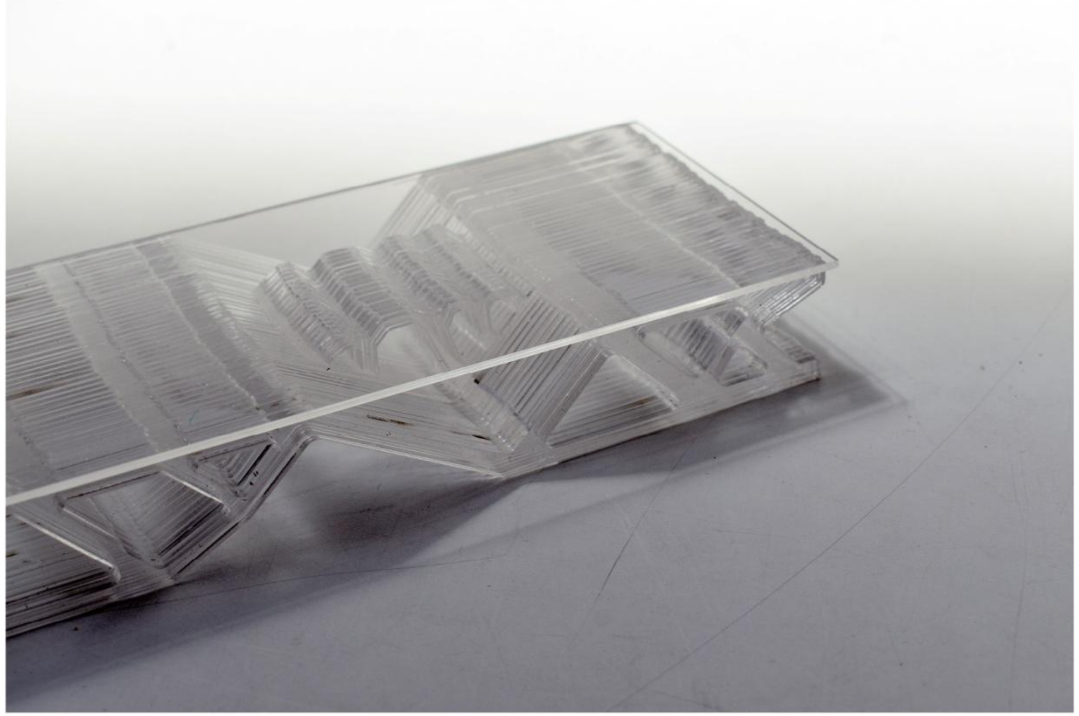
Just Glass.

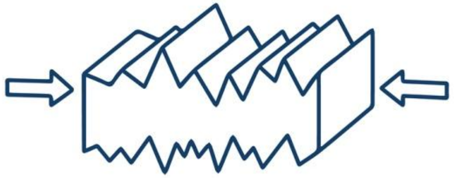
Development of a Topology Optimization Algorithm for a mass-customized cast glass component.



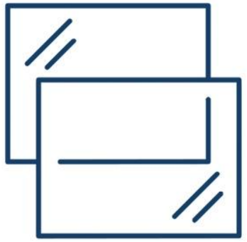
P5 Presentation
June 2022

Student: Anna Maria Koniari | **Mentor Team:** Dr. Faidra Oikonomopoulou & Dr. Charalampos Andriotis

Introduction



compressive strength



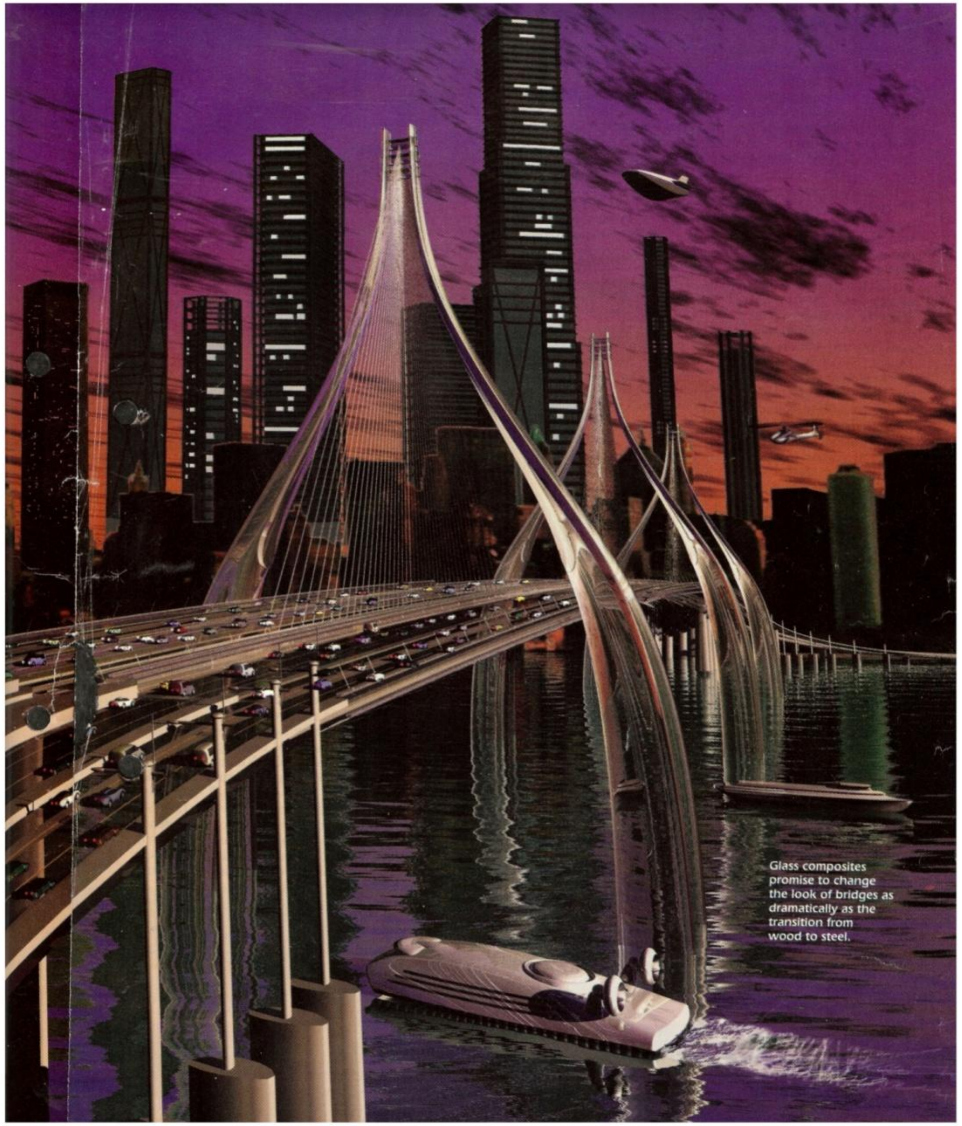
Spatial continuity

Young's modulus

Glass | Current situation

Apple Store, 5th Avenue, New York

Human-scale cast glass piece
Karen le Monte, Corning Museum of Glass

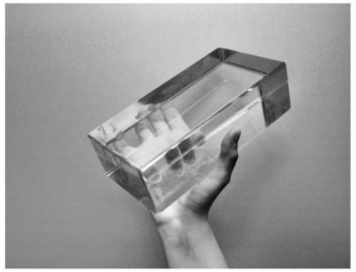

Glass composites promise to change the look of bridges as dramatically as the transition from wood to steel.

Monolithic glass structures | Limitations

Opposites of white' by Roni Horn displayed
in Kroller Moller museum

4 months

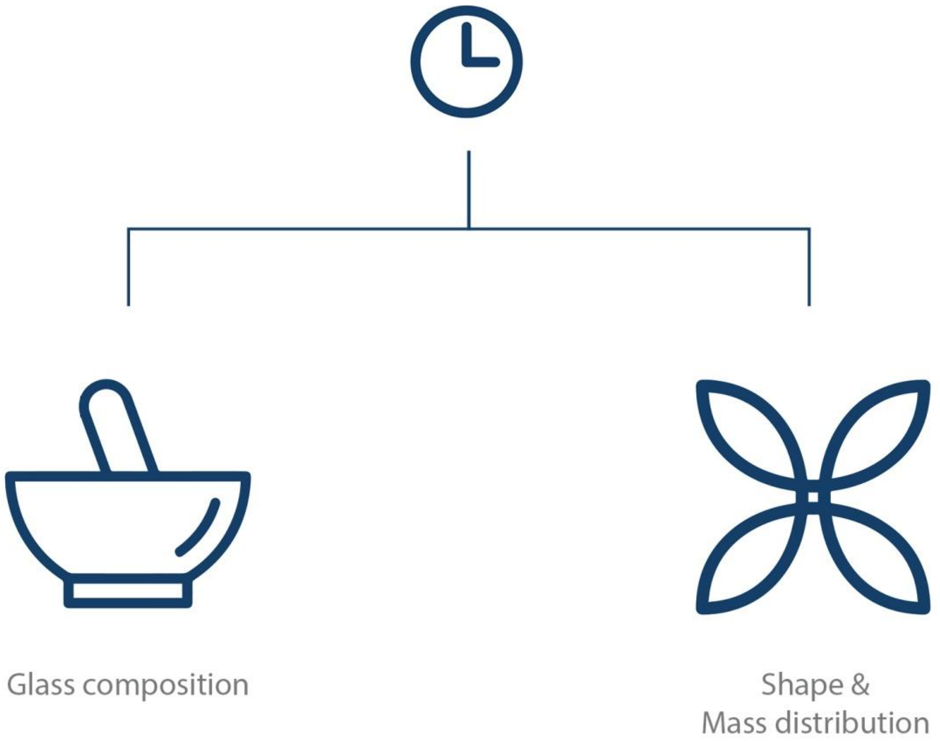
Monolithic glass structures | Limitations



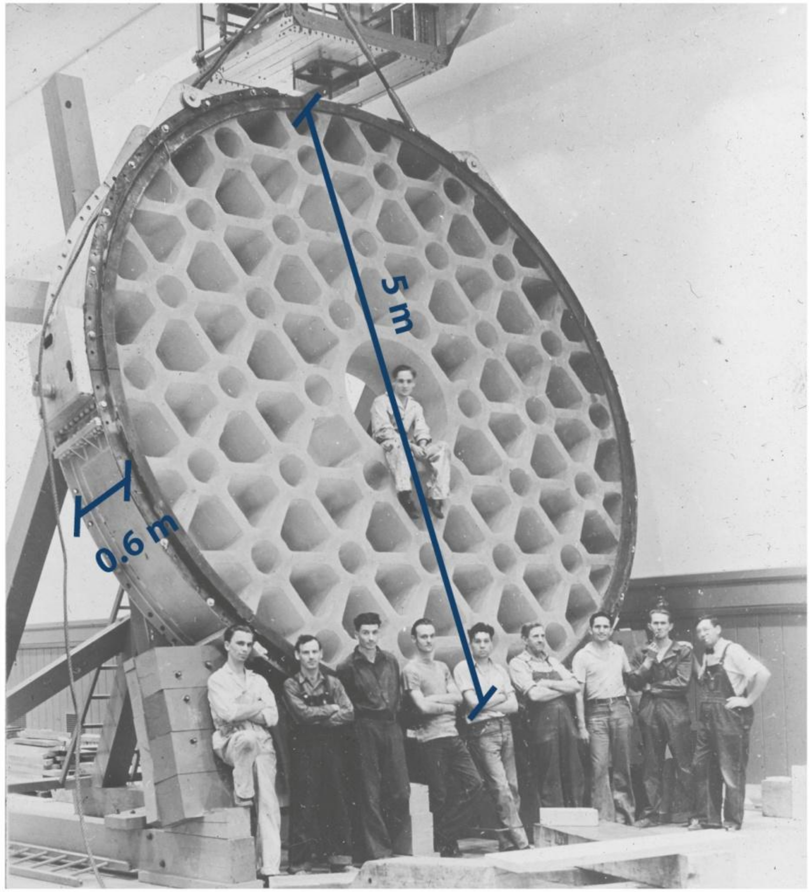
Crystal Houses, Amsterdam
(Oikonomopoulou, 2019)

8 - 38 hours

Cooling time | Factors



Borosilicate glass



Telescope mirror Hale 1

10 months

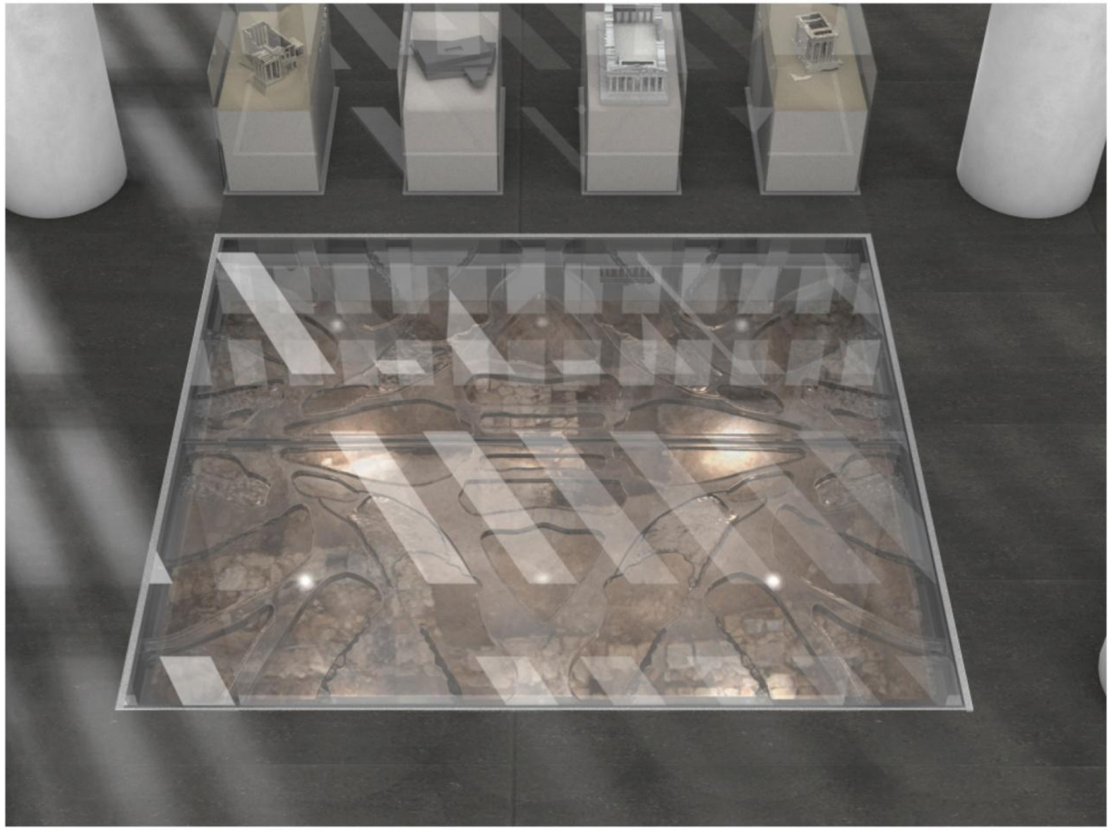
E6 Glass

Giant Magellan Telescope Mirror

3 months

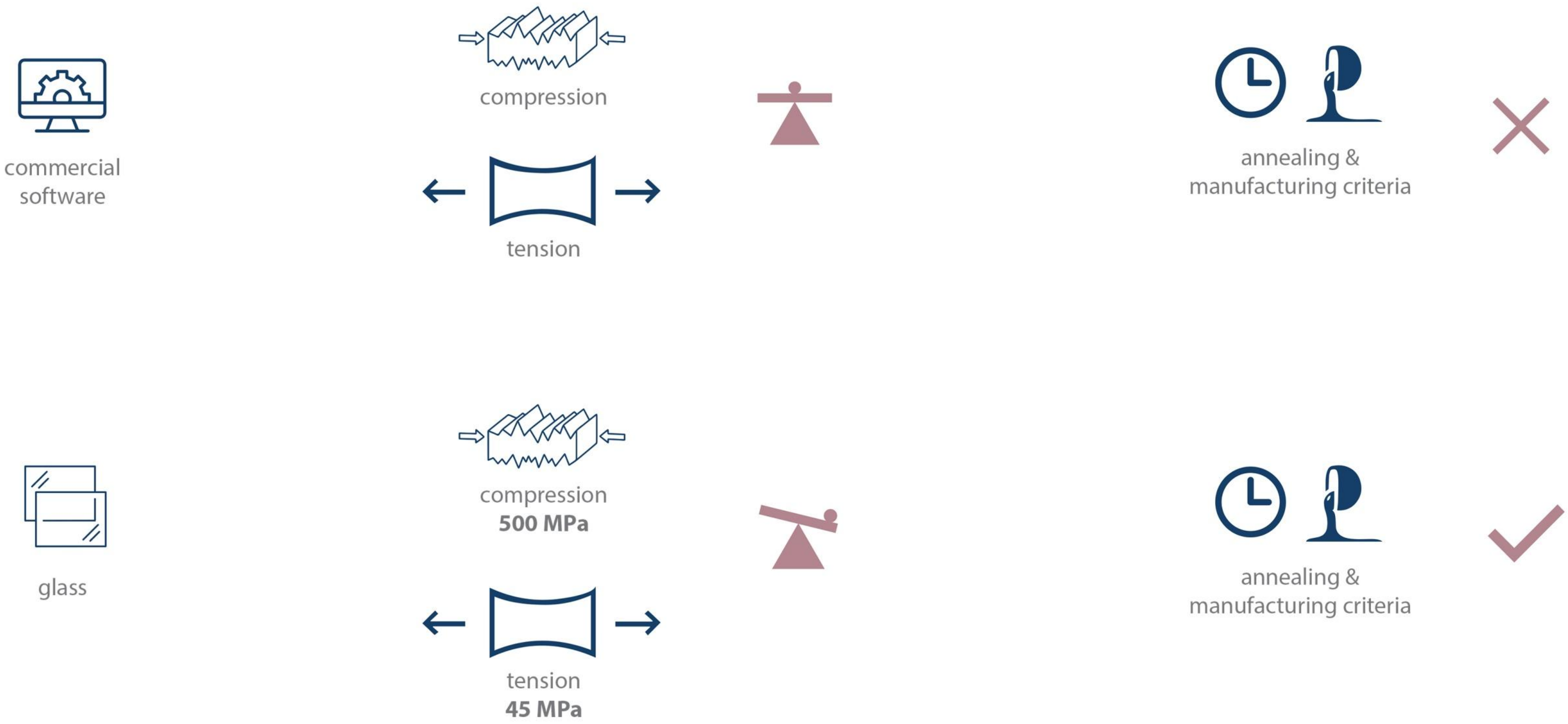
Glass & Topology Optimization

Topologically Optimized Cast Glass Grid Shell Nodes
Wilfried Damen (2019)



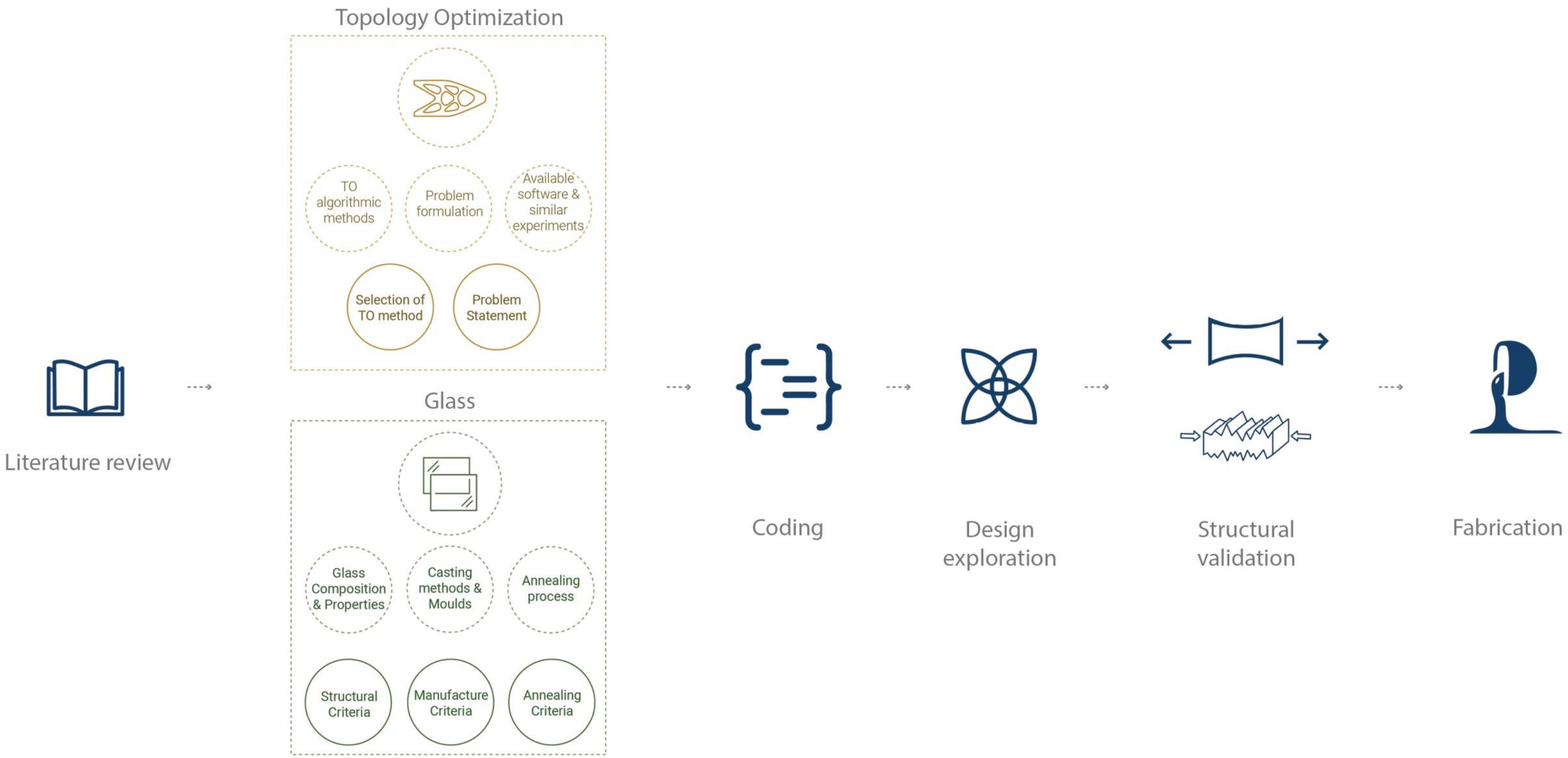
Glass Giants. Mass-optimized massive cast glass slab.
Iro-Maria Stefanaki (2020)

Glass & Topology Optimization | Limitations



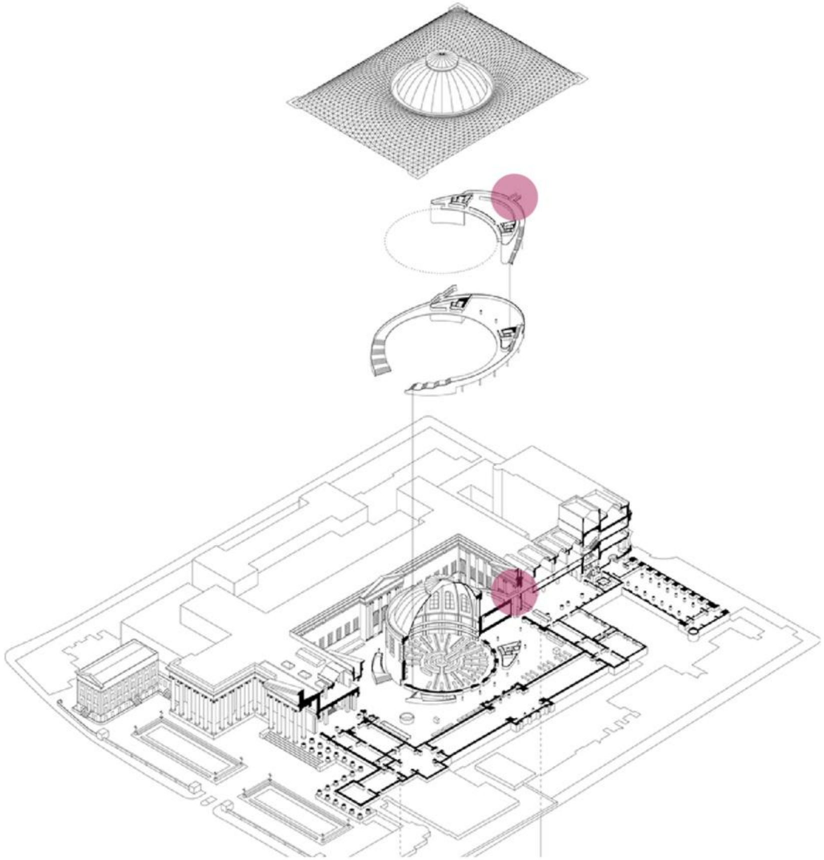
What are the main aspects and inherent limitations of composing a Topology Optimization algorithm for the design of massive cast glass structures which are time and cost efficient?

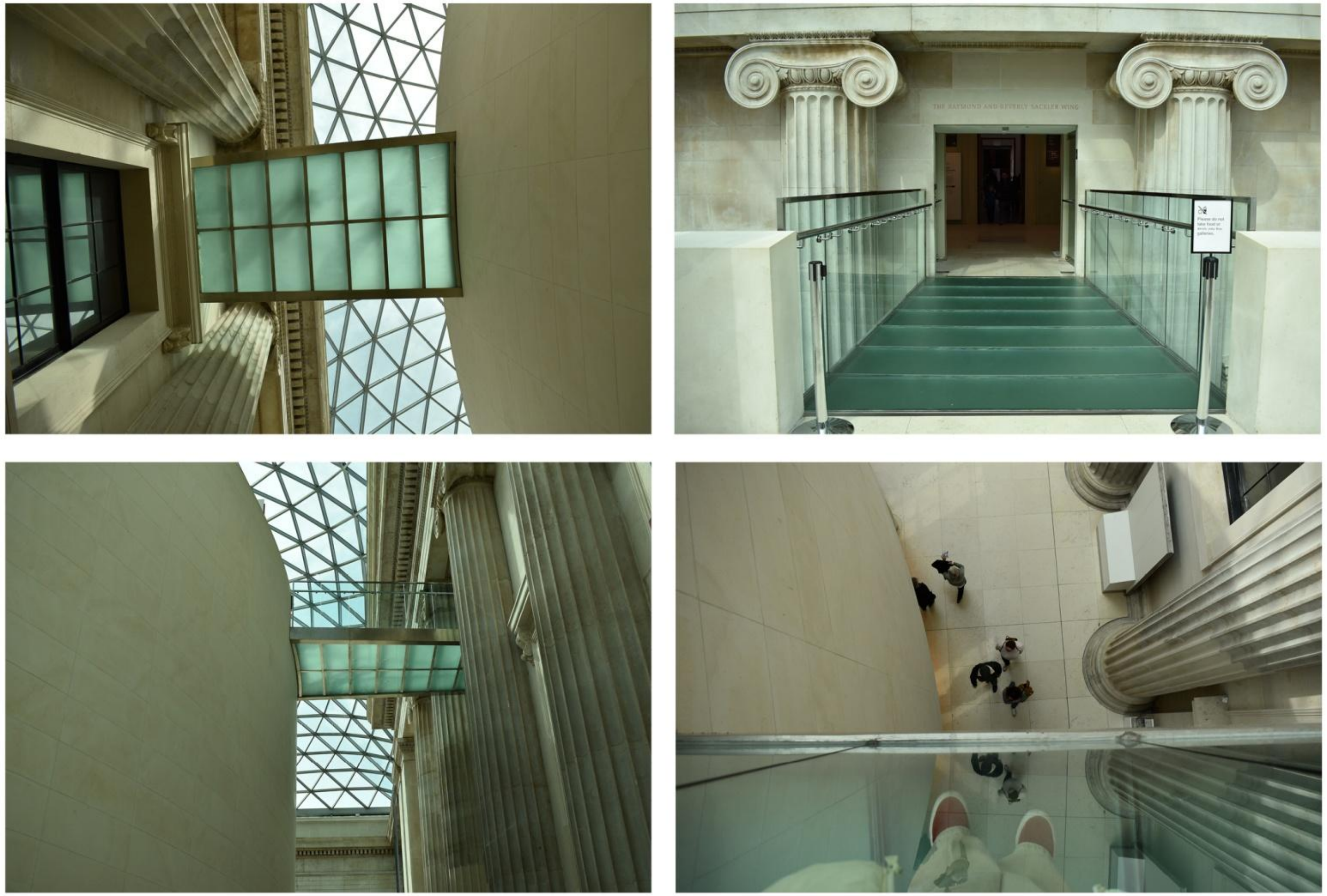
Methodology



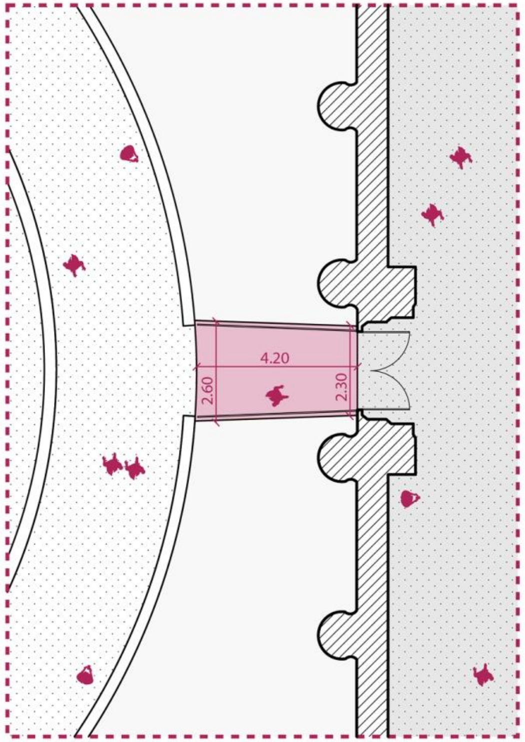
Case Study

Case Study | Location

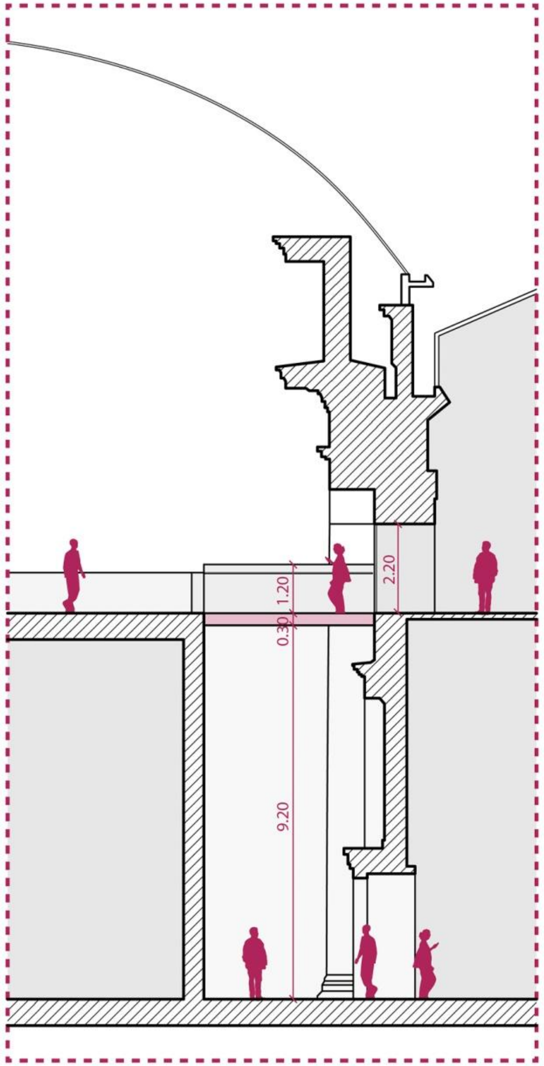




Glass slab inside the Great Court in the British Museum
Intervention by Foster + Partners (2000)



Plan

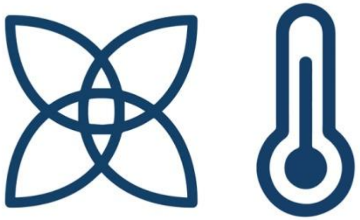


Section

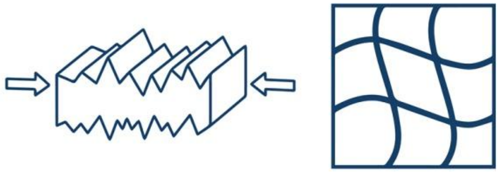
Glass

Glass composition | Borosilicate

annealing time



shaping behavior
melting temperature



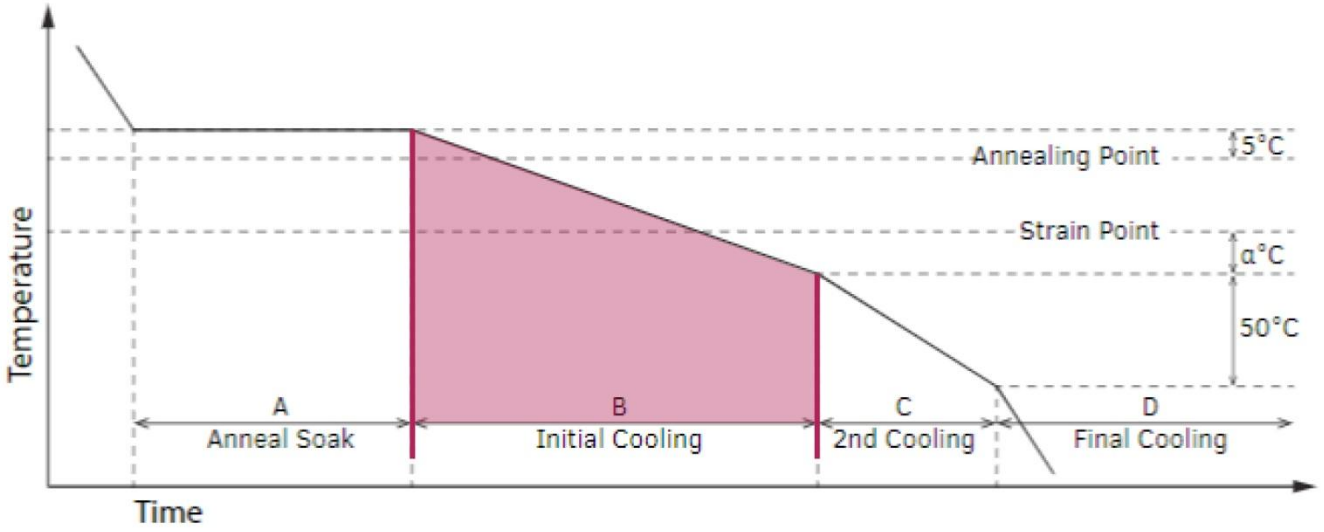
structural & mechanical
properties

Casting method

(Oikonomopoulou, 2019)

Kiln-casting

Cooling process



(Oikonomopoulou, 2019)

$$h = \frac{\sigma}{\boxed{\frac{E\,\alpha_{ex}\,\rho\,c_p}{(1-\mu)\,\lambda}} \times \boxed{d^2} \times b}$$

Moulds | 3d printed sand

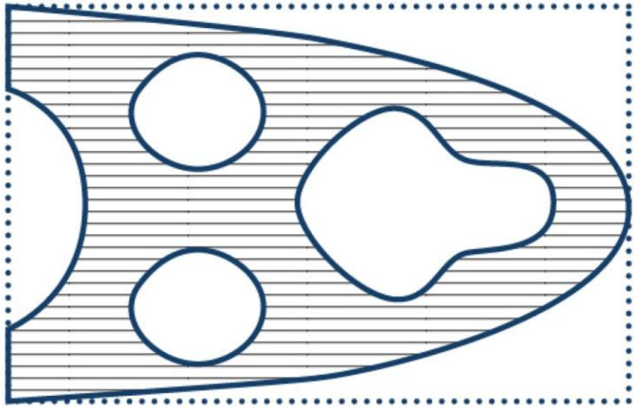
Arup/Davidfotografie.

Damen, W. (2019). Topologically Optimised Cast Glass Grid Shell Nodes

Topology Optimization

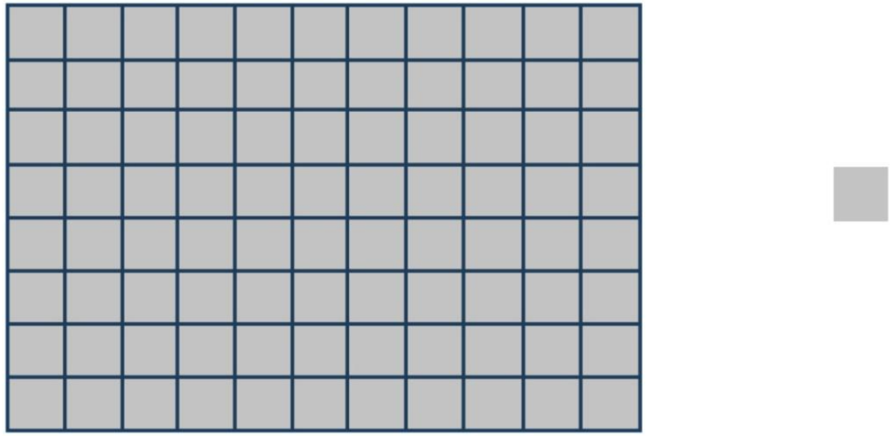
Definition

Initial shape

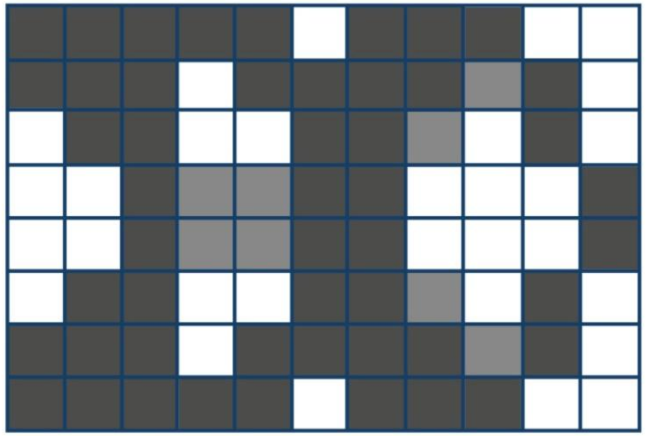


Shape after optimization

SIMP | Initial state



SIMP | Density interpolation



Artificial density - Full material

Artificial density - Void

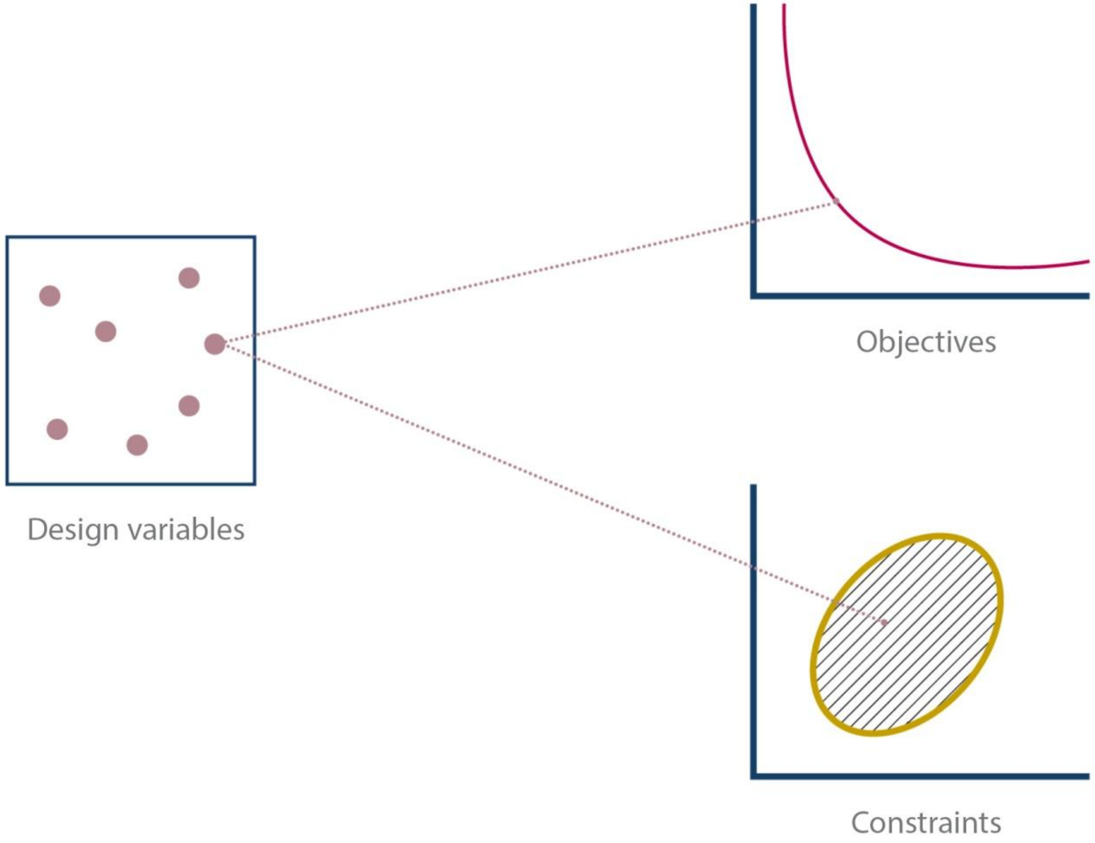
$$\min_x \ f(x), \ \ x(e)^p, \qquad 0 < x_{min} \le x(e) \le 1, \qquad x_{min} = 10^{-3}, \qquad p = 3$$

$$e \in \Omega_{des} = \Omega_{mat} \setminus \Omega_{nonmat}, \ \Omega_{mat} \subseteq \Omega \subseteq R^n, n = 2,3$$

$$\begin{aligned} \textit{subject to} \ \ g_i(x) &\le 0, \qquad i = 1,2,\dots,m \\ h_i(x) &= 0, \qquad i = 1,2,\dots,p \end{aligned}$$

Problem statement

Optimization problem



Objectives

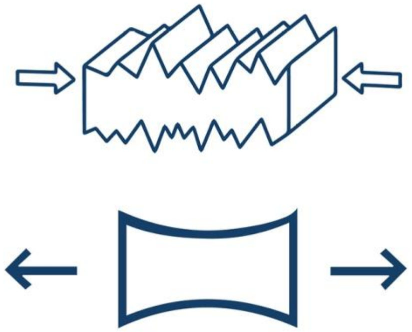
Compliance

$$\min_{x} \; C(x) = U^T K U$$

$$\textit{subject to:} \; \frac{V(x)}{V} \leq f$$

$$KU = F$$

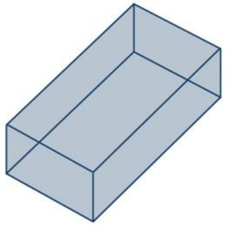
$$0 < x_{min} \leq x \leq 1$$



Stress

$$\min_{x} \; G(x)$$

$$\textit{subject to:} \; \int_{\Omega} x \, d\Omega \leq M_0$$



Volume

$$\min_{x} \; V(x) = \sum_{N} x_e V_e$$

$$\textit{subject to:} \; KU = F$$

$$\frac{C}{C_L} \leq 1$$

$$x_e^{(p-q)} \frac{\sigma_e}{\sigma_{Lt}} \leq 1, \qquad e = 1, \dots, N$$

$$0 < x_{min} \leq x \leq 1$$

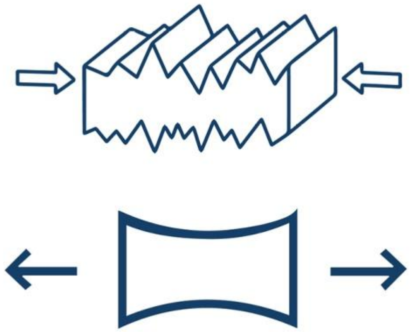
Compliance

$$\min_x \; C(x) = U^T K U$$

$$\text{subject to: } \frac{V(x)}{V} \le f$$

$$KU = F$$

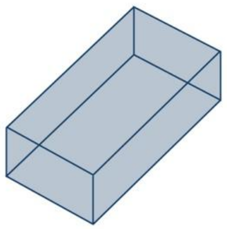
$$0 < x_{min} \le x \le 1$$



Stress

$$\min_x \; G(x)$$

$$\text{subject to: } \int_\Omega x \, d\Omega \le M_0$$



Volume

$$\min_x \; V(x) = \sum_N x_e V_e$$

$$\text{subject to: } KU = F$$

$$\frac{c}{c_L} \le 1$$

$$x_e^{(p-q)} \frac{\sigma_e}{\sigma_{Lt}} \le 1, \qquad e = 1, \dots, N$$

$$0 < x_{min} \le x \le 1$$

Constraints

Constraints

Objectives	Equilibrium*	Min element dimension/ filtering*	Volume	Compliance	Deflection	Principal stresses	Drucker - Prager	Annealing & Manufacturing (dmax)
Volume	✓	✓		✓	✓	✓	✓	✓
Compliance	✓	✓	✓		✓	✓	✓	✓

Constraints

Constraints

Objectives

	Equilibrium*	Min element dimension/ filtering*	Volume	Compliance	Deflection	Principal stresses	Drucker - Prager	Annealing & Manufacturing (dmax)
Volume	✓	✓		✓	✓	✓	✓	✓
Compliance	✓	✓	✓		✓	✓	✓	✓

$$\sum_{e=1}^{N} K_e(x_e)U_e = \sum_{e=1}^{N} f_e \Rightarrow \sum_{e=1}^{N} E(x_e)\, K_e\, U_e = \sum_{e=1}^{N} f_e$$

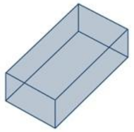
$$x_e = \frac{\sum_{j \in S_e} x_j\, w_j\, v_j}{\sum_{j \in S_e} w_j\, v_j}, \quad j \in S_e \ if \ r_j \le r_{min}, \quad r_j = \left\| x_j - \overline{x}^e \right\|, \quad r_{min} = \frac{d_{min}}{2} \quad w_j = \begin{cases} \frac{r_{min} - r_j}{r_{min}}, & if \ j \in \Omega_s \\ 0 & , \end{cases} \quad j \in \Omega_s \ if \ r_j = dist(e,j) \le r_{min}$$

Constraints

Constraints

	Equilibrium*	Min element dimension/ filtering*	Volume	Compliance	Deflection	Principal stresses	Drucker - Prager	Annealing & Manufacturing (dmax)
Volume	✓	✓		✓	✓	✓	✓	✓
Compliance	✓	✓	✓		✓	✓	✓	✓

Objectives



$$V(x) = \int_{\Omega_{des}} x(e)\, d\Omega_{des}, \quad 0 < x_{min} \leq x(e) \leq 1, \quad e \in \Omega_{des} = \Omega_{mat} \subseteq \Omega \subseteq R^n, n = 2 \qquad \frac{V(x)}{V} \leq f$$

$$\frac{c(x)}{c_L} \leq 1, \quad c(x) = \sum_{e=1}^{N} U_e^T E(x_e) K_e U_e, \ c_L = a_c c_0$$

Constraints

Constraints

Objectives

	Equilibrium*	Min element dimension/ filtering*	Volume	Compliance	Deflection	Principal stresses	Drucker - Prager	Annealing & Manufacturing (dmax)
Volume	✓	✓		✓	✓	✓	✓	✓
Compliance	✓	✓	✓		✓	✓	✓	✓

$$v_k^e < \frac{1}{500} l, \qquad k = 1,2,3,4, \qquad e \in \Omega_{des} = \Omega_{mat} \subseteq \Omega \subseteq R^n, n = 2$$

$$x_e^{(p-q)} \left(\frac{\sigma_{comp,e}}{\sigma_{comp,lm}} \right) \leq 1, \qquad e = 1,2, \dots, N$$

$$x_e^{(p-q)} \left(\frac{\sigma_{ten,e}}{\sigma_{ten,lm}} \right) \leq 1, \qquad e = 1,2, \dots, N$$

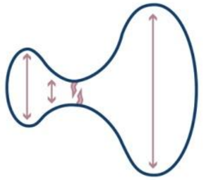
$$\sigma^{eq} = \frac{s+1}{2s} \sqrt{3J_{2D}} + \frac{s-1}{2s} I_1$$

$$x_e^{(p-q)} \frac{\sigma^{eq}}{\sigma_{ten,lm}} \leq 1$$

Constraints

Constraints

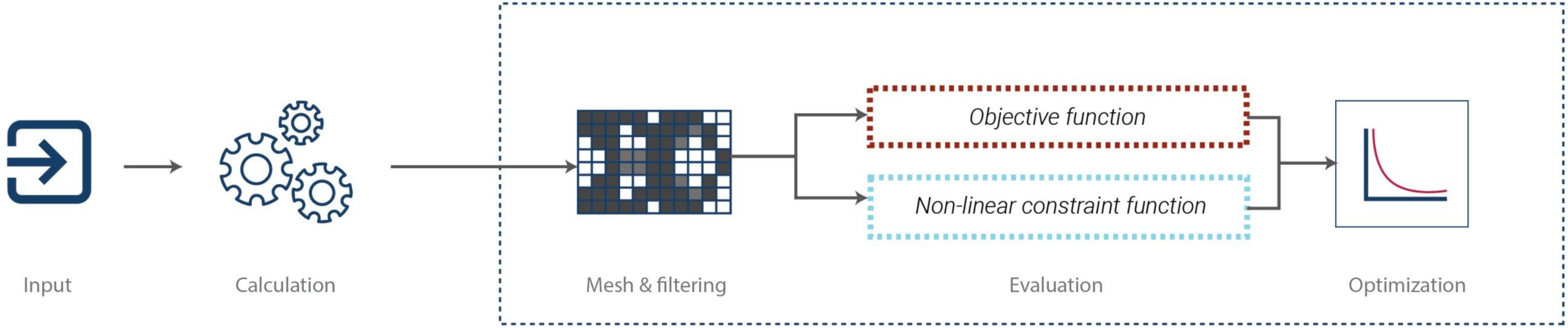
Objectives	Equilibrium*	Min element dimension/ filtering*	Volume	Compliance	Deflection	Principal stresses	Drucker - Prager	Annealing & Manufacturing (dmax)
Volume	✓	✓		✓	✓	✓	✓	✓
Compliance	✓	✓	✓		✓	✓	✓	✓



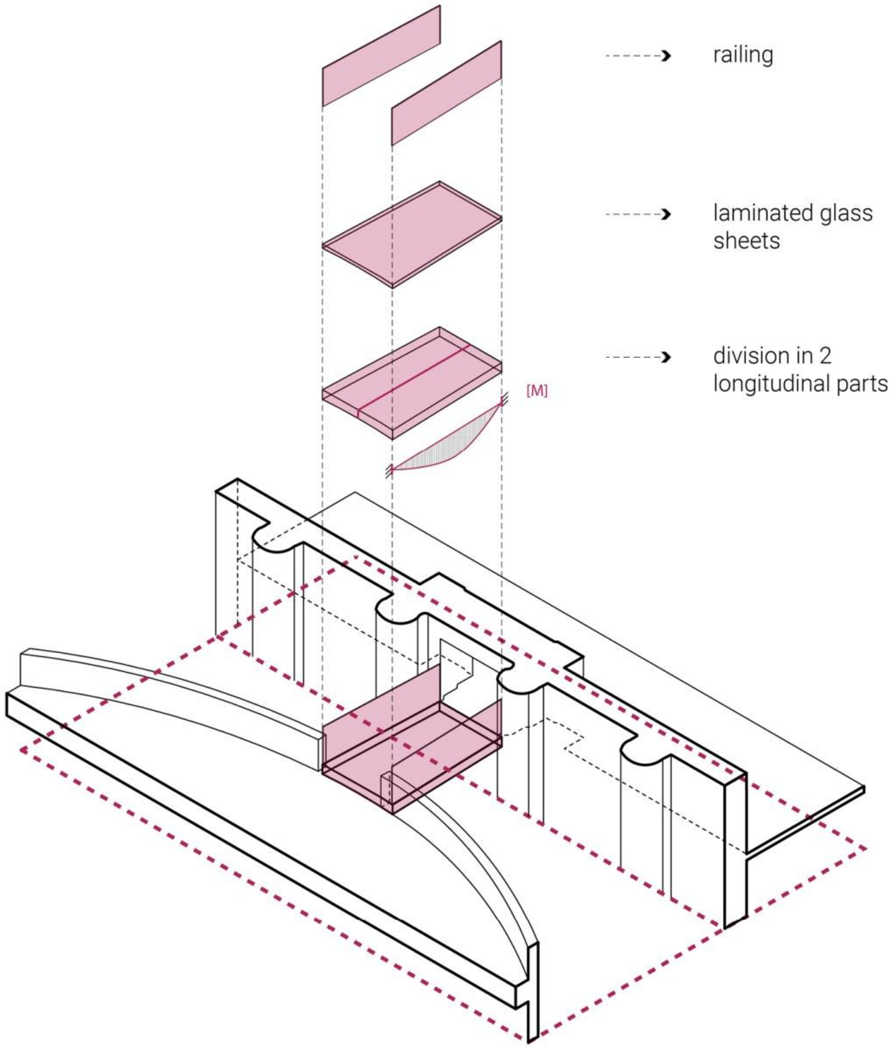
$$d_{max} = \begin{cases} 2 \times d_{min}\,, & if\ T_{ann}(2 \times d_{min}) \leq T_{annmax} \\ \sqrt{\dfrac{T_{annmax} \times \sigma_{res}}{\Delta T \times \dfrac{E \times \alpha_{ex}}{1-\mu} \times \dfrac{\rho \times c_p}{\lambda} \times b}}\,, & if\ T_{ann}(2 \times d_{min}) > T_{annmax} \end{cases}$$

Case Study Application

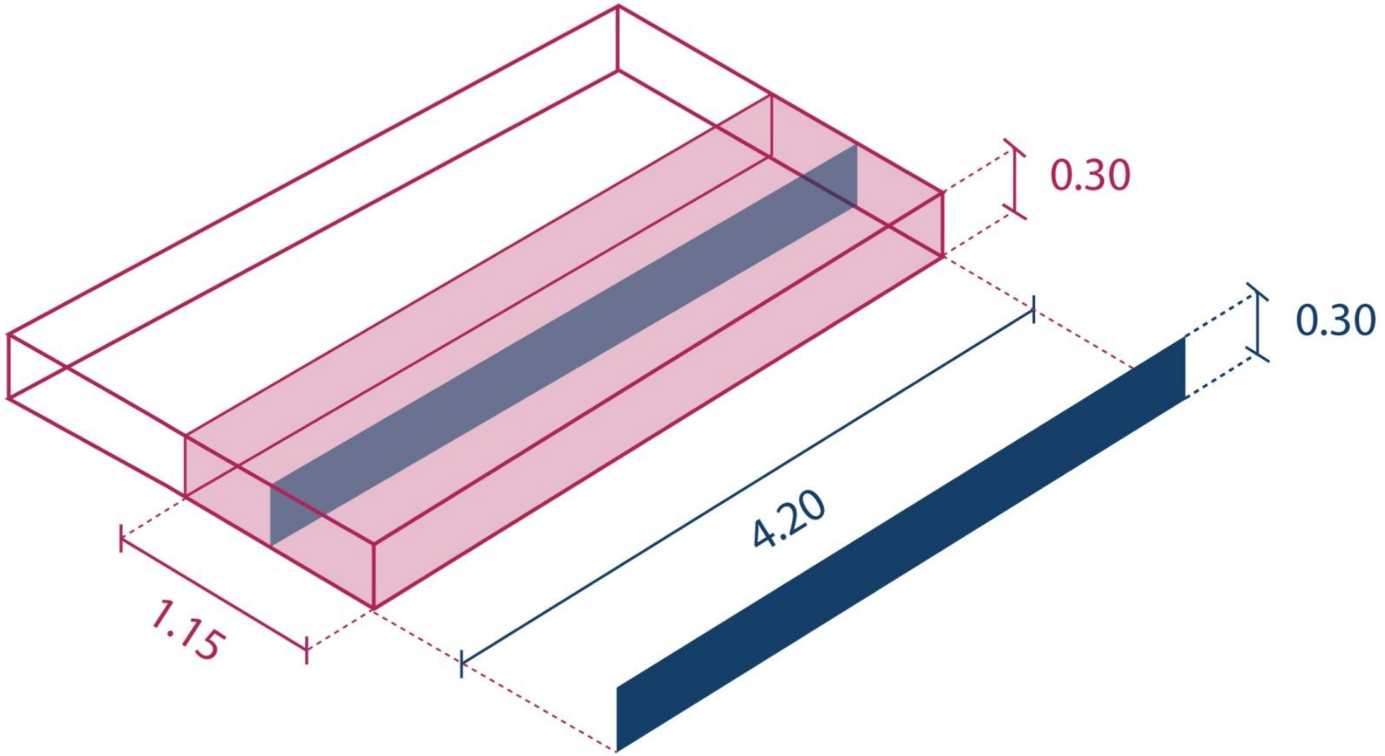
First iterations


Objective function
Non-linear constraint function
Input
Calculation
Mesh & filtering
Evaluation
Optimization

Design guidelines

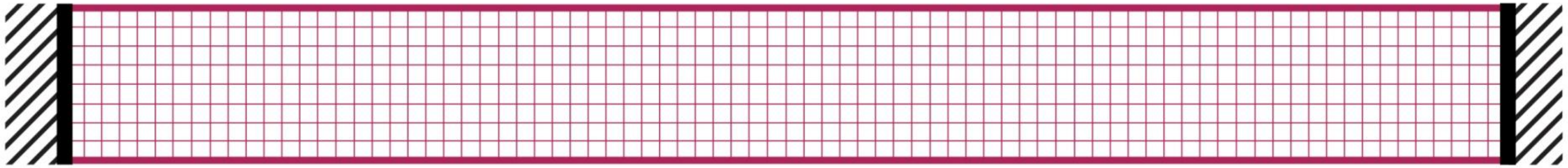


Design domain

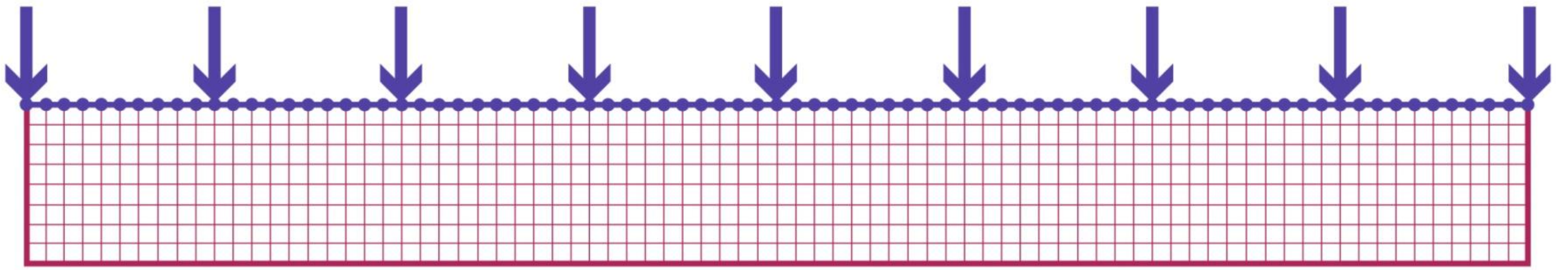



0.30
0.02
4.20

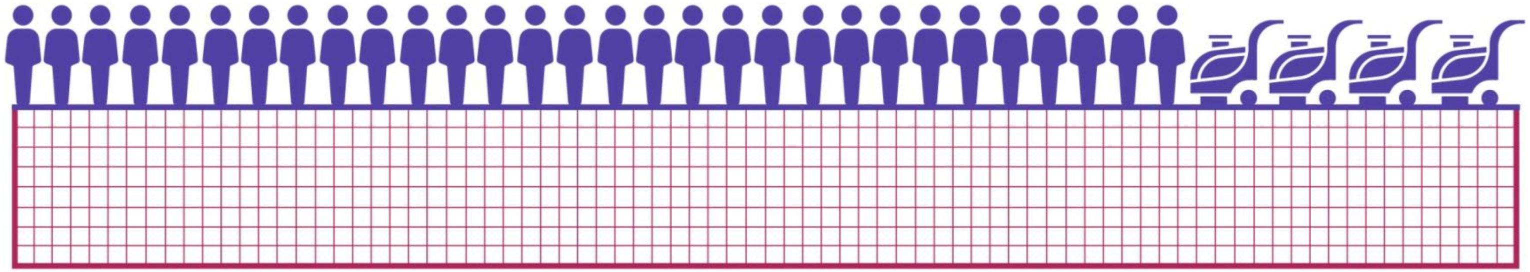
Supports



Loads



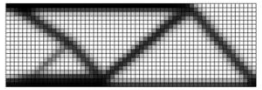
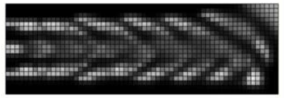
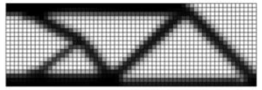
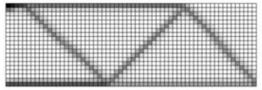
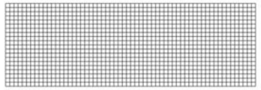
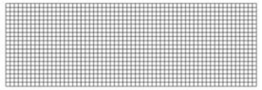
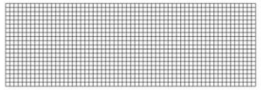
Loads



Calibrating

Constraints

Objectives

	Equilibrium*	Min element dimension/ filtering*	Volume	Compliance	Deflection	Principal stresses	Drucker - Prager	Annealing & Manufacturing (dmax)
Compliance	✓	✓						
Volume	✓	✓						

First iterations

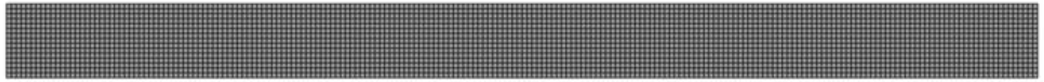
Formulations

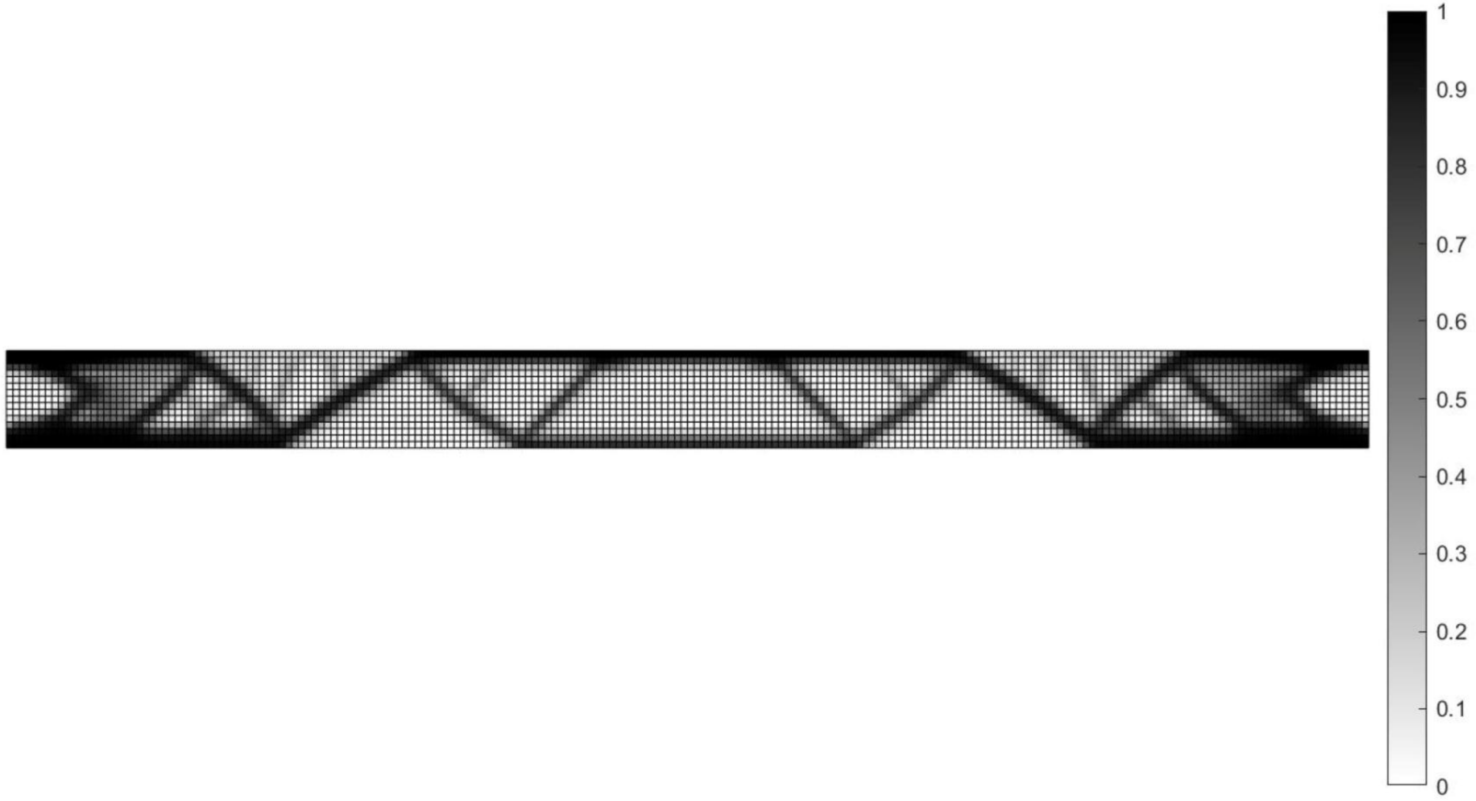
Constraints

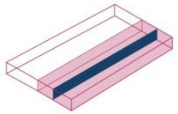
Objectives	Equilibrium*	Min element dimension/ filtering*	Volume	Compliance	Deflection	Principal stresses	Drucker - Prager	Annealing & Manufacturing (dmax)
Compliance	✓	✓	✓		✓	✓	✓	✓
Volume	✓	✓		✓	✓	✓	✓	✓

First iterations

Compliance minimization



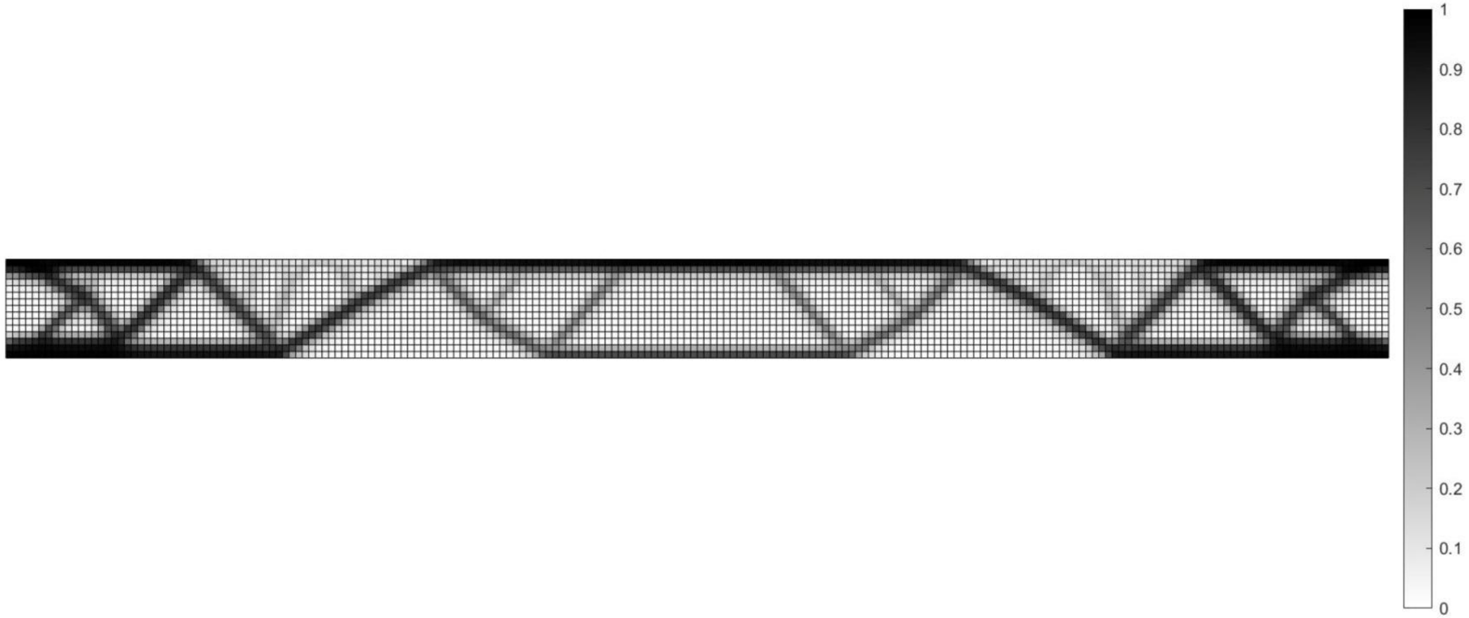


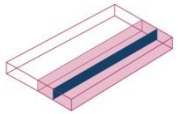


longitudinal section

First iterations

Volume minimization

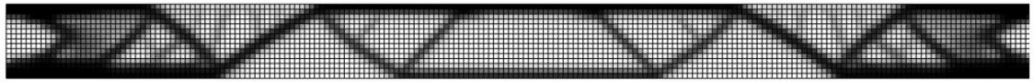




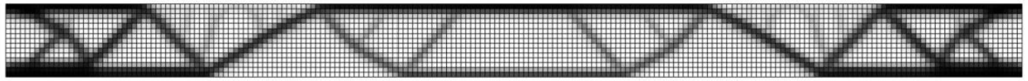
longitudinal section

First iterations

Comparison

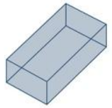


Compliance minimization



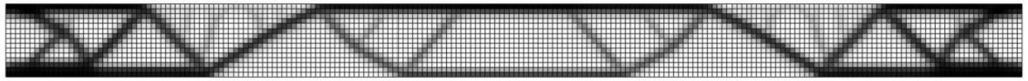
Volume minimization

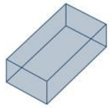
Comparison



1259.1

Compliance minimization





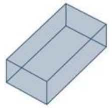
935.8

Volume minimization

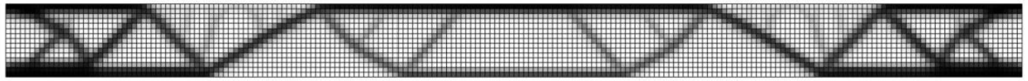
First iterations

Comparison

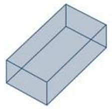
Compliance minimization



1259.1



Volume minimization



935.8

First iterations

Case Study Application
Design exploration

Overview

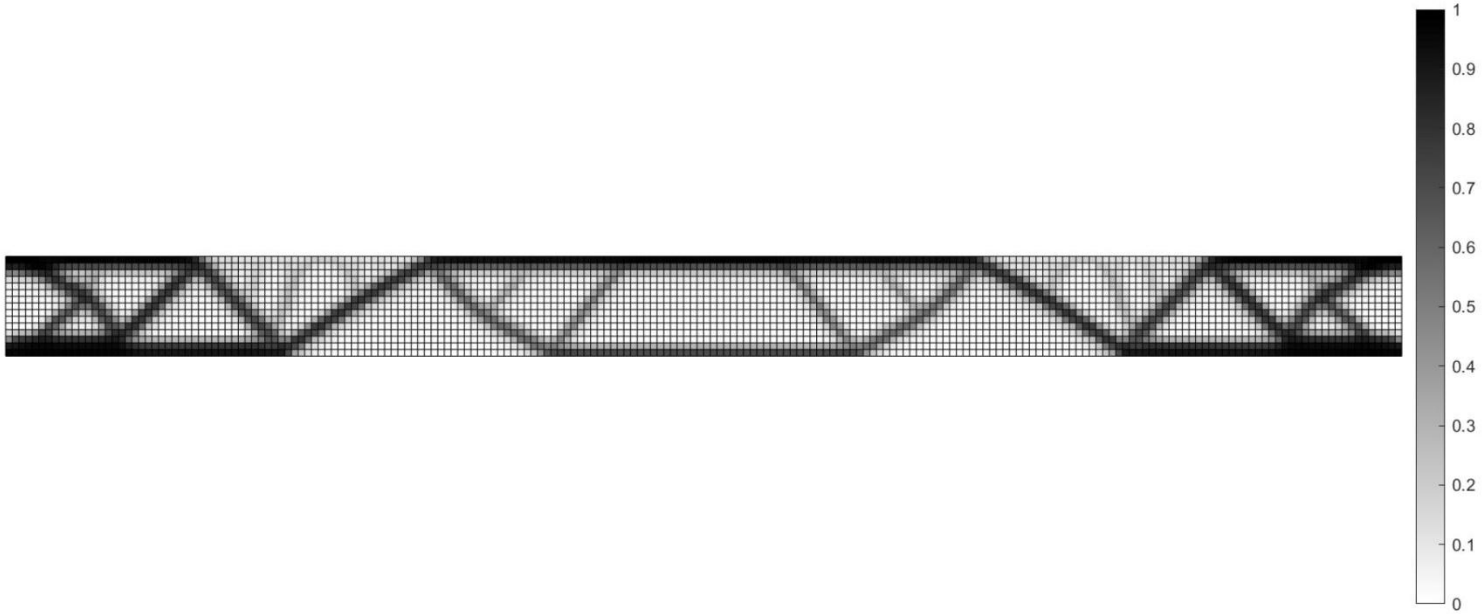
Glass composition

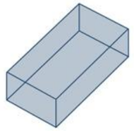
Design domain

Fabrication method

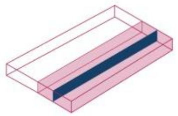
Boundary conditions

Soda lime



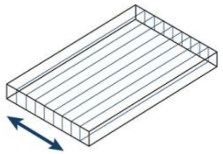


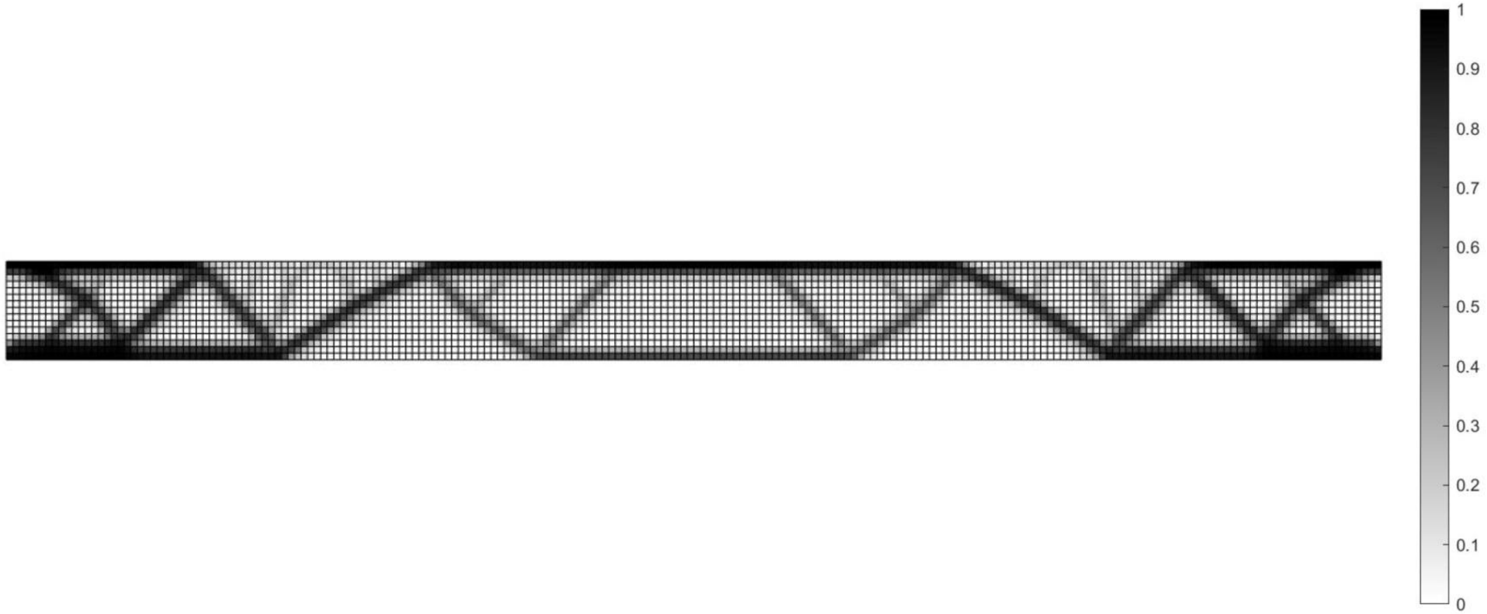
937.3

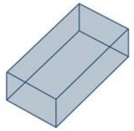


longitudinal section

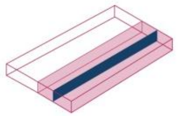
Stacking / Edge supports / Borosilicate





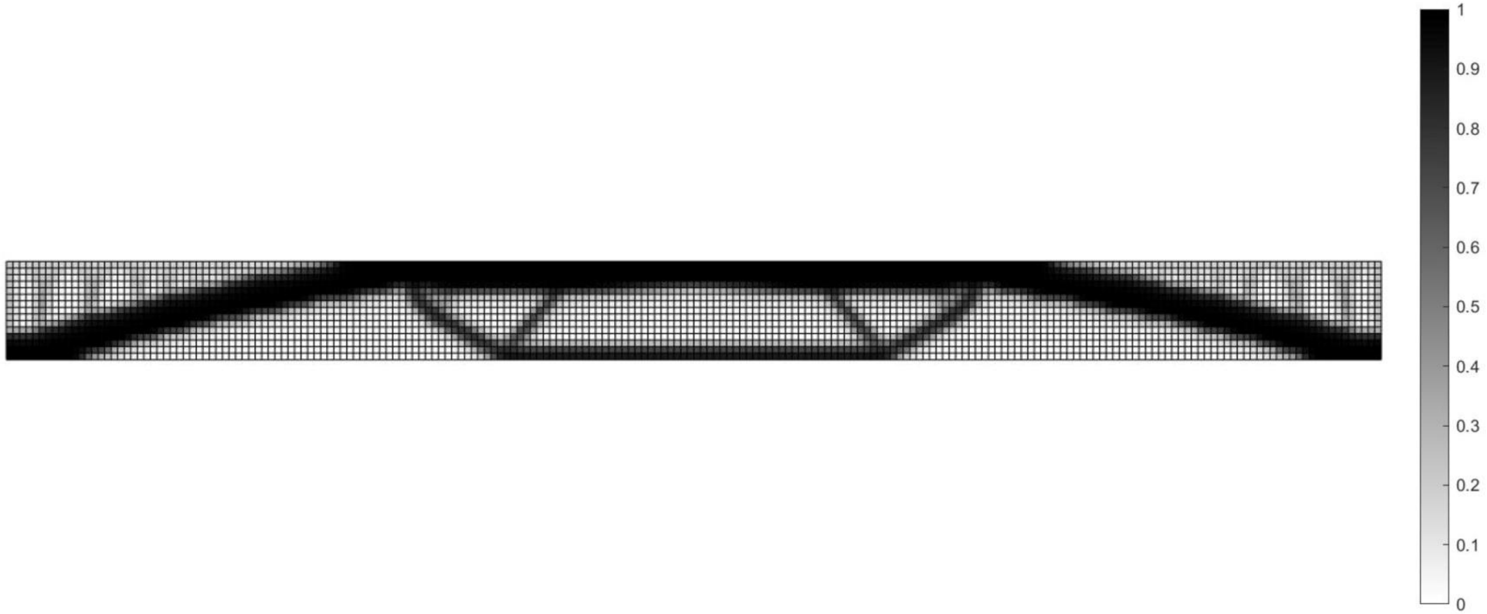


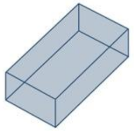
914.7



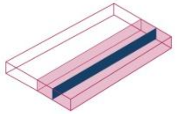
longitudinal section

Casting / Point supports / Borosilicate



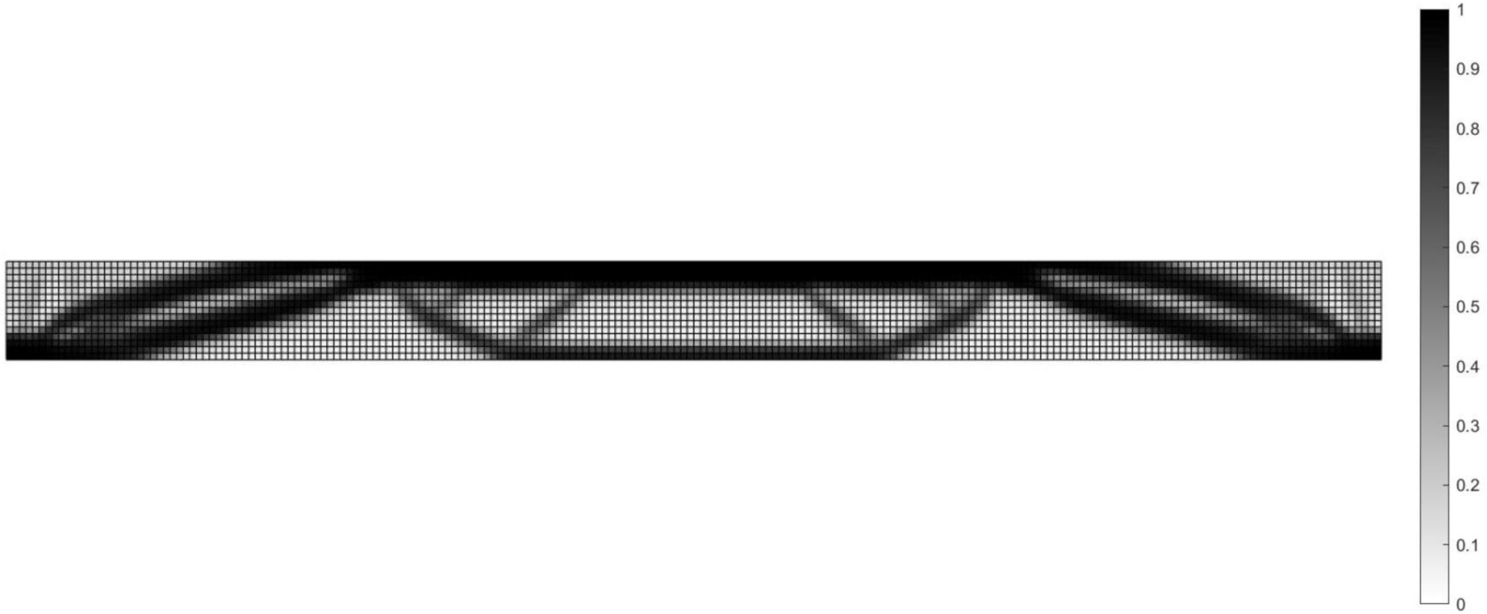


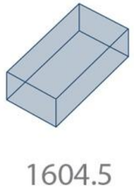
1500.2

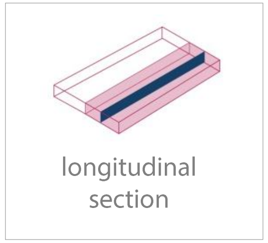


longitudinal section

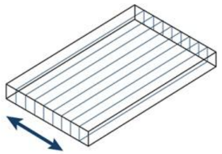
Casting / Point supports / Soda lime



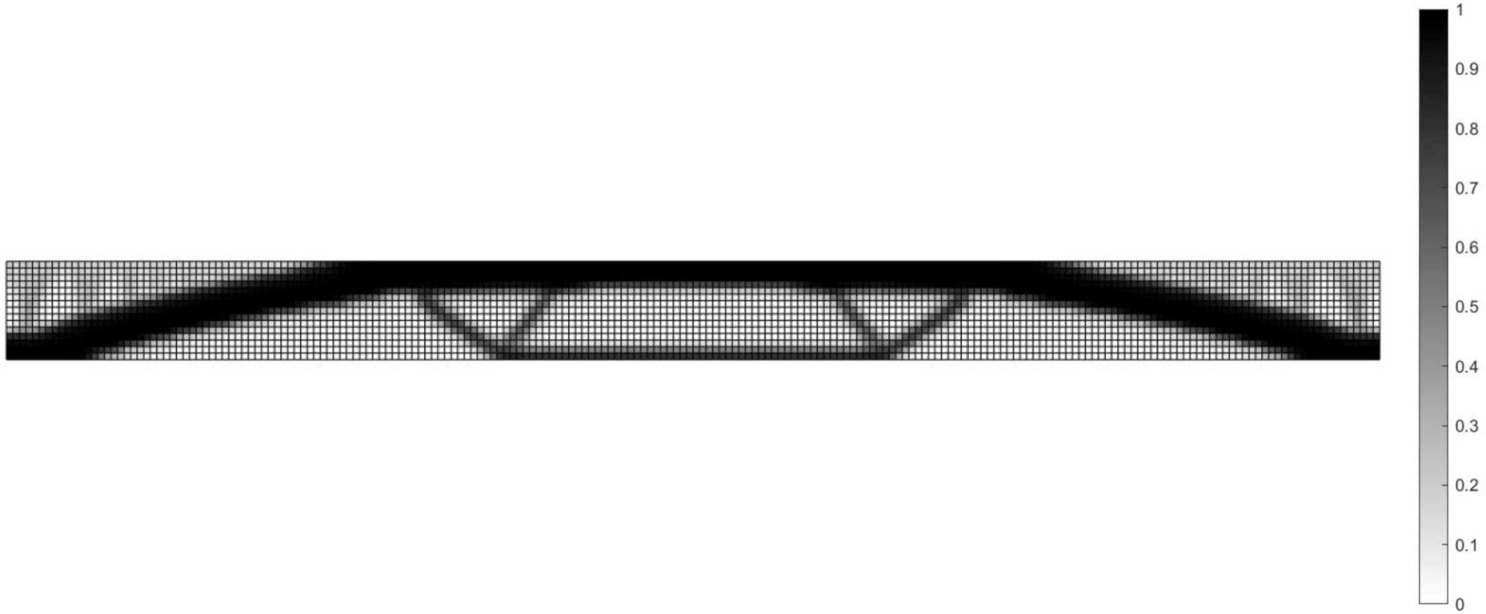


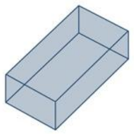


Stacking / Point supports / Borosilicate

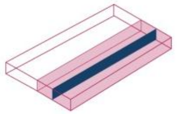


Borosilicate



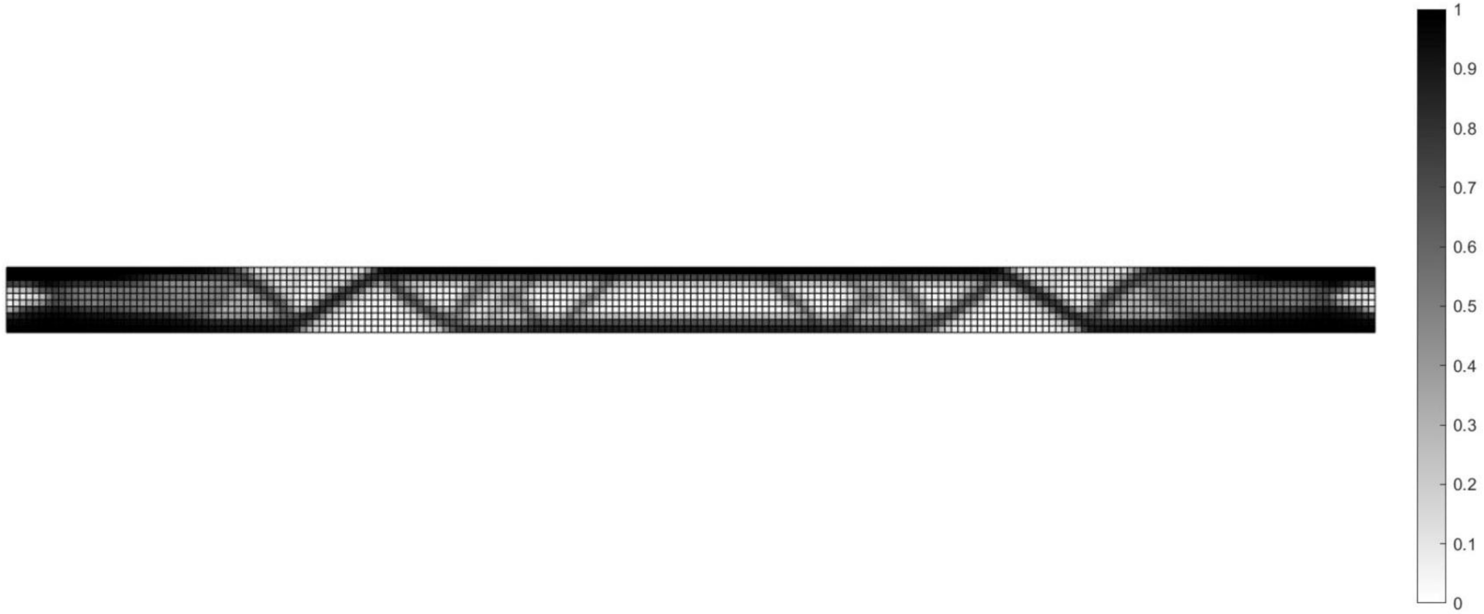


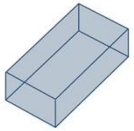
1381.8



longitudinal section

Casting / Edge supports / Borosilicate

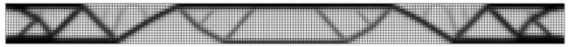
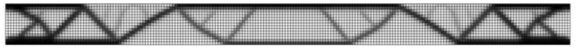
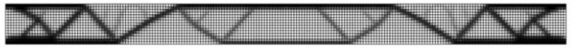
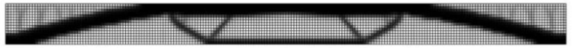
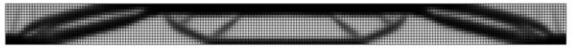
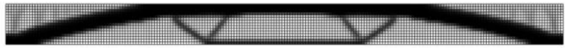
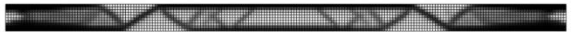




1066.1

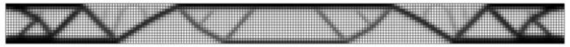
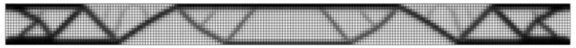
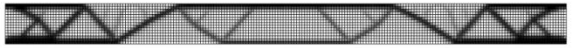
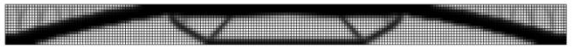
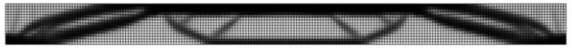
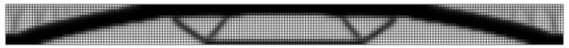
Comparison

Variations

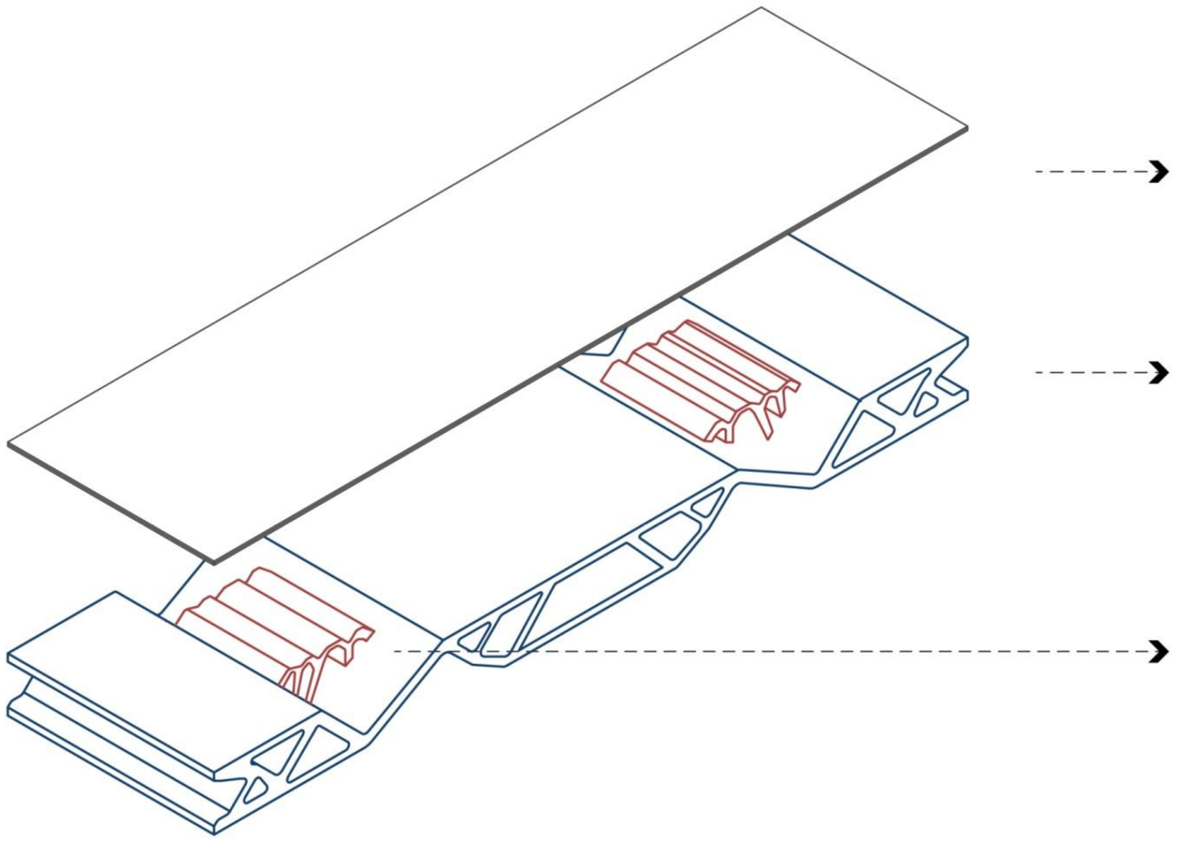
	Result	Volume
Casting Edge Supports Borosilicate		**935.8**
Casting Edge Supports Soda Lime		**937.3**
Stacking Edge Supports Borosilicate		**914.7**
Casting Point Supports Borosilicate		**1500.2**
Casting Point Supports Soda lime		**1604.5**
Stacking Point Supports Borosilicate		**1381.3**
Casting Edge Supports Borosilicate (20cm)		**1066.1**

Final selection

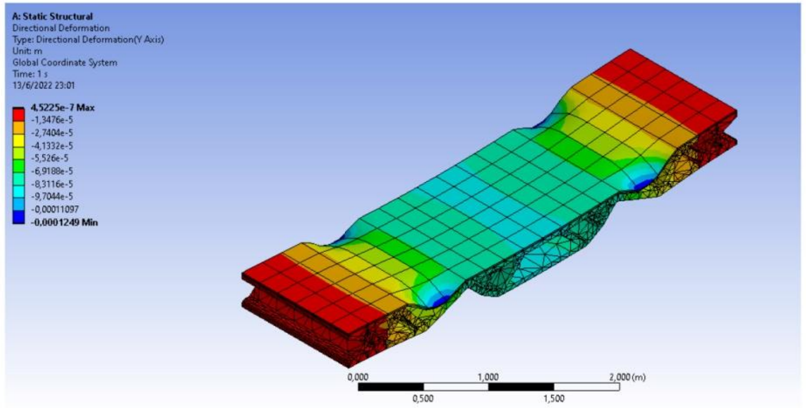
Variations

	Result	Volume
Casting Edge Supports Borosilicate		**935.8**
Casting Edge Supports Soda Lime		**937.3**
Stacking Edge Supports Borosilicate		**914.7**
Casting Point Supports Borosilicate		**1500.2**
Casting Point Supports Soda lime		**1604.5**
Stacking Point Supports Borosilicate		**1381.3**
Casting Edge Supports Borosilicate (20cm)		**1066.1**

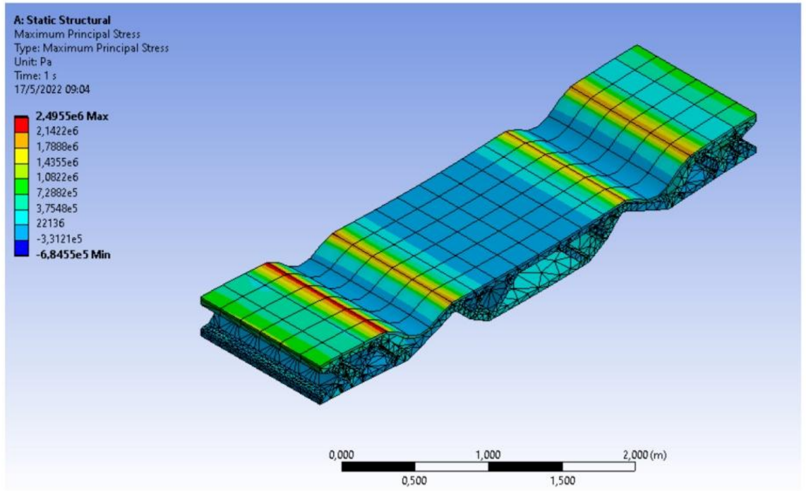
Case Study Application
Final Design


laminated float
glass sheets
main glass part
(densities > 0.6)
secondary glass part
(0.2 < densities < 0.6)

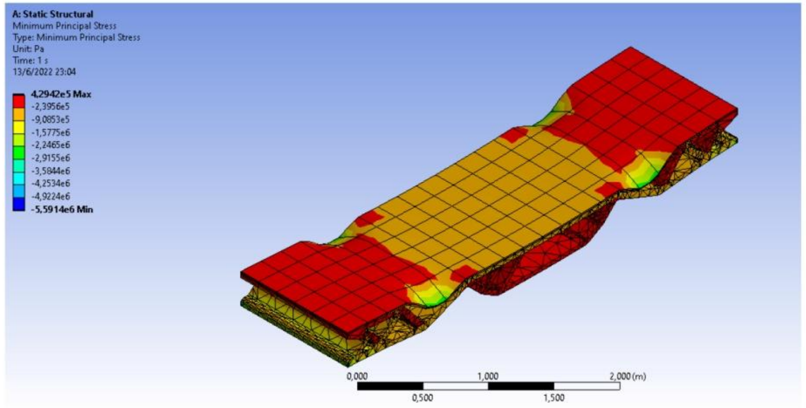
Structural verification



Displacement



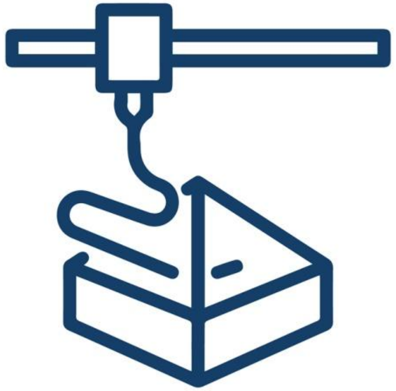
Maximum principal stress



Minimum principal stress

Fabrication

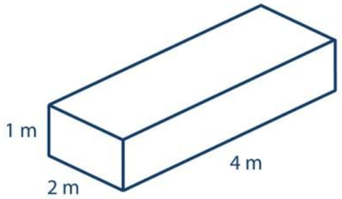
Mould fabrication



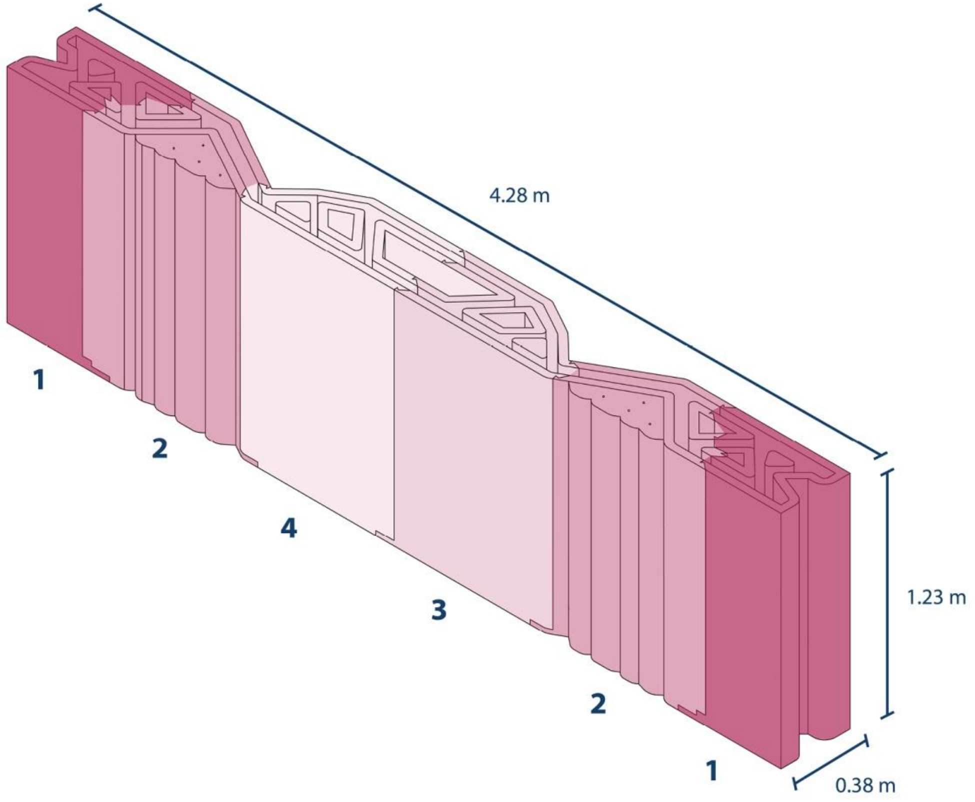
3d printed sand

Damen, W. (2019). Topologically Optimised Cast Glass Grid Shell Nodes

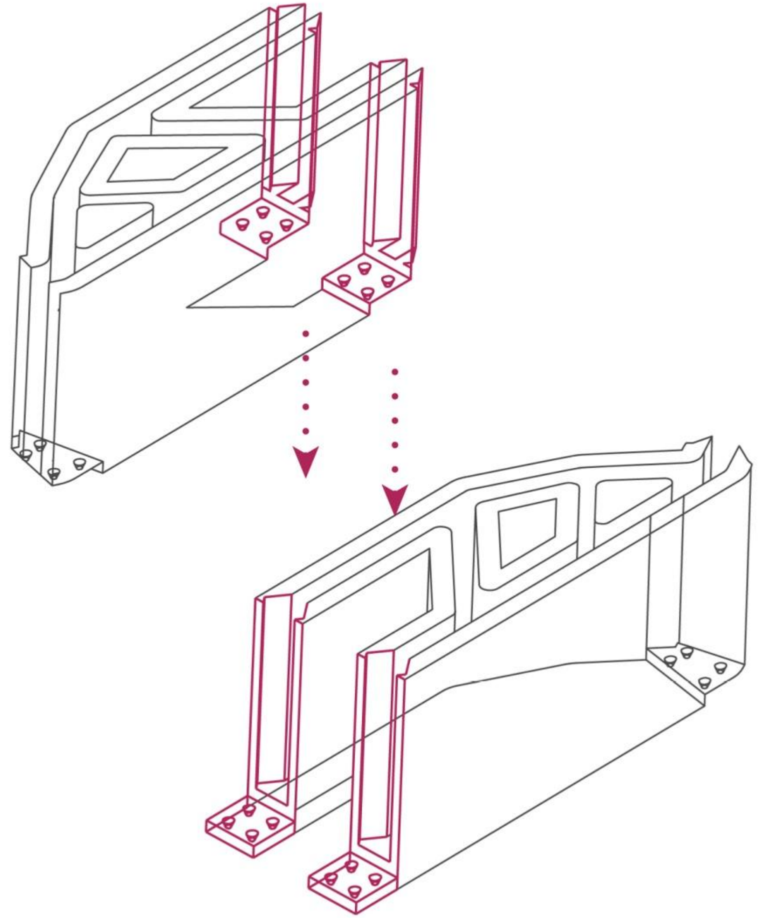
Mould fabrication | Division



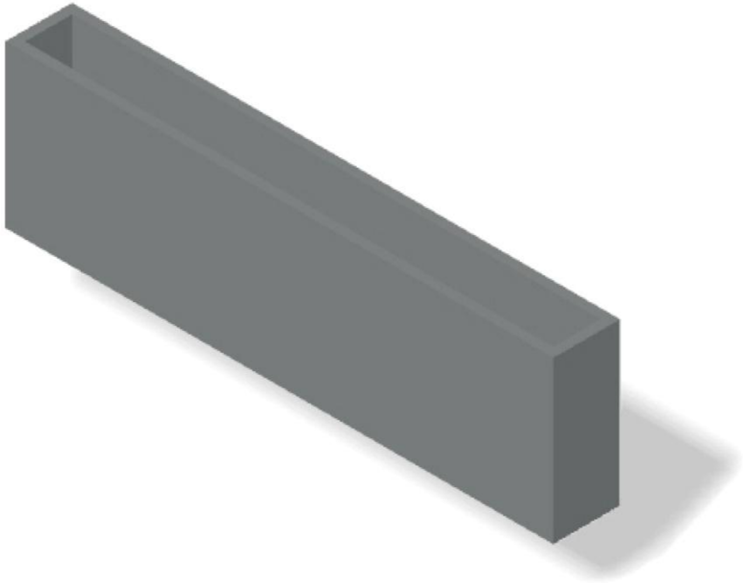
Maximum mould size

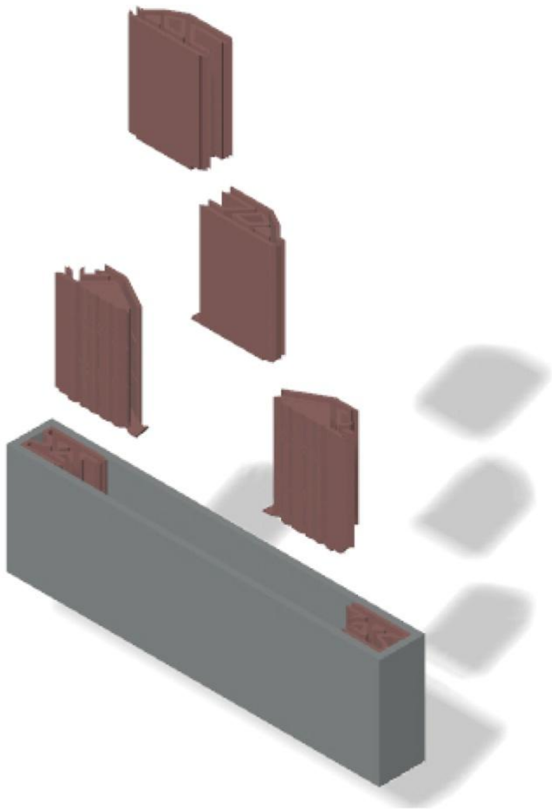


Mould division



Concrete mould base

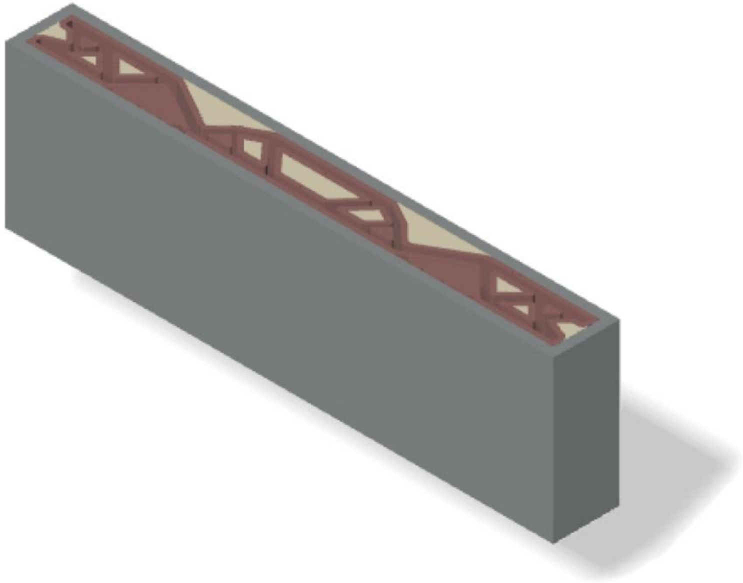


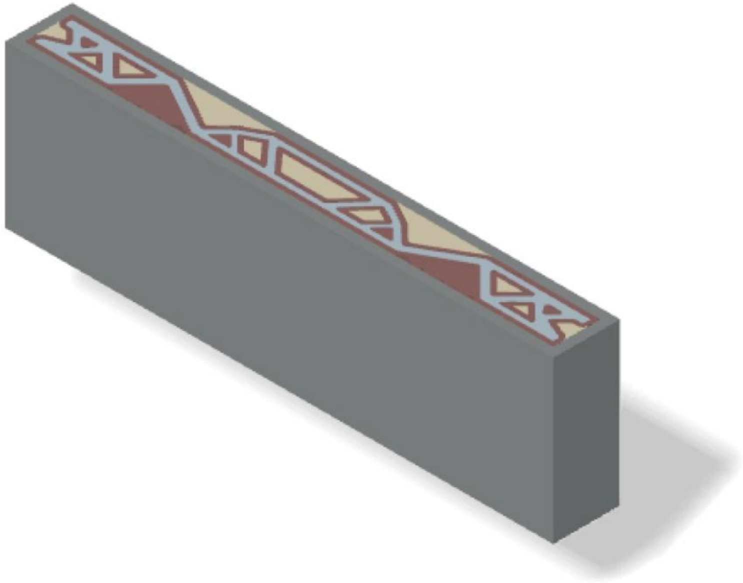


Placement of the
3d printed sand pieces

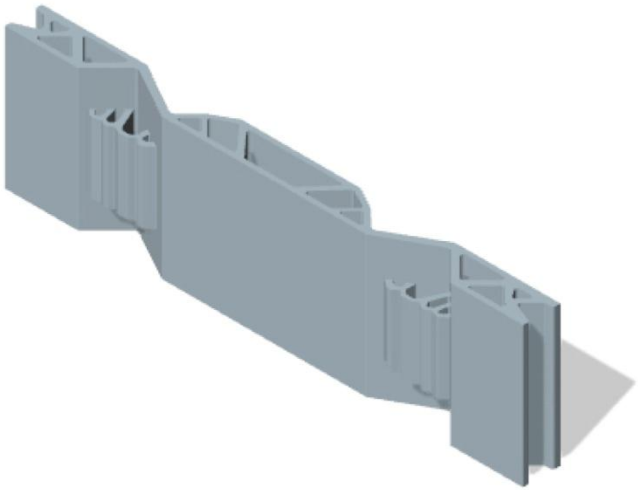
Fabrication III

Infill with sand





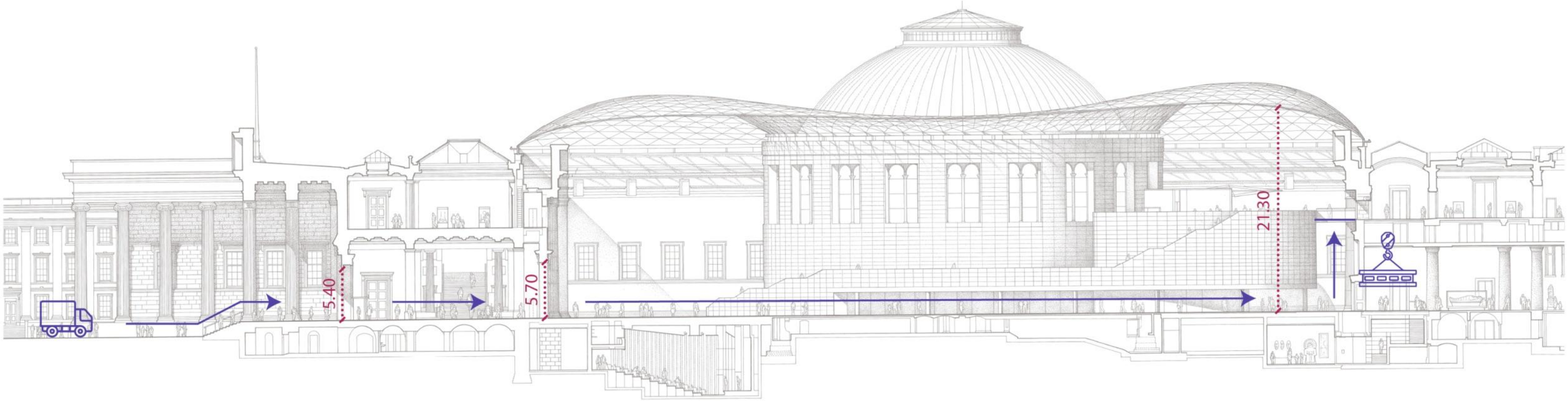
Casting with kiln-casting technique



Mould removal
Post-processing

Building integration

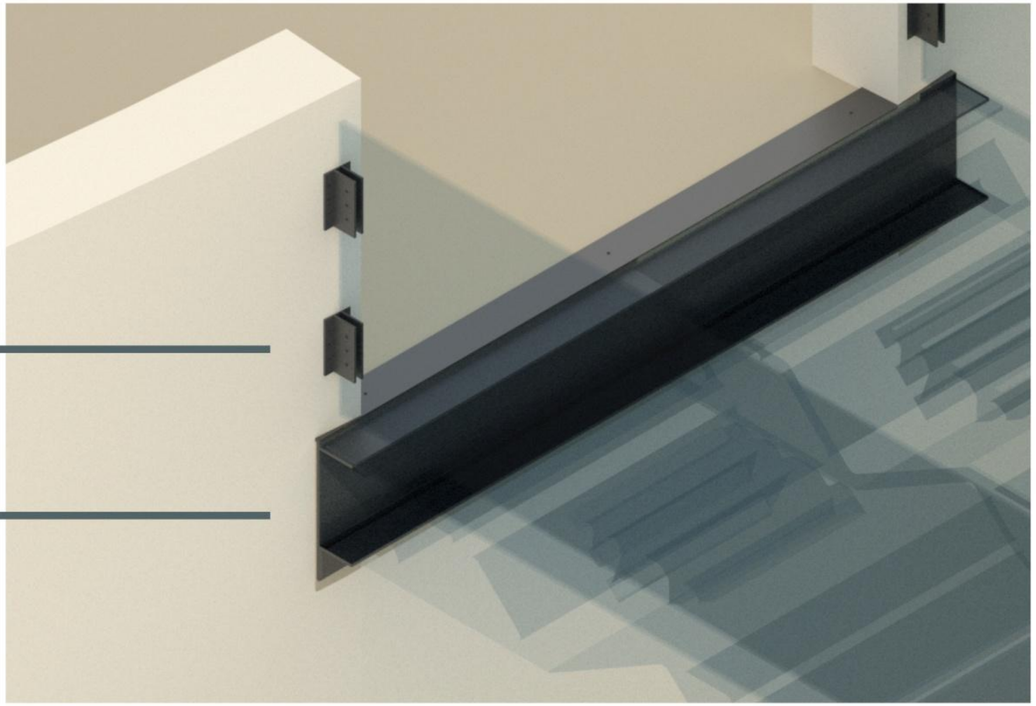
Transport



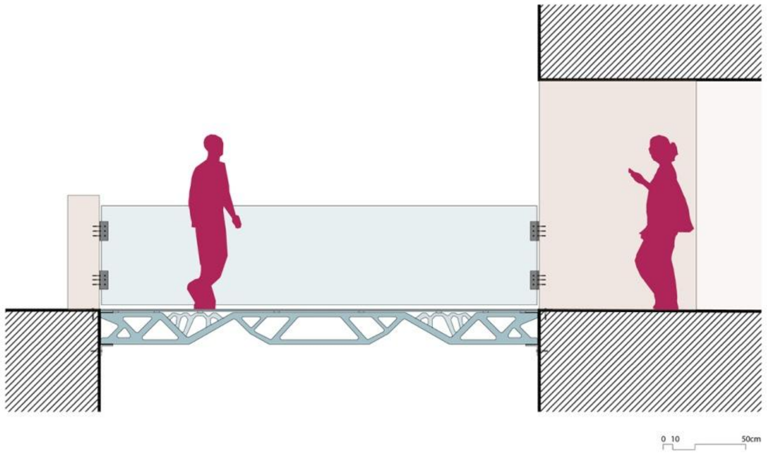
Installation

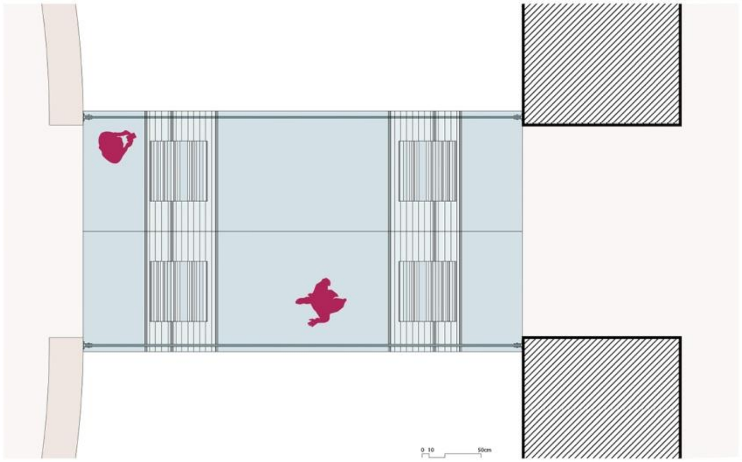
Metal frame
(railing)

Metal clamp
(slab & float glass sheets)



Installation IV: Final result





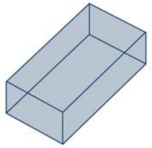
Reflection

Tool evaluation

Annealing time reduction

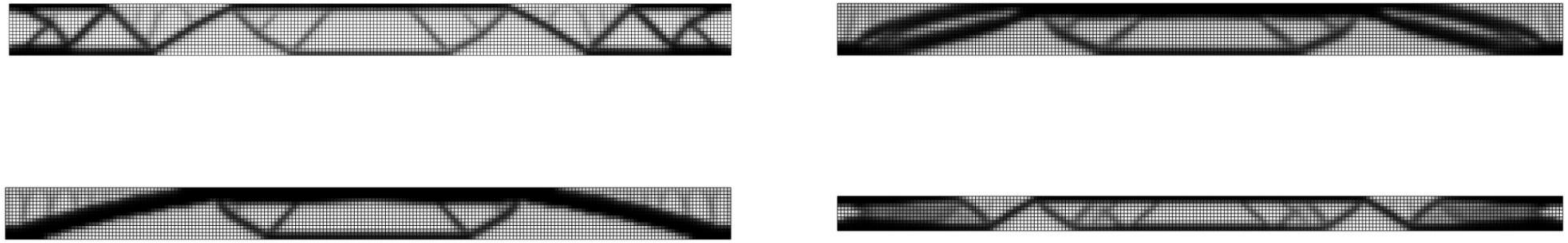
80%

Volume reduction



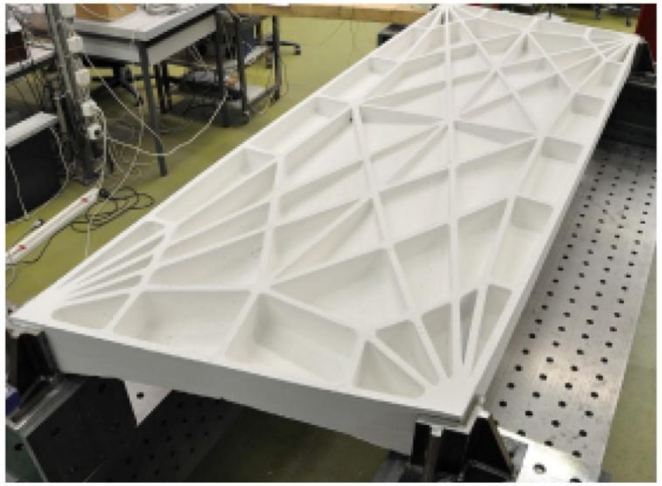
70%

Design exploration

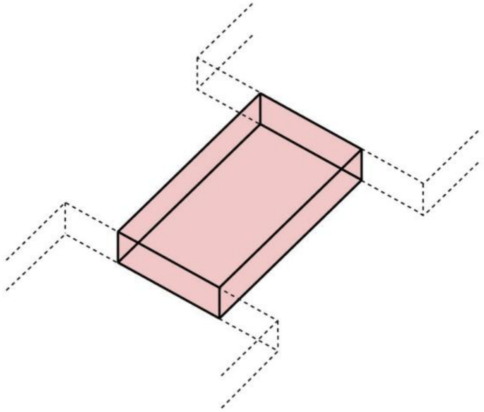


Further applicability

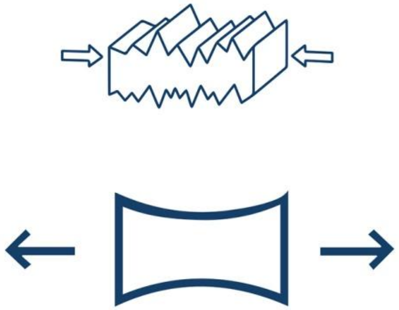
Unreinforced concrete



3d printed sand



3-dimensional code



Testing of structural performance