as expected. A data set (which can be found in Appendix B), based on the case study data, is used to evaluate the performance. The data set contains one vehicle, four stations with in total ten underground tanks and seven time periods. Five types of product are distributed. The model is implemented in Python and solved using Gurobi Optimizer version 8.1.1 (Gurobi Optimization, LLC, 2019). The verification experiments are executed on a computer with a 3.50 GHz 4-core processor and 32 GB of RAM.

For each verification experiment, the behaviour of the model is predicted. The KPIs are calculated for each solution and used to confirm if the model behaves as it should. The result for the total travel distance can, for example, be expected to be higher or lower in some situation. Finding a solution can be expected to be infeasible as well. Results of the verification experiments are shown in table 3.4.

Table 3.4: Overview of verification experiments and results (distance = total travel distance)

#	Description	Expected behaviour	Result	Ok?
1	Described data set + all constraints	distance >0 avg. stock >0% vehicle util. >0% avg. stops/trip >0 # dry runs = 0	distance = 541 avg. stock = 39.3% vehicle util. = 67,0% avg. stops/trip = 2.33 # dry runs = 0	Pass
2	Run without inventory constraints	distance = 0 # dry runs > 0	distance = 0 # dry runs = 80	Pass
3	Run without vehicle capacity constraints	distance <541 vehicle util. >67,0%	distance = 468 vehicle util. = 100,5%	Pass
4	Run without demand	distance = 0	distance = 0	Pass
5	Run without vehicles	Infeasible	Infeasible	Pass
6	2 vehicles with station restrictions	distance >541	distance = 641	Pass
7	2 stations and small vehicle capacity	∑u >1	∑u >1	Pass
8	Double station tank capacity	distance <541 avg. stock <39,3%	distance = 391 avg. stock = 26,4%	Pass
9	Multiple vehicles with small time windows	distance >541 avg. stops/trip <2.33	distance = 765 avg. stops/trip = 1.2	Pass
10	Set initial inventory levels to safety stock level	distance >541	distance = 810	Pass

The performed experiments are discussed in more detail below.

1. **Described data set + all constraints**
   This instance serves as benchmark for the other experiments. Values for the total travel distance and KPIs are expected to be higher than zero, what is confirmed by running the model.
2. **Run without inventory constraints**

Constraints (2-5) are removed in this experiment to create a situation without inventory constraints. Since this removes the incentive to visit a station, the expected value for the objective function is zero, which is confirmed by the found solution.

3. **Run without vehicle capacity constraints**
In this instance, constraint (11) is removed from the model, which leads to vehicles with unlimited capacity in the compartments. The total travel distance is expected to be lower than in experiment one and the vehicle utilisation is expected to increase, since a vehicle can deliver more fuel during one trip which decreases the total number of trips. The experiment confirms this result.

4. **Run without demand**
When the demand for each time period and tank is set to zero, the total travel distance is expected to be zero as well, because the inventory level does decrease below the minimum level. The result is as expected.

5. **Run without vehicles**
Without vehicles, a solution cannot be found because the model is infeasible. This is the expected model behaviour.

6. **Two vehicles with station restrictions**
This instance has two vehicles which can both only visit two of the four stations. The two vehicles can visit all stations. Since less stations can be combined in one trip, the value for the total travel distance is expected to increase. This is confirmed by the experiment. Another model characteristic that has been checked is to see if stations are only visited by vehicles that are able to visit the station according to the station restrictions.

7. **Two stations and small vehicle capacity**
With this experiment it is checked that, if multiple routes are driven by a vehicle during the same day, these routes are driven consecutively. This means that the service start times for the different routes increase in time, taking service and travel times into account. The test situation is created by lowering the vehicle's capacity, with as result that vehicles need to drive more often during a time period.

8. **Double station tank capacity**
If the capacity of the under ground tanks is doubled, it is expected that stations need to be visited less often, which results in a lower value for the total travel distance. The average stock level is expected to decrease, because the total capacity increases.

9. **Use multiple vehicles with small time windows**
If the time window of multiple vehicles is set to the maximum route duration for routes with one stop, it is expected that the total travel distance increases, because less stations can be combined in one trip. This also results in a lower average of stops per trip. The model behaves as expected.

10. **Set initial inventory levels to safety stock level**
The initial inventory level is set to safety stock level in this experiment, which means that there is no initial stock and that the tanks need to be replenished more often. Therefore, a increased value for the total travel distance is expected and confirmed by the experiment.

3.2.4. Validation of the model

To check *if the model is the right model* if the model is right, the solution found by the model for an instance with four stations, seven time periods and one vehicle is interpreted. Figure 3.3 shows a map of the depot and stations that are visited during the time period. The differently coloured lines show the trips made by vehicles. With the plot, it can be verified that all routes start and end at the depot.

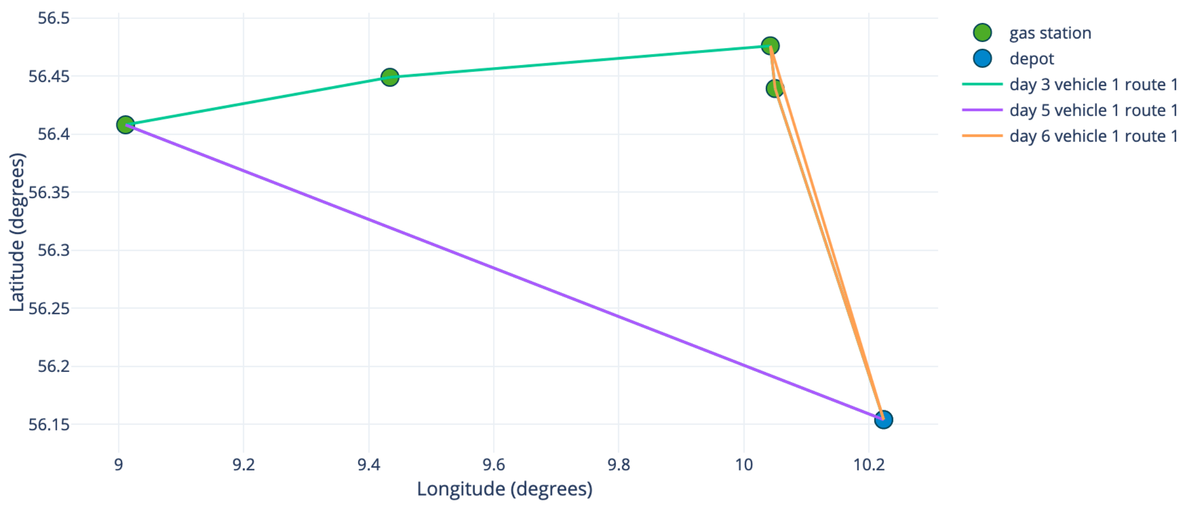

Figure 3.3: Visualisation of a possible solution for four stations and seven time periods

The map in Figure 3.3 shows that some stations are visited multiple times during the time horizon of seven days. To verify if this is necessary, the inventory levels are plotted for a tank of one of these stations in Figure 3.4. The graph shows the inventory level at the end of each day. Since the delivery time during the day is not taken into account, the maximum amount of fuel that can be delivered is the empty capacity of the tank minus the demand for that day, to guarantee that the planned amount can be delivered to the station. In the hypothetical case that a station is replenished before any fuel is sold, the dark blue line shows the maximum inventory level at this moment.

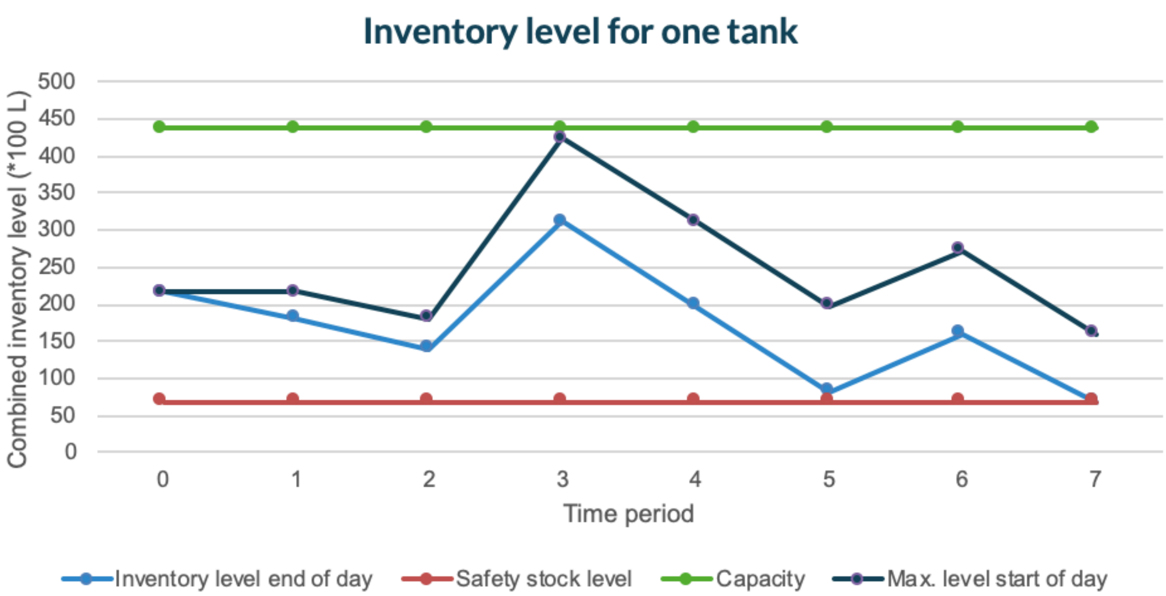

Figure 3.4: Inventory levels for one tank

Figure 3.5 shows the inventory levels for all tanks in the validation experiment combined. The safety stock level and maximum capacity are shown as well. As can be seen, the inventory level starts at the initial inventory level at day zero and declines till the safety stock level at the end of time period seven. This can be explained by the fact that the model strives to minimise the total travel distance, while the total travel distance increases when fuel is delivered. Since the inventory level must be higher than the safety stock level and at the same time fuel is consumed, fuel must be delivered. However, since delivery fuel increases the objective, the fuel delivered will be kept to a minimum.

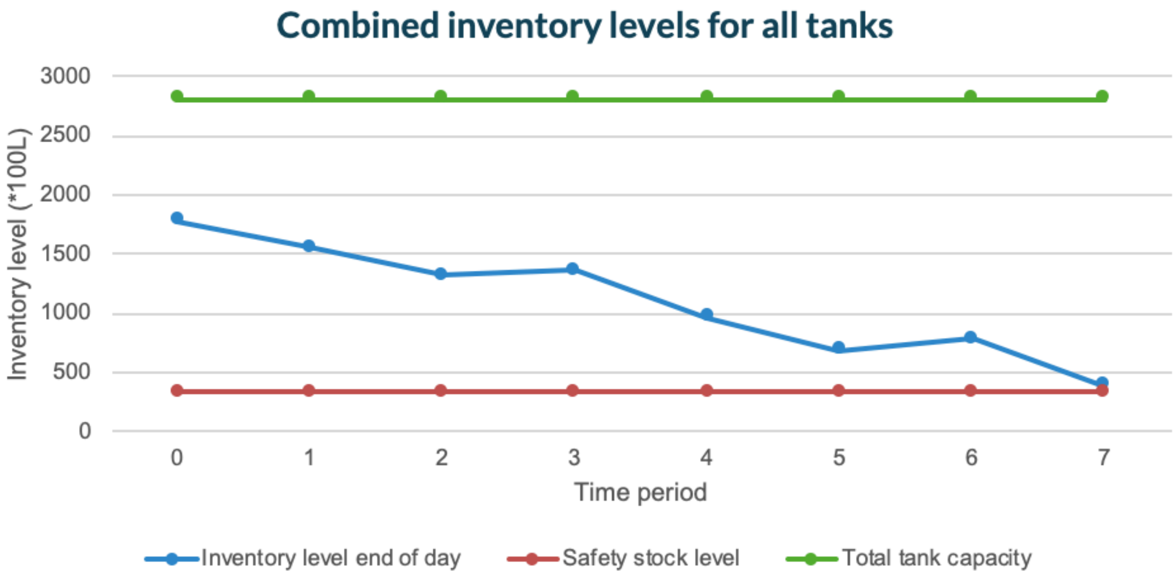

Figure 3.5: Inventory levels of all tanks combined per time period

3.2.5. Model variation 1: soft constraints on inventory

As explained in Section 3.1, a model variation with soft constrains on inventory level is proposed to determine the trade-off between travel distance and dry runs. The variant is implemented by introducing model variable E and penalty costs p, which is a fixed value that can be varied to investigate the trade-off between total travel distance and the occurrence of dry runs. Moreover, the objective function (1) is replaced by equation (30) and constraints (2) are replaced by constraints (31). This will allow the inventory to be below the safety stock level. If this occurs, however, a penalty is imposed in the objective function.

The new variable E is defined as:

$$E_{it}^{p} = \begin{cases} 1 & \text{if the inventory of product } p \text{ at location } i \text{ is lower than the safety stock during day } t \\ 0 & \text{otherwise} \end{cases}$$

$$MIN \sum_{t\in T} \sum_{r\in R} \sum_{k\in K} \sum_{(i,j)\in A} d_{ij}\, x_{ijrt}^{k} + p \sum_{t\in T} \sum_{p\in P} \sum_{i\in N} E_{it}^{p} \tag{30}$$

$$L\, E_{it}^{p} \geq I_{it}^{p} - Ls_{i}^{p} \forall p \in P, \forall i \in N, \forall t \in T \tag{31}$$

3.2.6. Model variation 2: minimising travel distance and inventory

Model variant two is used to determine the trade-off between travel distance and inventory, by minimising inventory levels and travel distance simultaneously. This is done by replacing the objective function (1) by equation (32). A factor f is introduced to weigh the costs of inventory relative to the costs of transportation.

$$MIN \sum_{t\in T} \sum_{r\in R} \sum_{k\in K} \sum_{(i,j)\in A} d_{ij}\, x_{ijrt}^{k} + f \sum_{t\in T} \sum_{p\in P} \sum_{i\in N} I_{it}^{p} \tag{32}$$

3.2.7. Model variation 3: including service rate

If the service time depends on the delivery quantity, a service rate can be used. To implement the service rate in the model, parameter Sr_i is introduced for the service rate and constraints (15) and (16) are replaced by constraints (33 - 35). Constraints (33) adds the service rate Sr_i multiplied with the delivered quantity to the station to the time between two deliveries. Constraints (34) and (35) add time before the start of a route, representing the time it takes to fill up a vehicle at the depot.

$$S_{irt}^{k} + St_{i} + Sr_{i} \sum_{m\in M} \sum_{p\in P} q_{irt}^{kmp} + t_{ij} - L(1 - x_{ijrt}^{k}) \leq S_{jrt}^{k}$$

$$\forall k \in K, \forall (i,j) \in A, \forall r \in R, \forall t \in T \tag{33}$$

$$S_{0rt}^{k} \geq Sr_{0} \sum_{i\in N} \sum_{m\in M} \sum_{p\in P} q_{irt}^{kmp} \qquad \forall r \in R, \forall k \in K, \forall t \in T \tag{34}$$

$$S_{0st}^{k} + L(1 - u_{rst}^{k}) \geq S_{n+1,rt}^{k} + Sr_{0} \sum_{i\in N} \sum_{m\in M} \sum_{p\in P} q_{ist}^{kmp} \qquad \forall r,s \in R, r < s, \forall k \in K, \forall t \in T \tag{35}$$

3.3. Simplified MILP model

The exact model presented in section 3.2 contains many variables which makes the model grow exponentially in terms of complexity with large problem instances. To simplify this model, a new model is proposed with predefined routes. This means that all possible routes are generated prior to the optimisation. Routes with a duration longer than the maximum work shift of a vehicle are not added to the set of possible routes. Time constraints are simplified as well, since the service start time is neglected and there is only checked if the sum of the duration of routes fits within the operating schedule of the driver of a vehicle.

3.3.1. Defining the routes

The predefined routes that are used in the simplified MILP model are generated prior to the optimisation. These routes consist of a set of stations that are visited, a travel time and a travel distance. The travel distance of the route is based on starting at the depot, visiting all stations and returning to the depot. The travel distance is based on visiting stations in the sequence that minimises travel distance. Route duration is based on the travel times between nodes and fixed service time at the depot and stations.

Algorithm 1: Defining the routes

```
Result: A list of routes with a set of stations, route duration and travel distance
input: list of stations with distances, travel times and service times;
create set with max. number of stops per trip = [1, 2, 3 .. n];
foreach max. number of stops per trip do
    foreach possible station sequence do
        calculate route distance and duration;
        if route distance is shortest for set of considered stations then
            set route characteristics for considered set of stations;
        end
    end
end
```

3.3.2. Mathematical formulation

The problem can now be formulated as follows. Let $V = (0, 1, ..., n)$ is a set of nodes and R is the set of possible routes. For each route, the travel distance d_r and sum of travel time and service time t_r are known. The stations are represented by nodes $N = (1, ..., n)$ and the depot is defined by node 0 or $n + 1$, depending on whether it is the initial or final node in a trip. Parameter $x_{ir} \in \{0, 1\}$ is used to set if node i is visited during route r. Each node has a fixed service time St_i, which is added to the travel time t_r or a route prior to the optimisation. Set $T = (1, ..., n)$ defines the time periods in the planning horizon.

The stations have one or multiple underground tanks which store one type of product p. Set P contains all products, which are stored at the depot. Each underground tank has a maximum capacity Lc_i^p, a safety stock Ls_i^p and an initial inventory level at the start of the planning horizon Li_i^p. Ld_{it}^p gives the demand for each product at each station per day t.

Fuel is transported by a heterogeneous fleet of vehicles. Each vehicle k has multiple compartments m with maximum capacity Q^{km}. Let K and M denote the set of vehicles and compartments. Station restrictions are included in the model by variable δ_i^k, which equals one if and only if station i can be visited by vehicle k. Operating hours of vehicles are represented by a time window $[a_t^k, b_t^k]$, where a_t^k is the start time and b_t^k the end time of the schedule for vehicle k during day t. Within the daily schedule, a vehicle can perform multiple trips, which all start and end at the depot. A vehicle can drive a specific route one time per time period. L is an arbitrary large constant. An overview of the parameters and variables is given below.

Parameters

d_r	distance route r
t_r	travel time for route r
x_{ir}	1 if station i is included in route r, otherwise
δ_i^k	1 if station i can be visited by vehicle k, 0 otherwise
Ls_i^p	safety stock level for product p at station i
Lc_i^p	tank capacity for product p at station i
Li_i^p	inventory level at start of first time period
Ld_{it}^p	demand for product p at station i during day t
Q^{km}	capacity of compartment m of vehicle k
a_t^k	start time workday for vehicle k
b_t^k	end time workday b
L	arbitrary large value

Variables

$$y_{rt}^k = \begin{cases} 1 & \text{if vehicle } k \text{ drives trip } r \text{ during day } t \\ 0 & \text{otherwise} \end{cases}$$

$$z_{irt}^k = \begin{cases} 1 & \text{if vehicle } k \text{ visits station } i \text{ during trip } r \text{ on day } t \\ 0 & \text{otherwise} \end{cases}$$

$$w_{rt}^{kmp} = \begin{cases} 1 & \text{if product } p \text{ is loaded into compartment } m \text{ of vehicle } k \text{ during trip } r \text{ on day } t \\ 0 & \text{otherwise} \end{cases}$$

q_{irt}^{kmp} = quantity of product p in compartment m of vehicle k delivered to location i during trip r on day t

I_{it}^p = inventory level of product p at station i at the end of day t

The model with predefined routes can be stated as:

$$MIN \sum_{t \in T} \sum_{r \in R} \sum_{k \in K} d_r \, y_{rt}^k \tag{1}$$

Subject to

$$I_{it}^p \geq Ls_i^p \qquad \forall p \in P, \forall i \in N, \forall t \in T \tag{2}$$

$$I_{i,t-1}^p + \sum_{r \in R} \sum_{k \in K} \sum_{m \in M} q_{irt}^{kmp} \leq Lc_i^p \qquad \forall p \in P, \forall i \in N, \forall t \in T \tag{3}$$

$$I_{i0}^p = Li_i^p \qquad \forall p \in P, \forall i \in N \tag{4}$$

$$I_{it}^p = I_{i,t-1}^p + \sum_{r \in R} \sum_{k \in K} \sum_{m \in M} q_{irt}^{kmp} - Ld_{it}^p \forall p \in P, \forall i \in N, \forall t \in T \tag{5}$$

$$z^k_{irt} \leq \delta^k_i \qquad \forall k \in K, \forall i \in N, \forall r \in R, \forall t \in T \qquad (6)$$

$$z^k_{irt} = y^k_{rt}\, x_{ir} \qquad \forall k \in K, \forall i \in N, \forall r \in R, \forall t \in T \qquad (7)$$

$$q^{kmp}_{irt} \leq L\, z^k_{irt} \qquad \forall k \in K, \forall p \in P, \forall m \in M, \forall i \in N, \forall r \in R, \forall t \in T \qquad (8)$$

$$\sum_{i \in N} q^{kmp}_{irt} \leq w^{kmp}_{rt}\, Q^{km} \qquad \forall k \in K, \forall m \in M, \forall p \in P, \forall r \in R, \forall t \in T \qquad (9)$$

$$\sum_{p \in P} w^{kmp}_{rt} \leq 1 \qquad \forall k \in K, \forall m \in M, \forall r \in R, \forall t \in T \qquad (10)$$

$$\sum_{r \in R} y^k_{rt}\, t_r \leq b^k_t - a^k_t \qquad \forall k \in K, \forall t \in T \qquad (11)$$

$$y^k_{rt} \in \{0, 1\} \qquad \forall k \in K, \forall r \in R, \forall t \in T \qquad (12)$$

$$z^k_{irt} \in \{0, 1\} \qquad \forall k \in K, \forall i \in N, \forall r \in R, \forall t \in T \qquad (13)$$

$$w^{kmp}_{rt} \in \{0, 1\} \qquad \forall k \in K, \forall m \in M, \forall p \in P, \forall r \in R, \forall t \in T \qquad (14)$$

$$q^{kmp}_{irt} \geq 0 \qquad \forall k \in K, \forall m \in M, \forall p \in P, \forall i \in V, \forall r \in R, \forall t \in T \qquad (15)$$

$$I^p_{it} \geq 0 \qquad \forall p \in P, \forall i \in N, \forall t \in T \qquad (16)$$

The objective function (1) minimises the total number of kilometres driven by all vehicles during the full time horizon. Constraints (2) and (3) ensure that inventory levels for fuel in underground tanks stay above the safety stock level and below the maximum capacity. Constraints (4) set the initial inventory level for each product at each station and constraints (5) set the daily inventory level, taking into account the demand and deliveries. Station restrictions are imposed by constraints (6), which means that some stations cannot be accessed by all vehicles. Constraints (7) and (8) are used to link x and q to z, which means that a vehicle can only drive to a station and deliver fuel when it is visited by that vehicle during route r in a certain time period. Constraints (9) ensure that compartment capacities can not be exceeded and constraints (10) make sure that only one type of product is loaded in a compartment. Constraints 11 limit the duration of all routes driven by one vehicle during one time period to the available time in the work schedule of the driver of the vehicle. Lastly, constraints (12-14) are used to define y, z and w as binary values and constraints (15-16) enforce the variables to be non-negative.

3.3.3. Verification of the simplified model

The experiments and criteria for an experiment to pass are the same as the verification experiments described in Section 3.2.3, except for the fact that the same objective value is expected for experiment 1 as found in Section 3.2.3. The solver finds an exact solution for the verification data set, so the expected objective function is the same for the full MILP model as for the simplified MILP model.

Table 3.5: Verification results for the simplified MILP model (distance = total travel distance)

#	Description	Expected behaviour	Result	Ok?
1	Described data set + all constraints	distance = 541 avg. stock >0% vehicle util. >0% avg. stops/trip >0 # dry runs = 0	distance = 541 avg. stock = 43,1% vehicle util. = 67,0% avg. stops/trip = 2.33 # dry runs = 0	Pass
2	Run without inventory constraints	distance = 0 # dry runs >0	distance = 0 # dry runs = 80	Pass
3	Run without vehicle capacity constraints	distance <541 vehicle util. >67,0%	distance = 468 vehicle util. = 100,5%	Pass
4	Run without demand	distance = 0	distance = 0	Pass
5	Run without vehicles	Infeasible	Infeasible	Pass
6	2 vehicles with station restrictons	distance >541	distance = 641	Pass
7	2 stations and small vehicle capacity	multiple route use	multiple route use	Pass
8	Double station tank capacity	distance <541 avg. stock <39,%3	distance = 391 avg. stock = 25,0%	Pass
9	Multiple vehicles with small time windows	distance >541 avg. stops/trip <2.33	distance = 772 avg. stops/trip = 1.5	Pass
10	Set initial inventory levels to safety stock level	distance >541	distance = 864	Pass

3.3.4. Validation of the simplified model

Figure 3.6 shows the trips that are driven for the verification result of the simplified model. The same routes as in the verification results of the full MILP model are driven (as shown in Section 3.2.4). Moreover, the total travel distance of 541 km is the same for the verification result of both models. This shows that the simplified model can generate solutions that are comparable to the results found with the full MILP model and that the model is working correctly.

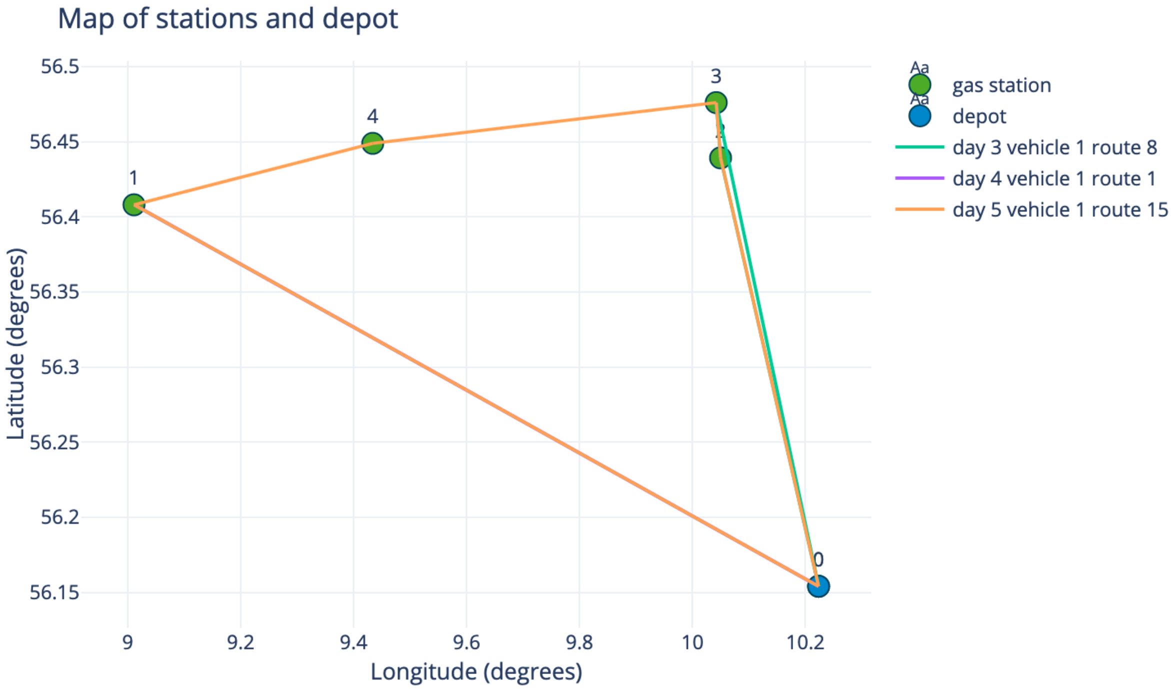

Figure 3.6: Visualisation of trips that are driven for the result of the simplified MILP model

3.3.5. Model variation 1: soft constraints on inventory

As explained in Section 3.1, a model variation with soft constrains on inventory level is proposed to determine the trade-off between travel distance and dry runs. The variant is implemented by introducing model variable E and penalty costs p, which is a fixed value that can be varied to investigate the trade-off between total travel distance and the occurrence of dry runs. Moreover, the objective function (1) is replaced by equation (30) and constraints (2) are replaced by constraints (31). This will allow the inventory to be below the safety stock level. If this occurs, however, a penalty is imposed in the objective function.

The new variable E is defined as:

$$E_{it}^{p} = \begin{cases} 1 & \text{if the inventory of product } p \text{ at location } i \text{ is lower than the safety stock during day } t \\ 0 & \text{otherwise} \end{cases}$$

$$MIN \sum_{t \in T} \sum_{r \in R} \sum_{k \in K} d_r \, y_{rt}^{k} + p \sum_{t \in T} \sum_{p \in P} \sum_{i \in N} E_{it}^{p} \tag{17}$$

$$L \, E_{it}^{p} \geq I_{it}^{p} - Ls_{i}^{p} \qquad \forall p \in P, \forall i \in N, \forall t \in T \tag{18}$$

3.3.6. Model variation 2: minimising travel distance and inventory

Model variant two is used to determine the trade-off between travel distance and inventory, by minimising inventory levels and travel distance simultaneously. This is done by replacing the objective function (1) by equation (32). A factor f is introduced to weigh the costs of inventory relative to the costs of transportation.

$$MIN \sum_{t \in T} \sum_{r \in R} \sum_{k \in K} d_r \, y_{rt}^{k} + f \sum_{t \in T} \sum_{p \in P} \sum_{i \in N} I_{it}^{p} \tag{19}$$

3.4. Heuristic

The previously discussed planning methods are solved using a solver, which is only able to find a solution within reasonable time for small problem instances (as appears from the case study in Chapter 4). Therefore, a decomposition heuristic is presented in this chapter, used to find solutions for real-life size instances. A decomposition approach is used to divide the large and complex problem into smaller sub problems, based on the decomposition of the decision process of the supplier. Decomposition approaches have proven to be an effective method to solve rich VRPs (as appears from Section 2.4 in the literature review). This approach, however, leads to lower quality solutions, since the decisions are not optimised simultaneously. Therefore, a local improvement procedure is used to improve the initial solution found by the decomposition heuristic. Candidate solutions are based on visiting stations earlier. The phases of the decomposition heuristic are shown in Figure 3.7 and discussed below. The heuristic procedure used to create an initial solution is described in in Section 3.4.6. Lastly, the improvement procedure is can be found in Section 3.4.7.

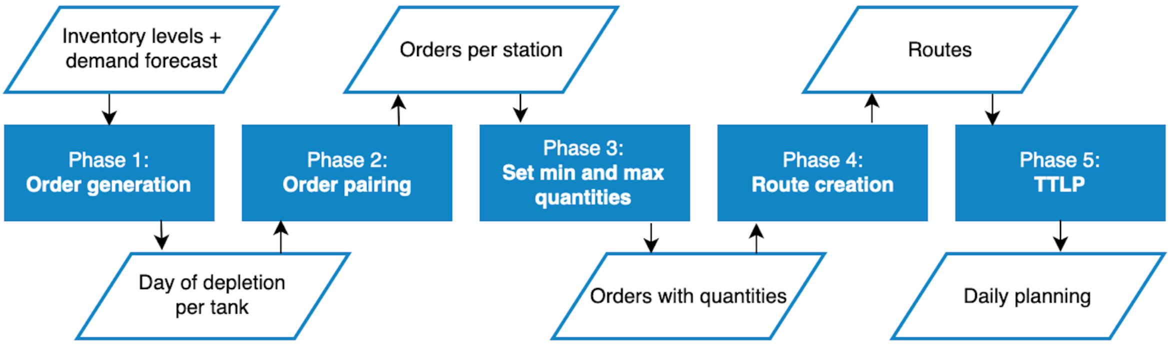

Figure 3.7: Phases of decomposition heuristic with input and output data

- **Phase 1: Order generation** (Section 3.4.1)
  In the first phase, an order is generated per underground tank. An order consists of the station index i, the product index p and the day the tank depletes t. A station is assumed to have maximum one tank per product, so the combination of the station index and the product index define an underground tank.

- **Phase 2: Order pairing** (Section 3.4.2)
  The following phases considered the ”current” day and combines, per station, all orders for this station during the future time periods. The output is a list of stations, with per station a set of tanks that are expected to deplete during future time periods in the considered time horizon.

- **Phase 3: Set min and max quantities based on the simultaneous dry run inventory policy** (Section 3.4.3)
  Minimum and maximum delivery quantities are determined in the third phase, according to the firstly in this research reported *simultaneous dry run* inventory policy. The maximum delivery quantity is set to the remaining tank's capacity, where the minimum quantity is set to a level for each tank at the considered station, that the next delivery to this station can be postponed for as long as possible.

- **Phase 4: Route creation** (Section 3.4.4)
  The list of stations that need to be visited during the considered time period, as determined in phase 2, are combined into routes in the fourth phase. A route generation model is used to create the routes with the lowest total travel distance for the considered day.

- **Phase 5: Tank Truck Loading Problem (TTLP)** (Section 3.4.5)
  In the final phase, quantities and routes are assigned to vehicles, while taking capacities, station

restrictions and driver schedules into account. The delivery quantity is bounded by the earlier determined minimum and maximum quantity.

3.4.1. Phase 1: Order generation

In the first phase, orders are generated for the remaining time periods in the considered time horizon. An order consists of an underground tank, represented by a station and product index, and the day the tank depletes. The pseudo code for the algorithm is presented below.

Algorithm 2: Order generation

```
Result: List with orders for tank on depletion day
initialisation: load inventory levels and demand forecast;
set t_c = current considered time period, T = last day time horizon;
foreach underground tank at station i with product p do
    for t in [t_c, T] do
        inventory level = inventory level end previous day - demand during t;
        if inventory level < safety stock then
            add tank to order list with depletion day t;
        end
    end
end
```

3.4.2. Phase 2: Order pairing

The second phase considers the "current" day and combines, per station, all generated orders from the previous phase. The algorithm is designed as follows:

Algorithm 3: Order pairing

```
Result: Per station, a list of orders for the full time horizon
set t_c = current considered time period, T = last day time horizon;
step 1. filter order list on orders for current day;
foreach order generated in phase 1 do
    if order depletion day = t_c then
        add order to order list for today;
    end
end
step 2. combine orders for the same station;
foreach generated order with depletion day t in [t_c + 1, T] do
    if station is in order list for today then
        add order to current day order list;
    end
end
```

3.4.3. Phase 3: Determine min and max delivery quantities

In the third phase, minimum and maximum quantities are determined to be delivered at the station. To set the quantities, the *simultaneous dry run* inventory policy is applied. The concept, that is also used by the company in the case study, considers all underground tanks at a station and sets the minimum delivery quantities to a value that postpones the next delivery as much as possible. For determining the maximum order quantity, the maximum level policy is applied, which means that the maximum delivery is equal to the remaining capacity in the tank. The working of the concept is shown in the figure below.

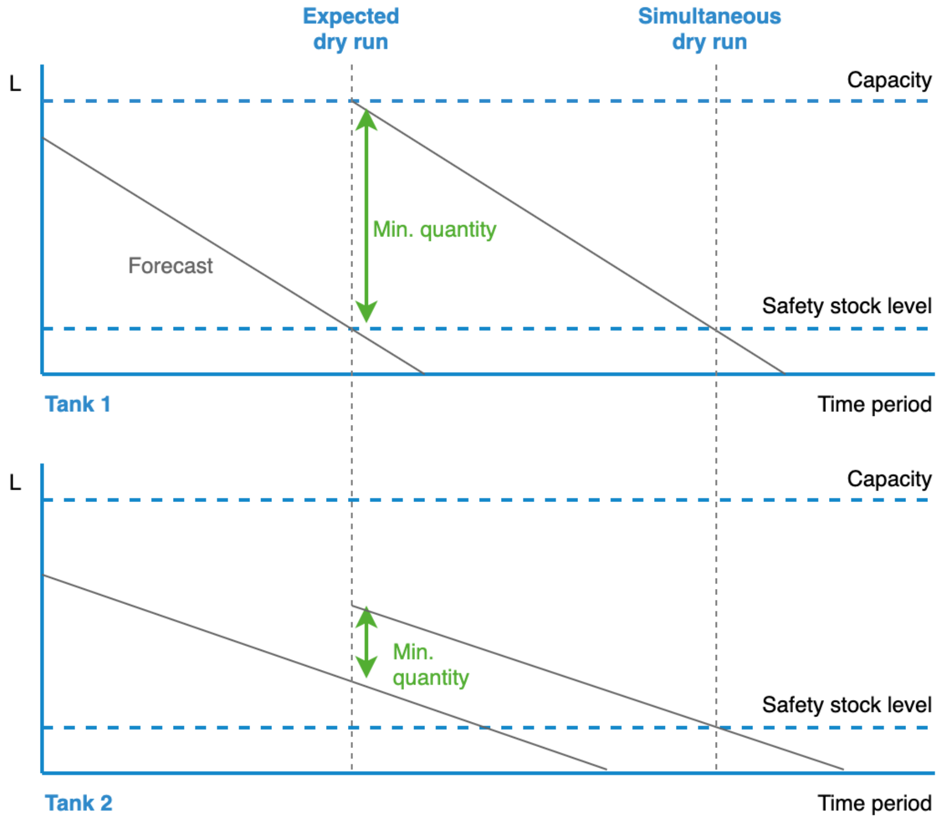

Figure 3.8: Visualisation of the simultaneous dry run inventory policy

Figure 3.8 shows the inventory levels for two tanks at the same station. The capacity and safety stock level are indicated by dashed lines and a simplified forecast is shown by the grey line. As can be seen, tank one is expected to run dry first. In this research, dry run is the moment when the inventory level drops below the safety stock level. The station needs to be replenished during the day that tank one runs dry at the latest. To determine the minimum delivery amounts for this day, the tank with the highest expected demand is filled to the maximum capacity. For the other tanks at the same station, the minimum delivery quantity is set in a way that the next delivery moment, at the next simultaneous dry run, is postponed as much as possible. The figure shows how setting the minimum delivery quantities, in green, lead to a simultaneous dry run in the future.

The pseudo code for determining the minimum and maximum delivery quantities is presented by Algorithm 4.

Algorithm 4: Determine minimum and maximum delivery quantities based on the simultaneous dry run inventory policy

```
Result: Minimum and maximum quantities for each order
set t_c = current considered time period, T = last day time horizon;
foreach station in current day order list do
    step 1. set maximum quantities;
    foreach order for the considered station do
        set maximum delivery quantity to the remaining capacity in the tank;
    end
    step 2. determine second depletion day;
    set current second depletion day = 100;
    foreach order for the considered station do
        foreach t in [t_c, T] do
            inventory level = inventory level end previous day - demand during t;
            if inventory level < safety stock then
                if t < current second depletion day then
                    current second depletion day = t;
                end
            end
        end
    end
    step 3. calculate consumption till second depletion day;
    foreach order for the considered station do
        foreach t in [t_c, second depletion day] do
            consumption for tank = current value + demand for day t
        end
    end
    step 4. per day, add the calculated consumption to the minimum delivery quantity;
    until the vehicle capacity is reached;
    set vehicle full = false;
    while vehicle full is false do
        foreach t in [t_c, second depletion day] do
            foreach order for the considered station do
                if minimum quantity for tank is lower than consumption till second depletion day
                 then
                    if combined minimum quantities are higher than vehicle capacity then
                        set vehicle full = true;
                    else
                        add consumption to minimum quantity for tank in order
                    end
                end
            end
        end
    end
end
```

As can be seen in Algorithm 4, minimum quantities are not set directly, since this may lead to infeasible solutions in case there is no vehicle with enough capacity to transport the minimum capacities to the station. Therefore, step 4 adds the expected demand per day to the value for the minimum delivery quantity of the order, until the capacity of the vehicle is reached (with station restrictions taken into account). This ensures that the next visit to the station is postponed for as much as possible, even when it is not feasible to deliver quantities that lead to simultaneous dry run.

3.4.4. Phase 4: Route creation

Now the minimum and maximum delivery quantities are determined for the stations that need to be visited during the considered time span, routes can be created for these stations. A route generation model is presented in this section, with the objective to minimise the total travel distance for the considered day.

All possible routes are generated prior to the optimisation, in the same way as described in Section 3.3.1. Only routes that can be driven by one of the vehicles are considered, taking station restrictions into account. For these routes in set R, with index r, the travel distance d_r are determined. If station i is visited during route r, parameter x_{ir}, which is zero by default, is set to 1. $C_{r,max}$ is the total capacity of the vehicle with the largest transport capacity that can drive route r. This capacity will be used to ensure that the created routes are feasible, because some stations have station restrictions and can therefore not be visited by each vehicle. $q^p_{ir,min}$ follows from the previous phase and is set to the minimum delivery quantities of product p to station i during route r. Moreover, the model uses the following variables:

$$y_r = \begin{cases} 1 & \text{if vehicle } r \text{ is selected} \\ 0 & \text{otherwise} \end{cases}$$

$$z_{ir} = \begin{cases} 1 & \text{if route } r \text{ is selected to visit station } i \\ 0 & \text{otherwise} \end{cases}$$

The route creation model can now be stated as:

$$MIN \sum_{r \in R} y_r \, d_r \tag{1}$$

$$z_{ir} \leq x_{ir} \qquad \forall i \in N, \forall r \in R \tag{2}$$

$$z_{ir} \leq y_r \qquad \forall i \in N, \forall r \in R \tag{3}$$

$$\sum_{r \in R} z_{ir} = 1 \qquad \forall i \in N \tag{4}$$

$$\sum_{i \in N} (q^p_{ir,min} z_{ir}) \leq 0.9 \, C_{k,min} \qquad \forall r \in R \tag{5}$$

The objective function (1) minimises the total travel distance for the considered time period. Constraints (2) make the choice to visit station i during route r only possible when the station is included in the predetermined route. Constraint (3) links variables y and z. Constraints (4) are used to ensure that each station is visited once. Combining stops in a route is limited by the sum of the minimum delivery quantities to 90% of the total capacity of the largest vehicle by constraints (5). The minimum delivery quantities $q^p_{ir,min}$ must be delivered, so the constraint ensures that the route can be driven with at least one vehicle. The capacity of the vehicle is decreased by 10%, because different types of product need to be transported by different compartments.

3.4.5. Phase 5: Tank Truck Loading Problem

Now the routes that have to be driven during the day are determined, together with minimum and maximum delivery quantities per tank at each visited station, the next step is to assign these routes and quantities to compartments of vehicles. A variant of the TTLP, which is also addressed by Cornillier et al. (2007), is developed to execute this assignment procedure. The TTLP maximises the actual delivery quantities, which leads to efficient vehicle utilisation. Both variants have in common that delivery quantities are maximised, within the minimum and maximum delivery quantities. The output of the model is the daily planning for the distribution company. The variant presented in this research also comprises a heterogeneous fleet of vehicles, station restrictions, multi commodity and driver schedule duration.

$q^p_{ir,min}$ and $q^p_{ir,max}$ follow from the previous phases and are the minimum and maximum delivery quantities of product p to station i during route r. δ^k_i is set to one if station i can be visited by vehicle k and 0 otherwise. Q^{km} is the capacity of compartment m of vehicle k. Time window $[a^k, b^k]$ represents the driver schedule of vehicle k and t_r represents the route duration. L is an arbitrary large value. Furthermore, the model uses the following variables:

$$y^k_r = \begin{cases} 1 & \text{if vehicle } k \text{ drives trip } r \\ 0 & \text{otherwise} \end{cases}$$

$$w^{kmp}_r = \begin{cases} 1 & \text{if product } p \text{ is loaded into compartment } m \text{ of vehicle } k \text{ during trip } r \\ 0 & \text{otherwise} \end{cases}$$

q^{kmp}_{ir} = quantity of product p in compartment m of vehicle k delivered to location i during trip r

The tank truck loading model can be stated as:

$$MAX \sum_{r\in R} \sum_{i\in N} \sum_{k\in K} \sum_{m\in M} \sum_{p\in P} q^{kmp}_{ir} \tag{1}$$

Subject to

$$\sum_{k\in K} \sum_{m\in M} q^{kmp}_{ir} \geq q^p_{ir,min} \qquad \forall p \in P, \forall i \in N, \forall r \in R \tag{2}$$

$$\sum_{k\in K} \sum_{m\in M} q^{kmp}_{ir} \leq q^p_{ir,max} \qquad \forall p \in P, \forall i \in N, \forall r \in R \tag{3}$$

$$\sum_{p\in P} \sum_{m\in M} q^{kmp}_{ir} \leq L\, \delta^k_i \qquad \forall i \in N, \forall k \in K \forall r \in R \tag{4}$$

$$\sum_{i\in N} q^{kmp}_{ir} \leq w^{kmp}_r\, Q^{km} \qquad \forall k \in K, \forall m \in M, \forall p \in P, \forall r \in R \tag{5}$$

$$\sum_{p\in P} w^{kmp}_r \leq 1 \qquad \forall k \in K, \forall m \in M, \forall r \in R \tag{6}$$

$$\sum_{k\in K} y^k_r = 1 \qquad \forall r \in R \tag{7}$$

$$\sum_{r\in R} (y^k_r\, t_r) \leq b^k - a^k \qquad \forall k \in K \tag{8}$$

$$w^{kmp}_r \leq y^k_r \qquad \forall k \in K, \forall m \in M, \forall p \in P, \forall r \in R \tag{9}$$

$$q^{kmp}_{ir} \geq 0 \qquad \forall k \in K, \forall m \in M, \forall p \in P, \forall r \in R \tag{10}$$

The objective function (1) maximises the total delivered quantity, to ensure efficient vehicle utilisation. Constraints (2-3) set the delivery quantity to be within the range previously determined by the minimum and maximum delivery quantities. Constraints (4) implement station restrictions and constraints (5) limit the delivery quantity per compartment to its capacity. The fact that a compartment can only be filled with one type of product is implemented with constraints (6). Constraints (7) ensure that each route is driven by a vehicle exactly one time and constraints (8) are used to limit the duration of all routes driven by a vehicle to the time available in the driver schedule. Constraints (9) link variables w and y. Lastly, constraints (10) are the non-negativity constraints.

3.4.6. Heuristic procedure

Since the planning of a day affects the inventory levels of the next day, and thus the planning for the next day, an dynamic programming approach is required to create the planning. The planning must be created per day, with the exact delivery quantities from the previous day known. Figure 3.9 gives an overview of the heuristic procedure.

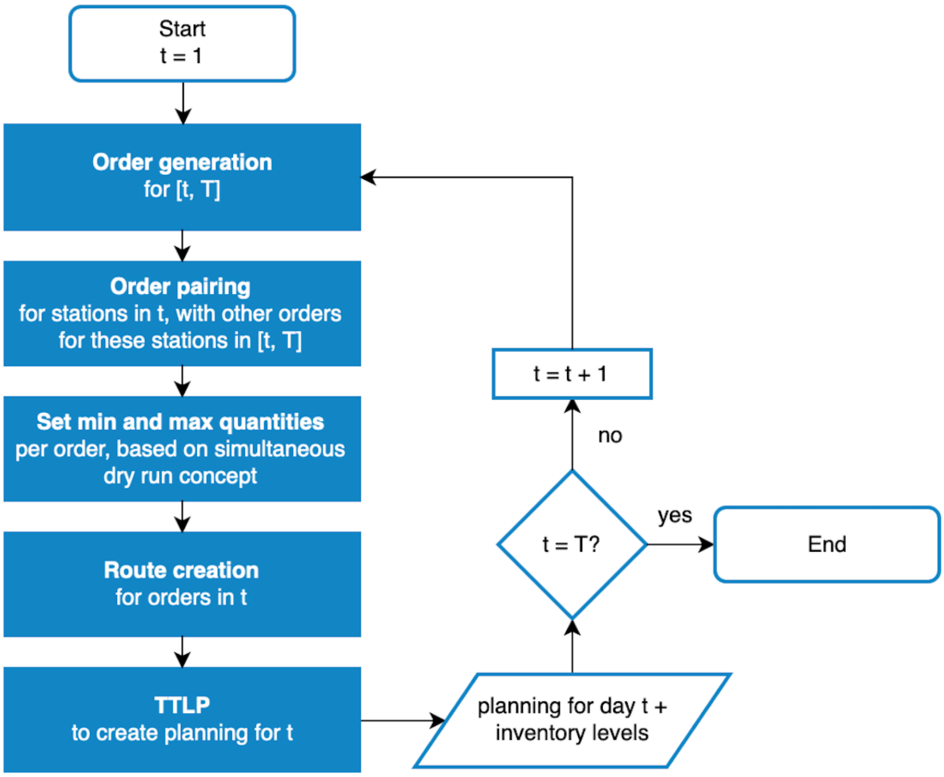

Figure 3.9: Heuristic procedure, with T = days in time horizon

To create the planning for the full time horizon, the planning for each time period are made separately, based on all information available. To create the planning for one time period, orders are generated on depletion day for each tank that depletes during the time horizon. In the second step, order pairing, the orders stations which have to be replenished during the considered day are combined with orders for other tanks at the same station from other time periods. Then, minimum and maximum quantities are determined based on the *simultaneous dry run* inventory policy. The fourth step is to combine stations into routes which need to be driven during the considered day. In the last step, the actual delivery quantities are determined by assigning the orders to vehicles.

Now the planning for the considered day is created, inventory levels are adjusted and the process is repeated for the next day untill the last day of the planning horizon is reached.

3.4.7. Local improvement procedure

For the initial solution, the decision to visit a station is based on the day one of the tanks deplete. This means that the routing is not optimised, since the planning is not based on minimising routing distances over days. It could, for example, happen that during three days only one station is visited. It would, from a routing perspective, be more efficient to visit these stations in the same trip. To improve the initial solution, a local improvement procedure is proposed, which is shown by Figure 3.10.

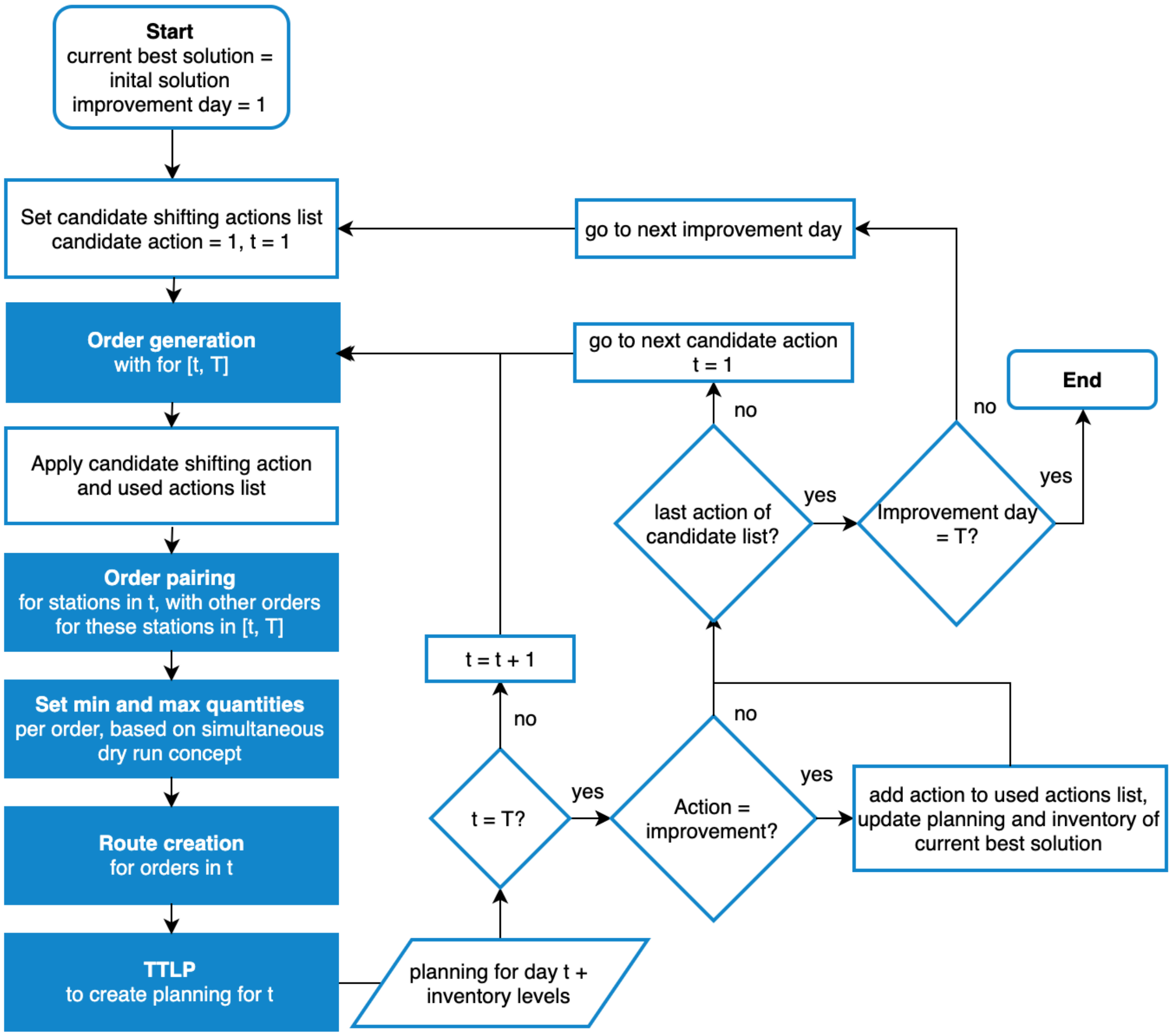

Figure 3.10: Overview of local improvement process

Stations can only be visited earlier than in the initial solution, since at the day of delivery in the initial solution, one of the tank depletes. Candidate solutions are, therefore, based on visiting a station a day earlier. The local improvement procedure evaluates if these candidate solutions lead to a decrease in total travel distance, compared to the current best solution. To evaluate a candidate solution, the planning needs to be recreated for the remaining time periods in the time horizon. Since this is an extensive process, the number of shifting actions is limited to perform each visit in the planning one day earlier.

The improvement procedure improves the planning day by day. For the currently considered day, called the improvement day, shifting actions are determined. These shifting actions are based on visiting a station during the considered day, that is originally scheduled to be visited during the next time period. These shifting actions are sorted on the number stops per trip in the original planning and the expected effect on the total mileage. The algorithm then evaluates the effect of each shifting action, by recreating the full planning for the remaining time periods [improvement day, last day time horizon]. If the action leads to an improved solution, the current best solution is updated with this action and the corresponding inventory level changes.

The design of the algorithms ensures that only feasible solutions are created, which is important because evaluating a solution is an extensive process. Therefore, it is not possible to evaluate a huge number of small changes, which happens in many other heuristics.

3.4.8. Verification of the heuristic

The heuristic is implemented in Python and experiments are executed on a computer with a 3.50 GHz 4-core processor and 32 GB of RAM. The experiments are discussed below.

Table 3.6: Results of heuristic verification experiments

#	Description	Expected behaviour	Result	Ok?
1	Described data set + normal configuration	distance >541 average stock >0% vehicle util. >0% # dryruns = 0	distance = 549 average stock = 46.3% vehicle util. = 98.6% # dryruns.= 0	Pass
2	Run without order generation	distance = 0 # dryruns >0	distance = 0 # dryruns = 23	Pass
3	Run without demand	distance = 0	distance = 0	Pass
4	Run without vehicles	Infeasible	Infeasible	Pass
5	2 vehicles with station restrictons	distance >541	distance = 641	Pass
6	Double station tank capacity	distance <541 average stock <39.3%	distance = 487 average stock = 24.6%	Pass
7	Multiple vehicles with small time windows	distance >541 stops/trip <2.33	distance = 685 stops/trip = 1.2	Pass
8	Set initial inventory levels to safety stock level	distance >541	distance = 864	Pass

1. **Described data set + normal configuration**
   With the normal configuration, the objective value is expected to be slightly higher than the value found by the MILP models. No dry runs are allowed.
2. **Run without order generation**
   If there are none orders generated, vehicles will not drive so the travel distance is expected to be zero and the number of dry runs to be higher than 0.
3. **Run without demand**
   No demand removes the incentive to drive with vehicles, so the travel distance is expected to be zero.
4. **Run without vehicles**
   No vehicles will lead to an infeasible solution.
5. **2 vehicles with station restrictions**
   This experiment is used to test the route selection procedure for the heuristic. Since the heuristic decomposes the problem into multiple phases, the route selection algorithm should only select routes that are feasible (routes that can be driven by at least one vehicle), otherwise the model is infeasible in a following phase. Moreover, the total travel distance is expected to increase, since some stations cannot be combined in a route any more.
6. **Double station tank capacity**
   Double tank capacity should lead to an decreased total travel distance, since more fuel can be

stored.

7. **Multiple vehicles with small time windows**
   If the time windows are small, less stations can be combined in a route, so the total travel distance is expected to increase.

8. **Set initial inventory level to safety stock level**
   If the initial level is set to the safety stock level, more fuel needs to be transported during the considered time horizon. Therefore, the total travel distance is expected to increase.

3.4.9. Verification of simultaneous dry run

Figure 3.11 shows the inventory and safety stock levels for the tanks at the same station. During the time horizon of 14 days, the station is replenished twice. As can be seen, tank two has the highest demand. At the moment of the first delivery, tank one and two are resupplied with a quantity that postpones the next delivery as much as possible. Tank 3 is not resupplied, which is logical because the inventory level is higher than the safety stock level during the full time horizon. These results confirm that the *simultaneous dry run* inventory policy leads to the desired effect.

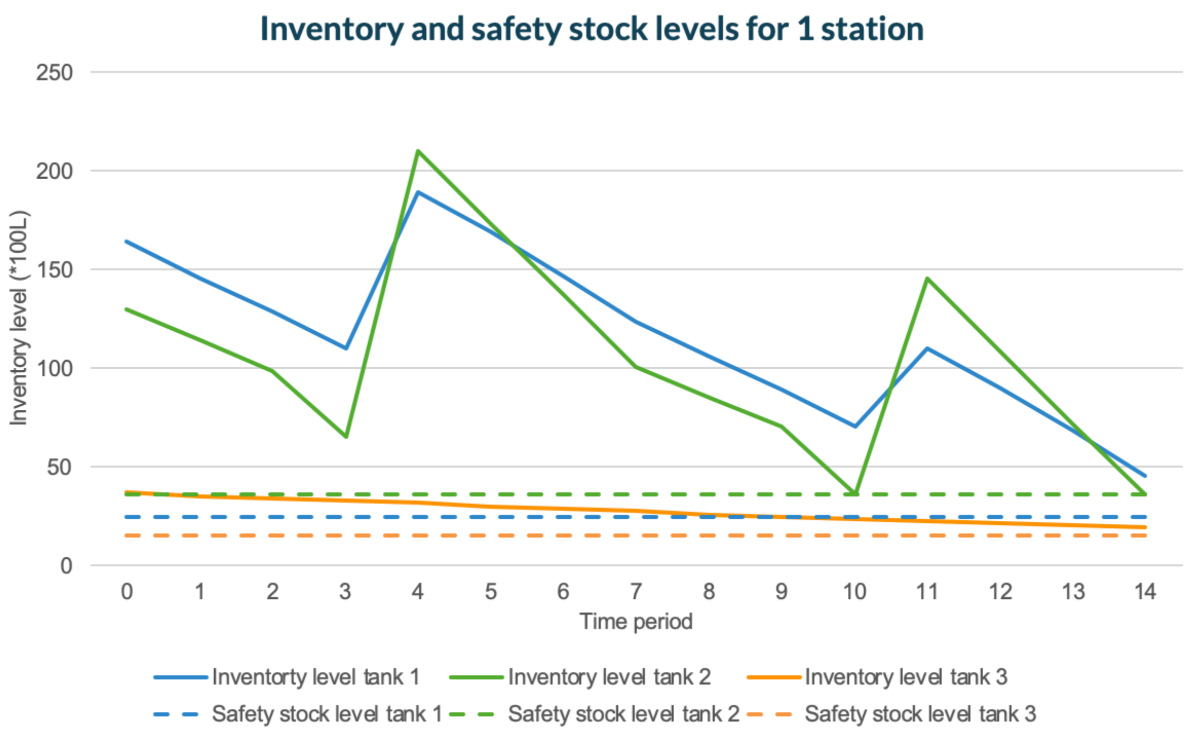

Figure 3.11: Inventory levels for the tank at one station

3.4.10. Heuristic validation by result interpretation

Table 3.7 shows the planning for the verification data set for both MILP models and the heuristic. The numbers between brackets are the stops in a route. As can be seen in the table, contains the initial solution found by the heuristic more trips with a lower number of stops, compared to the solution found by the MILP models. Therefore, the solution is improved by shifting visiting stations between days. This has the desired effect, since an improved solution can be found. The improved solution has the same number of trips as the solution found by the MILP models. The initial solution and improved solutions found by the heuristic are shown in Figures 3.12 and 3.13.

Table 3.7: Comparison of planning for data set S4-D7

Day	Solution MILP	Sol. simpl. MILP	Initial solution	Improved solution
1				
2				
3	(1,2,3,4)	(2,3)	(2)	(1,2)
4		(1)	(1)	
5	(1)	(1,2,3,4)	(3)	(3)
6	(2,3)		(2,4)	(1,2,4)
7			(1)	

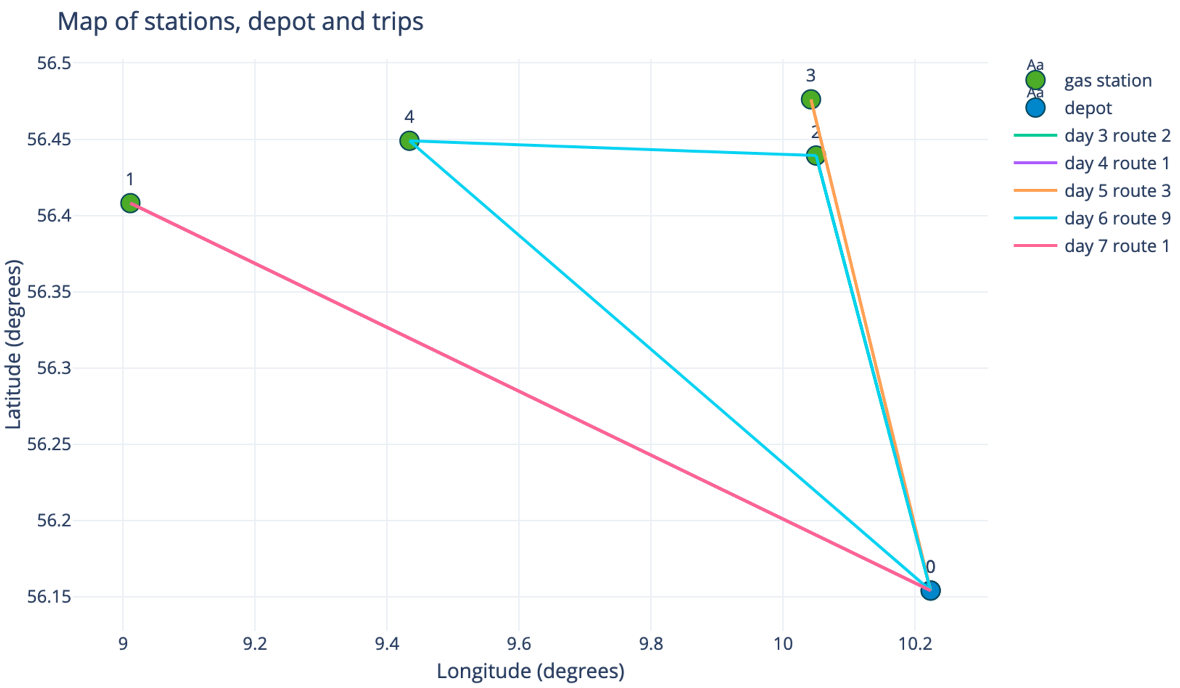

Figure 3.12: Initial solution found by heuristic for verification data set

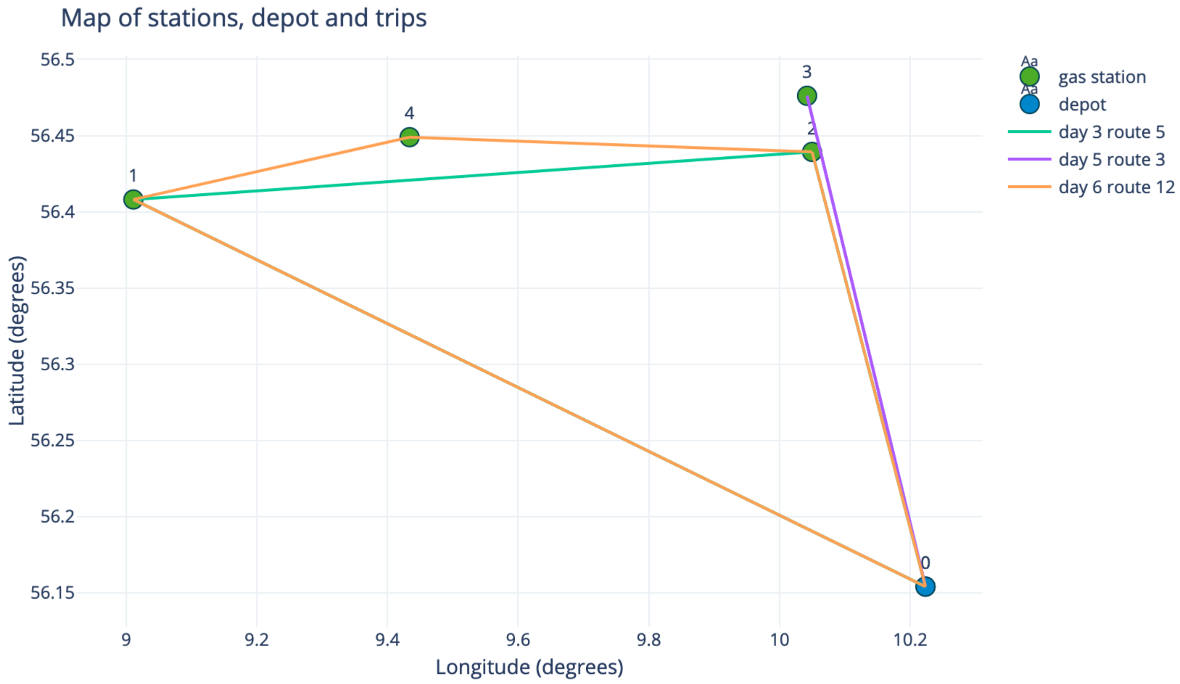

Figure 3.13: Improved solution found by heuristic for verification data set

3.4.11. Heuristic validation by company reference run

AMCS has performed its own simulations with the same data. The result found by the company are shown in table 3.8 and compared to the result found by the heuristic. Although the results cannot be compared one to one, because different algorithms are used to generate the solutions, the results indicate if the heuristic solution is feasible.

In terms of travel distance, the heuristic seems to be able to create a better solution compared to the result found by AMCS. On the other had, since the total delivered volume is lower, the total travel distance might only be lower for the considered time span and not for the long term. Furthermore, the number of stops per trip has decreased, while vehicle utilisation remains high, what indicates a more efficient planning. Unfortunately, the exact causes of the differences between the two results cannot be identified, so the observed effects are only an indication.

Table 3.8: Comparison of result created by AMCS and proposed heuristic for 59 stations and 14 days

KPI	AMCS Result	Heuristic result	Difference
Total distance (km)	12,398	10,877	-12.3%
Average inventory level (%)		43.6	
Trips	159	139	-12.6%
Stops	276	203	-26.4%
Average stops/trip	1.74	1.46	-15.9%
Delivered volume (L)	7,680,394	5,994,000	-22.0%
Volume/trip (L)	48,304	29,529	-38.9%
Vehicle utilisation (%)	97.8	96.7	-1.1%
Dryruns	2	0	

3.5. Concluding remarks on the planning methods

The second research question formulated in this research is ”*Which KPIs are relevant for assessing a planning method?*”. The objectives of the supplier, the decision maker, are analysed in this chapter to answer this question. The main objective, maximising profit, can be achieved by minimising costs and maximising revenue. To maximise revenue, dry run should be prevented and minimising costs can be achieved by minimising travel distance, minimising the average stock level and minimising the run time of the planning algorithm. Maximising vehicle utilisation and minimising the average number of stops per trip lead to a minimised travel distance. These quantitative objectives are the KPIs, which are used to evaluate the performance of the planning methods. The KPIs are calculated for each solution generated by each planning method in exactly the same way.

Research sub question three is stated as ”*Which planning methods can be developed for planning petrol station replenishment?*”. These planning methods are presented in this chapter. The three main planning methods are a full MILP model, a simplified MILP model and a heuristic approach. The literature review shows that these methods are a common and effective way to solve the PSRP. The presented main planning methods differ on computation time and quality of the generated solutions.

To determine trade-offs between dry run and travel distance and average inventory levels and travel distance, model variants are presented. These variants use a factor which can be varied to determine the effect of different objective cost ratios for these trade-offs. Moreover, a model variant is presented that shows how the main planning can be changed if the service time depends on the delivery quantity.

The heuristic is a combination of a decomposition heuristic with a local improvement procedure. The considered variant of the PSRP is rich, which makes it hard to find a solution for larger size instances. The decomposition heuristic divides the complex problem into smaller sub problems, based on the decision process of the supplier. This makes it easier to find a solution. Since the heuristic approach decomposes the problem into separate parts, inventory and routing decisions are not optimised simultaneously. To improve the initial heuristic solution, an improvement procedure is proposed, based on visiting stations a day earlier than scheduled. Since evaluating a candidate solution is an extensive process, only feasible solutions are considered.

4

AMCS Case study

The case study, which is used to evaluate the planning methods, is presented in this chapter. Firstly, the company AMCS is introduced in section 4.1, after which the characteristics of the case study are described. The case study is based on the real-life situation for a client of AMCS. Moreover, the currently used planning algorithms are presented in section 4.3, followed by the evaluation of the planning methods.

4.1. AMCS

AMCS is the worldwide market leader in software and vehicle technology for the waste, recycling and material resources industries. The company has more than 500 employees and 2450 customers around the world, who are being helped with reducing operating costs, increasing asset utilisation and improving customer service. In order to achieve these goals, AMCS has developed AMCS platform, a state of the art software platform built on the best practices and designed to meet the needs of the waste, recycling and resources industries (AMCS Group, 2019a). One of the focus areas of AMCS is routing optimisation, for which the company has several solutions for different routing problems. The solution for distribution of liquid fuels is called Oil Planner, a resource utility optimisation system with automatic planning.

4.1.1. AMCS Oil Planner

AMCS Oil Planner is an IT system developed to improve the efficiency of the distribution of all kinds of oil and petrol. The system covers the whole distribution process, from planning to distribution. Oil Planner is equipped with unique features to deal with oil and petrol distribution, such as tank level forecast, volume deviations, truck compartment planning and regional restrictions planning. The achievements experienced by the customers of AMCS, are (AMCS Group, 2019b):

- $5-20\%$ reduction in operational costs (driven km, time, trucks)
- $5-20\%$ reduction in CO_2 emissions
- $5-10\%$ reduction in number of vehicles
- $5-15\%$ increase of delivered oil and petrol per driven km
- $25-75\%$ less time spent on planning and resource scheduling
- Employee satisfaction and customer service levels improvements

4.2. Case study characteristics

The case study concerns the distribution network for one of AMCS' clients. The network consists of 59 petrol stations and one central depot, for which the distance and travel times between the locations are based on the road net and provided by AMCS. All stations have one to five tanks and there are four vehicles that supply the stations with fuel, which is sold to end customers at the petrol stations. Stations can have vehicle restrictions, which can be used to prevent trucks from blocking the road if there is not enough space. There are two brands with three products each (gasoline with octane number 92, gasoline with octane number 95 and diesel), so there are six products in total. Each station sells only one brand of fuels, so there are up to three different products distributed to a single station.

Figure 4.1: A truck with a semi-trailer (HMK Bilcon, 2019b)

Figure 4.2: Rigid trucks with drawbar trailers (HMK Bilcon, 2019a)

Two types of vehicles are used in the case study network, a truck with semi-trailer and rigid trucks with drawbar trailers (see Figures 4.1 and 4.2). The trucks are equipped with flow metres and the tanks have six to eight compartments. Because of the flow metres, the content of a compartment can be split over multiple visited stations.

An overview of the case study characteristics is shown in the table below.

Table 4.1: Overview of case study characteristics

Parameter	Value
Number of stations	59
Number of depots	1
Number of vehicles	4
Number of products	6 (2 brands, 1 brand per station)
Travel distances	Provided by AMCS in metres
Travel times	Provided by AMCS in seconds
Weekly consumption forecast per tank	Provided by AMCS in litres per day
Tanks per station	1 to 5
Tank capacity	6.000 - 100.300 L
Compartments per vehicle	6 to 8
Compartment volume	5.000 - 13.000 L
Operating hours vehicles	Time schedule provided by AMCS

Figure 4.3 gives an overview of the spatial distribution of the depot (in blue) and the gas stations (in green).

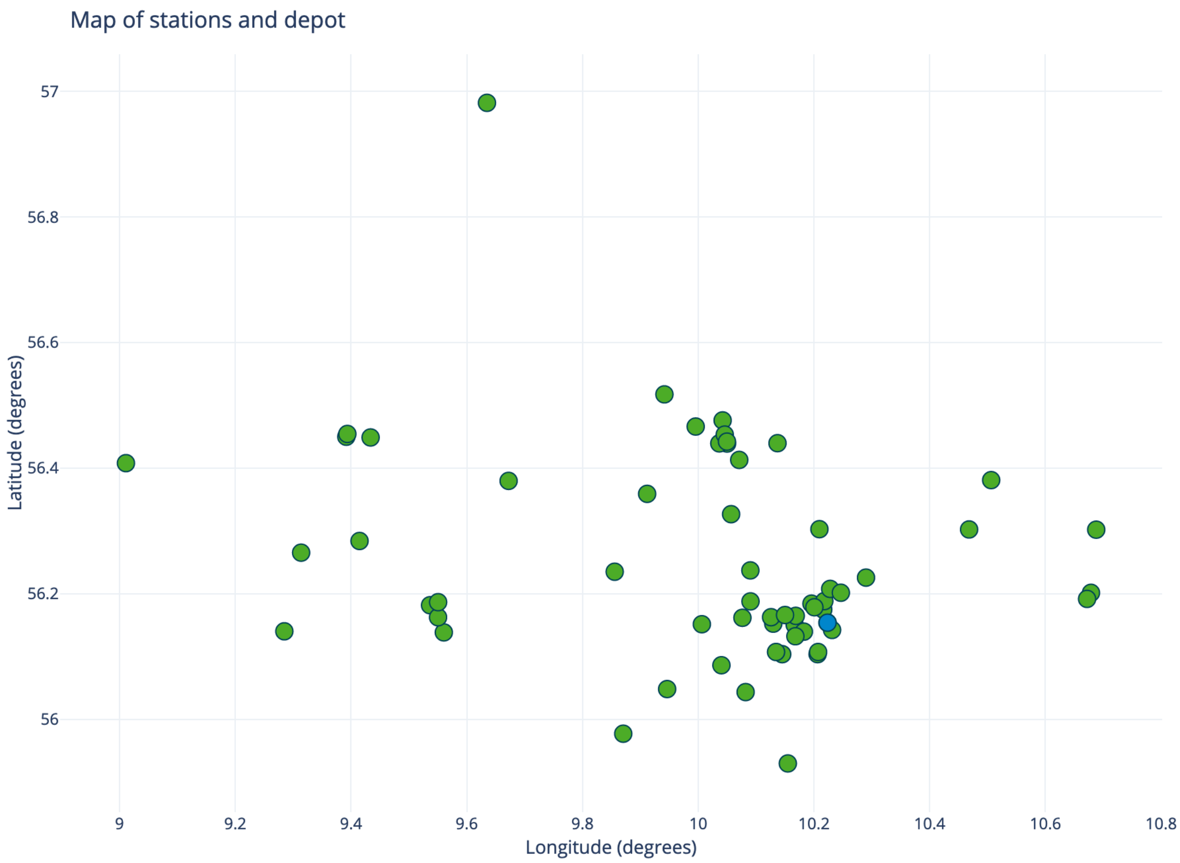

Figure 4.3: Map of stations (green dots) and depot (blue dot)

Furthermore, the stations are categorised by the distance to the depot, travel time to the depot, weekly demand and number of different product types. These categories are shown in Table 4.2.

Table 4.2: Categorisation of case study data

Category	**1**	**2**	**3**	**4**	**5**
Distance to depot (*1000 m)	0 - 25	25 - 50	50 - 75	75 - 100	100 - 125
Number of stations	27	15	12	3	2
Percentage of stations (%)	45.8	25.4	20.3	5.1	3.4
Travel time to depot (*300 s)	0 - 5	5 - 10	10 - 15	15 - 20	20 - 25
Number of stations	13	19	16	9	2
Percentage of stations (%)	22.0	23.2	27.1	15.3	3.4
Weekly demand (*100 L)	0 - 250	250 - 500	500 - 750	750 - 1000	1000+
Number of stations	5	24	14	10	6
Percentage of stations (%)	8.5	40.7	23.7	16.9	10.2
Number of product types	1	2	3		
Number of stations	4	9	46		
Percentage of stations (%)	6.8	15.3	78.0		

4.3. Current planning method

The planning method currently used by AMCS consists of several steps, which are described in the next sections. This section concludes with a schematic overview of the whole planning process. The order planning algorithms are executed over night.

4.3.1. Order generation

The order generation process generates orders for each station. The stock forecast and the available vehicles are the only input parameters for this algorithms. The orders are generated as follows:

1. Determine the exact expected depletion time for each tank at each station
2. Based on the depletion times for the tanks, set the expected depletion time for each station based on the earliest depletion time of its tanks
3. Determine the ideal order time, which is initially set at the expected depletion time and then shifted, taking into account:
   - The opening hours of the station
   - The operating hours of the available vehicles
   - The service time at the station
   - Driving time between the depot and the station
4. Determine smallest order amounts which push the next expected depletion as far as possible into the future. These amounts, called the target amounts, are:
   - For the tank with the highest consumption rate, the maximum delivery amount possible
   - For the other thanks, the exact amount needed to shift the next dry run of the tank to the moment when the tank with the highest consumption rate runs dry. This sets the expected dry run time for the next delivery at the same time for each tank, also called simultaneous dry run.
5. Determine the optional order amounts, which are the maximum amounts that could be delivered on top of the target amounts without exceeding the tank's capacity
6. Generate a time window for a possible delivery at the station, according to the following steps:
   - Find an open time frame for the station that contains the ideal order time and set the time window to this.
   - If the time window ends after order time, set it to end at order time.
   - If the time window begins before a minimal amount can be delivered without violating the tanks' capacities (because the stock first needs to be consumed), the time window's start time is changed to the earliest time when the minimum order amounts can be delivered feasibly. The minimum amount is defined as the target amount minus a maximum allowed percentage that defines how much can be differed from the intended amounts (typically 10 or 20 percent).

The result is an order for each station, with a target and optional amount for each tank at the station, a time window and an order time. Figure 4.4 visualises the concept of using target and optional amounts to push the simultaneous dryrun as far as possible in the future.

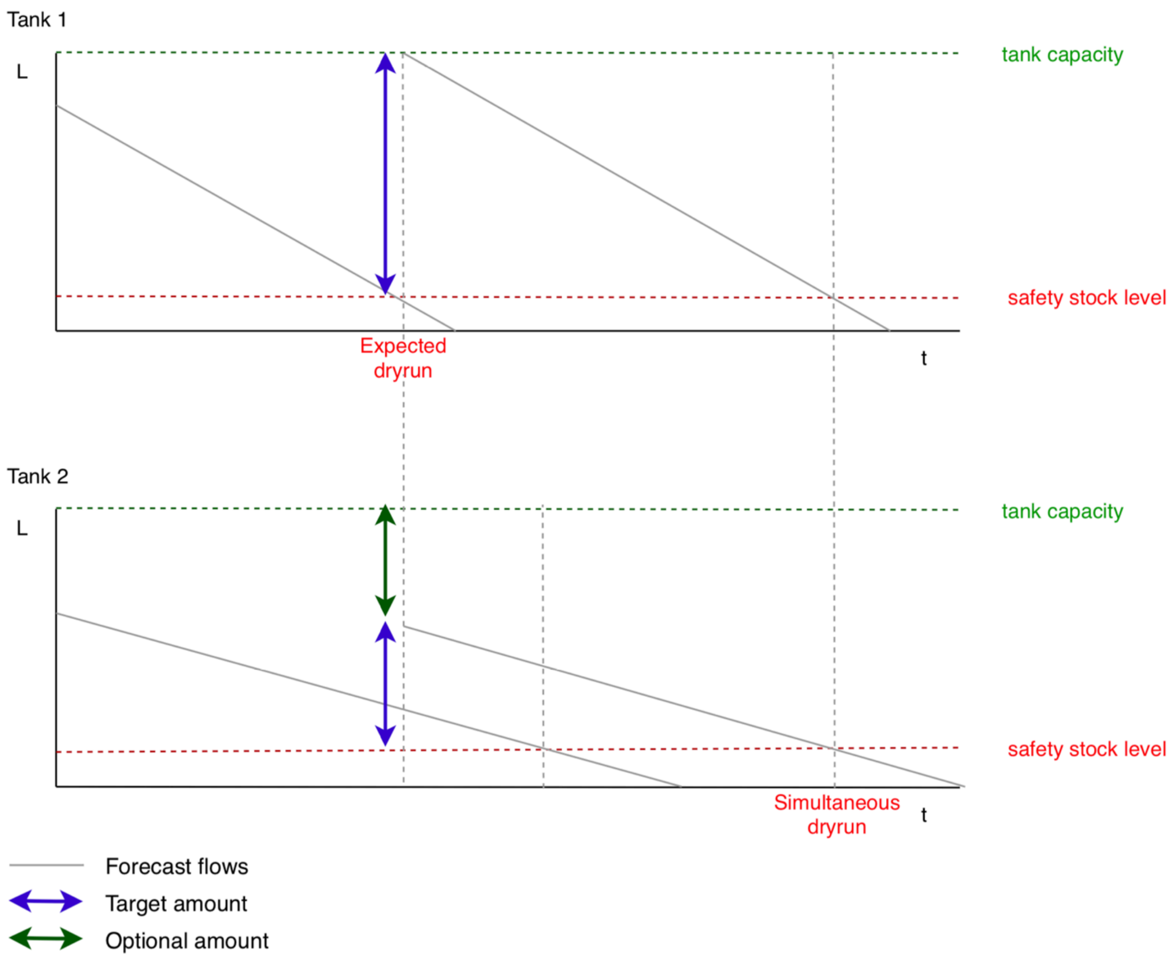

Figure 4.4: Visualisation of two tanks at one station with simultaneous dryrun as far as possible in the future

4.3.2. Order pairing

The next step after creating an order for each station, is pairing the orders. A paired order means that a truck visits multiple stations before it returns to the depot. The general order pairing algorithm works as follows:

- Find the "not full" orders for station within a station group. An order is considered "not full" when the order amounts are less than 90 % of a truck's capacity.
- Make a pairing with the "not full" orders with the earliest order times.
- Repeat the order generation process while considering the paired stations as one station. This results in new order amounts and a new time window.

Orders are generally only paired within the manually made groups. This pairing algorithm has several drawbacks.

- The network has to be thoroughly analysed to manually determine the station groups.
- If the order amounts for paired orders lead to simultaneous dry run, this will lead to the same paired orders in the future. This might mean that more multiple stop trips are generated than needed.
- When a paired order contains stations with high and low consumption rates, the station with low

consumption rates will be visited more frequently than necessary, since the combined expected depletion time is determined by the station with high consumption rates. It might be better to visit the station with low consumption rates with maximum order amounts, because the above described algorithm might lead to more trips than necessary.

To overcome the last mentioned drawback, some stations are given the ”priority” or ”dump” label. Stations with the ”priority” label must always be visited with the target amounts, to prevent unnecessary trips in the long run. Stations with the ”dump” label may always be visited to maximise the use of the vehicles' capacities. Next to the use of these labels, many other ”ifs” are used to determine which order pairings lead to less driven kilometres in the end.

4.3.3. Order planning

The last step in the planning process is to assign the generated orders to available trucks. There are a lot of ”empty trips” which represent stretches of operating hours of the different vehicles. Orders are planned on these trips. A greedy algorithm is used to assign the orders to compartments of the trucks. The target amounts of prioritised orders are firstly assigned to a compartment, followed by the target amounts of non-prioritised orders. Lastly, the optional amounts are used to fill up the vehicle.

4.3.4. Overview of the planning algorithm

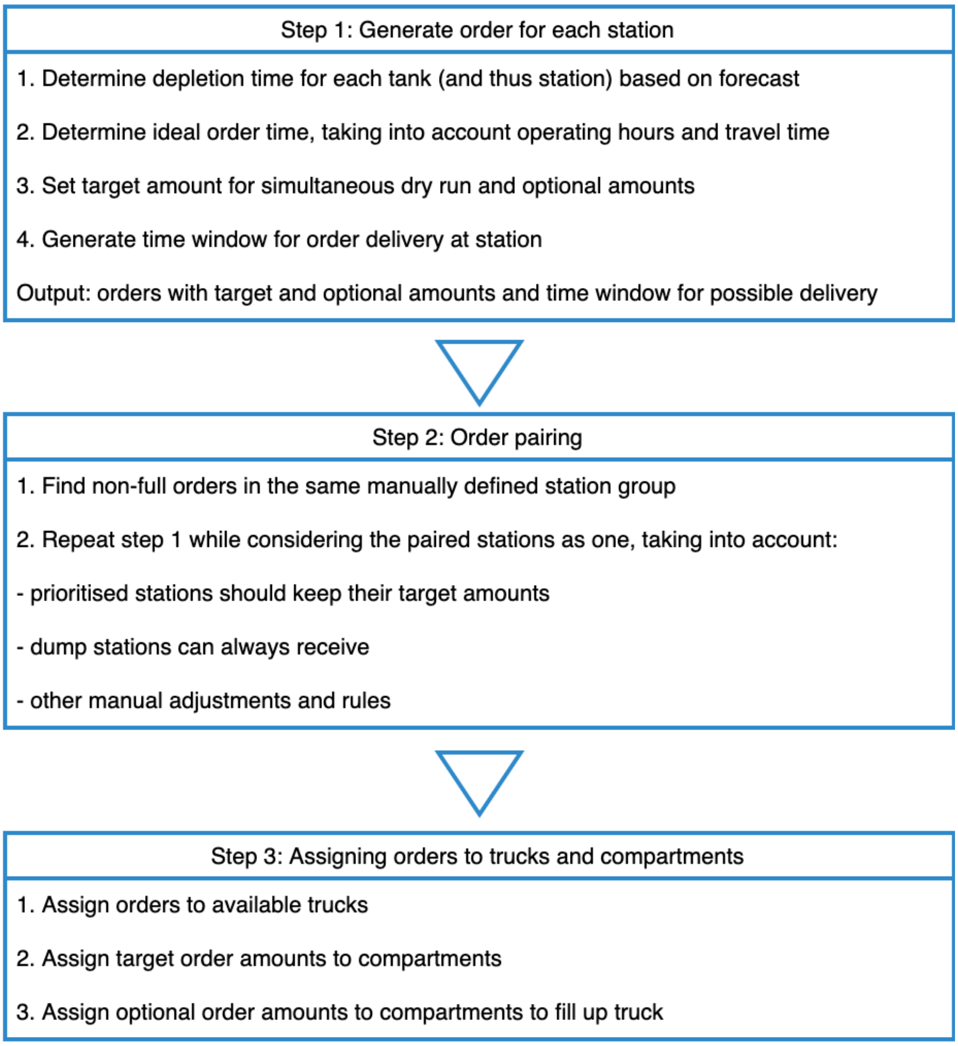

Figure 4.5: Overview of the planning algorithm

4.3.5. Concluding remarks on the planning algorithm

- Currently, vehicles are filled to the maximum, what possibly leads to the most efficient planning.
- Order pairing (and in the end vehicle routing) is based on manually defined groups and other empirically defined rules to generate desired behaviour in special situations, which means that the network has to be thoroughly analysed. It might be beneficial to automate a part of the order pairing process.

The current planning analysis is used to define recommendations for AMCS. These recommendations, which follow from the evaluation of the planning methods as well, are discussed in Section 4.6.

4.4. Evaluation of planning methods with the case study data

The planning methods presented in this research are evaluated with the case study. This section describes how the situation for the case study is implemented using the models.

4.4.1. Case study implementation

The case study data is slightly modified before the data could be used as input data for the mathematical models of the planning methods. Although the modifications are a relaxation of the real-life problem, the changes are made in such a way that the model remains as representative for the real-life situation as possible. The modifications are discussed below.

- **Underground tanks**
  The mathematical models only allow one tank of a product type per station. In the case study, there are stations with more tanks containing the same product type. In the models, the tanks with the same product at a station are considered as one. This means that the inventory levels and capacities are combined.
- **Capacity and inventory levels per tank**
  The case study data specify for each tank a capacity, maximum stock level, minimum stock level, empty level, an initial inventory level and a demand forecast per day. The maximum and minimum stock levels are set by the petrol company, where the minimum level is based on the capacity of the tank and the demand forecast. In the models, the capacity of each tank is set to the maximum stock level and the minimum stock level in the case study data is used as safety stock level. Initial inventory levels and the demand forecast per day are the same in the case study and in the model.
- **Driver shifts**
  Each vehicle is often used by two drivers during two work shifts in the case study. During such a shift, a driver can perform multiple trips. In the models, this situation is implemented by adding a vehicle twice. The time windows of the two vehicles differ, representing the driver schedules.
- **Station restrictions**
  Some stations cannot be visited by each vehicle in the case study. The normal configuration of vehicles is often too large to visit stations with restrictions. Therefore, the rigid truck without drawbar is used to visit stations with restrictions. Implementing this situation is done by reducing the number of compartments for this smaller vehicle that can visit all stations.
- **Accuracy of the units**
  The unites used in the case study data set are litres for quantities, seconds for time and meters for distances. The mathematical models model per 100 litres, 300 seconds and 1000 meters. This increases the speed of the solver to find a solution, which is seen as more important than the limited loss of accuracy. The effect of the loss of this accuracy on the results is limited.
- **Acceptable computation time**
  The planning methods must generate a planning within reasonable time. The planning is normally made over night. Starting as late as possible will give the more up-to-date data. At the other hand, there should be some time for manual adjustments before drivers get their schedule. Therefore, seven hours is seen as the maximum computation time. Using seven hours for creating the planning is only justified if this leads to significantly better solutions (based on personal communication with AMCS planning expert K. Hauge).
- **Maximum number of stops per trip**
  In the simplified MILP model and in the heuristic, routes are defined before the optimisation starts. The routes are generated for each combination of stations, for up to a number of stops per route. Verification the models in the previous chapter is performed with five stops per route for these two models, to create results that can be compared to the first presented MILP model. For the results

for the simplified model with case study data, there has been noticed that considering five stops per route significantly slows down the model relative to three stops per route, while the solution quality does not increase. This is due to the huge number of possible route combinations if five stops are allowed. Therefore, the number of stops per trip is limited to three in the case study experiments. According to AMCS planning expert K. Hauge, this is also representative for the real-life situation.

- **Service time**
  In the case study, there is a fixed service time at the terminal of fifteen minutes and a service rate of 1800 litres per minute. At a station, the fixed service time is ten minutes and the service rate is 900 litres per minute. Since the basic variants of the planning methods do not consider a service time that depends on the delivery quantity, the service time it would take to fully load and empty the vehicle with the largest capacity is taken as fixed service time for each route. This is done by adding the time it takes to fully empty and load the vehicle to the fixed service time at the depot. Fixed service time per station is added per stop.

4.4.2. Experimental setup

As appears from the results presented in Section 4.5, a solution for the MILP models can only be found by the solver for instances with a relatively small number of stations. The fourteen stations per subset are randomly selected, but same subsets of stations will be used to evaluate all models and model variations. This allows comparison of the results for different models. Doing multiple experiments instead of one limit the effect of spatial distribution and underground tank configuration on the results. The stations for a subset, with their own geographical distribution, are shown in Figure 4.6.

Figure 4.6: Sub sets of data set for experiments MILP model and simplified MILP model

4.5. Computational results

The mathematical models are implemented in Python and the MILP models are solved using Gurobi Optimizer version 8.1.1 (Gurobi Optimization, LLC, 2019). The experiments are executed on a computer with a 3.50 GHz 4-core processor and 32 GB of RAM. Results are generated for each sub set for each model. The heuristic is also ran with the full data set. The most relevant results are presented in this section, an overview of the solutions for all performed experiments is included in appendix C.

4.5.1. Evaluation of performance of the solver

Figure 4.7 shows the computation time versus the gap and objective value found by the solver for an experiment with 6 stations and 7 days. The graph shows that during 24 hours run time, the most optimal objective value is found within several minutes and does not improve any more. The gap quickly becomes smaller during the first minutes, steadily declines for a few hours, but after four hours, the decrease of the gap is marginal for the remaining hours. At the same time, the objective value does not improve any more after the first tens of minutes. For this reason, the maximum run time of experiments is set to two hours, even though a longer run time would be acceptable in practice.

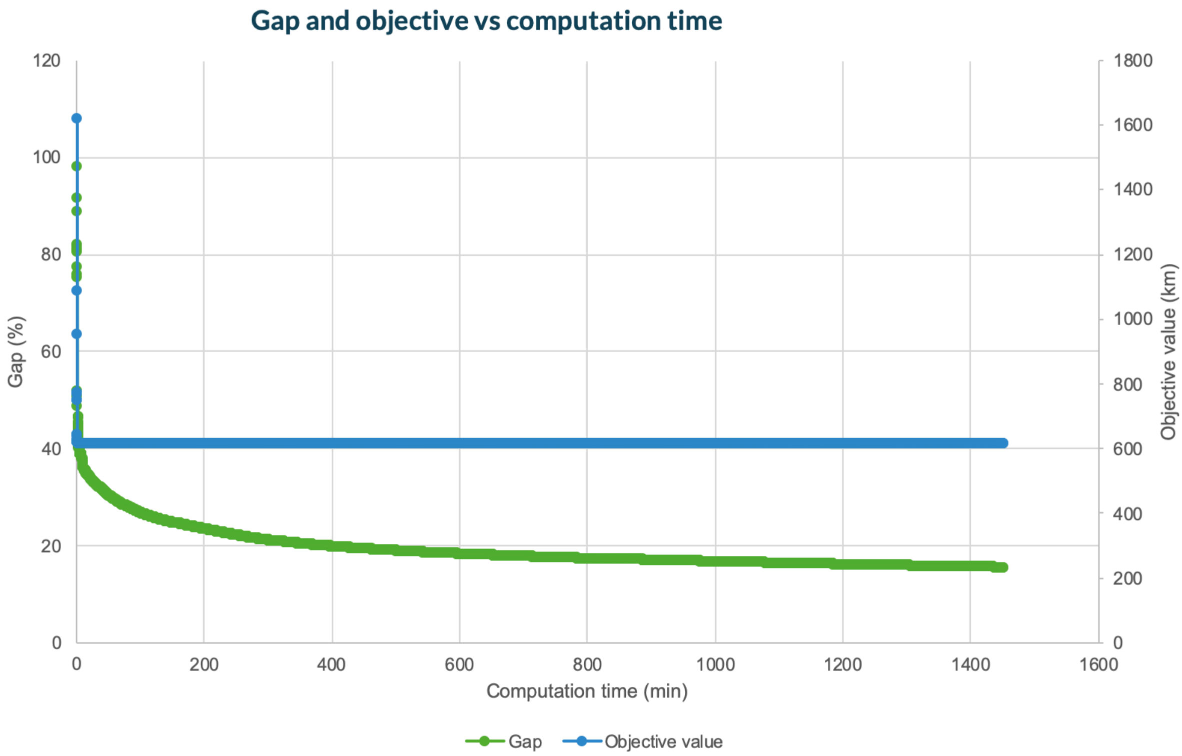

Figure 4.7: Performance of the solver

The effect of restarting the solver during the optimisation process is considered. Restarting the optimisation process of the solver does not lead to a significant effect. If runs with and without restarts are considered, the objective value is exactly the same and there is no significant difference in computation time. An explanation for this could be that, for MIP models, there is not much "warm information" available when the solver is restarted. Restarting the solver may be more suitable for solving LP models. This possible explanation is not verified.

4.5.2. Evaluation of performance of the heuristic

Figure 4.8 shows the gradual decrease in travel distance during the improvement phase of the heuristic. The results are based on an experiment with 10 stations and 7 days. Per day, there is evaluated what the effect is of visiting a station during the considered day that is originally scheduled to be visited tomorrow. This leads to the evaluation of 21 shifting actions, of which 8 actions lead to a decreased travel distance. For such small instances, the computation time of the heuristic is a few seconds.

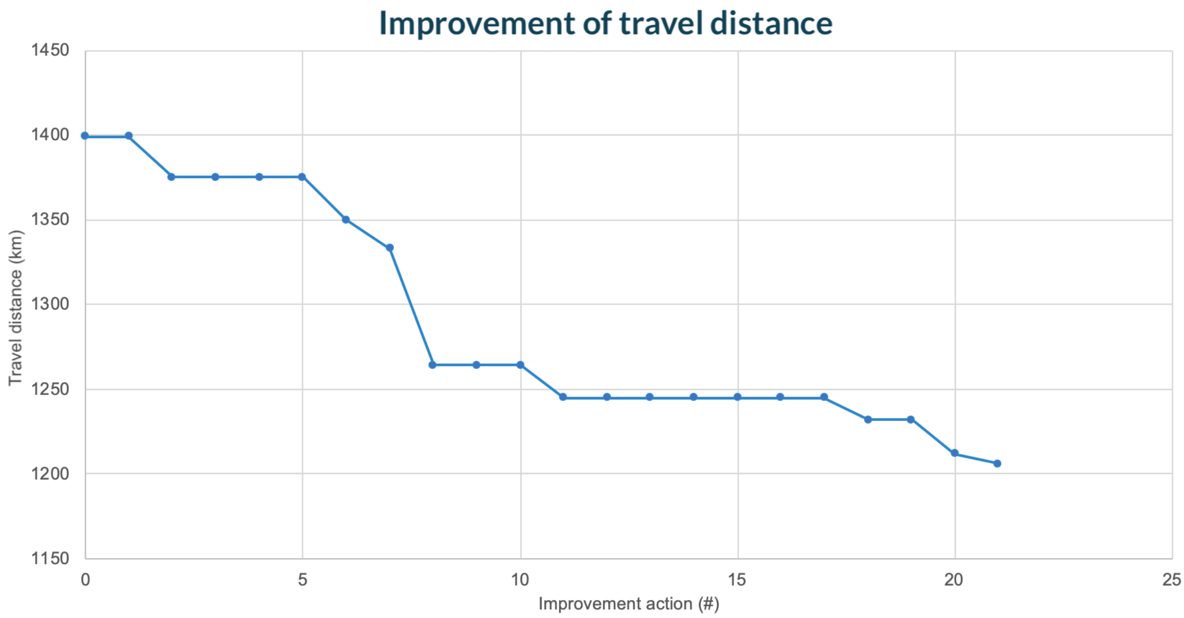

Figure 4.8: Gradual decline of travel distance during improvement phase

4.5.3. Distance and computation time comparison

Table 4.3 shows the distance and computation time for the basic variants of MILP models and the heuristic. The results are the average of performed experiments and the computation time is limited to two hours. Exact solutions can be found with the MILP models for instances with up to 10 stations and 5 days. Not exact solutions are found for instances with up to 12 stations and 7 days. A solution for 20 stations and 7 days is found with the full MILP model and not with the simplified MILP model. This can be explained by the fact that the large number of predefined routes makes it harder to find a solution.

In terms of computation time, the heuristic outperforms the MILP models. If an exact solution can be found within two hours, the solution is found faster by the simplified MILP model compared to the full MILP model.

The total travel distance for the solutions found by the simplified MILP model, compared to the full MILP model, are exactly the same for the experiments with four stations and slightly different for the other experiments. This can be explained by the fact that the simplified model uses predetermined routes. The average increase in travel distance for solutions created by the heuristic, compared to full MILP model, is 19.1%. If only experiments with seven days are considered, the average increase in travel distance is 15.6%.

Table 4.3: Computational results, with computation time (CT) and difference in distance relative to the full MILP model (Δ). Shown results are averages and S4-D5 means an experiment with 4 stations and 5 days. *not an exact solution **no solution found

	Full MILP model		Simplified MILP model			Heuristic		
Experiment	**Distance (km)**	**CT (s)**	**Distance (km)**	**Δ**	**CT (s)**	**Distance* (km)**	**Δ**	**CT (s)**
S4-D5	267	2	267	0.0%	2	304	13.8%	2
S4-D7	480	469	480	0.0%	90	550	14.6%	4
S6-D5	321	544	326	1.5%	42	413	28.8%	3
S6-D7	582*	7200	583*	0.1%	7200	690	18.5%	6
S8-D5	399	1955	412	3.2%	124	501	25.5%	5
S8-D7	753*	7200	747*	-0.8%	7200	849	12.7%	8
S10-D5	454*	7200	463	1.9%	2132	581	28.1%	6
S10-D7	873*	7200	857*	-1.8%	7200	984	12.7%	14
S12-D5	478*	7200	504*	5.4%	7200	606	26.8%	8
S12-D7	972*	7200	934*	-3.9%	7200	1063	9.4%	18
S20-D7	1376*	7200	-**	-	-**	1642	19.3%	39
S30-D7	-**	7200	-**	-	-**	2281	-	79
Average				**0.6%**			**19.1%**	

4.5.4. Results for larger instances

For larger instances, the MILP models can not be used to find solutions, even not when the limit on computation time is set to seven hours. Therefore, heuristic solutions are found for these instances. Table 4.4 shows the computational results for these real-life size instances. The initial solution found by the decomposition heuristic is improved during the improvement procedure by 4.6% on average. For the full data set, a solution is found within two hours, which proves that the heuristic is an effective method for creating solutions for real-life cases.

Experiments in which the *simultaneous dry run* inventory policy was not applied, showed a 22.3% increase in travel distance for an experiment with 20 stations and 14 days. An experiment with 30 stations and 14 days increased the travel distance by 18.2%. In these experiments, the minimum delivery amount was set to the demand of a few days. It was not possible to find a feasible solution for the full data set with this approach. This confirms that the proposed heuristic is an effective planning method.

Table 4.4: Computational results for larger instances. S20-D14 means 20 stations and 14 days.

Experiment	Initial solution (km)	Final solution (km)	Δ	CT Initial (s)	CT Improve proc. (s)
S20-D14	3931	3670	-6,6%	2	162
S30-D14	5588	5365	-4,0%	4	589
S40-D14	6965	6735	-3,3%	12	727
S50-D14	10267	9813	-4,4%	29	3747
S59-D14	11426	10877	-4,8%	14	5103

Figure 4.9 shows a visualisation of the solution for 59 stations and 14 days. Each coloured line represents a trip that is driven by a vehicle. Some stations are visited as only stop in a trip, what happens when the demand of one station is so high that it is not possible to combine stations in a trip.

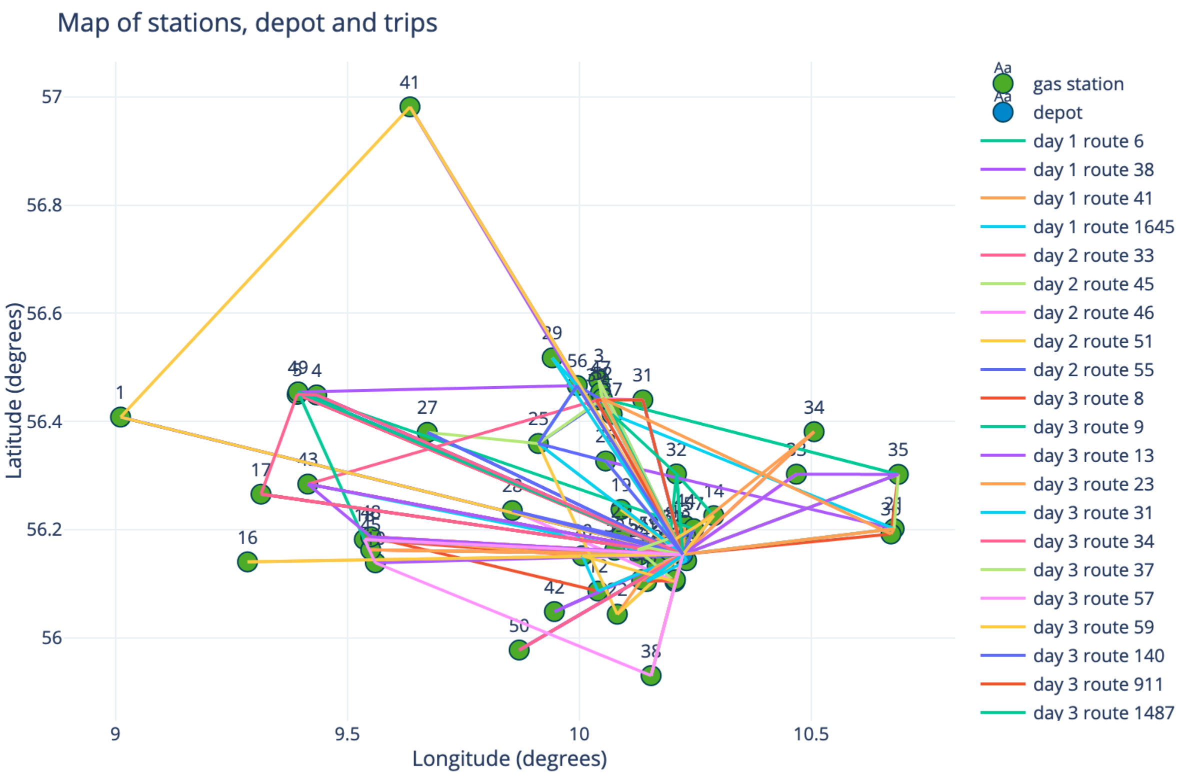

Figure 4.9: Impression of solution for S59-D14 (not all trips are shown in the legend)

4.5.5. Sensitivity analysis on route duration

Travel time and service time are assumed to be fixed in the mathematical models. In real-life, unexpected events and congestion can affect the duration of a trip. Table 4.5 shows the results for experiments in which the route duration is increased and decreased. The total travel distance changes less than one percent, when the rout duration is increased or decreased by 20%. The difference for the other KPIs is less than 2.5%. This shows that, for the full case study data set, the effect route duration on the solution is limited. There are enough hours available in the driver schedule to fulfil the service, even when the route duration is increased by 20%.

Table 4.5: Sensitivity analysis route duration. The results are based on the full data set with 59 stations and 14 days.

Route duration	**80%**	Δ	**100%**	Δ	**120%**
Total travel distance (km)	10,853	-0.2%	10,877	0.6%	10942
Average stock level (%)	44.4%	1.8%	43.6%	-0.1%	43.5%
Number of dry runs	0		0		0
Total delivered (L)	60,517	1.0%	59,944	0.3%	60118
Delivered per stop (L)	301	2.0%	295	0.8%	298
Vehicle utilisation (%)	95.5%	-1.2%	96.7%	-0.4%	96.3%
Trips	141	1.4%	139	1.4%	141
Stops	201	-1.0%	203	-0.5%	202
Average # stops per trip	1.43	-2.4%	1.46	-1.9%	1.43

4.5.6. Sensitivity analysis on safety stock

It can be assumed that a lower safety stock level leads to a decreased travel distance, since the moment that the inventory level drops below the safety stock level is postponed. This is confirmed by an experiment with 10% lower safety stock levels, of which the results are shown in Table 4.6. The total travel distance decreases by 2.3%.

Table 4.6: Sensitivity analysis safety stock level. The results are based on the full data set with 59 stations and 14 days.

Safety stock	**90%**	Δ	**100%**
Total travel distance (km)	10,625	-2.3%	10,877
Average stock level (%)	43.3%	-0.6%	43.6%
Number of dry runs	0		0
Total delivered (L)	59,709	-0.4%	59,944
Delivered per stop (L)	302	2.1%	295
Vehicle utilisation (%)	96.8%	0.0%	96.7%
Trips	138	-0.7%	139
Stops	198	-2.5%	203
Average # stops per trip	1.43	-1.8%	1.46

4.5.7. Comparison of planning methods on KPIs

Table 4.7 shows average KPIs for experiments with 4 stations and 7 days for the basic variants of the planning methods. There can be seen that, as found in the previous sections, the total travel distance for the solution generated by the heuristic is increased by 14.6%, while the computation time is significantly lower.

Furthermore, the total delivered quantity is much higher for the heuristic solution compared to the solution found with theMILP models. Higher total delivered quantity leads to higher values for the average stock level and vehicle utilisation. This can be explained by the fact that in the MILP models there is no incentive to maximise delivery quantities to the capacity of the vehicle, because inventory policy is based on maintaining a level higher than the safety stock level. For the long term, the heuristic solution is probably more efficient, since vehicle utilisation is higher and the number of stops per trip is lower. This is a strong indicator for more efficient transport as described in Section 3.1.

Table 4.7: Average KPIs for experiments with four stations and seven days. *relative difference with full MILP model

	Full MILP model	Simplified MILP	Δ*	Heuristic	Δ*
Total travel distance (km)	480	480	0.0%	550	14.6%
Average stock level (%)	38.6%	40.1%	4.0%	45.5%	18.0%
Number of dry runs	0	0		0	
Computation time (s)	461	90	-80.5%	5	-98.9%
Total delivered (L)	1379	1357	-1.6%	2086	51.3%
Delivered per stop (L)	171	168	-1.6%	294	72.1%
Vehicle utilisation (%)	81.0%	79.8%	-1.5%	86.4%	6.6%
Trips	4	4	0.0%	6	35.3%
Stops	8	8	0.0%	7	-12.5%
Average # stops per trip	1.95	1.95	0.0%	1.21	-37.9%

Figure 4.10 shows the average inventory levels for the data set with 4 stations and 7 days. As expected, the average inventory level is higher for the heuristic solution compared to the solution found with the MILP models. Furthermore, the average inventory level decreases towards the end of the time horizon. To show that this effect only happens at the end of the simulation, the inventory levels for a simulation with 14 days are shown by the yellow line. Average inventory levels for this experiment remain constant during the first 7 time periods.

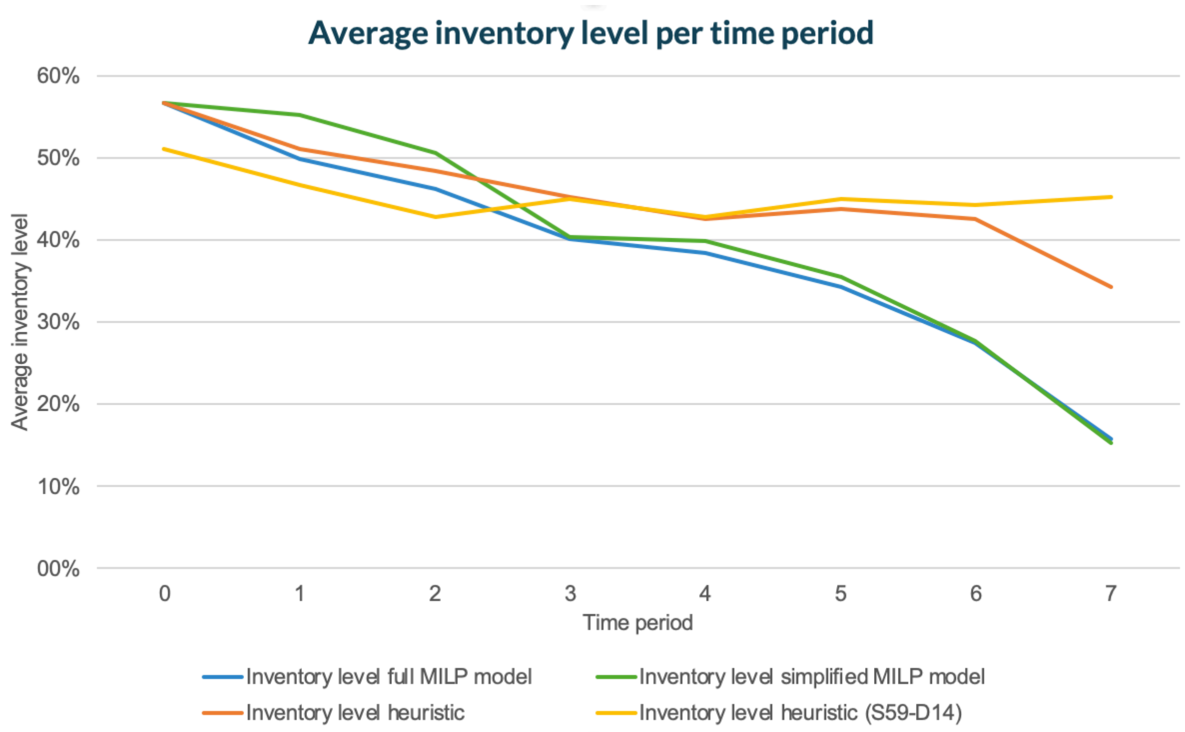

Figure 4.10: Average inventory level per time period

4.5.8. Trade-off between travel distance and dry runs

Model variant one of the MILP models, with soft constraints on inventory levels, is used to determine the trade-off between travel distance and dry run. In this research, dry run is the moment when the inventory level of a tank drops below the safety stock level, so allowing dry run increases the risk of selling out. On the other hand, however, allowing dry runs may decrease the total travel distance.

Average results for experiments with four stations, seven days and different penalty costs are shown in Figure 4.11. In the reference run (**), dry runs are not allowed. If dry run is allowed, the travel distance decreases. In general, lower penalty costs lead to a larger decrease in travel distance. The decrease in travel distance when comparing the results for a penalty of 25 and 0 is, however, relatively small compared to the large increase in number of dry runs. If dry runs are allowed, the penalty should not be much higher than 25 to prevent an unnecessarily high number of dry runs.

Penalty costs of 50 lead to an 15.6% decrease in distance and 0.75 dry runs, where penalty costs of 25 decreases the travel distance by 23.5%.

Figure 4.11: Travel distance and dry runs for different penalty costs. *the orange line is hidden below the yellow line **reference run, where dry run is not allowed

4.5.9. Trade-off between travel distance and average inventory level

Model variant two of the MILP models, where travel distance and inventory levels are minimised simultaneously, is used to determine the trade-off between travel distance and average inventory levels. The variant uses a cost factor to weigh distance costs versus inventory costs. Popović et al. (2011) and Vidović et al. (2014) also considered inventory and routing costs in their models. These authors used a costs of 2 euro per km and 1.09 euro per ton inventory costs per day. Experiments are performed for a data set of 4 stations and 7 days with the same factor ratio. Considering 7 time periods and modelling quantities per 100 L, this leads to a costs factor of 0.38 for the ratio between routing and inventory costs. The objective is then, in words, minimise the total travel distance plus 0.38 times the inventory level.

Figure 4.12 shows the average travel distances and inventory levels for experiments with different cost factors. Results are shown for the full MILP model and for the simplified MILP model. The inventory levels for the results of both models are different if a cost value of zero is applied, what can be explained by the fact that average inventory level is not minimised in that case. Furthermore, the general trend can be identified that a higher cost factor leads to an increase in travel distance and a decrease in average inventory level, up to a certain cost factor. Increasing the cost factor does then not lead to changed travel distance and inventory levels.

A cost factor of 0.1 leads to a 9.3% decrease in average inventory levels and a 6.1% increase in travel distance, where a cost factor of 0.3815 decreases the average inventory level by 13.7% and increases the travel distance by 38.4%.

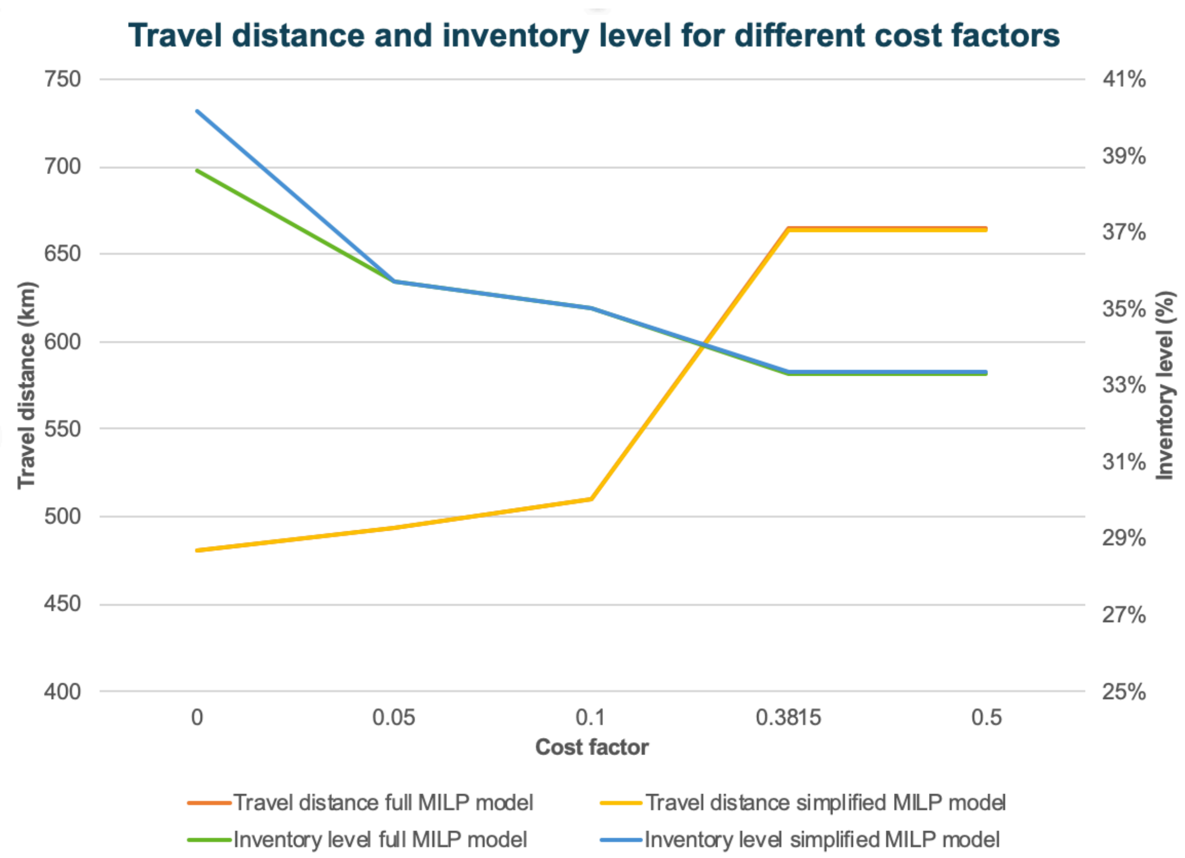

Figure 4.12: Inventory levels and distance of solutions found for different cost factors

To further explore the trade-off between travel distance and inventory levels, experiments are performed with only the objective to minimise travel distance or inventory levels, instead of minimising both objectives simultaneously. Results of experiments with one of the objectives and no other limitations are set to determine the minimum and maximum values for travel distance and inventory level. After determining these values, experiments are performed in which the average inventory level or the travel distance is limited to a value between the maximum and minimum value. The results of these experiments are plotted in Figure 4.13 for both MILP models. The result for cost factors 0.1 and 0.38 are shown as well.

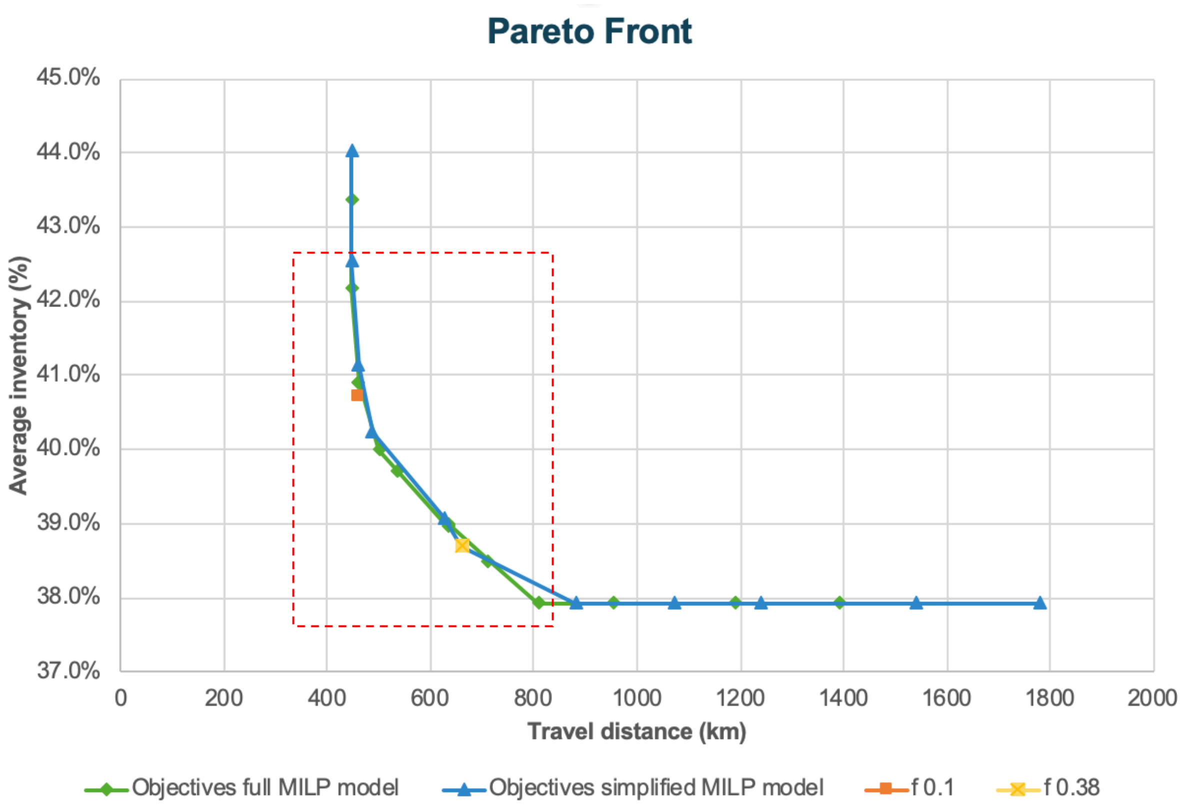

Figure 4.13: Pareto front (marked by the red square) for travel distance and average inventory level objectives

The figure shows that travel distance and inventory levels depend on each other, which is logical because having low inventory levels means that a station needs to be replenished more often. The figure also shows a red square. Solutions within this square are Pareto efficient and show the Pareto front. Solutions outside this square have an unnecessarily high increased value, since the other objective is not decreased. This can be seen for solutions with an inventory level just below 38%, where travel distance increases without further decreasing the average inventory level.

4.6. Recommendations for AMCS

If the heuristic solution is compared to the solution found by AMCS, the travel distance is 12.3% lower for the heuristic solution (as discussed in Section 3.4.11). These results cannot be compared one to one, because different algorithms are used to generate the solutions. It indicates, however, that the heuristic solution is efficient. Vehicle utilisation is high for both solutions. The average number of stops per trip is decreased by 15.9% in the heuristic solution, what indicates efficient planning. This could possibly be explained by the fact that, if a vehicle is not fully loaded, AMCS adds stations from a certain group to the trip to increase vehicle utilisation. The heuristic solution shows that, without adding extra stations, vehicle utilisation remains high.

Furthermore, one of the drawbacks of the planning algorithm used by AMCS, as identified in Section 4.3, is that stations are only combined into routes if the stations are in the same manually defined group. This requires a thorough analysis of the network for new clients. Next to the manually defined groups, stations have labels to determine if minimum delivery quantities must be delivered and if stations always can be visited to increase vehicle utilisation. The heuristic solution shows that high vehicle utilisation can be achieved without these customer specific settings. The heuristic does not use station categories, but optimises routing and truck loading per day. As this optimisation process only takes a few seconds, a recommendation for AMCS would be to do experiments with the same approach and see if this leads to improved solutions.

Moreover, in both experiments, the *simultaneous dry run* inventory policy is applied. The heuristic solution confirms this inventory policy to be an effective planning method. Another interesting area for AMCS to explore is allowing the inventory level to drop below the safety stock level. This research shows that if this is allowed it may result in a significant decrease in travel distance. Lastly, this research shows that in case inventory levels should be minimised, this can be achieved with higher travel distance. If multiple objectives are used simultaneously, the solution should be checked to be Pareto efficient.

4.7. Concluding remarks on the planning methods evaluation

From the computational results, it can be concluded that exact solutions can be found with the MILP models for instances with up to 10 stations and 5 days. Not exact solutions are found for instances with up to 20 stations and 7 days. In terms of computation time, the heuristic outperforms the MILP models. If an exact solution can be found within two hours, the solution is found faster by the simplified MILP model compared to the full MILP model. The average increase in travel distance for solutions created by the heuristic, compared to full MILP model, is 19.1%. If only experiments with seven days are considered, the average increase in travel distance is 15.6%.

For larger instances, including the full data set, the heuristic was able to find solutions within two hours. The travel distance of the initial decomposition heuristic solution is decreased during the improvement procedure by an average of 4.6%. For the full data set, the heuristic provides a better solution than the solution found by AMCS (as discussed in Section 3.4.11). The MILP models are not able to find solutions for larger instances, even not when the limit on computation time is set to seven hours.

The final sub research question is stated as "*How do the proposed methods perform according to the KPIs?*". To calculate the KPIs for the planning methods, the average result of multiple experiments is determined (as described in Section 4.4.2). This excludes the effect of underground tank configuration and spatial distribution of a sub set of stations on the KPIs. The total travel distance for the solution generated by the heuristic compared to the MILP models is 14.6% higher, while the computation time is significantly lower. The heuristic solution delivers more fuel with less stops per trip and higher vehicle utilisation, what is a strong indicator of more efficient transport. This results in higher average stock levels, compared to the MILP models.

Furthermore, if dry runs are allowed, it is advised to use a penalty cost for dry run in the objective function of not higher than 25. A dry run is seen as a moment when the inventory level drops below the safety stock level during a time period. If average inventory levels are minimised, there is a trade-off between inventory levels and travel distance. Optimising one objective will lead to deteriorated values for the other objective. Solutions with trade-offs should be checked on Pareto efficiency.

5

Conclusion and recommendations

The main research question is stated as "*What is an efficient planning method for the multi-period petrol station replenishment problem, in terms of computation time and solution quality?*". To answer this question, several planning methods are presented and evaluated according to the KPIs. The proposed planning methods are based on common methods to solve the PSRP, namely with MILP models and a decomposition heuristic. These models are an addition to the literature, because the combination of included real-life characteristics has not considered simultaneously before. This "richness" of the models, what makes the models more representative, is also the disadvantage of these models. The added constrains are adding complexity to the models too, what increases the time it takes for a solver to find a solution. A case study based on a petrol distributor is used to evaluate the performance of the models.

The MILP models can be used to find solutions for up to 20 stations and 7 days. The models can, however, not be used to find a solution for the real-life size problem. Therefore, a decomposition heuristic is presented, based on decomposing the decision process of the supplier. Decomposition heuristics have proven to be effective methods to solve the rich VRPs. Since the decisions of the supplier are not optimised simultaneously, a local improvement procedure based on visiting stations earlier is proposed. Delivery quantities are determined with the *simultaneous dry run* inventory policy. This new concept, which has been observed in practice, ensures that a station is visited as few times as possible.

The exact solutions found with the MILP models are used to evaluate the heuristic solution quality. In terms of computation time, the heuristic outperforms the MILP models. For experiments with seven days, the average increase in travel distance for heuristic solutions is 15.6%. At the same time, vehicle utilisation increases and the number of stops per trip decreases, indicating more efficient transport. A heuristic solution could be found within two hours for the full case study data set, consisting of 59 stations, 4 vehicles and 1 depot. Therefore, the proposed heuristic is an effective planning method for the multi-period petrol station replenishment problem. Presenting the heuristic is the answer to the main research question.

Moreover, MILP model variants are introduced to gain insight into trade-offs between KPIs. If inventory levels and travel distance are optimised simultaneously, it can be advised to check if the solution is Pareto efficient. In general, lower inventory levels require higher travel distance. If dry runs are allowed and penalised in the objective, what means that costs are added when the inventory level drops below the safety stock level, this could result in a decrease of 15.6% in total travel distance.

Future research in this area is encouraged. The value of the *simultaneous dry run* inventory policy could be further explored by considering the exact inventory level in the tank at the time of delivery, and adjusting maximum delivery quantities according to this remaining capacity. This allows to also replenish the quantity of fuel that has already been sold during the day till the moment of delivery. Moreover, the proposed local improvement procedure of the heuristic is based on recreating the planning for evaluating candidate solutions. If this process is sped up, more candidate solutions could be evaluated.

Another direction for further research is to simulate deliveries with unexpected events to investigate the effect of uncertainty on the distribution performance. Considering uncertainty can also be used to determine efficient safety stock levels.

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Appendix A - Scientific paper

Optimisation methods for the multi-period petrol station replenishment problem

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ABSTRACT

The petrol distribution problem is well-known in the literature as the Petrol Station Replenishment Problem (PSRP). This research concerns a "rich" version of the multi-period PSRP with real-life characteristics, to represent the complexity of the practical problem. The planning is optimised over multiple days, since stations do not need to be replenished every day. Mixed-Integer Linear Programming (MILP) models and a decomposition heuristic are proposed as planning methods, which are evaluated with a case study based on a real-life petrol distributor. Variants to these MILP models are proposed for the situations where the inventory is allowed to drop below the safety stock level, where inventory levels need to be minimised and where the service time depends on the delivery quantity. Moreover, to determine delivery quantities, the heuristic uses the new introduced *simultaneous dry run* inventory policy. An improvement procedure is applied to improve the initial heuristic solution. A commercial solver is able to find solutions for instances with up to 20 stations and 7 days for the MILP models. Exact solutions are found for instances up to 10 stations and 5 days. A heuristic solution was found for the full case study of 59 stations and 14 days, within the time limit of two hours.

1. Introduction

The PSRP, which is an extension of the well-known Capacitated Vehicle Routing Problem (CVRP), concerns the optimisation of the distribution of several petroleum products to a set of petrol stations over a given planning horizon. The products, stored in underground tanks, are delivered to petrol stations using a fleet of vehicles with multiple compartments. Maximum delivery amounts are limited by the available capacity of a storage tank at a station and the capacity of vehicles. The aim is to determine the most optimal route planning for the vehicles, according to the objective of maximising revenue by minimising costs or total travel distance (Benantar et al., 2016; Cornillier et al., 2009, 2012; Triki, Chefi, 2013). The aim of this research is to develop efficient planning methods for a new version of the PSRP, for which the main characteristics are discussed below.

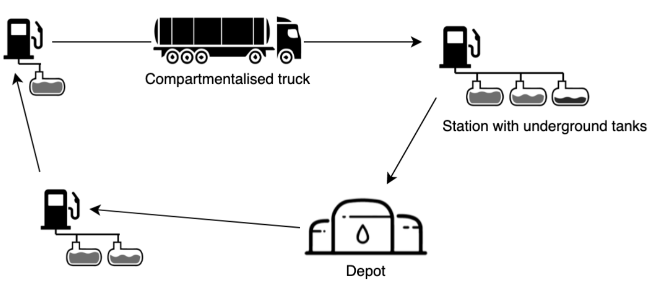

Figure 1: A trip of a vehicle to multiple stations with different numbers of underground tanks

The PSRP has traditionally been solved over a single day time span, mainly because of the complexity of the problem, what makes it an NP-hard problem (Al-Hinai and Triki, 2018). Because of the fact that stations do not have to be replenished every day, the problem is solved in this research for a time span of multiple days, since this leads to more efficient solutions (Triki and Al-hinai, 2016).

To decide when a station needs to be replenished, the Vendor-Managed Inventory (VMI) concept is applied. This means that the inventory levels at the stations are known by the supplier. The supplier makes then, per time period, the following decisions:

- Determine when stations are visited and which stations are combined in trips
- Determine which vehicles are used to perform the trips
- Determine the delivery quantities that are transported to each station
- Determine which compartments are used to deliver the fuel
- Determine for each trip the time a vehicle leaves and returns to the depot, while considering service times at the depot and at the stations

Solving the inventory and routing decision simultaneously leads to more efficient operations, since inventory and routing decisions are interrelated and these two factors are the main cost drivers of petrol supply chain costs (Vidović et al., 2014; Cordeau et al., 2015).

Petrol products are transported by compartmentalised vehicles. These vehicles are often assumed not to be equipped with flow meters (Macedo et al., 2011; Cornillier et al., 2007, 2008; Benantar et al., 2016; Popović et al., 2012; Vidović et al., 2014). The absence of flow meters limits the flexibility in utilisation of vehicles, since flow meters allow to split the load from one compartment over multiple vehicles (Coelho and Laporte, 2015). The case studied in this research assumes the availability of flow meters, which allows splitting loads.

Moreover, the PSRP is usually solved by deterministic models. However, in the real-life situation, the fuel consumption at stations cannot be known beforehand and is therefore stochastic. Uncertainty in actual fuel consumption can be taken into account by applying safety stock levels or emergency deliveries (Popović et al., 2012). In this research, a safety stock level is used to cope with uncertainty. The daily planning is usually made over night, with the latest inventory levels known. The safety stock ensures that the tank does not fully deplete during the next delivery day, what allows delivery during the full day.

The remainder of the paper is organised as follows. An overview of related literature is presented in Section 2, after which the problem is defined in Section 3. MILP models are proposed in Section 4 and the heuristic is presented in Section 5. The results of the evaluation of the models is discussed in Section 6 and Section 7 concludes with the main findings and recommendations for further research.

2. Literature review

The PSRP has received substantial attention in the literature over the last decades, after the problem was formulated for the first time by Brown and Graves (1981). A comprehensive overview of the history of the PSRP can be found in the papers published by Cornillier et al. (2012) and Benantar et al. (2016). Relevant and recent literature is discussed below.

Cornillier et al. (2007) developed an exact algorithm for the single day PSRP, where the number of stops per trip was limited to two. The problem is extended to multiple days by Cornillier et al. (2008), in which the researchers presented a MILP model with the objective to minimise costs with a penalty for overtime use of vehicles.

Popović et al. (2011) defined a MILP model for the PSRP which minimises both inventory and routing costs over a time span of multiple days. The number of stations that can be visited in a trip is limited to three and a station cannot be visited by more than one vehicle during a time period. The authors used a fixed consumption rate per tank.

In the paper published by Popović et al. (2012), a Variable Neighbourhood Search (VNS) heuristic is proposed, with a shaking procedure based on shifting deliveries between days on the planning horizon. In this research, fuel is transported by a homogeneous fleet of vehicles. Vidović et al. (2014) also used a homogeneous fleet of compartmentalised vehicles to distribute fuel and presented two MILP models which minimise inventory and routing costs. One of the models also includes vehicle fleet costs.

Li et al. (2014) presented a version of the PSRP with a fixed fleet of homogeneous vehicles. The stations are visited after a minimum delivery quantity can be delivered, which is set to improve vehicle utilisation .The problem is solved with a tabu search algorithm. The lower bounds of a reasonable sized problem are determined with Langrangian relaxation. The tabu search algorithm proved to be able to find solutions that are near optimal.

An exact model for a variant of the PSRP with multiple compartments and time windows is presented by Benantar et al. (2016). The time windows represent the scheduling horizon for each vehicle and there are two types of vehicles with different compartment sizes. The problem is solved using a MILP model and a heuristic.

2.1. Real-life case studies

Several researchers developed models to consider real-life cases. Ng et al. (2008) improved the distribution of petrol for a network in Hong Kong, in which the authors applied the VMI concept. The proposed approach helped the company to increase the delivery volume and decrease driver costs. Triki, Chefi (2013) used real-life cases to evaluate the performance of the solution methods proposed by the authors to solve the PSRP. Compared to a planning made by a human operator, a saving of 17.7 % was achieved with the most effective method. Triki et al. (2016) also investigated the PSRP for a distributor of petroleum products in Oman, in which a MILP model is presented that eventually can be used to prepare a bid for auctions of transportation procurement.

Li et al. (2014) considered a large petroleum company in China. The distribution of petroleum products in provinces was modelled as Inventory Routing Problem (IRP). A heuristic is developed, for which the solutions were proven to be near optimal. Kazemi and Szmerekovsky (2015) considered the petrol distribution in the United States. A MILP model was used to determine the optimal supply chain design, while considering multi-modal transportation methods when determining locations for the facilities. The case of distribution for an Algerian petroleum company has been evaluated by Benantar et al. (2016), with a model for the PSRP with compartmentalised vehicles and time windows. The method proposed by the authors outperformed the solution created by the company, in terms of number of vehicles and total travel distance.

The case studies mentioned in this sections show that developing MILP models and heuristics are suitable and effective methods for solving the PSRP and to gain insight in tactical and operational choices that need to be made. New trends like the possibility to equip vehicles with flow meters (Coelho and Laporte, 2015) and more interest in "richer" models promise even more effective methods for the future.

An overview of the problem characteristics considered in this research and in exact models presented by other researchers is shown in Table 1.

2.2. Decomposition heuristics

The combination of characteristics makes the considered version of the PSRP NP-hard, since most of the characteristics are already a NP-hard problem if considered separately (Li et al., 2014; Cornillier et al., 2008; Vidović et al., 2014; Al-Hinai and Triki, 2018). Decomposition heuristics are an effective method to solve rich and complex VRPs, because decomposing the complex problems into smaller sub problems makes it easier to find a solution. Decomposition heuristics can be seen as a class of matheuristics. Matheuristics is a category of heuristics that divides a problem into sub problems, of which at least one sub problem is solved using a MILP model. Another class of matheuristics are improvement heuristics, where MILP models are used to improve a

Table 1
Overview of related research, with time horizon (TH), single period (SP), multi-period (MP), single trip (ST), multiple trip (MT), multi-product (MP), vehicle use (VU), vehicle fleet (VF), homogeneous (HO), heterogeneous (HE), station restrictions (SR)

Paper	IRP	Stops/trip	TH	MP	VU	VF	Compartments	Split loads	Time Con.	SR
Azi et al. (2010)		unlimited	SP		MT	HO		✓	✓	
Benantar et al. (2016)		unlimited	SP	✓	ST	HE	✓		✓	✓
Coelho and Laporte (2015)	✓	unlimited	MP	✓	ST	HE	✓	✓		
Cornillier et al. (2008)	✓	max. 2	MP	✓	MT	HE	✓		✓	
Li et al. (2014)	✓	unlimited	SP		SU	HO			✓	
Macedo et al. (2011)		unlimited	SP		MT	HO		✓	✓	
Popović et al. (2012)	✓	max. 3	MP	✓	ST	HO	✓			
Al-Hinai and Triki (2018)		comp. + 1	MP	✓	ST	HO	✓			
Vidović et al. (2014)	✓	max. 3	MP	✓	ST	HO	✓			
This research	✓	unlimited	MP	✓	MT	HE	✓	✓	✓	✓

solution (Archetti and Speranza, 2014). The heuristic presented in this research is a decomposition heuristic, based on decomposing the decision process of the supplier. Since decisions are not optimised simultaneously, a local improvement procedure is proposed to improve the solution found by the decomposition heuristic.

Triki, Chefi (2013) has used a decomposition heuristic to solve the PSRP, in which inventory was not considered. The problem was decomposed into an assignment, routing and improvement procedure. A local search technique based on switching any two stations between service days was used to improve the solution found by the decomposition heuristic.

Cordeau et al. (2015) solved an IRP by decomposing the decision process of the vendor into a three-phase heuristic. The decisions that are made in the phases are the replenishment plan, the delivery sequence and which routes to drive. Inventory management has also been included by Cornillier et al. (2007), who decomposed the problem in a Tank Truck Loading Problem (TTLP) and a routing problem. The objective of the TTLP is to maximise the total delivery quantity, while considering vehicle and underground tank capacity constraints. The minimum delivery quantity per station was set to fulfil the demand and the maximum delivery quantity was set to the remaining capacity of the tank. Similar models are used in the heuristic presented by Cornillier et al. (2008), where the planning is constructed for each day of the planning horizon. The heuristic presented in this research uses the *simultaneous dry run* inventory policy to determine delivery quantities, which are set in a way that limits the number of visits to a station.

3. Problem definition

The problem can be formulated as follows. Let $G = (V, A)$ be a complete directed graph, where $V = (0, 1, ..., n)$ is a set of nodes and $A = \{(i, j) : i, j \in V\}$ is the set of arcs. Each arc has a travel distance d_{ij} and a travel time t_{ij}. The stations are represented by nodes $N = (1, ..., n)$ and the depot is defined by node 0 or $n + 1$, depending on whether it is the initial or final node in a trip. Each node has a fixed service time St_i. Set $T = (1, ..., n)$ defines the time periods in the planning horizon.

The stations have one or multiple underground tanks which store one type of product p. Set P contains all products, which are stored at the depot. Each underground tank has a maximum capacity Lc_i^p, a safety stock Ls_i^p and an initial inventory level at the start of the planning horizon Li_i^p. Ld_{it}^p gives the demand for each product at each station per day t.

Fuel is transported by a heterogeneous fleet of vehicles. Each vehicle k has multiple compartments m with maximum capacity Q^{km}. Let K and M denote the set of vehicles and compartments. Station restrictions are included in the model by variable δ_i^k, which equals 1 if station i can be visited by vehicle k. Operating hours of vehicles are represented by a time window $[a_t^k, b_t^k]$, where a_t^k is the start time and b_t^k the end time of the schedule for vehicle k during day t. L is an arbitrary large constant.

Furthermore, the list below gives an overview of the assumptions made:

- There is enough inventory at the depot to fulfil the replenishment plan.
- Inventory levels at stations are known by the supplier.
- Demand is considered as deterministic, based on a forecast. In real-life, demand is stochastic. To prevent stock out, a a safety stock level is determined for each tank at each station and the inventory level should always be higher than the safety stock level.
- A station can have only one underground tank per product.
- To transport the petrol products, a heterogeneous fleet of vehicles is used with compartments of known size. The vehicles are assumed to be equipped with flow meters.
- All vehicles are assumed to drive with the same speed, which means that they have the same travel time, given the same distance.
- Some stations can not be visited by all vehicles, because of spatial restrictions.
- Stations can be visited multiple times a day and 24/7, because it is assumed that a truck can always deliver the petrol products at the station.
- Vehicles can only be used within the operating hours, representing the work schedules of drivers.

4. Mathematical formulation

A MILP model and a simplification of this MIPL model are proposed to solve the problem. The models are defined in the next sections, after which the performance is evaluated in Section 6.

4.1. MILP model

Let variable x^k_{ijrt} be one if and only if vehicle k drives from i to j in trip r during day t. Variable y^k_{rt} is used to define if vehicle k drives trip r during day t. The same holds for variable z^k_{irt}, where i is added to define that station i is visited. If binary variable w^{kmp}_{rt} is one, product p is loaded into compartment m of vehicle k during trip r on day t. If trip s is driven after trip r by vehicle k during day t, binary variable u^k_{rst} is set to one. Variable q^{kmp}_{irt} is used for the delivery quantity of product p, loaded in compartment m of vehicle k and delivered to location i during trip r on day t. S^k_{irt} is the time vehicle k can start the service at station i during trip r on day t and I^p_{it} represents the inventory level of product p at station i at the end of day t.

Set R contains the trips that can be driven by vehicles, with $|R|$ chosen large enough to enable the maximum number of trips a vehicle can perform during a daily work schedule. The trip indices are used in increasing order, which means that $s > r$ if a vehicle performs trip s after trip r.

The problem can then be formulated as:

$$\text{Minimise} \sum_{t\in T} \sum_{r\in R} \sum_{k\in K} \sum_{(i,j)\in A} d_{ij}\; x^k_{ijrt} \tag{1}$$

Subject to:

$$I^p_{it} \geq Ls^p_i \qquad \forall p\in P, \forall i \in N, \forall t \in T \tag{2}$$

$$I^p_{i,t-1} + \sum_{r\in R}\sum_{k\in K}\sum_{m\in M} q^{kmp}_{irt} \leq Lc^p_i \qquad \forall p\in P, \forall i \in N, \forall t \in T \tag{3}$$

$$I^p_{i0} = Li^p_i \qquad \forall p \in P, \forall i \in N \tag{4}$$

$$I^p_{it} = I^p_{i,t-1} + \sum_{r\in R}\sum_{k\in K}\sum_{m\in M} q^{kmp}_{irt} - Ld^p_{it} \qquad \forall p\in P, \forall i \in N, \forall t \in T \tag{5}$$

$$z^k_{irt} \leq \delta^k_i \qquad \forall k\in K, \forall i\in N, \forall r \in R, \forall t\in T \tag{6}$$

$$x^k_{0jrt} = 1 \qquad \forall k \in K, \forall r\in R, \forall t \in T \tag{7}$$

$$x^k_{ijrt} - \sum_{i\in V} x^k_{jirt} = 0 \qquad \forall k\in K, \forall j\in V, \forall r\in R, \forall t\in T \tag{8}$$

$$z^k_{irt} = \sum_{j\in V} x^k_{ijrt} \qquad \forall k\in K, \forall i\in N, \forall r\in R, \forall t \in T \tag{9}$$

$$q^{kmp}_{irt} \leq L\, z^k_{irt} \qquad \forall k\in K, \forall p\in P, \forall m\in M, \forall i\in N, \forall r\in R, \forall t\in T \tag{10}$$

$$\sum_{i\in N} q^{kmp}_{irt} \leq w^{kmp}_{rt}\, Q^{km} \qquad \forall k\in K, \forall m\in M, \forall p\in P, \forall r\in R, \forall t\in T \tag{11}$$

$$\sum_{p\in P} w^{kmp}_{rt} \leq 1 \qquad \forall k\in K, \forall m\in M, \forall r\in R, \forall t\in T \tag{12}$$

$$S^k_{irt} \geq a^k_t \qquad \forall k\in K, \forall i\in V, \forall r\in R, \forall t\in T \tag{13}$$

$$S^k_{irt} \leq b^k_t \qquad \forall k\in K, \forall i\in V, \forall r\in R, \forall t\in T \tag{14}$$

$$S^k_{irt} + t_{ij} + St_i - L(1 - x^k_{ijrt}) \leq S^k_{jrt} \qquad \forall k\in K, \forall (i,j)\in A, \forall r\in R, \forall t\in T \tag{15}$$

$$S^k_{0st} + L(1 - u^k_{rst}) \geq S^k_{n+1,rt} \qquad \forall r,s\in R, r<s, \forall k\in K, \forall t\in T \tag{16}$$

$$\sum_{r\in R}\sum_{s\in R|s>r} u^k_{rst} \geq \sum_{r\in R} y^k_{rt} - 1 \qquad \forall k\in K, \forall t\in T \tag{17}$$

$$z^k_{irt} \leq y^k_{rt} \qquad \forall k\in K, \forall i\in N, \forall r\in R, \forall t\in T \tag{18}$$

$$x^k_{ijrt}, y^k_{rt}, z^k_{irt}, w^{kmp}_{rt}, u^k_{rst} \in \{0,1\} \qquad \forall k\in K, \forall m\in M, \forall p\in P, \forall i,j\in V, \forall r\in R, \forall t\in T \tag{19}$$

$$q^{kmp}_{irt}, S^k_{irt}, I^p_{it} \geq 0 \qquad \forall k\in K, \forall m\in M, \forall p\in P, \forall i\in V, \forall r\in R, \forall t\in T \tag{20}$$

The objective function (1) minimises the total number of kilometres driven by all vehicles during the full time horizon. Constraints (2) and (3) ensure that inventory levels for fuel in underground tanks stay above the safety stock level and below the maximum capacity. Constraints (4) set the initial inventory level for each product at each station and constraints (5) set the daily inventory level, taking into account the demand and deliveries. Station restrictions are imposed by constraints (6), which means that some stations cannot be accessed by all vehicles. Constraints (7) makes sure that all vehicles start and end at the depot. Constraints (8) represent the flow conservation constraints. Constraints (9) and (10) are used to link x and q to z, which means that a vehicle can only drive to a station and deliver fuel when it is visited by that vehicle in a certain time period. Constraints (11) ensure that compartment capacities can not be exceeded and constraints (12) make sure that only one type of product is loaded in a compartment.

Constraints (13-14) force the arrival time in node i to be within the work schedule of the vehicles. The start and end time of a trip must be within this time window. Constraints (15) take driving and service time into account, to calculate at what time the service can start at a node. Constraints (16-18) make sure that, in case a vehicle performs multiple trips during a time period, these trips are driven consecutively.

Lastly, constraints (19) define x, y, z, w and u as binary values and constraints (20) enforce the integer variables to be non-negative.

4.1.1. Valid inequalities

Valid inequalities are proposed to speed up the solution. Constraints (21) ensure that all vehicles return to the depot and constraints (22) strengthen the relationship between the binary variables x and y. Finally, (23) is used to link the constraint q and w.

$$\sum_{i \in V} x^k_{i,n+1,rt} = 1 \qquad \forall k \in K, \forall r \in R, \forall t \in T \quad (21)$$

$$x^k_{ijrt} \leq y^k_{rt} \qquad \forall k \in K, \forall i,j \in V, \forall r \in R, \forall t \in T \quad (22)$$

$$q^{kmp}_{irt} \leq L\, w^{kmp}_{rt} \qquad \forall k \in K, \forall m \in M, \forall p \in P, \forall i \in N, \forall r \in R, \forall t \in T \quad (23)$$

4.1.2. Model variation: soft constraints for dry run

A model variation is adopted in this section for the case that dry run is allowed, which means that the inventory level is allowed to drop below the safety stock level. Binary variable E is introduced, which is set to one if and only if the inventory of product p at location i is lower than the safety stock during day t. Moreover, a penalty cost p is added to the model, which is used to add costs to the objective in case the inventory level drops below the safety stock level. The objective function (1) is replaced by equation (24) and constraint (2) is replaced by constraint (25).

$$\text{Minimise} \sum_{t \in T} \sum_{r \in R} \sum_{k \in K} \sum_{(i,j) \in A} d_{ij}\, x^k_{ijrt} + p \sum_{t \in T} \sum_{p \in P} \sum_{i \in N} E^p_{it} \quad (24)$$

$$L\, E^p_{it} \geq I^p_{it} - Ls^p_i \qquad \forall p \in P, \forall i \in N, \forall t \in T \quad (25)$$

4.1.3. Model variation: inventory costs in the objective

For the situation that that inventory levels should be kept to a minimum, the objective function (1) should be replaced by equation (26). A factor f is introduced to weigh the costs of inventory relative to the costs of travel distance.

$$\text{Minimise} \sum_{t \in T} \sum_{r \in R} \sum_{k \in K} \sum_{(i,j) \in A} d_{ij}\, x^k_{ijrt} + f \sum_{t \in T} \sum_{p \in P} \sum_{i \in N} I^p_{it} \quad (26)$$

4.1.4. Model variation: service rate

In case that the service time depends on the delivery quantity, constraint (15) and (16) should be replaced by (27 - 29). Constraint (27) adds the service rate Sr_i, multiplied with the delivered quantity, to the time between visiting two stations. Constraint (28) and (29) add time before the start of a route, representing the time it takes to fill up a vehicle at the depot.

$$S^k_{irt} + St_i + Sr_i \sum_{m \in M} \sum_{p \in P} q^{kmp}_{irt} + t_{ij} - L(1 - x^k_{ijrt}) \leq S^k_{jrt} \qquad \forall k \in K, \forall (i,j) \in A, \forall r \in R, \forall t \in T \quad (27)$$

$$S^k_{0rt} \geq Sr_0 \sum_{i \in N} \sum_{m \in M} \sum_{p \in P} q^{kmp}_{irt} \qquad \forall r \in R, \forall k \in K, \forall t \in T \quad (28)$$

$$S^k_{0st} + L(1 - u^k_{rst}) \geq S^k_{n+1,rt} + Sr_0 \sum_{i \in N} \sum_{m \in M} \sum_{p \in P} q^{kmp}_{ist} \qquad \forall r,s \in R, r < s, \forall k \in K, \forall t \in T \quad (29)$$

4.2. Simplified MILP model

In the simplified MILP model, routes are generated prior to the optimisation. Per combination of stations, the sequence of visiting these stations that leads to the lowest travel distance are the order in which the stations are visited. Service time is included in the route duration. Variable x^k_{ijrt} is replaced by parameter $x_{ir} \in \{0,1\}$, which defines the stations that are visited in a route. Variable y^k_{rt} is then used to determine the routes r that are driven by vehicle k during time period t.

5. Heuristic

A decomposition heuristic, based on decomposing the decision process of the supplier, is considered to solve the rich version of the PSRP for real-life size instances. This approach does not lead to optimal solutions, since the decisions are not optimised simultaneously. To improve the initial solution created with the decomposition heuristic, an improvement procedure is proposed. The full heuristic procedure is discussed in Section 5.2.

5.1. Decomposition heuristic

To find a solution, the problem is divided into five phases, based on the decision process of the supplier. These phases are discussed in the next sections.

5.1.1. Phase 1 - Order generation

During the order generation phase, an order is generated for the time period when the inventory level drops below the safety stock level. An order is defined by the station index, product index and depletion day. Orders are generated with Algorithm 1.

Algorithm 1: Order generation

```
Result: List with orders for tank on depletion day
initialisation: load inventory levels and demand forecast;
set t_c = current considered time period, T = last day time horizon;
foreach underground tank at station i with product p do
    for t in [t_c, T] do
        inventory level = inventory level end previous day - demand during t;
        if inventory level < safety stock then
            add tank to order list with depletion day t;
```

5.1.2. Phase 2 - Order pairing

The second phase (Algorithm 2) considers the "current" day and combines, per station, all orders. The result is a list of stations and tanks that "run dry" (inventory level < safety stock level) during the considered time horizon.

Algorithm 2: Order pairing

Result: Per station, a list of orders for the full time horizon

set t_c = current considered time period, T = last day time horizon;
step 1. filter order list on orders for current day;
foreach *order generated in phase 1* **do**
    if *order depletion day* = t_c **then**
        add order to order list for today;
step 2. combine orders for the same station;
foreach *generated order with depletion day t in* $[t_c + 1, T]$ **do**
    if *station is in order list for today* **then**
        add order to current day order list;

5.1.3. Phase 3 - Set min and max quantities

In the third phase, minimum and maximum delivery quantities are set, based on the newly introduced *simultaneous dry run* inventory policy. All tanks at a station are considered and the minimum quantities are set to postpone the next visit to the station as much as possible. This minimum delivery quantity is the full remaining capacity for the tank with the highest consumption rate and for the other tanks it is the quantity necessary to ensure simultaneous dry run in the future (as can be seen in Figure 2. The maximum delivery quantity is set to the remaining capacity in the tank. Pseudo code is presented by Algorithm 3.

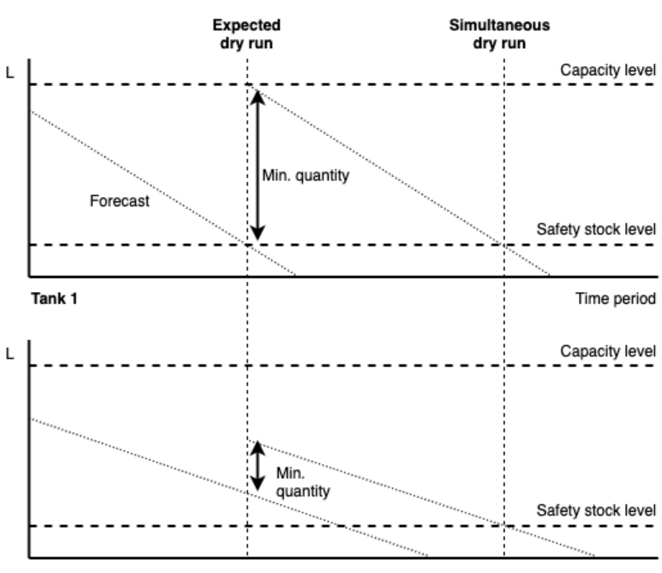

Figure 2: Minimum delivery quantities are set for two tanks at a station to ensure simultaneous dry run in the future.

Algorithm 3: Determine minimum and maximum delivery quantities based on the simultaneous dry run inventory policy

Result: Minimum and maximum quantities for each order

set t_c = current considered time period, T = last day time horizon;
foreach *station in current day order list* **do**
    step 1. set maximum quantities;
    foreach *order for the considered station* **do**
        set maximum delivery quantity to the remaining capacity in the tank;
    step 2. determine second depletion day;
    set current second depletion day = 100;
    foreach *order for the considered station* **do**
        foreach *t in* $[t_c, T]$ **do**
            inventory level = inventory level end previous day - demand during *t*;
            if *inventory level < safety stock* **then**
                if *t < current second depletion day* **then**
                    current second depletion day = *t*;
    step 3. calculate consumption till second depletion day;
    foreach *order for the considered station* **do**
        foreach *t in* $[t_c$*, second depletion day]* **do**
            consumption for tank = current value + demand for day t
    step 4. per day, add the calculated consumption to the minimum delivery quantity;
    until the vehicle capacity is reached;
    set vehicle full = false;
    while *vehicle full is false* **do**
        foreach *t in* $[t_c$*, second depletion day]* **do**
            foreach *order for the considered station* **do**
                if *minimum quantity for tank is lower than consumption till second depletion day* **then**
                    if *combined minimum quantities are higher than vehicle capacity* **then**
                        set vehicle full = true;
                    else
                        add consumption to minimum quantity for tank in order

In step 4 of the algorithm, vehicle capacity is taken into account. If it is not feasible to deliver the minimum delivery quantities with at least one vehicle, the vehicle needs to be visited again before the moment of simultaneous dry run. Minimum quantities are added per day to ensure that the next visit is postponed for as much days as possible.

5.1.4. Phase 4 - Route creation

The route creation model combines stations into a route. Binary variables y_r and z_{ir} are introduced to determine if route r is used. Moreover, with the minimum delivery quantity $q^p_{ir,min}$ following from the previous phase, the roue creation model can be stated as:

$$\text{Minimise} \sum_{r \in R} y_r * d_r \tag{30}$$

Subject to:

$$z_{ir} \leq x_{ir} \qquad \forall i \in N, \forall r \in R \tag{31}$$

$$z_{ir} \leq y_r \qquad \forall i \in N, \forall r \in R \tag{32}$$

$$\sum_{r \in R} z_{ir} = 1 \qquad \forall i \in N \tag{33}$$

$$\sum_{i \in N} (q^p_{ir,min} * z_{ir}) \leq 0.9\, C_{k,min} \qquad \forall r \in R \tag{34}$$

The objective function (30) minimises the total travel distance for the considered time period. Constraints (31) make the choice to visit station i during route r only possible when the station is included in the predetermined route. Constraints (32) link variables y and z. Constraints (33) are used to ensure that each station is visited once. Combining stops in a route is limited by the sum of the minimum delivery quantities to 90% of the total capacity of the largest vehicle by constraints (34). The minimum delivery quantities $q^p_{ir,min}$ must be delivered, so the constraint ensures that the route can be driven with at least one vehicle. The capacity of the vehicle is decreased by 10%, because different types of product need to be transported by different compartments.

5.1.5. Phase 5 - Tank Truck Loading

In the final phase, routes are assigned to trucks while maximising vehicle utilisation. This ensures efficient use of the vehicles. A variant of the Tank Truck Loading Problem (TTLP), which is also addressed by Cornillier et al. (2007), is developed to execute this assignment procedure. Values for $q^p_{ir,min}$ and $q^p_{ir,max}$ are previously determined. Binary variables y^k_r is introduced to determine which vehicle k drives a route r and w^{kmp}_r is introduced to set in which compartment product p is loaded. The model can then be stated as:

$$\text{Maximise} \sum_{r \in R} \sum_{i \in N} \sum_{k \in K} \sum_{m \in M} \sum_{p \in P} q^{kmp}_{ir} \tag{35}$$

Subject to:

$$\sum_{k \in K} \sum_{m \in M} q^{kmp}_{ir} \geq q^p_{ir,min} \quad \forall p \in P, \forall i \in N, \forall r \in R \tag{36}$$

$$\sum_{k \in K} \sum_{m \in M} q^{kmp}_{ir} \leq q^p_{ir,max} \quad \forall p \in P, \forall i \in N, \forall r \in R \tag{37}$$

$$\sum_{p \in P} \sum_{m \in M} q^{kmp}_{ir} \leq L * \delta^k_i \quad \forall i \in N, \forall k \in K \forall r \in R \tag{38}$$

$$\sum_{i \in N} q^{kmp}_{ir} \leq w^{kmp}_r * Q^{km} \qquad \forall k \in K, \forall m \in M, \forall p \in P, \forall r \in R \tag{39}$$

$$\sum_{p \in P} w^{kmp}_r \leq 1 \qquad \forall k \in K, \forall m \in M, \forall r \in R \tag{40}$$

$$\sum_{k \in K} y^k_r = 1 \qquad \forall r \in R \tag{41}$$

$$\sum_{r \in R} (y^k_r * t_r) \leq b^k - a^k \qquad \forall k \in K \tag{42}$$

$$w^{kmp}_r \leq y^k_r \quad \forall k \in K, \forall m \in M, \forall p \in P, \forall r \in R \tag{43}$$

The objective function (35) maximises the total delivered quantity, to ensure efficient vehicle utilisation. Constraints (36-37) set the delivery quantity to be within the range previously determined by the minimum and maximum delivery quantities. Constraints (38) implement station restrictions and constraints (39) limit the delivery quantity per compartment to its capacity. The fact that a compartment can only be filled with one type of product is implemented with constraints (40). Constraints (41) ensure that each route is driven by a vehicle exactly one time and constraints (42) are used to limit the duration of all routes driven by a vehicle to the time available in the driver schedule. Lastly, constraints (43) links variables w and y.

5.2. Heuristic procedure

Since the planning of a day affects the inventory levels of the next day, a dynamic programming approach is required to create the planning day by day. After each iteration, inventory levels are adjusted with the delivery quantities. An overview of this iterative process is shown by Figure 3.

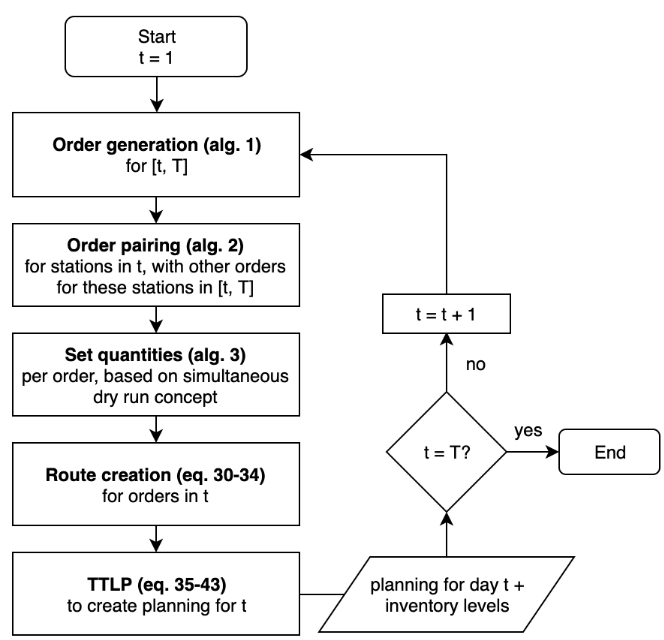

Figure 3: Heuristic procedure, with T days in time horizon

Table 2
Computational results, with computation time (CT) and difference in distance relative to the full MILP model (Δ). Shown results are averages and S4-D5 means an experiment with 4 stations considering 5 days. *not an exact solution **no solution found

Experiment	Full MILP model Distance (km)	CT (s)	Simplified MILP model Distance (km)	Δ	CT (s)	Heuristic Distance* (km)	Δ	CT (s)
S4-D5	267	2	267	0.0%	2	304	13.8%	2
S4-D7	480	469	480	0.0%	90	550	14.6%	4
S6-D5	321	544	326	1.5%	42	413	28.8%	3
S6-D7	582*	7200	583*	0.1%	7200	690	18.5%	6
S8-D5	399	1955	412	3.2%	124	501	25.5%	5
S8-D7	753*	7200	747*	-0.8%	7200	849	12.7%	8
S10-D5	454*	7200	463	1.9%	2132	581	28.1%	6
S10-D7	873*	7200	857*	-1.8%	7200	984	12.7%	14
S12-D5	478*	7200	504*	5.4%	7200	606	26.8%	8
S12-D7	972*	7200	934*	-3.9%	7200	1063	9.4%	18
S20-D7	1376*	7200	-**	-	-**	1642	19.3%	39
S30-D7	-**	7200	-**	-	-**	2281	-	79
Average				**0.6%**			**19.1%**	

Table 3
Computational results for larger instances, with computation time (CT). S20-D14 means 20 stations and 14 days.

Experiment	Initial solution (km)	Final solution (km)	Δ	CT Initial solution (s)	CT Improvement proced. (s)
S20-D14	3931	3670	-6.6%	2	162
S30-D14	5588	5365	-4.0%	4	589
S40-D14	6965	6735	-3.3%	12	727
S50-D14	10267	9813	-4.4%	29	3747
S59-D14	11426	10877	-4.8%	14	5103

For the initial solution, the decision to visit a station is based on the day that one of the tanks deplete. This means that routing is not optimised over the full time horizon and that the stations can not be visited later. To improve the initial solution, an improvement procedure is used with solution candidates that are based on visiting stations one time period earlier. To evaluate if the solution improves, the planning is recreated for the remaining days. Since this is an extensive process, the number of candidates is limited to the number of station visits.

6. Computational results

The mathematical models are implemented in Python and the MILP models are solved using Gurobi Optimizer version 8.1.1 (Gurobi Optimization, 2019). The experiments are executed on a computer with a 3.50 GHz 4-core processor and 32 GB of RAM. A data set based on a petrol distributor is used to evaluate the performance of the models. The full data set consists of one depot, 59 gas stations and 4 vehicles.

Table 2 shows the distance and computation time for solutions found with the MILP models and the heuristic. The results are the average of performed experiments and the computation time is limited to two hours. Exact solutions can be found with the MILP models for instances with up to 10 stations and 5 days. Not exact solutions are found for instances with up to 12 stations and 7 days. A solution for 20 stations and 7 days is found with the full MILP model and not with the simplified MILP model. This can be explained by the fact that the large number of possible routes makes it harder to find a solution.

In terms of computation time, the heuristic outperforms the MILP models. If an exact solution can be found within two hours, the solution is found faster by the simplified MILP model compared to the full MILP model.

The total travel distance for the solutions found by the simplified MILP model, compared to the full MILP model, are exactly the same for the experiments with four stations and slightly different for the other experiments. This can be explained by the fact that the simplified model uses predetermined routes. Moreover, the average increase in travel distance for solutions created by the heuristic, compared to full MILP model, is 19.1%. If only experiments with seven days are considered, the average increase in travel distance is 15.6%.

Table 3 shows the computational results for real-life size heuristic solutions. The initial solution found by the decomposition heuristic is improved during the improvement procedure by 4.6% on average. For the full data set, a solution is found within two hours, what proves that the heuristic is an effective method for solving real-life cases.

7. Conclusions

Planning methods for the multi-period PSRP with real-life characteristics are presented in this paper. Given these characteristics, the models become more representative. This adds complexity as well, what leads to longer computation time for a solver to find a solution to the MILP models. Therefore, a heuristic is presented, which is a combination of a decomposition heuristic and an improvement procedure. The decomposition heuristic uses the new introduced *simultaneous dry run* inventory policy to determine maximum and minimum delivery quantities. The models have been tested with a data set based on the real-life situation for a petrol distribution company. It is found that the MILP models can be used to find a solution within the two hour time limit for instances of up to 20 stations and 7 days. The heuristic can create the planning for the full case study with 59 stations and a considered time horizon of 14 day.

Future research in this area is encouraged. The value of the *simultaneous dry run* inventory policy could be further explored by considering the moment of delivery to increase the maximum delivery quantities. This allows to also replenish the quantity of fuel that has already been sold during the day till the moment of delivery. Another direction for further research could be to simulate deliveries with unexpected events, to determine the effect of uncertainty on the distribution performance.

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Appendix B - Verification and validation data set

Table 1: Travel distances (*1000 m)

From/To	0	1	2	3	4
0	0	107	39	43	82
1	109	0	86	89	36
2	39	85	0	5	48
3	44	88	5	0	51
4	83	36	48	51	0

Table 2: Travel times (*300 s)

From/To	0	1	2	3	4
0	0	23	11	14	18
1	23	0	18	19	9
2	11	18	0	3	11
3	14	19	3	0	13
4	18	8	12	13	0

Table 3: Station data, with i = station number, p = product index, levels are per 100L

i	p	Ls	Lc	Li	Ld-t1	Ld-t2	Ld-t3	Ld-t4	Ld-t5	Ld-t6	Ld-t7
1	3	28	186	141	19	18	39	47	43	42	33
1	2	15	188	131	11	11	8	8	8	8	12
2	5	30	289	213	28	26	31	31	31	35	35
2	6	68	438	216	36	40	112	114	114	111	93
3	5	35	289	202	37	37	39	37	37	42	44
3	6	66	513	386	39	43	89	86	88	91	94
3	4	15	94	52	8	8	9	9	9	10	11
4	5	28	288	174	23	23	25	28	28	29	32
4	6	36	414	215	19	22	31	34	32	31	32
4	4	15	113	51	6	6	5	5	6	6	6

Table 4: Vehicle characteristics

Characteristic	**Value**
Compartments [*100L]	70, 50, 70, 70, 70, 50, 70, 70
Time window	5:00 - 15:30 every day
Station restrictions	All stations can be visited

Appendix C - Results experiments

KPIs are only shown if an exact solution is found by the solver.

Full MILP model results

Experiment	Dist.	RT (s)	Gap (%)	IL (%)	# DR	Del.	Del/trip	VU (%)	# trips	# stops	Stops/trip
S4-D5-1	303	0	0.0%	44.2%	0	440	147	55.7%	2	3	1.50
S4-D5-2	272	1	0.0%	47.4%	0	666	167	84.3%	2	4	2.00
S4-D5-3	202	4	0.0%	43.8%	0	376	75	47.6%	2	5	2.50
S4-D5-4	292	3	0.0%	31.6%	0	954	159	72.3%	3	6	2.00
S4-D7-1	611	354	0.0%	40.8%	0	1045	149	66.1%	4	7	1.75
S4-D7-2	447	13	0.0%	43.5%	0	1672	209	79.2%	5	8	1.60
S4-D7-3	338	1041	0.0%	40.2%	0	962	120	91.6%	3	8	2.67
S4-D7-4	525	435	0.0%	29.9%	0	1836	204	87.0%	5	9	1.80
S6-D5-1	409	78	0.0%	34.8%	0	878	110	66.5%	3	8	2.67
S6-D5-2	278	16	0.0%	43.6%	0	1039	148	98.0%	2	7	3.50
S6-D5-3	305	2062	0.0%	38.8%	0	547	68	52.1%	3	8	2.67
S6-D5-4	291	21	0.0%	34.9%	0	1048	131	79.4%	3	8	2.67
S6-D7-1	777	7200	27.5%								
S6-D7-2	512	7200	6.4%								
S6-D7-3	482	7200	29.7%								
S6-D7-4	557	7200	10.6%								
S8-D5-1	406	328	0.0%	36.4%	0	1050	88	79.6%	3	12	4.00
S8-D5-2	386	82	0.0%	42.1%	0	1129	125	85.5%	3	9	3.00
S8-D5-3	333	5454	0.0%	40.2%	0	776	86	73.9%	3	9	3.00
S8-D5-4	470	7200	11.3%								
S8-D7-1	773	7200	33.8%								
S8-D7-2	712	7200	13.5%								
S8-D7-3	644	7200	45.0%								
S8-D7-4	883	7200	44.6%								
S10-D5-1	414	1754	0.0%	36.8%	0	1326	102	71.7%	4	13	3.25
S10-D5-2	401	7200	4.0%								
S10-D5-3	440	7200	24.1%								
S10-D5-4	560	7200	20.2%								
S10-D7-1	801	7200	41.7%								
S10-D7-2	771	7200	22.6%								
S10-D7-3	818	7200	48.4%								
S10-D7-4	1100	7200	44.5%								
S12-D5-1	423	7200	2.4%								
S12-D5-2	448	7200	9.6%								
S12-D5-3	446	7200	22.9%								
S12-D5-4	595	7200	23.9%								
S12-D7-1	824	7200	46.1%								
S12-D7-2	888	7200	31.0%								
S12-D7-3	877	7200	50.5%								
S12-D7-4	1299	7200	51.3%								

Description of abbreviations used in the table:

- S4-D5-1: 4 stations. 5 days. subset 1
- Dist = travel distance (km)
- RT = run time (s)
- IL = average inventory level (%)
- # DR = number of dry runs
- Del. = total delivered (*100 L)
- Del/trip = delivered per stop (*100 L)
- VU = vehicle utilisation (%)

Simplified MILP model results

Experiment	Dist.	RT (s)	Gap (%)	IL (%)	# DR	Del.	Del/trip	VU (%)	# trips	# stops	Stops/trip
S4-D5-1	303	1	44.0%	0	0.0%	372	124	47.1%	2	3	1.50
S4-D5-2	272	1	47.3%	0	0.0%	666	167	62.8%	2	4	2.00
S4-D5-3	202	4	43.3%	0	0.0%	513	103	64.9%	2	5	2.50
S4-D5-4	292	3	31.7%	0	0.0%	954	159	72.3%	3	6	2.00
S4-D7-1	611	190	42.1%	0	0.0%	1045	149	66.1%	4	7	1.75
S4-D7-2	447	6	44.0%	0	0.0%	1609	201	76.3%	5	8	1.60
S4-D7-3	338	75	41.1%	0	0.0%	944	118	89.9%	3	8	2.67
S4-D7-4	525	7200			14.1%						
S6-D5-1	413	22	35.8%	0	0.0%	799	114	60.5%	3	7	2.33
S6-D5-2	290	6	43.0%	0	0.0%	990	141	62.3%	3	7	2.33
S6-D5-3	307	132	38.6%	0	0.0%	526	75	50.1%	3	7	2.33
S6-D5-4	292	7	34.8%	0	0.0%	1040	149	78.8%	3	7	2.33
S6-D7-1	776	7200			16.2%						
S6-D7-2	512	7200			0.6%						
S6-D7-3	482	7200			13.1%						
S6-D7-4	560	7200			6.8%						
S8-D5-1	420	144	34.6%	0	0.0%	967	121	73.3%	3	8	2.67
S8-D5-2	392	23	41.7%	0	0.0%	1088	121	68.4%	3	9	3.00
S8-D5-3	357	204	39.3%	0	0.0%	722	80	68.8%	3	9	3.00
S8-D5-4	477	301	37.8%	0	0.0%	1590	159	86.0%	4	10	2.50
S8-D7-1	779	7200			19.6%						
S8-D7-2	707	7200			1.4%						
S8-D7-3	620	7200			19.5%						
S8-D7-4	882	7200			22.2%						
S10-D5-1	426	2132	35.9%	0	0.0%	1310	131	70.8%	4	10	2.50
S10-D5-2	419	217	44.4%	0	0.0%	1522	138	71.8%	4	11	2.75
S10-D5-3	447	259	38.2%	0	0.0%	1270	115	69.0%	5	11	2.20
S10-D5-4	558	642	38.7%	0	0.0%	2127	142	89.8%	6	15	2.50
S10-D7-1	798	7200			20.2%						
S10-D7-2	749	7200			6.1%						
S10-D7-3	857	7200			25.0%						
S10-D7-4	1023	7200			22.5%						
S12-D5-1	442	7200			0.2%						
S12-D5-2	526	922	39.9%	0	0.0%	1534	110	64.5%	5	14	2.80
S12-D5-3	453	865	40.6%	0	0.0%	1413	118	59.6%	6	12	2.00
S12-D5-4	594	2338	38.1%	0	0.0%	2560	142	88.3%	7	18	2.57
S12-D7-1	833	7200			21.0%						
S12-D7-2	937	7200			40.0%						
S12-D7-3	840	7200			23.7%						
S12-D7-4	1125	7200			27.0%						

Description of abbreviations used in the table:

- S4-D5-1: 4 stations. 5 days. subset 1
- Dist = travel distance (km)
- RT = run time (s)
- IL = average inventory level (%)
- # DR = number of dry runs
- Del. = total delivered (*100 L)
- Del/trip = delivered per stop (*100 L)
- VU = vehicle utilisation (%)

Heuristic solutions

Experiment	Dist	IL	DR	Deliv	Del/tr	VU	Tr	St	St/Tr	CI	IPT	IT
S4-D5-1	318	45.5%	0	558	186	78.6%	2	3	1.5	1	1	1
S4-D5-2	281	49.4%	0	1053	263	66.2%	3	4	1.3	1	1	1
S4-D5-3	215	42.4%	0	520	104	100.0%	2	5	2.5	1	1	1
S4-D5-4	403	40.7%	0	2160	360	90.8%	5	6	1.2	0	2	1
S4-D7-1	685	47.5%	0	1640	273	80.8%	5	6	1.2	1	2	1
S4-D7-2	549	47.1%	0	2296	328	87.3%	6	7	1.2	2	3	1
S4-D7-3	421	47.5%	0	1505	251	85.5%	5	6	1.2	0	2	1
S4-D7-4	546	40.1%	0	2901	322	91.8%	7	9	1.3	1	4	1
S6-D5-1	492	37.6%	0	1214	202	71.8%	4	6	1.5	2	1	1
S6-D5-2	403	47.2%	0	1883	269	88.8%	4	7	1.8	0	3	1
S6-D5-3	355	43.4%	0	1120	160	91.1%	4	7	1.8	1	2	1
S6-D5-4	402	41.8%	0	2160	309	90.8%	5	7	1.4	2	2	1
S6-D7-1	947	39.1%	0	2664	242	88.5%	7	11	1.6	2	4	1
S6-D7-2	580	44.3%	0	3163	288	99.8%	7	11	1.6	1	6	1
S6-D7-3	561	46.2%	0	2105	263	85.2%	7	8	1.1	0	3	1
S6-D7-4	670	40.4%	0	3391	283	91.7%	8	12	1.5	3	7	1
S8-D5-1	506	39.2%	0	1733	248	75.7%	5	7	1.4	2	2	1
S8-D5-2	520	45.7%	0	2096	233	88.1%	5	9	1.8	0	5	1
S8-D5-3	380	43.9%	0	1310	164	100.0%	4	8	2.0	1	3	1
S8-D5-4	596	40.9%	0	2355	236	99.0%	5	10	2.0	2	4	1
S8-D7-1	963	39.9%	0	3207	247	84.6%	9	13	1.4	5	5	1
S8-D7-2	805	45.2%	0	3663	262	84.4%	9	14	1.6	2	8	1
S8-D7-3	655	45.2%	0	2438	222	95.6%	7	11	1.6	0	6	1
S8-D7-4	969	39.0%	0	4674	292	98.2%	10	16	1.6	5	10	1
S10-D5-1	514	39.9%	0	2263	283	80.3%	6	8	1.3	2	3	1
S10-D5-2	542	44.9%	0	2413	241	91.1%	5	10	2.0	0	5	1
S10-D5-3	578	44.1%	0	2201	200	84.0%	7	11	1.6	1	5	1
S10-D5-4	682	41.6%	0	3145	242	99.2%	7	13	1.9	4	7	1
S10-D7-1	988	38.7%	0	3973	265	91.8%	10	15	1.5	1	8	1
S10-D7-2	872	44.8%	0	4649	273	94.1%	10	17	1.7	1	14	1
S10-D7-3	868	44.6%	0	3627	227	92.1%	10	16	1.6	1	11	1
S10-D7-4	1141	39.9%	0	5979	285	98.7%	13	21	1.6	6	17	1
S12-D5-1	511	37.0%	0	2402	240	85.2%	6	10	1.7	2	4	1
S12-D5-2	610	44.1%	0	2713	209	87.5%	6	13	2.2	3	9	1
S12-D5-3	581	46.5%	0	2389	199	91.2%	7	12	1.7	2	6	1
S12-D5-4	745	41.8%	0	4098	256	96.9%	9	16	1.8	2	10	1
S12-D7-1	1022	37.0%	0	4335	255	92.8%	11	17	1.5	3	11	1
S12-D7-2	1059	44.7%	0	5262	251	99.7%	11	21	1.9	4	19	1
S12-D7-3	874	46.0%	0	4012	236	91.4%	11	17	1.5	1	13	1
S12-D7-4	1298	42.0%	0	7183	276	85.9%	18	26	1.4	4	26	1

Description of abbreviations used in the table:

- S4-D5-1: 4 stations. 5 days. subset 1
- Dist = travel distance (km)
- IL = average inventory level (%)
- DR = number of dry runs
- Deliv = total delivered (*100 L)
- Del/trip = delivered per stop (*100 L)
- VU = vehicle utilisation (%)
- Tr = number of trips
- St = number of stops
- St/Tr = number of stops per trip
- CI = number of actions that led to improvement
- IT = computation time initial solution (s)
- IPT = computation time improvement procedure (s)