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**ICCS23 - 23<sup>rd</sup> International Conference on Composite Structures & MECHCOMP6 - 6<sup>th</sup> International Conference on Mechanics of Composites**

**Simultaneous temperature-strain measurement in a thin composite panel with embedded tilted Fibre Bragg Grating sensors**

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*FEUP-Faculty of Engineering, University of Porto, Portugal, 01-04 September 2020*

## Introduction

### Principle of Virtual Displacements for composite plates

$$\int_V \delta \epsilon^T \boldsymbol{\sigma} dV + \int_V \rho \delta \mathbf{u}^T \ddot{\mathbf{u}} dV = \int_V \delta \mathbf{u}^T \bar{\mathbf{t}} dV$$

$$\int_{\Omega} \int_A \delta \epsilon^T \boldsymbol{\sigma} d\Omega dz + \int_{\Omega} \int_A \rho \delta \mathbf{u}^T \ddot{\mathbf{u}} d\Omega dz = \int_{\Omega} \int_A \delta \mathbf{u}^T \bar{\mathbf{t}} d\Omega dz$$

$$\boldsymbol{\epsilon} = \begin{bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{xy} \\ \epsilon_{xz} \\ \epsilon_{yz} \\ \epsilon_{zz} \end{bmatrix} = \begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\ \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \\ \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \\ \frac{\partial w}{\partial z} \end{bmatrix} \quad \boldsymbol{\sigma} = \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \\ \sigma_{zz} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{16} & 0 & 0 & C_{13} \\ C_{12} & C_{22} & C_{26} & 0 & 0 & C_{23} \\ C_{16} & C_{26} & C_{66} & 0 & 0 & C_{36} \\ 0 & 0 & 0 & C_{55} & C_{45} & 0 \\ 0 & 0 & 0 & C_{45} & C_{44} & 0 \\ C_{13} & C_{23} & C_{36} & 0 & 0 & C_{33} \end{bmatrix} \begin{bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{xy} \\ \epsilon_{xz} \\ \epsilon_{yz} \\ \epsilon_{zz} \end{bmatrix}$$

\*J. N. Reddy and D. H. Robbins. "Theories and computational models for composite laminates". Appl. Mech. Rev., 47:147–165, 1994.

### PVD for partially coupled thermo-mechanical static problems

$$\int_{\Omega} \int_A \delta \epsilon^T (\boldsymbol{\sigma}_M - \boldsymbol{\sigma}_T) d\Omega dz = 0$$

$$\int_{\Omega} \int_A \delta \epsilon^T \boldsymbol{\sigma}_M d\Omega dz = \int_{\Omega} \int_A \delta \epsilon^T \boldsymbol{\sigma}_T d\Omega dz$$

$$\delta \mathbf{u} : \mathbf{K} \mathbf{u} = \mathbf{P}_T$$

$$P_{T,x} = \lambda_6 \int_A F_r \vartheta_A dz \int_{\Omega} \frac{\partial N_i}{\partial y} \vartheta_{\Omega} d\Omega + \lambda_1 \int_A F_r \vartheta_A dz \int_{\Omega} \frac{\partial N_i}{\partial x} \vartheta_{\Omega} d\Omega$$

$$P_{T,y} = \lambda_2 \int_A F_r \vartheta_A dz \int_{\Omega} \frac{\partial N_i}{\partial y} \vartheta_{\Omega} d\Omega + \lambda_6 \int_A F_r \vartheta_A dz \int_{\Omega} \frac{\partial N_i}{\partial x} \vartheta_{\Omega} d\Omega$$

$$P_{T,z} = \lambda_3 \int_A \frac{\partial F_r}{\partial z} \vartheta_A dz \int_{\Omega} N_i \vartheta_{\Omega} d\Omega$$

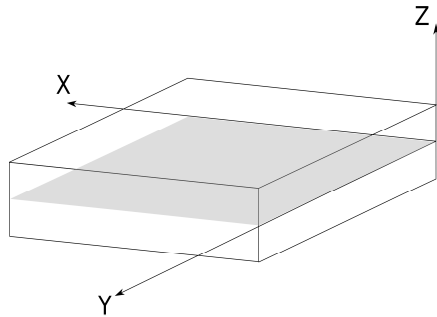
\*M. Cinefra, S. Valvano, and E. Carrera, "Heat conduction and thermal stress analysis of laminated composites by a variable kinematic mitc9 shell element," Curved and Layered Structures, vol. 2, pp. 301–320, 2015.

\*M. Cinefra, S. Valvano, and E. Carrera, "Thermal Stress Analysis of laminated structures by a variable kinematic MITC9 shell element," J. Therm. Stresses, vol. 39, no. 2, pp. 121–141, 2016

### Galerkin solution for Virtual Displacements

Approximate solution based on the generalized Galerkin method

2D approximation of **displacements** using the *thickness functions*



$$u(x, y, z) = \sum_{m=0}^n u_m(x, y)g_m(z)$$

$$v(x, y, z) = \sum_{m=0}^n v_m(x, y)g_m(z)$$

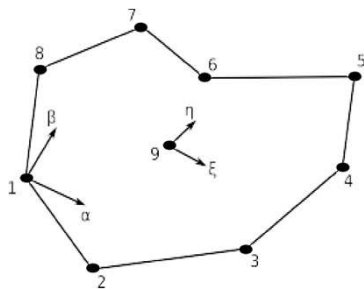
$$w(x, y, z) = \sum_{m=0}^n w_m(x, y)g_m(z)$$

\*K. Wahsizu, "Variational methods in elasticity and plasticity," Pergamon Press Ltd., Headington Hill Hall, Oxford OX3, UK, 1968.

### Finite Element Method

Approximation of **variables** in the mid-reference surface using the *Langrangian* shape functions:

$$u_m(x, y) = \sum_{i=1}^9 N_i(\xi, \eta) u_{mi}$$

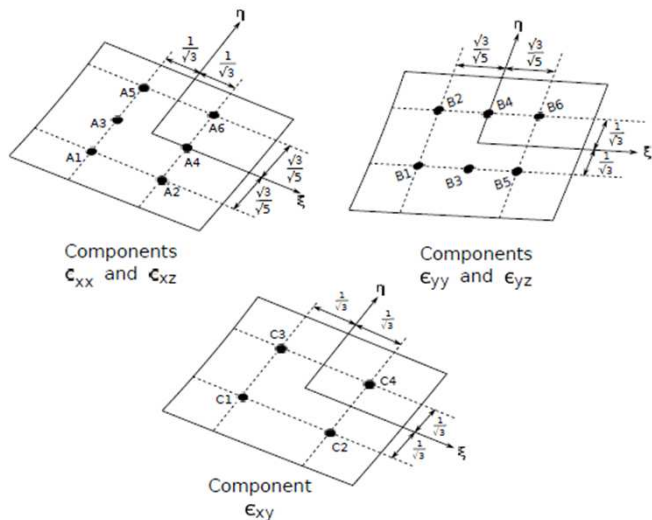


For example:

$$\epsilon_{xx} = N_{A1} \epsilon_{xxA1} + N_{B1} \epsilon_{xxB1} + N_{C1} \epsilon_{xxC1} + N_{D1} \epsilon_{xxD1} + N_{E1} \epsilon_{xxE1} + N_{F1} \epsilon_{xxF1}$$

### MITC

To overcome the problem of the **membrane and shear locking**, the strain components are calculated using a specific interpolation strategy:



## Higher-Order Layer-Wise Approach (LW)

### Legendre Polynomial

$$\mathbf{g}_d^k(z) = \mathbf{g}_0^k(z) + \mathbf{g}_1^k(z) + \mathbf{g}_2^k(z) + \mathbf{g}_3^k(z) + \mathbf{g}_4^k(z)$$

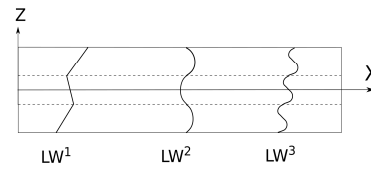
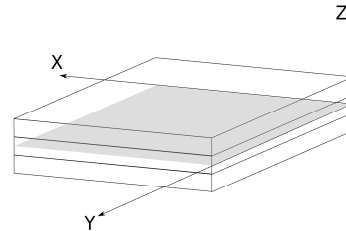
$$\mathbf{g}_0^k(z) = \left(\frac{1+\zeta}{2}\right) \text{ (top)}$$

$$\mathbf{g}_1^k(z) = \left(\frac{1-\zeta}{2}\right) \text{ (bottom)}$$

$$\mathbf{g}_2^k(z) = \frac{3(\zeta^2-1)}{2}$$

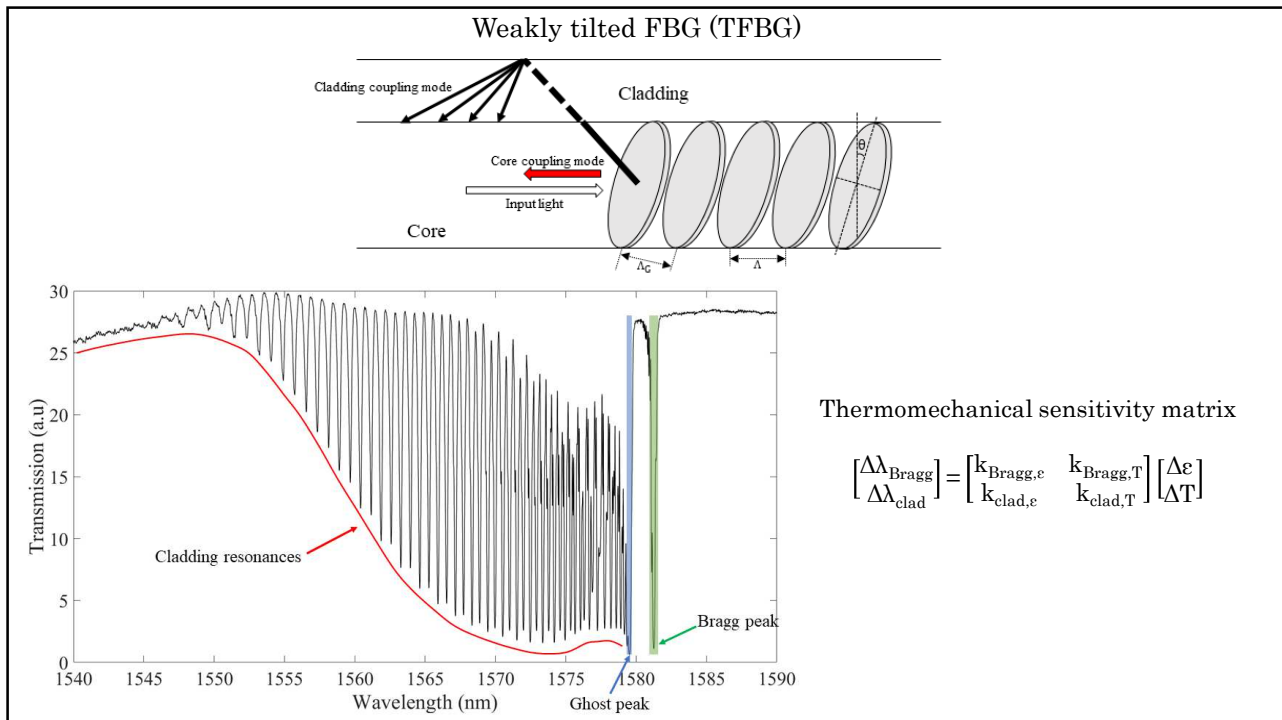
$$\mathbf{g}_3^k(z) = \frac{5\zeta(\zeta^2-1)}{2}$$

$$\mathbf{g}_4^k(z) = \frac{(35\zeta^4-42\zeta^2-1)}{8}$$



\*J. N. Reddy. An evaluation of equivalent single-layer and layerwise theories of composite laminates. Compos. Struct., 25:21-35, 1993.

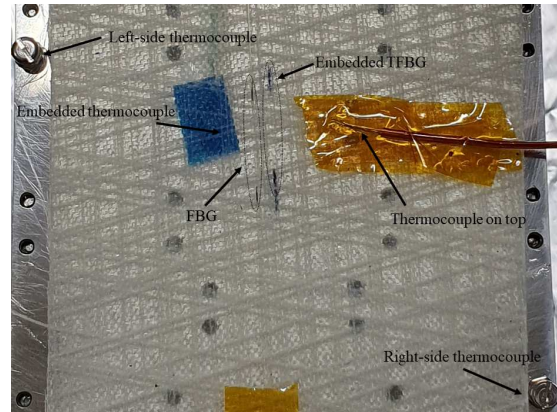
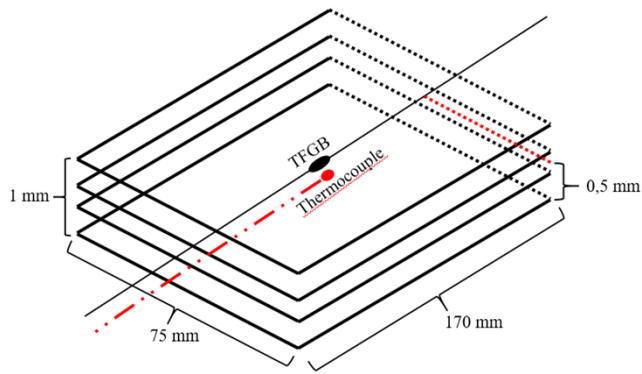
## Tilted Fibre Bragg Grating (TFBG) sensor



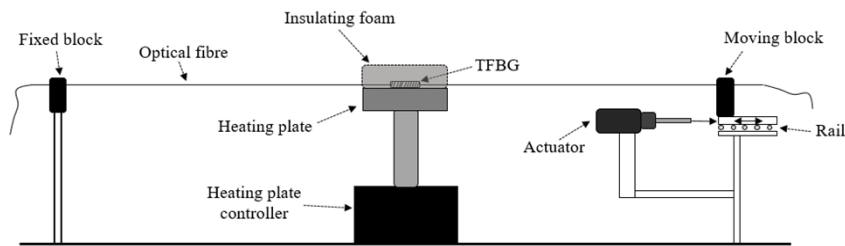
## Methodology

### Design and setup

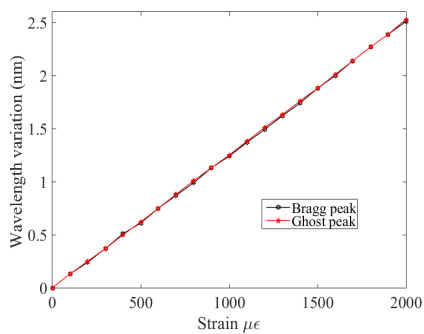
Glass Fibre/Epoxy resin plate with embedded TFBG sensor and thermocouple.



### TFBG calibration

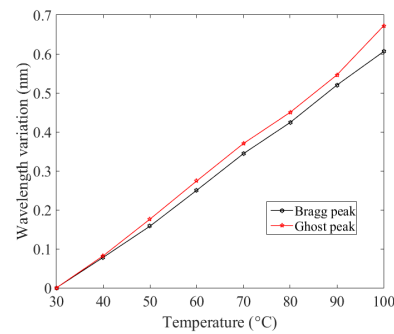


Thermal resolution  
 $swR = 4 \text{ pm} \Rightarrow TR = 8.2305 \text{ }^\circ\text{C}$



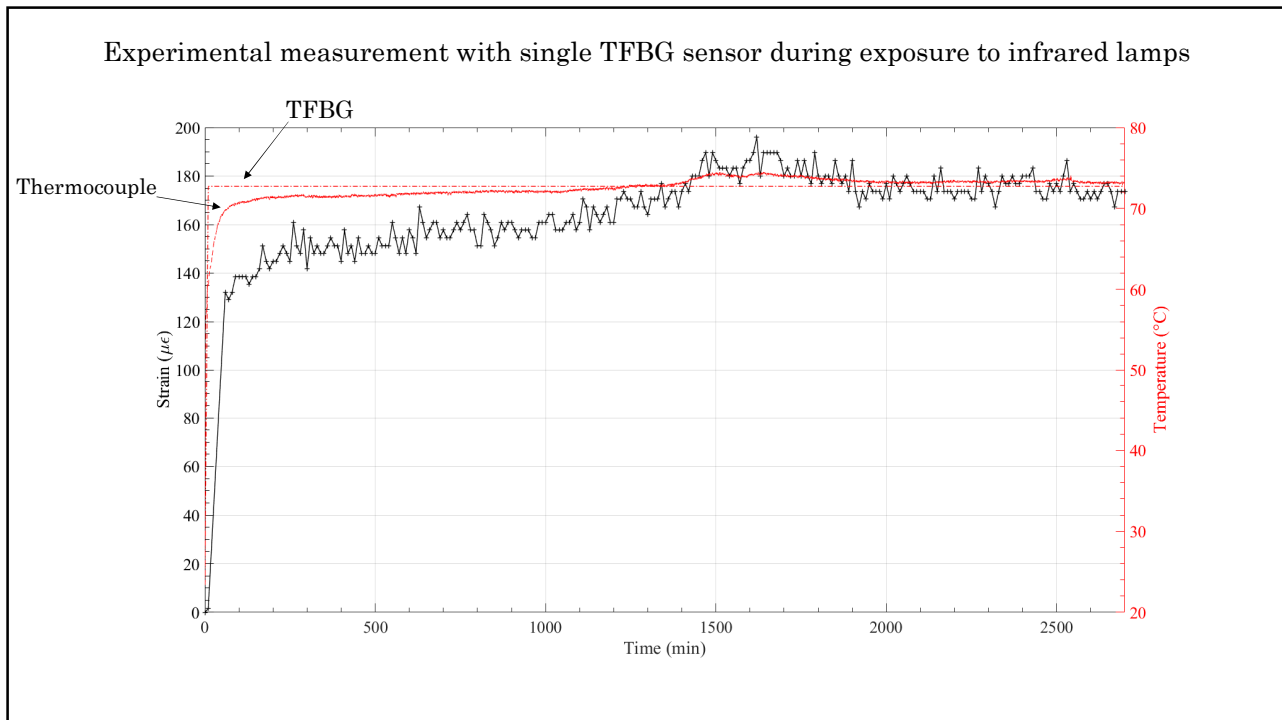
$$k_{\text{Bragg},\epsilon} = 1.255 \pm 0.004 \text{ pm}/\mu\epsilon$$

$$k_{\text{clad},i,\epsilon} = 1.255 \pm 0.006 \text{ pm}/\mu\epsilon$$



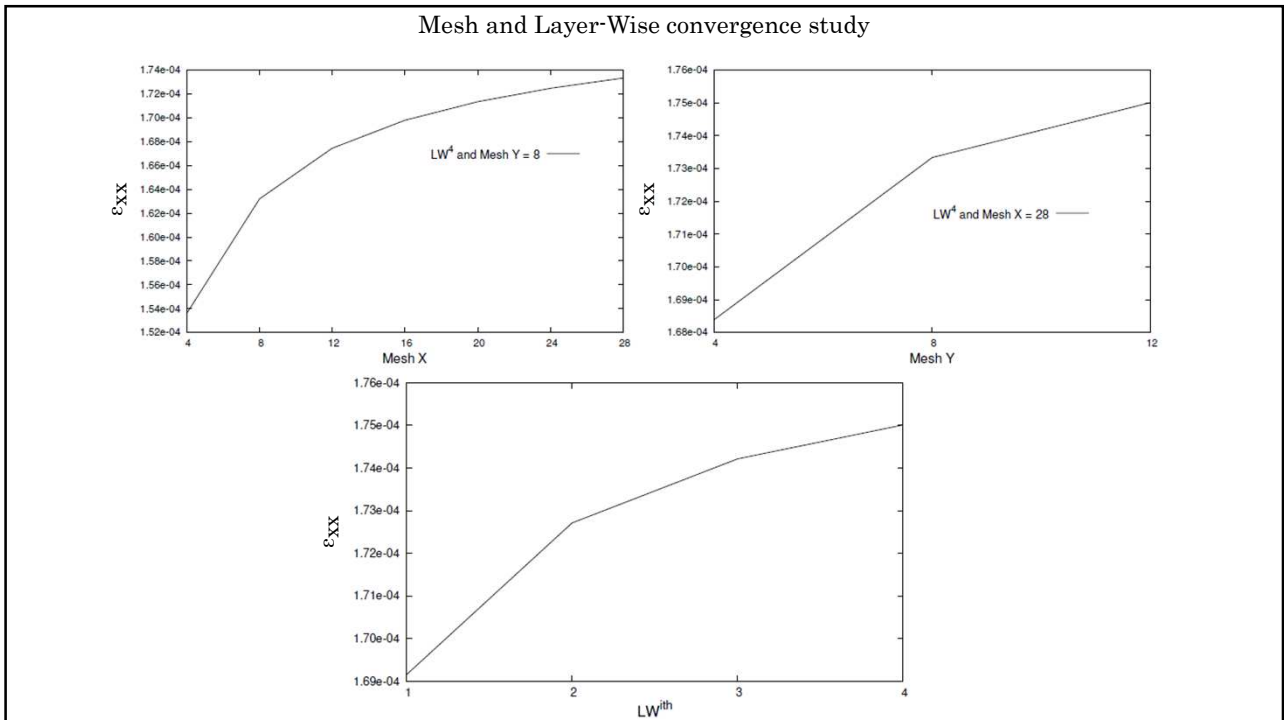
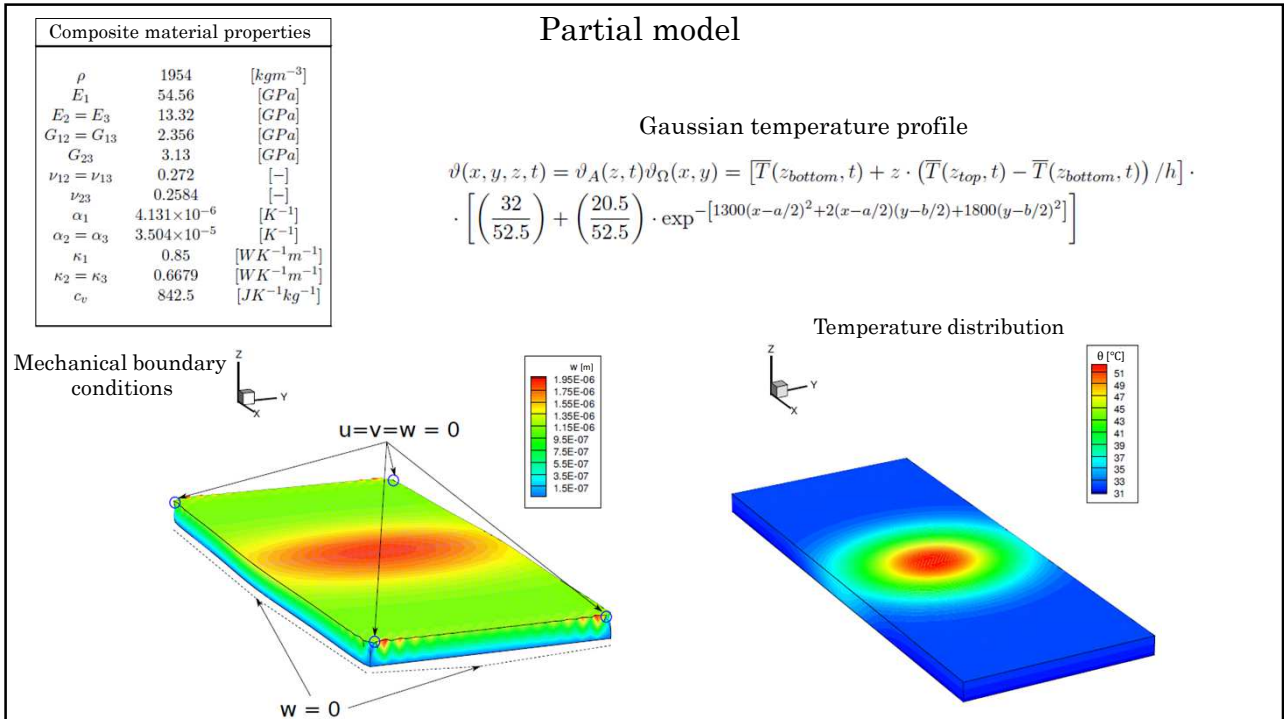
$$k_{\text{Bragg},T} = 9.114 \pm 0.007 \text{ pm}/^\circ\text{C}$$

$$k_{\text{clad},i,T} = 9.6 \pm 0.01 \text{ pm}/^\circ\text{C}$$



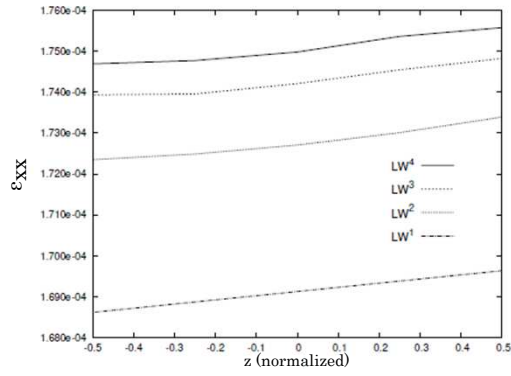
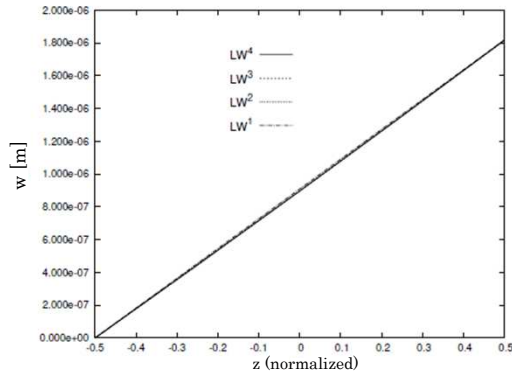
Modelling and numerical simulation





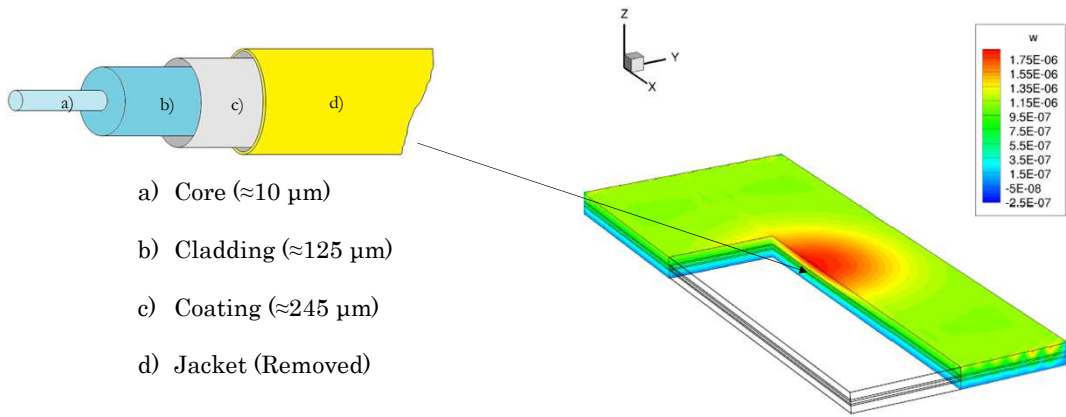
Error, transverse displacement ( $w$ ) and in-plane strain

Method	$\epsilon_{xx} \cdot 10^{-4}$	Error %
<i>Experimental</i>	1.754	
Without Correction	1.702	2.95
<i>Selective</i>	1.702	2.95
<i>MITC</i>	1.750	0.22



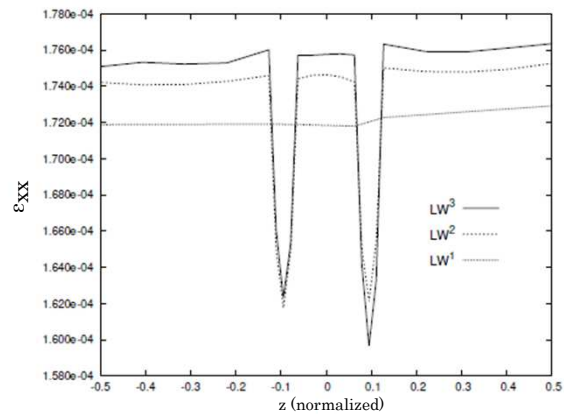
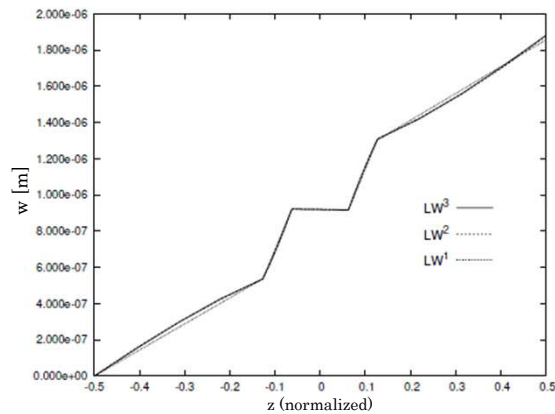
Full model

	Optical fiber core	Acrylate coating	
$\rho$	2300	1100	$[kgm^{-3}]$
$E_1 = E_2 = E_3$	73.1	3.1	$[GPa]$
$G_{12} = G_{13} = G_{23}$	31.5	1.7	$[GPa]$
$\nu_{12} = \nu_{13} = \nu_{23}$	0.16	0.36	$[-]$
$\alpha_1 = \alpha_2 = \alpha_3$	$5.5 \times 10^{-7}$	$7.8 \times 10^{-5}$	$[K^{-1}]$
$\kappa_1 = \kappa_2 = \kappa_3$	1.4	0.189	$[WK^{-1}m^{-1}]$
$c_v$	703	1360	$[JK^{-1}kg^{-1}]$



### Transverse displacement ( $w$ ) and in-plane strain

	FEM Simplified Model	FEM Full Model	Experimental TFBG	Experimental compensated
$3D$			1.754	1.711
$LW^3$	1.742	1.754		
$LW^2$	1.727	1.743		
$LW^1$	1.691	1.721		



### Conclusions

- The results regarding the strain in the Full model report a good matching with the measurements performed through the TFBG sensor.
- The proposed advanced plate element, with Layer-Wise kinematic, demonstrated important capabilities to implement real boundary conditions in order to reproduce experimental tests.
- The present numerical models reach accurate solutions with higher-order thickness polynomial expansions.
- The proposed method can be used as an effective and efficient numerical tool for the thermomechanical analysis of composite structures embedding optical fibre sensors.

Thank you.