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Review

A critical review on the vertical stiffness irregularity of railway ballasted track

Can Shi [a], Yu Zhou [a], Lei Xu [b], Xu Zhang [c,*], Yunlong Guo [d]

[a] College of Civil and Transportation Engineering, Shenzhen University, Shenzhen 518060, PR China
[b] School of Civil Engineering, Central South University, Changsha 410075, PR China
[c] School of Civil and Transportation Engineering, Guangdong University of Technology, Guangzhou 510006, PR China
[d] Faculty of Civil Engineering and Geosciences, Delft University of Technology, 2628 CN Delft, the Netherlands

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ABSTRACT

The dynamic performance of a railway track subjected to moving trains depends strongly on track support conditions. In reality, even for the well-constructed and well-maintained tracks, sleeper support stiffness and global track stiffness vary substantially along the track, which affects the train-track dynamic interactions, causing rapid track geometry degradation as well as the riding comfort and safety issues. Consequently, track stiffness irregularity (TSI, the spatial variation of track stiffness along the track) is important for railway construction and maintenance in addition to track geometry irregularities. So far, extensive research has been published on the TSI whereas the relevant issues have not been paid sufficient attention. In this paper, a summary and comments have been made in the field of TSI about the current research status and future trends from a critical point of view. Novel concepts of the critical values of TSIs and the integrated management of the track geometry and stiffness irregularities are proposed. The review presented in this work is valuable to advance the research on TSI and can help guide the design, construction and maintenance of railway tracks.

1. Introduction

Ballasted tracks having the advantages of low construction cost, high water conductivity, good elasticity and easy maintenance are widely used for rail networks all over the world. In a railway ballasted track system, the rails are fastened to sleepers and supported by the ballast layer and the subgrade, as illustrated schematically in Fig. 1. During train passage, these track components inevitably experience reversible and irreversible deformation resulting the dynamic and permanent displacement of the track. To ensure the trains running safely, smoothly and comfortably, the track should keep appropriate elasticity or deformation resistance to the train loads.

The vertical track stiffness reflects the resistance of a track to the vertical deformation induced by a wheel loading. It is an important index for the quality and condition assessment of railway tracks constructed with various types of material. Generally, the total track stiffness arises from two parts. One part is the stiffness of the track superstructures (rail, rail pads and sleepers), termed as the rail bending stiffness and the rail pad stiffness which are relatively easy to be controlled. The other part is the stiffness of the track substructures (i.e. ballast layer, subgrade and foundation), which can be evaluated using the sleeper support stiffness. The stiffness of these geotechnical substructures are significantly affected by many factors such as compactness and thickness of the ballast and the subgrade soil, contamination of the ballast, moisture content of the subgrade soil [1]. It is extremely difficult to keep these factors consistent along the track in engineering even on a short track section. In fact, the geotechnical properties of the substructures almost vary here and there as reported in [2] resulting in the inevitable variation of track stiffness and non-homogeneous stiffness along the track. Besides, the track stiffness at insulated joints, switches, crossings and transition zones are different to that in common track sections. The discrete sleeper supports to the rail and the voids under sleepers also lead to the variation of the vertical stiffness along the track [3]. Therefore, the ideal track with uniformly-distributed stiffness along the track is almost impossible despite careful design and construction procedures.

Nowadays, lots of field tests have already been carried out to measure the track stiffness, and the results have shown that the track stiffness varies significantly along the track even on a very short section where track support conditions are similar [4,5]. The significant

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* Corresponding author.
E-mail address: xuzhang@gdut.edu.cn (X. Zhang).

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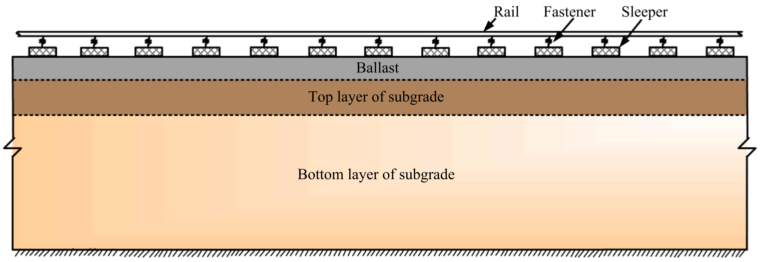
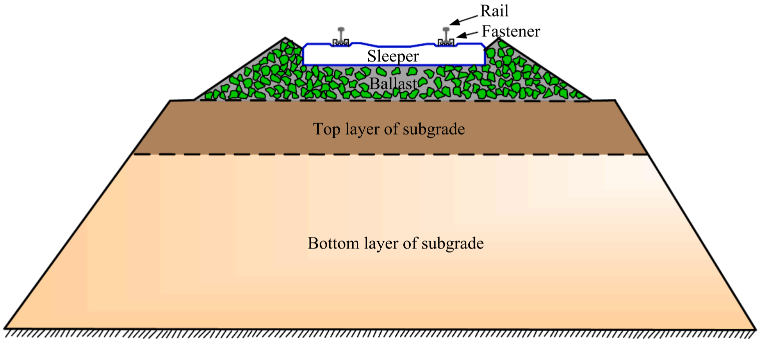


(a) longitudinal section



(b) cross section

Fig. 1. Schematic of a ballasted track system.

variation of track stiffness generates excessive wheel/rail contact forces giving rise to the degradation of track structures [6,7]. Under cyclic train loads, the differential settlement occurs to the track because of the inhomogeneous permanent deformation of the ballast layer and the subgrade. The differential track settlement will further deteriorate track geometry, in turn increasing the wheel/rail contact forces and then speeding up the track degradation rate, which have negative impacts on the track performance, train running stability and passengers' comfort [8]. Attention should be paid to the variation of the track stiffness along the track because of its distinct adverse effects to the train-track system.

The issue on the variation of railway track stiffness was early discussed by Dahlberg [1]. In that article, the main factors that induce the variation of track stiffness were analyzed and the possible solutions to smooth out track stiffness were presented. The results demonstrated that by the use of grouting or under-sleeper pads, the stiffness variation along the track can be modified, and accordingly the wheel/rail contact force can be considerably reduced. However, that article mainly focused on the track transition zones such as the bridge-embankment transition zone where the track stiffness difference is very large within a short distance. For common track sections, the variation of track stiffness along the track is difficult to deal with because the stiffness variation of a track with ideal or good track geometry may be hidden and not discovered until the track is loaded by a train.

Although a number of papers have been published in recent years to present the research results in this field, the issue on the variation of railway track stiffness has not attracted as much attention as that on the track geometry irregularities. As the stiffness variation has significant effects on the dynamic and long-term performance of the railway system, it is urgent to systematically summary and analyze the research in this field. This paper tries to present a critical review on the state-of-the-art research of this issue. Referring to the concept of track geometry irregularities, the variation of vertical track stiffness along the track will be termed as the track stiffness irregularity hereinafter. A concept of integrated track geometry and stiffness irregularities will be proposed to improve the understanding and guide the condition assessment of railway tracks.

This paper is structured into the following sections. In Section 2, the concepts of the track stiffness and stiffness irregularity will be discussed firstly. In Section 3, the measurement results of track stiffness in published studies will be summarized and analyzed to better understand the issue on track stiffness irregularity. In Section 4, the influence of track stiffness irregularity on the train-track system will be further discussed comprehensively. In Section 5, the research on the optimal track stiffness and the concept of acceptable level of track stiffness irregularity will be presented. In Section 6, the possible solutions to modify track stiffness irregularity are also discussed to help guide the design and the maintenance of railway tracks. Lastly, some conclusions will be drawn based on the literature review.

2. Concepts of track stiffness and stiffness irregularity

Since various definitions of the track stiffness were presented in the literatures, some widely-accepted definitions are discussed herein to understand their differences. The total track stiffness is governed by the incorporated effect of elastic track components (rail pads, ballast, subgrade and soil), the Young's modulus of the rail (E) and the rail moment of inertia (I). The concept behind this parameter is the beam theory with discrete elastic supports that considers the rail as a beam supported by springs. Each spring is supposed to recreate the effect of a sleeper and its support stiffness provided by the substructures beneath it. However, applying the discrete elastic support method requires solving a huge number of algebraic equations which is not practical and neither easy. Generally, the simplest form of track stiffness (k) is defined as the ratio of the force (F) acting on the rail to the produced rail deflection (z) at a given moment (t), where the force can be either a wheel load or an external load [9]:

$$k(t) = \frac{F(t)}{z(t)} \tag{1}$$

Considering the mechanical behaviours of some track components are nonlinear and the sleepers may also have voids beneath them, a larger rail deflection could be induced by low load magnitudes. To eliminate the void and slack behaviour, another definition of track stiffness is presented as the secant stiffness [10]:

$$k_{x-y} = \frac{F_y - F_x}{z_y - z_x} \tag{2}$$

where F_x is the selected boundary force and F_y is the maximum load. z_x and z_y are the corresponding rail deflections induced by F_x and F_y, respectively. Different values are chosen for the boundary force and the maximum load in various applications or standards. In the Eurobalt II project, the track stiffness was calculated in the load range of 10 kN to 70 kN [11], while in the Chinese standard it is recommended to determine the track stiffness according to the boundary force of 7.5 kN and the maximum load of 35 kN [12].

As the track stiffness presents the proportion between the load and the track deflection, the tangent stiffness, defined as the formula below, is also used by engineers to evaluate the quality of track.

$$k_{tng,y} = \left|\frac{dF(t)}{dz(t)}\right|_y \tag{3}$$

Besides, track modulus is sometimes also used instead of track stiffness to describe the same practical applications to remove the effects of the rail. The track modulus is defined as the applied force per unit length of each rail per unit deflection. Based on the theory of the beam model [13] on an elastic foundation, the relation between track modulus and track stiffness can be determined using the following equation [14]:

$$\mu = \frac{k^{4/3}}{(64EI)^{1/3}} \tag{4}$$

where u is the track modulus and k is the track stiffness. The symbol E is the Young's modulus of elasticity of the rail and I is the inertia moment of the rail. The difference between track modulus and track stiffness is that track stiffness includes the effect of the rail (EI) whereas track modulus relates only to the remainder of the superstructures (fasteners and sleepers) and the substructures (ballast, subgrade and subsoil). Track modulus, on the other hand, is a measure of the vertical stiffness of the track foundation [14] where everything except the rail is considered. The rail bending stiffness determines the relation between track modulus and track stiffness.

As described in the introduction, the vertical track stiffness affected by the infrastructure conditions and defects is uneven along the track. So the track stiffness (k) can be regarded as a function of the longitudinal position (x) on the track. Similar to the definition of track geometry irregularities [15], the track stiffness irregularity refers to the deviation of the vertical track stiffness from its average value, and can be expressed as

$$k_{ti} = k_{(x)} - k_0 \tag{5}$$

where $k_{(x)}$ is the vertical track stiffness at the position x, and k_0 is the nominal track stiffness which can be the target value in design or the average value of the measured track stiffness in a certain section.

3. Experimental testing and measurement of track stiffness

Over the past several decades, lots of railway inspection devices and trains have been developed in the world for standstill and continuous track stiffness measurement to investigate and quantify the track stiffness irregularity. The track stiffness can be generally measured by the apply-loading method, e.g. using the falling weight deflectometer (FWD) and the track loading vehicle (TLV), as well as by using different types of sensors-strain gauges, accelerometers, displacement transducers. More details of these measurement methods, principles and performance have been summarized in Refs. [16,17]. Instead of discussing the test methods, this section presents important aspects of the measurement results and the quantification of track stiffness irregularity.

3.1. Standstill measurement of track stiffness

The measurement of track stiffness with the standstill method implies that the measuring point is pre-positioning and the measurement is carried out by exerting a force to the rail or the sleeper. The track stiffness is calculated according to the applied force and the measured displacement of the rail or the sleeper.

Usually, quasi-static vertical forces are exerted to the track using hydraulic jacks, and the magnitudes of the applied forces can be measured accurately. Ebersohn et al. [18] and Read et al. [19] developed a track-loading vehicle with the hydraulic jack-loading set-up to measure the track stiffness. The track-loading vehicle can easily move to the desired position on the track, but the cost of the vehicle is high. Besides, the vertical quasi-static load exerted on the rail should be smaller than the static wheel load due to insufficient reaction force. Using traditional hydraulic loading devices, Oscarsson et al. [20,21] conducted measurements on a full-scale track to obtain stochastic values of the sleeper support stiffness. The results showed that the mean values of sleeper support stiffness at two different test sites were 255 MN/m and 186 MN/m, respectively. It indicated that the stiffness of the track substructures under different sleepers varied significantly. Summarily, by using the hydraulic loading devices, the static track stiffness can be accurately measured, but this standstill measurement is inconvenient to obtain the track stiffness at different locations.

Hand portable devices such as the FWD and the impact instrumented hammer are more suitable for measuring the dynamic track modulus/stiffness at a test site. These devices can be used conveniently, quickly and easily. By using an impact hammer, Kaewunruen and Remennikov [22,23] presented an effective method to obtain the dynamic moduli of the in-situ railway track components. The test results were applied for experimental modal analysis based on which finite element modelling was performed. Their results showed that the ballast layer stiffness was in the range of 150–470 MN/m, and the rail pad stiffness was within the range of 800–1500 MN/m. These values were in a good agreement with the stiffness used in track design.

Besides, many standstill measurements of track stiffness were performed with the FWD method [24]. Fig. 2 shows the FWD test results reported in Ref. [25]. Here, surface deflections of the subgrade, the sub-ballast and the ballast layer were measured over a 200 m-long section including a short reinforced section. The surface deflection of the subgrade is 0.3–1.4 mm. It is 0.6–2.5 mm for the sub-ballast and 1.3–3 mm for the ballast layer, which showed significant stiffness irregularities of the subgrade, sub-ballast and ballast layer along the track, respectively. And it can be deduced that in this section, the ballast layer and sub-ballast have more remarkable influence on track stiffness irregularity than the subgrade. It can also be known from Fig. 2 that the reinforcement in a short section effectively mitigated the track stiffness irregularities caused by the substructures.

Recently, multiple types of sensors and techniques have been used at trackside to assess the track performance when a train moves over [4–6,26,27]. The track stiffness can be calculated based on the measurement results obtained from sensors. However, the loads applied on the rail subjected to the moving trains vary dynamically and the accurate force is difficult to be measured. Thus, different approaches were proposed to calculate the values of track stiffness. According to the field measurement conducted by Priest and Powrie [28], the track modulus calculated with the inferred displacement basin test (DBT) method varied from 70.8 MN/m^2 to 136 MN/m^2 along the track, while the result varied from 111 MN/m^2 to 288 MN/m^2 if a modified beam based on an elastic foundation (BOEF) method was applied. Since the dynamic response of the ground is markedly different from the theoretical static response and the DBT method requires a number of sleeper deflections to be measured, the DBT method appears to be significantly affected by the viscoelastic response of the soil. However, the BOEF method only requires the maximum displacement directly under the wheel, and the maximum displacement in the field test is close to the theoretical static

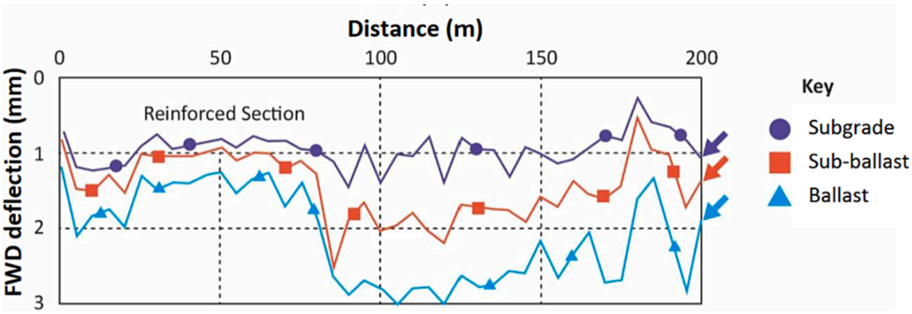


Fig. 2. Track component stiffness measured at Purfleet using a Falling Weight Deflectometer (figure reproduced from [25]).

response. Therefore, the BOEF method gives more reliable results and the track modulus calculated with the inferred DBT method is smaller.

Summarily, various standstill measurements have been carried out until now, and the track stiffness values measured at different sites have proved that the track stiffness experiences a high variation in the field. The statistical characteristics of the random track stiffness irregularity need to be further analyzed according to more field measurement results. Thus, the continuous measurement results on track stiffness are discussed as follow.

3.2. Continuous measurement of track stiffness irregularity

The standstill measurement of track stiffness is suitable for the related research purpose (e.g., obtaining parameters for numerical models), but it is difficult to obtain a large amount of data because the testing process is time-consuming and laborious. For the maintenance of a railway line, it requires a long-distance measurement. The inspection vehicles for continuous measurement of track stiffness have become a hot research area. There are two main approaches for measuring the track stiffness. Some researchers make use of a laser device to measure the vertical track deflections, and the forces applied by the inspection vehicles on the track can be obtained by assumption [17,29]. However, the track geometry irregularities might disturb the stiffness measurements since the laser devices in most cases measure a combination of the deflection due to track deformation and that due to track irregularities. Other researchers calculate the track stiffness based on the velocity and acceleration responses of the train, which eliminates the necessity to determine the axle loads [30,31].

Fig. 3 shows the track stiffness irregularity over a 25 m-long track section, which was measured by an inspection vehicle running at a speed of 1 km/h [25]. From Fig. 3, the track stiffness is smaller (about 140 kN/mm) nearby 149.807 km, which is most likely caused by the ballast crushing or hanging sleepers at that position. Besides, it can be seen that the track stiffness roughly varies periodically over the sleeper spacing (approximately 0.65 m). This indicates that the measured track stiffness at the position above a sleeper is larger than that between adjacent sleepers. The variation of track stiffness is about 5–20 kN/mm between adjacent sleepers, and the variation can be as large as 60 kN/mm on the whole section even when the small track stiffness nearby the position at 149.807 km is not considered. This result further confirms that the track stiffness irregularity is significantly affected by the discrete sleeper supports and the track substructures.

Besides, the deflection of 350 sleepers in a 200 m-long track section was measured by Milne et al. [33] using stand-alone micro-electro-mechanical-systems (MEMS) accelerometers, and the track modulus was calculated for three types of trains. A s shown in Fig. 4, the results are presented in terms of the support system modulus which includes the effect of the rail pads and the track substructures and represents the track modulus at each sleeper along the track. Fig. 4 shows continuous and distinct variations of the support system modulus in this section including 350 sleepers. Attributed by the combined effects of the rail pads and the substructures, the mean value of the support system modulus is about 25 MN/m^2 with the standard deviation of about 10 MN/m^2. The largest variation amplitude of the support system modulus occurs close to the No. 100 sleeper, which may be caused by the void under the sleeper.

3.3. Summary on measurement values of track stiffness

The total track stiffness represents the combined stiffness of supported layers, and it is determined by the stiffness of all track components. Therefore, complete acquisition of the stiffness characteristics of track components (especially those below the sleepers) is of great necessity to quantify the track stiffness. The measured results of the different track components' stiffness reported in published studies are summarized in Table 1. It can be found that the value of track stiffness in

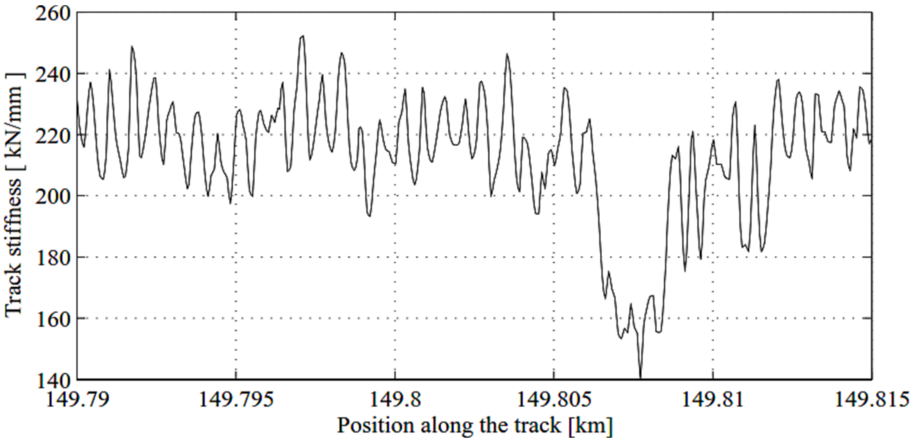


Fig. 3. Track stiffness measured from an inspection vehicle with a loaded axle of 18 t (figure reproduced from [32]).

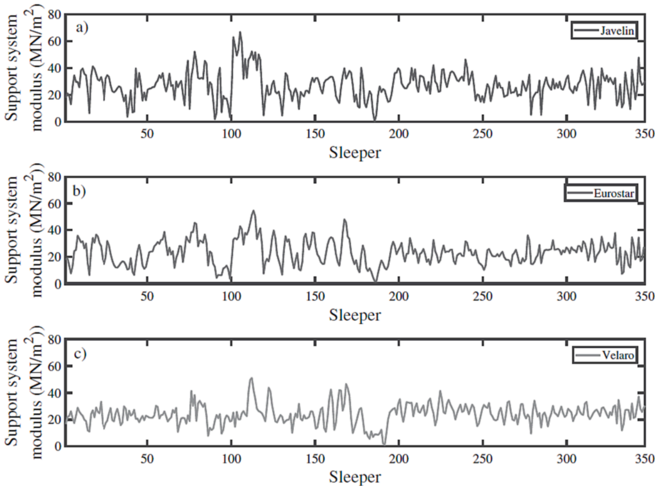


Fig. 4. Measured track modulus along 200 m track for three types of trains (figure reproduced from [33]).

existing studies varies substantially relying on the on-site track conditions. The typical values of subgrade modulus, ballast layer stiffness and overall track stiffness are within the range of 50–100 MPa, 100–200 kN/mm and 70–140 kN/mm, respectively.

The statistical distribution characteristic of track stiffness was further investigated by some researchers. Grossoni et al. [46] measured four sets of sleeper support stiffness using the FWD equipment, where the number of the measured sleepers was 155, 70, 80 and 81, respectively. A normal distribution curve was used to fit the statistical distribution of the measured support stiffness as shown in Fig. 5. By checking that the p-value of each set of data was not less than the 10% significance level, it was found that the normal probability distribution fitted to each dataset for four sites. Similar results can be found in [33].

Xu and Zhai [47] concludes that the sleeper support stiffness follows a normal distribution by analysing the numerical results of the sleeper support stiffness calculated with a discrete element model of ballasted track. Le Pen et al. [48] used the Weibull and Gamma cumulative distribution model to fit the distribution curves of track bed modulus (the load per unit length causing a unit displacement of the sleeper), which were deduced by the time histories of sleeper displacement under the moving train loads. The results showed that both distributions reflect key characteristics of the site data.

Summarily, these studies can help quickly establish track stiffness irregularity based on the probability distribution of track stiffness, and it can be used for simulating the reasonable values of track stiffness in the vehicle-track interaction simulations.

Table 1
Summary of the measurement results of track stiffness.

Track components' stiffness	Measurement method	Test results	Test site conditions	References
Subgrade stiffness (E_{V2} and K_{30})	Standstill measurement by using FWD	E_{V2}: 75–105 MPa	A mixed traffic railway line with the maximum speed of 220 km/h	Paixão et al. 2015 [34]
		E_{V2}: 48.1–254.2 MPa with an average value of 118.7 MPa K_{30}: 183.3–478.5 MN/m^2 with an average value of 249.0 MN/m^2	The high-speed railway between Daegu and Busan in South Korea	Kim and Park 2011 [35]
	Continuous measurement by using "Portancemetre"	E_{V2}:73.1–79 MPa	A full-scale railway track constructed in laboratory	Hosseingholia et al. 2009 [10]
	Hydraulic static loading	Modulus of subgrade: 32.7–61.1 MPa; Mean track stiffness: 36.9–50.0 kN/mm	A laboratory railways with accelerated fatigue testing facility	Woodward et al. 2014 [36]
Ballast layer stiffness	Hydraulic static loading	71.98–193.52 MN/m with the stochastic value of 125 MN/m±36.3 MN/m	A high-speed railway in China	Ma 2016 [37]
		Site A: 255 MN/m±16 MN/m Site B: 186 MN/m±22 MN/m	A conventional railway in Swedish	Oscarsson 2002 [21]
		110–167 MN/m	A full-scale track in laboratory which is laid on a rigid foundation	Liu et al. 2000 [38]
	Standstill measurement by using impact hammer	150–470 MN/m	The track line between Nebo and Hay Point Port in Australia	Kaewunruen and Remennikov 2007 [23]
	Standstill measurement by using wheel-axle drop	85–140 MN/m	A full-scale railway track in laboratory	Liu and Wang 2002 [39]
Sleeper support stiffness	Standstill measurement by using FWD	Site A: 44.4–143.8 kN/mm with stochastic values of 84.6 kN/mm±14.4 MN/m; Site B: 59.8–157.9 kN/mm with stochastic values of 110. kN/mm±16.2 MN/m	A conventional railway in UK	Grossoni et al. 2018 [40]
		28–70 kN/mm with the stochastic values of 45 kN/mm±8.3 kN/mm	The south of Leominster railway station	Brough et al. 2006[41]
		about 50 MN/m	A renewed or well-maintained ballasted track	Powrie and Pen 2016 [25]
Track stiffness	Continuous measurement by using video gauge	Site A: 19.9 kN/mm; Site B: 16.8–19.4 kN/mm; Site C: 27.6–61.4 kN/mm	Three track sites in UK.	Gallou et al, 2017 [42]
	Hydraulic static loading	74.4–93.4 kN/mm	A ballasted track constructed on the reinforced concrete floor in laboratory	Kim and Sung 2018 [43]
		100–140 kN/mm	A high-speed track constructed in laboratory	Estaire et al. 2017 [44]
	Continuous measurement based on wheel acceleration data	120–130 kN/mm	A high-speed ballasted track	Cano et al. 2016 [45]
Track modulus	Continuous measurement by using geophones	70.8–136 MN/m^2 with inferred DBT method; 111–288 MN/m^2 with modified BOEF method	A high speed railway line in UK	Priest and Powrie 2009 [28]

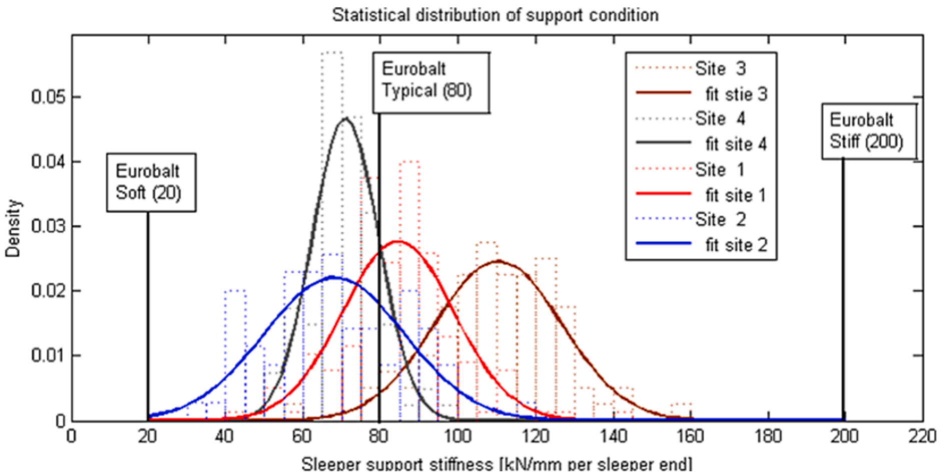


Fig. 5. Histograms and distribution curves of sleeper support stiffness (figure reproduced from [46]).

4. Influence of track stiffness irregularity on train-track system

In theory, a train is dynamically interacted with the track, and the dynamic loads triggered by a train could be significantly higher than the static load [49–51]. The dynamic performance of train-track system varies along the track even if the train moves with a constant speed on a straight track. Existing studies have indicated that the track quality, in terms of track geometry irregularities and track stiffness irregularity, has significant influence on the performance of wheel-rail interaction [52–55] and the long-term track degradation [56,57]. Fig. 6 shows an example of an improved vertical train-track dynamic interaction model considering not only the track geometry irregularity but also the track stiffness irregularity. The unevenness of stiffness comes from the randomness and variability of the parameters of the track components (i.e. fasteners, ballast and subgrade). However, how much the track stiffness irregularity deteriorates the train-track performance is still controversial. In this section, the influence of track stiffness irregularity on the train-track system is summarized and an emphasis is put on the discussion of the effects of stiffness irregularity on the deterioration of track structures.

4.1. *Influence of track stiffness irregularity on train-track interaction*

To reveal the effect of track stiffness irregularity on the performance of wheel-rail interaction, Li and Selig [58] presented the relation between the track stiffness irregularity and the dynamic responses of the rail and sleepers, and concluded that it has a direct influence on the train-track dynamic interaction. Lei and Zhang [59] suggested that the wheel unloading rate increased by up to 20% by changing the track stiffness. Choi et al. [60] illustrated the high-frequency component P_2 of the dynamic wheel-rail force was more remarkably affected by the track stiffness than the low-frequency component P_1. It indicates that keeping a constant vertical track stiffness is significantly important to reduce the high-frequency dynamic force P_2 for ballasted tracks. The results in Refs. [61–63] showed that the track stiffness irregularity contributed by the rail pad made the largest contribution to the increase of wheel-rail contact forces. Generally speaking, the track stiffness irregularity attributes to the increase of the wheel-rail contact force and then leads to the increase of wheel acceleration and wheel unloading rate. The result in Ref. [64] further indicated that the wheel load distribution factor was affected by the rail pad stiffness and the sleeper spacing. It can be deduced that the track stiffness influences the wheel load distribution on the track and therefore affects the dynamic performance of the track.

Whereas, the vibration induced by the track stiffness irregularity has insignificant influence on the ride quality of a train [65], because the track stiffness (e.g. 100 MN/m) is much higher than the stiffness of the primary and the secondary train suspension systems (e.g. 2–5 MN/m for the primary suspension and 0.1–0.5 MN/m for the secondary secondary). Hence, attention needs to be paid to the effect of track stiffness irregularity on the dynamic performance of track structures.

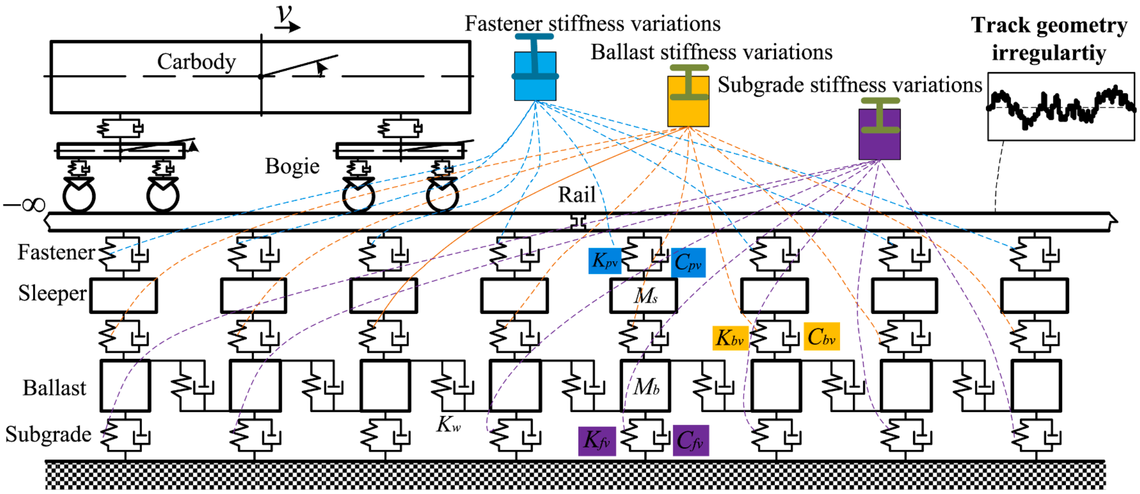


Fig. 6. A train-track interaction model including track stiffness irregularity.

It should be noted that these earlier studies generally used artificially assumed track stiffness along the track for a lack of measured data. Some researchers tried to use available track stiffness data measured at different sites together with numerical simulations [20,46]. Considering the time-consumption and high cost of obtaining the track stiffness from experiments, discrete element models as shown in Fig. 7 were established in [66,67] to determine the ballast layer stiffness. These studies indicate the discrete element method is useful to calculate the stiffness of the ballast layer and the sleeper support stiffness considering the inhomogeneous properties of the geomaterials.

4.2. *Influence of track stiffness irregularity on track structures*

To investigate the influence of track stiffness on the rail, Fallah et al. [68] established a track model with stochastically varying track stiffness and proved that the track stiffness irregularity significantly affects the rail bending moment. Besides, the track stiffness irregularity intensifies the wheel-rail contact forces causing the uneven wear of the rail head and rail corrugation. The studies in [69,70] showed that the rail fatigue crack propagation rate under variable moving train loads (based on the actual load history) was much faster than that under constant moving train loads. Nkundineza and Turner [71] also reported that higher track stiffness irregularity resulted in faster rail fatigue and reduced the rail life by up to 100 times comparing with that with constant track stiffness. The minimum number of fatigue loading cycles against the track modulus is illustrated in Fig. 8. It can be concluded that the effect of track stiffness irregularity must be considered in the prediction of rail fatigue life to obtain a scientific schedule for rail maintenance.

The material and conditions of the track substructures under various sleepers are not even, which causes the variation of the sleeper support condition along the track. The uneven supports to the sleepers increase the sleeper-ballast contact forces and intensify the deflections and vibration of sleepers [21,73]. Specifically, the results in Ref. [74] indicated that the variation of the ballast layer stiffness is one of the main sources influencing the bending moment of sleepers. As a result, the larger vibration of sleepers aggravates the degradation of ballast and reduces the sleepers' life. López Pita [75] presented that ballast layer stress may increase by up to 50% due to the track stiffness irregularity at two adjacent sleepers, as shown in Fig. 9. This additional stress in ballast layer accelerates the degradation of ballast particles and leads to track geometry deterioration, which in turn increases the dynamic wheel-rail contact force. Further, Grossoni et al. [76] reported that an increase of 10 kN/mm in the standard deviation of the vertical track stiffness corresponds to a relative increase of 22.2% in the settlement rate of the ballast.

Besides, the condition of sleeper-ballast interface is not even for a sleeper, which has great influence on the sleeper support stiffness. An example is shown in Fig. 10 [77]. The white and black parts in the figure indicated different pressure in ballast layer. Due to different loading conditions and ballast layer stiffness, the sleeper-ballast interaction load distributed randomly along the length of a sleeper, which could influence the dynamic behaviour of the sleeper and ballast particles. Meanwhile, the conclusion drawn in [78] showed that too soft support of ballast layer and too large variation of ballast layer stiffness under a sleeper might cause severe breakage and abrasion of ballast particles, especially the top ballast and that under the medium of the sleeper.

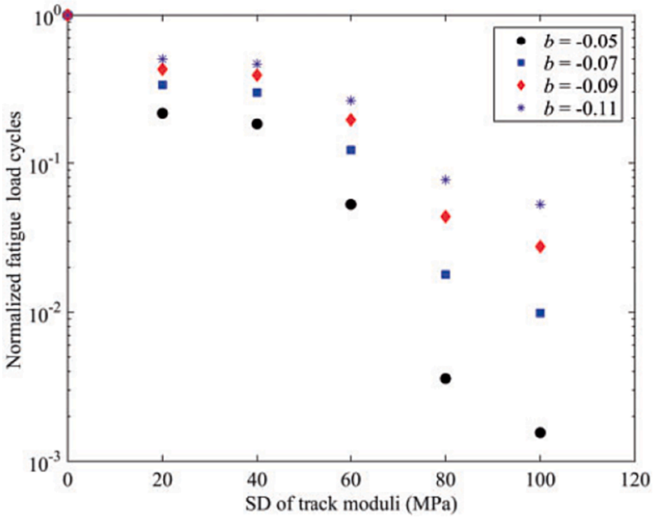


Fig. 8. Normalized rail fatigue loading cycles at different fatigue coefficient exponents (figure reproduced from [72]).

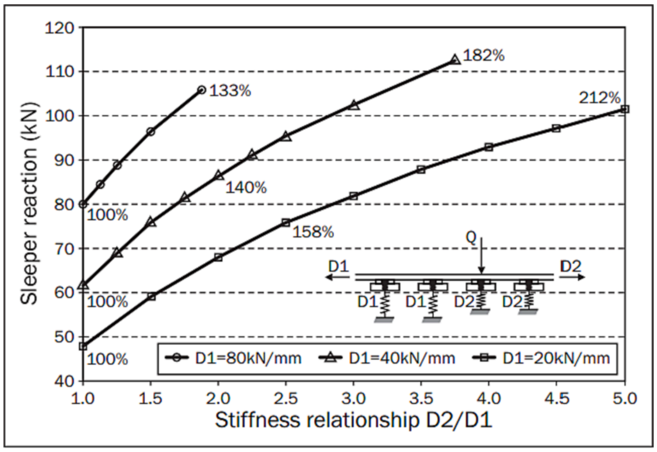


Fig. 9. Influence of track stiffness irregularity on the sleeper reaction (figure reproduced from [75]).

Summarily, track stiffness irregularity is an important factor affecting rail failure, and further studies about the rail fatigue life need to consider the effect of the track stiffness irregularity. Furthermore, it can be concluded that the influences of track stiffness irregularity on the degradation of ballast and sleepers have been investigated in literatures while few related literatures studied the effect of track stiffness irregularity on the fastener system.

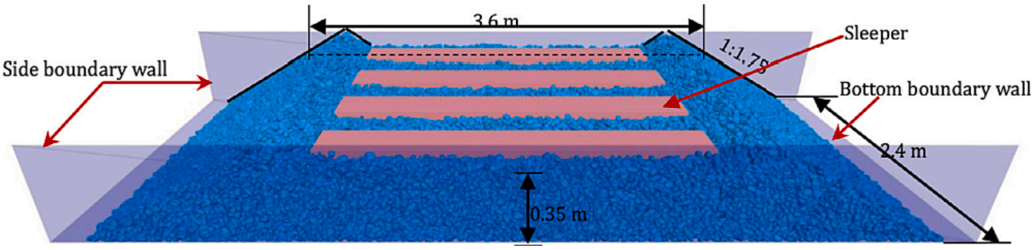


Fig. 7. Three-dimensional DEM model of ballast and sleeper (figure reproduced from [66]).

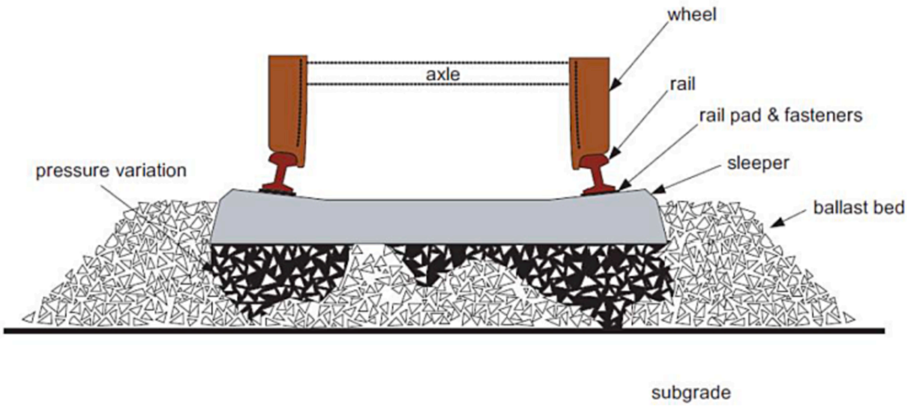


Fig. 10. Schematic of sleeper support condition (figure reproduced from [77]).

4.3. *Influence of track stiffness irregularity on track geometry deterioration*

Until now, lots of studies have been carried out about the track geometry deterioration, and a series of track settlement models have been proposed to predict the railway track geometry condition. In Refs [71,79–82], the track settlement models are usually expressed with the number of train passages, the magnitude of dynamic wheel loads and the qualities of track components. As the results shown in [83–89], the small track stiffness causes the increase of track settlement. The research conducted in [90] confirmed that a track section with the track modulus of 14 MPa requires more maintenance by 183% than that with a track modulus of 27 MPa. Therefore, the track stiffness is also an indispensable part for the models for predicting track settlement.

To study the relationship between track stiffness irregularity and track geometry deterioration, Frohling et al. [8] presented the predicted settlement for a track section in the conditions of varying track stiffness or constant track stiffness, as illustrated in Fig. 11. The results show that track settlement is sensitive to the variation of track stiffness, and constant track stiffness almost does not accelerate track geometry irregularity. The studies conducted in [8,40,62] also indicated that the track stiffness irregularity had significant influence on track geometry deterioration. According to these studies, the increase of track geometry irregularity is mainly caused by track differential settlement and the nonuniformity of track stiffness, and the railway sections with high track stiffness irregularities were difficult to be maintained.

A general physical mechanism was analyzed in [91,92] to explore the track geometry degradation affected by track stiffness irregularity. The results showed that the degradation of the track is mainly caused by a local strong amplification of the stress, strain fields and mechanical energy. After the train repeatedly runs on the track with suddenly-changed stiffness, additional dynamic loads gradually accelerate the differential track settlement. Typically, the track geometry deterioration happened very frequently where the track stiffness suddenly changed such as the embankment-bridge transition zones [93–95], switches and crossings [96]. The large wheel-rail forces applied to the track in these sections [97] cause rapid compaction and settlement of the ballast layer and the subgrade and result in fast permanent deformation of the track in the transition zone.

Summarily, the existing research indicates that the track stiffness irregularity plays an important role in track geometry deterioration. The effect of the track stiffness irregularity needs be taken into account in further studies to predict the settlement and geometry conditions of railway tracks and in the maintenance of tracks.

5. Optimal track stiffness and critical value of stiffness irregularity

In general terms, the track stiffness needs to be reasonable, not excessively too high, too low, or too variable. The inappropriate track stiffness may accelerate the degradation of track components and cause fast deterioration of track geometry. Considering the remarkable influence of track stiffness irregularity, it is necessary to determine the optimal track stiffness and the critical value of the track stiffness irregularity at the design stage or in service, and even to achieve a relatively homogenous stiffness along the track.

5.1. *Low or high track stiffness*

In theory, low track stiffness induces high bending moments and considerably large dynamic deflection of rails, as shown in Fig. 12, which may cause fast fatigue damage to the rail. A wider deflection bowl of rail means the dynamic train loads can be shared by more sleepers, and then reduces the force applied to sleepers [98] Hence, the stresses in

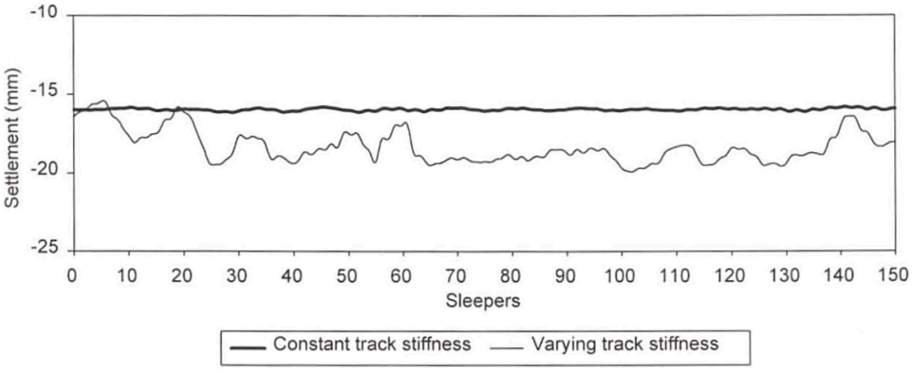


Fig. 11. Influence of track stiffness irregularity on track settlement (figure reproduced from [8]).

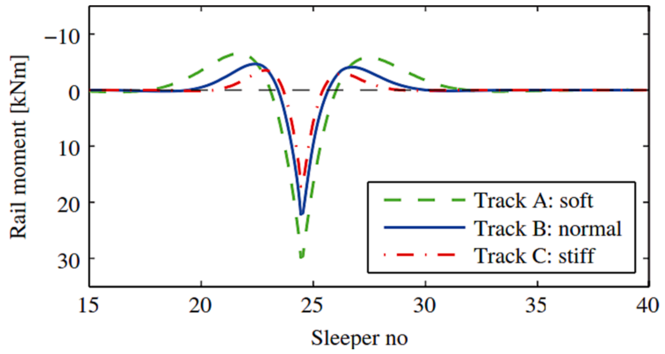


Fig. 12. Rail moment for the track with different stiffness (figure reproduced from [7]).

ballast layer or subgrade can be reduced. Consequently, lower track stiffness may be beneficial for ballast particles and subgrade by distributing the wheel loads more effectively from the loading points. However, an interesting result found in the international EUROBALT research project [87] was that the track settlement was directly affected by the low track stiffness because high shear strain occurred in the ballast layer in this circumstance. Overall, a too low value of track stiffness is usually associated with poor dynamic performance of the track system and a fast track settlement rate.

On the other hand, a high global track stiffness results in the increase of the dynamic wheel-rail interaction force, especially its high frequency components and the decrease of the rail displacement. Small rail displacement indicates that the wheel loads concentrate on the local superstructure causing high loading on the sleepers and ballast, which aggravates the deterioration of track components [98]. What's more, too high stiffness values are not conducive to the energy dissipation and then accelerate track deterioration. The ballast layer and subgrade located at stiff area experience large compression-expansion stress under the moving train loads compared with that on the soft track section [9,85]. Thus, a too high a value of track stiffness also results in fast deterioration of the track.

5.2. *Optimal track stiffness*

As discussed above, a too low value of track stiffness causes considerable increase to the bending moment of rail and results in fast track settlement rate. A too high value of track stiffness enlarges dynamic wheel-rail forces and thus accelerates the deterioration of track structures. Therefore, the track stiffness shall be within reasonable range not only to prevent large deflection of rail but also to guarantee the longevity of track components. To obtain better dynamic performance of a train-track system, the necessity to identify the optimal range of track stiffness is imperative [84,99]. Fig. 13 shows the relation between track stiffness and the deterioration rate of track geometry [89]. It can be seen that the track geometry has relatively low deterioration rate in the stiffness range of 50–90 kN/mm and the optimal track stiffness is about 75 kN/mm from the perspective of track geometry deterioration. An optimal track stiffness of 70–80 kN/mm was also proposed in [83] by accounting for the influence of track stiffness on track maintenance cost. Similar results can be found in Ref. [85].

Table 2 lists the optimal range of track stiffness according to several studies that have been undertaken. The optimal range is related to particular line circumstances and qualifications, for example, the value can be dependent on the type of traffic (i.e. heavy freight trains, high-speed trains, passenger trains, and mixed traffic, etc.) and the construction cost.

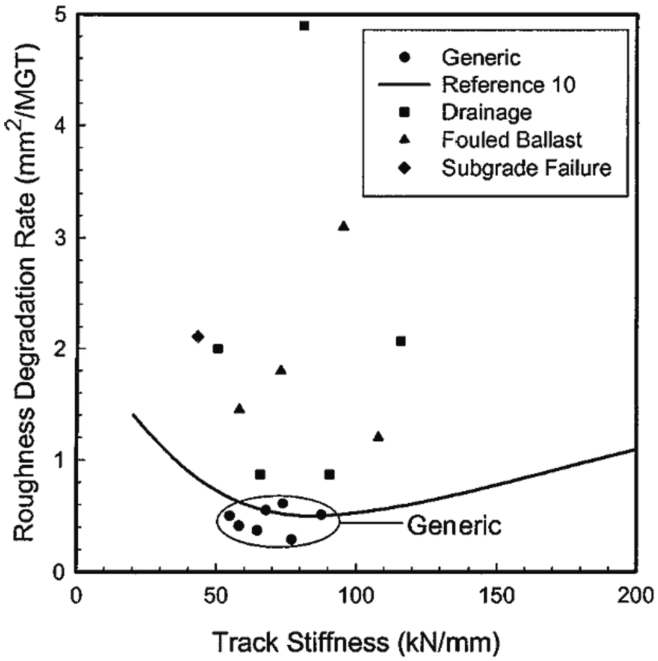


Fig. 13. Illustration of optimal track stiffness (figure reproduced from [89]).

Table 2
Optimal track stiffness.

Optimal track stiffness	Remarks	References
70–80 kN/mm	For high speed line	Pita et al. 2004 [83]
50–100 kN/m	For freight traffic	Sussman et al. 2001 [89]
80–130 kN/mm	Lower value for medium speed, higher value for high speed	Hunt and Wood 2005 [9]
65–100 kN/mm	Railway tracks with a limit speed of 160 km/h	Burrow et al. 2009 [99]
55 kN/mm	A minimum value to ensure track performance for existing main lines	Selig and Li 1994 [14]
60 kN/mm per sleeper end	A minimum value for track construction	Network Rail 2005 [100]

5.3. *Critical value of track stiffness irregularity*

The analysis in the previous section indicates significant influence of track stiffness irregularity on track performance. In theory, an ideal railway track with constant support stiffness along the track is desirable. However, building a line with a constant track stiffness is not practical. Instead of setting a constant target for the track stiffness, it is more practical to investigate and identify critical values of track stiffness irregularities for different railway tracks to achieve better train-track dynamic and long-term performance. According to the studies conducted by Lei et al. [101], following the increase of the difference in track stiffness ratio, the dynamic responses of track structures also increased. The ratio of track stiffness at adjacent sleepers should less than 5 times to protect the stability of track structures. The results in the EUROBOLT project showed that the variation of subgrade stiffness should be limited to less than 10% of the mean value. However, the critical value of track stiffness irregularity is still lack of recommendations at present.

Further systematic research should be carried out to quantify acceptable levels of track stiffness irregularity. Research in this field should consider not only the dynamic performance of the train-track interaction system but also the long-term performance of track structures and the track geometry deterioration. The critical values of track stiffness irregularity can be measured in terms of the percentage of the optimal track stiffness, the standard deviation of the track stiffness, the gradient of the track stiffness along the track in a given length or other indexes. Referring to the evaluation methods of the track geometry irregularities, some complex indicators similar with the track quality index (TQI) also can be developed to evaluate the stiffness condition of a track section.

It is worthy being pointed out that the stiffness irregularity of an operational railway track evolves temporally and spatially due to the stiffness variations of the track components in service, which means the track stiffness irregularity should be monitored and managed to ensure satisfied track conditions. Since the rail deflection can be easily measured and recorded by the inspection vehicles or trains, the vertical stiffness along the track can be estimated using the rail deflection data with the continuous measurement methods. So, the critical value of track stiffness irregularity can be applied to conveniently evaluate whether or not the track stiffness of an operational railway meets requirements. Thereby, the track stiffness irregularity can be evaluated to guide the maintenance plans as well as the management of track geometry irregularities. That means an integrated management of track geometry and stiffness irregularities can be established to improve the maintenance of railway tracks given the distinct effects of both the track geometry irregularities and the track stiffness irregularity on the train-track system. Moreover, the critical value of track stiffness irregularity also can be adopted to guide the track design in transition zones to achieve better stiffness transition.

6. Solutions to track stiffness irregularity

Since the serious problems are associated with track stiffness irregularity, the severity of the track stiffness irregularity should be alleviated to ensure the long-term service performance of the track. Thus, a summary of current solutions to reduce the stiffness irregularity along the track is summarized and presented in this section.

6.1. Solutions to track stiffness irregularity in common sections

To solve the problems induced by the track stiffness irregularity, the main factors and reasons affecting the track stiffness irregularity need be investigated firstly. Aimed at this, Shi et al. [102] studied the macroscopic and the mesoscopic characteristics of sleeper support stiffness using a hybrid discrete-continuum approach. The results showed that the sleeper support stiffness is considerably related to the contact conditions among ballast particles. Besides, the parametric studies conducted in [103] indicated that subgrade modulus is the most significant factor affecting the track stiffness. However, remedial solutions to deal with the subgrade modulus of an existing track are complicated and very expensive [104]. Therefore, the most feasible maintenance activity is to modify the stiffness of the ballast layer to achieve more uniform track stiffness distribution.

Nowadays, the maintenance activities of tamping and dynamic track stabilising are commonly adopted to modify the contact conditions among ballast particles. The dynamic track stabilising can provide relatively homogeneous consolidation throughout the entire ballast layer [105] by simultaneously applying static vertical loads and lateral vibration to the track, as shown in Fig. 14. The vertical loads and the lateral vibration make rearrangement of ballast particles and cause permanent deformation of the entire ballast layer. Thus, the homogenizing compaction and stiffness improvement are achieved by the dynamic track stabilization. The effectiveness of maintenance activities on enhancing the ballast compactness and supporting stiffness was also proved in Refs. [106,107].

Additionally, using grouting [108,109], polymers [110,111], tire-derived aggregates (TDA) [112,113] and under sleeper pads (USP) [114], under ballast mats (UBM) [114,115] are also common methods to alleviate the stiffness irregularity along the track. By using grouting and polymers, the ballast assemblies are bonded together to form a cemented aggregate structure. Compared with the conventional ballast bed, there are more contacts in the bonded ballast and the number of contacts further increases when ballast glue is used. The field tests carried out by Xiao et al [109] showed that the sleeper support stiffness increased from 220 kN/mm to 313.9 kN/mm when the dosage of the polyurethane was 33 kg/m^3, and it further increased to 371.6 kN/mm when the dosage of the polyurethane was 48 kg/m^3. The effectiveness of using polyurethane was also proved by Woodward et al. [36], and the result indicated that track stiffness significantly increased by 40% with the application of polyurethane.

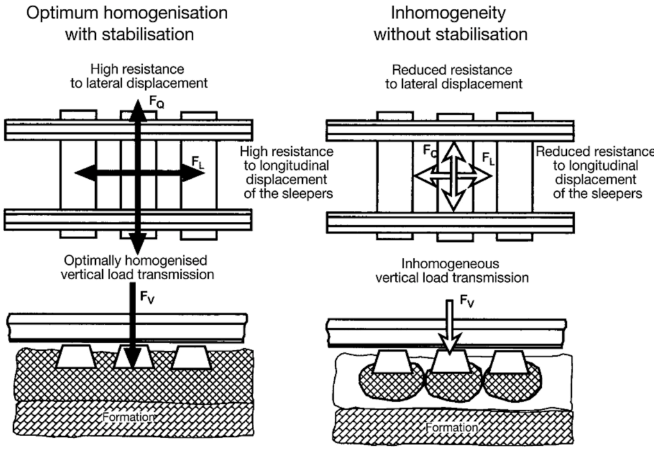


Fig. 14. Homogenizing compaction of ballast achieved by dynamic track stabilization(figure reproduced from [105]).

Contrary to the effect of polyurethane bonding, the installation of elastic elements into the track can reduce track stiffness because of the relatively small stiffness of the elastic elements. According to the numerical simulations of the ballast assembly in a steel box presented by Kumar et al. [114], the sleeper support stiffness without USP was 2580 MPa/m and it decreased to 330 MPa/m when the USP was installed. Fig. 18 presented by Le Pen et al. [116] shows that the application of USPs had apparent effects on reducing the standard deviation of track geometry. As discussed previously, the track stiffness irregularity has distinct influence on the deterioration of the track geometry, it can be understood here that the improvement of the track geometry in Fig. 15 benefited from the alleviation of the track stiffness irregularity with the application of USPs.

Besides, the use of waste TDA mixed into ballast is considered as an economic and environmentally friendly solution to extend the service life of the railway track. Fig. 16 shows the influence of the crumb rubber (C.R.) on the stiffness of the ballast layer. The increase of the percentage of C.R. in the mixed ballast leads to reduction of the ballast stiffness. Similar results are presented in Refs. [117–119]. However, the use of high amount (such as 20%–30% by volume) of crumb rubber particles showed excessive reduction of ballast stiffness, which could lead to fast deterioration of track geometry. Thus, 10% rubber was defined as the optimum amount in [115] to reduce track stiffness and improve the ballast performance. The field test carried out by Esmaeili et al. [117] presented that the ballast stiffness decreased by 31% with the TDA amount of 5% and by 75% with 10% TDA, respectively.

6.2. Solutions to track stiffness irregularity in transition zones

Some railway lines are about hundreds of kilometres or even thousands of kilometres long passing through a variety of topography and landforms. Inevitably, transition sections among the subgrade, bridges and tunnels (i.e. subgrade-tunnel, subgrade-bridge, bridge-tunnel, ballasted-ballastless tracks, etc) are built according to the actual conditions of the railway lines. This induces an abrupt change of track stiffness,

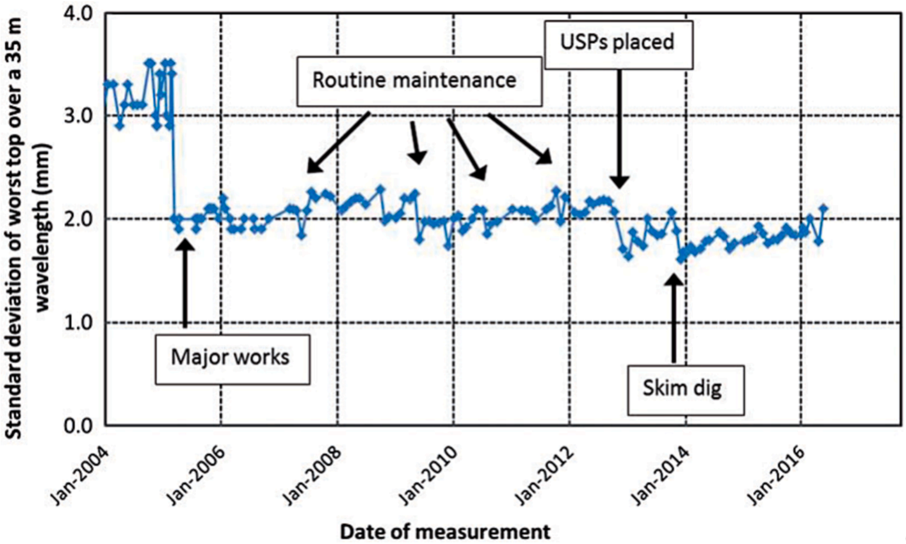


Fig. 15. Standard deviation for the worst 35 m wavelength vertical top (figure reproduced from [116]).

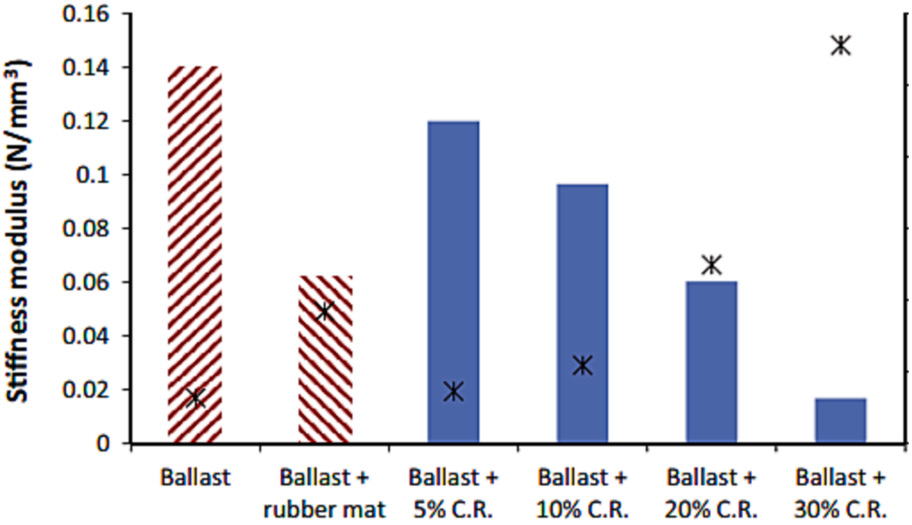


Fig. 16. Effects of rubber mat and rubber particles on ballast layer stiffness. (figure reproduced from [115]).

which is significantly affected by various types of substructures, in the transition zones. Earlier studies have reported estimations of vertical track stiffness in transition zones. The value ranges between 20 kN/mm and 120 kN/mm in Ref. [120] and between 100 kN/mm and 200 kN/mm in Ref. [121]. The stiffness of the track on the abutment is over two times higher than that on the backfill. As the discussion above indicated the close relation between the track stiffness irregularity and the track performance deterioration, the localized track deterioration is more likely to occur in the transition zones.

To improve the uniformity of the track stiffness in transition zones so as to improve the track performance under the long-term train loads, the wedge-shaped backfills constructed by well-compacted layers of selected granular materials are commonly used in embankment-bridge transition zones. The materials applied in wedge-shaped structures should have the characteristics of higher deformation moduli and are less sensitive to plastic deformation than the geomaterials generally utilized in embankments. Thus, layers of cement-bound granular mixtures (CBGM) and unbound granular material (UGM) are frequently adopted to construct wedge-shaped backfills in transition zones, as shown in Fig. 17. Studies in Ref [122–124] proved that a transition zone between two track sections can alleviate sudden changes of track stiffness along the track. However, the higher stiffness values still exist on the CBGM layers when compared with the ones at adjacent unbound granular material (UGM) layers. Fig. 17 shows the measured E_{V2} values along alignment 2&3 ranged from 100 to 500 MPa. Similar result can be found in Ref. [34].

Therefore, other solutions, such as the use of elastomeric products, reinforcement with geogrids or geotextiles and chemical treatment of ballast, are still recommended to construct a tailor-made transition zone with the tolerable track stiffness irregularity. Installation of USPs into the track is one of the commonly used solutions to adjust the track stiffness, and the effectiveness of USPs easing the inconsistent track stiffness in the transition zone is proved in Refs. [125–128]. Fig. 18 shows the vertical displacement of the first axle of a train passing a transition zone calculated by Ribeiro et al. [129]. By comparing the two displacement profiles presented in the figure, it can be observed that there was an increase in the vertical displacement of the axle by nearly 3 times at UP1 where USPs were installed. It indicates that the installation of USPs significantly reduces the track stiffness. However, the axle displacement changes abruptly in three sleeper-spans when entering or exiting the track section with USPs, which indicates the installation of USPs at UP1 results in large track stiffness irregularity and does not

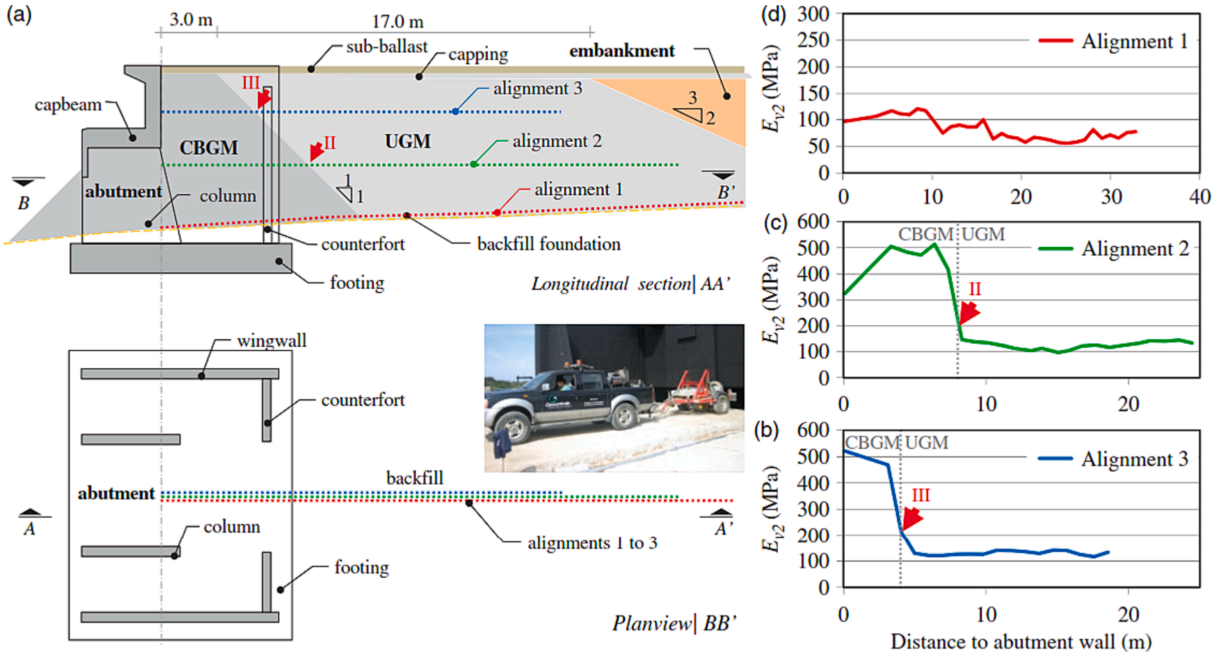


Fig. 17. Measured stiffness at a transition zone with constructed wedge-shaped backfills (figure reproduced from [34]).

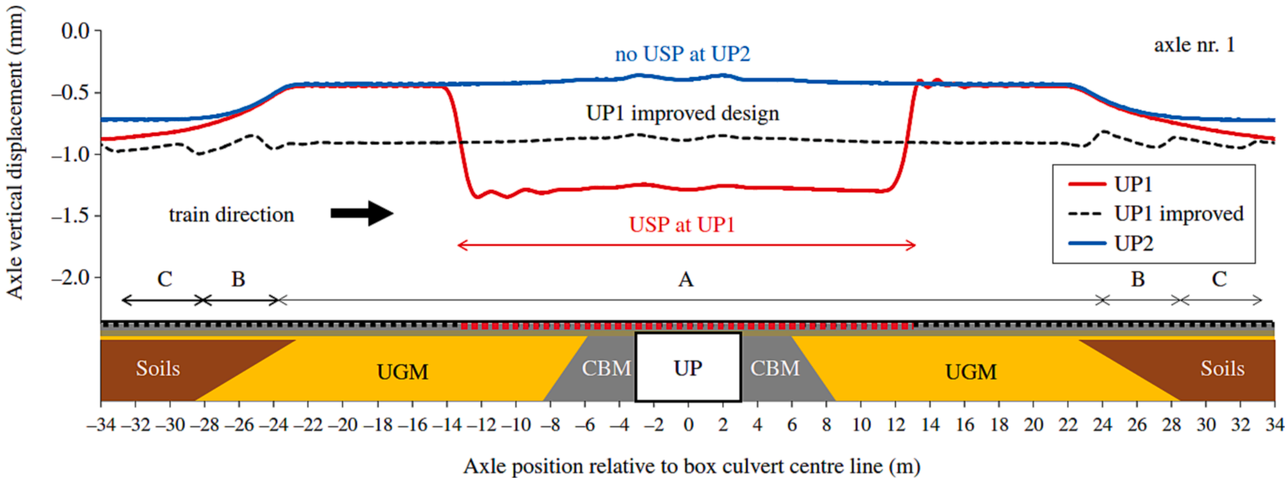


Fig. 18. Vertical displacement of the first axle of a train crossing transition zones (figure reproduced from [129]).

provide gradual variation of the track vertical stiffness. Therefore, the stiffness of USPs should be carefully determined for each transition zone before installation.

Furthermore, the influence of a combination of polyurethane and assistant rails to improve the track stiffness distribution in transition zones was investigated by Zhao et al. [130]. Fig. 19 shows the vertical static track stiffness in the ballasted-ballastless transition zone measured in the field. The result indicated that the vertical stiffness of the ballastless track is about twice as that of the ballasted track and there was a sudden change in the track stiffness between the two types of tracks. By bonding the ballast with various amount of polyurethane along the track, the gradual transition from low track stiffness to high stiffness was achieved in the transition zone. In addition, adding assistant rails spreads the vertical track stiffness more evenly.

Although these solutions are beneficial to diminish the track stiffness irregularity in transition zones and to alleviate the harm of uneven track stiffness to train operations and track structures, problems caused by stiffness irregularity still exist in the long-term train operation [125]. Therefore, it is recommended to propose better solutions to smooth out the track stiffness irregularity in the future, and to further study the effects of these solutions on the long-term track performance.

7. Conclusions

Extensive literatures have been published in recent years to the investigate the problems related to track stiffness irregularity. Focusing on the following four important aspects on the track stiffness irregularity, i.e. the measurement results, its influence, the critical values and the solutions, a summary and a critical review on the existing research have been discussed in this paper.

Field and laboratory measurement results have indicated significant difference among the track stiffness values at various test sites. The minimum track stiffness found in the existing studies is 16.8 kN/mm whereas the maximum value reaches 140 kN/mm. Continuous measurement results further show distinct track stiffness irregularities for railway tracks, e.g. up to 60 kN/mm (track stiffness) or 60 MN/m^2 (track modulus) in a few tens of meters-long track sections. Some preliminary statistical results show that the track stiffness follows the normal

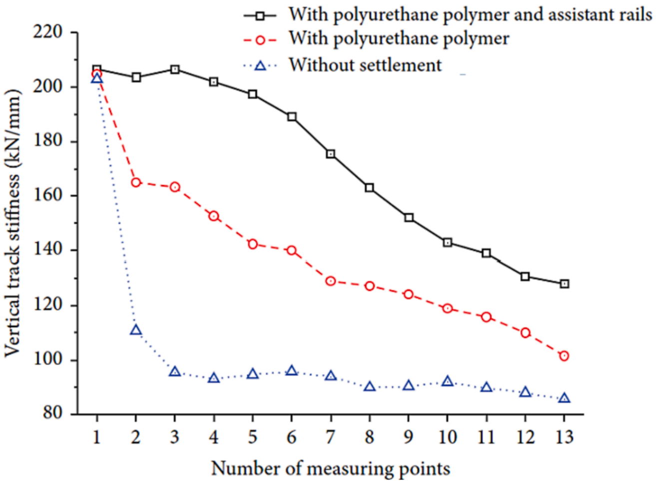


Fig. 19. Vertical track stiffness in a transition zone (figure reproduced from [130]).

distribution or the Weibull and Gamma distribution. These studies help incorporating the track stiffness irregularity in the train-track dynamic interaction simulations.

The existing research has proved that track stiffness irregularity has significant influence on the performance of the train-track system. The track stiffness irregularity affects the train-track dynamic interaction and causes load distribution variation on adjacent sleepers, which in turn accelerates the degradation and settlement of the track substructures and deteriorates the track geometry irregularities. It is recommended that future research on the long-term deterioration and predicting the fatigue life of track structures should better take into account the effects of the track stiffness irregularity.

Regarding the important role of track stiffness in the train-track system, the optimal track stiffness suggested in the published studies is within the range of 50–130 kN/mm for different railway lines. The allowable range of track stiffness regarding to the characteristics of faster and heavier rail transportation still needs to be studied in follow-up research. The critical values of the track stiffness irregularities for various railways should be systematically investigated in the future considering the remarkable influences of the track stiffness irregularity and very few suggestions on the critical values are found in the present literatures. With the application of the critical values of the track stiffness irregularities, the stiffness conditions of the operational tracks can be conveniently evaluated. The research in this field can be extended to the establishment of an integrated management of track geometry and stiffness irregularities. It is beneficial to improve railway maintenance and extend the service life of tracks.

Additionally, the countermeasures applied to mitigate the track stiffness irregularities are also summarized, which could help researchers and engineers to choose more suitable modification techniques. More in-depth studies are still needed to investigate the influence of the mitigation measures on the long-term service performance of the tracks.

CRediT authorship contribution statement

Can Shi: Conceptualization, Funding acquisition, Writing – original draft. **Yu Zhou:** Writing – original draft. **Lei Xu:** Supervision, Writing – review & editing. **Xu Zhang:** Conceptualization, Funding acquisition, Supervision, Writing – review & editing. **Yunlong Guo:** Writing – review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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