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PERSPECTIVE

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Perspective

Recent advances and perspectives in GNSS PPP-RTK

Pengyu Hou[1,2,3], **Jiuping Zha**[1], **Teng Liu**[1,4] **and Baocheng Zhang**[1,5,*]

[1] State Key Laboratory of Geodesy and Earth's Dynamics, Innovation Academy for Precision Measurement Science and Technology, Chinese Academy of Sciences, Wuhan, People's Republic of China
[2] College of Earth and Planetary Sciences, University of Chinese Academy of Sciences, Beijing, People's Republic of China
[3] Department of Geoscience and Remote Sensing, Delft University of Technology, Delft, The Netherlands
[4] Hubei Luojia Laboratory, Wuhan, People's Republic of China
[5] (German) International Science and Technology Cooperative Offshore Center for Geodesy Frontier Technology, Wuhan, People's Republic of China

E-mail: b.zhang@whigg.ac.cn

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Abstract

Precise point positioning-real-time kinematic (PPP-RTK), otherwise known as integer ambiguity resolution-enabled precise pointing positioning, has attracted much attention in recent years and has become state-of-the-art in the global navigation satellite system (GNSS) high-precision positioning community. This work reviews several PPP-RTK methods, outlines a set of PPP-RTK applications, and presents possible future developments. According to the parameterization considered, we clarify the PPP-RTK models into a distinct-clock category and two common-clock categories (common-clock-1 and common-clock-2), in which several ionosphere-free PPP-RTK models can be cast. Compared with the ionosphere-free PPP-RTK model, we emphasize the advantages of the undifferenced and uncombined (UDUC) formulation and recommend the common-clock-1 UDUC PPP-RTK model since it is optimal, flexible, and widely applicable. Based on what kinds of parameters can be estimated by PPP-RTK models, we outline the PPP-RTK applications in several aspects, including position-based applications, time transfer, atmospheric retrieval, and GNSS bias estimation. Despite the huge advances in GNSS PPP-RTK, future research should improve PPP-RTK performances in harsh environments and apply PPP-RTK to mass markets.

Keywords: global navigation satellite system (GNSS), integer ambiguity resolution-enabled precise point positioning, PPP-RTK, undifferenced and uncombined (UDUC), distinct-clock model, common-clock model, ionosphere-free PPP-RTK

(Some figures may appear in colour only in the online journal)

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[*] Author to whom any correspondence should be addressed.

1. Introduction

With the advancement of positioning measurement systems, knowledge of our location is nowadays taken for granted by people worldwide. In 1923, Close [1] emphasized the importance of positioning measurement and established a new fundamental benchmark for ensuring a stable one-dimensional leveling operation, while nowadays, the three-dimensional position can be precisely determined. This is largely attributed to the advent of the global positioning system (GPS) [2] and the expanding family of global navigation satellite systems (GNSSs) [3]. GNSS, a global infrastructure that provides positioning, navigation, and timing services, has been applied to a wide range of fields, such as deformation monitoring [4, 5], precision agriculture [6, 7], intelligent driving [8, 9], and aviation [10, 11].

Concerning GNSS precise positioning, two technologies have taken center stage for a long time: precise point positioning (PPP) and real-time kinematic (RTK). PPP can achieve absolute positioning accuracy at a centimeter-to-millimeter level, as it takes advantage of precise satellite orbits, satellite clocks, and other corrections estimated by globally-distributed GNSS stations [12–14]. On the other hand, RTK relies on a regional network to carry out relative positioning, thereby reducing or even eliminating space-correlated errors, such as ionospheric and tropospheric delays [15–17]. More attractively, RTK fully exploits the high-accuracy GNSS phase observables as it can fix the double-differenced integer ambiguities [18, 19]. In contrast to RTK, PPP usually lacks precise atmospheric corrections and estimates undifferenced float ambiguities [20, 21]; this is why PPP requires tens of minutes to converge, but RTK can provide instantaneous centimeter-level positioning [22, 23]. However, the reliance on a regional network implies the difficulty in providing RTK service for thousands of users in a wide area. These features seem to separate PPP and RTK into two unbridgeable divisions, that is, PPP servers for global users with a long convergence time [24, 25], while RTK provides a regional positioning solution in real-time [26, 27]. However, one question is can PPP achieve RTK-like positioning relying on a regional network? Or can RTK service be extended to a large scale and even a global level like PPP?

A new concept of PPP-RTK proposed by Wübbena *et al* [23] answered the question in the affirmative. PPP-RTK, defined as the integer ambiguity resolution-enabled PPP, extends the PPP by providing users, in addition to the satellite orbits and clocks, the satellite phase biases, and optionally, the atmospheric delays [28]. Correction of satellite phase biases paves the way for single-receiver integer ambiguity resolution, thereby accelerating the convergence [29, 30]. When additionally correcting atmospheric delays estimated in a regional network, the RTK-like positioning is achievable [31–33]. PPP-RTK is more promising than RTK since PPP-RTK broadcasts individual error correction in state space representation (SSR) [34] instead of observable space representation (OSR) adopted by RTK [35]. Compared with OSR, SSR separates various error sources for a precise description and can lead to much lower bandwidth for transmission [23, 35]. In this sense, PPP-RTK combines the advantages of both PPP and RTK, making it state-of-the-art in GNSS positioning community [36–42].

Attracted by the superiority of PPP-RTK, researchers in recent years have made great efforts to investigate the PPP-RTK principle, develop various PPP-RTK methods, and apply PPP-RTK to a wide range of fields. This work aims to review these and point out possible future developments. The remainder of this work proceeds as follows. In section 2, we introduce the principle and properties of PPP-RTK. Then, we review several PPP-RTK methods in section 3, where we also present the recent progress in PPP-RTK algorithms. Following that, we introduce the PPP-RTK applications in section 4. Finally, we conclude this study and provide insights into the PPP-RTK future in section 5.

2. Principle and properties of PPP-RTK

2.1. Principle

The concept of PPP-RTK involves a network and a user side. The network side first collects GNSS observation data in a global, regional, or local network. A computation center then processes the data to generate a variety of corrections, including satellite orbits, satellite clocks, satellite phase biases, satellite code biases, and atmospheric delays. After encoding, the network side broadcasts these corrections to users through a satellite link or the internet. PPP-RTK users receive the products estimated on the network side and correct their GNSS observables. Based on these error-corrected undifferenced GNSS observables, single-receiver PPP-RTK users can achieve high-precision positioning with integer ambiguity resolution.

Among PPP-RTK corrections, the satellite phase biases are crucial since they play the role in recovering the integer nature of phase ambiguities on the user side. These biases are a result of small delays because of imperfections and/or physical limitations in satellite hardware. Moreover, these hardware-induced biases could be stable in a certain term, differing in GNSS systems and frequencies [43].

2.2. Properties

To better understand the principle and advantages of PPP-RTK, we summarize the following five properties of it.

(a) Global, regional, local, or even mixed networks are feasible. PPP relies on a global network to calculate satellite orbits and clocks, whereas PPP-RTK corrections, including orbits and clocks, can be determined in a regional network [44–46]. This implies that one can deploy GNSS stations in a regional area where the PPP-RTK service is covered instead of on a global scale. Moreover, one can even estimate some products (e.g. satellite orbits) in a global network and generate others (e.g. satellite clocks) in a regional network [47–49]. This reflects the higher flexibility of PPP-RTK over PPP.

(b) Broadcast ephemeris is possible. In contrast to PPP, which requires precise satellite orbits, PPP-RTK can use broadcast ephemeris to realize high-accuracy positioning in regional networks. This is because satellite clocks estimated in regional networks can absorb the satellite orbit errors. Due to the spatial correlation, the orbit errors in user observables can be compensated by correcting satellite clocks [23, 30].

(c) Multiple accuracy levels are achievable. One can utilize only code-related corrections for low-accuracy applications to realize meter-level positioning. With the integration of corrected phase observables, the centimeter-to-millimeter-level positioning is attainable within tens of minutes by PPP. To accelerate the convergence, the satellite phase biases are corrected to conduct integer ambiguity resolution-enabled positioning [50–52]. By further introducing atmospheric corrections estimated in a regional network, rapid and accurate positioning is ultimately achievable [53–55].

(d) PPP-RTK is compatible with PPP and RTK. If we only provide users with satellite orbits and clocks, the user positioning is then actually the PPP. RTK terminal can also take advantage of PPP-RTK corrections as the SSR corrections can be transformed into OSR data through a proper algorithm [39, 56, 57].

(e) PPP-RTK services are user-friendly. PPP-RTK server broadcasts SSR corrections through a uni-directional communication link, which is ideally feasible to provide services for an unlimited number of users [23]. The uni-directional communication link protects user privacy since users do not need to upload personal information to the PPP-RTK server. The precise description of each error source allows for robust integrity monitoring [58, 59], which is promising for emerging industries such as intelligent driving.

3. PPP-RTK methods

There exist several PPP-RTK methods, differing in the choices of parameterization. Teunissen and Khodabandeh [28] unified these methods into one theoretical framework and classified them into distinct-clock PPP-RTK and common-clock PPP-RTK. This section reviews these methods and further presents the advances in the extension of PPP-RTK models.

3.1. Distinct-clock PPP-RTK model

We must bear in mind that PPP-RTK corrections (e.g. satellite clocks and phase biases) are not the original quantities but the biased ones [60, 61]. This is because the original observation equations are rank-deficient. For this, we select some parameters as the datum (or the S-basis) and estimate the linear functions of the original parameters [62–64]. Due to non-unique datum selection, several methods exist that formulate different estimable parameters [28, 65, 66].

The distinct-clock PPP-RTK model, proposed by Teunissen *et al* [33], parameterizes different clocks for different observable types. This distinct-clock concept was initially introduced by de Jonge [67], and Odijk [68] applied this concept to RTK. Specifically, distinct-clock PPP-RTK parameterizes one common clock for code observables and two different clocks for two phase observables in the dual-frequency case. Via these phase clocks estimated on the network side, PPP-RTK users can recover the integer nature of ambiguities, thereby accelerating the convergence by resolving these integer ambiguities with a high success rate. We note that one should be aware of the fact that although distinct-clock PPP-RTK works without additional phase bias products, the phase clocks absorb the phase biases.

One potential advantage of the distinct-clock PPP-RTK model is that it lumps the biases with the clocks, implying that it models the biases as time-variant parameters [69]. Indeed, many studies have shown that GNSS biases, especially receiver code biases, may exhibit remarkable variations [70, 71]. Furthermore, researchers also revealed that these variations are related to temperature changes [72, 73]. Hence, considering the biases as time-constant parameters may degrade the PPP-RTK performance. Fortunately, distinct-clock PPPP-RTK avoids this problem due to the lump of clocks and biases.

3.2. Common-clock PPP-RTK model

In contrast to the distinct PPP-RTK model, the common clock PPP-RTK model parameterizes only one clock for all observable types and estimates additionally the code and phase biases. Since the phase biases are also linearly correlated with ambiguities, one can parameterize them in different ways [38]. This results in two common-clock PPP-RTK models: the common-clock-1 model and the common-clock-2 model.

Zhang *et al* [74] proposed the common-clock-1 PPP-RTK model based on undifferenced and uncombined (UDUC) GNSS observables. Concerning the linear correlation between the phase biases and ambiguities, the common-clock-1 model selects a subset of ambiguities as the datum. As a result, the common-clock-1 model provides the satellite phase biases lumped with ambiguities, based on which the users can formulate integer-estimable double-differenced ambiguities. Moreover, ambiguities on the network side are also constructed in a double-differenced form, enabling integer ambiguity resolution to improve the precision of products.

Alternatively, one can select the phase biases as the datum when addressing the rank-deficiency problem, yielding the common-clock-2 PPP-RTK model. In this way, the original products for user ambiguity resolution are real-valued ambiguities estimated on the network side. One can directly transmit these ambiguities to users for ambiguity resolution-enabled positioning or extract the fractional parts of the ambiguities with a proper algorithm [22]. However, due to the real-valued formulation of ambiguities on the network side, the common-clock-2 model cannot directly conduct network integer ambiguity resolution without further operations [75].

Since the common-clock-2 model lumps the phase biases with ambiguities, it has to model the phase biases as time-invariant parameters, as the ambiguities are typically considered continuous as long as no cycle slips occur. This implies that the performance of the common-clock-2 PPP-RTK model would be degraded in a situation where the phase biases exhibit remarkable variations. However, this is not a problem for the common-clock-1 model since it separates the biases and ambiguities. The common-clock-1 model is even more flexible than the distinct-clock model as it can describe the realistic dynamic characteristics of the biases by designing a proper dynamic model [76–78], whereas the distinct-clock model can only model the phase biases as time-variant parameters.

We conclude that the three PPP-RTK models differ in the datum they select to construct the full-rank models, thus resulting in different estimable parameters. It is worth noting that, as also demonstrated by Teunissen and Khodabandeh [28], the estimable parameters of the three models can be transformed through a proper datum transformation [62, 64]. This implies that the three models are equivalent if one adopts an identical stochastic model of observables and considers an identical dynamic model to describe the characteristics of all parameters, e.g. the biases.

3.3. Ionosphere-free PPP-RTK model

Since the ionosphere-free combination has obtained huge success in PPP, many studies also established the PPP-RTK models based on the ionosphere-free combination. There exist several ionosphere-free PPP-RTK models, including the integer recovery clock (IRC) model, the decoupled satellite clock (DSC) model, and the uncalibrated phase delay/fractional cycle bias (UPD/FCB) model.

Laurichesse *et al* [79] developed the IRC model, while Collins *et al* [69] proposed the DSC model. Since the IRC and DSC models are essentially the same [28], we review them together. The IRC/DSC model adopts the same datum of the distinct-clock model and thus can be considered as an ionosphere-free version of distinct-clock PPP-RTK. This implies that the IRC/DSC model also avoids the adverse effects of time-variant GNSS biases. However, from another perspective, the IRC/DSC PPP-RTK loses the potential opportunity to strengthen the model by properly considering the dynamic characteristics of biases.

Ge *et al* [22] proposed the UPD/FCB model, which is an ionosphere-free version of the common-clock-2 model. Since the UPD/FCB model parameterizes the real-valued ambiguities, it originally provided the ambiguity-float products until Geng *et al* [75] proposed an ambiguity-fixed UPD/FCB method. To ensure the consistency of the products, the UPD/FCB model requires a fractional operation, which extracts only the fractional part of the ambiguities and discards the integer parts. This operation is permitted as it still guarantees the integer nature of user ambiguities. However, one should be careful that the use of this nonlinear fractional operator changes the stochastic properties of the user-corrected observables, as demonstrated by Teunissen and Khodabandeh [28].

Ionosphere-free PPP-RTK enjoys a high computational efficiency due to the elimination of ionospheric delays. This implies, however, that the ionosphere-free PPP-RTK cannot provide users the ionospheric corrections, which are essential for rapid positioning. One should also be aware of the fact that the ionosphere-free approach (in the dual-frequency case) starts with four observables (two code and two phase) and ends up with two observables (one ionosphere-free code and one ionosphere-free phase), while only one independent parameter, the ionospheric delay, gets eliminated in this process [61]. This results in a loss of information compared to that provided by the original four observables. To remedy this loss of information, one has to add the third observable, which is the difference between the wide-lane phase and the narrow-lane code observables [61].

Moreover, since the ionosphere-free model combines observables at different frequencies (at least two), it fails in formulating double-differenced ambiguities at each frequency, to which the integer ambiguity resolution can be directly applied. Alternatively, ionosphere-free PPP-RTK usually fixes the wide-lane ambiguities and then fixes the narrow-lane ambiguities. On the other hand, ionosphere-free PPP-RTK does not generate products at each frequency but the combination of frequencies, e.g. wide-lane and narrow-lane products. This results in a cumbersome process when extending the ionosphere-free PPP-RTK to multi-GNSS and multi-frequency cases [80–82].

3.4. UDUC PPP-RTK model

Facing the multi-GNSS and multi-frequency trend, the UDUC formulation becomes an attractive choice. This concept of UDUC formulation has been proposed for a long time [83, 84]. Its advantages have already been recognized in PPP [85–88]. Teunissen *et al* [33] developed a distinct-clock UDUC PPP-RTK model, while Zhang *et al* [74] proposed a common-clock-1 UDUC model. In recent years, some studies also transformed the ionosphere-free PPP-RTK models reviewed above to a UDUC formulation [89, 90]. We here summarize several advantages when formulating the PPP-RTK model based on UDUC observables.

(a) It enables a unified functional model and a simplest stochastic model. One can formulate a unified functional model with an arbitrary number of systems and frequencies. The stochastic model is simplest since it avoids mathematical correlations, which may exist in combined and/or differenced observables.
(b) It allows for strengthening the model to the best extent. Since it preserves all original parameters, a strongest model is possible by imposing a proper constraint on each parameter.
(c) It generates the products with the best consistency. It calculates all of the products, including satellite clocks,

satellite biases, and atmospheric delays, by one estimator, making the estimates consistent.

(d) It provides directly observable-specific-bias corrections. It estimates biases at each frequency instead of a combination of them, thus promising for multi-frequency applications.

(e) It ensures a straightforward integer ambiguity resolution process. Since it formulates double-differenced integer ambiguities at each frequency, one can directly conduct integer ambiguity resolution.

We remark that one may argue that UDUC PPP-RTK requires much more computational resources than the differenced or combined methods. However, studies have clarified that this is not a fact since we can eliminate the parameters of no interests (e.g. receiver-related parameters) by reducing the normal matrix instead of performing an *a-priori* elimination at the observation level [38]. This implies that UDUC PPP-RTK can be as efficient as the differenced or combined methods.

3.5. Extended PPP-RTK models

Since the first proposal of PPP-RTK, great efforts have been taken to modify and extend the PPP-RTK models. We here review several advances in the extension of the GNSS PPP-RTK models.

3.5.1. Ionosphere-fixed, ionosphere-weighted, and ionosphere-float PPP-RTK. As we pointed out, PPP-RTK is not limited to global networks but also regional and local networks. It is feasible to apply one model to all types of networks, whereas a better choice is to consider distinct features of ionospheric delays in different networks. In a local network where the inter-station distances are only several kilometers, one can consider that the ionospheric delays at all stations are identical, yielding the ionosphere-fixed PPP-RTK model [33]. Concerning a regional network where the inter-station distances range from tens to hundreds of kilometers, one can impose a zero-mean weighted constraint on the between-receiver single-differenced ionospheric delays. This yields the ionosphere-weighted PPP-RTK model [91] that describes the uncertainty of the zero-mean constraint by a proper stochastic model [92, 93]. When the inter-station distance further increases, one parameterizes the ionospheric delays without any constraints, formulating the ionosphere-float PPP-RTK model [74].

We remark that the ionosphere-weighted model can be considered a general model that unifies the ionosphere-fixed and ionosphere-float models. In the case of setting the weight of zero-mean ionospheric constraint as infinite, the ionosphere-weighted model is converted to the ionosphere-fixed model. Considering another extreme case where we set the weight of the constraint as zero, the ionosphere-weighted model is reduced to the ionosphere-float model. We also note that the ionosphere-float model is equivalent to the ionosphere-free model in the sense that eliminating the ionospheric delays in the parameter domain is identical to estimating them without any constraints. Of particular note, here, the ionosphere-free model is referred to as the one that includes the third observable described in section 3.3, which ensures the same information as that contained by the ionosphere-float model.

3.5.2. Single-, dual-, and multi-frequency PPP-RTK. No matter what methods are adopted, PPP-RTK primarily served dual-frequency users. Facing the mass-market demands and the increasing number of GNSS frequencies, single- and multi-frequency PPP-RTK were proposed.

With the aid of ionospheric corrections, single-frequency integer ambiguity resolution is achievable with a single receiver. Experiments using single-frequency observables collected by a geodetic receiver showed a positioning accuracy at the centimeter level [94, 95], while the low-cost single-frequency PPP-RTK can also achieve a positioning accuracy of better than one decimeter [96]. To improve the performance of single-frequency PPP-RTK, contributions have been made to integrate PPP-RTK with other systems, e.g. the inertial navigation system [97]. Recently, researchers even realized single-frequency PPP-RTK with smartphones [98].

Although dual-frequency ionosphere-free PPP-RTK can be extended to multi-frequency cases, a more attractive way is to adopt the UDUC PPP-RTK model. In the multi-frequency case, the UDUC PPP-RTK provides the observable-specific satellite phase biases on each frequency [99]. Additionally, it has to provide the satellite code biases for the observables on the third frequency and above. Experiments have shown that multi-frequency PPP-RTK can accelerate convergence and improve positioning accuracy [48, 54, 100].

3.5.3. Frequency division multiple access (FDMA) PPP-RTK. The essential idea of PPP-RTK is integer ambiguity resolution, which is straightforward for code division multiple access systems (e.g. GPS), where all satellites share the same frequencies. However, the FDMA system, namely, the GLONASS, faces significant challenges in integer ambiguity resolution as it adopts different frequencies to identify satellites, resulting in inter-frequency biases (IFBs) and failing in formulating double-differenced ambiguities.

To conduct FDMA PPP-RTK, many studies contributed to calibrating the IFBs in advance [101, 102], thereby preventing IFBs from undermining the integer property of ambiguities. Considering that an all-inclusive IFB look-up table requires a heavy workload, studies found that one can avoid the adverse effects of IFBs through careful re-parameterization, provided that the receivers deployed in the network are homogeneous [103]. In a network consisting of heterogeneous receivers, some studies introduced external ionospheric corrections to ensure the integer nature of ambiguities [104, 105], while an alternative way is to estimate the IFBs [106, 107].

Most of the preliminary studies on FDMA PPP-RTK focused on dealing with the IFBs, whereas investigations on integer ambiguity resolution models were limited for the FDMA system. Teunissen [108] addressed this problem and proposed a new integer-estimable FDMA model that ensures the integer nature of FDMA ambiguities and guarantees a high success rate with partial integer ambiguity resolution. Many studies then applied this integer-estimable FDMA model to RTK and achieved great positioning performances [109–112]. Based on the integer-estimable FDMA model, Zhang *et al* [113] proposed an FDMA PPP-RTK solution and achieved fast FDMA integer ambiguity resolution in both homogeneous and heterogeneous networks. Recently, Teunissen and Khodabandeh [114] further generalized the integer-estimable theory for frequency-varying systems, including the FDMA GNSS system and the terrestrial interferometric sensory systems.

3.5.4. Phase-only PPP-RTK. To circumvent the unmodeled code-related errors, for instance, code IFBs and code multipath, one can exclude the code observables and use only the phase observables. This phase-only concept was first applied to relative positioning, in which a long convergence time is required due to the exclusion of code observables [115]. As integer ambiguity resolution is an effective way to accelerate convergence, many studies investigated the phase-only integer ambiguity resolution and applied it to RTK positioning [116, 117]. The results showed that phase-only RTK can achieve ambiguity-fixed positioning in two epochs [118, 119].

Hou *et al* [120] formulated a multi-frequency phase-only PPP-RTK model based on UDUC GNSS observables. This phase-only PPP-RTK model provides satellite phase biases only for the user observables on the third frequency and above, as the satellite phase biases on the first two frequencies are lumped with the satellite clocks. Results showed that phase-only PPP-RTK achieved successful integer ambiguity resolution in two epochs by using a regional network where the inter-station distances are approximately 100 kilometers. More attractively, the phase-only PPP-RTK model outperforms the classical code-plus-phase model in remarkable code multipath cases, which have been identified in the second-generation BeiDou navigation satellite system [121].

4. PPP-RTK applications

PPP-RTK can be applied to a wide range of fields. It is almost impossible to list all kinds of PPP-RTK applications in a short-review article. We here provide one perspective that introduces different PPP-RTK applications corresponding to each kind of parameter the PPP-RTK models estimate. In addition to the position parameters, PPP-RTK also estimates the clocks, the atmospheric delays, and the biases, which can be applied to various fields.

4.1. Position-based applications

The initial goal of PPP-RTK is to rapidly provide users with a high-accuracy position, which constitutes the basic information in many industrial and scientific fields. Regarding industrial fields, PPP-RTK has been applied to surveying and mapping [122], intelligent transportation systems [123], precision agriculture [124], orbit determination of low-orbit satellites [59], and so on. In emerging industries, such as intelligent driving and unmanned aircraft systems, PPP-RTK can contribute to one essential part of the integrated system [125]. Concerning the scientific fields, the precise position provided by PPP-RTK can be used for geodesy and geodynamics analysis, such as the seismic deformation [126], the earth plate movement [127], and the sea-level changes [128].

4.2. Time transfer

PPP-RTK provides both satellite clock and receiver clock estimates. The satellite clock is one of the essential products for high-precision positioning, while the receiver clock can be used for time transfer. Generally, integer ambiguities were not fixed in GNSS time transfer, typically, the PPP-based time transfer method [129–131]. Some studies achieved integer ambiguity resolution-enabled time transfer using single-differenced observables [132]. The advent of PPP-RTK makes integer ambiguity resolution possible with single-receiver UDUC observables. This motivated the researchers to apply the PPP-RTK to time transfer and investigate the impact of integer ambiguity resolution on time transfer [133–135].

4.3. Atmospheric retrieval

GNSS has proven to be an effective sensor for monitoring space atmosphere, including two essential components: ionosphere and troposphere. There are several ways to extract ionospheric delays based on GNSS, for instance, the carrier-to-code leveling method and the PPP method [71]. However, these methods discard the integer nature of ambiguities, implying a limited accuracy of the estimates. For this, some studies applied the ionosphere-free PPP-RTK to estimate satellite phase biases, based on which the ambiguities can be fixed at a stand-alone receiver. Then, one can extract ionospheric delays from ambiguity-resolved UDUC phase observables [136]. In contrast to this two-step method, the UDUC PPP-RTK directly estimates ionospheric delays at all stations by constructing a network model with the integer nature of ambiguities being remained [91]. Regarding the tropospheric delay, both ionosphere-free PPP-RTK and UDUC PPP-RTK can extract it directly [45, 137]. However, the UDUC PPP-RTK can simultaneously extract both ionospheric and tropospheric delays, leading to more consistent estimates.

4.4. GNSS bias estimation

GNSS biases, originating from both satellite and receiver ends and identified in both code and phase observables, are one of the most intricate error sources in GNSS, especially in the current multi-frequency and multi-GNSS stage. For a better use of GNSS biases estimated by PPP-RTK, we first discuss the representation of these biases. Previous studies usually represent the GNSS biases in a differenced or combined form, for instance, the widely-used differential code biases [138] and the wide-lane and narrow-lane phase biases estimated in ionosphere-free PPP-RTK [22]. However, the community recently recognized that a more convenient and

Table 1. Different PPP-RTK models and their properties.

Model	Parameterization	Ionosphere	Biases	References
Distinct-clock	IRC/DSC	Ionosphere-free	Time-variant	[69, 79]
	UDUC	Estimated	Time-variant	[33]
Common-clock-1	UDUC	Estimated	Realistic	[74]
Common-clock-2	UPD/FCB	Ionosphere-free	Time-invariant	[22, 75]

attractive way is to provide the biases for each observable type, thereby simplifying the fusion of multi-frequency and multi-GNSS observables [139]. Ionosphere-free PPP-RTK adopted this observable-specific-bias representation by imposing additional constraints to the differential biases [140], while the UDUC PPP-RTK directly formulates the observable-specific-bias parameters in the model [30]. These biases estimated by PPP-RTK are essential for PPP-RTK itself to achieve high-precision positioning. In addition, timing and ionospheric applications can utilize these biases products for *a-priori* calibration since the estimable receiver clocks and ionospheric delays contain the biases [71, 130].

5. Conclusions and outlook

This work introduced the principle of GNSS PPP-RTK, reviewed several PPP-RTK methods, and outlined various PPP-RTK applications in different fields. Table 1 concludes these methods and identifies their differences and similarities. Among these methods, we recommend the common-clock-1 UDUC PPP-RTK method since it is optimal, flexible, and widely applicable. It is optimal since the common-clock-1 parameterization separates the clocks and biases and thus allows a most robust model by considering the realistic dynamic characteristics of the biases. In contrast, the distinct-clock model moves to one extreme where the biases are considered as time-variant parameters, while the common-clock-2 model goes to another extreme where the biases are modeled as time-invariant parameters. The common-clock-1 UDUC PPP-RTK is also flexible since it directly provides observable-specific-bias corrections and allows for a straightforward fusion of multi-frequency and multi-GNSS observables. The common-clock-1 UDUC PPP-RTK is widely applicable since it simultaneously estimates all parameters, including ionospheric delays, which can be applied to a large number of fields.

Despite the huge advances in GNSS PPP-RTK methods and the blooming applications, some limitations still exist that restrict the use of PPP-RTK. These limitations include the low availability of PPP-RTK service in harsh environments and the difficulty in applying PPP-RTK to mass markets. Since PPP-RTK relies only on GNSS signals, its services are vulnerable and even inaccessible in harsh environments, e.g. urban cities and canyons [141, 142]. Although some initial studies have integrated PPP-RTK with other systems to improve the positioning [96, 123], optimal integration of PPP-RTK with other systems should be further investigated. Future research should also focus on PPP-RTK using mass-market devices (e.g. smartphones), which is challenging since some specific biases originated from mass-market devices destroy the integer nature of ambiguities [143, 144].

Data availability statement

No new data were created or analysed in this study.

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ORCID iD

Baocheng Zhang https://orcid.org/0000-0001-5006-1432

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