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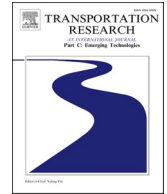
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A holding control strategy for diverging bus lines

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ABSTRACT

Holding has been extensively used as control strategy to regulate public transport operations, especially to maintain even headways and prevent buses of the same line to bunch up. Applying holding to multiple lines requires however to deal with the transition between corridor and branching segments. In this study, we introduce a holding criterion for network configurations with lines that operate jointly along a common corridor and then diverge to individual branches serving different urban areas. The proposed holding decision rule accounts for all different passenger groups in the overlapping segment and considers the transition to individual line operation. The holding rule is evaluated using simulation for different demand levels and compositions and is compared with state-of-the-art control schemes for a real-world network. Results show that the proposed multi-line control yields performance improvements along the shared transit corridor as well as at the line level. The performance of the control scheme is affected by the demand composition and we provide indications regarding the conditions under which multi-line control is advisable.

1. Introduction

Real-time control is essential for maintaining a high level of service in a transit network. Long and unpredictable travel times, bus bunching and delays are some of issues that occur daily due to the inherent variability of travel times and passenger demand. The effects of these phenomena can be mitigated by utilizing available Information and Communication Technology (ICT), which allows monitoring operations in real-time and acting dynamically to tackle potential disruptions.

Depending on the source of stochasticity, operators may focus on different parts of the network, applying different types of control (Ibarra-Rojas et al., 2015). Considering control at the stop level, a stop can be skipped or the dwell time at the bus stop can be extended beyond the required time for boarding and alighting operations. This latter strategy, known as holding, is popular among operators due to its implementation simplicity, and is an extensively researched topic.

While holding has been thoroughly investigated for single line control, research has largely overlooked the potential interactions between different lines. In modern urban networks, shared transit corridors, characterized by multiple overlapping lines, are designed to increase the joint frequency especially in busier segments as well as offer direct services with fewer transfers, serving high demand areas are common. Apart from applying network design principles, the performance of shared transit corridors can also be addressed at

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the tactical planning and timetable design phase (Ceder et al., 2001; Guihaire and Hao, 2010).

In scientific literature as well as in practice, holding has mostly focused on regulating the operation of a single line. Lately, research on the topic has been extended to real-time control of shared transit corridors (Argote-Cabanero et al., 2015; Fabian and Sánchez-Martínez, 2017; Hernández et al., 2015). These studies concluded that cooperation between lines can improve the overall performance of the network. However, the analysis area is still limited to the route segment where lines overlap. While the studies have focused on regulating the operation of the overlapping segment, control needs arguably to extend beyond the corridor boundaries in order to improve overall network-wide effects. In particular, local decisions may prove counter-productive if the downstream consequences for the line branches are neglected.

In previous work we have examined the case of lines merging to a shared transit corridor (Laskaris et al., 2019a,b). In the merging case, passengers can be satisfied by all lines. In the case of a network with diverging lines, there are two coexisting groups of passengers with conflicting interests within the shared transit corridor: (i) passengers who are indifferent between the two lines and are thus affected only by the joint corridor headway, and (ii) passengers who wait for a specific line and are thus affected only by the operations of the respective line.

In this study, we extend our network control formulation to networks with diverging lines. This induces an increased complexity due to the need for handling different passenger groups and their potentially conflicting interests. To our knowledge, this is the first work that explicitly accounts for the transition from joint to individual line operation and explores its effects on the network and the travel time it inflicts for each passenger group separately. The problem in this study is a deterministic optimization problem and treats the transit operation as deterministic. We compare the proposed network control to single line control and we analyze the performance under different demand compositions to determine the demand patterns for which coordinated control is superior to single line control. Results reveal that the performance of the coordinated control depends mostly on the number of passengers traveling within the corridor, suggesting to apply a specific control strategy depending on the estimated or predicted origin-destination passenger flows.

The remainder of the paper is structured as follows: in Section 2 related work is reviewed, followed by Section 3, where the methodology is presented. The experimental setup is described in Section 4 and the results are discussed in Section 5. Finally, in Section 6 conclusions are drawn.

2. Literature review

2.1. Single line holding control

According to the spatial classification introduced by Eberlein et al. (2001), holding strategies belong to the family of station strategies, together with stop skipping strategies. The main elements of holding control are the holding criterion and the stops where control should be applied (Cats et al., 2011). As far as the criterion is concerned, Zolfaghari et al. (2004) categorized the criteria based on the solution approach, differentiating between rule-based and optimization models. A final classification depends on the characteristics of the line; the criterion may focus on reducing headway variability or minimizing passenger cost (Ibarra-Rojas et al., 2015).

In early studies, Barnett (1974) introduced a model to minimize the waiting time of passengers at stops. Abkowitz and Lepofsky (1990) instructed vehicles to be held until a certain time threshold was reached. Hickman (2001) formulated an analytical holding model accounting for stochastic travel times. Fu and Yang (2002) compared headway regulation considering the succeeding vehicle only and considering both succeeding and preceding vehicles. They found the second strategy to be more effective and concluded that vehicles arriving early at control stops should be held for a time between 0.6 and 0.8 times the planned headway. Zhao et al. (2003) used an agent-based approach for vehicles and stops in order to minimize passenger travel times.

The objective function of the passenger cost was gradually expanded to include different components such as the in-vehicle delay or accounting for passengers that were denied from boarding either because of capacity constraints (Zolfaghari et al., 2004) or because of boarding limits (Delgado et al., 2009). Daganzo (2009) formulated dynamic holding control based on the forward headway (i.e. distance from the preceding bus) in order to maximize commercial speed. Daganzo and Pilachowski (2011) extended the work of Daganzo by including the backward headway (i.e. from the succeeding vehicle). Xuan et al. (2011) proposed a set of holding strategies that incorporate the headways to both the preceding and the following vehicle providing higher headway stability and better schedule adherence. Cats et al. (2011) combined the headways to both the succeeding and the preceding vehicle with a term that limits the maximum allowed headway. In a simulation-based comparison, the strategy proved superior to other holding strategies. Bartholdi III and Eisenstein (2012) did not follow a predefined headway but let headways be self-coordinated to eliminate bunching in the event of large disturbances. Berrebi et al. (2015) minimized the sum of squared headways to determine the holding time and managed to reduce the passenger waiting time. Berrebi et al. (2018) compared different holding methods from literature and practice, assessing their performance based on the headway instability and the mean holding time. They concluded that the prediction based methods have the best trade-off between holding time and headway regularity. Sánchez-Martínez et al. (2016) included dynamic changes in running times and demand in their holding optimization model.

Wu et al. (2017) proposed an ad hoc bus propagation model accounting for overtaking and the dynamic passenger queue swapping when buses form platoons. Their model was combined with different holding strategies (schedule and headway based) to minimize headway variability concluding that overtaking can help when travel variability is high, a finding in line with the work of Schmöcker et al. (2016). Asgharzadeh and Shafahi (2017) extended the mathematical model of Zolfaghari et al (2004) accounting for the waiting time of the passengers on board, with the objective of minimizing the total passenger waiting time. Zhang and Lo (2018) introduced a two way looking self-equalizing holding method applied for both deterministic and stochastic running times. Their control method

resulted in convergence to a common headway at applications with deterministic running times and to reduced headway variability at applications with stochastic running times. As all holding strategies also increase the travel time of the trips, [Gkiotsalitis and Cats \(2019\)](#) introduced a time window based approach with which the holding times of all buses are calculated within given predefined time windows and optimized based on the minimization of passenger in-vehicle time and schedule-related constraints. Recently, [Wang and Sun \(2020\)](#) introduced a multi-agent deep reinforcement framework, with which vehicle are treated as agents and learn global holding strategies to restore headway regularity with minor interventions.

Holding has also been combined with other control strategies to reinforce its performance. For instance, [Sáez et al. \(2012\)](#) combined holding with stop skipping, while [Chandrasekar et al., \(2002\)](#) and [Koehler et al. \(2018\)](#) combined holding with transit signal priority to minimize the total delay of passengers on board and at stops. [Nesheli and Ceder \(2017\)](#) tested combinations of holding, stop skipping, boarding limits and speed adjustment in order to minimize total passenger travel time and increase the number of direct transfers. [Sirmatel and Geroliminis \(2018\)](#) introduced a hybrid model predictive control to regularize headways and improve speed of transit vehicles. Finally, in the works of ([Laskaris et al., 2019b, 2020a, 2020b](#)) the synergy between holding for regularity and Cooperative ITS-based driver advisory systems to mitigate the number of stop at traffic lights has been studied and proven to be feasible and effective for both objectives. In addition, the hybrid C-ITS based control strategy was shown to reduce Transit Signal Priority requests.

2.2. Multiline holding control

Extending beyond a single line, the first category of holding rules that take into account vehicles originating from lines other than the controlled one is to regulate transfers at a single common stop ([Abkowitz et al., 1987](#)). [Dessouky et al. \(2003\)](#) introduced transfer time as a component of the total time subject to the minimization of which holding is calculated. [Hadas and Ceder \(2010\)](#) applied holding in order to maximize the number of direct transfers. [Gavriilidou and Cats \(2018\)](#) formulated a controller that optimally trades off single line regularity and multiline synchronization based on the minimization of the generalized travel cost while considering different passengers information (passengers on board, crowding, capacity limitations).

Cooperation between lines on a shared transit corridor has been shown to be beneficial for the operators, since it increases both their revenue and service rate ([Chen et al., 2010](#)). [Hernández et al., \(2015\)](#) applied holding control comparing different operations schemes. [Argote-Cabanero et al. \(2015\)](#) extended the work of [Xuan et al. \(2011\)](#) from single line holding control to multiline control. [Fabian and Sánchez-Martínez \(2017\)](#) compared schedule-based holding and headway-based holding strategies for a multi-branch light

Table 1
Notation.

<i>Sets</i>	
I	set of lines
J_i	set of stops served by line i
K_i	set of trips of line i
<i>Stop sets</i>	
C	Set of all stops of the shared transit corridor
B_i	Set of all branch stops served by line i
<i>Time related variables</i>	
$t_{ijk}^{arrival}$	arrival time at stop j of trip k of line i in [time units]
t_{ijk}^{dwell}	dwell time at stop j of trip k of line i in [time units]
t_{ijk}^{exit}	exit (departure) time at stop j of trip k of line i in [time units]
$t_{j-1,j}^{riding}$	scheduled riding time between stops $j-1$ and j in [time units]
$t_{j-1,j}^{riding}$	actual riding time between stops $j-1$ and j in [time units]
t_{ijk}^{hold}	holding time at stop j of trip k of line i in [time units]
$h_{j,k,k-1}$	actual headway at stop j between trips k and $k-1$ of line i in [time units]
\hat{h}_i	planned headway of line i in [time units]
\hat{h}^{join}	planned joint headway in [time units]
t_{ijk}^{wait}	waiting time at stop j of trip k of line i in [time units]
t_{ijk}^{inveh}	in vehicle time at stop j of trip k of line i in [time units]
t_{ijk}^{travel}	travel time at stop j of trip k of line i in [time units]
<i>Passenger related labels and variables</i>	
o	origin stop
d	destination stop
$\lambda_{o,d}$	arrival rate between origin o and destination d in [passengers per hour]
q_{ijk}	passengers on board after completion of dwell time on trip k of line i at stop j in [passengers]

rail network, finding that holding based on the headway of the shared corridor outperforms schedule-based control. Recently, [Seman et al., \(2019\)](#) proposed an optimization-based multiline holding control strategy to regulate the headways at a shared transit corridor aiming at minimizing the waiting time of passengers at common stops, including passengers seeking a specific line and passengers that failed to board due to capacity constraints.

2.3. Synthesis

Apart from few exceptions, holding strategy has hitherto mainly focused on single line operations. The interaction of multiple bus lines on shared transit corridors and the control of their operation has been a topic of recent research. Some of the existing single line criteria and the proposed solutions have been extended for multiline networks such as the works of [Argote-Cabanero et al, \(2015\)](#) and [Seman et al. \(2019\)](#), which are follow-up studies of [Xuan et al. \(2011\)](#) and [Koehler et al. \(2018\)](#), respectively. The main focus is on regulating the shared transit corridor and utilize the joint operation and the increased supply offered on these route segments. However, the interaction between the corridor and the remaining parts of the networks where lines continue operating individually has not been addressed. Moreover, the effect of the control strategies on the different passenger groups within and outside of the shared transit corridor has not yet been investigated.

The contribution of this work is therefore twofold: First, it formulates a holding criterion strategy that can be applied to any configuration of branch and trunk networks accounting for the dynamics between single and multiline operation, as well as all different passenger groups interacting in this network type. Second, it provides insight into the passenger segmentations under which multiline control is beneficial.

3. Methodology

3.1. Notation

The notation employed in this paper is given in [Table 1](#):

3.2. Network configuration

We consider a network consisting of a shared corridor with a number of consecutive stops common to different lines until a diverging stop. All common stops belong to the set $C = \{c_1, c_2, \dots, c_n, \dots, c^{split}\}$. After the last common stop c^{split} the lines split and serve different sets of stops. All stops served solely by line i belong to the branch stop set $B_i = \{b_{i1}, b_{i2}, \dots, b_{in}\}$. All stops served by line i constitute a set J_i , which is the union of the shared transit corridor stop set C and the branch stop set B_i , served by line i ($J_i = C \cup B_i$). An illustrative representation of this network type is sketched in [Fig. 1](#).

3.3. Assumptions

The following assumptions are taken into consideration:

- **Passengers do not perform transfers in this network configuration:** As explained in the previous section, the first leg of the route is common for both lines. At this set of stops, passengers travel either until or beyond the diverging stop. We assume that both passenger groups in the corridor, one travelling within and another from corridor to branches, choose the vehicle that arrives first to the stop and minimizes their travel time ([Chriqui and Robillard, 1975](#); [Marguier and Ceder, 1984](#)). Following a rational behavior, the second passenger group chooses the first bus from the line that travels to their destination refraining from unnecessarily splitting their trip into two segments, one within the corridor until the diverging stop and another beyond that.
- **Historical data for the demand of the lines are available:** Demand for each of the different network parts affects the control decision. Therefore, we assume that historical data for the demand profiles (e.g. boarding and alighting passengers at each stop) of

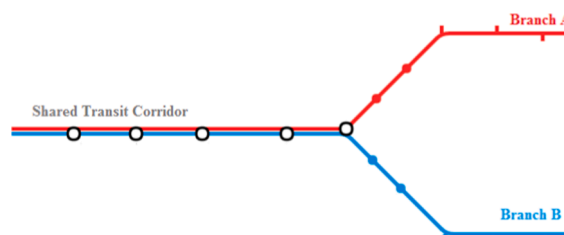


Fig. 1. Schematic network configuration.

the lines are retrievable, so that the expected passengers waiting at stops and on board can be estimated/predicted. The need of historical data can be partially lifted by introducing an accurate prediction model relying both on passenger counting and historical averages or fully lifted in systems, where the actual number of passengers waiting at stops and are on board can be continuously monitored.

- **Medium to high frequency lines:** We assume that the lines have headways of up to 12 min, hence they can be considered high frequency lines. Consequently, passengers are assumed to arrive randomly at the stops and not accounting for the specific vehicles dispatching timetable.
- **Capacity constraints are not binding:** We assume demand to be below vehicle capacities. Denied boarding is therefore not considered, and consequently we do not consider in this study crowding related performance indicators.
- **The joint headway has been set at the tactical planning phase:** we consider that line schedules have taken into account their joint operation at the terminal. Vehicles are dispatched in a fashion that provides a high joint frequency at the corridor while the headway of each line is respected.
- **AVL data are available in real time:** The locations of the vehicles are available in real time to monitor the progression of the actual headway between vehicles and to calculate their expected headways and the holding time needed when applying the control logic.
- **We consider the relationship between the boarding passengers and the passengers on board with holding time to be linear.** Passenger activity is directly affected by the travel time variation and any control action. The number of passengers waiting at stops and on board varies based on the departure times and the control actions taken prior to the current stop. In the current study a myopic approach is followed, examining the effect of holding time on the passengers at the current stop.

3.4. Holding criterion

3.4.1. Passenger groups

In a diverging fork there are three different passenger groups: i) Passengers travelling within the shared transit corridor (c), ii) passengers travelling within the branch (b) and iii) from the corridor to the branch (cb). At their origin stop, the latter group is affected by the vehicle sequence of the line that satisfies their final destination rather than the joint headway of the corridor as shown in Fig. 2. For this reason, the holding time has to account for the regularity of the joint headway as well as each single line regularity.

The total demand along line i with N stops expressed in arrival rates is given by Eq. (1):

$$\sum_{m=1}^N \sum_{n=m+1}^N \lambda_{m,n} = \sum_{m=c_1}^{c_n} \sum_{n=m+1 \in C}^{c_n} \lambda_{m,n}^c + \sum_{m=c_1}^{c^{split}} \sum_{n=m+1 \in C \cup B_i}^{b_{in}} \lambda_{m,n}^{cb} + \sum_{m=c^{split}}^{b_{in}} \sum_{n=m+1 \in B_i}^{b_{in}} \lambda_{m,n}^b \tag{1}$$

The arrival rates of each group with origin m and destination n are denoted respectively as $\lambda_{m,n}^c$, $\lambda_{m,n}^{cb}$ and $\lambda_{m,n}^b$. For the sake of simplicity, let:

$$\begin{aligned} \sum_{m=1}^N \sum_{n=m+1}^N \lambda_{m,n} &= \Lambda \\ \sum_{m=c_1}^{c_n} \sum_{n=m+1 \in C}^{c_n} \lambda_{m,n}^c &= \Lambda^c \\ \sum_{m=c_1}^{c^{split}} \sum_{n=m+1 \in C \cup B_i}^{b_{in}} \lambda_{m,n}^{cb} &= \Lambda^{cb} \\ \sum_{m=c^{split}}^{b_{in}} \sum_{n=m+1 \in B_i}^{b_{in}} \lambda_{m,n}^b &= \Lambda^b \end{aligned} \tag{2}$$

where Λ^c is the sum of the arrival rates generated from stop $c \in C$ of the corridor and having a destination within the corridor, Λ^{cb} is the

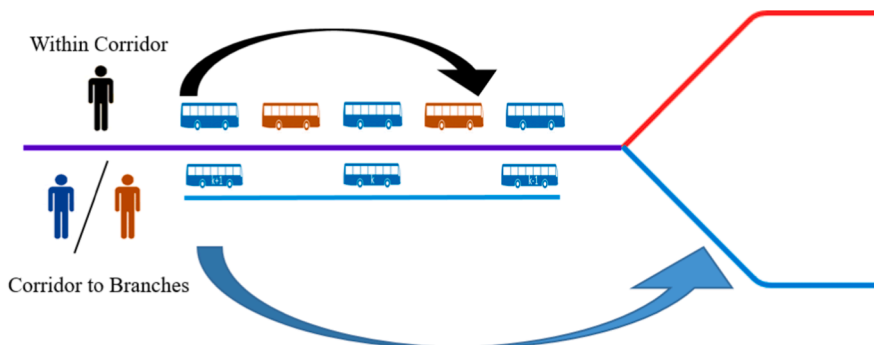


Fig. 2. Waiting time of passengers in the shared transit corridor depends on their final destination.

sum of arrival rates with origin $c \in C$ and destination stop $b_i \in B_i$ within the branch of line i , and Λ^b is the sum of arrival rates with origin and destination stops that both belong to the branch stop set of line i ($b_i \in B_i$). Eq. (1) can be now written in a more compact form as:

$$\Lambda = \Lambda^c + \Lambda^{cb} + \Lambda^b \quad (3)$$

3.4.2. Travel time

The holding time is derived from the minimization of a weighted cost function consisting of waiting time at stops t^{wait} and in-vehicle delay t^{inveh} given by Equation (4). Weight terms β^{wait} and β^{inveh} are introduced for waiting time and in-vehicle delay, respectively, to reflect the impedance as perceived by the passengers:

$$t_{ijk}^{travel} = \beta^{wait} t_{ijk}^{wait} + \beta^{inveh} t_{ijk}^{inveh} \quad (4)$$

The objective of the holding criterion is the minimization of the weighted travel cost function and its components for all passenger groups. The decision variable is holding time (for a vehicle dwelling at a certain stop). Each of the following sections is dedicated to each of the travel time function components and the derivation of the holding criterion.

3.4.3. Waiting time terms

Following the assumptions of high frequency lines and random passenger arrivals at stops, the number of passengers waiting at a given stop is estimated through the sum of the arrival rates generated at the stop multiplied by the actual headway, and the total passenger waiting time is assumed to be half of the actual headway multiplied by the sum of the arrival rates at the stop. When a control action is triggered, the passenger waiting time differs from what the same passengers would have experienced in the corresponding uncontrolled case. We calculate the passenger waiting time due to holding as the difference between waiting time when holding is applied $t^{wait,H}$ and when isn't $t^{wait,0}$:

$$t^{wait} = t^{wait,H} - t^{wait,0} \quad (5)$$

Each waiting time term consists of the waiting time from the preceding vehicle p and the succeeding vehicle s :

$$\begin{aligned} t^{wait,x} &= t^{wait,px} + t^{wait,sx} \\ x &= \{0, H\} \end{aligned} \quad (6)$$

The waiting time of trip k of line i at stop j from the succeeding and the preceding vehicles when no holding is applied are shown in the following formulas:

$$t_{ijk}^{wait,p0} = \frac{(t_{ijk}^{exit} - t_{ijk-1}^{exit})^2}{2} \Lambda_j \quad (7)$$

$$t_{ijk}^{wait,s0} = \frac{(t_{ijk+1}^{exit} - t_{ijk}^{exit})^2}{2} \Lambda_j \quad (8)$$

where $k-1$ is the preceding trip and $k+1$ is the succeeding trip. When a vehicle is instructed to hold the waiting time from the preceding and the succeeding vehicles are expressed by Eqs. (9) and (10):

$$t_{ijk}^{wait,pH} = \frac{\left((t_{ijk}^{exit} + t_{ijk}^{hold}) - t_{ijk-1}^{exit} \right)^2}{2} \Lambda_j \quad (9)$$

$$t_{ijk}^{wait,sH} = \frac{\left(t_{ijk+1}^{exit} - (t_{ijk}^{exit} + t_{ijk}^{hold}) \right)^2}{2} \Lambda_j \quad (10)$$

After substituting the corresponding waiting times in Eq. (6), then waiting time can be expressed as a function of holding time:

$$t_{ijk}^{wait}(t_{ijk}^{hold}) = \Lambda_j (t_{ijk}^{hold})^2 + \left\{ \Lambda_j \left[(t_{ijk}^{exit} - t_{ijk-1}^{exit}) - (t_{ijk+1}^{exit} - t_{ijk}^{exit}) \right] \right\} t_{ijk}^{hold} \quad (11)$$

In this network configuration, two passenger groups coexist on the shared transit corridor and have different objectives: passengers travelling within the corridor can board on any line while passengers travelling to the branches wait for a vehicle from the line that serves their final destination. Thus, the first group depends on the regularization of the joint headway on the corridor while the second on the regularization of the headway of the desired line. We therefore consider two different waiting time terms at each stop of the

shared transit corridor c . The first term takes into account all vehicles that serve the current stop regardless of the line, and the second considers only the vehicles of the same line as the one served by the current vehicle. The first term $t_{ck}^{wait, joint}$ calculates the passenger waiting time for the passenger group travelling within the shared transit corridor, while the second term $t_{ick}^{wait, line}$ represents the same quantity, however subject to vehicles pertaining to the same line i as that of the current vehicle k . The two terms are given in Eqs. (12) and (13) respectively:

$$t_{ck}^{wait, joint}(t_{ick}^{hold}) = \Lambda_c^c (t_{ick}^{hold})^2 + \left\{ \Lambda_c^c [(t_{ck}^{exit} - t_{ck-1}^{exit}) - (t_{ck+1}^{exit} - t_{ck}^{exit})] \right\} t_{ick}^{hold} \quad (12)$$

$$t_{ick}^{wait, line}(t_{ick}^{hold}) = \Lambda_c^{cb} (t_{ick}^{hold})^2 + \left\{ \Lambda_c^{cb} [(t_{ick}^{exit} - t_{ick-1}^{exit}) - (t_{ick+1}^{exit} - t_{ick}^{exit})] \right\} t_{ick}^{hold} \quad (13)$$

where t_{ck}^{exit} is the predicted exit (departure) time of any trip k regardless of the line from a shared transit corridor stop c .

3.4.4. Projection to the final common stop

Passengers beyond the shared transit corridor are affected only by the single line performance independently from the regularization of headways in the common part. In order to account for the transition from the joint operation to the single operation, along the corridor we ensure that vehicles will initiate their operation with the least headway variability. Hence, to ensure a smooth transition we regulate the expected headway with which the vehicles will enter to the branches. We compute the predicted departure from the last common stop, where the transition to the individual operation is made. The arrival time of all vehicles of the same line is projected to the final common stop. Projection to the last common stop is obtained by summing the scheduled riding times between the last recorded stop s that a vehicle has visited with the departure time from this stop, as formulated in Eq. (15). After vehicle trajectories are projected from their respective last visited stop s to the last common stop c^{split} , and the headways of the preceding and the succeeding vehicles with respect to the current one are determined, the predicted departure time from the last common stop is regulated based on the expected waiting times at the last common stop. The passengers that are affected by this term are those travelling on the branch, expressed by Λ^b . The expected waiting time at the last common stop expressed as a function of holding time is given by Eq. (14):

$$\tilde{t}_{i,c^{split},k}^{wait}(t_{ick}^{hold}) = \Lambda_{c^{split}}^b (t_{ick}^{hold})^2 + \left\{ \Lambda_{c^{split}}^b \left[\left(\tilde{t}_{i,c^{split},k}^{exit} - \tilde{t}_{i,c^{split},k-1}^{exit} \right) - \left(\tilde{t}_{i,c^{split},k+1}^{exit} - \tilde{t}_{i,c^{split},k}^{exit} \right) \right] \right\} t_{ick}^{hold} \quad (14)$$

$$\tilde{t}_{i,c^{split},k}^{exit} = t_{isk}^{arrival} + t_{isk}^{dwell} + \sum_{l=m}^{c^{split}} \tau_{l,l+1}^{riding} \quad (15)$$

3.4.5. In-vehicle time

In vehicle time due to holding is the product of holding time t_{ijk}^{hold} of trip k of line i at a stop j and the passengers on board q_{ijk} of the trip k of line i at a stop j :

$$t_{ijk}^{inveh} = q_{ijk} t_{ijk}^{hold} \quad (16)$$

3.4.6. Total travel time

The total travel time t^{travel} consists of the waiting time for the different passenger groups and the in-vehicle time (Eq. (16)). The waiting time is composed of the waiting time experienced by passengers travelling within the corridor (given by Eq. (12)), from corridor to branch (given by Eq. (13)) and the expected waiting time for passengers travelling within the branch (given by Eq. (14)). By plugging in all the different terms into Eq. (5), we obtain the travel time at a corridor stop c expressed as a function of holding time t^{hold} :

$$\begin{aligned} t_{ick}^{travel}(t_{ick}^{hold}) &= \beta^{wait} t_{ick}^{wait}(t_{ick}^{hold}) + \beta^{inveh} t_{ick}^{inveh}(t_{ick}^{hold}) = \\ &= \beta^{wait} \Lambda_c (t_{ick}^{hold})^2 + \\ & t_{ick}^{hold} \beta^{wait} \left\{ \Lambda_c^c [(t_{ick}^{exit} - t_{ck-1}^{exit}) - (t_{ck+1}^{exit} - t_{ck}^{exit})] + \Lambda_c^{cb} [(t_{ick}^{exit} - t_{ck-1}^{exit}) - (t_{ck+1}^{exit} - t_{ck}^{exit})] \right\} + \\ & \Lambda_{c^{split}}^b \left[\left(\tilde{t}_{i,c^{split},k}^{exit} - \tilde{t}_{i,c^{split},k-1}^{exit} \right) - \left(\tilde{t}_{i,c^{split},k+1}^{exit} - \tilde{t}_{i,c^{split},k}^{exit} \right) \right] \right\} + \beta^{inveh} q_{ick} t_{ick}^{hold} \end{aligned} \quad (17)$$

The optimal holding time is then calculated by taking the first derivative subject to holding time and setting it equal to zero, and solving the resulting equation with respect to holding time t_{ick}^{hold} with the constraint that $t_{ick}^{hold} \geq 0$:

$$\begin{aligned}
\frac{\partial t_{ick}^{travel}(t_{ick}^{hold})}{\partial t_{ick}^{hold}} &= 0 \rightarrow \\
\rightarrow \frac{\partial [\beta^{wait} t_{ick}^{wait}(t_{ick}^{hold}) + \beta^{inveh} t_{ick}^{inveh}(t_{ick}^{hold})]}{\partial t_{ijk}^{hold}} &= 0 \rightarrow \\
\rightarrow 2\beta^{wait} \Lambda_c^{hold} + \beta^{wait} \{ \Lambda_c^c [(t_{ick}^{exit} - t_{ck-1}^{exit}) - (t_{ck+1}^{exit} - t_{ick}^{exit})] + \Lambda_c^{cb} [(t_{ick}^{exit} - t_{ck-1}^{exit}) - (t_{ck+1}^{exit} - t_{ick}^{exit})] + \\
\Lambda_{jplit}^b [(\tilde{t}_{i,jplit,k}^{exit} - \tilde{t}_{i,jplit,k-1}^{exit}) - (\tilde{t}_{i,jplit,k+1}^{exit} - \tilde{t}_{i,jplit,k}^{exit})] \} + \beta^{inveh} q_{ick} &= 0 \rightarrow \\
\rightarrow 2\beta^{wait} \Lambda_c^{hold} = -\beta^{wait} \{ \Lambda_c^c [(t_{ick}^{exit} - t_{ck-1}^{exit}) - (t_{ck+1}^{exit} - t_{ick}^{exit})] + \Lambda_c^{cb} [(t_{ick}^{exit} - t_{ck-1}^{exit}) - (t_{ck+1}^{exit} - t_{ick}^{exit})] + \\
\Lambda_{csplit}^b [(\tilde{t}_{i,csplit,k}^{exit} - \tilde{t}_{i,csplit,k-1}^{exit}) - (\tilde{t}_{i,csplit,k+1}^{exit} - \tilde{t}_{i,csplit,k}^{exit})] \} - \beta^{inveh} q_{ick} &\rightarrow \\
\rightarrow t_{ick}^{hold} = \frac{\beta^{wait} \Lambda_c^c [(t_{ick}^{exit} - t_{ck-1}^{exit}) - (t_{ck+1}^{exit} - t_{ick}^{exit})] - \beta^{wait} \Lambda_c^{cb} [(t_{ick}^{exit} - t_{ck-1}^{exit}) - (t_{ck+1}^{exit} - t_{ick}^{exit})]}{2\beta^{wait} \Lambda_c} &+ \\
\frac{\Lambda_{jplit}^b [(\tilde{t}_{i,jplit,k}^{exit} - \tilde{t}_{i,jplit,k-1}^{exit}) - (\tilde{t}_{i,jplit,k+1}^{exit} - \tilde{t}_{i,jplit,k}^{exit})]}{2\beta^{wait} \Lambda_c} - \frac{\beta^{inveh} q_{ick}}{2\beta^{wait} \Lambda_c} &\rightarrow \\
t_{ick}^{hold} = \frac{\Lambda_c^c [(t_{ck+1}^{exit} - t_{ick}^{exit}) - (t_{ick}^{exit} - t_{ck-1}^{exit})]}{\Lambda_c} + \frac{\Lambda_c^{cb} [(t_{ick+1}^{exit} - t_{ick}^{exit}) - (t_{ick}^{exit} - t_{ick-1}^{exit})]}{\Lambda_c} &+ \\
+ \frac{\Lambda_{jplit}^b [(\tilde{t}_{i,csplit,k+1}^{exit} - \tilde{t}_{i,csplit,k}^{exit}) - (\tilde{t}_{i,csplit,k}^{exit} - \tilde{t}_{i,csplit,k-1}^{exit})]}{\Lambda_c} - \frac{\beta^{inveh} q_{ick}}{2\beta^{wait} \Lambda_c} &
\end{aligned} \tag{18}$$

The first two terms regulate the departure from the current corridor stop c : the first one considers all vehicles in the shared transit corridor, regardless of the line they serve, while the second considers vehicles of the same line i as the current vehicle. The third term regulates the predicted departures at line level from the diverging stop, to ensure that the lines will continue to their branch stops with low headway variability. For the third term, the predicted departure time from the diverging stop c^{split} is estimated by summing the scheduled riding times between the current stop of each vehicle and the diverging stop. Finally, the holding time calculated is adjusted based on the ratio between the number of passengers on board and the number of remaining passengers downstream expressed through the corresponding arrival rates.

3.4.7. Weights

As shown in Eq. (18), we consider the contribution of each term to be weighted based on the respective passenger volume. We also introduce a weighting factor based on the position of the current stop subject to the last common stop, c^{split} , to ensure a smoother transition from joint operation to single line operation. The second weighting factor is estimated in terms of stops between the current and the last common stop. The first two weights regulate the headways of vehicles within the corridor and therefore share the same position weight multiplied by a parameter $\alpha = 0.5$ to ensure that the two terms are equally important when calculating the holding time. Parameter α reflects the magnitude of multilane control subject to the length of the shared transit corridor. Depending on number and the characteristics of the line and the importance of the shared transit corridor in the network, the regulation of joint operation might be heavily prioritized over single line operation, giving α a higher value. In contrast, a line that is not interacting substantially with the remaining lines of a shared corridor will focus on regulating the single line operation with a lower value for parameter α . The calibration of parameter α for different multilane networks will be investigated in future work.

$$\begin{aligned}
\theta_1 &= \frac{\Lambda_c^c}{\Lambda_c} + (\alpha) \left(1 - \frac{1}{c^{split} - c} \right) \\
\theta_2 &= \frac{\Lambda_c^{cb}}{\Lambda_c} + (1 - \alpha) \left(1 - \frac{1}{c^{split} - c} \right) \\
\theta_3 &= \frac{\Lambda_{csplit}^b}{\Lambda_c} + \left(\frac{1}{c^{split} - c} \right)
\end{aligned} \tag{19}$$

The final holding criterion for every stop on the shared corridor is given in Eq. (20):

$$t_{ick}^{hold} = \max \left\{ \theta_1 \frac{[(f_{ck+1}^{exit} - f_{ick}^{exit}) - (f_{ick}^{exit} - f_{ck-1}^{exit})]}{2} + \theta_2 \frac{[(f_{ick+1}^{exit} - f_{ick}^{exit}) - (f_{ick}^{exit} - f_{ick-1}^{exit})]}{2} + \theta_3 \frac{[(\tilde{f}_{i,c^{split},k}^{exit} - \tilde{f}_{i,c^{split},k-1}^{exit}) - (\tilde{f}_{i,c^{split},k+1}^{exit} - \tilde{f}_{i,c^{split},k}^{exit})]}{2} - \frac{\beta^{inveh} q_{ick}}{2\beta^{wait} \Lambda_c}, 0 \right\} \quad (20)$$

In its current form, the final holding time is the sum of: (i) holding needed to regulate the joint operation, (ii) the single line operation at the shared transit corridor, and; (iii) the transition to the branches. Moreover, thanks to the ratio of the occupancy onboard the vehicle over the sum of arrival rates of the current and the remaining stops of the network, the calculated holding time is adjusted to the passengers that are experiencing the control action. The formulated criterion can be applied in all different network configurations with a shared transit corridor and lines merging or diverging to branches. Holding time is calculated subject to the current stop and the next stop set (branch or corridor) that succeeds after the current stop. In the following sections, the applicability of the holding criterion for the diverging branches, merging branches and corridor is detailed.

3.4.8. Diverging branch criterion

After the shared transit corridor, a single line criterion is applied to maintain control on each branch, derived from the shared corridor holding criterion in Eq. (20) considering only the remaining demand downstream. At the diverging branch, the demand within corridor Λ^c and from corridor to branch Λ^{cb} is zero and the total demand corresponds to the demand within the branch Λ^b . Therefore, the weights of the terms for regulating the joint operation and the expected headway at the last common stop are zero ($\theta_1 = 0, \theta_3 = 0$) and the weight for regulating the line is equal to 1 ($\theta_2 = 1$). For each stop b_i which belongs to the branch ($b_i \in B_i$), the holding criterion is given in Eq. (21):

$$t_{ib,k}^{hold} = \max \left\{ \frac{(f_{ib,k+1}^{exit} - f_{ib,k}^{exit}) - (f_{ib,k}^{exit} - f_{ib,k-1}^{exit})}{2} - \frac{\beta^{inveh} q_{ib,k}}{2\beta^{wait} \Lambda_{b_i}}, 0 \right\} \quad (21)$$

This single line criterion was already introduced by Laskaris et al. (2016) and it is therefore a special case of the criterion in Eq. (20) and can be applied on a single line or at diverging branches.

3.4.9. Merging fork network criterion

In a merging fork network, lines operate individually and then merge to a shared transit corridor. Again there is a stop set $B_i = \{b_{i1}, b_{i2}, \dots, b_{in}\}$ and a stop set $C = \{c^{merg}, c_1, \dots, c_n\}$ with all the common stops starting with the first common stop c^{merg} . Therefore, the holding criterion on merging branches accounts for the regularity of the line and the transition from the independent operation of the lines to common operation. At merging branches at each stop b_i ($b_i \in B_i$), the criterion takes the following form:

$$t_{ib,k}^{hold} = \max \left\{ \begin{aligned} & \theta_1 \frac{[(f_{ib,k}^{exit} - f_{ib,k-1}^{exit}) - (f_{ib,k+1}^{exit} - f_{ib,k}^{exit})]}{2} + \\ & + \theta_2 \frac{[(\tilde{f}_{i,c^{merg},k}^{exit} - \tilde{f}_{i,c^{merg},k-1}^{exit}) - (\tilde{f}_{i,c^{merg},k}^{exit} - \tilde{f}_{i,c^{merg},k+1}^{exit})]}{2} - \\ & \frac{\beta^{inveh} q_{ib,k}}{2\beta^{wait} \Lambda_{b_i}}, 0 \end{aligned} \right\} \quad (22)$$

The first term regulates the departure of vehicle k of line i from current branch stop b_i , subject to its preceding and succeeding vehicle. At branch stops, vehicles may also be held in order to ensure evenly spaced arrivals between vehicles of different lines at the shared transit corridor. Hence, the second term regulates the predicted departure from the first common stop c^{merg} between consecutive vehicles regardless of the line. The holding time to fulfill the two objectives is calculated through the two aforementioned terms. The contribution of each term to the total holding time is related to the share of the demand affected by the corresponding control action and the position of the current branch stop b_i from the first common stop c^{merg} , as reflected by the assigned weights. The passenger demand taken into account is the demand within the branch from the current branch stop b_i up the end of the branch at the merging stop c^{merg} , the demand generated along the branch with destination at the shared transit corridor, and the corridor demand for all the remaining stops beyond the first common stop c^{merg} .

Once the lines merge to the shared transit corridor, and assuming that they will operate under said conditions until the end of their route, the holding criterion focuses solely on the regularity of the joint operation and the total demand of the network corresponds to the demand travelling within the shared transit corridor:

$$\Lambda_c = \Lambda_c$$

The criterion for each stop along the shared transit corridor c takes the following form:

$$t_{ick}^{hold} = \max \left\{ \frac{(t_{ick+1}^{exit} - t_{ick}^{exit}) - (t_{ick}^{exit} - t_{ick-1}^{exit})}{2} - \frac{q_{ick}}{2\beta^{wait}\Lambda_c}, 0 \right\} \tag{23}$$

The merging fork network criteria were originally introduced in the work of Laskaris et al. (2019a,b) and they are limited to this specific network configuration. The current form of the criterion (Eq. (20)) is general enough to be applied for any part of a branch and trunk network, including therefore merging fork networks as a specific special case.

4. Experimental setup

4.1. Case study

The routes of lines 176 and 177 of the bus network of the city of Stockholm, Sweden, are structured in a fork network configuration, consisting of a common set of stops and two branches, each served by one line (Fig. 3). The two lines run together between Mörby and Ekerö, and have distinctive branches between Ekerö and either Solbacka or Skärvik. There are 24 common stops, all located in the municipality of Solna, providing connections with the commuter train, light rail and subway. Line 176 has 19 branch stops and line 177 has 12 branch stops. Because of their layout, they provide an ideal ground for evaluating the proposed holding rule for the westbound direction. The frequency of the lines is set to 10 min with a joint frequency of 5 min at the shared transit corridor and the vehicles depart alternately from the common terminal at Mörby.

4.2. Scenarios

4.2.1. Actual demand

The first scenario set is intended to assess the performance of the network for the actual observed hourly demand. The demand of lines 176 and 177 and the different segments is given in Table 2. It can be observed that the majority of the demand travels within the common part. The second largest group contains the passengers that travel from the shared transit corridor to the branch, while the smallest share of the demand of each line travels within the branches.

Apart from the actual demand, sensitivity analysis is performed considering a demand increase of 50%. The additional scenario with increased demand is added to assess the effect of the demand on the holding criterion and its performance.

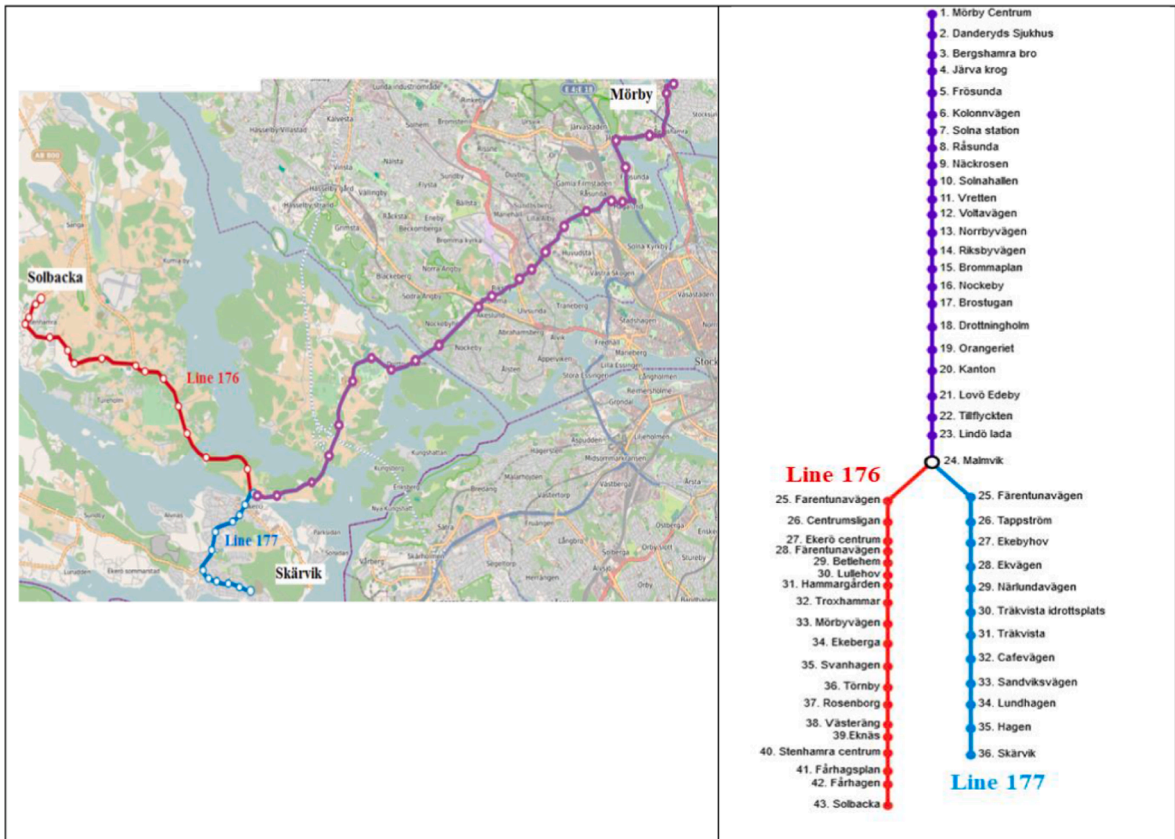


Fig. 3. Lines 176 and 177 in Stockholm, Sweden.

Table 2
Demand Segmentation for Lines 176 and 177.

	Line 176		Line 177	
	Passengers/hour	% demand	Passengers	% demand
Total Demand	148	100	143	100
Demand Generated on Corridor	137	92.57	137	95.8
Demand Generated on Branch	11	7.43	6	4.2
Demand within Corridor	108	72.97	108	75.52
Demand Corridor to Branch	29	19.59	29	20.28
Demand within Branch	11	7.43	6	4.2

4.2.2. Control strategies

A no-control scenario is used as a benchmark for both demand levels. During this scenario, the vehicles depart immediately after the completion of dwell time determined by boarding and alighting operations. Additionally, two control schemes are applied to compare their performance on this network type. The first is a single-line holding criterion that aims at attaining even headways from the preceding and the succeeding vehicle and at the same time limits the maximum allowed headway to a share of the planned headway. The single-line criterion is introduced by [Cats et al. \(2011\)](#) and it has outperformed other holding rule-based strategies. The second control scheme is based on the multiline holding criterion introduced in this paper in Equation (20). The aim of the comparison of the two criteria is to assess the strengths and the weaknesses of each scheme on the performance of the lines and the different passenger groups. The scenarios are summarized in [Table 3](#). The No-Control scenario is denoted with NC, the single line control as Even Headway (EH) and the multiline control named Cooperative Passenger Cost (CPC).

4.2.3. Demand composition scenarios

In addition, with a second set of scenarios we examine the effect of different demand compositions on the performance of the multiline control. For this scenario set 25 additional scenarios with different demand compositions are tested, while maintaining the total demand unchanged. The scenarios are summarized in [Table 4](#). Each row corresponds to a different share of passengers travelling from the corridor to branch, each column to a different share of passengers travelling within branch and each cell to the passengers travelling within the corridor. The index of each scenario is given in parenthesis. The distribution of the passengers over stops within a certain category follows the actual demand pattern as much as possible.

Again, all demand profiles are tested for the same control schemes as the actual demand set (NC, EH and CPC) and evaluated based on the differences in travel time.

4.3. Simulation tool

To simulate the test network, we employ the simulation software Busmezzo. Busmezzo is a transit simulator embedded in the mesoscopic traffic simulator Mezzo ([Burghout et al., 2005](#); [Toledo et al., 2010](#)). Busmezzo has been used to simulate bus operations and different control strategies, including holding for single-line and transfer synchronization ([Cats et al., 2012, 2011](#); [Gavriilidou and Cats, 2018](#)), and short-turning ([Leffler et al., 2017](#)). The simulator represents the passengers as agents, assisting in tracking the individual passenger paths in the network and evaluate the effect of the control on each passenger group. BusMezzo explicitly accounts for the relationship between holding time and the resulting number of boarding passengers (and consequently the number of on-board passengers). This is done by admitting more passengers during the prolonged dwell time in case more passengers have been generated during the elapsed time. Since passenger boarding may in itself prolong the dwell times, this might be repeated until no further passengers board. Simulation includes a warm-up period and a cool-down period ([Cats and Hartl, 2016](#)). When a vehicle arrives at a stop, the simulator firstly identifies if the current stop is a corridor or a branch stop. In case of a corridor stop, the latest departure and the expected arrivals from all overlapping lines are retrieved together with the passenger demand to execute the holding time needed.

Busmezzo is stochastic by design, and a certain number of replications is needed to attain statistically significant results. The trips of both lines serve first the corridor stops. Vehicles are dispatched without any disruption and there is no trip chaining; hence the sources of stochasticity are all endogenous and pertain to running times and passenger demand. Fifty replications are conducted for each scenario and the weighted travel time is used as reference variable, as expressed in Equation (1). Among all scenarios and for a 5% statistical error (student t-value: 2.01) the maximum number of replications for statistically significant results is found to be 11, therefore the number of replications is considered sufficient.

Table 3
Actual Demand Scenarios.

	Actual Demand (100)	Peak Demand (150)
No Control (NC)	NC_100	NC_150
Even Headway (EH)	EH_100	EH_150
Cooperative Passenger Cost (CPC)	CPC_100	CPC_150

Table 4
Demand segmentation scenarios.

		Share of Passengers Travelling within Branch (%B)				
		5	10	15	20	25
Share of Passengers Travelling from Corridor to Branch (%CB)	5	(1) 90%	(3) 85%	(6) 80%	(10) 75%	(15) 70%
	10	(2) 85%	(5) 80%	(9) 75%	(14) 70%	(19) 65%
	15	(4) 80%	(8) 75%	(13) 70%	(18) 65%	(22) 60%
	20	(7) 75%	(12) 70%	(17) 65%	(21) 60%	(24) 55%
	25	(11) 70%	(16) 65%	(20) 60%	(23) 55%	(25) 50%

5. Results

5.1. Actual demand scenario

5.1.1. Shared transit corridor performance

The results of the performance of each control scheme at the corridor are summarized in Table 5. The first index for the performance of the lines at the corridor is the coefficient of variation (CV) of the joint headway. Since the joint headway is defined by the difference between consecutive departures of trips at the corridor stops, it is clear from the results that CPC outperforms the benchmark and the EH control, obtaining the lowest variability among the scenarios. The differences between control schemes are similar in the two demand scenarios. According to the Transit Capacity and Quality of Service Manual the level of bunching is estimated as the share of trips the actual headway of which deviates more than 50% from the planned headway (Transit capacity and quality of service manual, 2003). For the shared transit corridor this applies to the planned joint headway. Bunching at the corridor decreases significantly with multi-line control as all vehicles from different lines are taken into consideration by the criterion. Single-line control neglects that in a set of consecutive stops it is not only vehicles from same line that may arrive in a platoon, but also vehicles from different lines might be present. The findings from the regularity indices are also reflected in the travel time components. Similarly, both strategies reduce the waiting time at stops for both demand levels, in correspondence to the reduction of headway variability. In-vehicle time is slightly increased with EH compared to the NC scenario, which is expected as holding strategies tend to prolong the time spent at stops to regulate operations. CPC manages to maintain in-vehicle time at the same level as under NC. The weighted travel time is a sum of waiting time and in-vehicle time and for both demand levels it is the lowest under the CPC strategy.

5.1.2. Line performance

As one may expect, applying single-line control is more effective at single-line level compared to joint control. The results summarized in Table 6 show that for both lines, EH outperforms CPC under all scenarios.

Recall that with EH the same criterion is applied throughout the line, while with CPC the criterion adjusts to the stop set, from multi-line (Eq. (20)) to single-line criterion (Eq. (21)) as the vehicle progresses from the corridor to the branch. When limiting the perspective to the individual line level, the single-line control is the most suitable choice as it results in the lowest values in terms of both regularity indices and passenger travel times. The multi-line control performs better than no control with satisfactory results in terms of its ability to mitigate bunching.

The case study network consists of two lines of different length. It can be observed from the results that the longer line (Line 176) achieves better performance compared to Line 177 under CPC. The extent to which CPC outperforms NC decreases as demand increases. This can be explained by the nature of the multi-line criterion. At each stop the holding time depends on the share of the demand at the remaining downstream stops. As the vehicle approaches the end of the corridor, the criterion handles the transition from joint to single-line operation. During this transition, a loss of performance is observed which is recovered gradually as vehicles progress along the branch. Furthermore, we observe a similar phenomenon to the one observed in the merging fork case: one of the lines sacrifices individual performance for larger total performance gains, from a passenger-centric perspective (Laskaris et al., 2019a). As shown in Fig. 4, the level of variability with CPC for line 176 shows an upward trend compared to EH until the 35th stop, where CPC starts to effectively regulate the line-specific vehicles headway. On the other hand, for line 177 there are not enough stops in order to

Table 5
Performance indicators of the shared transit corridor.

	Shared Transit Corridor	CV of Joint Headway	Level of Bunching	Waiting time per passenger [sec]	In vehicle time per passenger [sec]	Weighted travel time per passenger [sec]
1_100	NC	0.507	0.397	155.4	194.3	505.2
	EH	0.421	0.307	153.6	196.1	503.3
	CPC	0.391	0.283	151.7	194.7	498.2
1_150	NC	0.512	0.378	150.5	214.1	515.1
	EH	0.468	0.341	149.1	215.8	513.9
	CPC	0.424	0.315	148.9	214.4	512.1

Table 6
Performance indicators at the single-line level.

		Line 176					Line 177				
		CV of Headway	Bunching	Waiting time per passenger [sec]	In vehicle time per passenger [sec]	Weighted Travel Time per passenger [sec]	CV of Headway	Bunching	Waiting time per passenger [sec]	In vehicle time per passenger [sec]	Weighted Travel Time per passenger [sec]
Actual Demand	NC	0.27	0.10	314.7	148.0	777.4	0.33	0.14	320.9	157.1	799.0
	EH	0.17	0.01	310.8	148.8	770.4	0.17	0.01	312.5	158.6	783.6
	CPC	0.21	0.03	311.1	148.3	770.5	0.21	0.03	314.4	158.0	786.7
Peak Demand	NC	0.38	0.21	320.5	160.6	801.5	0.40	0.25	333.3	172.0	838.6
	EH	0.20	0.03	314.2	161.5	789.8	0.24	0.07	321.3	173.4	816.0
	CPC	0.25	0.07	314.9	160.8	790.6	0.34	0.16	326.5	173.5	826.6

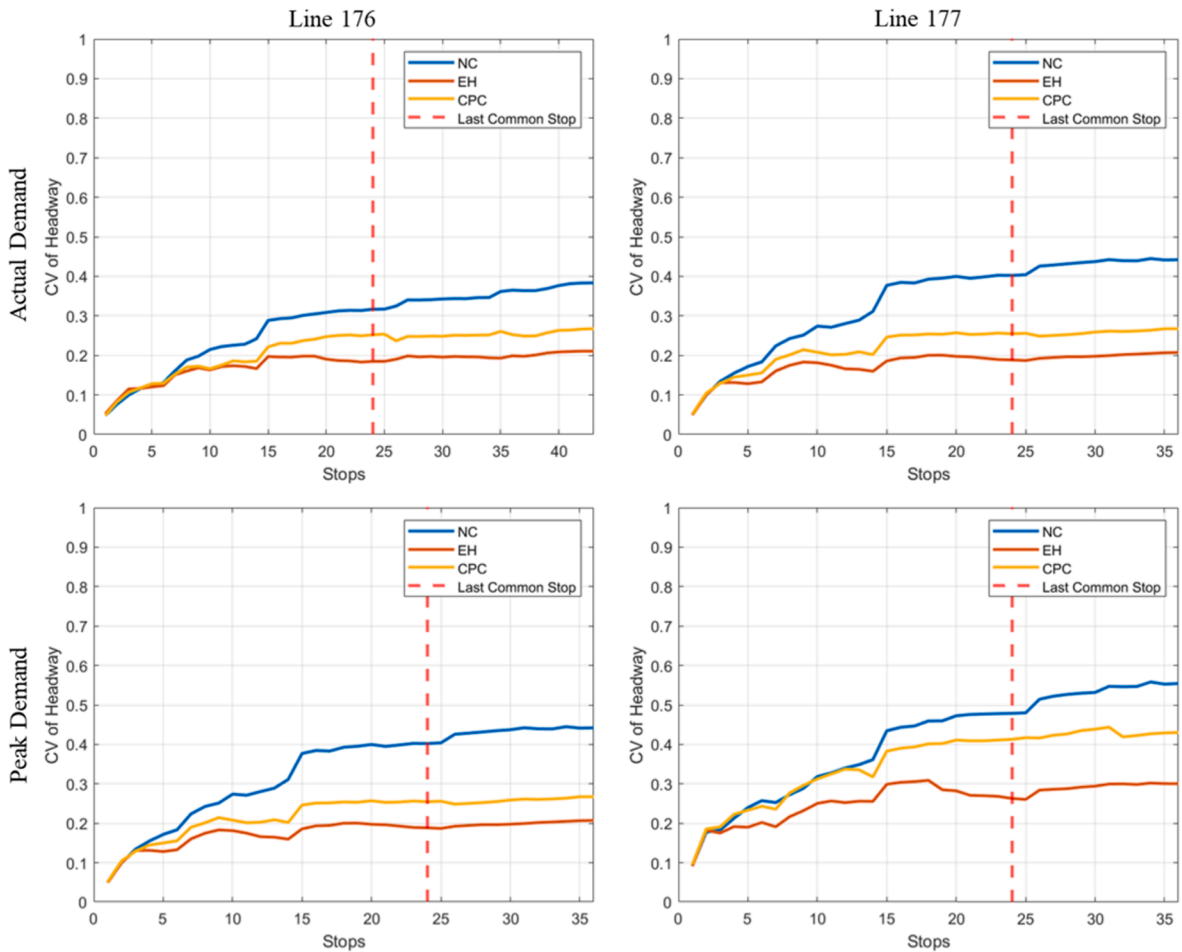


Fig. 4. CV of Headway per stop.

restore the variability of the headway. Especially at the peak demand scenario, the performance of line 177 is severely affected by regularization of the joint operation along the shared corridor and consequently it exhibits poor performance in terms of headway variability. Although there is a tendency to recover, line 177 does not manage to fully recover from the loss of regularity due to the limited length of the line beyond the divergence stop.

5.1.3. Vehicle trip travel times

The 90th percentile of vehicle trip travel times and its variability is used as a measure of performance by the operator. The duration of the trip is essential for a robust timetable design and at the tactical planning phase for dimensioning the fleet and the driving roster. The histograms of vehicle travel times for the actual demand and the peak demand are depicted in Fig. 5. It can be observed that CPC results in less variable travel times under the actual demand profile and yields better performance for the longer line (line 176). Specifically under standard demand conditions (demand level 100), CPC results in an average travel time of 6349sec with a standard deviation of 95sec, outperforming EH, which reports an average travel time and standard deviation of 6376 sec and 100 sec respectively. When demand increases (scenario 150), variability affects all control scenarios (NC, EH, CPC), and EH becomes the best alternative. Based on the results, vehicle scheduling with CPC has no high fleet requirements at demand level 100 due to low variability in travel time. However, this is not the case for the scenario set with demand level 150, where variability increases especially for line 177 and may require additional vehicles to be dispatched.

5.1.4. Passenger travel times

Fig. 6 shows the differences in passenger travel time for the two components of the passenger cost and as a sum of the weighted travel time under each strategy compared to the NC case. With EH, reductions are achieved in waiting times in both scenarios. While the two schemes perform similarly in terms of waiting time savings for the actual demand, the reduction in waiting time with EH is double compared to CPC at the peak demand scenario. CPC outperforms EH in terms of in-vehicle delay. The gains in in-vehicle delay are even higher in the peak demand scenario, where EH's performance deteriorates. Because of this great reduction in in-vehicle time, CPC outperforms EH in the overall travel time per passenger for both demand levels.

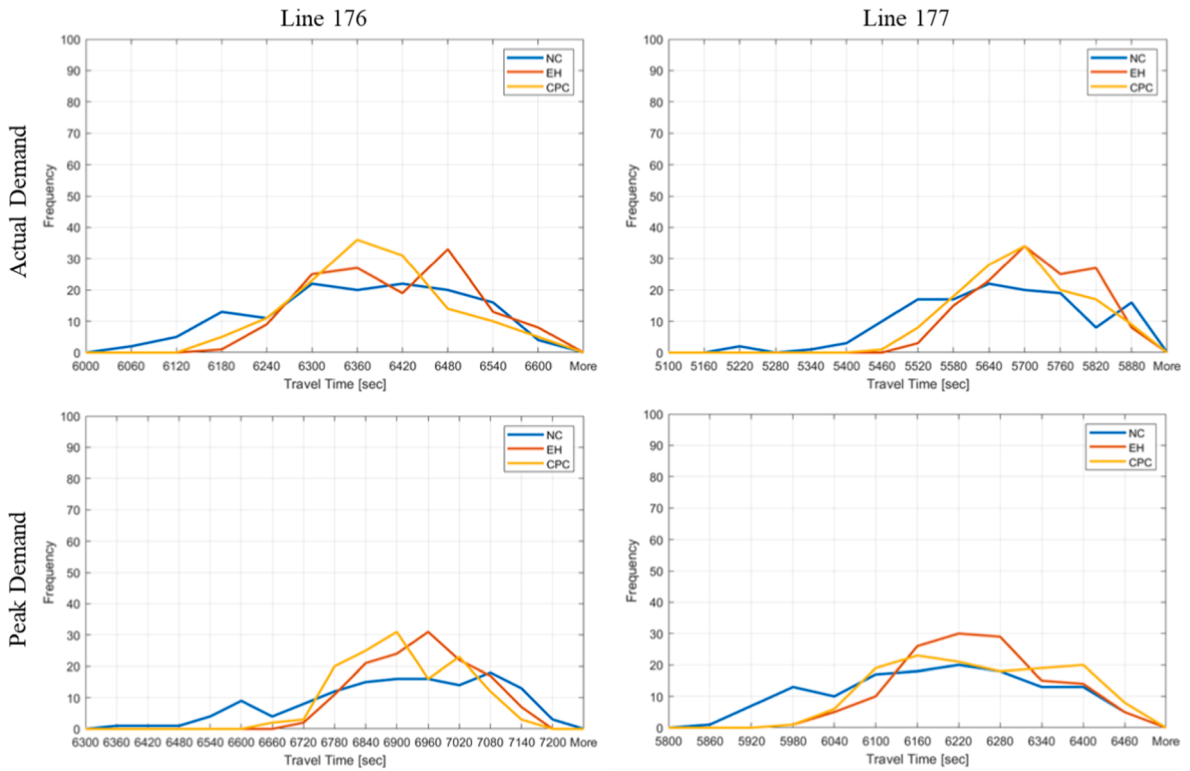


Fig. 5. Vehicle Trip Travel Time Distribution.

5.2. Demand scenarios

In the second set of scenarios, we investigate the effect of the demand composition on the different passenger groups. We focus on comparing the travel time of each passenger group under single-line versus multi-line control. The difference in passenger cost between CPC and EH is reported for each passenger group (Tables 7–9) and for the network in total (Fig. 7 and Table 10). A color scale from green to red is used to characterize the performance of the multi-line control compared to single-line control and each figure is scaled based on the range of values for the highest and the lowest difference in passenger cost between single- and multi-line control. Green color corresponds to greater gains from multiline control while red from single line control.

5.2.1. Shared transit corridor

Table 7 summarizes the differences in passenger cost for the passengers travelling within the shared transit corridor. Since the demand for this part of the network can be satisfied by more than one line, multi-line control results in lower passenger costs in the majority of the scenarios. As expected, CPC is more effective when the majority of the demand is concentrated to the shared transit corridor. Specifically, the gains are higher when the share of passengers travelling within the shared transit corridor exceeds 70% of the total demand. The results are sensitive to the increase in demand from the corridor to branch, due to the fact that they have a conflicting objective. This can be observed in the scenarios with a high share of passengers travelling from the corridor to a branch where CPC reports marginal losses compared to EH. Holding time with CPC is a sum of holding times weighted by the corresponding demand share. Passengers travelling within the corridor benefit from regulating the joint operation while passengers travelling beyond the corridor benefit from line regularization, therefore the size of the group impacts the control decision. The size of the branch passenger group does not affect the performance of CPC for the corridor passengers.

5.2.2. Corridor to branch

The group of passengers that travels from corridor to branch is generally benefited by single-line operation. The cost comparison of this passenger group is shown in Table 8. Single-line control results in lower passenger cost compared to multi-line control, with the latter having marginal differences from the passenger cost of EH at the few scenarios that manages to achieve better results. This can be explained by the fact that they experience the control actions for the transition of the line from corridor to branch and a prolonged travel time due to holding to regulate the joint operation. CPC has the best performance for a 10% demand for both branch and corridor to branch.

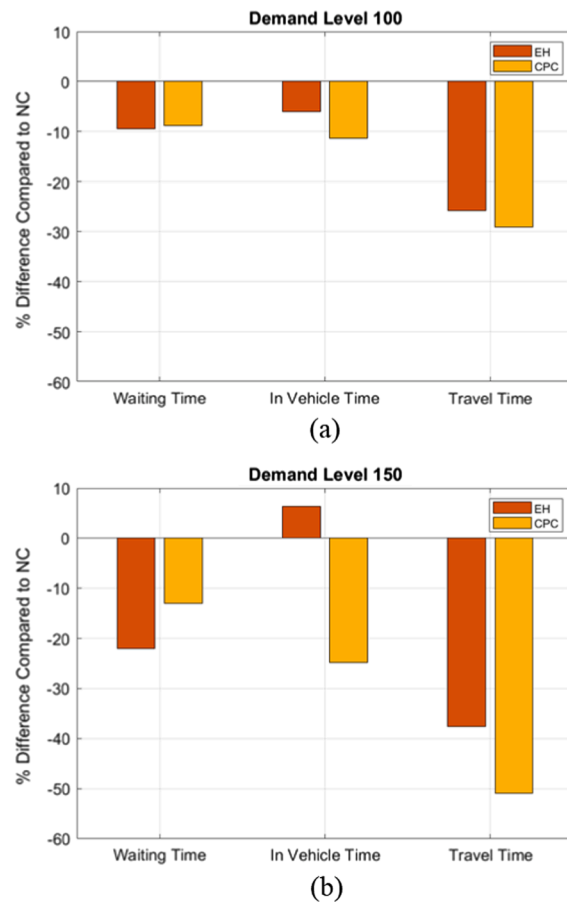


Fig. 6. Passenger cost differences compared to NC for (a) actual demand and (b) peak demand.

Table 7

% Difference between EH and CPC passenger cost for passengers travelling within the corridor.

		Share of passengers travelling within the Branch %B				
		5	10	15	20	25
Share of passengers travelling from Corridor to Branch %CB	5	-0.54	-0.85	-0.71	-0.36	-0.85
	10	-0.37	-1.00	-0.57	-0.71	-0.74
	15	-0.48	-0.59	-0.67	-0.46	-0.35
	20	-0.33	0.08	-0.25	-0.26	-0.34
	25	-0.22	0.05	0.06	-0.34	-0.17

Table 8

% Difference between EH and CPC passenger cost for passengers travelling from corridor to branch.

		Share of passengers travelling within the Branch %B				
		5	10	15	20	25
Share of passengers travelling from Corridor to Branch %CB	5	0.62	0.30	0.46	0.16	0.86
	10	0.21	-0.28	0.58	0.33	-0.05
	15	0.02	0.12	-0.17	0.47	0.15
	20	0.32	0.50	0.13	0.29	0.12
	25	0.31	0.04	0.21	0.11	-0.04

Table 9

% Difference between EH and CPC passenger cost for passengers travelling within the branch.

		Share of passengers travelling within the Branch %B				
		5	10	15	20	25
Share of passengers travelling from Corridor to Branch %CB	5	1.10	-1.57	-1.22	-0.72	-1.64
	10	-2.27	-3.06	0.42	-1.84	-2.26
	15	0.86	0.91	0.43	0.41	-0.88
	20	0.51	0.40	-0.08	0.33	-0.30
	25	-2.30	0.48	0.97	2.29	1.90

5.2.3. Branch

The results of passenger cost for the branch demand follow the same pattern as the one observed for the corridor group. As shown in Table 9, CPC performs significantly better when traversing passengers from corridor to branch constitute up to 10% of the total demand, regardless the size of the passenger group travelling within the branch. Although control schemes on this part of the network focus only on line headway regularization, CPC starts to regulate the line headway for the transition to their individual operation via the projection term. The importance of the projection term is relative to the share of branch demand beyond the diverging stop. A low share of passengers travelling from the corridor to a branch keeps distinct the objectives of regulating the corridor and the branch, resulting to substantial benefits for CPC. Multi-line control is recommended when the demand within the corridor is more than 50% of the total demand.

5.2.4. Total demand

The results for the travel cost of the total demand summarize under which demand scenario is each control scheme most effective. As observed in Fig. 7 there is a clear pattern based on the demand composition. The multi-line control is most effective for scenarios where more than 60% of the demand is concentrated at the shared transit corridor regardless of the share of the group of passengers travelling within the branch. The performance is mostly affected by the passengers travelling between stops sets, as they influence the magnitude of multi- and single-line control at the corridor. When this group amounts to 25% of the total demand, CPC is steered away from regulating joint operation to single line too abruptly, reducing its performance on this network part and making the single-line control strategy more adequate since it does not switch objectives along the route.

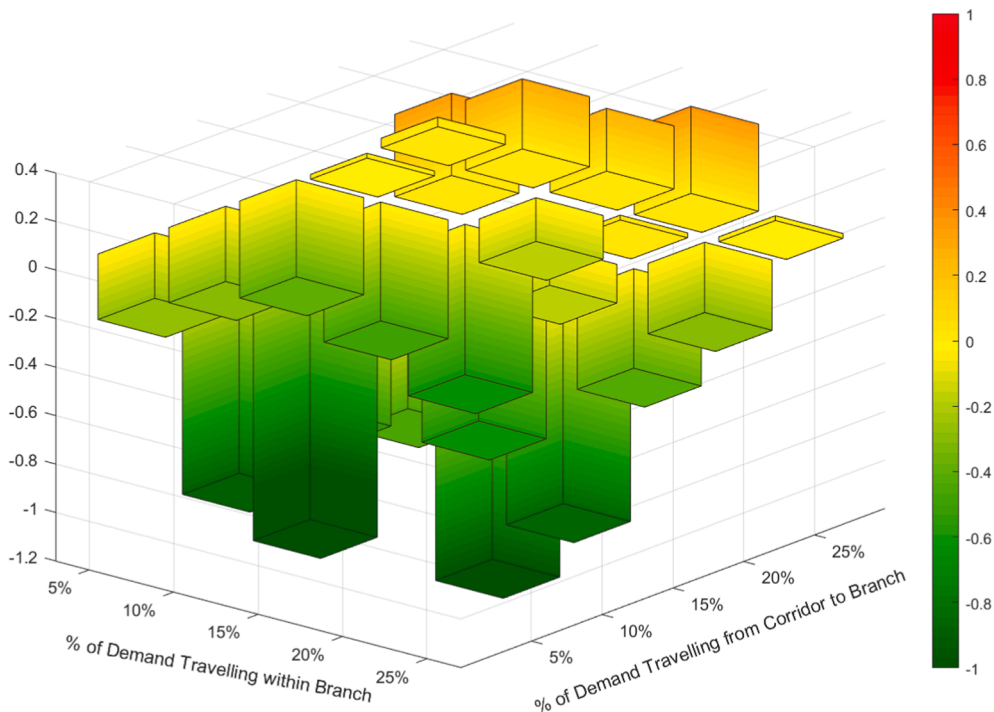


Fig. 7. Difference in network passenger cost between single line and multiline control.

Table 10

% Difference between EH and CPC passenger cost for passengers for the total demand.

		Share of passengers travelling within the Branch %B				
		5	10	15	20	25
Share of passengers travelling from Corridor to Branch %CB	5	-0.27	-0.90	-0.54	-0.45	-0.98
	10	-0.31	-1.20	-0.25	-0.61	-0.86
	15	-0.40	-0.49	-0.62	-0.16	-0.41
	20	-0.02	0.34	-0.18	0.02	-0.29
	25	0.04	0.29	0.26	0.36	-0.02

6. Conclusions

In this study we introduced a multi-line holding criterion for diverging fork networks based on the minimization of passenger travel times under holding. The criterion regularizes the joint headway and the line headway at the shared transit corridor and the expected departure from the last common stop accounting for all different passenger groups and adjusting holding time to the number of passengers that experience the control action. We evaluated the criterion using simulation for a case study of two lines in Stockholm, Sweden under different demand levels and compositions.

The proposed holding rule, Cooperative Passenger Cost (CPC), manages to substantially reduce passenger cost by achieving savings in waiting time and large reductions in in-vehicle time as the criterion adjusts holding time according to the expected passengers on board and passengers waiting at the downstream stops. At the single-line level, it is found to regulate the operation of the shared transit corridor with decent performance. However, the individual operation of one line is partially sacrificed in order to regulate the joint operation. Furthermore, a transition period to shift from joint control to single line control is required. Depending on the length of the branches, this may yield a loss of performance around the last common stop.

When should the multi-line control rule be applied? This has been shown to depend on the demand composition. When the majority of the demand consists of passenger groups that do not interact (i.e. within corridor and within individual branches) CPC is likely to outperform single-line control. However, when a high share of the passengers travels from the corridor to one of the branches – in our case study when their share exceeds 20% – single-line control is recommended.

Further research should involve additional tests on networks with a greater number of lines and different operating schemes to assess the performance of the criterion in terms of cost for the operator. Additionally, historical data may be replaced by real-time data allowing online estimation and predictions of passenger arrivals at stops and bus travel times. This will improve the quality of the input data needed and will improve the effectiveness of the criterion. The criterion should be tested in networks with multiple branches prior and after the shared transit corridor. In this network type, the interaction between all different passenger groups met in merging and diverging fork networks can be explored and transfers are introduced. Therefore, a transfer criterion will be included in order to allow synchronization over regularity based on the difference of passenger cost of the two criteria. Finally, it is important to remark that the decision rule proposed in this study does not explicitly address the dependence between passengers, waiting at stops and on-board, and holding time. In future work, the problem of regulating high frequency transit lines in branch and trunk networks will be revisited adopting more complex and sophisticated modelling approaches.

CRedit authorship contribution statement

Georgios Laskaris: Conceptualization, Methodology, Writing - original draft. **Oded Cats:** Writing - review & editing, Supervision. **Erik Jenelius:** Writing - review & editing, Supervision. **Marco Rinaldi:** Software, Writing - review & editing. **Francesco Viti:** Writing - review & editing.

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