Evaluating Linear and Nonlinear Model Predictive Control for Reducing Cross-coupling Effects in Helicopter Flight

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***Abstract*—Model predictive control is an optimal, model-based control method that has the powerful capability of directly including input and output constraints. Next to this, it is known that helicopters are hard to fly with its complex, unstable and highly coupled dynamics. With the introduction of the concept of handling qualities, guidelines for helicopter and flight control system design were set in the ADS-33 document to improve the ease of controlling rotorcraft. In order to improve helicopter handling qualities, this paper investigates whether linear and nonlinear MPC are suitable for online application to helicopters to reduce cross-coupling effects. This was investigated by evaluating its performance on the cross-coupling requirements of the ADS-33 handling quality document. It was found that both linear and nonlinear MPC are very effective to reduce cross-coupling effects even when disturbances or prediction model errors are present. The model predictive controller could reduce the off-axis coupling response by around 99% compared to the uncontrolled helicopter. Furthermore, it performed 90% to 99% better than a PID controller in most coupling cases.**

***Index Terms*—cross-coupling effects, flight control, handling qualities, helicopters, model predictive control.**

NOMENCLATURE

ADS	Aeronautical Design Standard
DOF	Degree of Freedom
LMPC	Linear Model Predictive Control
MPC	Model Predictive Control
NLMPC	Nonlinear Model Predictive Control
PID	Proportional Integral Derivative
TA&T	Target Acquisition and Tracking
β	sideslip angle
$\delta_{lon}, \delta_{lat}$	longitudinal and lateral stick displacement
Δt_s	simulation sampling time
ϵ	error in the derivatives of the prediction model
λ_0	non-dimensional uniform inflow velocity
$\lambda_{0_{tr}}$	tail rotor non-dimensional uniform inflow velocity
σ	standard deviation of ε
$\theta_0, \theta_{1s}, \theta_{1c}, \theta_{0_{tr}}$	helicopter control inputs: collective pitch angle, longitudinal cyclic pitch angle, lateral cyclic pitch angle and tail rotor collective pitch angle
ϕ, θ, ψ	fuselage Euler angles
ε	simulation model uncertainty or disturbance
D_{MPC}	estimated derivative used in the prediction model
D_{actual}	actual helicopter derivative
e	tracking error
$\bar{e}$	tracking error vector along the prediction horizon
h	altitude
i	prediction horizon time step
K	feedback gain
k	control time step
N	prediction horizon
N_u	control horizon
n_z	normal acceleration
p, q, r	helicopter body angular rates
pk	subscript peak
Q	tracking error weight matrix
$\bar{r}$	reference state vector along the prediction horizon
ref	subscript reference
t	time
$trim$	subscript value at trim
u	control input vector
u, v, w	helicopter velocity along the body axes
$\bar{u}$	control input vector along the prediction horizon
x	state vector
x, y, z	helicopter coordinates in the Earth reference frame
$\bar{x}$	predicted state vector along the prediction horizon

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I. INTRODUCTION

COMPARED to fixed-wing aircraft, helicopters are highly versatile vehicles that can be used to execute a diverse range of commercial and military missions mainly due to its extreme maneuverability in low- and high-speed flight, vertical take-off and landing capabilities and the ability to hover. However, these great capabilities come with the fact that they are very difficult to control: they have fast, complex dynamics, are inherently unstable and its motion is highly coupled. Not only does this increase the workload of the pilot tremendously, it is also the cause of many fatal accidents [1]. With the introduction of flight control systems and fly-by-wire in helicopters in the 90's-00's, the flying characteristics of the helicopter could be adjusted to the pilot's needs to make the helicopter easier and safer to fly [2], [3]. Furthermore, handling quality requirements were set up in order to serve as a guideline for desired flight characteristics to improve the ease of controlling an aircraft [4]. However, to this day helicopters remain hard to fly and not accessible to the general public. Therefore, designing flight control systems in order to improve the helicopter handling qualities and safety is an important but challenging task.

At the same time Model Predictive Control (MPC) is emerging as a promising model-based optimal control technique with the powerful capabilities of including constraints on inputs and outputs, and including an objective function directly in the control algorithm. Furthermore, MPC has the advantage of being able to take into account future information of the system and the environment. This allows MPC to deal efficiently with time delays, non-minimum phase behaviour and to anticipate on future events [5]. Therefore, MPC offers an easy way to directly incorporate technical specifications, safety limits and performance bounds into the helicopter flight control design and to calculate the optimal control input based on a customized objective function and future information of the flight dynamics and flight condition.

On the other hand the optimization process in MPC brings along a big computational burden. Even though optimization methods and computer power are rapidly improving, the real time application of MPC to fast dynamic systems such as helicopters is still in development. Furthermore, when the theoretical and unpractical Lyapunov stability modifications are not implemented to the MPC problem, the MPC problem has to be stabilized by means of tuning. This can be time consuming and requires expertise as no structured tuning approach exists. Especially for nonlinear MPC, the computational burden and stability matter can become critical [6].

MPC was originally used in the 80's for industrial processes in areas as refining, petrochemicals and pulp and paper but is now making its way into other applications such as electronics, medicine, energy and environment and the automotive and aerospace industry [7]–[9]. With the rising popularity of MPC new possibilities for improving helicopter flight are emerging [10]. Research has been performed on MPC applied to helicopters from the 00's onwards where mainly tracking tasks but also other tasks were investigated such as formation flying [11], object avoidance [12], [13], flying in autorotation [14] and for defining control limits corresponding to flight envelope limits [15], [16] or load limits [17]. It has been demonstrated by Liu et al. (2012) that MPC has excellent tracking performance for flying a pirouette maneuver showing that the controller can handle the extremely coupled lateral and longitudinal dynamics [18]. Furthermore, the square maneuver performed by Liu et al. (2010) tests the MPC controlled helicopter's ability to fly forwards, backwards and sideways [19]. Here, flying the square trajectory was performed within 10 cm of the reference trajectory in a small-scaled flight test. It was also shown that by using robust MPC, the controller can deal with bounded external disturbances [20] and with constant wind gusts [18]. However, most previous research on MPC applied to helicopters focused on application in simulation. Only few research tested the controller experimentally in a mechanical set-up with limited Degrees of Freedom (DOF) [21]–[23] or in a small-scaled flight test with an unmanned aerial vehicle [18], [24]–[26].

In short, it can be seen that there is a clear need for helicopters to achieve good handling qualities such that helicopters will be easier to fly and maneuver. One of the biggest reasons it is so hard to fly a helicopter is because of the many cross-coupling effects in its dynamics. Therefore, this is also a big aspect in the handling quality requirements specified in the "ADS-33 Aeronautical design standard performance specification: handling qualities requirements for military rotorcraft" [27]. With MPC having numerous advantages and making its way into the aerospace industry, it is being applied to helicopters in multiple researches. In this research, it will be investigated how MPC can be used for helicopter flight control to reduce cross-coupling effects and achieve better handling qualities. Therefore, the objective of this research is:

to investigate whether linear and nonlinear MPC are suitable for online application to helicopters to reduce cross-coupling effects by evaluating its performance on the cross-coupling handling quality requirements of the ADS-33 document.

On one hand, it will be investigated how well Linear Model Predictive Control (LMPC) and Nonlinear Model Predictive Control (NLMPC) are able to reduce cross-couplings on the handling quality rating scale, compared to an uncontrolled helicopter and compared to a Proportional Integral Derivative (PID) controlled helicopter. On the other hand, it will be investigated how sensitive the MPC controllers are to prediction model errors when reducing cross-coupling effects. Furthermore, the similarities and differences between linear and nonlinear MPC will be analyzed.

This paper will first clarify the methodology used to fulfill the research objective in Section II. Section III shows the model predictive control design that will be analyzed. Next, Section IV presents the results of the cross-coupling requirement simulations after which the results of the sensitivity analysis will be presented in Section V. Finally, the findings of this paper and recommendations for future work will be stated in Section VI.

II. Methodology

In this section the method used for answering the research objective will be described. First the cross-coupling requirements that will be investigated will be explained. After this, the simulation set-up of the cross-coupling requirement simulations and the sensitivity analysis will be stated. Then, the uncertainty implemented in the simulation model for the cross-coupling requirement simulations will be introduced. Furthermore, the error implemented in the prediction model for the sensitivity analysis will be presented. Next, the nonlinear and linear helicopter model used for the simulations will be introduced. Finally, the PID controller used to compare the MPC controller to will be presented.

A. Cross-coupling Requirements

First, some background information on cross-coupling effects and the requirements defined by the Aeronautical Design Standard (ADS) will be given. After this, the cross-coupling test cases used for the simulations will be presented.

1) Background: When for example a step input is given in the collective stick of the helicopter, a change in height is the helicopter's primary dynamic response. However, due to the helicopter's complex dynamics many secondary, off-axis responses arise as well: because of the change in collective

TABLE I
PRIMARY AND SECONDARY RESPONSES FOR EACH INPUT AXIS [28].

Input \ Response	Pitch θ	Roll ϕ	Heave w	Yaw ψ
Longitudinal cyclic θ_{1s}	primary response	due to lateral flapping	desired in forward flight	negligible
Lateral cyclic θ_{1c}	due to longitudinal flapping	primary response	descent with roll angle	undesired
Collective input θ_0	due to longitudinal flapping	due to lateral flapping and sideslip	primary response	due to change in torque requires tail rotor thrust
Tail rotor collective $\theta_{0_{tr}}$	negligible	due to tail rotor thrust and sideslip	undesired	primary response

TABLE II
CROSS-COUPLING REQUIREMENTS SPECIFIED BY THE ADS-33 FOR OFF-AXIS DYNAMIC RESPONSES [29].
* NO CURRENT REQUIREMENTS.

Input \ Response	Pitch θ	Roll ϕ	Heave w	Yaw ψ
Pitch θ (Longitudinal cyclic θ_{1s})	X	$\Delta\phi_{pk}/\Delta\theta_4$ hover and fwd flight	flight path response not objectionable in forward flight	* yaw response due to rotor torque changes in aggressive pitch manoeuvres
Roll ϕ (Lateral cyclic θ_{1c})	$\Delta\theta_{pk}/\Delta\phi_4$ hover and fwd flight	X	* thrust/torque spikes in rapid roll reversals	$\Delta\beta/\Delta\phi$ ratios in fwd flight
Heave w (Collective input θ_0)	$\Delta\theta_{pk}/\Delta n_{z_{pk}}$ in fwd flight	* $\Delta\phi_{pk}/\Delta n_{z_{pk}}$	X	$r/\left\lvert\dot{h}\right\rvert$ ratios in hover
Yaw ψ (Tail rotor collective $\theta_{0_{tr}}$)	* pitching moments due to sideslip in fwd flight	dihedral effect on roll control power	not objectionable in hover	X

input, there is a change in torque of the main rotor which will cause the helicopter to yaw. In order to counter this yaw motion, the pedal needs to be used to generate a counter-acting moment coming from the tail thrust. Similarly, when an input is given to one of the other control inputs, the helicopter responds with a primary on-axis response and some secondary responses in the off-axis degrees of freedom. An overview of the primary and secondary responses of each control input is given in Table I where it can be seen that many cross-coupling effects are caused by lateral or longitudinal flapping of the rotor blades or by changes in the rotor torque. These off-axis responses are often referred to as inter-axis coupling, input-output coupling or cross-coupling effects. They are mostly undesired as they increase the workload of the pilot immensely even for straightforward tasks such as maintaining hover.

Therefore, requirements on the amount of cross-coupling effects in helicopter flight are widely described in the ADS-33 handling qualities document [27]. Here, the ADS-33 puts requirements on the amount of off-axis response present such that the helicopter has good handling qualities. In this way, the ADS-33 provides a way to objectively measure cross-coupling effects and handling qualities and serves as a guidance for the design of the helicopter and its flight control systems. Here, handling qualities are defined as *"those qualities or characteristics of an aircraft that govern the ease and precision with which a pilot is able to perform the tasks required in support of an aircraft role"* by Cooper and Harper (1969) [30]. For most cross-coupling effects, the document has defined a certain parameter indicating the amount of off-axis response compared to the amount of on-axis input given. Hence, when flying the helicopter and giving a step input in one of the controls, this parameter that resembles the amount of off-axis response should remain within the required limits in order to have a certain level of handling qualities. In order to specify these limits, level 1, 2 and 3 handling quality boundaries for these parameters were defined based on Cooper-Harper ratings of flight tests. This rating scale subjectively measures the ease of controlling an aircraft by letting the pilot answer a series of questions about flying the maneuver to then categorize the maneuver in a level of handling quality [30]. Here, level 1 is the best level with excellent to fair handling qualities requiring no to minimal pilot workload to perform the maneuver. Level 2 captures the maneuvers with aircraft characteristics with minor to very objectionable but tolerable deficiencies. Level 3 indicates the worst level of handling qualities where major deficiencies are present in the aircraft characteristics and an extensive workload is required to fly the maneuver. These boundaries can then be used as design requirements or just as indicative guidelines. The cross-coupling requirements specified in the ADS-33 document for off-axis responses are summarized in Table II with its respective parameter representing the amount of cross-coupling.

2) Test Cases: There are 10 cross-coupling requirements that will be tested which are formulated in the ADS-33 in Section 3.3.9 page 12 on interaxis coupling for hover and low speed flight and 3.4.5 page 17 on interaxis coupling for forward flight. The hover and low speed flight requirements will be performed at 0 knots flight speed and the forward flight requirements will be simulated at 80 knots or 41 m/s flight speed. For all these requirements, an excitation in one of the control inputs is given after which the off-axis response will be measured by means of a predefined cross-coupling parameter that scales with the off-axis response. The cross-coupling criteria for hover and low speed flight and for forward flight that will be tested are presented below and will be explained more thoroughly in Section IV.

For hover and low speed flight:

1) yaw due to collective for aggressive agility
2) pitch due to roll coupling for aggressive agility
3) roll due to pitch coupling for aggressive agility
4) pitch due to roll coupling for target acquisition & tracking
5) roll due to pitch coupling for target acquisition & tracking

For forward flight:

6) pitch attitude due to collective control
   a) small collective inputs
   b) large collective inputs
7) pitch due to roll coupling for aggressive agility
8) roll due to pitch coupling for aggressive agility
9) pitch due to roll coupling for target acquisition & tracking
10) roll due to pitch coupling for target acquisition & tracking

Both time (for aggressive agility) and frequency (for target acquisition and tracking) requirements are set out in the ADS-33 for pitch and roll coupling as coupling handling qualities are not only task but also frequency dependent. "A pilot may be less tolerant of large amounts of coupling at high frequency for an aggressive-precision task but may find the same amount acceptable for a non-aggressive low precision task." as discussed by Blanken et al. (1997) [31]. Therefore, the frequency domain criteria is needed in order to also capture the short-term coupling response that corresponds to high precision, agile tracking tasks.

For the time domain requirements, the control input that will be given in order to excite the on-axis response will mostly be a step input of plus or minus 10% of the control input range given one second after the simulation started. This usually leads to a significant and fast change in the on-axis attitude. In some simulation cases, which will be mentioned, the step input is smaller than the 10% change because of helicopter limits. The control input that will be given for the frequency domain requirements will be explained in Section IV-E.

B. Simulation Set-up

This section will discuss the control and model set-ups used for the cross-coupling requirement simulations and the sensitivity analysis. An overview of the models used as simulation and prediction model for the cross-coupling simulations and the sensitivity analysis can be found in Figure 1.

1) Cross-coupling Requirement Simulations: The effectiveness of MPC to reduce cross-coupling effects during helicopter flight will be evaluated by investigating its performance on the 10 cross-coupling requirements set out by the ADS-33 document for hover and forward flight. The performance of reducing cross-coupling effects will be measured by means

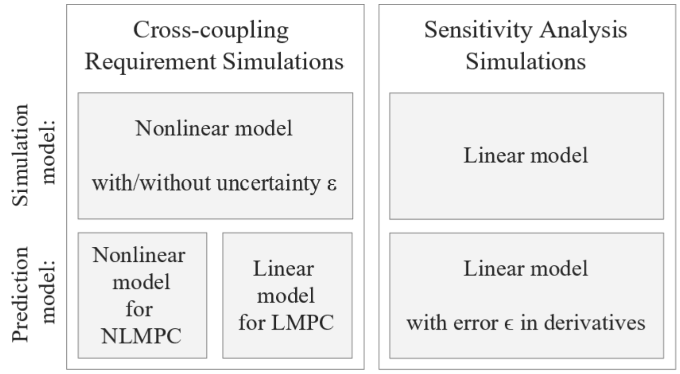

Fig. 1. Overview of simulation and prediction model set-up for the cross-coupling requirement and sensitivity analysis simulations.

of the cross-coupling parameter defined in the ADS-33 and the handling quality level it corresponds to. This will be done in a simulation of the BO-105 helicopter where each of the cross-coupling cases will be tested for the helicopter with nonlinear MPC applied to it, with linear MPC applied to it, the helicopter without controller and the PID controlled helicopter. In this way, the performance of the MPC controllers can be compared to the uncontrolled helicopter and to a conventional control technique. Furthermore, the linear and nonlinear MPC controller can be compared to each other. In this simulation the objective of the controllers will be to minimize the off-axis attitude responses when simulating both a positive and negative step in the on-axis control input. The position of the helicopter and the on-axis response will be uncontrolled. In the uncontrolled simulations, the on-axis and relevant off-axis attitude will be uncontrolled. The off-axis attitude that is not part of the cross-coupling case will be controlled to remain constant using the simple PID controller from Section II-F e.g. yaw attitude in the pitch due to roll coupling case.

The simulation will use the nonlinear, 8 DOF helicopter model ran at 100 Hz as simulation model which has to represent the actual helicopter dynamics. Furthermore, the MPC controllers also use a helicopter model in order to predict the future states of the helicopter. The same nonlinear 8 DOF model is used as prediction model for the nonlinear MPC controller whereas the linear MPC controller will use the linearized 8 DOF model. Both models will be explained further in Section II-E. In order to be able to compare the performance of NLMPC with LMPC without the bias of NLMPC having a perfect future state prediction, an uncertainty is added to the simulation model. Hence, the 4 control configurations will be tested in a simulation with and without uncertainty added to the simulation model. In this way, not only an unbiased comparison can take place but also a more realistic behaviour of the helicopter can be simulated as the uncertainty will be implemented as a disturbance in the main rotor thrust. More on this uncertainty that is added to the simulation model can be found in Section II-C.

2) Sensitivity Analysis Simulations: The robustness or sensitivity of MPC to prediction model errors will be investigated by evaluating the decoupling performance of the MPC controllers when a mismatch or error is present in the prediction model. In order to be able to systematically implement an error in the prediction model, the linear prediction model will be used. In this way the error can be applied to one of the relevant derivatives in the state and input matrix. To reduce the model mismatch between the simulation and prediction model, the linear model is also used for the simulation. The implementation of the fixed error in the prediction model will be explained in Section II-D

The aim of the sensitivity analysis is twofold. First of all, for each cross-coupling case the important derivatives will be identified by means of implementing a fixed error in every prediction model derivative relevant to the cross-coupling case, one at a time. Then, cross-coupling requirement simulations are performed and the cross-coupling parameters are measured. Based on the change in cross-coupling parameter and if the controller still has level 1 handling qualities the derivatives which alter the handling qualities of the MPC controlled helicopter the most can be found. This information is important as to known which prediction model derivative needs to be of high accuracy in order to still have level 1 handling qualities. Secondly, once the important derivatives have been identified they will be investigated further by varying the error that is implemented and measuring how this affects the cross-coupling parameter. This information gives understanding to how sensitive these derivatives are to errors and what kind of errors are most performance degrading (over/underestimating, changing sign, etc.). It must be noted that in this research only the influence of one error at a time will be investigated as to pinpoint the important derivatives. The robustness to multiple errors at the same time is beyond the scope of this research.

C. Introducing the Uncertainty

An uncertainty will be implemented in the nonlinear simulation model for the cross-coupling simulations for two reasons. Firstly and most importantly, the error is introduced in order to remove the positive bias of the nonlinear MPC controller. Secondly, the addition of the uncertainty into the helicopter model adds more realistic dynamics as the uncertainty that is added acts as a disturbance to the main rotor thrust. Without the uncertainty, the nonlinear MPC would have a perfect prediction model which is unrealistic and yields an unfair comparison of the nonlinear MPC with the linear MPC. Furthermore, it was decided to introduce the uncertainty in the simulation model instead of in the prediction model in order to have a consistent implementation for both the linear and nonlinear MPC, maintaining comparability. This entails that there is also a disturbance introduced in the helicopter dynamics which will be noticeable in the behavior of the helicopter but not unwanted.

The uncertainty ε is introduced as a time-varying random variable with normal distribution $\varepsilon \sim \mathcal{N}(\sigma, 0)$ with a standard deviation of σ and zero mean [32]. It is applied to the main rotor thrust coefficient as the thrust force is the main aerodynamic force acting on the helicopter, affecting the motion in all degrees of freedom, and is also very hard to predict. Hence, adding an uncertainty in the thrust coefficient in the model is realistic. It is applied according to Equation 1 so

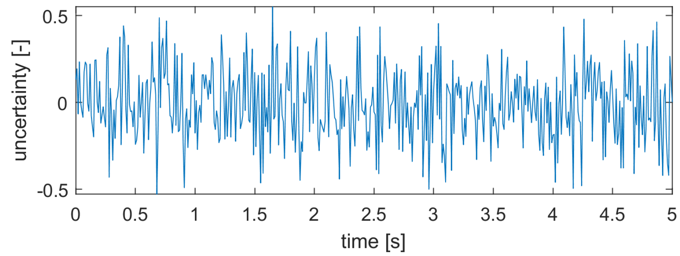

Fig. 2. A 5 second trial of the uncertainty ε with $\sigma = 0.2$ over time.

that C_T is being decreased or enlarged with ε multiplied with the original thrust coefficient. As can be seen, the uncertainty varies with time: each simulation time step Δt the uncertainty ε changes. As the uncertainty is randomly generated each time step, every simulation is different. Therefore, a series of 6 simulations, called trials, are ran where the cross-coupling results are linearly averaged.

$$C_T = C_T \cdot (1 + \varepsilon(\Delta t)) \tag{1}$$

For the simulations, a standard deviation of $\sigma = 0.2$ is chosen which means that 68% of the generated uncertainties will be within $[-0.2, 0.2]$ and 95% will be within $[-0.4, 0.4]$. In Figure 2, one can see a trial of this randomly generated uncertainty over 5 seconds.

D. Introducing the Sensitivity Analysis Error

The error will be implemented in the prediction model of the MPC controller in the elements of the state matrix A and input matrix B of the linear helicopter model. More specifically, it will be implemented in the relevant elements only e.g. for yaw due to collective coupling the error will be implemented in the derivatives of the yaw acceleration so $\frac{\partial \dot{r}}{\partial u}, \frac{\partial \dot{r}}{\partial v}, \ldots$ in the A matrix and $\frac{\partial \dot{r}}{\partial \theta_0}, \frac{\partial \dot{r}}{\partial \theta_{1s}}, \ldots$ in the B matrix. Here, a simplified notation will be used such that for example the derivative $\frac{\partial \dot{r}}{\partial u}$ will be noted as $\dot{r}_u$.

The error ϵ will be implemented to the actual derivative in a dimensionless manner as can be seen in Equation 2. Here, the estimated derivative D_{MPC}, so the derivative with error used by the MPC controller, will be equal to the actual derivative D_{actual} plus a fraction ϵ of the actual derivative. An overview of how the error value influences the proportions between the actual and the MPC derivative can be found in Equation 3.

$$D_{MPC} = D_{actual}(1 + \epsilon) \tag{2}$$

$$\begin{aligned}
\epsilon < -1: &\quad \text{sgn}(D_{MPC}) = -\text{sgn}(D_{actual}) \\
\epsilon = -1: &\quad D_{MPC} = 0 \\
-1 < \epsilon < 0: &\quad |D_{MPC}| < |D_{actual}| \\
\epsilon = 0: &\quad D_{MPC} = D_{actual} \\
\epsilon > 0: &\quad |D_{MPC}| > |D_{actual}|
\end{aligned} \tag{3}$$

In order to find out how large such an error realistically could be when modeling a helicopter, data from Pavel (1996), considered as estimated derivatives, was compared to data from the NASA model of Heffley et. al (1979), considered as actual derivatives [33], [34]. Here, it could be seen that most errors are within -1 and 0, hence underestimating the actual derivative in absolute value. It is only for a few cases that a greater positive or negative error occurs but still around an absolute value of 1. Furthermore, some outliers were spotted with errors of ± 30. However, these only occur when the actual derivative is almost zero. As will be clear later from the results of the sensitivity analysis, the accuracy of these derivatives barely influence the MPC performance at all.

Based on an error analysis of the data from Pavel (1996) and Heffley et. al (1979), it was chosen to first find the important derivatives by applying an error of 10 and -10 to all of the relevant derivatives one by one and measuring the cross-coupling parameters [33], [34]. After this a range of errors from -10 to 10, so $\epsilon = -10, \ -9, \ -8, \ -7, \ -6, -5, \ -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$, will be applied to the most important derivatives in order to have an individual analysis.

E. Helicopter Model

The 8 DOF nonlinear model of the BO-105 helicopter used for the simulations in this research was developed at the TU Delft and consists out of 6 helicopter body DOFs and 2 rotor inflow DOFs, one for the main rotor and one for the tail rotor [33], [35]. The rotor inflow dynamics is added because the hingeless rotor system of the BO-105 causes the rotor and body dynamics to be highly coupled [36]. Furthermore, the body dynamics takes into account the forces and moments from the main rotor, tail rotor, fuselage, horizontal tail and vertical tail. The helicopter's motion will be described by a total of 14 states and will be controlled by 4 control inputs namely the main rotor collective, the longitudinal cyclic, the lateral cyclic and the tail rotor collective as seen in Equation 4 and 5 respectively.

$$x = [u \ v \ w \ p \ q \ r \ \psi \ \theta \ \phi \ x \ y \ z \ \lambda_0 \ \lambda_{0_{tr}}]' \tag{4}$$

$$u = [\theta_0 \ \theta_{1s} \ \theta_{1c} \ \theta_{0_{tr}}]' \tag{5}$$

The linear 8 DOF model of the system is obtained by linearizing the nonlinear model around a certain trim condition (x_{trim}, u_{trim}) using perturbation linearization [37][p. 563]. The linear model then approximates the nonlinear model at and around this trim condition. The more the helicopter state deviates from the trim condition or the more nonlinear the helicopter behaves at this trim condition, the worse the linear approximation will be.

Furthermore, some physical boundaries are imposed on the control inputs because of actuator limits. Firstly, the control inputs are bounded by upper and lower limits. The data for these limits of the BO-105 helicopter is retrieved from Prouty (2002) [37]. Secondly, the rate of change in each control input is limited. No rate limits were found for the BO-105 so the rate data for the Bell 412 helicopter from Voskuijl et al. (2010) was used [38]. The input ranges and input rate limits of the BO-105 helicopter model can be found in Table III.

F. PID Controller Design

In order to be able to compare the performance of the MPC controller with a controlled helicopter, a simple Proportional Integral Derivative controller will be implemented. This PID controller uses control rules based on the error between the reference state and the actual state, the integral of this error and the gradient of this error. For the cross-coupling simulations, only the attitude of the helicopter will be controlled. Therefore, the PID rules, which can be seen in Equation 6-8, are implemented to θ_{1s_0}, θ_{1c} and $\theta_{0_{tr_0}}$ only [36]. Here, the $K_{...}$'s are the gains that were tuned using the Ziegler-Nichols method and fine-tuned using trial and error. The final values of the gains can be seen in Table IV. Furthermore, the integral term in these PID rules is taken in discrete time over an interval of $t - 5\Delta t$ to t where t is the current time and Δt_s is the simulation time step. As can be seen, the inputs are solely dependent on the on-axis tracking error e.g. θ_{1s} depends on $\theta - \theta_{ref}$ only.

$$\theta_{1s} = \theta_{1s_{trim}} + K_{\theta_1}(\theta - \theta_{ref}) + K_q q + K_{\theta_2} \sum_{t-5\Delta t_s}^{t} (\theta - \theta_{ref})\Delta t \tag{6}$$

$$\theta_{1c} = \theta_{1c_{trim}} + K_{\phi_1}(\phi_{ref} - \phi) + K_p p + K_{\phi_2} \sum_{t-5\Delta t_s}^{t} (\phi_{ref} - \phi)\Delta t \tag{7}$$

$$\theta_{0_{tr}} = \theta_{0_{tr_{trim}}} + K_{\psi_1}(\psi - \psi_{ref}) + K_r r + K_{\psi_2} \sum_{t-5\Delta t_s}^{t} (\psi - \psi_{ref})\Delta t \tag{8}$$

Similar to the MPC controller, only the relevant DOFs will be tracked in a simulation. The inputs for the uncontrolled DOFs are then set to the trim value instead of applying the PID rule. Furthermore, the inputs calculated by the PID controller are limited to their respective maximum or minimum boundary value as there are physical constraints on the control inputs.

TABLE III
INPUT RANGE AND RATE LIMITS.

Limit	Value [deg]	Limit	Value [deg]	Limit	Value [deg·s]
$\theta_{0_{min}}$	-0.2	$\theta_{0_{max}}$	15.0	$\Delta\theta_{0_{max}}$	16.0 $\cdot\Delta t$
$\theta_{1s_{min}}$	-6.0	$\theta_{1s_{max}}$	11.0	$\Delta\theta_{1s_{max}}$	28.8 $\cdot\Delta t$
$\theta_{1c_{min}}$	-5.7	$\theta_{1c_{max}}$	4.2	$\Delta\theta_{1c_{max}}$	16.0 $\cdot\Delta t$
$\theta_{0_{tr_{min}}}$	-8.0	$\theta_{0_{tr_{max}}}$	20.0	$\Delta\theta_{0_{tr_{max}}}$	32.0 $\cdot\Delta t$

TABLE IV
PID CONTROLLER GAINS FOR THE SIMULATIONS.

Gain	Value [-]	Gain	Value [-]	Gain	Value [-]
K_{θ_1}	3	K_{ϕ_1}	0.55	K_{ψ_1}	16
K_{θ_2}	11.2	K_{ϕ_2}	40	K_{ψ_2}	170
K_q	0.8	K_p	-0.35	K_r	1.9

III. MODEL PREDICTIVE CONTROL

This section will first introduce the concept of linear and nonlinear model predictive control for reference tracking after

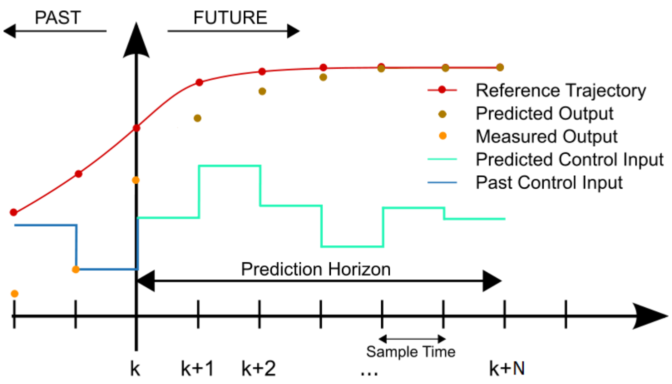

Fig. 3. The concept of MPC in discrete time for reference tracking [39].

which the MPC controller design used for the simulations will be presented.

A. Introduction to MPC

MPC is a type of model-based, optimal control where at each time step, k, an optimal control input sequence $\bar{u}_k = [u_k,\ u_{k+1},\ \ldots,\ u_{k+N-1}]$ is computed online over a future time horizon, the prediction horizon N, by solving an open-loop optimization problem that has knowledge of the system model [5]. The optimization uses the current state of the system as initial state and a model of the system to compute the future states along the prediction horizon in order to optimize a desired objective function. Then, only the first control input in this optimal control input sequence u_k is applied to the system. At the next time step, the prediction horizon of the optimization problem shifts one step forward, to $k+1$, and the next optimal control sequence $\bar{u}_{k+1} = [u_{k+1},\ u_{k+2},\ \ldots,\ u_{k+N}]$ is computed.

In Figure 3, one can see the concept of MPC explained in discrete time for a reference tracking problem. In a reference tracking problem, the objective function of the optimization is to minimize the error $\bar{e} = [e_{k+1},\ \ldots,\ e_{k+N}]$ between the reference trajectory $\bar{r} = [r_{k+1},\ \ldots,\ r_{k+N}]$ and the predicted output trajectory $\bar{x} = [x_{k+1},\ \ldots,\ x_{k+N}]$. Then, the optimization problem consists of computing the optimal control input over the prediction horizon such that the tracking error is minimized and the constraints are met.

A distinction can be made between linear and nonlinear model predictive control. The difference lies in the use of a linear or nonlinear objective function, constraints and prediction model. If one these elements is nonlinear, the controller is considered a nonlinear MPC controller [5]. Nonlinearity often comes with non-convexity which can cause the optimization problem to have multiple local optima and which also increases the complexity of solving the optimization problem. Therefore, NLMPC usually has an increased computation time and can cause the optimization solution to become suboptimal. However, also the fidelity of the model plays a big roll in the closed-loop performance as the algorithm optimizes the error between the predicted state and the reference state over the prediction horizon. When MPC with a linear prediction model is applied to a highly nonlinear system, the prediction model might not be of sufficient fidelity. A discussion on how this influences the results of the cross-coupling requirement simulations is held in Section IV-F4. Furthermore it must be noted that in this report use is made of a quadratic objective function with positive definite weight and of a constraint with an absolute value function which are nonlinear but convex functions. Nevertheless, when the linear prediction model is used the controller will still be considered a linear MPC controller as the objective function, constraints and prediction model are still convex.

B. Controller Design

The MPC design used for the simulations will be presented in this section including its objective function, constraints, the prediction models and tuning parameters.

1) Objective Function: The goal of the controller in the cross-coupling requirement simulations is to reduce the off-axis response when an on-axis input is given. In order to achieve this, the MPC controller is going to track a constant trim reference signal for the off-axis responses only. Then, the objective of the MPC controller in the cross-coupling requirement simulations is to minimize the error between the state and the reference signal for the off-axis states. A quadratic objective function will be used to minimize the tracking error with weight Q and reference trajectory r as can be seen in Equation 9.

$$\underset{\bar{u}_k,\ \bar{x}_k}{\text{minimize}} \quad \sum_{i=0}^{N} \left\{ \left(x_{k+i} - r_{k+1}\right)' \, Q \, \left(x_{k+i} - r_{k+1}\right) \right\} \tag{9}$$

Here, the weight Q changes depending on the cross-coupling case. For example, if the requirement for pitch due to roll cross-coupling is being simulated, the pitch and yaw angle will be tracked whereas the roll angle won't be controlled. For this case Q will be equal to $\text{diag}(0,0,0,0,0,0,1,0,1,0,0,0,0,0)$. The reference trajectory of the pitch and yaw angle will be the trim value of the respective angle. It must be noted that only the attitudes (ψ, θ, ϕ) will be controlled and not the angular rates (p, q, r) or angular accelerations $(\dot{p}, \dot{q}, \dot{r})$. This would yield steady-state offsets if no integral term would be added. Furthermore, it is only the attitude that is the direct state that needs to be controlled.

2) Constraints: One of the big advantages of model predictive control is that it can incorporate soft and hard constraints on inputs and states directly in the controller. Hence, some physical boundaries on the input range and input rates are imposed because of actuator limits. Firstly, the input range is limited for each control input by $u_{min} = [\theta_{0_{min}}\ \ \theta_{1s_{min}}\ \ \theta_{1c_{min}}\ \ \theta_{0_{tr_{min}}}]'$ and $u_{max} = [\theta_{0_{max}}\ \ \theta_{1s_{max}}\ \ \theta_{1c_{max}}\ \ \theta_{0_{tr_{max}}}]'$. Secondly, the rate of change in each control input is limited by $\Delta u_{max} = [\Delta\theta_{0_{max}}\ \ \Delta\theta_{1s_{max}}\ \ \Delta\theta_{1c_{max}}\ \ \Delta\theta_{0_{tr_{max}}}]'$. The values of the limits used in the simulations can be seen in Table III. These limits are implemented according to Equation 10 and 11 and hold over the entire prediction horizon and for all control inputs. The state variables are not bounded by upper and lower limits but are constraint by the dynamics of the helicopter.

$$u_{min} < u_{k+i} < u_{max} \qquad \text{for } i = 1, 2, \dots, N \tag{10}$$

$$|u_{k+i} - u_{k+i-1}| < \Delta u_{max} \qquad \text{for } i = 1, 2, 3, \dots, N \tag{11}$$

3) Prediction Model: The 8 DOF BO-105 helicopter model described in Section II-E will be used as prediction model in the MPC controller. Depending on whether linear or nonlinear MPC will be implemented, the linear or nonlinear 8 DOF model will be used as prediction model. It must be noted that by using the nonlinear model as prediction model, the optimization of the MPC controller becomes non-convex. More on the differences between NLMPC and LMPC can be found in IV-F4.

4) Tuning Parameters: First of all, the controller will have a sampling time of 0.03 s. With a simulation sampling time of 0.01 s this means the controller calculates a new control input every 3 simulation time steps. In the remaining steps, the control input is kept the same as the previously calculated input. Next, a constant prediction horizon N of 5 control time steps (0.15 s) is used. In order to reduce the computation time, a control horizon N_u of 3 control steps (0.09 s) was selected. Hence, after 3 control time steps the control input of the last step is fixed for the remaining steps in the prediction horizon.

5) Complete MPC Formulation: To summarize the MPC controller design that is used in the cross-coupling simulations and the sensitivity analysis simulations, the complete MPC optimization problem is presented in Equation 12. The optimization problem will be solved in Matlab 2020b with the fmincon-function using sequential quadratic programming as optimization algorithm which is a smooth nonlinear optimization method. Here, the trim control inputs are used as initial value. It must be noted that for each simulation individual components can change such as the model when using LMPC or NLMPC or implementing the error from the sensitivity analysis, or the weight Q when a different cross-coupling case is tested.

$$\begin{aligned}
&\underset{\bar{u}_k,\, \bar{x}_k}{\text{minimize}} && \sum_{i=1}^{N} \left\{ \left(x_{k+i} - r_{k+1}\right)' \, Q \, \left(x_{k+i} - r_{k+1}\right) \right\} \\
&\text{subject to:} && x_{k+i} = f(x_{k+i-1}, u_{k+i-1}) \quad \text{for } i = 1, 2, ..., N \\
& && u_{min} < u_{k+i} < u_{max} \quad \text{for } i = 0, 1, ..., N-1 \\
& && |u_{k+i} - u_{k+i-1}| < \Delta u_{max} \quad \text{for } i = 0, 1, ..., N-1 \\
&\text{with:} && x = [u \; v \; w \; p \; q \; r \; \psi \; \theta \; \phi \; x \; y \; z \; \lambda_0 \; \lambda_{0_{tr}}]' \\
& && u = [\theta_0 \; \theta_{1s} \; \theta_{1c} \; \theta_{0_{tr}}]'
\end{aligned} \tag{12}$$

IV. Cross-coupling Requirement Simulations

This section will present the results and analysis of the cross-coupling requirement simulations for all 10 cross-coupling cases. For each coupling case the cross-coupling parameter results for one simulation setting will be shown. In general, the results of the other settings are comparable and will therefore be discussed briefly in the overview tables in Section IV-F. Furthermore, a demonstration of how to calculate the cross-coupling parameter will be presented for

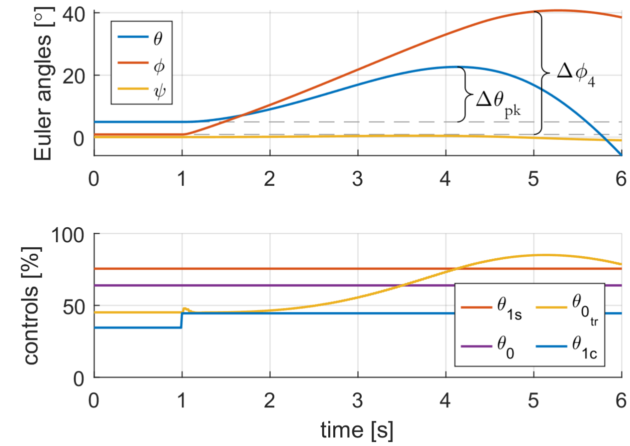

Fig. 4. Pitch due to roll requirement simulation of the uncontrolled helicopter for 80 knots for a positive (right) lateral cyclic step input.

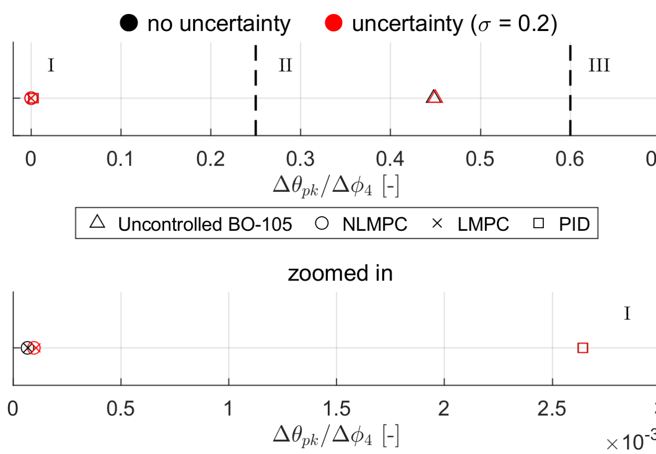

Fig. 5. Pitch due to roll requirement results for 80 knots for a positive (right) lateral cyclic step input.

pitch due to roll for both the time and frequency domain requirement. Moreover, an off-axis rate response analysis will be performed for pitch due roll coupling as an example in order to analyze and compare the coupling reduction performance of the PID and MPC controller.

A. Pitch due to Roll Coupling

For both pitch due to roll and roll due to pitch coupling the ADS33 states that *"The ratio of peak off-axis attitude response from trim within 4 seconds to the desired (on-axis) attitude response from trim at 4 seconds, $\Delta\theta_{pk}/\Delta\phi_4$ ($\Delta\phi_{pk}/\Delta\theta_4$), following an abrupt lateral (longitudinal) cockpit control step input, shall not exceed ± 0.25 for Level 1 or ± 0.60 for Level 2. Heading shall be maintained essentially constant."* [27]. Therefore, a step input of ±10% the control range is given in the lateral cyclic at $t = 1$ s as can be seen in Figure 4. In this Figure a demonstration is given on how to calculate the cross-coupling parameter of the uncontrolled helicopter using Equation 13. A $\Delta\theta_{pk}/\Delta\phi_4$ of 0.45 was obtained.

$$\begin{aligned}
&\text{if a step input is given at } t = 0 \text{ s} \\
&\Delta\theta_{pk} = (\max |\theta| \text{ before } t = 4 \text{ s}) - \theta_{trim} \\
&\Delta\phi_4 = \phi(t = 4 \text{ s}) - \phi_{trim}
\end{aligned} \tag{13}$$

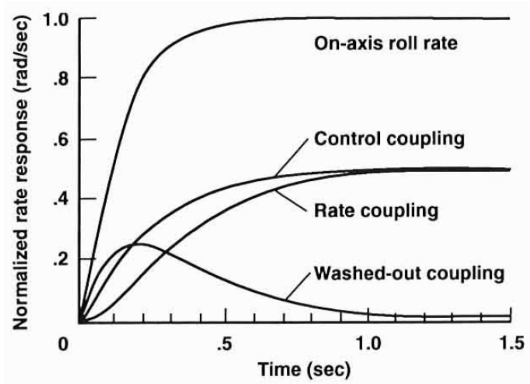

Fig. 6. On- and off-axis rate responses to a lateral cyclic input [31].

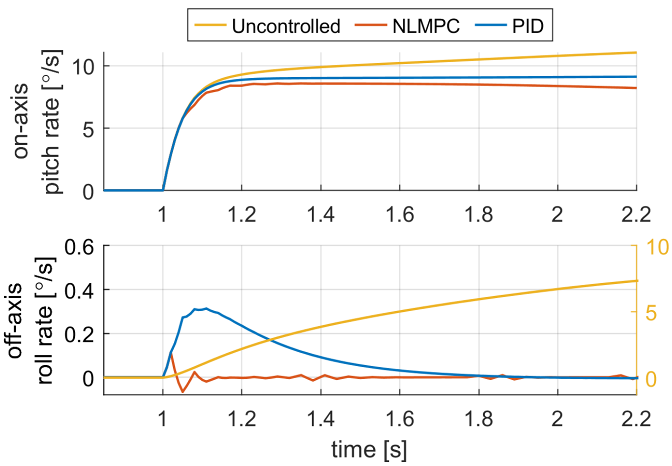

Fig. 7. Pitch due to roll coupling on/off-axis response analysis for 80 knots for a positive (right) lateral cyclic step input.

The cross-coupling parameter results for all control configurations for 80 knots flight with a positive lateral cyclic step input can be seen in Figure 5. It can be seen that the cross-coupling parameter is reduced significantly when the helicopter is being controlled, going from level 2 to level 1 with plenty of margin. When zooming in to 10^{-3} one can see that NLMPC reduces the off-axis response the most with shortly after that the LMPC controller. The PID controller also performs great but cannot surpass the MPC performance. Moreover, it can be seen that the uncertainty doesn't seem to have much of effect to the coupling reduction performance for all control set-ups.

As to investigate the off-axis rate response of the different control set-ups and to indicate the difference between the PID and MPC coupling reduction behaviour, the pitch and roll rate responses for a step input in the lateral cyclic at $t = 1$ s are investigated. The different types of off-axis rate responses defined by Blanken et al. (1997) can be seen in Figure 6. Here, the ideal off-axis rate response is the response with no coupling so with a rate staying as close to zero as possible. In Figure 7 it can be seen that the uncontrolled helicopter shows an off-axis rate response with control coupling. When the controllers are introduced, the off-axis response reduces significantly, eliminating most cross-coupling effects. The PID

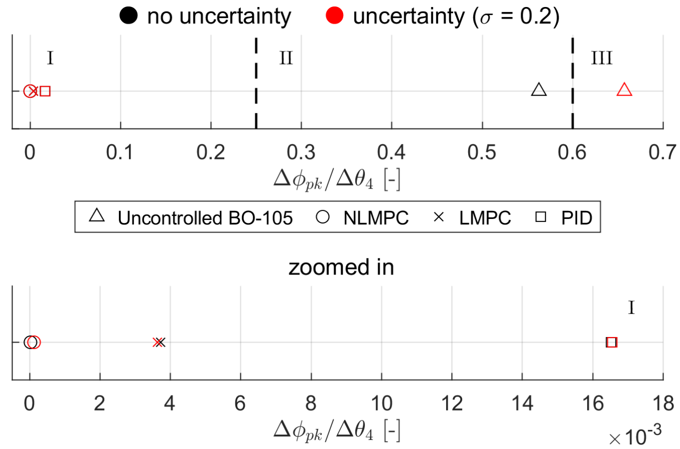

Fig. 8. Roll due to pitch requirement results for 80 knots for a positive (up) longitudinal cyclic step input.

controller shows a small and quick washed-out coupling response whereas the MPC controller reduces the off-axis rate even more and faster, showing a response with quasi no coupling.

B. Roll due to Pitch Coupling

The requirement for roll due to pitch coupling is very similar to the pitch due to roll coupling requirement and is therefore already explained in Section IV-A. The computation of the cross-coupling parameter can be seen in Equation 14.

$$\begin{aligned} &\text{if a step input is given at } t = 0 \text{ s} \\ &\Delta\phi_{pk} = (\max\ |\phi| \text{ before } t = 4 \text{ s}) - \phi_{trim} \\ &\Delta\theta_4 = \theta(t = 4 \text{ s}) - \theta_{trim} \end{aligned} \tag{14}$$

In Figure 8 one can see that again the controllers reduce the handling qualities from level 3 or 2 to level 1. When zooming in to 10^{-3} it can be seen that NLMPC performs best at minimizing the roll angle, almost completely eliminating the cross-coupling effects. Close after NLMPC comes LMPC and then the PID controller. Again, the uncertainty barely has an effect on the cross-coupling parameter results with controller. Without controller, the uncertainty degrades the handling qualities to level 3.

C. Yaw due to Collective Coupling

The ADS-33 states that *"The yaw rate response to abrupt step collective control inputs with the directional controller fixed shall not exceed the boundaries specified in Figure 11. The directional controller may be free if the rotorcraft is equipped with a heading hold function. Pitch and roll attitudes shall be maintained essentially constant. ... Oscillations involving yaw rates greater than 5 deg/sec shall be deemed objectionable."* [27]. The yaw rate boundaries that are referred to can be seen in Figure 9. Here, r_1 is defined as the largest peak of yaw rate by magnitude between the start of the step input and 3 seconds after the step input. Furthermore, $\dot{h}(3)$ is the value of $\dot{h}$ at 3 seconds after the step input. Finally, r_3 is equal to $r(3) - r_1$ for $r_1 > 0$ and to $r_1 - r(3)$ for $r_1 < 0$

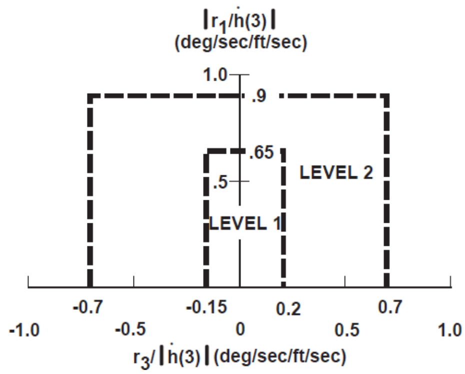

Fig. 9. Yaw due to collective coupling requirement [27]

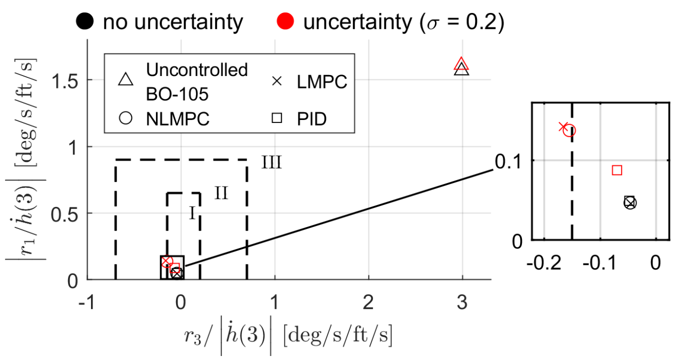

Fig. 10. Yaw due to collective requirement results for hover for a positive (up) collective step input.

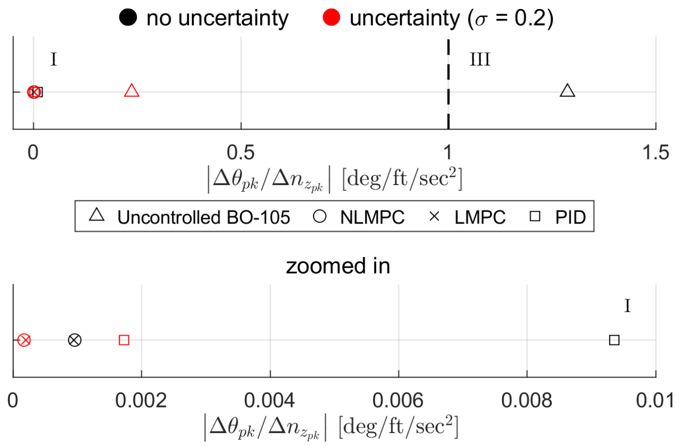

Fig. 11. Pitch due to collective requirement results for 80 knots for a small, positive (up) collective cyclic step input.

where $r(3)$ is the yaw rate at 3 seconds after the step input. The complete computation of the cross-coupling parameters can be seen in Equation 15.

$$\begin{aligned}
&\text{if a step input is given at } t = 0 \text{ s}\\
&\dot{h}(3) = \dot{h}(t = 3 \text{ s})\\
&r_1 = \max |r| \text{ before } t = 3 \text{ s}\\
&\text{if } r_1 > 0 : r_3 = r(t = 3 \text{ s}) - r_1\\
&\text{if } r_1 < 0 : r_3 = r_1 - r(t = 3 \text{ s})
\end{aligned} \tag{15}$$

The results of the yaw due to collective requirement simulations for hover for a positive input can be seen in Figure 10. Here, again the handling qualities are improved from level 3 to level 1 when a controller is introduced. However, when the uncertainty is present the results of the linear and nonlinear MPC controllers are both located just over the border of the level 1 boundary. Nevertheless, the result of the PID controller with uncertainty remains in level 1.

This rather large performance difference can be explained by the fact that the MPC uses the prediction model of the helicopter which now has a mismatch with the disturbed simulation model. Furthermore, this coupling case is significantly more vulnerable to the mismatch as the uncertainty is applied to the thrust coefficient which is directly related to the collective input. Moreover, when the positive input is given the thrust of the helicopter increases, as opposed to the negative input, causing the disturbance in the thrust coefficient to have more effect. As can be seen in Table V, the MPC controllers are still in the level 1 zone for the negative input case. Hence, it can be concluded that for yaw due to collective coupling the MPC controllers are sensitive to this uncertainty in the main rotor thrust. However, one could improve the robustness of the MPC controller to this kind of model mismatches by implementing robust MPC.

D. Pitch due to Collective Coupling

The requirement for pitch due to collective coupling is split in a requirement for small collective inputs (<20% rotor torque change) and large collective input (>20% rotor torque change). For small collective inputs the ADS-33 says that *"the peak change in pitch attitude from trim, $\Delta\theta_{pk}$, occurring within the first 3 seconds following a step change in collective causing less than 20% torque change, shall be such that the ratio $|\Delta\theta_{pk}/\Delta n_{z_{pk}}|$ is no greater than 1.0 deg/ft/sec^2, where $\Delta n_{z_{pk}}$ is the peak incremental normal acceleration from 1 g flight."* [27]. For large collective inputs, the ratio $|\Delta\theta_{pk}/\Delta n_{z_{pk}}|$ should be no greater than 0.5 deg/ft/sec^2 for a positive collective input and no greater than 0.25 deg/ft/sec^2 for negative collective inputs. The computation of the cross-coupling parameter can be seen in Equation 16.

$$\begin{aligned}
&\text{if a step input is given at } t = 0 \text{ s}\\
&\Delta\theta_{pk} = (\max |\theta| \text{ before } t = 3 \text{ s}) - \theta_{trim}\\
&\Delta n_{z_{pk}} = (\max |\dot{w}| \text{ before } t = 3 \text{ s}) - \dot{w}_{trim}
\end{aligned} \tag{16}$$

The cross-coupling results for pitch due to small collective inputs can be seen in Figure 11. Here, it is clear that again the handling qualities are improved from level 3 or 2 to level 1 when a controller is applied. When zooming in to 10^{-3} it can be seen that both NLMPC and LMPC have a very small cross-coupling parameter, almost completely eliminating the off-axis response. The PID controller also improves the handling qualities a lot but still has a larger cross-coupling parameter than the MPC controllers.

What is remarkable about these simulations is that the simulation with uncertainty has, for all control set-ups, significantly better coupling reduction performance. This can be explained by the random behaviour of the uncertainty that is

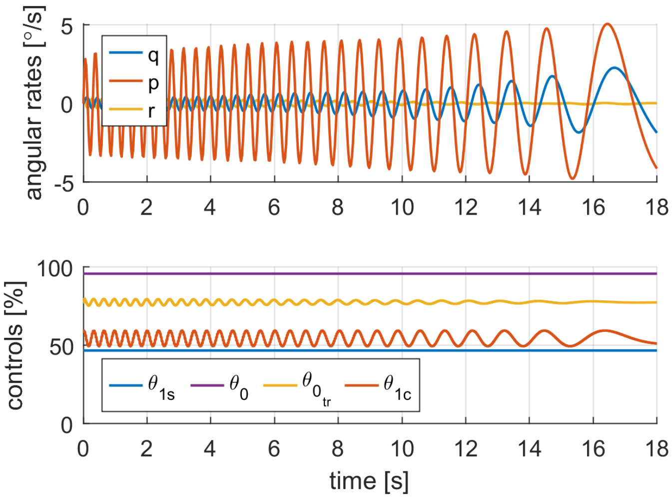

Fig. 12. Pitch due to roll frequency requirement simulation of the uncontrolled helicopter for 80 knots for a positive (right) lateral cyclic step input.

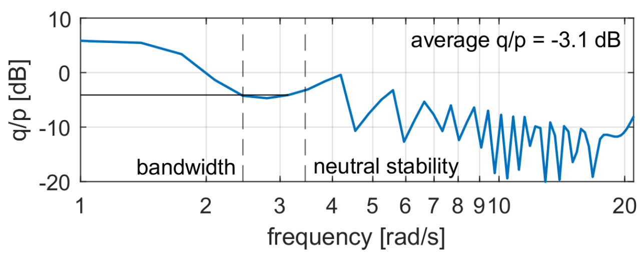

Fig. 13. q/p frequency response of the uncontrolled helicopter for 80 knots for a positive (right) lateral cyclic step input (corresponding to Figure 12).

implemented in the thrust coefficient and that changes each simulation time step. This causes $\dot{w}$ and hence $n_{z_{pk}}$ to change each time step as well, yielding a very large $\Delta n_{z_{pk}}$. Therefore, in this coupling case the cross-coupling parameter does not give a proper indication of the off-axis response compared to on-axis input. That is, the cases with uncertainty cannot be compared to the undisturbed cases. Still, the same performance proportions are found for the uncontrolled, NLMPC, LMPC and PID controlled helicopter with uncertainty as compared to the results without uncertainty.

E. Pitch due to Roll and Roll due to Pitch Coupling for Target Acquisition and Tracking

The ADS-33 states that the pitch due to roll (q/p) and roll due to pitch (p/q) coupling parameters should not exceed the boundaries indicated in Figure 14 where *"the average q/p and average p/q are derived from ratios of pitch and roll frequency responses. Specifically, average q/p is defined as the magnitude of pitch-due-to-roll control input (q/δ_{lat}) divided by roll-due-to-roll control input (p/δ_{lat}) averaged between the bandwidth and neutral-stability (phase = -180 deg) frequencies of the pitch-due-to-pitch control inputs (θ/δ_{lon}). Similarly, average p/q is defined as the magnitude (p/δ_{lon}) divided by (q/δ_{lon}) between the roll-axis (ϕ/δ_{lat}) bandwidth and neutral stability frequencies."* [27]. Here, the bandwidth is defined as the lesser of the phase bandwidth, which is the frequency corresponding to -135° phase, and gain bandwidth,

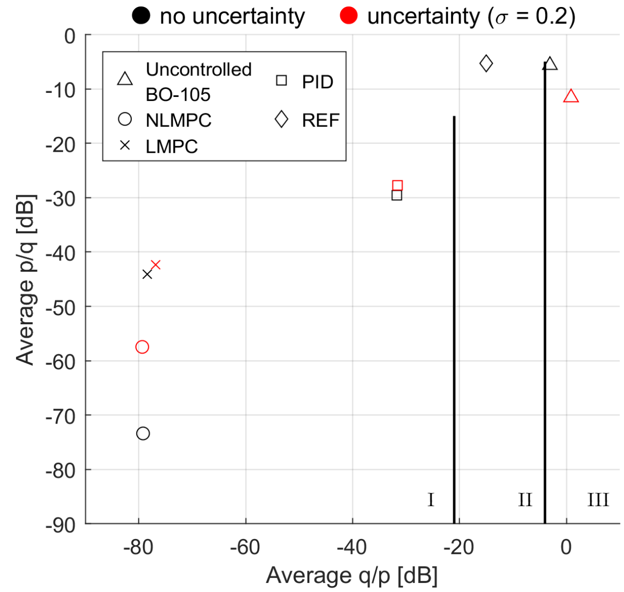

Fig. 14. Average p/q over average q/p for 80 knots [31].

which is the frequency corresponding to the magnitude at neutral stability with a margin of 6 dB added to it. For the calculation of the pitch and roll bandwidth it was assumed that δ_{lon} and δ_{lat} are equivalent to θ_{1s} and θ_{1c} respectively. As the limits set by the ADS-33 are not perfectly clear, the limits for q/p will be set to -21 dB for level 1/2 and -4 dB for level 2/3 and for p/q to -10 dB for level 1/2 and -5 dB for level 2/3.

As a demonstration for the frequency parameter calculations, the simulation of the pitch due to roll frequency requirement for the uncontrolled helicopter in 80 knots flight is shown in Figure 12. Here, a frequency sweep was given in the lateral cyclic input from 20 rad/s to 0.5 rad/s for 18 s. The longitudinal cyclic and collective were kept constant whereas the tail rotor collective was controlled by a PID controller in order to maintain a constant yaw angle. As can be seen, the on-axis roll rate is oscillating with the lateral input, inducing the off-axis pitch rate to oscillate as well but with a slightly smaller amplitude. By calculating the frequency response of the pitch rate divided by the roll rate using the fast Fourier transform algorithm, the q/p gain can be obtained. Here, the gain of q/p gives an accurate indication of the amount of off-axis pitch rate response compared to on-axis roll rate. As can be seen in Figure 13, the average q/p gain between the pitch bandwidth and neutral stability frequency was found to be -3.1 dB. This means that for a roll rate amplitude of 10°/s the pitch rate amplitude would be 7°/s on average for frequencies between 2.4 and 3.4 rad/s.

The average p/q over average q/p for 80 knots results for the different control set-ups can be seen in Figure 14. Here, the uncontrolled helicopter has level 3, at the border of level 2, handling qualities. When the controllers are introduced the handling qualities go to level 1. The PID controller brings the amount of cross-coupling back to around -30 dB for both pitch due to roll and roll due to pitch coupling, with and without uncertainty. This indicates that for a roll (pitch) rate amplitude of 10°/s the pitch (roll) rate amplitude would be 3°/s

TABLE V
OVERVIEW OF THE CROSS-COUPLING HANDLING QUALITY LEVEL RESULTS.

Cross-coupling case	Condition	BO-105		NLMPC		LMPC		PID	
		σ = 0	σ = 0.2	σ = 0	σ = 0.2	σ = 0	σ = 0.2	σ = 0	σ = 0.2
Pitch d.t. roll	0 kn, +ve input	III	III	I	I	I	I	I	I
	0 kn, -ve input	II	II	I	I	I	I	I	I
	80 kn, +ve input	II	II	I	I	I	I	I	I
	80 kn, -ve input	III	III	I	I	I	I	I	I
Roll d.t. pitch	0 kn, +ve input	III	III	I	I	I	I	I	I
	0 kn, -ve input	III	III	I	I	I	I	I	I
	80 kn, +ve input	II	III	I	I	I	I	I	I
	80 kn, -ve input	II	II	I	I	I	I	I	I
Yaw d.t. collective	+ve input	III	III	I	II	I	II	I	I
	-ve input	III	III	I	I	I	I	I	I
Pitch d.t. collective	small, +ve input	III	I	I	I	I	I	I	I
	small, -ve input	III	I	I	I	I	I	I	I
	large, +ve input	III	III	I	I	I	I	I	I
	large, -ve input	III	III	I	I	I	I	I	I
Pitch d.t. roll for TA&T	0 kn	II	II	I	I	I	I	I	I
	80 kn	III	III	I	I	I	I	I	I
Roll d.t. pitch for TA&T	0 kn	II	II	I	I	I	I	I	I
	80 kn	II	I	I	I	I	I	I	I

on average. The MPC controllers go even further to about -80 dB for q/p indicating a pitch rate amplitude of only 0.002°/s for a roll rate amplitude input of 10°/s. For p/q NLMPC goes to -75 dB without and -57 dB with uncertainty whereas LMPC goes to about -45 dB for both with and without uncertainty.

F. Overview of the Cross-coupling Results

This section will first present an overview of the handling quality levels of each cross-coupling case. Next, a comparison of the cross-coupling parameter of NLMPC with the uncontrolled helicopter and of NLMPC and LMPC with the PID controller will be made for both the simulations with and without uncertainty. Lastly, a comparison of the performance of linear and nonlinear MPC will be presented.

1) Overview of Handling Quality Levels: An overview of the cross-coupling handling quality level results can be seen in Table V. Here, the uncontrolled helicopter mostly has level 3 or 2 handling qualities. Once a controller is introduced, the handling qualities are improved to level 1. This indicates that all controllers succeed very well at reducing the cross-coupling effects in order to have good handling qualities. Even with uncertainty added to the simulation model, the controllers are able to obtain level 1 handling qualities. The only exception is the NLMPC controller for the yaw due to collective case for a positive collective input which obtained level 2 handling qualities with the uncertainty. This exception will be further explained when looking at Table VII.

2) Comparison of the Cross-coupling Parameter (σ = 0): In Table VI a comparison of the cross-coupling parameters in percentage increase can be seen for the simulations without uncertainty. First of all, the NLMPC results are compared to the uncontrolled helicopter results where a negative percentage indicates a reduction of cross-couplings. Next, the NLMPC and LMPC are compared to the PID controller by indicating how much percent the MPC cross-coupling parameter is increased with respect to the PID cross-coupling parameter. Here, the positive values are indicated in red and indicate the PID controller is better at reducing couplings than MPC. It must be noted that for the yaw due to collective case, the $r_3/\left|\dot{h}(3)\right|$ parameter is used for the percentages as this was the limiting parameter for most cases.

First of all, it can be seen that the NLMPC reduces coupling by about 99.9% for almost all cross-coupling cases which is remarkably high. It indicates that the off-axis response can be almost entirely eliminated by introducing the MPC controller. Furthermore, when comparing the MPC to the PID controller almost all cases have much better cross-coupling reduction than the PID controller. Percentages of about 90% and 99% better than the PID controller are achieved for NLMPC whereas the LMPC has slightly lower percentages especially for roll due to pitch.

The roll due to pitch case for hover and a positive input even has the PID controller performing better than LMPC. This degradation of the LMPC performance happens because of the mismatch between the linear prediction model and nonlinear simulation model. It was found that at some point in the simulation the linear model estimates the roll and pitch rate to be of opposite sign as the actual nonlinear model causing the controls to change drastically, decreasing the coupling reduction performance. Nevertheless, the handling qualities of LMPC still remain far within the level 1 zone.

Next to this, the yaw due to collective case with a negative input seems to have a better cross-coupling parameter with PID controller. Furthermore, for a positive input the cross-coupling parameter for MPC is only 3 to 5 percent better than the PID controller which is much lower than in the other

TABLE VI
COMPARISON OF THE CROSS-COUPLING PARAMETER RESULTS IN PERCENTAGE INCREASE FOR THE SIMULATIONS WITHOUT UNCERTAINTY.

Cross-coupling case	Condition	NLMPC compared to BO105 [%]	NLMPC compared to PID [%]	LMPC compared to PID [%]
Pitch d.t. roll	0 kn, +ve input	-99.99	-97.71	-56.74
	0 kn, -ve input	-99.98	-98.35	-92.57
	80 kn, +ve input	-99.99	-97.47	-97.39
	80 kn, -ve input	-99.99	-98.95	-98.97
Roll d.t. pitch	0 kn, +ve input	-99.99	-98.99	58.20
	0 kn, -ve input	-100.00	-99.54	-92.70
	80 kn, +ve input	-100.00	-99.86	-77.47
	80 kn, -ve input	-99.96	-97.66	-73.36
Yaw d.t. collective	+ve input	-98.44	-3.13	-5.42
	-ve input	-98.13	4.40	6.25
Pitch d.t. collective	small, +ve input	-99.93	-89.73	-89.89
	small, -ve input	-99.93	-90.13	-89.98
	large, +ve input	-99.92	-89.37	-89.90
	large, -ve input	-99.94	-90.59	-90.11
Pitch d.t. roll for TA&T	0 kn	-99.95	-99.29	-99.45
	80 kn	-99.98	-99.58	-99.54
Roll d.t. pitch for TA&T	0 kn	-99.96	-99.53	-99.53
	80 kn	-99.96	-99.36	-81.21

TABLE VII
COMPARISON OF THE CROSS-COUPLING PARAMETER RESULTS IN PERCENTAGE INCREASE FOR THE SIMULATIONS WITH AN UNCERTAINTY OF $\sigma = 0.2$.

Cross-coupling case	Condition	NLMPC compared to BO105 [%]	NLMPC compared to PID [%]	LMPC compared to PID [%]
Pitch d.t. roll	0 kn, +ve input	-99.97	-95.01	-69.54
	0 kn, -ve input	-99.90	-90.89	-87.17
	80 kn, +ve input	-99.98	-96.35	-95.98
	80 kn, -ve input	-99.97	-96.46	-96.09
Roll d.t. pitch	0 kn, +ve input	-99.98	-98.82	60.42
	0 kn, -ve input	-99.98	-98.10	-90.39
	80 kn, +ve input	-99.98	-99.25	-78.07
	80 kn, -ve input	-99.82	-90.62	-64.22
Yaw d.t. collective	+ve input	-94.81	121.77	137.05
	-ve input	-97.26	5.85	13.72
Pitch d.t. collective	small, +ve input	-99.93	-90.12	-88.52
	small, -ve input	-99.93	-89.57	-88.49
	large, +ve input	-99.91	-88.59	-89.74
	large, -ve input	-99.96	-89.57	-89.24
Pitch d.t. roll for TA&T	0 kn	-99.76	-97.07	-95.61
	80 kn	-99.99	-99.59	-99.45
Roll d.t. pitch for TA&T	0 kn	-99.83	-98.04	-98.23
	80 kn	-99.49	-96.73	-81.39

cases. This can be explained by the fact that this parameter relies on the yaw rate response instead of the yaw angle. It is the only cross-coupling parameter depending on the angular rate instead of attitude. Since the MPC controller is focusing solely on minimizing the attitude error, aggressive yaw rate motions are induced causing the cross-coupling parameter to take up higher values. The PID controller is not that aggressive because of the differential term. The results for this case could be improved by adding a term to the objective function that directly minimizes the yaw rate.

3) Comparison of the Cross-coupling Parameter (σ = 0.2): In Table VII one can see the comparison of cross-coupling parameters in percentage increase for the simulations with uncertainty applied to the thrust coefficient. In general, it can be seen that the absolute percentages are only slightly lower than the absolute percentages of the simulations without uncertainty. This indicates that the MPC controllers are robust to this disturbance, preserving the coupling reduction performance.

Here, the yaw due to collective coupling case seems to be the exception. With uncertainty, the handling qualities for the

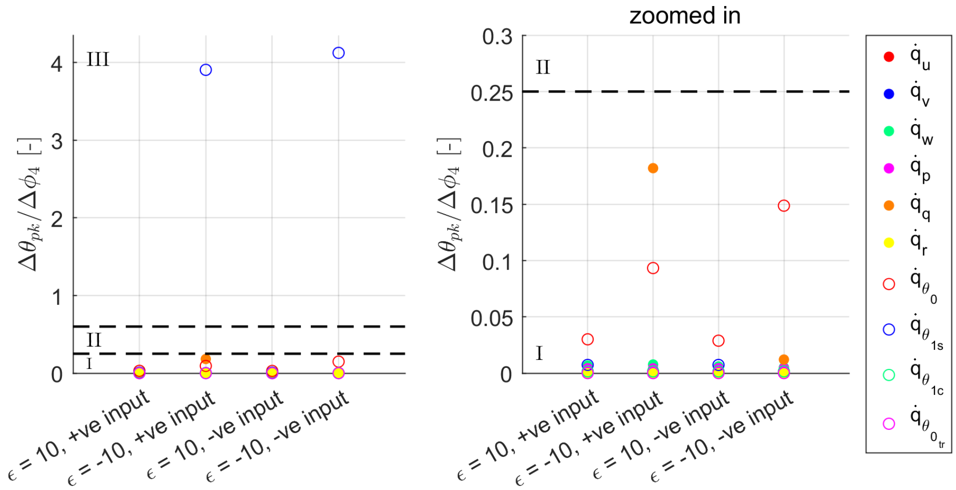

Fig. 15. Pitch due to roll requirement sensitivity analysis for 80 knots for a positive (right) and negative (left) lateral cyclic and for a positive and negative error implemented in one of the derivatives.

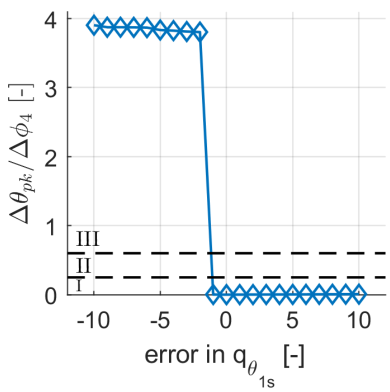

Fig. 16. Analysis of error in $\dot{q}_{\theta_{1s}}$-derivative for pitch due to roll coupling at 80 knots for a positive input.

positive input are even decreased to level 2 as said before. As the uncertainty is implemented in the thrust coefficient, which greatly influences the rotor torque, the yaw coupling is directly influenced. With this poorly estimated main rotor torque in the MPC prediction model, the MPC controller is unable to reduce the couplings in the yaw axis sufficiently. Also when comparing the MPC controllers to the PID controller, which does not rely on a prediction model, it is clear that the PID controller performs much better. A solution to this deteriorated performance of the MPC due to the highly influential disturbance could be to implement robust model predictive control. This will improve the performance of MPC to unmeasured disturbances but at the cost of decreased overall performance.

Next to this, the yaw due to collective coupling case seems to be the case with the least reduction of cross-couplings compared to the uncontrolled helicopter. This was also seen for the results without uncertainty as the yaw rate instead of angle is measured in the parameter.

4) Comparison of Linear and Nonlinear MPC: As discussed before in Section III-A, the difference between linear and nonlinear MPC in this report lies in the use of the nonlinear or linear prediction model in the MPC algorithm. On one hand, the nonlinearity in the optimization scheme comes with non-convexity and hence multiple local optima and a heavier computational burden. On the other hand, also the fidelity of the prediction model plays a roll in the closed-loop performance. Here, the linear prediction model might fall short as the linearization of the nonlinear system around a trim point only approximates the system at and around this trim point. The more the helicopter state deviates from the trim condition, the worse the linear approximation will be. Also, the more nonlinear the helicopter behaves at this trim condition, the worse the linear approximation will be.

In the cross-coupling results in this chapter it can be seen that both linear and nonlinear MPC perform very well at reducing couplings, even with an uncertainty applied in the simulation model. The performance difference between linear and nonlinear MPC for these simulations is very small. In most cases the nonlinear controller performs slightly better than the linear controller or has almost similar performance as the linear controller. This is an indication that the fidelity of the linear model is sufficient for the cross-coupling simulations to be used as prediction model. This can be explained by means of two reasons based on properties specific to the cross-coupling simulations. First of all, the model mismatch stays small because of the use of a very short prediction horizon which prevents the accumulation of error along the horizon. Secondly, as the reference trajectory that is tracked is the trim condition around which is linearized, the state stays relatively close to the linearization point which also limits the linear model mismatch.

There is one case, the roll due to pitch coupling case for hover with a positive input, where the linear controller performs worse than the PID controller. This was due to a linear model mismatch where the linear model predicted that $\dot{p}, \dot{q} > 0$ whereas the actual, nonlinear model states that $\dot{p}, \dot{q} < 0$, resulting in a sudden change of controls which is not present in the NLMPC and PID simulations. Nevertheless, the fidelity is still good enough to have level 1 handling qualities.

Overall it can be concluded that the differences in cross-coupling reduction performance between LMPC and NLMPC are so small they do not noticeably deteriorate the handling qualities and can be assumed to be non existent. As linear MPC has the advantage of having a shorter computation time and no suboptimal solutions, linear MPC is preferred over nonlinear MPC in order to reduce cross-coupling effects.

V. Sensitivity Analysis

This section will present the results of the sensitivity analysis simulations for all 10 cross-coupling cases. First, the sensitivity analysis of the pitch due to roll coupling case will be worked out as an example. Next, the final results of all coupling cases will be introduced in an overview table.

A. Pitch due to Roll Coupling

The sensitivity analysis for pitch due to roll coupling for 80 knots can be seen in Figure 15. Here, each dot represents the value of the cross-coupling parameter when the error of 10 or -10 is implemented in the corresponding derivative as indicated in the legend. It can be seen that the only derivative that gets the handling qualities out of the level 1 zone when an error, namely a negative error, is applied is the change in pitch acceleration due to longitudinal cyclic derivative $\dot{q}_{\theta_{1s}}$. When zooming in to the level 1 zone, it can be seen that also negative errors in $\dot{q}_q$ and $\dot{q}_{\theta_0}$ increase the cross-coupling parameter. Nevertheless, the handling qualities for these derivatives stay within level 1. It is notable that these three derivatives are also the ones with largest absolute value in the A and B matrix as can be seen in the pitch acceleration derivatives in Equation 17. This is also highly logical as the elements with the largest absolute value influence the dynamics of that degree of freedom the most.

From implementing this large error, it was found that $\dot{q}_{\theta_{1s}}$ is the important derivative for this coupling case, bringing the handling qualities from level 1 to level 3. Therefore, a more elaborate individual analysis is performed varying the error implemented in $\dot{q}_{\theta_{1s}}$. This individual analysis can be seen in Figure 16 and shows that once the error gets smaller than -1, so when the estimated derivative changes sign, the handling qualities jump from level 1 to level 3. Physically this is logical because if the change in pitch acceleration due to longitudinal cyclic input is estimated to be of opposite sign, then pulling the cyclic stick up would be causing the helicopter to pitch down. Hence, when the MPC prediction model has this physically incorrect and influential derivative, the resulting optimal control input cannot reduce the cross-coupling effects sufficiently in closed-loop. Nevertheless, positive errors seem to barely have an effect on the handling qualities when implemented to $\dot{q}_{\theta_{1s}}$.

B. Overview of the Sensitivity Analysis

An overview of the important derivatives for each cross-coupling case can be seen in Table VIII together with some characteristics of how the error influences the cross-coupling parameters. For example, when it says ϵ <-1, it means that the handling qualities are degraded to level 2 or 3 only for errors smaller than -1. Furthermore, 'symmetrical' means the error in the derivative influences the handling qualities in a symmetric way: when the absolute value of the error increases the cross-coupling parameter increases and hence the handling qualities decrease. When 0 or 80 knots is stated in the characteristics this means the handling qualities are only affected negatively for this flight speed. Furthermore, the actual values of the derivatives at 80 knots can be seen in Equation 17.

$$\begin{bmatrix} \dot{p}_u & \dot{p}_v & \dot{p}_w & \dot{p}_p & \dot{p}_q & \dot{p}_r \\ \dot{q}_u & \dot{q}_v & \dot{q}_w & \dot{q}_p & \dot{q}_q & \dot{q}_r \\ \dot{r}_u & \dot{r}_v & \dot{r}_w & \dot{r}_p & \dot{r}_q & \dot{r}_r \end{bmatrix} = \\ = \begin{bmatrix} 0.1 & -0.1 & -0.2 & -17.4 & 4.5 & 0.4 \\ 0.1 & 0.0 & 0.2 & 1.5 & -4.0 & 0.0 \\ 0.0 & 0.3 & -0.2 & -2.8 & 1.5 & -1.4 \end{bmatrix}$$

$$\begin{bmatrix} \dot{p}_{\theta_0} & \dot{p}_{\theta_{1s}} & \dot{p}_{\theta_{1c}} & \dot{p}_{\theta_{0_{tr}}} \\ \dot{q}_{\theta_0} & \dot{q}_{\theta_{1s}} & \dot{q}_{\theta_{1c}} & \dot{q}_{\theta_{0_{tr}}} \\ \dot{r}_{\theta_0} & \dot{r}_{\theta_{1s}} & \dot{r}_{\theta_{1c}} & \dot{r}_{\theta_{0_{tr}}} \end{bmatrix} = \\ = \begin{bmatrix} 4.3 & -8.7 & 159.6 & 9.0 \\ 23.5 & -49.8 & 4.6 & 0.0 \\ 4.6 & 8.4 & 21.8 & -22.5 \end{bmatrix} \tag{17}$$

Similar to the pitch due to roll coupling analysis, it is in general noticeable that the important derivatives are the derivatives that either have a relatively large value in the state-space matrix (Equation 17) or that experience a large change from trim throughout the cross-coupling simulations. Again, this is quite logical as the product of the derivative and the deviation of the state from trim determines the acceleration of that degree of freedom. Hence, when an error is present in the derivative with a large value, the mismatch between the estimated and actual motion increases. Being able to deduct which derivatives are important from the state-space matrices enables to extend the results of this BO-105 sensitivity analysis to other helicopters as well.

It can also be seen in this overview that the important derivatives are mostly control derivatives from matrix B. Furthermore, mostly negative errors, at least smaller than -1, degrade the handling qualities to level 2 or 3 whereas the positive errors barely change the cross-coupling effects in most cases. For the control derivatives this is highly logical because the error smaller than -1 indicates the derivative changes sign, meaning that the controls would be working in the opposite direction. For example, if the $\dot{r}_{\theta_{0_{tr}}}$ derivative is of opposite sign, the tail rotor force would be pointing the opposite direction. For the pitch damping derivative $\dot{q}_q$, the opposite sign is degrading the handling qualities because this is an important stability derivative for the phugoid Eigenmotion. When the sign is estimated incorrectly, the Eigenmotion of the helicopter is majorly affected.

Besides the control derivatives and the pitch damping derivative that degrade when negative errors are implemented, there are the $\dot{p}_p$ and $\dot{p}_u$ derivatives which are important for the roll due to pitch coupling for both positive and negative errors. Here, the roll damping derivative $\dot{p}_p$ is characteristic for the roll subsidence Eigenmotion and is therefore also important to be accurate regardless of the sign. Furthermore,

TABLE VIII
OVERVIEW AND CHARACTERISTICS OF THE IMPORTANT DERIVATIVES FOR EACH CROSS-COUPLING CASE.

Cross-coupling case	Important derivatives	Characteristics	Cross-coupling case	Important derivatives	Characteristics
Pitch d.t. roll	$\dot{q}_{\theta_{1s}}$	$\epsilon <$-1	Pitch d.t. roll for TA&T	$\dot{q}_{q}$	$\epsilon \leq$-8
				$\dot{q}_{\theta_0}$	$\epsilon \leq$-4, 80 kn
				$\dot{q}_{\theta_{1s}}$	$\epsilon \leq$-1
Roll d.t. pitch	$\dot{p}_{u}$	symmetrical, 0 kn	Roll d.t. pitch for TA&T	$\dot{p}_{p}$	$\sim$ symmetrical
	$\dot{p}_{p}$			$\dot{p}_{\theta_{1c}}$	$\epsilon \leq$-1
	$\dot{p}_{\theta_{1c}}$	$\epsilon \leq$-1		$\dot{p}_{\theta_{0_{tr}}}$	$\epsilon \leq$-6, 80 kn
Yaw d.t. collective	$\dot{r}_{\theta_0}$	symmetrical			
	$\dot{r}_{\theta_{1c}}$	$\epsilon \leq$-3			
	$\dot{r}_{\theta_{0_{tr}}}$	$\epsilon \leq$-1			
Pitch d.t. collective	$\dot{q}_{\theta_{1s}}$	$\epsilon \leq$-1			

the $\dot{p}_u$ derivative is a coupling derivative which couples the lateral and longitudinal motion when the rotor is tilting and a forward velocity change occurs. Hence, the tilting forward during the roll due to pitch maneuver creates this large change in forward velocity u, giving this derivative more importance in the helicopter dynamics.

As the error in the derivative was found to mostly stay within -1 and 1 in Section II-D, it can be concluded that the MPC controller is robust to these model errors and keeps having level 1 handling qualities. However, when the absolute error increases and specially when the errors gets smaller than -1, the performance of the MPC controller deteriorates to level 2 or 3 handling qualities. This could be solved by implementing robust MPC which improves the performance when an unmeasured error or disturbance is present.

VI. Conclusion and Recommendations

This work investigated whether linear and nonlinear model predictive control are suitable for online application to helicopters to reduce cross-coupling effects by evaluating its performance on the cross-coupling handling quality requirements of the ADS-33 document. The cross-coupling requirements were tested in simulation by implementing a step in one control input and measuring the cross-coupling parameter which represents the amount of off-axis response.

It was found that both linear and nonlinear MPC are able to reduce the off-axis response of the tested cross-coupling cases by around 99% compared to the uncontrolled helicopter bringing all handling quality levels from level 2 or 3 to level 1. Here, handling qualities of level 1 indicate having minimal pilot workload and desired aircraft characteristics. Also the PID controller is able to bring the handling qualities from level 2 or 3 to level 1. However, when comparing the MPC to the PID controller almost all MPC cases have 90% to 99% better cross-coupling reduction than the PID controller which can be explained by the optimal and model-based behaviour of the MPC controllers. Where the PID controller shows a washed-out coupling off-axis rate response, the MPC controllers almost eliminate all coupling showing a quasi decoupled off-axis rate response.

When a disturbance is introduced in the simulation model, the cross-coupling reduction performance is only slightly less, keeping level 1 handling qualities for most coupling cases. This indicates that MPC is robust to this disturbance. Only the yaw due to collective coupling case with uncertainty for a positive collective input gives level 2 handling qualities for the MPC controllers. However, this can be explained by the poorly estimated yaw coupling in the prediction model because of the unknown disturbance in rotor thrust and by the cross-coupling parameter that is based on the yaw rate instead of yaw angle which is optimized for. This could be solved by implementing a robust MPC controller or adapting the objective function to also minimize the yaw rate.

Furthermore, the differences in performance between linear and nonlinear MPC for the cross-coupling simulations are so small they do not noticeably degrade the handling qualities and can be assumed to be non-existent. As linear MPC has the advantage of having a shorter computation time and no suboptimal solutions, linear MPC is preferred over nonlinear MPC in order to reduce cross-coupling effects.

In addition, it was examined how sensitive MPC is to prediction model errors when reducing cross-coupling effects by implementing a fixed error in the relevant derivatives of the linear prediction model and measuring the performance change. It was found that the derivatives sensitive to errors are the derivatives that either have a relatively large value in the state-space matrix or that experience a large change from trim throughout the simulation. These derivatives were mainly control derivatives. After individual analysis of the important derivatives it was found that mostly negative errors smaller than -1 degrade the handling qualities to level 2 or 3 whereas the positive errors barely change the cross-coupling effects in most cases. For the control derivatives this is highly logical because the error smaller than -1 indicates the derivative changes sign, meaning that the controls would be working in the opposite direction according to the prediction model. As the error in the derivative was found to mostly stay within -1 and 1, it can be concluded that the MPC controller is robust to these model errors and keeps having level 1 handling qualities. Nevertheless, when the absolute error increases and specially when the errors gets smaller than -1, the degradation in performance could be solved by implementing robust MPC which improves the performance when an unmeasured error or disturbance is present.

As a recommendation for future work it is suggested to test the established controller more elaborately by extending the test cases with more flight speeds and by evaluating the performance to a disturbance implemented in other parts in the model. Besides this, robust MPC could be implemented in order to improve the robustness to both model errors and disturbances in the simulation model. By implementing robust MPC, one increases the robustness going at the cost of the overall performance. Therefore, this trade-off between robustness and performance should be investigated.

References

[1] H. S. A. T. of IHSTI-CIS, "Helicopter accidents: Statistics, trends and causes," International Helicopter Safety Team - Commonwealth of Independent States, Louisville, Kentucky, USA, Tech. Rep., 03 2016.

[2] H. Huber and P. Hamel, "Helicopter flight control: State of the art and future directions," in *Nineteenth European Rotorcraft Forum*. Cernobbio (Como), Italy: European Rotorcraft Forum, 09 1993.

[3] G. Padfield, "Helicopter handling qualities and control: is the helicopter community prepared for change?" in *Royal Aeronautical Society Conference on Helicopter Handling Qualities and Control*, Royal Aerospace Establishment. London: Controller HMSO London, 11 1988.

[4] D. G. Mitchell, D. B. Doman, D. L. Key, D. H. Klyde, D. B. Leggett, D. J. Moorhouse, D. H. Mason, D. L. Raney, and D. K. Schmidt, "Evolution, revolution, and challenges of handling qualities," *Journal of Guidance, Control, and Dynamics*, vol. 27, no. 1, pp. 12–28, 2004.

[5] J. Rawlings, D. Mayne, and M. Diehl, *Model Predictive Control: Theory, Computation, and Design*, 2nd ed. Santa Barbara, California: Nob Hill Publishing, 11 2017.

[6] Y.-G. Xi, D. Li, and S. Lin, "Model predictive control — status and challenges," *Acta Automatica Sinica*, vol. 39, p. 222–236, 03 2013.

[7] S. Qin and T. A. Badgwell, "A survey of industrial model predictive control technology," *Control Engineering Practice*, vol. 11, no. 7, pp. 733–764, 07 2003.

[8] D. Hrovat, S. Di Cairano, H. E. Tseng, and I. V. Kolmanovsky, "The development of model predictive control in automotive industry: A survey," in *2012 IEEE International Conference on Control Applications*, 11 2012, pp. 295–302.

[9] S. Vazquez, J. Leon, L. Franquelo, J. Rodriguez, H. Young, A. Marquez, and P. Zanchetta, "Model predictive control: A review of its applications in power electronics," *Industrial Electronics Magazine, IEEE*, vol. 8, pp. 16–31, 03 2014.

[10] U. Eren, A. Prach, B. B. Koçer, S. V. Rakovic, E. Kayacan, and B. Acikmese, "Model predictive control in aerospace systems: Current state and opportunities," *Journal of Guidance, Control, and Dynamics*, vol. 40, no. 7, pp. 1541–1566, 2017.

[11] H. Chung and S. Sastry, "Autonomous helicopter formation using model predictive control," in *Collection of Technical Papers - AIAA Guidance, Navigation, and Control Conference*, vol. 1, Keystone, CO, USA, 12 2006, pp. 459–473.

[12] B. J. N. Guerreiro, C. Silvestre, and R. Cunha, "Terrain avoidance nonlinear model predictive control for autonomous rotorcraft," *Journal of Intelligent and Robotic Systems: Theory and Applications*, vol. 68, no. 1, pp. 69–85, 2012.

[13] S. Salmah, S. Sutrisno, E. Joelianto, A. Budiyono, I. E. Wijayanti, and N. Y. Megawati, "Model predictive control for obstacle avoidance as hybrid systems of small scale helicopter," in *Proceedings of 2013 3rd International Conference on Instrumentation, Control and Automation, ICA 2013*, 2013, pp. 127–132.

[14] K. Dalamagkidis, K. P. Valavanis, and L. A. Piegl, "Nonlinear model predictive control with neural network optimization for autonomous autorotation of small unmanned helicopters," *IEEE Transactions on Control Systems Technology*, vol. 19, no. 4, pp. 818–831, 2011.

[15] C. Bottasso Luigi and P. Montinari, "Rotorcraft flight envelope protection by model predictive control," *Journal of the American Helicopter Society*, vol. 60, 04 2015.

[16] W. Greer and C. Sultan, "Helicopter ship landing envelope for model predictive control," in *75th Annual Vertical Flight Society Forum and Technology Display*, vol. 4, Pennsylvania, USA, 05 2019, pp. 2670–2678.

[17] C. E. Mballo and J. V. R. Prasad, "Helicopter maneuver performance with active load limiting," in *the 45th European Rotorcraft Forum*, Warsaw, Poland, 09 2019.

[18] C. Liu, W. Chen, and J. Andrews, "Tracking control of small-scale helicopters using explicit nonlinear mpc augmented with disturbance observers," *Control Engineering Practice*, vol. 20, no. 3, pp. 258–268, 2012.

[19] C. Liu, W. Chen, and J. Andrews, "Model predictive control for autonomous helicopters with computational delay," in *UKACC International Conference on Control 2010*. Coventry, UK: IET, 09 2010, pp. 1–6.

[20] M. H. Maia and R. K. H. Galvao, "Robust constrained predictive control of a 3dof helicopter model with external disturbances," *ABCM Symposium Series in Mechatronics*, vol. 3, pp. 19–26, 2008.

[21] A. Dutka, A. Ordys, and M. Grimble, "Non-linear predictive control of 2 dof helicopter model," in *the 42nd IEEE Conference on Decision and Control*, vol. 4, Hawaii, USA, 12 2003, pp. 3954 – 3959.

[22] M. Gulan, P. Minarcik, and M. Lizuch, "Real-time stabilizing mpc of a 2dof helicopter," in *Proceedings of the 22nd International Conference on Process Control, PC 2019*, 2019, pp. 215–221.

[23] M. Mehndiratta and E. Kayacan, "Receding horizon control of a 3 dof helicopter using online estimation of aerodynamic parameters," *Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering*, vol. 232, no. 8, pp. 1442–1453, 2018.

[24] M. T. Frye, Chunjiang Qian, and R. D. Colgren, "Receding horizon control of a 6-dof model of the raptor 50 helicopter: robustness to changing flight conditions," in *Proceedings of 2005 IEEE Conference on Control Applications, 2005. CCA 2005.*, 08 2005, pp. 185–190.

[25] J. Du, Y. Zhang, and T. Lü, "Unmanned helicopter flight controller design by use of model predictive control," *WSEAS Transactions on Systems*, vol. 7, no. 2, pp. 81–87, 2008.

[26] K. Kunz, S. M. Huck, and T. H. Summers, "Fast model predictive control of miniature helicopters," in *2013 European Control Conference, ECC 2013*, 2013, pp. 1377–1382.

[27] M. Höfinger, "Ads-33e-prf - aeronautical design standard, performance specification, handling qualities requirements for military rotorcraft," 11 2005.

[28] S. Taamalllah, "Small-scale helicopter automatic autorotation," Ph.D. dissertation, Delft University of Technology, 2015.

[29] G. D. Padfield, *Helicopter Flight Dynamics*, 2nd ed. Blackwell Publishing, 01 2007.

[30] G. E. Cooper and R. P. J. Harper, "The use of pilot rating in the evaluation of aircraft handling qualities," NASA TN-D5153, Tech. Rep., 4 1969.

[31] C. L. Blanken, C. J. Ockier, H.-J. Pausder, and R. C. Simmons, "Rotorcraft pitch-roll decoupling requirements from a roll tracking maneuver," in *50th American Helicopter Society Annual Forum 1994*. Washington, DC: the American Helicopter Society. Inc., 07 1997.

[32] D. P. Loucks and E. van Beek, *Modeling Uncertainty*. Cham: Springer International Publishing, 2017, pp. 301–330.

[33] M. Pavel, *Six Degrees of Freedom Linear Model for Helicopter Trim and Stability Calculation*, ser. Memorandum. Delft University of Technology, Faculty of Aerospace Engineering, 12 1996.

[34] R. K. Heffley, W. F. Jewell, J. M. Lehman, and R. A. Vanwinkle, "A compilation and analysis of helicopter handling qualities data. volume 1: Data compilation," NASA, Tech. Rep. NASA-CR-3144, 08 1979.

[35] J. Van Der Vorst, "Advanced pilot model for helicopter manoeuvres," Master's thesis, Delft University of Technology, 1998.

[36] M. Pavel, "On the necessary degrees of freedom for helicopter and wind turbine low-frequency mode modelling," Ph.D. dissertation, Delft University of Technology, 2001.

[37] R. W. Prouty, *Helicopter performance, stability, and control*. Krieger Publishing, 10 2002.

[38] M. Voskuijl, G. Padfield, D. Walker, B. Manimala, and A. Gubbels, "Simulation of automatic helicopter deck landings using nature inspired flight control," *The Aeronautical Journal*, vol. 114, no. 1151, pp. 25–34, 2010.

[39] M. Behrendt, https://creativecommons.org/licenses/by-sa/3.0), accessed: 2020-01.