Quantum Networks
Exploring Scalability, Topology, and Error Correction

Choi, Hyeongrak; Davis, Marc G.; Iñesta, Álvaro G.; Englund, Dirk

Publication date
2024

Document Version
Final published version

Published in
Proceedings of the Optical Fiber Communications Conference and Exhibition, OFC 2024

 OFC 2024 © Optica Publishing Group 2024

Quantum Networks: Exploring Scalability, Topology, and Error Correction

Hyeongrak Choi[1,*], Marc G. Davis[1], Álvaro G. Iñesta[2], Dirk Englund[1]

[1] *Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA*
[2] *QuTech, Delft University of Technology, Lorentzweg 1, 2628 CJ Delft, The Netherlands*
[*] *choihr@mit.edu*

Abstract: We introduce Quantum Tree Networks, a k-ary tree topology for scalable, error-corrected entanglement routing. Using sublinear qubit overhead and network-level simulations, we demonstrate efficient routing and congestion avoidance. © 2023 The Author(s)

1. Introduction

In quantum networks, entanglement is distributed to two dedicated end nodes by generating short-distance entanglement and performing entanglement swapping over the path connecting them [1]. Intermediate nodes in the network perform various tasks, including heralded entanglement generation [2], storing quantum states until a success signal is received, selecting entangled qubits, and recovering fidelities using purification or quantum error correction [3]. Quantum networks operated as above are close to the circuit-switched network, where the channel usage of one party blocks the use by others.

Current and near-future quantum networks are memory-limited and poses congestion challenges in circuit-switched networks, where the number of entanglement routing paths exceeds the available qubits in a router. To address this, we aim to develop scalable quantum networks that support simultaneous multi-flow entanglement routing. Scalable networks should meet three conditions: (1) enable error-corrected entanglement flow proportional to the number of users, (2) operate without congestion, and (3) require sublinear quantum memories per end node.

In this work, we propose a quantum tree network (QTN) that satisfies these conditions and overcomes the limitations of conventional mesh networks. The QTN architecture allows for optimal entanglement routing without the need for time-consuming multipath-finding algorithms [4].

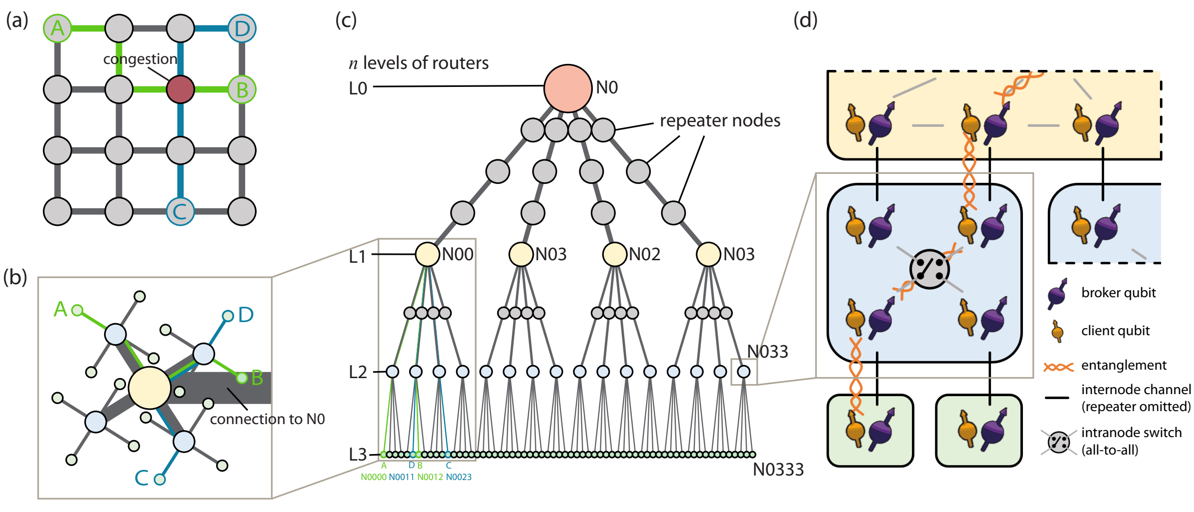

Fig. 1. (a) Quantum mesh network with congested nodes highlighted in red. (b) Architectural view of QTN with node size and edge width representing allocated qubits. (c) k-ary quantum tree network ($k = 4$) with labeled layers and repeater chains for inter-layer connections. (d) Quantum router architecture utilizing broker-client qubits for robust and scalable entanglement distribution. All-to-all intranode connectivity assumed.

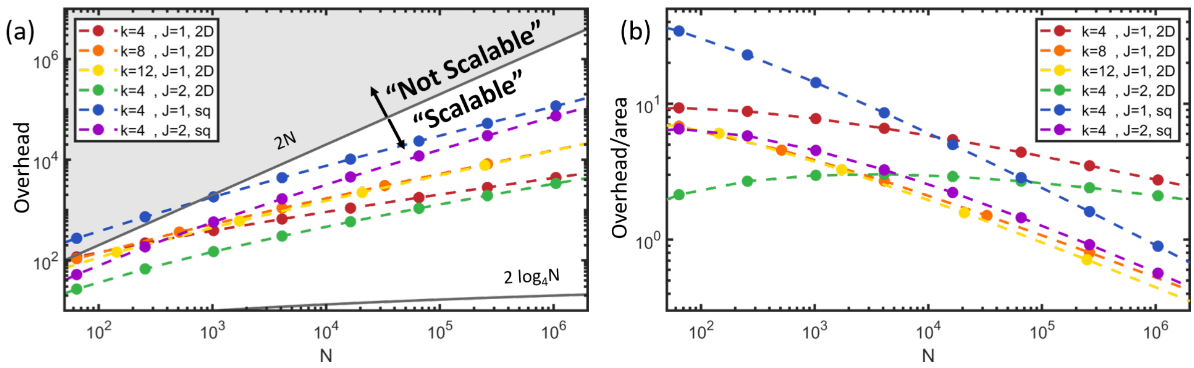

Fig. 2. (a) The resource overhead of quantum tree networks (QTNs) for user numbers. $J = 1$ QTNs with $k = 4, 8$, and 12 deployed on a 2D surface exhibit lower overheads (red, orange, and yellow markers), while $J = 2$ encoded QTN shows a faster scaling but smaller overheads (green markers). Deploying QTNs with square-lattice end nodes (blue and purple markers) leads to faster scaling and larger overhead. (b) The resource overheads are presented in terms of qubits per user per elementary link length squared. Larger k QTNs (orange and yellow) scale better with N compared to smaller k QTNs (red), and square lattice QTNs (blue and purple) scale better than surface-covering QTNs (red and green). Small k, surface-covering QTNs are more suitable for dense networks with aggregated users, while large k, square lattice QTNs are preferable for sparse networks.

2. Architecture and overhead

The QTN architecture is based on a hierarchical tree structure, where end nodes are the leaves of a tree (Fig. 1). The communication between end nodes is facilitated through routers located at internal nodes of the tree. This hierarchical arrangement allows for efficient and scalable entanglement routing in quantum networks. A key advantage of the QTN architecture is its scalability. The overhead, defined as the qubit-per-node ratio, scales sublinearly with the number of end nodes. Specifically, the overhead scaling for any k-ary tree is given by $\mathcal{O}\left(N^{\log_k a_k} \cdot \log_k N\right)$, where N represents the number of end nodes and a_k denotes the growth rate of channel length as we move from the leaves to the root node.

We examine the resource overhead of a deployed quantum tree network (QTN). The resource overheads for different QTN configurations are plotted in Figure 2. QTNs with larger k values and square lattice configurations exhibit better scaling with N. Surface-covering QTNs with smaller k values are more suitable for dense networks, while square lattice QTNs with larger k values are better for sparse networks. The exponent J in the analysis represents the encoding rate, which influences the resource scaling for error correction. The resource overhead of the error-corrected QTNs can be expressed as $\sim \mathcal{O}(N^{\log_k a_k} \cdot (\log_k N)^J)$. The specific value of J depends on the error correction code used. For example, for the linear-scaling CSS codes, $J = 1$, and for the 2D surface code, $J = 2$.

3. Outlook

While our quantum tree network architecture provides a promising avenue for scalable quantum networking, it is worth considering several directions for future research. One intriguing possibility is exploring alternative routing schemes such as Hamiltonian routing or teleportation-based methods [5,6]. There may be potential for further efficiency improvements by incorporating new architectural elements like quantum data centers or quantum random access memory systems [7,8]. These additions could expand the utility of the network from mere entanglement distribution to more complex quantum information processing tasks. Moreover, future work could benefit from the inclusion of percolation-assisted network operation, an approach that has demonstrated distance-independent rates [4, 9, 10]. Additionally, optimizing the QTN routing algorithms could offer further efficiency gains, and hybrid mesh-tree network structures might provide a balanced approach to scalability and resilience.

In terms of network security and resilience, the topology does face vulnerabilities to router-wise attacks or random failures. Future work might explore the use of q-redundant routers in a Clos network configuration for enhanced resilience. By incorporating redundancy, the network's reliability could be exponentially improved, thus making it a more robust platform for quantum communication.

Another prospective direction is the fault-tolerant operation of the network, particularly in the context of distributed quantum computing. Recent work suggests potential methods for achieving fault-tolerance with minimal resource overhead [11], which would be a critical milestone for realizing large-scale, reliable quantum networks.

Note: The preprint version of the full paper is available on *arXiv* [12].

References

1. M. Ruf, N. H. Wan, H. Choi, D. Englund, and R. Hanson, "Quantum networks based on color centers in diamond," J. Appl. Phys. **130**, 070901 (2021).
2. H. Bernien, B. Hensen, W. Pfaff, G. Koolstra, M. S. Blok, L. Robledo, T. H. Taminiau, M. Markham, D. J. Twitchen, L. Childress *et al.*, "Heralded entanglement between solid-state qubits separated by three metres," Nature **497**, 86–90 (2013).
3. L. Jiang, J. M. Taylor, K. Nemoto, W. J. Munro, R. Van Meter, and M. D. Lukin, "Quantum repeater with encoding," Phys. Rev. A **79**, 032325 (2009).
4. M. Pant, H. Krovi, D. Towsley, L. Tassiulas, L. Jiang, P. Basu, D. Englund, and S. Guha, "Routing entanglement in the quantum internet," npj Quantum Inf. **5**, 1–9 (2019).
5. A. Bapat, A. M. Childs, A. V. Gorshkov, and E. Schoute, "Advantages and limitations of quantum routing," PRX Quantum **4**, 010313 (2023).
6. D. Devulapalli, E. Schoute, A. Bapat, A. M. Childs, and A. V. Gorshkov, "Quantum routing with teleportation," arXiv preprint arXiv:2204.04185 (2022).
7. J. Liu, C. T. Hann, and L. Jiang, "Quantum data center: Theories and applications," arXiv preprint arXiv:2207.14336 (2022).
8. S. Xu, C. T. Hann, B. Foxman, S. M. Girvin, and Y. Ding, "Systems architecture for quantum random access memory," arXiv preprint arXiv:2306.03242 (2023).
9. H. Choi, M. Pant, S. Guha, and D. Englund, "Percolation-based architecture for cluster state creation using photon-mediated entanglement between atomic memories," npj Quantum Inf. **5**, 104 (2019).
10. A. Patil, M. Pant, D. Englund, D. Towsley, and S. Guha, "Entanglement generation in a quantum network at distance-independent rate," npj Quantum Inf. **8**, 1–9 (2022).
11. H. Choi, F. T. Chong, D. Englund, and Y. Ding, "Fault tolerant non-clifford state preparation for arbitrary rotations," arXiv preprint arXiv:2303.17380 (2023).
12. H. Choi, M. G. Davis, Á. G. Iñesta, and D. R. Englund, "Scalable quantum networks: Congestion-free hierarchical entanglement routing with error correction," arXiv preprint arXiv:2306.09216 (2023).