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Preconditioners for Multi-Screen Scattering

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***Abstract*—In this contribution, a well-conditioned method for the modelling of scattering by so-called multi-screens or PEC sheets including junctions is introduced. The method starts from the *inflated* screen approach by Claeys and Hiptmair. We introduce a Calderón preconditioner and a suitable discretisation scheme. The resulting scheme contains many more DoFs than strictly required. We will show how almost all redundancy can be removed without significant loss of effectiveness of the method.**

I. Introduction

In [3], a variational formulation for the scattering by multi-screens has been introduced and analysed. The problem is posed on the jump space: the quotient-space of the multi-trace space with all single-trace (non-radiating) currents. In [2] a discretisation for this quotient-space is presented, and a strategy to reduce the number of degrees of freedoms (DoFs) is proposed. In this contribution, we construct a Calderón preconditioner that works for the quotient-space discretisation. We explore ways to reduce the representation of the quotient-space such that preconditioners can be constructed efficiently. It turns out that the choice that seems most straightforward does not result in the desired efficiency, and extra care is required in choosing these spaces.

II. Equations and Discretisation

We define simple screens as open surfaces in $\mathbb{R}^3$ without junctions. For orientable simple screens Γ_p and Γ_q, with their chosen normals n_p and n_q, the single layer operator is

$$T_{pq}j = -\, n_p \times i\kappa \int_{\Gamma_q} \frac{e^{-i\kappa|x-y|}}{4\pi|x-y|} j(y)dy + n_p \times \frac{1}{i\kappa} \operatorname{grad} \int_{\Gamma_q} \frac{e^{-i\kappa|x-y|}}{4\pi|x-y|} \operatorname{div} j(y)dy, \quad (1)$$

with $x \in \Gamma_p$. Consider a multi-screen Γ containing a single junction γ. Three simple screens $\Gamma_i, i = 1, 2, 3$ meet at the junction. We decompose Γ as

$$\Gamma = \cup_{i=1}^{3} \Gamma_{i,i+1} \quad (2)$$

with $\Gamma_{i,j} = \Gamma_i \cup \Gamma_j$. The normal on $\Gamma_{i,i+1}$ is chosen 'outward', i.e. such that near the junction it points away from other screens. Each simple screen Γ_i appears once as the 'front' and once as the 'back' of the multi-screen (Fig. 1). The simple screens Γ_i are meshed with target element size h, with the meshes matching up along γ. The simple screens $\Gamma_{i,j}$ inherit this mesh.

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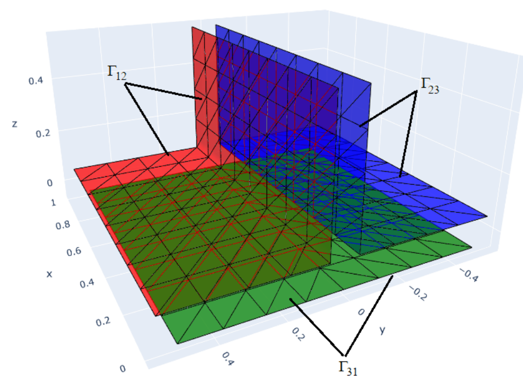

Fig. 1. Inflated screen. The discrete multi-trace contains two DoFs per edge not on the junction, and three DoFs per edge on the junction.

Let $U_{i,i+1}$ be the Raviart-Thomas spaces on $\Gamma_{i,i+1}$ subject to the condition $m \cdot f = 0$ on $\partial\Gamma_{i,i+1}$, with m the bi-normal on $\partial\Gamma_{i.i+1}$ (the tangent to $\Gamma_{i,i+1}$ at $\partial\Gamma_{i,i+1}$, normal to $\partial\Gamma_{i,i+1}$). The discrete multi-trace space is built as the direct product of these spaces, i.e. $U = \prod_{i=1}^{3} U_{i,i+1}$. A variational description of the scattering problem reads: Find $j \in U$ such that, for all $k \in U$:

$$\ll n \times k, Tj \gg_{\Gamma} = -\frac{1}{\eta} \ll n \times k, n \times e^{inc} \gg_{\Gamma} . \quad (3)$$

The double parentheses denote the multi-screen duality pairing

$$\ll u, v \gg_{\Gamma} = \sum_{i=1}^{3} < u, v >_{\Gamma_{i,i+1}} . \quad (4)$$

Note that (3) does not yield a unique solution. Fortunately, for regular incident fields, Fredholm compatibility is ensured. Moreover, all solutions of (3) radiate the same field, so the representation converged to by the iterative solver does not affect physical quantities.

The matrix for (3) is severely ill-conditioned, leading us to look for an efficient preconditioner. Inspired by the success of Calderón preconditioning, the space $V = \prod_{i=1}^{3} V_{i,i+1}$ is defined as the space of Buffa-Christiansen functions [1], dual to U and with basis $(g_n)_{n=1}^{N}$. Denote the standard basis of U by $(f_n)_{n=1}^{N}$. The following preconditioned system is proposed:

$$\mathbf{N}^{fg}\mathbf{T}_{gg}\mathbf{N}^{gf}\mathbf{T}_{ff}\mathsf{x}^{f} = \mathbf{N}^{fg}\mathbf{T}_{gg}\mathbf{N}^{gf}\mathsf{e}_{f} \quad (5)$$

with $(\mathsf{T}_{ff})_{m,n} = \ll n \times f_m, Tf_n \gg_\Gamma$, $\mathsf{N}^{gf} = (\mathsf{N}_{fg})^{-1}$, $(\mathsf{N}_{fg})_{m,n} = \ll n \times f_m, g_n \gg_\Gamma$, etc. Note that N_{fg} is block-diagonal, subordinate to the partitioning (2) of Γ.

Unfortunately, this strategy is flawed. In fact, for simple screens, the range of $\mathsf{N}^{gf}\mathsf{T}_{ff}$ coincides with the non-radiating subspace of V, which in turn coincides with the nullspace of $\mathsf{N}^{fg}\mathsf{T}_{gg}$. This implies the preconditioned system matrix is zero. To fix this, a block-diagonal bilinear form is introduced:

$$\tilde{t}(k, j) = \sum_{i=1}^{3} < n \times k_{i,i+1}, Tj_{i,i+1} >_{\Gamma_{i,i+1}} \tag{6}$$

with $j = (j_{i,i+1})_{i=1}^3$ and $k = (k_{i,i+1})_{i=1}^3$. As opposed to the *full* single layer bilinear form, this block-diagonal form gives rise to invertible matrices. The nullspace of the system does not grow upon application of the preconditioner. The preconditioned system becomes

$$\mathsf{N}^{fg}\tilde{\mathsf{T}}_{gg}\mathsf{N}^{gf}\mathsf{T}_{ff}\mathsf{x}^f = \mathsf{N}^{fg}\tilde{\mathsf{T}}_{gg}\mathsf{N}^{gf}\mathsf{e}_f \tag{7}$$

where the full single layer operator T_{gg} has been replaced by the block-diagonal single layer operator $\tilde{\mathsf{T}}_{gg}$.

III. Reduction of Degrees of Freedom

Numerical examples demonstrate that this preconditioner is highly effective in reducing the number of iterations required for the approximate solution of the scattering problem. However, the space on which the problem is posed contains a very large subspace of non-radiating currents. As explained above, this redundancy does not lead to incorrect solutions, but it does lead to unnecessary computations, which is not desirable. In the following, it is investigated how the multi-trace space can be reduced whilst retaining the efficiency of the preconditioner.

It is tempting to try to reduce the DoFs to the absolute minimum, and choose them so that its span only intersects the single-trace subspace at 0. The saw-tooth reduction (Fig. 2) is such a reduction. Unfortunately, this results in a residual branch of eigenvalues piling up at zero, and a number of iterations that increases as the mesh parameter tends to zero.

The most economic discretisation that leads to Calderón preconditioning with the expected iteration count is one where the full finite element space on Γ_{12} is retained, together with the finite elements on $\hat{\Gamma}_{23}$, where $\hat{\Gamma}_{23}$ is the union of all triangles in Γ_{23} that either are not on Γ_2 or have a vertex in common with the junction. Roughly speaking, this amounts to extending Γ_3 with a single strip of elements from Γ_2 beyond the junction. Fig. 2 shows the two reduced discretisations. The primal space for the strip reduction is $\hat{U} = U_{12} \times \hat{U}_{23}$, with U_{12} and $\hat{U}_{23}$ the finite element spaces spanned by standard bases of RWG functions attached to *internal* edges of the meshes on Γ_{12} and $\hat{\Gamma}_{23}$, respectively.

IV. Numerical Example

Consider a plane wave $e^{inc}(x) = (1, 0, 0)^T \exp\left(-i\frac{2\pi}{\lambda}x_3\right)$ with $\lambda = 2\pi$ meter. This field illuminates the structure in Fig. 1. The problem is solved with the strip and saw-tooth reduced quotient-space boundary element method. The far fields for these solutions agree (Fig. 3). The number of iterations for a tolerance of $2e-5$ for the preconditioned saw-tooth reduced method is much smaller than that for the unpreconditioned unreduced solver and depends only mildly on h. It is reduced by another factor of 2 by the preconditioned strip reduced method (Fig 3).

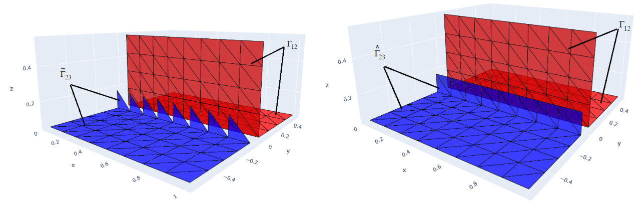

Fig. 2. Two strategies to reduce the degrees of freedom in the multi-trace space. Left (**saw-tooth mesh**): the RWG space on this mesh is a complement of minimal dimension of the non-radiating subspace. Right (**strip mesh**): DoFs at edges in the overlapping strip are redundant for description of the jump space but required in building an efficient preconditioner.

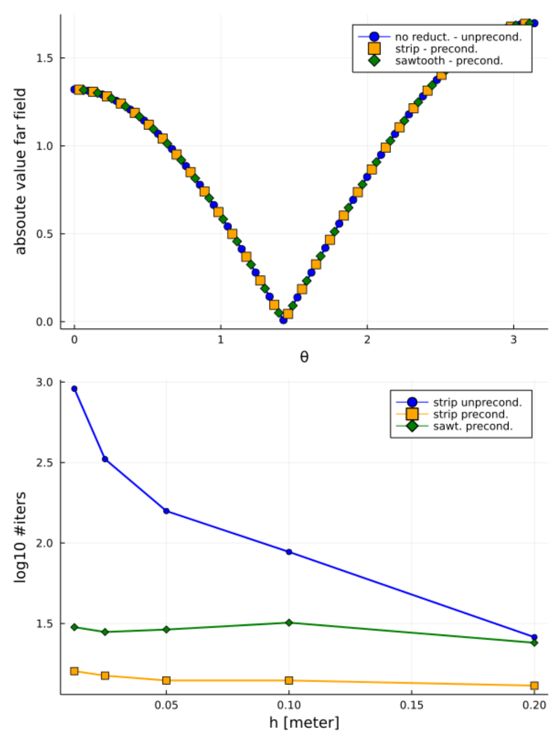

Fig. 3. Top: far fields for the unpreconditioned, unreduced inflated screen method and the precondition strip and sawtooth-reductions agree exactly. Bottom: number of iterations required for GMRES convergence. At $h = 0.0125$ the strip reduction outperforms the sawtooth reduction by a factor of 2.

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