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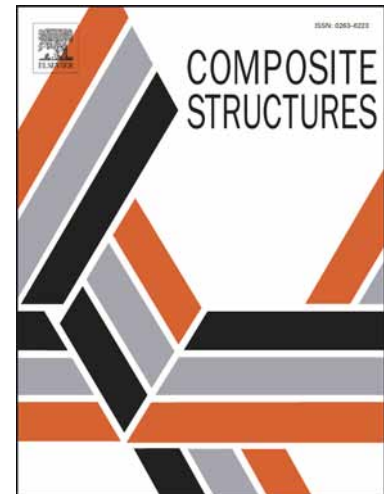
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**Fibre bridging effect on the Paris relation for Mode I fatigue delamination  
growth in composites with consideration of interface configuration**

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**Abstract:**

Fibre bridging can significantly enhance delamination resistance making the use of a single Paris resistance curve to determine fatigue crack growth insufficient. An empirical Paris-type relation has been developed in a previous study to take fibre bridging into account in fatigue delamination growth. This relation was developed by correlating the Paris constants  $C$  and  $n$  to the amount of fibre bridging. This paper provides a further investigation on the interface configuration effect on fatigue delamination growth, illustrating the significance of fibre bridging. The results demonstrated that more bridging fibres can be generated in a multidirectional interface, making both  $\log(C)$  and  $n$  significantly depend on fibre bridging. Thus, the method proposed in the previous study is further extended to take into account of the interface configuration effect.

**Keywords:** Fatigue; Delamination; Fibre bridging; Interface configuration;

Polymer-matrix composites (PMCs)

## 1. Introduction

Characterizing and addressing fatigue damage growth in composite materials is an important task for the application of these materials in advanced aircraft.

Delamination, a damage mechanism that occurs between adjacent layers, has been demonstrated to be one of the most important and common failure types in composite structures. This kind of damage can propagate under cyclic loading and finally result in catastrophic failure of composite components during their service-life.

Composite structural design is commonly based on the concept of *no delamination growth*, leading to artificial limits on the application, and to weight reduction potential of these materials [1-2]. To overcome these limits, reliable methods should be developed according to in-depth understanding of delamination growth behavior under both quasi-static and fatigue loadings.

Extensively theoretical and experimental studies [1-13] have been conducted to characterize delamination initiation and propagation in composites under different loading conditions. Two test standards, ASTM 5528 and ASTM D6115, have been established and widely employed in mode I quasi-static and fatigue delamination studies. In the data reduction, the concept of strain energy release rate (*SERR*) in fracture mechanics has demonstrated to be a reasonable similitude parameter to determine delamination growth in composite materials [12-13]. Particularly, the critical *SERR*  $G_c$  and resistance curve (*R*-curve) have been maturely applied to quantify interlaminar crack initiation and propagation under monotonic loading

[14,15]. Relevant numerical models have been built upon either  $G_c$  or the  $R$ -curve to predict delamination growth with sufficient accuracy [16,17].

Over the years, the Paris relation and its variations have been widely employed to determine fatigue delamination growth in composite materials, with consideration of various factors, including stress ratio  $R=P_{min}/P_{max}$ , temperature, moisture, etc [8-13].

Only a limited number of papers have been published on the phenomenon of fibre bridging and its effect on fatigue delamination growth in composite materials, even though people know it exists. Fibre bridging can hinder delamination propagation in composites by tying fracture surfaces and enhance the interlaminar fracture resistance.

Hwang et al [18] investigated the fibre bridging effect on fatigue delamination growth using width-tapered double cantilever beam (DCB) specimens. Fatigue crack growth was observed to decrease with increase fibre bridging. Hojo et al [19] developed a dedicated test program,  $G_{max}$ -constant test, to directly quantify the fibre bridging contribution to fatigue delamination growth. The representation and interpretation of experimental data, in terms of  $da/dN$  against a constant  $G_{max}$ , apparently demonstrated that fibre bridging can significantly inhibit delamination growth. This test program was further developed and recommended to obtain a fatigue resistance curve in composite laminates without fibre bridging [20]. Khan et al [21] explored the fibre bridging contribution to the  $SERR$  under fatigue loading and observed that this phenomenon had no effect on the stress ratio  $R$ , according to experimental data from fatigue tests with and without removal of bridging fibres. Recently, a series of experimental studies on fibre bridging were carried out by Yao et al [6,7] to have

better understanding of this phenomenon and its effects on fatigue delamination growth. Apart from the observation of fatigue delamination decrease due to fibre bridging, the authors highlighted that bridging has a significant effect on the form of the Paris relation as well as the resistance curve distribution. Thus, it is insufficient to employ only one Paris resistance curve to predict fatigue delamination growth behavior. Furthermore, the amount of bridging fibres generated under fatigue loading differs from that under quasi-static loading, invalidating the application of the quasi-static fracture toughness to normalize fatigue delamination data in composite materials [6]. To accurately describe fatigue delamination growth with fibre bridging, Yao et al [7] subsequently proposed a Paris-type relation based on correlating the coefficients in the Paris relation with the amount of fibre bridging, see Eq.(1). For the unidirectional interface, the exponent  $n$  is only a function of the stress ratio  $R$ . However, the coefficient  $\log(C)$  depends on stress ratio  $R$  as well as the amount of fibre bridging.

$$\frac{da}{dN} = C(a - a_0, R)\Delta G^{n(R)} \quad (1)$$

Despite the fact that technically the dependence of  $C$  on the crack length  $a$  violates similitude, it is worth highlighting that Eq.(1) is the first empirical model at present that can be directly applied to predict fatigue delamination propagation in composite materials with large-scale fibre bridging. Referring to quasi-static delamination growth, the amount of bridging fibres significantly depends on the interface configuration. Generally, more bridging fibres seem to appear in delamination growth at multidirectional interfaces, as compared to the unidirectional interface. As a result,

the interlaminar resistance of the multidirectional interfaces is much higher, as shown in Fig.1. Therefore, ply orientation apparently plays an important role in determining delamination propagation in composite materials.

Taking the quasi-static delamination study as a reference, two questions arise here : what is the ply orientation effect on the fibre bridging under fatigue loading, and what is its influence on the formulation of Eq.(1). These questions need to be carefully answered in order to have better understanding of the bridging effect on fatigue delamination growth, as well as to improve the empirical model. The aim of this paper, therefore, is to reveal the interface configuration effect on the Paris-type relation and the effect of fibre bridging in fatigue delamination growth.

## 2. The mechanism of fibre bridging

Fibre bridging is a common shielding mechanism in crack growth in composite materials. Its presence can bridge the fracture surfaces, enhance interlaminar resistance and prohibit crack propagation. Bao et al [22] provided an insight with their discussion on the bridging phenomenon. They highlighted that all bridging mechanisms can be physically represented by traction-separation relations. A general form of these relations can be summarized as

$$\frac{\sigma}{\sigma_0} = \chi \left( \frac{\delta}{\delta_0} \right) \quad (2)$$

where  $\sigma_0$  is the strength, and  $\delta_0$  is the maximum separation in the bridging region; The dimensionless function  $\chi$  demonstrates the shape of the traction-separation law and relates to the bridging mechanism significantly.

Fibre bridging in mode I delamination growth in composite laminates, i.e. cross-over

bridging, is attributed to misaligned or inclined fibres. In agreement with the J-integral concept, the total *SERR* for crack advancing under quasi-static loading can be explicitly determined as

$$G_{IC} = G_{IC0} + \int_0^{\delta_0} \sigma(\delta) d\delta \quad (3)$$

where  $G_{IC0}$  is the fracture toughness of crack initiation;  $\sigma(\delta)$  represents the bridging law and  $\delta_0$  is the maximum separation of the fibre bridging region. Taking the derivative of Eq.(3) with respect to the crack separation  $\delta$ , the bridging law in quasi-static crack growth can be determined as [14]

$$\sigma(\delta) = \frac{\partial G_{IC}}{\partial \delta} \quad (4)$$

However, this method cannot be directly applied to evaluate the traction-separation law in fatigue crack growth [23]. In addition, it is worth highlighting that fibre bridging effect in fatigue delamination is not the same as it in quasi-static, invalidating the use of the quasi-static fracture toughness to normalize fatigue delamination results directly [6]. To investigate this difference in fatigue delamination growth, Zhang et al [24] tried to correlate the quasi-static delamination resistance with fatigue delamination resistance, via a *compliance approach*. According to this correlation, they proposed a modified Paris relation for mixed-mode I/II fatigue delamination growth. Stutz et al [25] introduced a *semi-experimental method* to quantify the bridging laws in mode I quasi-static and fatigue delamination growth. It demonstrated that the traction-separation relations are load dependent. The relation under monotonic loading differs from that under cyclic loading. Donough et al [23] applied an *inverse approach* to determine the bridging laws under monotonic and



fatigue loadings. An obvious difference was observed in the bridging laws as well, except for the maximum bridging stress. With consideration of bridging, a new scaling parameter was introduced into the *Forman equation* in order to characterize fatigue delamination growth under different stress ratios in composite laminates. The aforementioned studies are important to understand the fibre bridging mechanisms as well as its effect on fatigue delamination growth. The concept of bridging law is a promising approach to physically interpret the bridging mechanism and its effects on delamination growth in composite materials.

However, little attention has ever been put into the ply orientation effect on fatigue delamination growth as well as its effect on fibre bridging. Therefore, the objective of this paper is to investigate the fatigue delamination growth behavior in multidirectional interfaces with large-scale fibre bridging. Experiments were conducted on DCB specimens with 45//45 interface first. A linear regression analysis methodology was subsequently employed in fatigue data reduction. The empirical Paris-type relation, Eq.(1), was finally extended to take into account of interface effect on fatigue delamination growth.

### **3. Fatigue delamination experiments**

#### **3.1 Material and specimen geometry**

Multidirectional DCB specimens were manufactured and fatigue tested to investigate the delamination growth behavior with fibre bridging. The composite laminates were produced by a hand-lay-up of 32 thermosetting unidirectional carbon/epoxy prepreg layers of M30SC/DT120 with a nominal cured laminates thickness of 5.0 mm. The

stacking sequence for DCB specimens with 45//45 interface was designed as  $[(\pm 45/0_{12}/\mp 45)]/[(\pm 45/0_{12}/\mp 45)]$ , taking into account of avoiding crack jumping, and minimizing both residual thermal stress after curing and non-uniform energy release rate distributions across the width of the crack front during delamination propagation. A 12.7 $\mu\text{m}$  Teflon film was inserted in the middle plane of laminates during the hand-lay-up process to act as an initial delamination. The laminates were cured in vacuum in an autoclave at a pressure of 6 bars and curing temperature of 120 $^{\circ}\text{C}$  for 90 minutes. After curing, the laminates were C-scanned to detect potential imperfections. Then the plate was cut by a diamond coated saw into 25 mm width beams with 200 mm length, and only these samples were tested where the C-scan did not reveal any obvious imperfections. A pair of aluminum blocks, 25 mm width by 20 mm length with 6 mm thickness, was adhesively bonded onto each specimen at the side of the Teflon insert for load introduction.

### 3.2. Experimental procedure

Fatigue tests were conducted on a 10kN hydraulic MTS machine under displacement control at a frequency of 5Hz with a stress ratio  $R=0.5$ . In order to have real-time monitoring of the delamination growth during the entire test, photographs of the crack extension at one side of the specimen, which has been coated with white typewriter correlation fluid in advance to enhance the visibility of crack front, were automatically recorded at the maximum displacement at regular intervals, using a digital camera controlled by a computer system. The corresponding information of load, displacement and cycle number were automatically stored in an Excel file

enabling data evaluation after the test.

Prior to the first fatigue test performed on each DCB specimen, they were quasi-statically loaded to create a 2-3mm onset sharp crack as a natural crack tip for the subsequent fatigue test. Then, the DCB specimens were fatigue tested multiple times, but at the same stress ratio  $R=0.5$ , to obtain fatigue delamination resistance data with different amounts of fibre bridging. In each test, the delamination growth gradually decreased with the decrease in *SERR* until it nearly retarded. Subsequently, the test was repeated with increased displacements keeping the stress ratio the same. This sequence was repeated several times until the maximum displacement capacity of the test machine was reached. With this test procedure, multiple delamination resistance curves were obtained, with each curve representing the resistance equivalent to a specific fatigue pre-crack length, i.e. delamination length at which point that fatigue test was initiated.

According to the Paris relation, all experimental fatigue results were presented in terms of crack growth rate  $da/dN$  against *SERR* range  $\Delta G$ , see Eq.(5).

$$\frac{da}{dN} = C(\Delta G)^n = C \left[ (\sqrt{G_{max}} - \sqrt{G_{min}})^2 \right]^n \quad (5)$$

The corresponding maximum and minimum *SERR* in fatigue loading, i.e.  $G_{max}$  and  $G_{min}$ , were calculated with the Modified Compliance Calibration (MCC) method, recommended in ASTM D5528, as shown in Eq. (6).

$$G_I = \frac{3P^2 C^{(2/3)}}{2A_1 B h} \quad (6)$$

where  $C$  is the compliance of DCB specimen.  $B$  is the specimen width and  $h$  is the thickness of specimen.  $A_I$  is the slope of the curve in the graph where  $a/h$  is plotted

against  $C^{1/3}$ .

The 7-point Incremental Polynomial Method, recommended in ASTM E647, was employed to determine the delamination growth rate  $da/dN$ .

#### 4. Results and discussion

DCB specimens with 45//45 interface were fatigue tested via the aforementioned test procedure to obtain a series of resistance curves, taking into account of fibre bridging effect on delamination growth. In total, two multidirectional DCB specimens were fatigue tested, labeled as MD\_1 and MD\_2 respectively. As a comparison, the fatigue delamination results for the unidirectional DCB specimens with different amounts of fibre bridging at the same stress ratio were reported in the previous study [7].

Fig.2 shows the experimental fatigue results for the multidirectional DCB specimens. Not surprisingly, the resistance curves substantially shift from left to right in the resistance graph with the development of fibre bridging. They finally converge into a single curve, representing the saturation of bridging. It is worth noting that a large number of people only paid attention to the most left resistance curves and used them for fatigue delamination prediction in composite materials. They actually ignored the bridging effect on fatigue resistance, i.e. the evident decrease shift of the resistance curves with increasing crack length. This phenomenon elucidates that fatigue crack growth behavior is not identical at a given *SERR* range in case of delamination growth with fibre bridging. Consequently, it is invalid to employ any single resistance curve, shown in Fig.2, to determine fatigue delamination growth with fibre bridging. In particular, the application of the most left curves for engineering design can cause

significantly conservative results and limit the weight saving potential. The use of the most right ones cannot guarantee safety. To solve these problems, the resistance curves located in-between must be carefully considered in the characterization of the fatigue delamination growth behavior.

The shift in resistance curves also indicates that the threshold can significantly increase with the development of fibre bridging and finally remain constant at a high level in fatigue delamination growth. The threshold here was calculated by the best-fit Paris relation of the resultant data at a fatigue crack growth rate  $da/dN = 1 \times 10^{-10}$  m/Cycle, as recommended in ASTM E647. The 95% confidence interval of the threshold was determined in accordance with a regression analysis on the experimental data. For the sake of revealing the interface configuration effect, the threshold for fatigue delamination growth in the unidirectional DCB specimens was also calculated based on the results reported in the previous study [7].

The threshold values for both unidirectional and multidirectional interfaces are illustrated in Fig.3. It clearly shows that threshold significantly depends on the crack extension  $a-a_0$ , equivalent to a certain amount of bridging fibres, in each interface.

The threshold increases with delamination propagation, and eventually reaches a plateau level, like a *threshold R-curve*. The initial threshold values for both interfaces are identical, indicating that the bridging at the initiation delamination is not significant and the initial delamination behavior is predominantly determined by the matrix property. However, with fatigue delamination propagation, the difference in threshold becomes more obvious. The plateau for 45//45 interface is almost 30%

higher than that for 0//0 interface, evidently indicating more bridging fibres are generated in delamination growth at the multidirectional interface under fatigue loading. These observations are indeed consistent with experimental quasi-static delamination results. The fracture toughness is the same in the onset delamination at both interfaces, as shown in Fig.1. However, more fibre bridging develops during the delamination propagation at the multidirectional interface, constituting the corresponding plateau fracture toughness for 45//45 interface about 70% higher than that for 0//0 interface [6].

As discussed in the previous studies [6,7], fibre bridging is the main reason for the shift in the resistance curves shown in Fig.2, as well as the *threshold R-curves* shown in Fig.3. Bridging fibres can constrain the fracture surfaces and periodically store and release strain energy in the uploading and unloading parts of a cycle. As a result, fatigue delamination growth decreases significantly as fibre bridging develops.

The amount of bridging fibres generated in fatigue delamination growth is demonstrated to be interface configuration dependent. Therefore, the explicit form of the Paris-type relation for the multidirectional interface may be different from the unidirectional interface. In the light of this difference, the methodology, used in the previous study [7], was subsequently applied to the fatigue data analysis and presented in the following section.

##### **5. Fibre bridging effect on the Paris relation in the multidirectional interface**

Linear regression analysis was employed to investigate the bridging effect on the Paris relation, Eq.(5), in accordance with the fatigue data of the 45//45 interface.

Specifically, the Paris relation was linearized for the regression analysis on the exponent  $n$  first. The correlation between parameter  $n$  and the amount of fibre bridging, represented by crack extension  $a-a_0$ , was established, as shown in Fig.4. As a comparison, the corresponding correlation from literature [7] for the unidirectional interface at the same stress ratio is added in Fig.4. An obvious difference is observed. Instead of the constant exponent  $n$  for the unidirectional interface,  $n$  linearly decreases with fibre bridging and finally becomes constant, indicating fibre bridging has reached saturation in the multidirectional interface. This difference in  $n$  mainly attributes to the different severities in the amount of bridging fibres generated in the distinct interfaces. More bridging fibres can be located in the wake of a crack front in the multidirectional interface, as compared to the unidirectional interface. This is rationalized by a higher plateau of *threshold R-curve* shown in Fig.3. Additionally, a lower slope of the Paris resistance curve means a larger energy dissipation increase in  $\Delta G$  at the same magnitude of  $da/dN$ . For a longer crack extension  $a-a_0$ , more bridging fibres can be generated in the wake of the crack front. As a result, more energy should be dissipated in the bridging fibres during delamination growth, reflected by a larger increase in  $\Delta G$  at the same magnitude of  $da/dN$ . The values of exponent  $n$  are summarized in the Appendix.

In the second step, the correlation shown in Fig.4 was substituted into the regression analysis for the coefficient  $\log(C)$ . A correlation between  $\log(C)$  and crack extension  $a-a_0$  was obtained, as shown in Fig.5. The counterpart correlation from previous study [7] for the unidirectional interface is added as a comparison. Different from the

tri-linear relationship between  $\log(C)$  and crack extension  $a-a_0$  observed for 0//0 interface, there is an initially linear decrease of  $\log(C)$  with delamination growth until the crack extension is about 20mm in the 45//45 interface. After this, the values of  $\log(C)$  increase apparently and finally become constant as the saturation of fibre bridging has been reached. The values of  $\log(C)$  are summarized in the Appendix.

In mathematics,  $\log(C)$  is the intersection point between the linearized Paris relation and the ordinate, as shown in Fig.6. Recalling the previous study on fatigue delamination growth [7], a  $\log(C)$  decrease in the unidirectional interface can be explained by the shift phenomenon observed in the resistance graph at a fixed exponent  $n$ . However, it is far more complicated in case of fatigue delamination growth in the multidirectional interface. The shift can lead to decrease of  $\log(C)$  as well. However, the decrease in the exponent  $n$  can cause an apparent increase of  $\log(C)$ , as schematically shown in Fig.6. As a result, the magnitude of  $\log(C)$  for the multidirectional interface is indeed a balance of these two opposite effects on the y-intercept. At the beginning of delamination growth, the shift possibly plays a dominant role in determining  $\log(C)$ , resulting in the decrease of  $\log(C)$ . However, in subsequent delamination growth, the change of exponent  $n$  possibly has more influence on  $\log(C)$  and causes an increase of it. That this parameter becomes constant, is attributed to the fibre bridging having fully developed.

According to the above analysis and discussion, the prediction model of Eq.(1), can be further extended to take into account the interface configuration in the Paris-type power law formulation as



$$\frac{da}{dN} = C(a - a_0, R)\Delta G^{n(a-a_0, R)} \quad (7)$$

In the equation, both  $\log(C)$  and  $n$  depend on crack extension  $a-a_0$  as well as stress ratio  $R$ . Hence, the ply orientation is implicitly incorporated in the fitting of both  $C$  and  $n$  to  $a-a_0$  and  $R$ . With consideration of these correlations, the fibre bridging contribution to fatigue delamination growth can be characterized appropriately.

## 6. Verification

A verification of Eq.(1) in determining fatigue delamination growth in the unidirectional interface with fibre bridging under different stress ratios has been completed in the previous study [7]. Good agreement between predictions and experiments was obtained, demonstrating the accuracy of the model in fatigue delamination prediction of the unidirectional interface.

To demonstrate the accuracy of the extended relationship of Eq.(7) in the prediction of fatigue delamination growth with fibre bridging in the multidirectional interface, this model is similarly verified here with the same data sets used to correlate  $\log(C)$  and  $n$  with fibre bridging in Section 5. It is necessary and meaningful to conduct this verification as providing the first attempt to show the accuracy and reliability of the present method. In order to take into account scatter in the fatigue data, the 95% confidence interval of the experimental results was calculated in accordance with the regression analysis.

Fig.7 gives the prediction results for fatigue delamination growth in the 45//45 interface. In three of the total fifteen cases, the predictions are located outside the 95% confidence interval or the slope of the prediction differs from the experimental data.

The main reason for this discrepancy is derived from the inaccurate correlations as well as the large scatter commonly found in fatigue test. However, in most cases, the predicted results are located in or really close to the 95% confidence interval and the predicted slopes are the same to the experimental results, demonstrating the robustness of the method presented here. Thus, one can conclude that the proposed model can be applied to predict fatigue delamination growth accounting for fibre bridging.

### **7. Discussion on the present method**

This paper conducted an investigation on the interface configuration effect on the Paris relation using DCB specimens with 45//45 interface. It demonstrated that fibre bridging significance is ply orientation dependent in fatigue delamination growth. More bridging fibres can be located in the wake of a crack front of multidirectional interfaces, as compared to the unidirectional 0//0 interface.

The objective of this paper is to propose a robust empirical method for the engineering purposes. To achieve this, the authors tried to investigate the fibre bridging phenomenon based on its consequence in fatigue delamination growth, instead of its mechanisms. A linear regression analysis on the Paris curve-fitting parameters revealed that both coefficient  $\log(C)$  and exponent  $n$  are fibre bridging dependent for the 45//45 interface. This differs from the results obtained for the 0//0 interface, in which only the parameter  $\log(C)$  depends on the fibre bridging. According to this information, the Paris-type relation, Eq.(1), was further extended to implicitly take into account the interface configuration, see Eq.(7).

It is worth keeping in mind that delamination can not only occur in interfaces with the same ply orientation, 0//0 and 45//45 interface used in the authors' study for example, but also in interfaces with different ply orientations, such as 0//45, 45// -45, 0//90 and 45//90 interfaces. From the present study, it is concluded that the specific correlations, illustrated in Fig.4 and Fig.5, can be different for other interfaces. Thus, extra fatigue experiments should be performed to quantify these relationships. With a substitution of these relationships into Eq.(7), the fatigue delamination growth behavior can be appropriately determined for a given interface configuration. Such extension should highlight that Eq.(7) is indeed a general formula to determine fatigue crack growth accounting for fibre bridging. Its form is pertinent to the correlations. In the perspective of engineering, the proposed model can be applied to determine fatigue crack growth in composite laminates with excellent accuracy, as shown in Fig.7. However, in the perspective of physics, it cannot provide enough information on the mechanisms of bridging. Referring to quasi-static delamination, the concept of bridging laws is a promising way for future study.

Threshold is another important parameter for characterizing fatigue crack growth behavior, as well as for composite structure design. The increasing threshold induced by fibre bridging can be appropriately evaluated via the Paris-type relation of Eq.(7).

In case of delamination growth with fibre bridging, the concept of a *threshold R-curve* can be used to determine the limit under which there is no crack growth. In addition, the shape of the *threshold R-curve* is interface configuration dependent. The threshold information seems very useful in fatigue crack data interpretation. For example, Jones

et al illustrated how the use of a threshold in the *Hartman-Schijve* equation improves the correlation in fatigue delamination characterization [26]. According to these authors, the *Hartman-Schijve* relation, incorporating a threshold increase, is a reasonable choice for the interpretation of fatigue delamination with fibre bridging. However, in the authors' opinion, more evidence should be provided to consolidate this conclusion in future studies.

Just as mentioned in the previous study [7], several factors can significantly affect the fibre bridging significance. These are, among others, the stress ratio  $R$ , the mixed-mode I/II ratio, the interface configuration, the specimen thickness and the loading type. To have better understanding of all these factors on fibre bridging, a large number of fatigue experiments should be conducted and provide necessary information for developing the empirical as well as the physical models. Based on the analysis of fibre bridging effect on fatigue resistance, a general approach has been developed based on the Paris-type formula, Eq.(7), to predict crack growth including fibre bridging. This method seems convenient for the application in engineering, as it does not require complex derivations or calculations. It can also be used as a reference to investigate other influencing factors on fatigue delamination growth with fibre bridging.

## 8. Conclusions

Interface configuration has an important effect on the amount of bridging fibres generated in fatigue delamination growth. Similar to quasi-static delamination growth, more bridging fibres can be present in multidirectional interfaces. Fibre bridging has

significant effects on fatigue delamination growth in composite materials, leading to a shift in the resistance curves as well as to an increase in the threshold. The concept of the *threshold R-curve*, therefore, was proposed to quantify the threshold increase. It was observed that the threshold is interface independent in initial delamination, but significantly depends on ply orientation in crack propagation.

Hence, the interface configuration has an obvious influence on the correlations between the coefficients of the Paris relation and the amount of fibre bridging. These correlations are more complicated for multidirectional interfaces, as compared to delamination growth in the unidirectional interface. The exponent  $n$  linearly decreases until fibre bridging has fully developed, while the magnitude of  $\log(C)$  linearly decreases first, followed by an obvious linear increase until fibre bridging reaches saturation. With a substitution of these correlations, a Paris-type relation is finally proposed to take fibre bridging into account in fatigue delamination growth. There is a good agreement between predictions and experimental results, demonstrating the reliability and accuracy of the Paris-type relation. Therefore, the empirical method proposed in the previous study [7] was further extended to take interface configuration into account.

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#### Appendix

The values of  $\log(C)$  and exponent  $n$  are summarized as

#### List of figure captions

Fig.1 Quasi-static *R-curves* for different interfaces [6]

Fig.2 Fatigue resistance curves with different amounts of fibre bridging

(a) Specimen MD\_1;      (b) Specimen MD\_2.

Fig.3 Threshold R-curve

Fig.4  $n$  vs.  $a-a_0$  in the different interfaces

Fig.5  $\log(C)$  vs.  $a-a_0$  in the different interfaces

Fig.6 Schematic of  $\log(C)$  increase with the exponent  $n$  decrease

Fig.7 Prediction results and experimental data for multidirectional DCB specimens

(a) MD\_1\_prediction\_1;      (b) MD\_1\_prediction\_2;

(c) MD\_2\_prediction\_1;      (d) MD\_2\_prediction\_2.

List of table captions

Table 1  $\log(C)$  and exponent  $n$  of specimen MD\_1

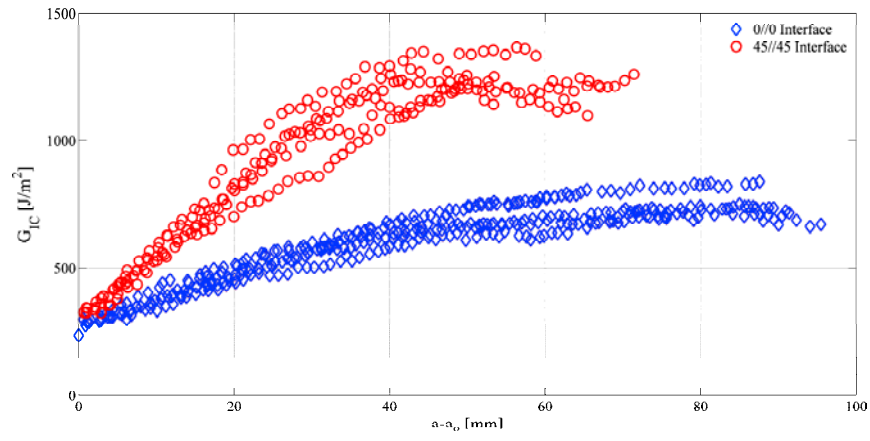
Table 2  $\log(C)$  and exponent  $n$  of specimen MD\_2

Table 1  $\log(C)$  and exponent  $n$  of specimen MD\_1

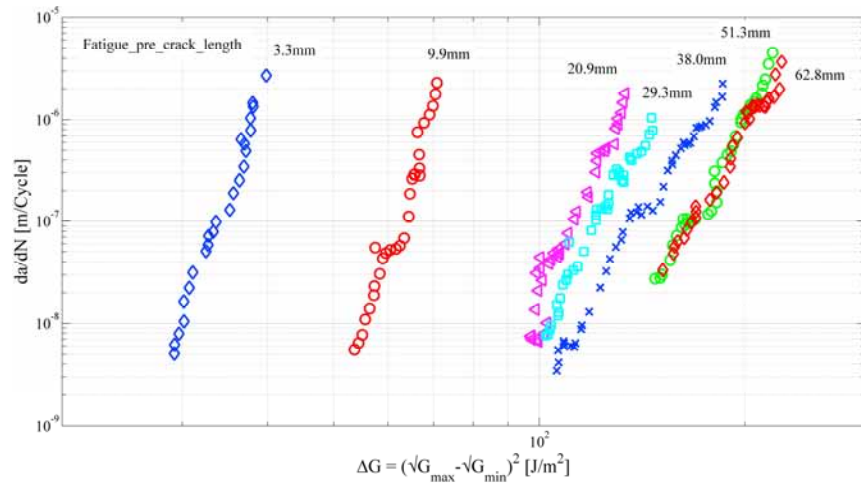
Crack extension [mm]	$n$	$\log(C)$
3.3	$18.82 \pm 1.29$	$-36.97 \pm 0.05$
9.9	$20.51 \pm 1.82$	$-39.23 \pm 0.07$
20.9	$15.73 \pm 1.27$	$-38.30 \pm 0.05$
29.3	$13.0 \pm 0.71$	$-34.52 \pm 0.03$
38.0	$10.99 \pm 0.38$	$-30.96 \pm 0.03$
51.3	$12.54 \pm 0.70$	$-31.73 \pm 0.04$
62.8	$11.35 \pm 0.60$	$-31.80 \pm 0.03$

Table 2  $\log(C)$  and exponent  $n$  of specimen MD\_2

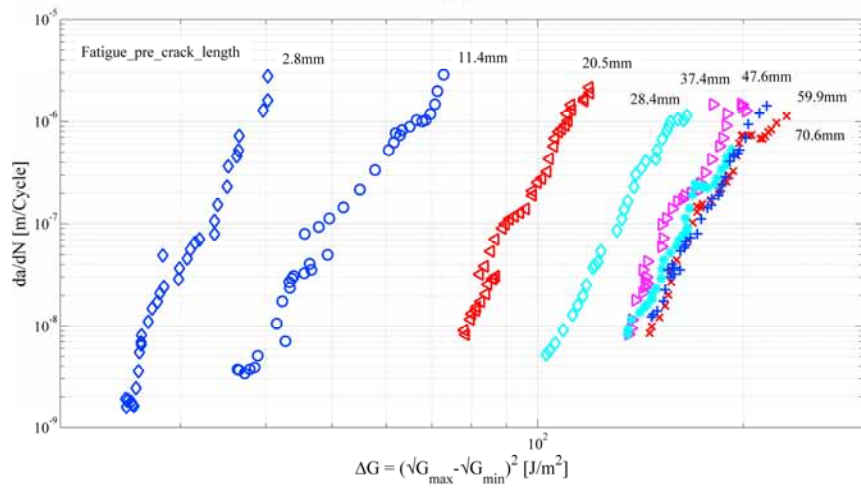
Crack extension [mm]	$n$	$\log(C)$
2.8	$14.17 \pm 0.99$	$-36.20 \pm 0.14$
11.4	$9.49 \pm 0.53$	$-37.12 \pm 0.27$
20.5	$12.99 \pm 0.61$	$-37.24 \pm 0.05$
28.4	$12.35 \pm 0.47$	$-35.42 \pm 0.04$
37.4	$12.60 \pm 0.97$	$-31.59 \pm 0.06$
47.6	$10.86 \pm 1.02$	$-32.01 \pm 0.06$
59.9	$12.10 \pm 0.76$	$-31.77 \pm 0.03$
70.6	$12.45 \pm 0.41$	$-31.96 \pm 0.02$



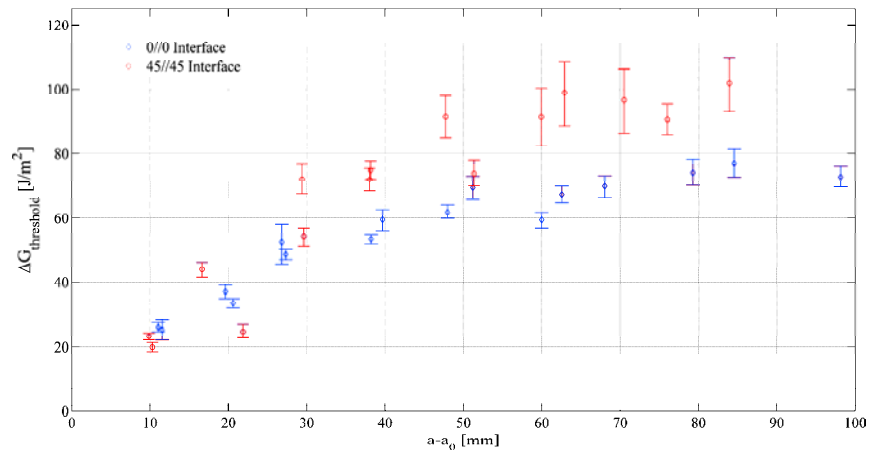
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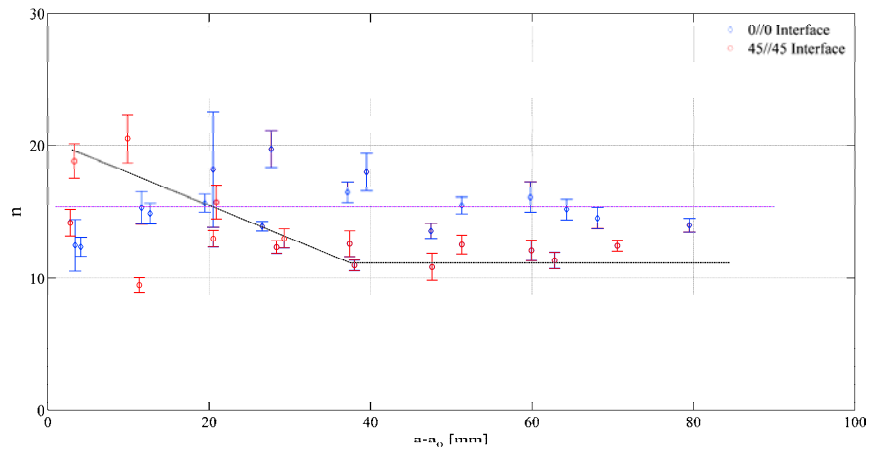


(a)

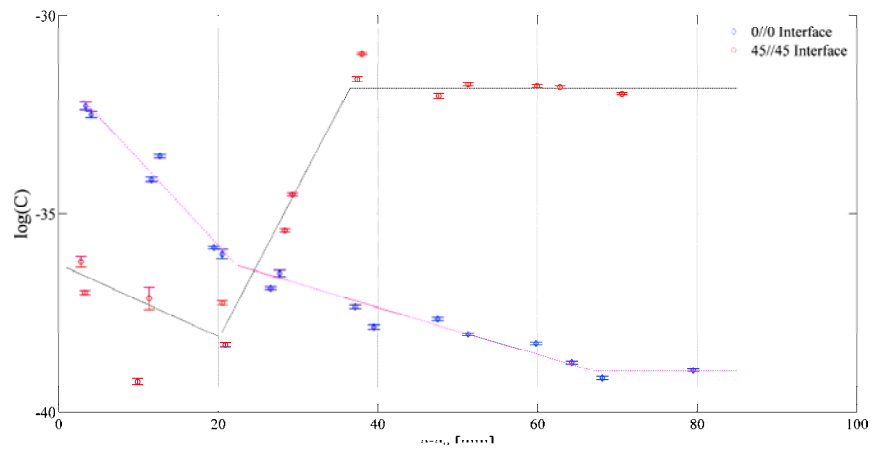


(b)

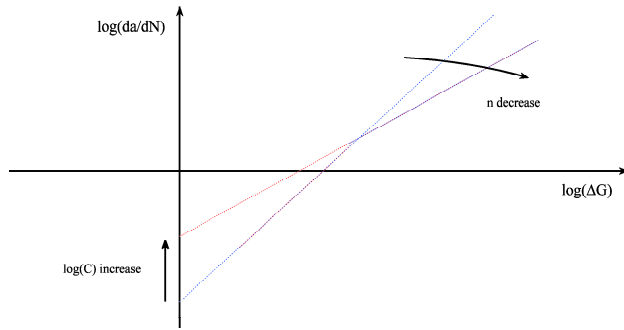




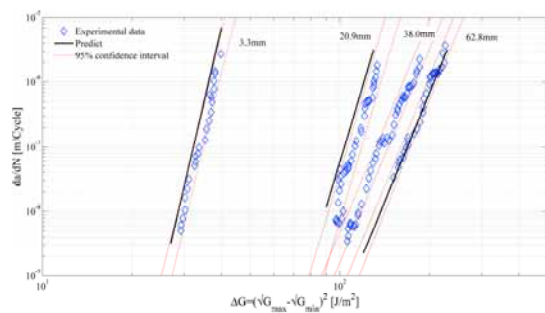




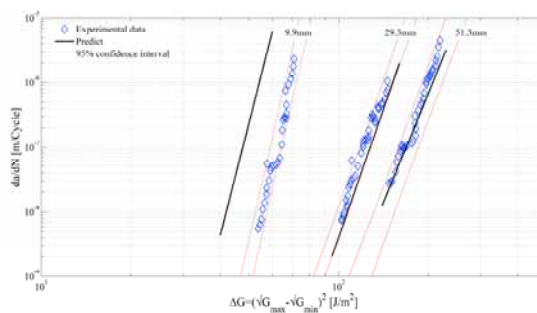
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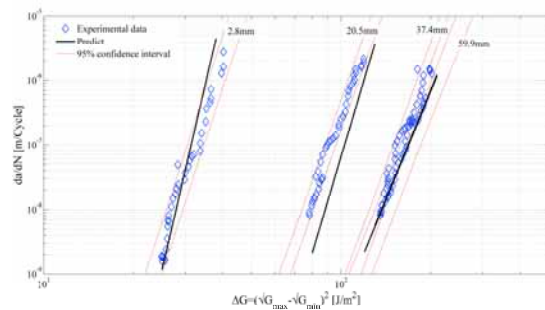
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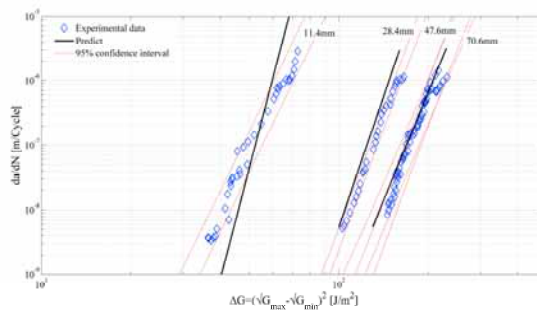
(a)



(b)



(c)



(d)

ACCEPTED MANUSCRIPT