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Integrated route choice and assignment model for fixed and flexible public transport systems

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ABSTRACT

The recent technological innovations in various ICT platforms have given rise to innovative mobility solutions. Such systems could potentially address some of the inherent shortcomings of a line/schedule based public transport system. Previous studies either assumed that flexible on-demand services are used as an exclusive door-to-door service or offered as a feeder connection to high-capacity public transport services. However, users may combine line/schedule based public transport systems (Fixed PT) and on-demand services (Flexible PT) so that their travel impedance is minimized. To this end, we propose a multimodal route choice and assignment model that allows users combining Fixed and Flexible PT or use them as individual modes while demand for these services is endogenously determined. The model takes into account the dynamic demand–supply interaction using an iterative learning framework. Flexible public transport can be used to perform any part of the trip, ranging from a first/last mile service to an exclusive direct door-to-door connection. The developed model is implemented in an agent based simulation framework. The model is applied to a network centered around the city of Amsterdam, The Netherlands. Scenarios where Fixed PT and Flexible PT are offered as mutually exclusive modes or can be combined into a single journey, are analysed. Results indicate that Flexible PT is predominantly used for covering <30% of the trip length, indicating that it is mainly used as an access or egress mode to Fixed PT. This results with an overall increase in the share of public transport trips. Also, the average waiting time of Flexible PT users when used in combination with Fixed PT are lower than the scenario where each of them is used as an exclusive mode.

1. Introduction

The emergence of innovative mobility solutions, brought about by ICT advancements, is set to change the public tranport landscape. Emerging mobility solutions offer on-demand services picking up and dropping off passengers from a pre-defined set of stops (stop-to-stop service) or between selected locations (door-to-door service) either controlled by a central dispatching unit (such as an app-based vehicle-travel request matching service) or as a competing fleet of vehicles with drivers having the discretion to accept or reject travel requests. Travelers may use these on-demand services to travel from their origin to destination or combine it with traditional line/schedule-based services. Fixed and flexible services may not only co-exist within a given urban area as alternative, mutually-exclusive modes but may also be combined by passengers along a given journey. Fixed and flexible services may thus not only compete for market shares but also complement each other and potentially serve different parts of the journey which they are best suited depending on their characteristics such as speed, capacity and availability. From this perspective, it is important

to understand the potential to combine on-demand services and line/schedule-based public transport services and the dynamic interaction between the demand (users) and supply (services). To this end, a model is developed for the integrated public transport route choice of users allowing for the combination of on-demand service (Flexible PT) and line/schedule-based public transport service (Fixed PT) along a single trip.

System analysis of a combined Fixed PT and Flexible PT comprise of two major components: Route choice modelling and Assignment (network loading). The Fixed PT comprises of a line/schedule-based service (such as train, tram, bus, or metro). Service network of Fixed PT involves route allignment and service frequencies. The Flexible PT comprises of a fleet of vehicles offering on-demand services to passengers along with their operational strategy. This is followed by the *Route choice modelling* phase in which the travel options of users are modelled. In the *Assignment* phase, passenger demand is distributed over the choice alternatives. The assignment procedure' is performed for the service network over several iterations (iterative network loading) until a steady-state (equilibrium) is attained. We study and classify the existing literature based on the modelling approaches that have been used for the Route choice and Assignment phase of service design.

A large number of studies have used analytical, mathematical programming, or simulation methods to model the assignment of travel requests to on-demand services. Notable works that used an analytical approach include Wilson et al. (1976) and Potter (1976). They modelled the assignment problem as an Integrated Dial-a-Ride Problem (IDARP) and used a passenger utility maximisation approach and modelled demand responsive services as feeder to fixed route service. Mathematical programming approach involve solving the assignment problem as an optimization problem by assigning travel requests to a fleet of on-demand vehicles (Posada et al., 2017; Häll et al., 2009; Salazar et al., 2018). Posada et al. (2017) and Häll et al. (2009) solved the assignment problem as Integrated Dial-A-Ride Problem (IDARP) and assigned travel requests of on-demand service to coordinate with the service of Fixed PT. Salazar et al. (2018) used a flow optimization model for assigning the travel requests while maximising the social welfare. Liaw et al. (1996) and Hickman and Blume (2001) solved for the combination of static and dynamic version of Dial-a-Ride Problem where part of the travel requests are known before the planning stage. However, such analytical models often fail to capture the real-time system dynamics.

Simulation and agent-based simulation methods mitigate this issue to an extent. Works that used simulation methods for on-demand service design include Edwards et al. (2011) and Horn (2004). Edwards et al. (2011) introduced the concept of network inspired transportation system (NITS) that routes passengers analogous to routing packets through a telecommunications network. Horn (2004) used a simulation model for planning journeys combining fixed route services and demand responsive services. The journey could be carried out by a single mode which includes walk, taxi, or fixed route service. The fixed route service included conventional services such as bus and lightrail and demand responsive modes. However, they considered an exogenous demand that was fixed throughout the assignment process. Neumann and Nagel (2013) presented an evolutionary algorithm for the design of an optimal paratransit service network. They designed the paratransit services as a competing mode with a Fixed PT service. Atasoy et al. (2015) designed an on-demand service in which a list of travel options is given to passengers in real-time. The travel options include using taxi service (single passenger with door-to-door service), shared taxi service (multiple passengers with door-to-door service), or minibus (multiple passenger with fixed routes but flexible schedules). Maciejewski and Nagel (2013) and Maciejewski (2016) designed a framework for implementing dynamic transport services in an agent-based simulation framework. Hörl (2016) implemented an autonomous taxi service in competition with a Fixed PT service. The autonomous taxi service were modelled as a fleet of vehicles controlled by a central dispatching unit offering door-to-door service to passengers. The studies mentioned so far modeled on-demand transport in isolation with the demand for this services considered to be externally defined and independent of the level of service offered or as an alternative that fully substitutes public transport.

Another line of research has considered Fixed PT and Flexible PT as part of a joint passenger transport by introducing a flexible service as feeder to the high-capacity fixed route network such. Notable works include Potter (1976), Uchimura et al. (2002), Aldaihani et al. (2004), Vakayil et al. (2017), Shen et al. (2017), Moorthy et al. (2017), Ma (2017), Charisis et al. (2017), Wen et al. (2018), Stiglic et al. (2018), Lee et al. (2004), Cayford and Yim (2004). Vakayil et al. (2017) designed an autonomous mobility on demand as a first/lst mile option when mass transit services are available. Their results indicated a 50% reduction in vehicle miles travelled of mobility on-demand vehicles when integrated with mass transit. Shen et al. (2017) investigate the case of autonomous vehicles serving as first/last mile problem during morning peak for a public tranport sysem in Singapore. They suggested replacing low demand bus routes with shared autonomous vehicles. Ma (2017) presented a dynamic vehicle dispatching and routing algorithm for shared services in coordination with an existing public transport network. The objective was to attain optimal passenger-vehicle assignment. Wen et al. (2018) designed an integrated autonomous vehicle and public transport system. The autonomous services were designed to provide first/last mile connections to rail services and efficient mobility in low-density sub-urban areas. Aldaihani et al. (2004) presented an anlytical tool to determine the optimal number of zones to provide demand responsive services. The on-demand services either transfers passengers to a fixed route line or transports them from their final stop to their destination. Cayford and Yim (2004) designed a demand responsive system as a feeder service to a fixed route system for the city of Milbrae, California. Results showed that the demand responsive service is a feasible solution for downtown feeder system. Li and Quadrifoglio (2011) and Lee and Savelsbergh (2017) investigated the deployment of demand responsive services at a zonal level. Li and Quadrifoglio (2011) developed an anlytical model based on continuous approximations to address the optimal zone design problem. Assuming a two vehicle operation in each zone, the objective was to arrive at an optimal number of zones by considering level of service and operating cost. Lee and Savelsbergh (2017) considered a zone with multiple transfer points. They found out that the benefits of a more flexible system are substantial but depends on characteristics such as passneger and station density. More recently, joint optimisation of capacity and headway of on-demand transit services with autonomous buses were carried out by Chen et al. (2019) and Dai et al. (2020). While Chen et al. (2019) solved the joint design as a mixed integer linear programming model with a

Table 1
Comparitive summary of reviewed literature based on modelling approach and Flexible PT design.

Paper	Modelling approach	Flexible PT operation	Demand for Flexible PT	Objective
Wilson et al. (1976)	Analytical	Feeder to Fixed PT	Exogenous	Assignment of travel requests
Potter (1976)	Analytical	Feeder to Fixed PT	Exogenous	Assignment of travel requests
Posada et al. (2017)	Mathematical programming	Feeder to Fixed PT	Exogenous	Optimal assignment of travel requests
Salazar et al. (2018)	Mathematical programming	Feeder to Fixed PT	Exogenous	Maximise social welfare
Quadrifoglio and Li (2009) and Li and Quadrifoglio (2010)	Analytical and simulation modelling	Exclusive mode	Exogenous	Determine the range of demand for Flexible PT operation
Neumann and Nagel (2013)	Agent based simulation	Competing mode with Fixed PT	Exogenous	Determine a set of optimal routes for the services
Horn (2004)	Mathematical programming and simulation	Integrated design with Fixed PT	Exogenous	Planning of multi-leg journeys with Fixed PT and Flexible PT
Atasoy et al. (2015)	Mathematical programming and simulation	Exclusive mode	Exogenous	Planning of journeys with multiple Flexible PT services such as taxi, shared taxi and mini bus
Maciejewski and Nagel (2013), Maciejewski (2016), and Hörl (2016)	Agent based simulation	Exclusive mode	Exogenous	Assignment of travel requests
Li and Quadrifoglio (2011)	Analytical and simulation	Feeder service to Fixed PT network	Exogenous	Determine the optimal number of zones for Flexible PT operation
Aldaihani et al. (2004)	Analytical method	Feeder service to Fixed PT network	Exogenous	Determine the optimal nmber of zones for Flexible PT operation
Dai et al. (2020)	Analytical and simulation	Exclusive mode	Exogenous	Joint optimisation of headway and capacity

homogeneous autonomous shuttle fleet, Dai et al. (2020) formulates the problem as an integer nonlinear programming model considering a heterogeneous fleet of autonomous and human driven buses. However, both the studies considered an exogenous demand for the services without considering integration with a line and schedule based public transport.

Neumann and Nagel (2013), Kalpakcı and Ünverdi (2016), and Aldaihani and Dessouky (2003) considered on-demand services in competition with a Fixed PT network. The on-demand services were modelled as paratransit services which operates on fixed routes but with no fixed schedules. The objective of these studies were to determine a set of optimal set of routes for the paratransit services.

From a demand perspective, determining the feasible conditions to operate a demand responsive service has been studied by Quadrifoglio and Li (2009) and Li and Quadrifoglio (2010). They developed analytical and simulation models to this end and the results indicated that the switching point between a demand responsive and a fixed route service is in the range from 10–50 customers/mi^2/hr.

A comparitive summary of the reviewed literature based on the modelling approach, operation of Flexible PT, and the objective is shown in Table 1. As can also be seen from the table, while these studies shed light on the interaction between fixed and flexible services and related design variables, all of them have considered fixed and flexible services as mutually exclusive. Consequently, the assignment process for fixed and flexible services was performed in isolation as passengers either had to choose between fixed and flexible services or had to combine them in a shuttle setting. In all cases, passengers' ability to choose the best sequence of legs connecting their origin and destination including possibly combining fixed and flexible services within a given journey was not accounted for in the model. From a demand perspective, most of the studies designed fixed and flexible services while considering demand as exogenous. In other words, none of the studies allowed users to choose between flexible services as first/last mile service or exclusive door-to-door service from their origin to destination.

In this study, a route choice model that allows users to combine fixed and flexible passenger services or use them as individual modes, with endogenous demand is proposed. This enables examining the share of the trip that users choose to travel using each of these services, as well as the share of travelers choosing to do so. Furthermore, the locations where users choose to interchange between these services can be identified. All these choice dimensions become therefore an integral outcome of the integrated assignment model as opposed to part of the case settings or model assumptions. The route choice model is integrated in an agent-based transport assignment framework. The model allows combined journey with fixed and flexible service as well as a journey that consists exclusively of fixed or flexible transport; with endogenous demand. The model, implemented in MATSim, is applied to a network centered around the city of Amsterdam and provides insights on how an individual ride-hailing service will perform and its potential market share as a self-standing mode as well as in combination with the existing Fixed PT. Measures of performance of the system in terms of the passenger travel time, impact of fleet size and thus level of service (in particular waiting times) on the number of passenger trips performed using Flexible PT and their respective share of travel demand and fleet utilisation metrics is analysed under scenarios where Fixed PT and Flexible PT services are offered as mutually exclusive modes or can be combined into a single journey. This in turn may facilitate the efficient design of an integrated public transport system consisting of both Fixed PT and Flexible PT.

The paper is structured as follows. Section 2 presents the Methodology developed starting by defining the integrated public transport journey of a user. Section 3 details the application of the model which includes the case study description and the simulation scenarios. This is followed by Section 4 which presents the results and analysis. Section 5 concludes the paper and suggests directions for future research.

2. Methodology

This section presents the overall methodology developed. First, **System components** and **Integrated public transport route** of a user are defined. Second, the model developed for generating and evaluating public transit travel alternatives and the assignment procedure are described. Third, we provide details on model implemenation. Each of these points are detailed in the following sub-sections.

2.1. Definitions

2.1.1. System components
We first introduce the key system elements.

Network: Refers to the network which comprise of the sub-networks of road and fixed public transport. The sub-network of fixed public transport involve the route network for public transport modes (e.g. train, tram, metro, bus) with their respective stop locations.

Demand: Comprise of passengers with a set of origin and destination points in the network. In this study it is assumed that the passengers have full knowledge about the network and schedules of the Fixed PT system.

Supply: Comprise of the modes available to each user for travelling from their origin to their destination. The modes available are:

Fixed PT: Comprise of line-based services that follow a pre-defined route and schedule operated by a fleet of vehicles.
Flexible PT: On-demand services picking up passengers from their origin or a Fixed PT stop and dropping them off at their destination or a Fixed PT stop, operated by a fleet of vehicles. The fleet of vehicles are controlled by a central dispatching unit which assigns incoming travel requests to vehicles in real-time. Individual rides are offered by the on-demand services and all travimulations were...' should start el requests are generated in real-time i.e. no pre-booking is allowed.

Walk: Passengers may choose to walk from their origin to a Fixed PT stop, from a Fixed PT stop to their destination, or even from their origin to their destinaion.

Bike: Passengers may choose to bike from their origin to a Fixed PT stop, from a Fixed PT stop to their destination, or from their origin to their destinaion.

Car: Privately owned vehicles that may be used between the trip origin and destination.

2.1.2. Integrated public transport route

Integrated public transport route refers to the trip a passenger makes from an origin to a destination using one or more public transport services. It may thus consist of a Fixed PT service (or a combination of several modes - e.g. bus and metro), a Flexible PT service, or the combination thereof. Depending on the type of service used in the journey, integrated public transport routes are classified into one of the following types.

1. Walk/Bike + Fixed PT + Walk/Bike: A passenger walks/bikes from his/her origin to a Fixed PT stop, waits for a Fixed PT service, makes a Fixed PT trip to another Fixed PT stop, and finally walks/bikes from that stop to his/her destination.

2. Walk/Bike + Fixed PT + Flexible PT: A passenger walks/bikes from his/her origin to a Fixed PT stop, waits for a Fixed PT service, travels with Fixed PT trip to another Fixed PT stop, books a Flexible PT service, waits for the Flexible PT service, and finally makes a Flexible PT trip to his/her destination.

3. Flexible PT + Fixed PT + Walk/Bike: A passenger calls a Flexible PT service from his/her origin, waits for the Flexible PT service, makes the Flexible PT trip to a Fixed PT stop, waits for a Fixed PT service, makes a Fixed PT trip to another Fixed PT stop, and finally walks/bikes from that stop to his/her destination.

4. Flexible PT + Fixed PT + Flexible PT: A passenger books a Flexible PT service from his/her origin, waits for the Flexible PT service, makes the Flexible PT trip to a Fixed PT stop, waits for a Fixed PT service, makes a Fixed PT trip to another Fixed PT stop, books a Flexible PT service, waits for the Flexible PT service, and finally makes a Flexible PT trip to his/her destination.

5. Flexible PT: A passenger orders a Flexible PT service from his/her origin, waits for the Flexible PT service, rides the Flexible PT trip to his/her destination.

Note that route composition options 1–4 are composed of three legs, whereas the fifth option consists of a single leg. A spatio-temporal representation of the different types of routes is given in Fig. 1. The following section describes the method developed to generate, for each user, a set of integrated public transport routes and the process by which a particular route is chosen.

2.2. Integrated passenger transport assignment

For each origin–destination pair, first a set of integrated public transport routes (choice set) are generated in the module *Choice set generation*. Then for each of these options, a utility value is computed - which is a function of attributes of that particular route- in the module *Scoring of choice alternatives*. Finally, the origin destination demand is iteratively assigned to those paths based on the computed utility values in the *Assignment* module. The following sub-sections describe each of these modules.

2.2.1. Combined route choice set generation

A choice alternative here refers to the sequence of stops and services for realising the integrated public transport trip. A choice for a given origin–destination pair consists of a path connecting the origin and destination, transfer points if any, and the modes used to travel each leg of the path. Consider a route of a person traveling from origin location O to destination location D. It is assumed in this study that passengers search for Fixed PT stops in a circle (symmetric around their origin and desination). Let r be the radius (Euclidean distance) with which transit users search for Fixed PT stops from their origin and destination. Let d be the maximum distance (Euclidean distance) that a transit user chooses to walk to reach from their origin to a transit stop or from a transit stop to their destination. Thus r and d define the catchment area by on-demand transport and active modes, respectively. Based on the search radius r and walking distance limit d, the following sets are defined:

1. $S(r)_O$ = set of all Fixed PT stops within the radius r from origin O = $\{S_O^1, S_O^2, S_O^3, ..., S_O^m\}$
2. $S(r)_D$ = set of all Fixed PT stops within the radius r from destination D = $\{S_D^1, S_D^2, S_D^3, ..., S_D^n\}$
3. $S(d)_{O,walk/bike}$ = set of all Fixed PT stops in S_O with walking or biking distance from origin O to the stop, less than or equal to d
4. $S(d)_{D,walk/bike}$ = set of all Fixed PT stops in S_D with walking/biking distance from destination D to the stop, less than or equal to d
5. S_{OD} = set of all origin destination transit stop pairs = $\{(S_O^1, S_D^1), (S_O^1, S_D^2), ...(S_O^i, S_D^j), ...(S_O^m, S_D^n)\}$ where $S_O^i \in S_O(r)$ and $S_D^j \in S_D(r)$

We refer to the origin destination pair as OD demand pair and origin destination tansit stop pair as OD transit pair. Following the process of generating these sets per OD demand pair, a set of feasible paths is generated by examining the feasibility of alternative route composition. If the stop S_O^i is in the set $S(d)_{O,walk/bike}$, then the path between origin and S_O^i is assigned the walk/bike mode. If the stop S_O^i is not in the set $S(d)_{O,walk/bike}$, then the path between origin and S_O^i is assigned the Flexible PT mode. Similarly the path between S_D^j and destination is also assigned a mode. The path between S_O^i and S_D^j is assigned the Fixed PT mode. The path chosen for the Fixed PT part is the shortest path (based on generalised travel cost) between the two stops using the Fixed PT network. The Fixed PT routing between an OD transit stop pair is based on Rieser (2010). The path obtained with modes assigned to it, is stored as a choice alternative for the respective OD demand pair. The assigned OD transit stop pair $\{(S_O^i, S_D^j)\}$ is then removed from the set S_{OD} and a new pair is chosen at random. This process of assigning modes is repeated for each of the OD transit stop pairs in S_{OD}. If either or both of the sets $S_O(r)$ and $S_D(r)$ are empty, then the path between origin and destination is assigned to Flexible PT. In this case the OD
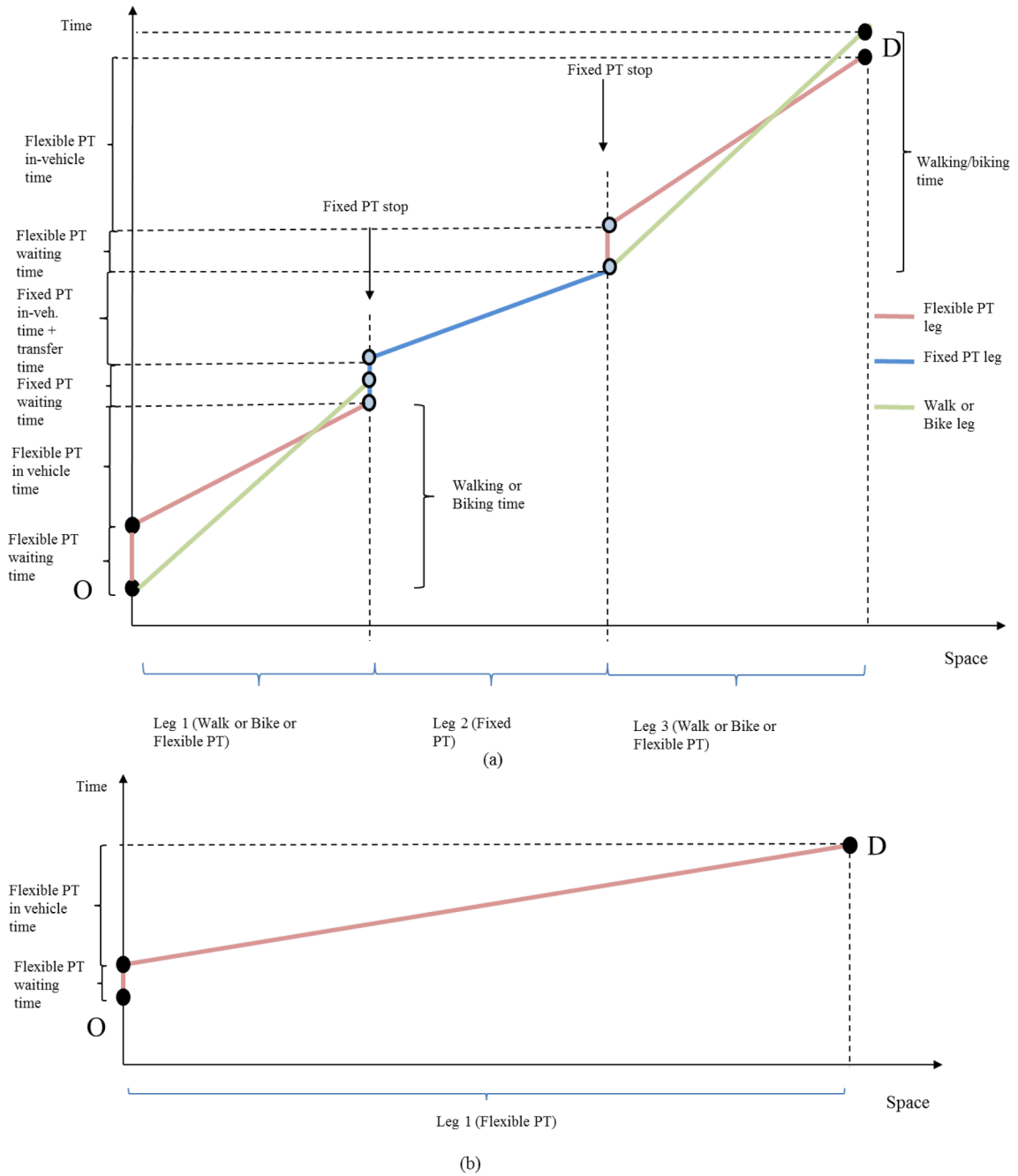
Fig. 1. Spatio-temporal representation of integrated public transport route composition options.

demand pair will have a single choice. Note that in any case assigning Flexible PT to the entire trip between O and D will be one of the alternative. An illustration of all the possible integrated public transport routes of a passenger with sets $S(r)_O$, $S(r)_D$, $S(d)_{O,walk/bike}$, and $S(d)_{D,walk/bike}$ is shown in Fig. 2. We assume a connected Fixed PT network which implies that it is possible to reach a Fixed PT stop from any other Fixed PT stop in the network. The Fixed PT network in the figure indicates a schematic network and Fixed PT links may correspond to a direct line or a combination of lines involving interchanges.

Consider the origin–destination transit stop pair (i, ii). Following the process described above, the segment from O to i will be assigned walk/bike mode, the portion i to ii will be assigned Fixed PT mode, and the portion ii to D will be assigned Flexible PT mode. The public transit journey path e represents the trip covered entirely by Flexible PT. Table 2 provides the pseudocode for the algorithm for generating the choice set for the OD pair. Set C represents the choice set of integrated public transport routes for the origin–destination pair including the modes assigned to each of the journey legs.
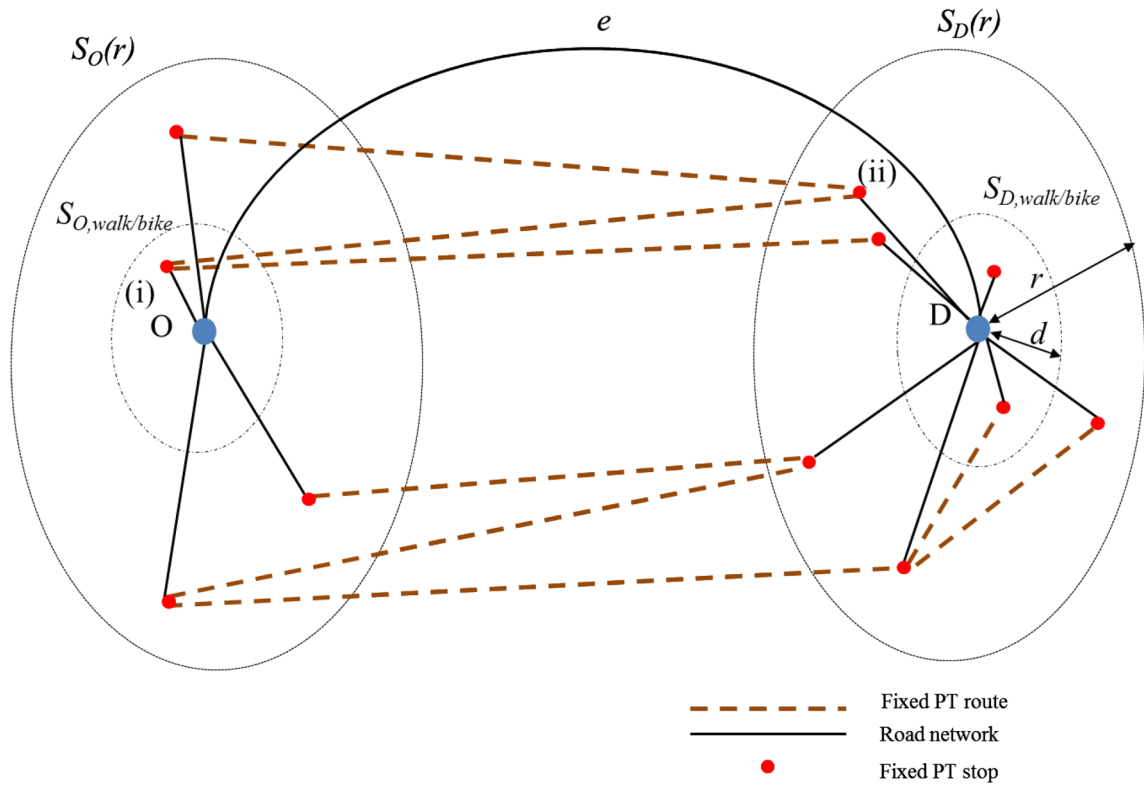
Fig. 2. Illustration of integrated public transport routes with Fixed PT stop sets between origin O and destination D.

Table 2
Pseudocode for generation of choice set for an origin destination pair.

Step 1:	Consider an origin destination pair
Step 2:	Define $S_O(r)$, $S_D(r)$, $S(d)_{O,walk/bike}$, $S(d)_{D,walk/bike}$, Initiate $C = \emptyset$ and $k = 1$
Step 3:	$C_k = \{O, \text{Flexible PT}, D\}$
Step 4:	if $(S_{OD} = \emptyset)$
	Stop
	else
	$k++$
	Randomly select a pair from S_{OD}
	Let the pair be (a, b)
	if $((a \in S(d)_{O,walk/bike}) \wedge (b \in S(d)_{D,walk/bike}))$
	$C_k = \{O, \text{walk or bike}, a, \text{Fixed PT}, b, \text{walk or bike}, D\}$
	Go to *Step 5*
	else
	if $((a \notin S(d)_{O,walk/bike}) and (b \notin S(d)_{D,walk/bike}))$
	$C_k = \{O, \text{Flexible PT}, a, \text{Fixed PT}, b, \text{Flexible PT}, D\}$
	Go to *Step 5*
	else
	if $(a \notin S(d)_{O,walk/bike})$
	$C_k = \{O, \text{Flexible PT}, a, \text{Fixed PT}, b, \text{walk or bike}, D\}$
	Go to *Step 5*
	else
	$C_k = \{O, \text{walk or bike}, a, \text{Fixed PT}, b, \text{Flexible PT}, D\}$
	Go to *Step 5*
Step 5:	Remove (a, b) from S_{OD}
Step 6:	Go to *Step 4*

2.2.2. Scoring of route choice alternatives

Once a choice set has been generated for an *OD* pair, the next task is to distribute the demand over the alternatives available. To this end, it is essential to evaluate each of the choice alternatives in the choice set. A utility function is developed which scores each choice alternatives based on the following attributes: number of transfers involved in the choice alternative, mode specific attributes

such as fare, in-vehicle time and waiting time. The utility value of alternative i, U_i is given by,

$$U_i = \beta_{travel}^{walk/bike} \cdot t_{travel}^{walk/bike} + \beta_{transfer} \cdot N_{transfer} + \sum_{m=Fixed\ PT, Flexible\ PT} [\beta_{wait}^m \cdot t_{wait}^m + \beta_{travel}^m \cdot t_{travel}^m + \beta_{money} \cdot \gamma^m \cdot d^m]$$

(1)

where,

$t_{travel}^{walk/bike}$ is the total walking/biking time for alternative i

t_{wait}^m is the waiting time for mode m

$t_{inveh.}^m$ is the total in vehicle time of mode m

γ^m is the fare per unit distance for mode m

d^m is the total distance travelled with mode m

βs are behavioral route choice parameters

$N_{transfer}$ is the total number of transfers between public transport services, regardless of their mode of operations

The computation of travel times and the waiting times of the user, with Fixed PT and Flexible PT is described as follows. The in-vehicle time of both the Fixed PT and Flexible PT is calculated using Dijkstra's algorithm using the length of each link in the network and mode specific speed. The waiting time of Fixed PT is computed from the schedule of the Fixed PT based on the arrival time of the user at the Fixed PT stop. The waiting time of Flexible PT is determined by the Flexible PT dispatching algorithm that assigns user travel requests to vehicles based on minimising their waiting time. The waiting time is computed as the time required for the Flexible PT vehicle to reach the user's pick-up location from its current location. The network travel time between an origin and destination travel time is computed using Dijkstra's algorithm. The travel time experienced by all users include the effect of congestion in the network.

2.2.3. Iterative network loading

Once the utilities of all the choice alternatives have been computed, the demand is assigned to the choice alternative with the lowest travel utility (*all-or-nothing* assignment). The *all-or-nothing* assignment implemented in this study is described as follows. Fig. 3 shows a network with origin O and destination D and the sets $S_O(r)$, $S_D(r)$, $S(d)_{O,walk/bike}$, and $S(d)_{D,walk/bike}$ as described in the previous section. The number of possible paths for the user to travel from O to D is 3 as shown in Table 3 which also shows the individual travel time components and utility. The utility value of each path is computed as per Eq. (1). The demand between O and D is then assigned to the path with the highest utility value.

2.3. Model implementation

The proposed model is embedded in a multi-agent transport simulation framework. It is implementd and integrated in the open source software MATSim (Horni et al., 2016). During the course of the iterative assignment, agents may undertake different strategies to alter their travel plans while making their trip from origin to destination based on the service experienced on past days. In this study, the strategies available to an agent are: changing the route of travel, changing the mode of travel, changing the departure time from an activity, and selecting a plan with the best score. An overview of the overall modelling framework with the developed multimodal route choice model is depicted in Fig. 4. User assignment for the combined Fixed and Flexible PT takes place in the
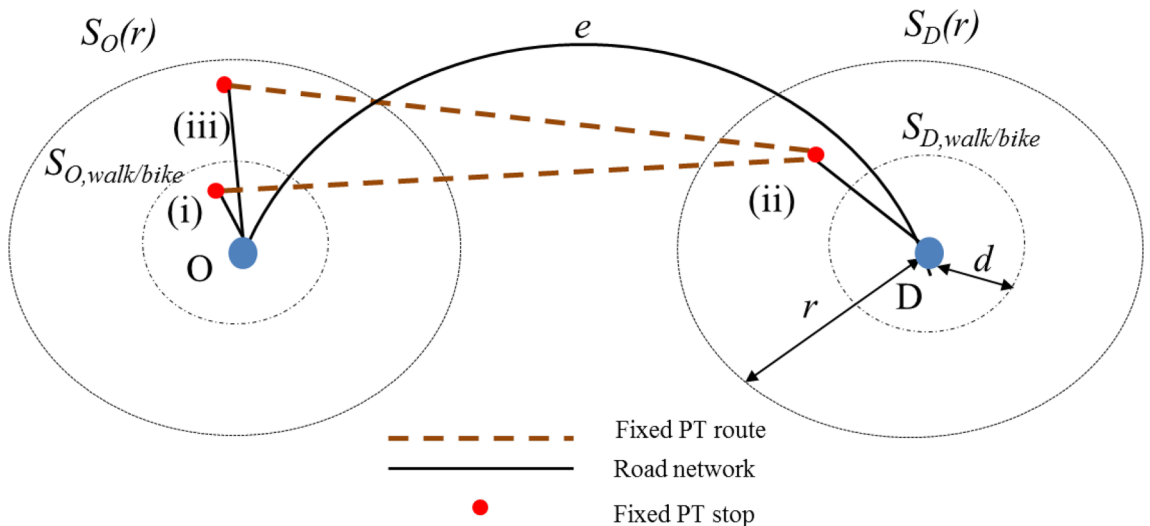

Fig. 3. Network for *all-or-nothing* assignment illustration.

Table 3
Travel time components of integrated routes of Fig. 3.

| Path index	Nodes in sequential order	Travel time components	Utility
I	$\{O, (i), (ii), D\}$	Walking or biking time from O to (i)	U_I
		Fixed PT waiting time at (i)	
		Fixed PT in-vehicle time from (i) to (ii)	
		Flexible PT waiting time at (ii)	
		Flexible PT in-vehicle time from (ii) to D	
II	$\{O, (iii), (ii), D\}$	Flexible PT waiting time at O	U_{II}
		Flexible PT in-vehicle time from O to (iii)	
		Fixed PT waiting time at (iii)	
		Fixed PT in-vehicle time from (iii) to (ii)	
		Flexible PT waiting time at (ii)	
		Flexible PT in-vehicle time from (ii) to D	
III	$\{O, D\}$	Flexible PT waiting time at O	U_{III}
		Flexible PT in-vehicle time from O to D	
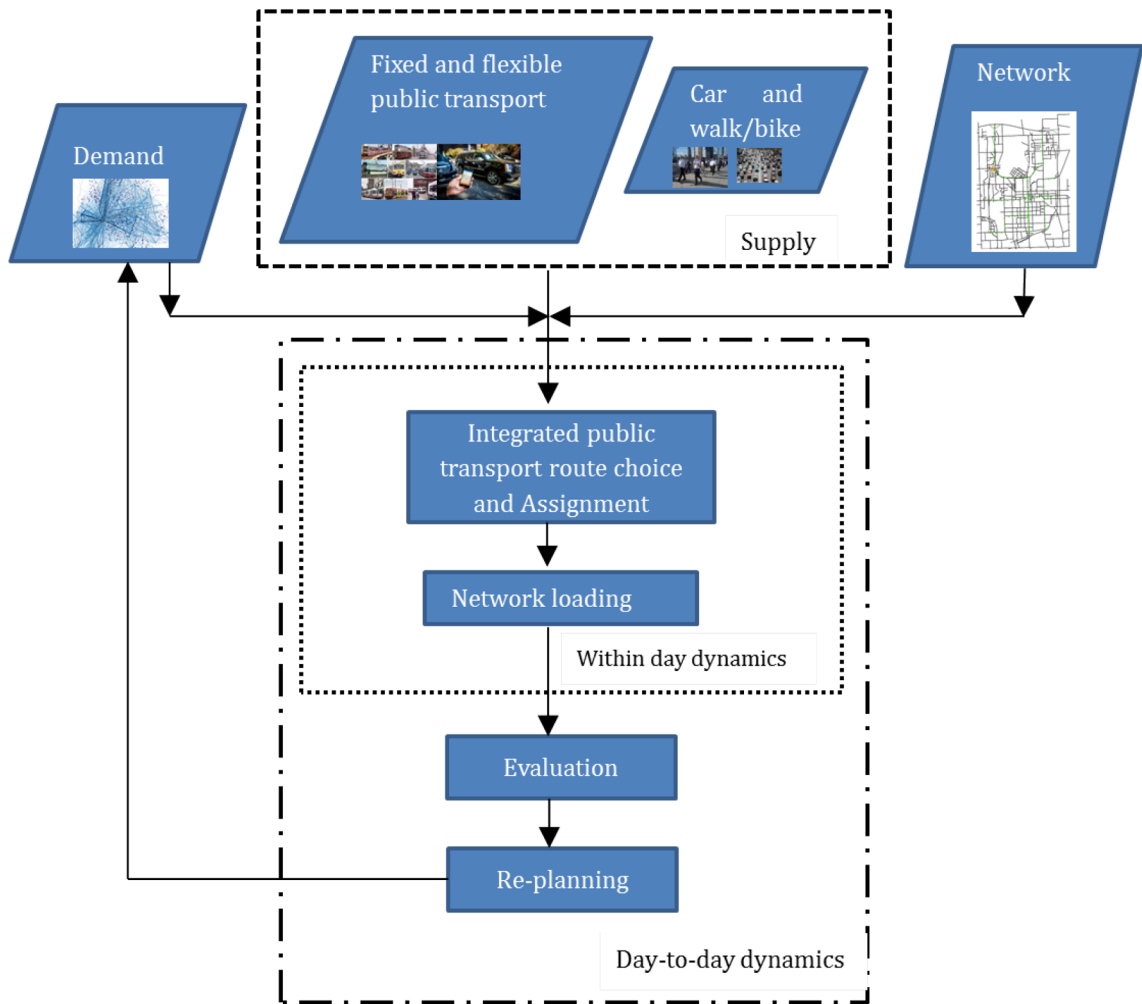
Fig. 4. Overall modelling framework.

Integrated public transport route choice and Assignment module. The assigned users and vehicles are loaded to the network and simulated in the *Network Loading* module. The *Integrated public transport route choice and Assignment* and the *Network Loading* module comprise the daily dynamics of the system. Following the simulation, the users evaluate their executed travel plan based on the travel time experienced on the network in the *Evaluation* module. Travel time uncertainty such as a long waiting time and a possible

increase in travel time due to congestion, is incorporated in the **Evaluation** module. For instance, if a user has to wait long for a service or experiences increased in-vehicle time due to congestion, this would lead to an increase in the overall travel time for that particular travel plan. Subsequently, that particular travel plan would receive a low score in the **Evaluation** module and the probability of that plan being selected in the following iterations diminishes. Consequently, modes of travel that consistently or even occasionally underperform due to inherent service uncertainty are less likely to be selected by users at equilibrium. The users make adjustments to their travel plans based on the strategies mentioned above in the *Re-planning* module. The selection of a strategy is based on a Logit function which is the default option in MATSim. The *Evaluation* and *Re-planning* modules entail the day-to-day dynamics of the system. The model has been coded in Java, the documentation and source code of which are available on github[1].

3. Application

3.1. Network

The proposed integrated route choice model is applied to the network centered around Amsterdam, The Netherlands (Fig. 5) and extends to the national airport, Schiphol, located at the south-east of the study area. The objective of the application is to gain insights into the implications of combining the Fixed PT service (which includes metro, tram, and bus) and Flexible PT service in Amsterdam. The network consists of 17, 375 nodes, 31, 502 links, and 2, 517 public transport stops and includes train, tram, bus, and metro. The demand comprise of an activity based travel demand data with each agent performing a series of activity and travel leg for an entire day of simulation. It consists of 168, 103 agents (representing 20% of the population), performing 556, 437 trips and is adopted from the national activity-based demand model, Albatross (Arentze et al., 2000). Simulating a fraction of the population is found to provide meaningful simulation results as shown in Bischoff and Maciejewski (2016). The following modes are considered by the model: car, Fixed PT, walk, and bike.

3.2. Model settings

We introduce the various aspects of model settings under the following sub-sections, namely *Mode and route choice*, *Flexible PT dispatching algorithm*, and *Modal attributes*.

3.2.1. Mode and route choice

We calibrated the mode specific parameters by performing a sensitivity analysis on the alternate specific constants. We set the target modal share for all the modes to reflect a realistic modal split for the city of Amsterdam and arrived at values for the alternate specific constants that generated a realistic modal share at equilibrium. The marginal utility of performing an activity (β_{dur}), marginal utility of time spent by traveling (β_{travel}) for all the modes, marginal utility of arriving late for an activity ($\beta_{latear.}$) have been set to $+6$ utilities/hour, -6 utilities/hour, and -18 utilities/hour respectively. The behavioral parameter values were set to the default values set in MATSim following the calibration guidelines provided in Horni et al. (2016). Marginal utility of money (β_{money}) is set to -0.685 utilities/€(based on the Dutch value of time). The radius of the catchment area for on-demand and active modes, r and d were set to 1000 m and 500 m, respectively, in accordance wih the average access and egress distances assumed for active modes.

3.2.2. Flexible PT dispatching algorithm

In this study, the Flexible PT system comprises of a fleet of vehicles controlled by a central dispatching unit that assigns vehicles to travel requests in real time. The vehicle dispatching algorithm has been adopted from Maciejewski (2016). The Flexible PT offers taxi-like individual door-to-door service and does not allow sharing. A vehicle that has been assigned a request by the dispatching unit, drives to the pick-up location, picks up the passenger, drives to the drop-off location, and drops the passenger. It then stays at the drop off location till further requests are assigned.

3.2.3. Modal attributes

The cost of the Fixed PT has been set according to the values provided by the incumbent public transport service provider. The Flexible PT is assumed to be 10 times as expensive as Fixed PT which is a reasonable assumption for taxi-like door-to-door service in Amsterdam. The distance based fares of Fixed PT ($\gamma^{FixedPT}$) and Flexible PT ($\gamma^{FlexiblePT}$) used in the model are 0.154 €/km and 1.54 €/km, respectively. The capacity of a Fixed PT vehicle is 100 whereas Flexible PT offers a taxi-like door-to-door service with a capacity of a single passenger. The speed of walk and bike modes were set to the default values in MATSim which are 3 km/hour and 15 km/hour, respectively.

3.2.4. Model calibration

In the absence of real data, we calibrated the utility functions following the calibration guidelines in MATSim. We methodically investigated the alternate specific constants of the available modes by fixing the marginal utility of traveling term to a fixed value as suggested in the calibration guideline for MATSim. We set the modal share for the modes to a realistic value for our case study area and obtained the ASCs which yielded the desired modal share. The obtained set of values was then fixed throughout the simulation

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[1] https://github.com/Jishnuns/IntegratedFixedFlexibleRouteChoiceAssignment.git

Fig. 5. Road and public transport network of Amsterdam.

runs.

3.3. Simulation scenarios

The simulation scenarios are summarised in Table 4. The scenarios are based on the type of public transport services offered and whether the combination of fixed and flexible services in a single trip is possible. In the **Base scenario**, the modes available to the user are car, walk, bike, and Fixed PT. In scenario **Fixed PT or Flexible PT**, a fleet of vehicles is introduced which offers Flexible PT. Fixed and flexible services are mutually exclusive in this scenario. Finally, in scenario **Fixed PT + Flexible PT**, in addition to the modes available in the previous scenario, users may also combine Fixed PT and Flexible PT when travelling from their origin to their destination.

From a planning perspective, it is important to investigate the effects of the fleet size of Flexible PT on the level-of-service offered to users and on system performance in general. To this end, a sensitivity analysis is conducted with respect to the fleet size of Flexible PT for both the second and third scenarios. The fleet sizes are equivalent to 0.1, 0.5, 1, 2, 3, 5, and 10 percentages of the simulated agent population.

The simulations were run in the Dutch national supercomputer, Cartesius. The simulation timestep is 1s and the running time of a

Table 4
Scenario description with mode choice of users.

Index	Scenario	Car	Walk	Bike	Fixed PT only	Fixed PT and Flexible PT	Flexible PT only
I	**Base Scenario**	Y	Y	Y	Y	N	N
II	**Fixed PT or Flexible** PT	Y	Y	Y	Y	N	Y
III	**Fixed PT + Flexible PT**	Y	Y	Y	Y	Y	Y

Table 5
Modal share per mode for the scenarios in percentages.

Scenario	Fleet size	Car	Walk	Bike	Fixed PT only	Fixed PT and Flexible PT	Flexible PT only	Total PT	Vehicle-km($\times 10^6$)
Base Scenario	NA	28.35	30.33	22.15	19.17	NA	NA	19.17	2.49
Fixed PT or Flexible PT	0.5	18.75	29.61	14.98	19.34	NA	17.32	36.66	2.35
	1	16.10	28.49	13.18	17.68	NA	24.56	42.24	2.77
	2	13.40	23.57	10.88	12.08	NA	40.06	52.14	3.67
	3	12.36	19.94	10.66	9.15	NA	47.88	57.03	4.60
	5	12.28	18.45	10.66	8.43	NA	50.19	58.62	4.53
	10	12.07	17.87	10.39	7.94	NA	51.73	59.67	4.62
Fixed PT + Flexible PT	0.5	19.17	29.24	11.84	11.39	8.55	19.81	39.75	2.34
	1	17.83	28.72	12.30	8.20	9.53	23.43	41.16	2.78
	2	15.17	25.20	10.87	3.01	13.63	32.12	48.76	3.54
	3	12.30	20.52	8.12	0.87	18.51	39.69	59.07	4.17
	5	9.18	16.09	5.03	0.19	23.78	45.73	69.70	4.91
	10	8.54	15.26	4.94	0.16	24.55	46.55	71.26	4.88

simulation run till convergence is approximately 32 and 42 h for scenario **Fixed PT or Flexible PT** and **Fixed PT + Flexible PT** respectively. In order to account for stochasticity in the results, 10 runs for each simulation instance was carried out and the key performance indices were averaged over these runs.

4. Results and analysis

The results are analysed with respect to *Modal usage*, *Service performance*, and *Fleet utilisation* related to the number of trips performed per mode, travel times for users, and fleet utilisation of Flexible PT, respectively. The results provide insights on the performance of a Flexible PT service and its market share, as a self-standing mode as well as in combination with a Fixed PT. The following sub-sections discuss these results in detail.

4.1. Modal usage

Table 5 shows the number of trips per mode and the vehicle-km travelled for the three scenarios considered. From the table, it becomes evident that with the increase in fleet size of Flexible PT, there is a steady increase in the number of trips with only Flexible PT in the scenario **Fixed PT or Flexible PT** and for trips with only Flexible PT as well as trips combining Flexible PT and Fixed PT in
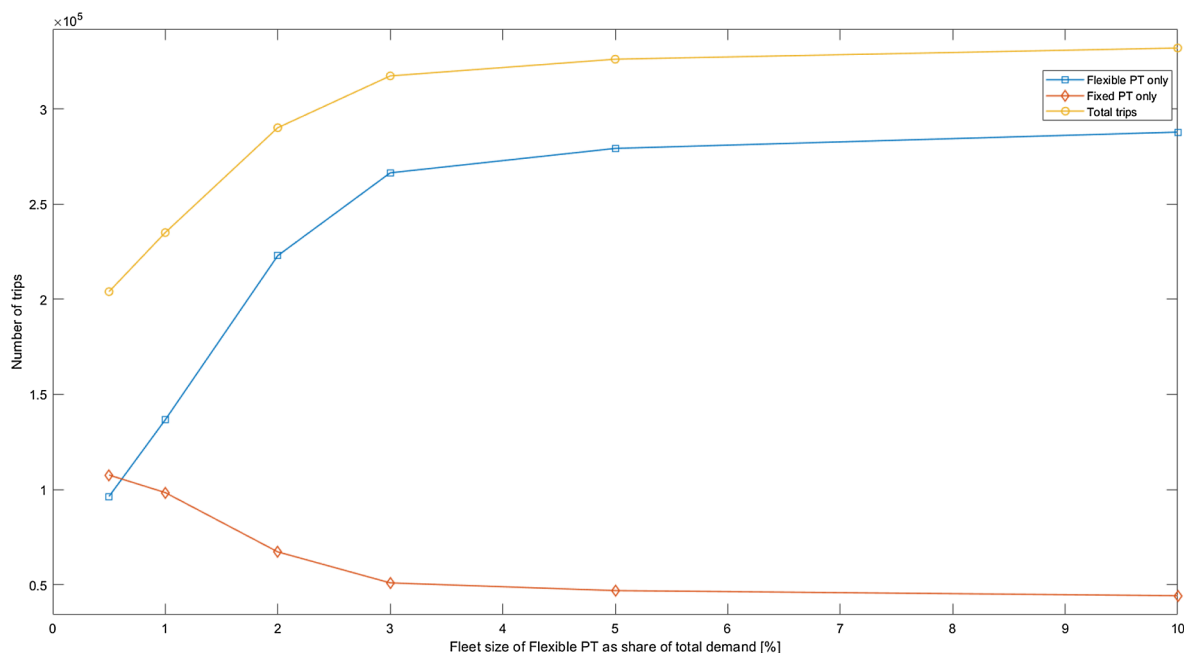

Fig. 6. Number of PT trips versus fleet size of Flexible PT for scenario Fixed PT or Flexible PT.
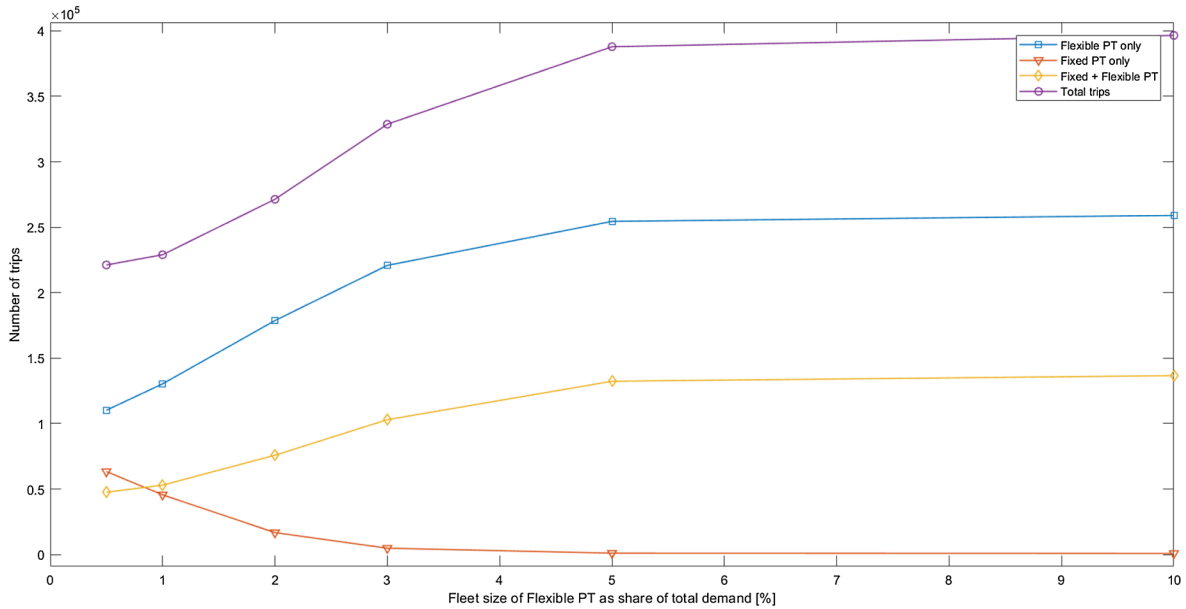
Fig. 7. Number of PT trips versus fleet size of Flexible PT for scenario Fixed PT + Flexible PT.

the scenario **Fixed PT + Flexible PT**. To further understand this trend, we look at the plots of number of PT trips versus Flexible PT fleet size for scenarios **Fixed PT or Flexible PT** and **Fixed PT + Flexible PT** as shown in Figs. 6 and 7. It can be seen from the Fig. 6 that the variation of number of trips by Flexible PT only is super-linear till a fleet size that is equivalent to 2% of the population after which the variation becomes sublinear with the number of trips stabilising around fleet size of 3%. Similary from Fig. 7, it can be observed that the increase in the number of trips by Flexible PT only and trips combining Fixed PT and Flexible PT is sub-linear till 5% of fleet size after which the number of trips stabilises. This trend can be further explained by examining the average waiting time plots for scenarios **Fixed PT or Flexible PT** and **Fixed PT + Flexible PT** (Fig. 8). The waiting time plots indicate a similar trend, indicating that the increase in number of trips for Flexible PT is primarily governed by the waiting time reductions. It can be also seen
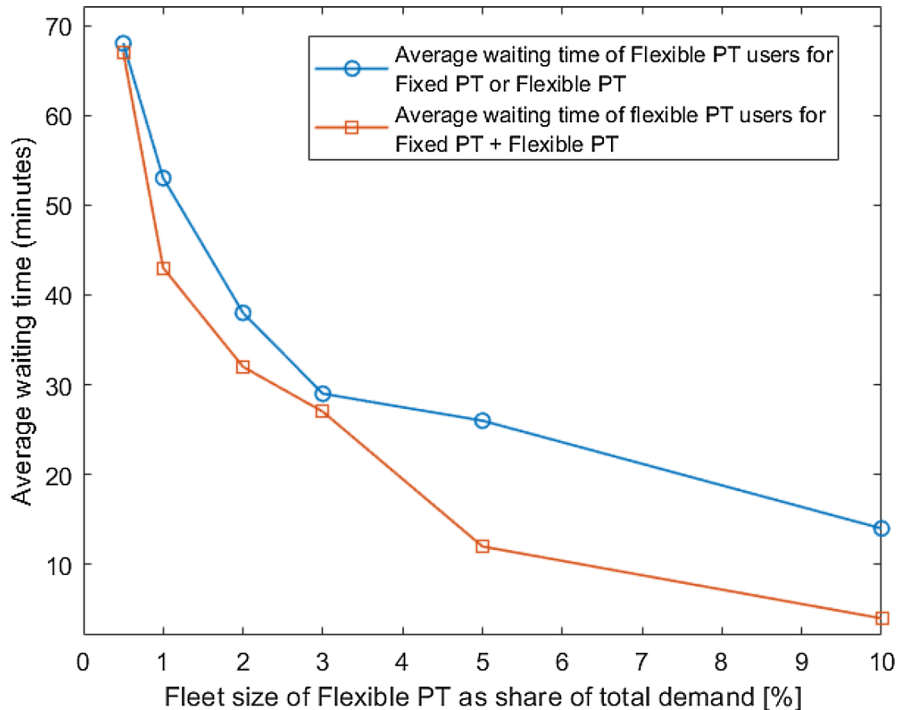

Fig. 8. Average waiting time vs fleet size of Flexible PT for scenarios Fixed PT or Flexible PT and Fixed PT + Flexible PT.
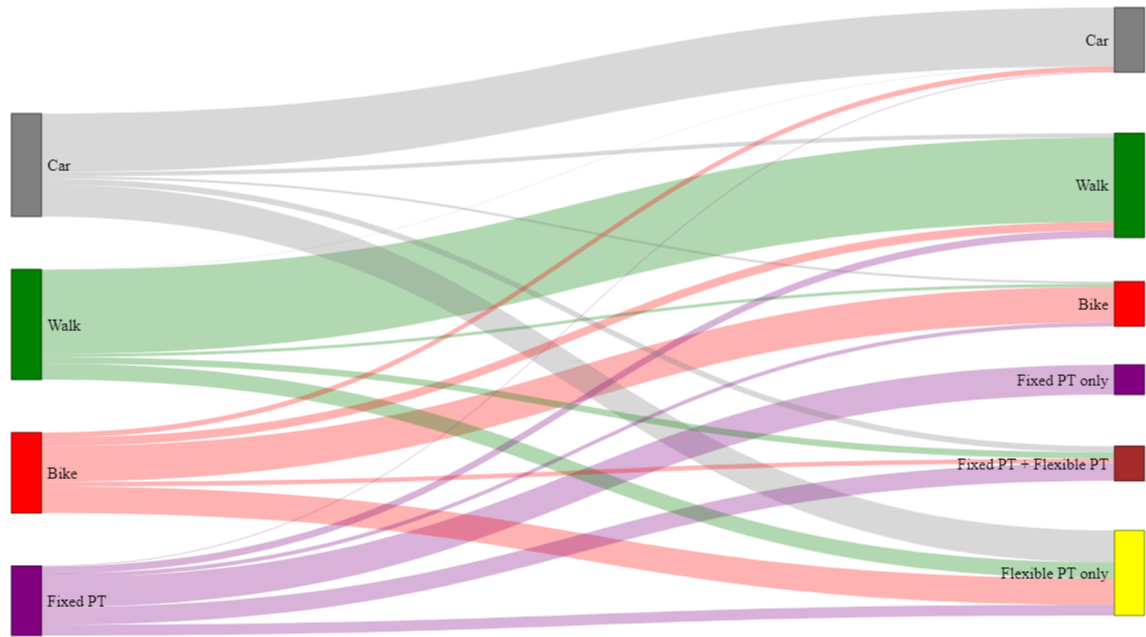
Fig. 9. Migration plot for Base case to scenario Fixed PT + Flexible PT, fleet size = 1%.

from Table 5 that there is an overall increase in the share of PT for the scenario **Fixed PT + Flexible PT** of about 12% (for a fleet size of 10% of the total demand) compared to the scenario **Fixed PT or Flexible PT** indicating that the integration of Fixed and Flexible PT makes the service more attractive. There is also a steady decrease in mode share of car and active modes (walk and bike) for scenarios **Fixed PT or Flexible PT** and **Fixed PT + Flexible PT** compared to the **Base Scenario**.

We further analyze modal shift by plotting the migration pattern when making the transition from the **Base Scenario** to **Fixed PT + Flexible PT** scenario (indicating the number of people shifting from the modes in **Base Scenario** to the modes in scenario **Fixed PT + Flexible PT**). As can be seen in Fig. 9 that the trips with Fixed PT in the base case is absorbed by Flexible PT and the combination of Fixed PT and Flexible PT in scenario **Fixed PT + Flexible PT**. It can be also seen that the share of combined Fixed PT and Flexible PT trips comes predominantly from Fixed PT users from the **Base Scenario**. Most of the users who switch to Flexible PT, have previously traveled by car. A considerable portion of car users in the **Base Scenario**, switch to Flexible PT in the scenario **Fixed PT + Flexible PT**. There is also a considerable shift from the active modes (Walk and Bike) to Flexible PT. Most of the previous Fixed PT users choose the combination of Fixed PT and Flexible PT, Flexible PT only, or stay with Fixed PT only in the scenario **Fixed PT + Flexible PT**.

The total vehicle-kms travelled for scenario **Fixed PT or Flexible PT** steadily increases for fleet sizes of up to 3% after which it starts to stabilize. A similar trend is observed for scenario **Fixed PT + Flexible PT** albeit the vehicle-km steadily continues to increase for fleet sizes of up to 5%, after which it remains at the same level. This could be explained by the increase in the mode share of Flexible PT when possible to use it as either an exclusive mode or in combination with Fixed PT; and the decrease in modal share of active modes. The increase in mode share for Flexible PT translates into additional time spent by vehicles en-route to picking up passengers from their origins. Also, the decrease in mode share of active modes implies a shift from non-motorised to motorised modes. This can also be observed in Fig. 9, which shows that Flexible PT attracts a considerable portion from trips previously performed by active modes. These factors results in additional vehicle-kms travelled as fleet size of Flexible PT increases. Beyond a fleet size of 3% for scenario **Fixed PT or Flexible PT** and 5% for scenario **Fixed PT + Flexible PT**, the increase in the modal share becomes marginal and thus does not induce further increase in vehicle-kms.

Further, we analyze how passengers combine Fixed PT and Flexible PT in a trip in scenario **Fixed PT + Flexible PT**. For all the trips combining Fixed and Flexible PT in **Fixed PT + Flexible PT**, Fig. 11 plots the absolute frequency distribution for the ratio of Flexible PT trip lenghts to the total trip length. It can be seen from Fig. 11 that among the trips that combine Fixed PT and Flexible PT, majority of the trips use Flexible PT to cover only about 10–20% of their trip lengths, indicating that Flexible PT is mostly used as an access or egress mode to Fixed PT. This trend is especially pronounced for larger fleet sizes of Flexible PT. As the fleet size increases, there is an overall increase in the number of trips combining Fixed and Flexible PT. At the same time, this increase in the number of trips is skewed towards using Flexible PT for travelling less than 30% of their total trip length. Fig. 10 shows a spatial representation of the transfer points between Fixed PT and Flexible PT for Flexible PT fleet size = 1% of the total demand. It can be seen from the Fig. 10 that transfers between Fixed PT and Flexible PT and vice versa take place throughout the network and in combination with all modes (such as bus, tram, metro, and train). We cluster the transfer points based on the volume of transfer as shown in Fig. 10. While stops with fewer than 100 transfers are spread throughout the network, high volume transfer stops correspond to metro stations and public transport interchange locations within the ring area. Central station is the most popular transfer location between Fixed PT and Flexible PT.
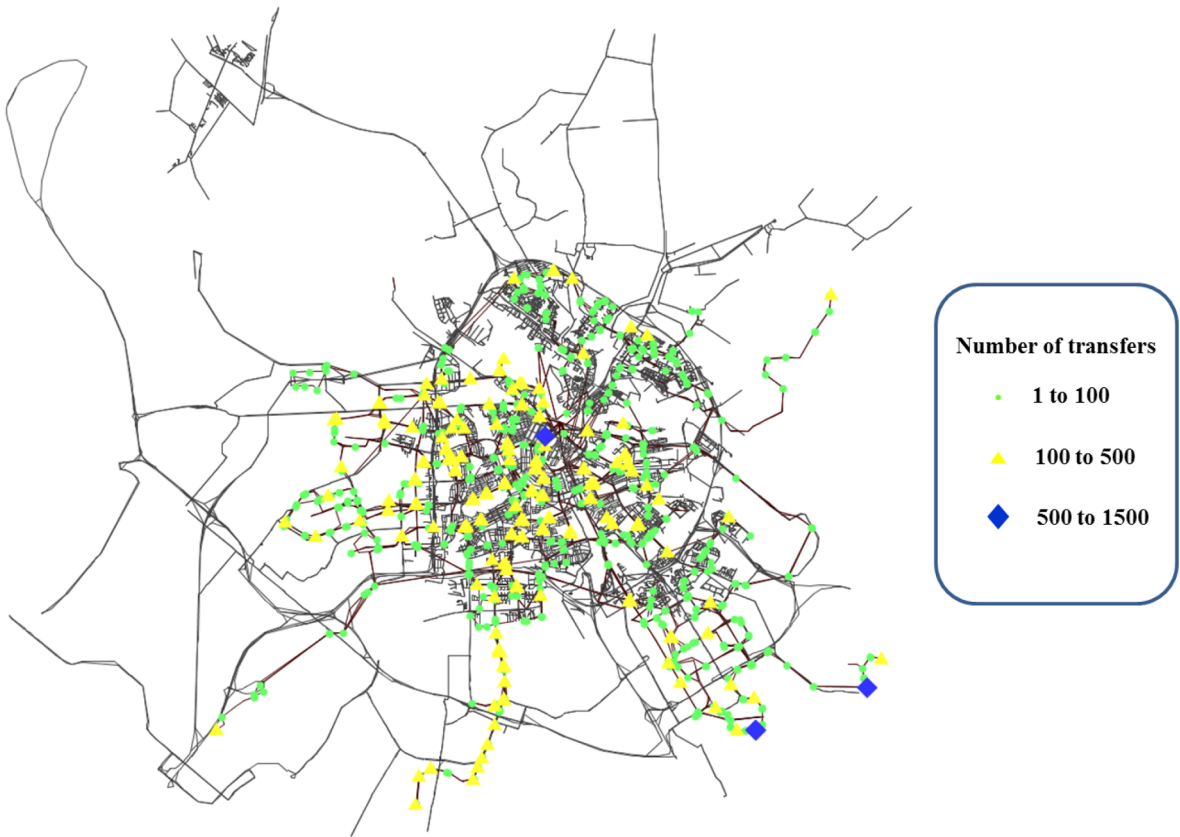
Fig. 10. Transfer points between Fixed PT and Flexible PT.

4.1.1. Fare sensitivity analysis

In order to understand the effect of fare of Flexible PT on the modal share we perform a sensitivity analysis with varying fare for Flexible PT. We vary the fare of Flexible PT relative to the fare of Fixed PT. The ratio of fare of Fixed PT to Flexible PT considered are 1:1, 1:2, 1:5, 1:10 (considered in this study), 1:25, and 1:50. Fig. 12 shows how the modal shares of Flexible PT, Fixed PT + Flexible
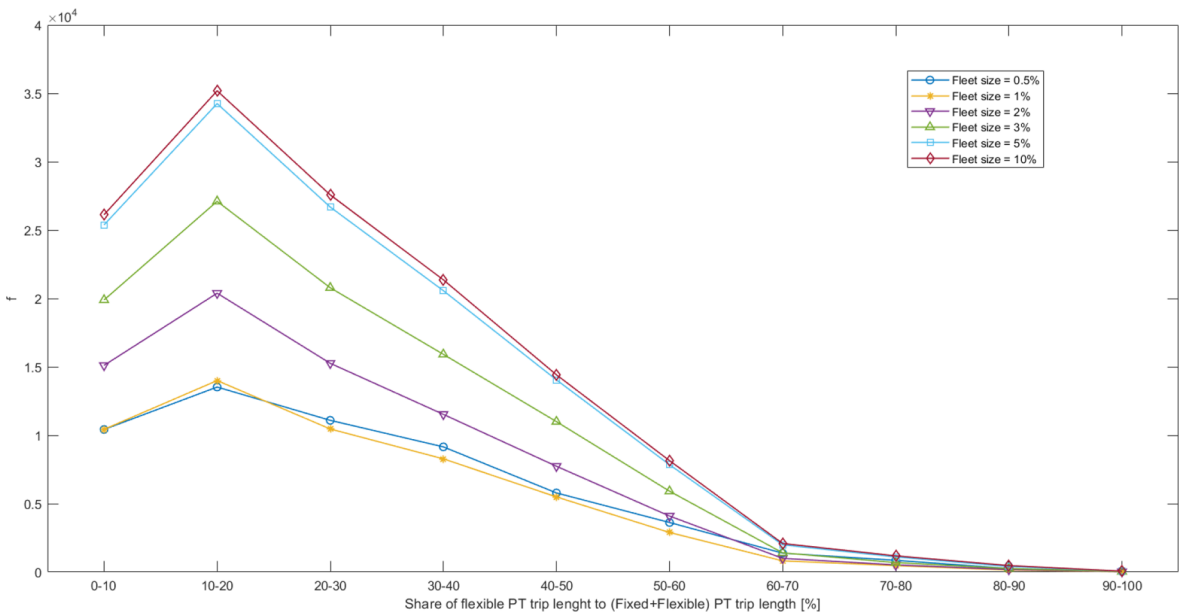

Fig. 11. Frequency distribution of fraction of Flexible PT trip length in trips combining Fixed and Flexible PT.
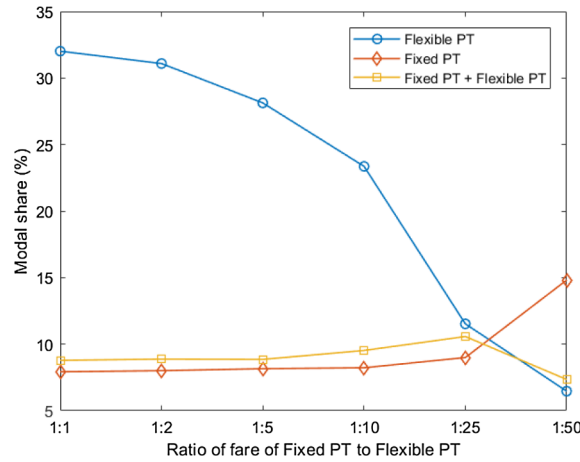
Fig. 12. Modal share of public transport alternatives under different fare settings in the scenario **Fixed PT + Flexible PT**.

PT, and Fixed PT under varying fare ratios. As can be seen from the figure, the modal share of Flexible PT decreases monotonically as the fare increases. The variation is sub-linear until a fare ratio of 1:10 after which the variation becomes super-linear. The share of Flexible PT + Flexible PT does not show considerable variation till a ratio of 1:5, after which the share increases monotonically up to a ratio of 1:25 and then decreases monotonically from 1:25 to 1:50. This implies that as the fare ratio increases, Fixed PT + Flexible PT becomes more attractive compared to Flexible PT, hence attracting passengers from Flexible PT. However, beyond a ratio of 1:25, using Flexible PT both as exclusive door-to-door travel and in the combination with Fixed PT becomes prohibitive for many users and hence the share of both Flexible PT and Fixed PT + Flexible PT decreases beyond the ratio of 1:25. The share of Fixed PT increases substantially beyond the ratio of 1:25, as it becomes more attractive compared to Flexible PT and Fixed PT + Flexible PT and hence attracts passengers from both these modes.

4.2. Service performance

Figs. 13 and 14 plot the cumulative relative frequency distribution for travel time components of waiting time and total travel time, respectively, for Flexible PT users in the **Fixed PT + Flexible PT** scenario. As can be seen from Fig. 14, an increase in fleet size of Flexible PT results in an overall increase in the fraction of short trips (travel time of up to 20 min). To further understand the effect of fleet size of Flexible PT on the travel time of its users, we look at the relative frequency distribution of average waiting time of its users. From Fig. 13 it becomes evident that the increase in fleet size leads to an overall increase in the fraction of trips with waiting time shorter than 10 min (shift from 40% for Fleet size = 0.1% to more than 90% for fleet size = 10%). With the increase in fleet
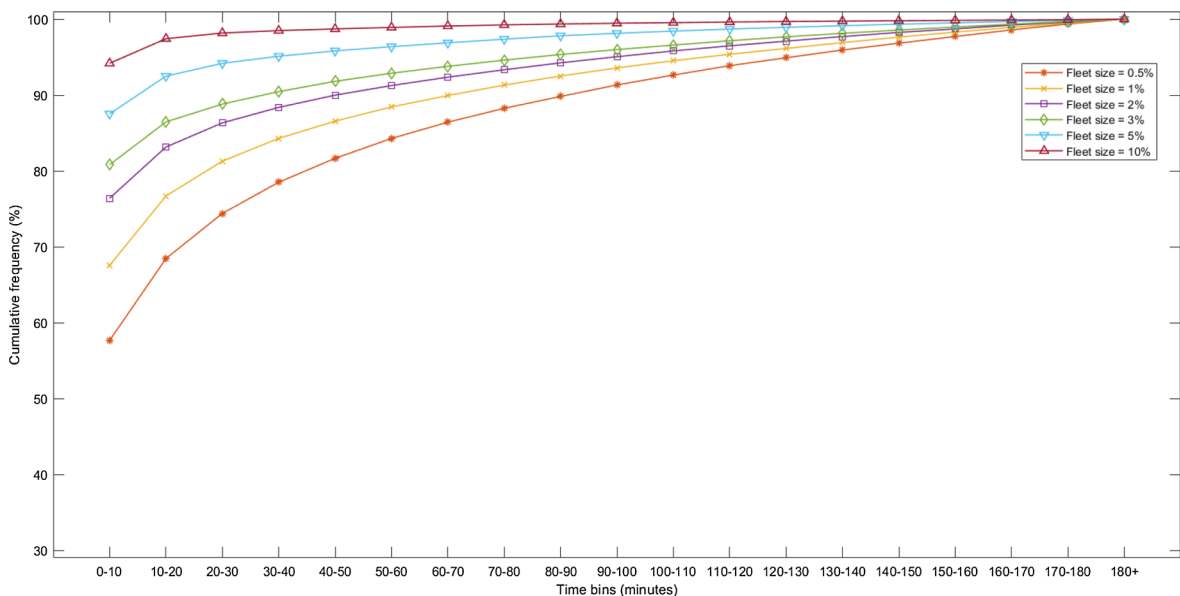

Fig. 13. Cumulative frequency distribution of waiting time for Flexible PT for scenario Fixed PT + Flexible PT.
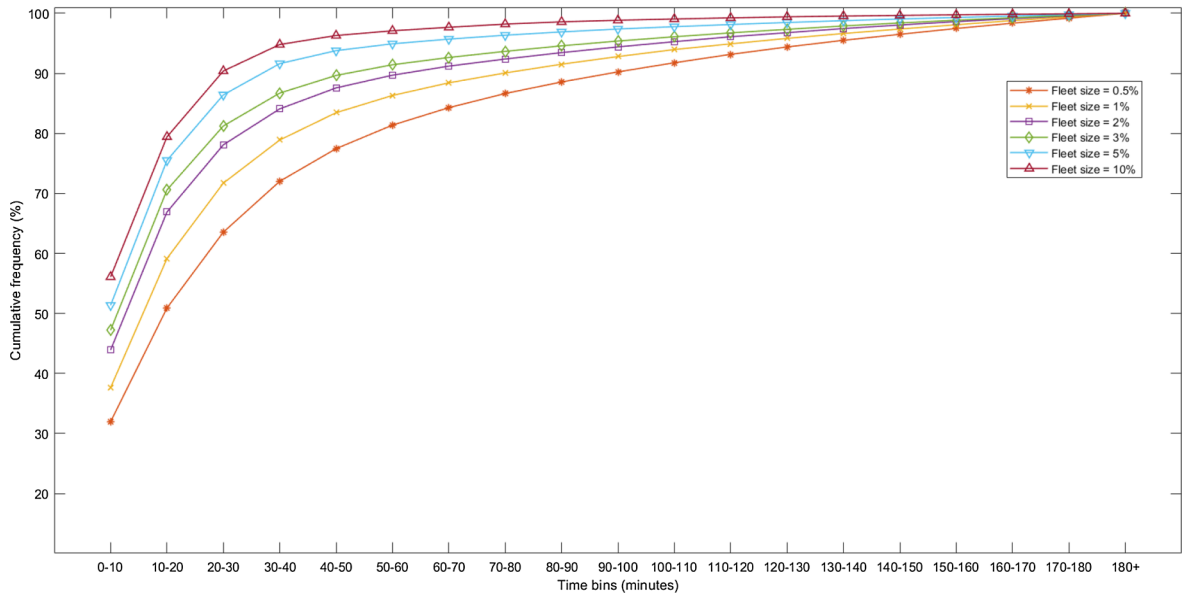
Fig. 14. Cumulative frequency distribution of total travel time for Flexible PT for scenario Fixed PT + Flexible PT.

size, the fraction of trips with travel time 0–10 min increases whereas the fraction of trips for all other in-vehicle time range (>10 min) decreases. This overall gain in the percentage of shorter trips can be explained from the overall reduction in the number of active mode trips (bike and walk) when shifting from the **Base Scenario** to scenario **Fixed PT** + **Flexible PT** (Table 5 and Fig. 9). These trips are attracted by Flexible PT as the fleet size increases and hence leads to an overall increase in the number of shorter trips.

4.3. Fleet utilisation

This section discusses the fleet utilisation of Flexible PT under the second and third scenarios. The performance indices considered in assessing the fleet utilisation are: *Idle ratio*, *Empty drive ratio*, and *Passenger drive ratio* defined as follows.

- *Idle ratio* is the fraction of time all the vehicles spend without being assigned a request, to the total time the vehicles are in service
- *Empty drive ratio* is defined as the fraction of time the vehicles spend driving in the network without a passenger on-board
- *Passenger drive ratio* is defined as the fraction of time all the vehicles spend driving in the network with a passenger on-board

We examine the breakdown of *Idle ratio*, *Empty drive ratio*, and *Passenger drive ratio* for the scenarios **Fixed PT or Flexible PT** and **Fixed PT** + **Flexible PT**, as displayed in Figs. 15 and 16 respectively. The *Passenger drive ratio* decreases and *Idle ratio* increases in both scenarios with increase in fleet size of Flexible PT. This indicates that the vehicles spend more time in the network without being assigned a request and spend less time transporting passengers as the fleet size increases. In particular, the fleet of Flexible PT remains largely underutilised when the fleet size is equivalent to 10% of the travel demand.

5. Conclusion

We developed an integrated multimodal route choice and assignment model that allows users to combine conventional line-based and on-demand passenger transport services so that their travel impedance is minimized. The model is implemented in an agent-based simulation framework which incorporates the day-to-day learning of users and was applied for the network based on the city of Amsterdam. Results are presented and discussed for scenarios where Fixed PT and Flexible PT are either mutually exclusive or facilitate their combination. Key performance indicators related to modal usage, service performance, fleet utilisation, and impact of fleet size and thus level of service on the number of passenger trips are discussed. Potential applications of the model include identifying locations for transfers between Fixed PT and Flexible PT to support interchange facility design and assessing the performance and level of service of Flexible PT services as first/last mile under various Fixed PT service attributes such as frequency.

Results indicate that Flexible PT is mainly used to cover <30% of the trip length, when the two modes of operations can be combined within a single passenger journey. Most of the users combining Fixed and Flexible PT services are otherwise using solely Fixed PT in the base case. Sensitivity analysis with respect to fleet size of Flexible PT indicate that no significant gains in level-of-service are made when increasing the fleet size beyond 5% of the travel demand. Fleet utilisation results indicates that the fleet of Flexible PT remains increasingly underutilised beyond a fleet size of 5% of the travel demand. This indicates a need for better relocation strategies for the fleet of Flexible PT.
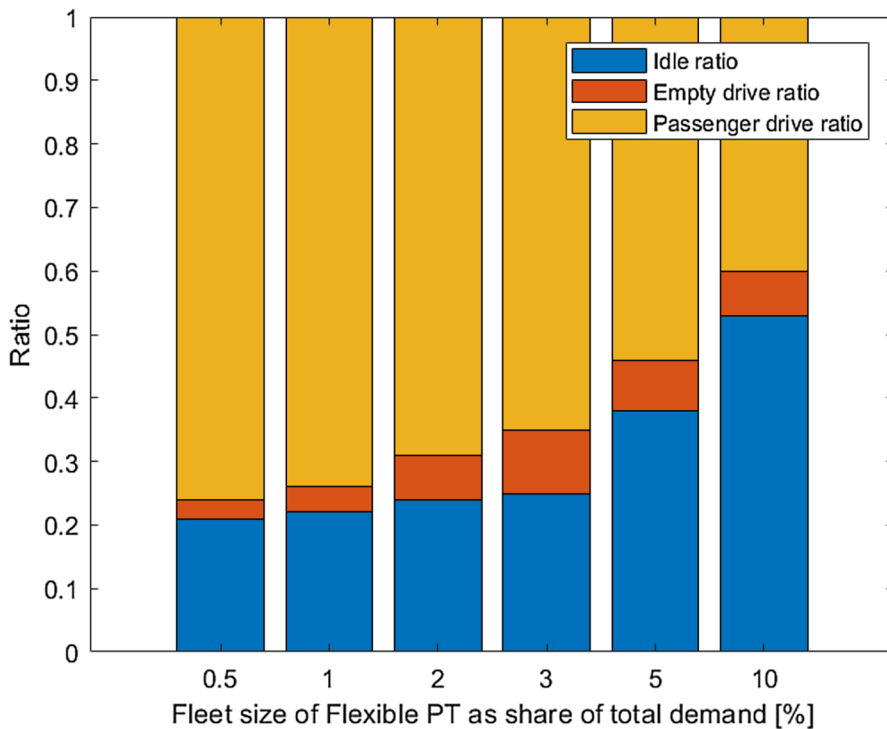
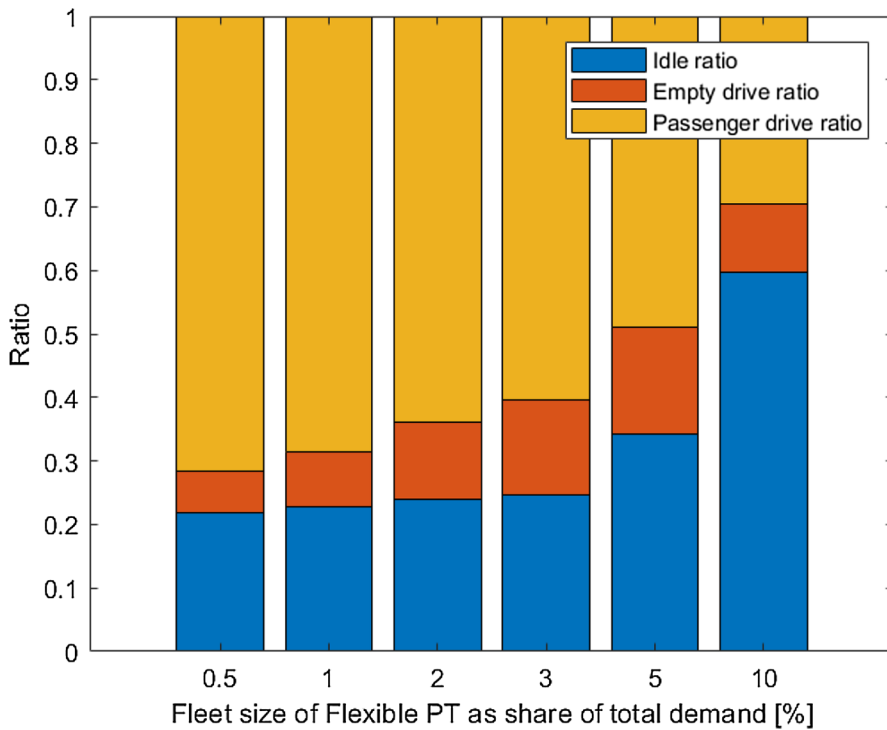**Fig. 15.** Flexible PT fleet utilisation for scenario **Fixed PT or Flexible PT**.

Fig. 16. Flexible PT fleet utilisation for scenario **Fixed PT + Flexible PT**.

The application of the model to the area centered around Amsterdam shows that the model is scalable for large-scale real-world applications. Hence the study provides a model allowing for the evaluation of fixed and flexible passenger services in contexts where on-demand services are expected to interact with conventional line-based services.

The limitations of the assignment model used in this study can be addressed in future research by employing a dynamic and stochastic assignment which accounts for correlations among public transport alternatives (e.g. in the form of path size logit). Furthermore, further research may include accommodating other modal combinations such as shared on-demand services, park and ride, kiss and ride, and car-sharing while considering supply adaptation in response to prevailing demand patterns. Future work may also enrich the model by incorporating the value of time and reliability in the behavioral modelling of passengers' preferences towards on-demand transport (Alonso-González et al., 2020).

CRediT authorship contribution statement

Jishnu Narayan: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Data curation, Writing - original draft, Visualization. **Oded Cats:** Conceptualization, Methodology, Writing - review & editing, Supervision, Project administration, Funding acquisition. **Niels van Oort:** Supervision, Writing - review & editing, Funding acquisition. **Serge Hoogendoorn:** Supervision, Writing - review & editing, Project administration, Funding acquisition.

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Appendix A. Supplementary material

Supplementary data associated with this article can be found, in the online version, at https://doi.org/10.1016/j.trc.2020. 102631.

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