Design of fibre-polymer composite structures – European Technical Specification: Combined stresses

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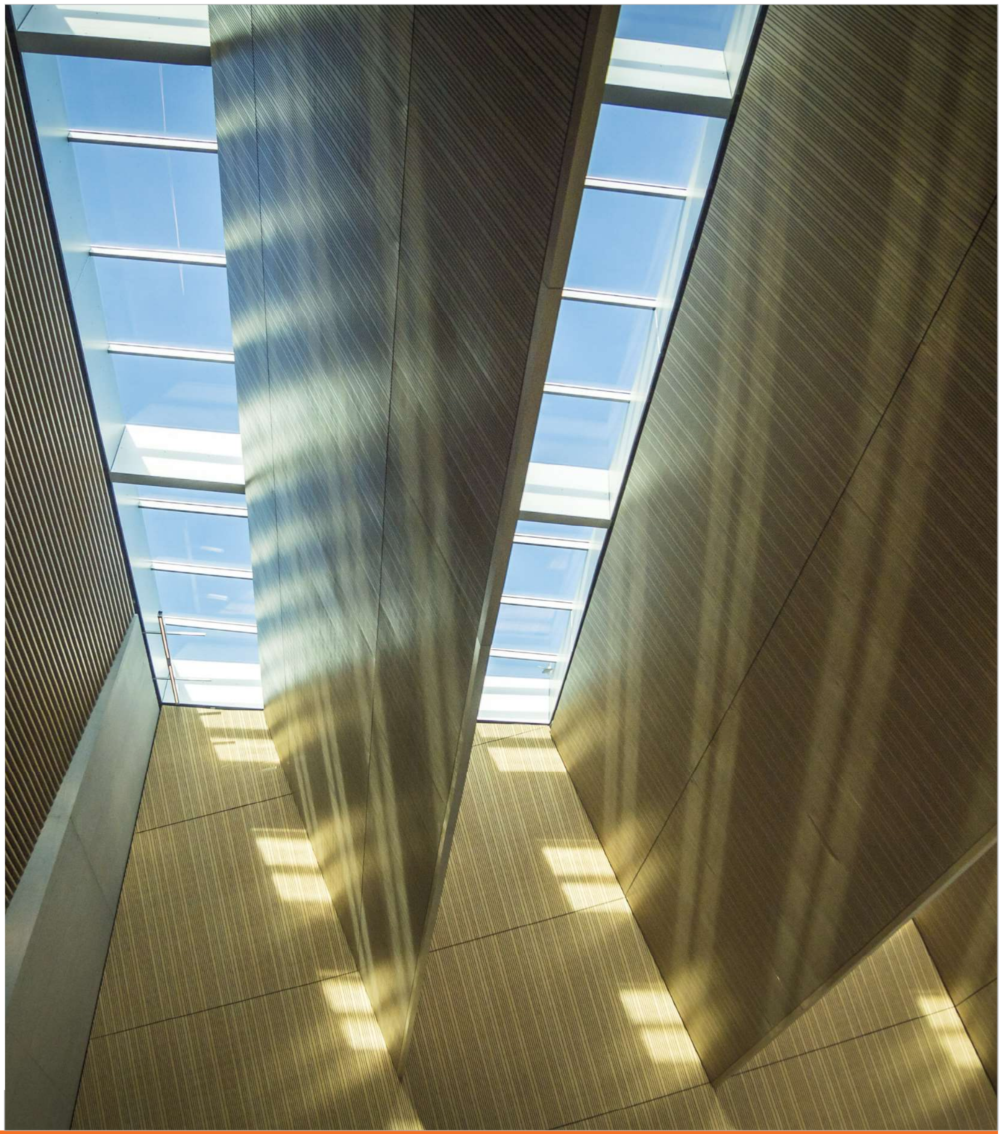

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DESIGN OF FIBRE-POLYMER COMPOSITE STRUCTURES – EUROPEAN TECHNICAL SPECIFICATION: COMBINED STRESSES

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Abstract: *It was essential in the European Technical Specification for the Design of Fibre-Polymer Composite Structures (prCEN/TS 19101) to provide an Ultimate Limit State design procedure for a failure criterion for multi-ply laminates subjected to in-plane combined actions, which will give a generically applicable and a simple design procedure. This paper will discuss the rationale for the project team (of Working Group 4 to CEN/TC250) deciding that the resistance formula is to satisfy a linear interaction failure criterion.*

Keywords: Eurocode; combined stresse; laminates; ULS design

1. Introduction

SAMPE conference paper [1] has a helpful introduction to the CEN Technical Specification for the '*Design of Fibre-Polymer Composite Structures*' [2], by presenting its scope, content together with the main principles and their background. FprCEN/TS 19101 [2] was prepared by Project Team (WG4.T2) with essential technical and scientific support from the wider Working Group 4 (WG4: *Fibre-reinforced Polymer Structures*) to the structural Eurocode committee CEN/TC250. Following an inquiry consultation with the National Standards Bodies (NSBs) version FprCEN/TS 19101 [2] of the Technical Specification was submitted to CEN in November 2021 to prepare for a Formal Vote process, commencing in April 2022. A successful outcome of the vote will be for publication of the TS in January 2023, together with a set of worked examples and a comprehensive commentary document [3]. In this paper, the abbreviation TS is used for CEN/TS 19101 [2]. Several of the composite manufacturing processes permitted by the TS are pultrusion, filament winding, hand lay-up, resin transfer moulding, resin infusion moulding, and vacuum-assisted resin transfer moulding. Chapter 4 in [4] has an introduction to composite laminates of fibre-reinforced polymers made by these and other processes.

The basis of design in the TS is developed in accordance with the general rules given in prEN 1990 [5], supplemented by provisions that are specific to fibre-polymer composites. From the rules for limit state design, the design value of resistance, R_{d}, can be calculated from:

$$R_{\mathrm{d}} = \frac{1}{\gamma_{\mathrm{Rd}} \cdot \gamma_{\mathrm{m}}} R\{\eta_{\mathrm{c},i} . X_{\mathrm{k},i}; a_{\mathrm{d}}; \sum F_{\mathrm{Ed}}\} \tag{1}$$

where: γ_{Rd} is a partial factor accounting for the uncertainty in the resistance model, and for geometrical deviations, if these are not modelled explicitly, according to 4.4.6 of the TS; γ_{m} is a partial factor for a material accounting for the unfavourable deviations of the representative material from their characteristic values; $R\{\ldots\}$ denotes the output of the resistance model; $\eta_{\mathrm{c},i}$ is the conversion factor accounting for effects of temperature and moisture, effects of ageing of materials, according to 4.4.7 of the TS; $X_{\mathrm{k},i}$ represents the characteristic values of material (defined as 5% fractiles); a_d denotes the design values of geometrical parameters; F_Ed denotes the design values of actions used in the assessment of the design value of the effect of actions; i is for the i^th material property. Note that sub-clauses 4.4.6 and 4.4.7 are not reproduced herein.

Ultimate Limit State (ULS) verifications for laminates, profiles and sandwich panels are described in four sub-clauses in Section 8 *Ultimate limit states* and in formative Annex C *Buckling of orthotropic laminates and profiles*. Relevant to this paper is sub-clause 8.2 *Ultimate limit states of laminates* that provides the necessary ULS verifications for balanced symmetrical laminates in cases of in-plane axial, shear and bending stresses, out-of-plane tensile and bending stresses, interlaminar shear stresses, and, for the topic of this paper, in-plane combined stresses.

2. Rationale for having Formula (8.18) for Combined Stresses

Clearly, the TS has the requirement to include section-level design procedures for known modes of failure for ULSs of composite laminates, which are thin-walled [4] and can be flat or curved. What is proposed in the TS involves also laminate- or ply-level procedures. These are suitable for the evaluation of moulded, laminated structures of monocoque or stiffened shell forms, that owing to the complex stress distributions cannot be designed at the section-level and are therefore designed using finite element outputs and a laminate- or ply-level failure criterion [4].

The first stage towards the publication of a new Eurocode is the preparation of a '*Prospect*' by the Joint Research Council. WG4 to CEN/TC250 drafted a second version of a '*Prospect*' report [6], following a NSBs inquiry consultation of the first version. In [6] sub-clause 6.3.1 presents provisions for ULS verifications at the two levels of ply and laminate, but with a different underlying modelling approach and formulae than for the provisions in sub-clause 8.2 of the TS [2]. A '*Prospect*' approach is however in the more elaborate procedure of Annex B7.7 of the TS.

Based on the findings by leading academic and developers of software/numerical codes associated with the premier World-Wide Failure (WWF) exercises [7], it is recognized that, even today, it is not practical to specify a single formula (or theory) to represent the failure of laminates that are subjected to in-plane combined stresses. The conventional, yet complex approach that designers may apply is to establish the resistances of laminates by employing classical lamination theory or higher-order theory to analyse the stress states inside multi-ply laminates subjected to increasing loads up to their design values. As each analysis proceeds there are continual checks at the ply-level for failure using one of the recognized ply-level failure criteria [4, 7]. The laminate's ULS resistance can be established either by first ply failure or last ply failure (when the laminate has ultimately failed). For background details on the application of finite element analyses to numerically predict resistances of laminated plates and shells subjected to combined stresses you can consult Section 5.2 in [4], with sub-section 5.2.6 covering initial failure and progressive damage of laminates to their ultimate failure.

Appendix I below gives four edited extracts from the TS [2]; note that the commentary [3] has technical and scientific information on these paragraphs. One extract is from the informative Annex B *Indicative values of material properties for preliminary design*. Its sub-clause B.7.7 recommends the application of six well-established failure criteria at the ply-level, namely: Maximum Stress; Maximum Strain; Tsai-Hill; Tsai-Wu; Puck; Hashin. It is noted that paragraph B.7.7(2) defines the closed form formula to the Tsai-Hill failure criterion and B.7.7(3) is similar in presenting the formula to the Tsai-Wu failure criterion. No other ply-level failure criterion formula is given in the TS. It is recognised that this complex approach aims to predict the failure of multi-ply laminates more precisely and reliably [4, 7]. The choice of which failure criterion to use can be informed by previous structural analysis experience and/or because of the modelling options available in finite element software (e.g. ABAQUS, ANSYS, Altair HyperWorks™, etc.). What today cannot support the decisions made by designers (and thereby code writers) are the gaps in physical test results and in theoretical developments to enable the Project Team to define precisely which modelling approach and/or failure criterion/criteria is/are the most reliable. This can be seen as a relevant on-going finding and weakness from the pioneering contributions comprising the WWF exercises [7], which were started in 1992. It is observed that one of the main challenges, in addition to having a reliable combined stress criterion at ply-level concerns the establishment of degradation models after first ply failure has occurred.

Towards the TS's preparation by Project Team WG4.T2 is noteworthy that in the '*Prospect*' report [6], sub-clause 6.3.1 provided three laminate-level approaches for ULS verification. Paragraph 6.3.1.2(1) recommends an analysis that corresponds to informative Annex B, B.7.7 of the TS [2]. Whereas, for preliminary design of balanced symmetrical laminates having glass reinforcement and uniaxial loading only, paragraph 6.3.1.2(2) offers a design criterion based on direct or shear strain limits. This approach was not deemed acceptable to go into sub-clause 8.2 [2]. Paragraph 6.3.1.2(3) in the '*Prospect*' is for design by testing, which is discussed for fibre-polymer composite laminates and structures in sub-section 5.1.12 in [4].

Following discussions within the Project Team and, also, via consultations with WG4 members it was recognised that the (informative) Annex B design approach (summarized above) does not provide designers with a quick, non-complex and practical procedure to calculate the resistance of laminates subjected to in-plane combined stresses. In the absence of a more general and consensual interaction failure criterion combining both in-plane and out-plane stresses a linear interaction failure criterion is proposed in 8.2.9 [2]. This sub-clause is given in **Appendix I**, which defines terms and cross-links to other paragraphs in clause 8.2 for the determination of eight different design values of resistance that can be required in specific verifications. It is expected that Formula (8.18) in 8.2.9 is providing a safe (conservative) design solution at the laminate-level. To enable the adoption of a more reliable failure criterion (which is likely to be non-linear, e.g., see [8]), targeted research is needed, including the verification and calibration with test results from relevant physical testing of laminates subjected to varying combined stresses.

The combined stress requirement in Formula (8.18) (see **Appendix I**) is for the three stress components from the actions of axial tensions or compression in the x direction, in-plane shearing and axial tension or compression in the y direction. The x direction is defined as the principal load direction, which coincides with the orientation direction of the laminate with the highest direct stiffness and direct strengths; also referred to as the 0° direction [2, 4].

To introduce how to use Formula (8.18), the first linear-interaction term is given next:

$$\left|\frac{\sigma_{\mathrm{x,t,Ed}}}{f_{\mathrm{x,t,d}}} \text{ or } \frac{\sigma_{\mathrm{x,c,Ed}}}{\min\{f_{\mathrm{x,c,d}}, f_{\mathrm{x,cr,d}}\}}\right| \tag{2}$$

In Eq. (2) the numerators are for the calculated stress in the x direction of the laminate, which depending on the effect of actions from the design load cases can be either tension, subscript t, or compression, subscript c,. The denominator is for the required design value, which is $f_{\mathrm{x,t,d}}$ by 8.2.2.1(1), or either $f_{\mathrm{x,c,d}}$ by 8.2.2.2(2) or $f_{\mathrm{x,cr,d}}$ by 8.2.2.2(3) and Annex C.4.

To establish the design value of the tensile strength Eq. (1) is written as:

$$f_{\mathrm{x,t,d}} = \frac{\eta_c}{\gamma_{\mathrm{m}}\gamma_{\mathrm{Rd}}} f_{\mathrm{x,t,k}} \qquad (3)$$

where $f_{\mathrm{x,t,d}}$ is the characteristic value of the tensile strength in the x direction of the laminate, which is determined using standard coupon testing (in accordance with EN ISO 527) with the batch results analysed for the characteristic value using the procedure in Annex D of prEN 1990 [5]. For establishing $f_{\mathrm{x,c,d}}$, again Eq. (3) can be used, on this occasion with the characteristic value of the compressive strength, $f_{\mathrm{x,c,k}}$, from testing by EN ISO 14126, replacing $f_{\mathrm{x,t,k}}$.

To determine $f_{\mathrm{x,cr,d}}$ (the design value of the critical buckling compressive stress in the x direction of the laminate under uniform compression) the form of Eq. (1) is now:

$$f_{\mathrm{x,cr,d}} = \frac{1}{\gamma_{\mathrm{m}}\gamma_{\mathrm{Rd}}} \chi_{\mathrm{x,c}} f_{\mathrm{x,cr,k}} \qquad (4)$$

where: $f_{\mathrm{x,cr,k}}$ is the characteristic value of the critical buckling compressive stress in the x direction determined analytically using formative Annex C, C.4.2.1 *Compression for flat laminates* (refer to **Appendix I** for extracts from Annex C *Buckling of orthotropic laminates and profiles*), and considering the appropriate values of the conversion factor, η_{c}, for the relevant material properties (defined in 4.4.7 of the TS). Note that characteristics values for the relevant moduli of elasticity (i.e., E_{11}, E_{22} and G_{12}) are the mean values determined using the same ISO standards as for direct strengths. $\chi_{\mathrm{x,c}}$ is the buckling reduction factor for compression in the x direction to consider the effect of imperfections in elastic post-buckling regime. Paragraph 8.2.2.2(4) states that for flat laminates (for which $\chi_{\mathrm{x,c}}$ (or $\chi_{\mathrm{y,c}}$) ≥ 1,0), $\chi_{\mathrm{x,c}}$ or $\chi_{\mathrm{y,c}}$ may be taken as 1,0. Guidance in the TS is that given $\chi_{\mathrm{x,c}}$ (or $\chi_{\mathrm{y,c}}$) is < 1,0 for curved laminates the buckling reduction factor for such laminates can be determined by testing, in accordance with prEN 1990, Annex D [5] and/or by numerical modelling, which should be verified by testing [4].

Although the linear interaction Formula (8.18) has not been, and cannot be verified because of the lack of test data, there is consensus amongst WG4 experts that such a linear interaction failure formula for laminates experiencing in-plane combined stresses should give, on the safe side, a more conservative strength prediction than an interaction formula of higher-order (e.g. a quadratic interaction formulae), such as could be offered by way of the three interaction formulae presented in [8], which have not been verified. Formula (8.18) is therefore offered in 8.2.9 for a quick, non-complex and practical procedure that designers can adopt to carry-out ULS designs of laminates subjected to any combination of in-plane stresses.

3. Application

For the laminate-level failure criterion of Formula (8.18) rectangular fibre-polymer composite plates are to satisfy the conditions of Annex C, C.4.1 (**Appendix I**) and the displacement boundary conditions of a closed form formula (see, e.g., Figure C.1). When subjected to compression stress in either x or y direction or in both directions, and/or an in-plane shear stress a characteristic strength in Formula (8.18) can be for elastic buckling modes of failure (i.e. $f_{\mathrm{x,cr,k}}$ by C.4.2.1, $f_{\mathrm{y,cr,k}}$ by C.4.2.1 or $f_{\mathrm{xy,cr,k}}$ or C.4.2.2). This application of a linear-interaction in a failure criterion is novel, design case specific and owing to lack of test data has not been verified.

To gain an insight into the application of Formula (8.18), let's consider the three effect of actions $\sigma_{\mathrm{x,t,Ed}}$, $\sigma_{\mathrm{y,t,Ed}}$ and $\tau_{\mathrm{xy,Ed}}$, with stress states that ensures elastic buckling is not going to happen. Table B.8 in the TS [2] gives the indicative (characteristic) tensile strengths $f_{\mathrm{x,t,k}}$ and $f_{\mathrm{y,t,k}}$ as 400

MPa for a balanced bidirectional laminate of continuous glass fibre reinforcement (volume fraction of 50%) in an epoxy matrix. Because Table B.8 does not report a characteristic in-plane shear strength we take $f_{\mathrm{xy,v,k}}$ = 50 MPa. To simplify the presentation, it is assumed that $\eta_{\mathrm{c}}/(\gamma_{\mathrm{m}}\cdot\gamma_{\mathrm{Rd}})$ = 1,0 (unfactored). For this laminate example, Table 1 reports nine different limit combinations of in-plane combined stresses where Formula (8.18) equals 1,0 to signal ULS failure. Practically, these stress combinations translate into a three-dimensional failure envelop, where all combinations bounded within the envelop means the laminate does not fail.

Note that by applying sub-clauses 4.4.5 to 4.4.7 of the TS [2], the value of $\eta_{\mathrm{c}}/(\gamma_{\mathrm{m}}\cdot\gamma_{\mathrm{Rd}})$ for material failure of composite laminates is not 1,0, and may be estimated to lie in the bounded range of (0,6·0,6)/(1,23·1,4) = 0,2 (with coefficient of variation V_{x} known and 0,15 for specifying γ_{m}) to (1,0/(1,07·1,4) = 0,67 (V_{x} = 0,05 and known for γ_{m}). When applying Formula (8.18) in design there will be reductions made to the stress magnitudes, such as to those reported in Table 1.

Table 1: Combinations of $\sigma_{\mathrm{x,Ed}}$, $\sigma_{\mathrm{y,Ed}}$ and $\tau_{\mathrm{xy,Ed}}$ (in MPa) that with Formula (8.18) equal to 1.0 are for ULS failure.

$\sigma_{\mathrm{x,t,Ed}}$	$\sigma_{\mathrm{y,t,Ed}}$	$\tau_{\mathrm{xy,Ed}}$	$\sigma_{\mathrm{x,t,Ed}}$	$\sigma_{\mathrm{y,t,Ed}}$	$\tau_{\mathrm{xy,Ed}}$	$\sigma_{\mathrm{x,t,Ed}}$	$\sigma_{\mathrm{y,t,Ed}}$	$\tau_{\mathrm{xy,Ed}}$
400	0	0	160	160	10	40	40	40
0	400	0	130	130	16,25	83	0	40
200	200	0	80	80	30	0	0	50

4. Acknowledgements

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Appendix 1. Extracts from Design of Fibre-polymer Composite Structures (CEN/TC 250: FprCEN/TS 19101:2022) [2]

Extracts from the TS are given using Cambria font type with accompanying notes using Calibra font type. NOTES have been removed unless essential. There are four extracts comprising:

- Paragraphs 8.2.9(1) and (2) for design at ULS of laminates subjected to combined stresses.
- Parargraph B.1 on 'use' of Annex B presenting indicative values of material properties for preliminary design; this annex is informative.
- Paragraph B.7.7 on empirical-based failure criteria for plies.
- Annex C and relevant paragraphs, namley C.1(1), C.2(1), C.3(1) and C.3(2), C.4(1) and C.4(2), and C.4.2.1(1) to introducde elastic buckling of orthotropic laminates; for the load case of uniform compression paragraph 8.2.2.2(3) determines $f_{\mathrm{x,cr,d}}$ in Formula (8.118). Extract from C.4.2.1(1) is incomplete because there's not space to reproduce everything.

8.2.9 Combined Stresses

(1) The resistance of laminates subjected to combined stresses may satisfy a linear interaction failure criterion (which represents a conservative approximation for in-plane stresses). For laminates subjected to in-plane stresses the linear interaction failure criterion should be defined as in Formula (8.18):

$$\left|\frac{\sigma_{\mathrm{x,t,Ed}}}{f_{\mathrm{x,t,d}}} \text{ or } \frac{\sigma_{\mathrm{x,c,Ed}}}{\min\{f_{\mathrm{x,c,d}}, f_{\mathrm{x,cr,d}}\}}\right| + \left|\frac{\tau_{\mathrm{xy,Ed}}}{\min\{f_{\mathrm{xy,v,d}}, f_{\mathrm{xy,cr,d}}\}}\right| + \left|\frac{\sigma_{\mathrm{y,t,Ed}}}{f_{\mathrm{y,t,d}}} \text{ or } \frac{\sigma_{\mathrm{y,c,Ed}}}{\min\{f_{\mathrm{y,c,d}}, f_{\mathrm{y,cr,d}}\}}\right| \qquad (8.18)$$

where: $\sigma_{\mathrm{x,t,Ed}}$ is the design value of the axial tensile stress in the x direction of the laminate; $f_{\mathrm{x,t,d}}$ is the design value of the tensile strength in the x direction of the laminate (8.2.2.1); $\sigma_{\mathrm{x,c,Ed}}$ is the design value of the axial compressive stress in the x direction of the laminate; $f_{\mathrm{x,c,d}}$ is the design value of the compressive strength in the x direction of the laminate (8.2.2.2(2)); $f_{\mathrm{x,cr,d}}$ is the design value of the critical buckling compressive stress in the x direction of the laminate under uniform compression (8.2.2.2(3)); $\tau_{\mathrm{xy,Ed}}$ is the design value of the in-plane shear stress of the laminate; $f_{\mathrm{xy,v,d}}$ is the design value of the in-plane shear strength of the laminate (8.2.3(2))); $f_{\mathrm{xy,cr,d}}$ is the design value of the critical buckling shear stress of the laminate under in-plane shear loading (8.2.3(3)); $\sigma_{\mathrm{y,t,Ed}}$ is the design value of the axial tensile stress in the y direction of the laminate; $f_{\mathrm{y,t,d}}$ is the design value of the tensile strength in the y direction of the laminate (8.2.2.1); $\sigma_{\mathrm{y,c,Ed}}$ is the design value of the axial compressive stress in the y direction of the laminate; $f_{\mathrm{y,c,d}}$ is the design value of the compressive strength in the y direction of the laminate (8.2.2.2(2)); $f_{\mathrm{y,cr,d}}$ is the design value of the critical buckling compressive stress in the y direction of the laminate under uniform compression (8.2.2.2(3)).

(2) As an alternative to 8.2.9(1), the resistance of laminates subjected to combined stresses (including in-plane and out-of-plane directions) may be determined by testing, and/or by analytical formulae using the approach given in Annex B, or numerical modelling, both appropriately verified.

Note that there is not space in this paper to priovide the paragraphs, given in brackets (e.g (8.2.2.1) or (8.2.2.2(3)) for the determination of the design values in Formula (8.18).

Annex B (informative) **Indicative values of material properties for preliminary design**

B.1 Use of this annex

(1) This informative Annex provides supplementary guidance to that given in the Note to 4.3.2(1) and Clause 5 for the physical and mechanical properties of fibres, resins, core materials, composite plies and laminates that can be used for the preliminary design of fibre-polymer composite structures.

B.7.7 Failure criteria for plies

(1) Empirical failure criteria, which have been developed to represent experimental data for failure of single plies of composite laminates under plane stress conditions, may be used.

NOTE: Well-established failure criteria for plies of composite laminates include Maximum Stress, Maximum Strain, Tsai-Hill, Tsai-Wu, Puck and Hashin.

Annex C (normative) **Buckling of orthotropic laminates and profiles**

C.1 Use of this annex

(1) This Normative Annex contains additional provisions to Clause 8 for estimating the elastic buckling resistances of orthotropic laminates and profiles.

C.2 Scope and field of application

(1) This Normative Annex applies to orthotropic laminates and profiles, providing formulae to estimate their elastic buckling resistances. The member types and loading cases covered in this annex are:

- Subclause C.4 is for orthotropic flat laminates with different boundary conditions and under various loading cases.

C.3 General

(1) In general, flexural stiffnesses should be calculated using Classical Laminate Theory (CLT). For orthotropic, symmetric and balanced laminates (e.g., walls of pultruded profiles), when mechanical properties are determined at the laminate level, such stiffnesses should be calculated from Formulae (C.1) to (C.4):

$$D_{11} = \frac{\eta_{\mathrm{c}} \cdot E_{\mathrm{x.c.k}} \cdot t^3}{12\left(1-\upsilon_{\mathrm{xy,k}} \cdot \upsilon_{\mathrm{yx,k}}\right)} \quad \text{(C.1);} \quad D_{12} = \upsilon_{\mathrm{xy,k}} \cdot D_{11} \quad \text{(C.2)}$$

$$D_{22} = \frac{\eta_{\mathrm{c}} \cdot E_{\mathrm{y.c.k}} \cdot t^3}{12\left(1-\upsilon_{\mathrm{xy,k}} \cdot \upsilon_{\mathrm{yx,k}}\right)} \quad \text{(C.3);} \quad D_{66} = \frac{\eta_{\mathrm{c}} \cdot G_{\mathrm{xy.k}} \cdot t^3}{12} \quad \text{(C.4)}$$

where: D_{11}, D_{12}, D_{22} and D_{66} are the longitudinal, coupling, transverse and shear flexural stiffness, respectively; t is the wall thickness (laminate, flange or web); $E_{\mathrm{x,c,k}}$ and $E_{\mathrm{y,c,k}}$ are the characteristic values of the elastic moduli in compression in the x and y directions; $G_{\mathrm{xy,k}}$ is the characteristic value of the in-plane shear modulus; $\nu_{\mathrm{xy,k}}$ and $\nu_{\mathrm{yx,k}}$ are the characteristic values of major and minor Poisson's ratios, respectively.

(2) When the in-plane moduli of a composite laminate in a given direction is significantly different from the flexural moduli in the same direction, the flexural moduli should be considered in Formulae (C.1) to (C.3).

C.4 Elastic buckling of orthotropic laminates

C.4.1 Scope

(1) Subclause C.4 provides formulae to estimate the critical elastic buckling stresses of flat rectangular laminates that have orthotropic in-plane elastic constants, a balanced symmetrical lamination configuration, width-to-thickness ratio higher than 20 and length-to-width ratio higher than 5, for specific boundary conditions.

NOTE 1: The formulae in subclause C.4 are for elastic critical buckling stresses (bifurcation) of geometrically perfect laminates.

NOTE 3: For flat laminates having width-to-thickness ratio higher than 20 and length-to-width ratio lower than 5, the formulae in subclause C.4 provide conservative estimates of elastic critical buckling stresses.

(2) The critical elastic buckling stresses of laminates (bifurcation) having (i) width-to-thickness ratio lower than 20, or (ii) curvature should be determined by numerical modelling.

C.4.2 Orthotropic symmetrical laminates

C.4.2.1 Compression

(1) The characteristic value of the critical buckling compressive stress of a laminate under in-plane compression loading for the different boundary conditions illustrated in Figure C.1, $f_{i,\mathrm{cr,k}}$, should be calculated from Formulae (C.5) to (C.6):

— Both edges simply supported (SS) (Figure C.1a):

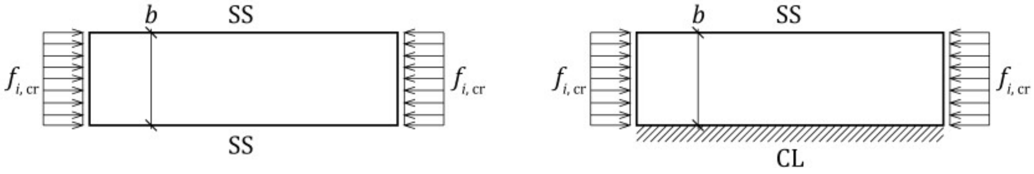
$$f_{i,\mathrm{cr,k}} = \frac{\pi^2}{t \cdot \mathrm{b}^2}\left[2\sqrt{D_{11} \cdot D_{22}} + 2(D_{12} + 2 \cdot D_{66})\right] \tag{C.5}$$

— One edge simply supported (SS) and one edge clamped (CL) (Figure C.1b):

$$f_{i,\mathrm{cr,k}} = \frac{\pi^2}{t \cdot \mathrm{b}^2}\left[3{,}13\sqrt{D_{11} \cdot D_{22}} + 2{,}33(D_{12} + 2 \cdot D_{66})\right] \tag{C.6}$$

where: b is the width of the laminate (perpendicular to the compressive stress direction)

NOTE: In Formulae (C.5) and (C.6) i is either for the x or y direction of the laminate (i.e., longitudinal or perpendicular to the laminate width).



a) Simply-supported (SS) - (SS)   b) Simply-supported (SS) – Clamped (CL)

Figure C.1 — *Orthotropic laminate under in-plane compression with different boundary conditions for the edge(s): Simply Supported (SS), Free (Free) or Clamped (CL).*

To use Annex C, C.4.2.1 with the buckling reduction factor for compression, $\chi_{\mathrm{x,c}}$ or $\chi_{\mathrm{y,c}}$ = 1,0, the designer must be designing with laminates that are flat and of constant thicknesses, and are without geometrical curvature or with significant geometrical or other imperfections. For valid geometries, the rectangular plates will be relatively thin compared to their edge lengths, with the minimum edge length/thickness ratio defined in C.4.1. $f_{\mathrm{x,cr,k}}$, or $f_{\mathrm{y,cr,k}}$, are obtained using the formulae in C.4.2.1 with the relevant longer edges' displacement boundary conditions, see Figure C.1. If the shape of compressed laminates is not rectangular then the designer can determine elastic critical buckling resistances using an appropriate numerical methodolgy.