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# Bayesian inference for deep excavations

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**ABSTRACT:** In the Observational method Ab Initio approach observational feedback is used to optimize a structural design to field conditions that are found to be uncertain in the design phase. As a deep excavation takes place, the supporting retaining wall deflections can be observed by inclinometers which serve as an extra source of information on the structure's performance. Although the Observational method can be beneficial for both safety and economic point of view, limited deep excavations have been executed via this design strategy. This is mainly due to the lack of specification on how measurement-processing can be used to support engineers to assess the structure's safety. This study introduces a methodology for real-time measurement-processing with the use of Bayesian updating. In this methodology retaining wall deflections are used to update the unique field conditions of the construction site. This methodology is applied to a measurement set gathered at the construction of a deep excavation in Groningen, The Netherlands, to demonstrate its potential to supplement the Observational method.

## 1 INTRODUCTION

The design of a geotechnical structure needs to deal with the uncertainties in the heterogeneous and dynamic field conditions that are unique for each construction site (Hicks & Nutall, 2012). In the Observational method Ab Initio approach a flexible design and construction plan is made to allow anticipation to observational data (Peck, 1969). This data is gathered throughout different construction phases to give feedback on the performance of the geotechnical design. This way the design can be optimized to the in-situ conditions as they appear, which can be beneficial from both safety and economic point of view (Powderham & Nicholson 1996). It is believed that the design method is especially suitable for deep excavations in the soft soil conditions of The Netherlands, because of a staged construction sequence and non-brittle failure mechanisms (Nicholson et al. 1999, Korff et al. 2013).

Despite its potentials, the Observational method Ab Initio approach has few applications. A valuable design approach is presented by the CIRIA guideline C760 on retaining walls from the UK (Gaba et al.

2017). The guideline proposes the use of a traffic light system as a basis for decision making. However, due to the integral approach of the Observational method many concerns are raised on how to actually quantify the safety of the structure being built (Patel et al. 2007). This quantification is required to justify interventions and to avoid and reduce the impact of any unforeseen event. In order to do so, it is necessary to subtract crucial information from the observational data.

In this paper a method is shown for real-time data interpretation with the use of Bayesian updating (Ang & Tang, 2007). In the Bayesian update the predicted retaining wall performance, as assessed by a computational model in the design phase, is combined with the information obtained from observations during construction. By describing both sources of information via probability density functions different uncertainties in both the design and construction phase can be weighted in the outcome of the update. As construction continues a Bayesian update can be performed each time new observations are available. Consequently, the retaining wall behaviour as predicted by the computer model can be re-assessed throughout the

different excavation stages.

This paper is structured as follows. First, the principles of the Bayesian updating method are introduced. The method is then demonstrated using a case study of a deep excavation in The Netherlands. This is followed by a more general evaluation of the proposed method, to finally conclude on the value of the Bayesian update in the context of the Observational method for the use of deep excavations in soft soil conditions.

## 2 BAYESIAN UPDATING

### 2.1 Monitoring wall displacements

Spross et al. 2014 promoted the potential of Bayesian inference by presenting a fictive case of a rock pillar. They illustrated the use of Bayesian updating to find a deformation modulus in accordance with a fictive measurement set. To show general validity of his method, they concluded that more case studies of different geotechnical kinds are needed. This work was used in the underlying study to look into the potential of Bayesian updating for processing retaining wall displacements for deep excavations. Often strict SLS requirements are set on the maximum allowable deflection of the retaining wall. Therefore, the performance of the retaining wall can be directly related to the movements that occur during sequential excavation (Gaba et al. 2017).

At the beginning of construction there is only one source of information available on the expected deformations of the wall, namely the prediction made by a computational model. This prediction is based on the model input that represents the engineers' assumptions of the initial field conditions. Inevitably, those assumptions carry uncertainty, the magnitude of which depends on site-investigation efforts. The goal of Bayesian updating is to reduce these initial uncertainties by adding a second source of information, which is the monitored wall displacement. Each time measurements are taken, this observational data is combined with the prediction to form a new updated prediction on the expected deformations. This combination is done by the equations of the Bayesian update (Ang & Tang, 2007) that requires the quantification of all the uncertainties in both the design, construction and monitoring of the retaining wall.

### 2.2 Predictive uncertainty

The prediction contains (1) uncertainty in the model input and (2) inaccuracy of the computational model used (De Wolf, 2019). The uncertainty in the model input follows directly from the heterogeneity of soil strength properties and is commonly described by a normal distribution (CUR, 2008). Other input uncertainties can be taken into account as well, for example the natural fluctuations of water levels. The impact of the variance of each input parameter can be first accessed by a sensitivity analysis. Each parameter is then assigned with a sensitivity score that represents the impact of its variance on the modelled outcome. Because each construction phase is different, the sensitivity score varies per phase. Hence, during different construction phases, different soil parameters can become relevant or irrelevant for the structure's stability.

Consequently, the variance of the prediction on the structure's performance can be quantified by means of a Monte Carlo simulation. For this simulation only the parameters with significant sensitivity scores should serve as stochastic input. The Monte Carlo simulation then returns an output distribution with properties  $\mu_{MC}$  and  $\sigma_{MC}$  representative for the uncertainty in the expectation of retaining wall displacements (Fig. 1). For calculations performed with the Spring model, this is typically a lognormal distribution (De Wolf, 2019).

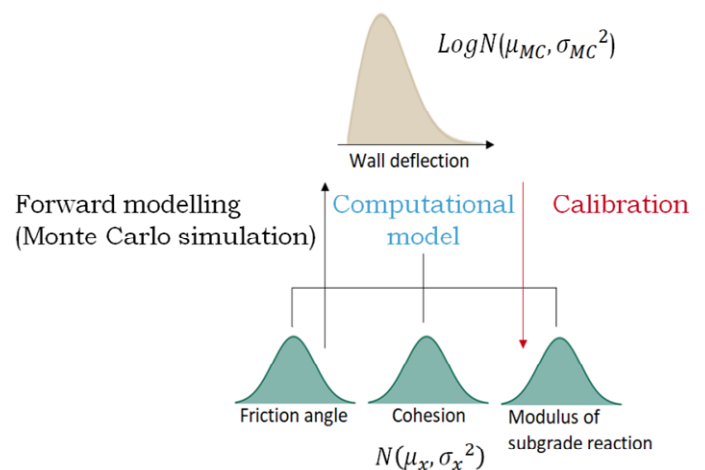


Figure 1. Principle of Monte Carlo simulation

To recognize the inaccuracy of the calculation model to predict retaining wall displacements an additional model error is added to the output distribution of the Monte Carlo simulation. For the Spring model, an absolute error of 10% is added in

accordance with the Dutch CUR standards. Finally, the properties of the lognormal distribution can be described by mean  $\mu_d$  and standard deviation  $\sigma_d$ .

### 2.3 Measurement uncertainty

Measurements of the retaining wall displacements have an uncertainty introduced by:

(1) The inherent variability between the  $n$  measurement devices:  $\sigma_{inh}^2$ .

(2) Error of the measurement device:  $\sigma_{m.e.}^2$ . Both these factors can be combined via:

$$\zeta^2 = \ln[\sigma_{inh}^2 + \sigma_{m.e.}^2] \quad (1)$$

with  $\zeta^2$  being the variance of the measurement distribution. Other errors can be added as well to equation (1) (Spröss et al. 2014). The mean of the measured dataset is indicated by  $\bar{x}$ .

Note that the natural logarithm in equation (1) is used to transform the normally distributed variances to their lognormal equivalent. This is done in order to combine these measurements with the lognormally distributed prediction in the Bayesian update.

### 2.4 Bayesian update

Once the first construction phase has been completed the measurement data obtained is added to the original model prediction with initial properties  $\mu_d, \sigma_d$ . The outcome of the Bayesian update, that follows from applying equations (2) and (3) (Ang & Tang, 2007), is therefore indicated by the superscripts  $\mu_d'$  and  $\sigma_d'$ .

The Bayesian update can be repeated after the second construction phase has finished: the  $\mu_d'$  and  $\sigma_d'$  obtained after processing the first measurement set will be updated again with a second measurement set, leading to  $\mu_d''$  and  $\sigma_d''$ . This can be repeated each construction phase such that the model prediction is updated with all the information on hand.

$$\mu_d' = \frac{\mu_d \left( \frac{\zeta^2}{n} \right) + \sigma_d^2 \ln \bar{x}}{\left( \frac{\zeta^2}{n} \right) + \sigma_d^2} \quad (2)$$

$$\sigma_d' = \sqrt{\frac{\sigma_d^2 \left( \frac{\zeta^2}{n} \right)}{\left( \frac{\zeta^2}{n} \right) + \sigma_d^2}} \quad (3)$$

### 2.5 Calibration

Each updated prediction is associated with a new set of input parameters for the model. This parameter set can be found by calibration, which is in principle the opposite of a Monte Carlo simulation (Fig. 1): Given a new range of displacements, the calibration is looking for the representative stochastic input distributions. As more observations are added, ideally, the range of possible displacements decreases. This implies that the initially assumed parametric uncertainty decreases as well. Consequently, the calibrated parameter distributions can be used to assess the safety of the build geotechnical structure via the principles of Eurocode 7 (Vrijling, 2015).

The above described methodology is summarized by Figure 2.

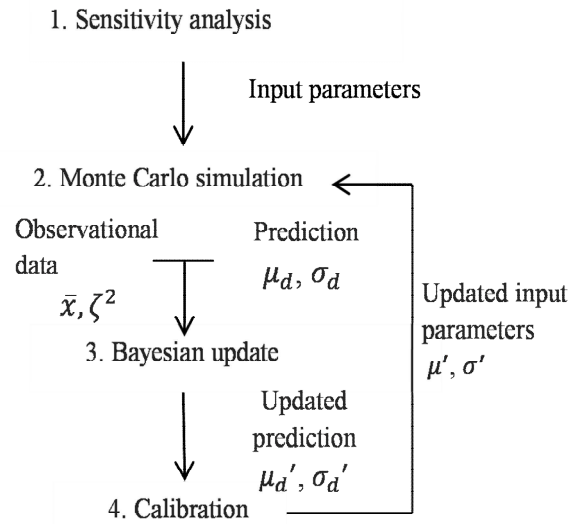


Figure 2. Outline of the methodology.

## 3 CASE STUDY

### 3.1 Project description

The methodology is demonstrated by means of a case study that concerns the construction of a 2-layered basement in the north of The Netherlands. Due to the geological history, strong heterogeneity is expected at the construction site. Additional uncertainty is introduced by basing the design on a rather simplified 2D spring model (Fig. 3), whereas the asymmetric shape of the deep excavation (Fig. 4) might actually lead to a more favourable distribution of strut forces (Fuentes et al. 2018).

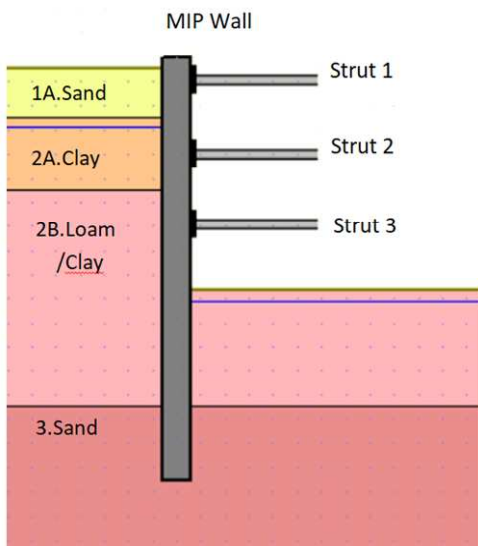


Figure 3. Implemented design with stratigraphy.

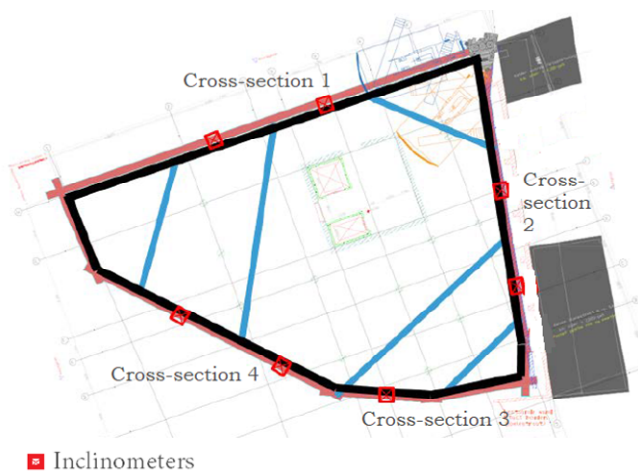


Figure 4. Top view of the building pit.

The implemented design with 3 layers of struts was based on a limit state design and a soil profile taken from the site-investigation (Table 1 and Fig. 3). To avoid any installation problems, a Mixed in Place (MIP) wall was selected. During the excavation phases, see Table 2, no difficulties were obtained. In fact, the actual displacements in the field were kept very limited. For cross-section 4, the observed maximum deflection was less than 5 mm. In this case study, the data of the 2 inclinometers of cross-section 4 are analysed with the goal to find representative parameters for loamy clay layer 2B. In the site-investigation no laboratory tests were performed. Instead, Table 2B of the Dutch National Annex NEN9997-1 was used to derive the characteristic soil parameters from CPT data. Therefore, the coefficients of variations (COV) are

adopted from the Annex as well to stochastically describe the uncertainty of these parameters. Their values are stated in Table 1.

Table 1. Characteristic values for soil layers.

Soil parameters				
	1A.Sand	2A.Clay	2B.Loam/clay	COV
$\gamma_{sat}$ [kN/m <sup>3</sup> ]	19	19	21.5	5%
$\varphi$ [°]	30	22.5	28	10%
$\delta$ [°]	30	22.5	28	10%
$c$ [kPa]	-	-	2.5	20%
$OCR$ [-]	-	-	3.0	20%
$k_I$ [kN/m <sup>3</sup> ]	1.2E+04	4.0E+03	6.0E+03	20%

Table 2. Construction phasing.

Phase #	Description
1	Installation of MIP wall at +7.6 m.
2	Excavation +6.1 m, installation 1 <sup>st</sup> layer of struts.
3	Excavation +3.52 m, installation 2 <sup>nd</sup> layer of struts.
4	Excavation +0.6 m, installation 3 <sup>rd</sup> layer of struts.
5	Excavation: final depth -2.0 m.
6	Installation of concrete floor.
7	Construction of basement floors, stepwise removal of struts.

### 3.2 Sensitivity analysis

At first a sensitivity analysis is performed to see what parameters dominate the structural response. To take into account the possible underestimation of the structural force distribution, it was chosen to vary the stiffness modulus  $EI$  with 20% alongside the soil strength parameters.

The results of the sensitivity analysis are presented in Figure 5. The friction angle of top layer 1A only has significant influence to the wall deformation for the first part of the excavation. It can be seen that the  $EI$  becomes relevant once a certain depth of excavation has been passed. The strength properties of layer 2B contribute to the structure's performance throughout the whole excavation process. Its friction angle has an increased dominant impact during construction phases 4 and 5.



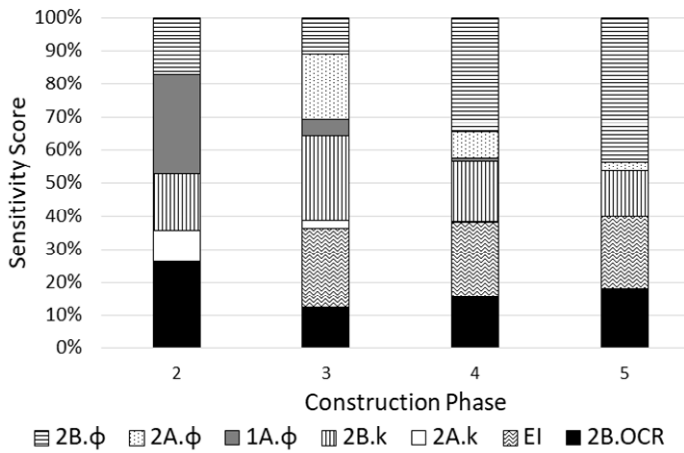


Figure 5. Result of the sensitivity analysis.

### 3.3 Bayesian update

Next, the Bayesian update is performed. The inclinometer error  $\sigma_{m.e.}$  is set to be 1.36 mm (De Wolf, 2019). This is a rather large standard deviation compared to the actual observations that were less than 5 mm. Therefore the variance of this error in the Bayesian update is reduced for each construction phase via equation 4.

$$\zeta^2 = LN \left[ \sigma_{inh.}^2 + \frac{\sigma_{m.e.}^2}{n_{phase} - 1} \right] \quad (4)$$

Figure 6 illustrates the Bayesian updates throughout the excavation phases 2 to 5. It can be seen that the shape of each update gets smaller in the process. As more observational data is added, the incorrectness of the originally assumed parameter

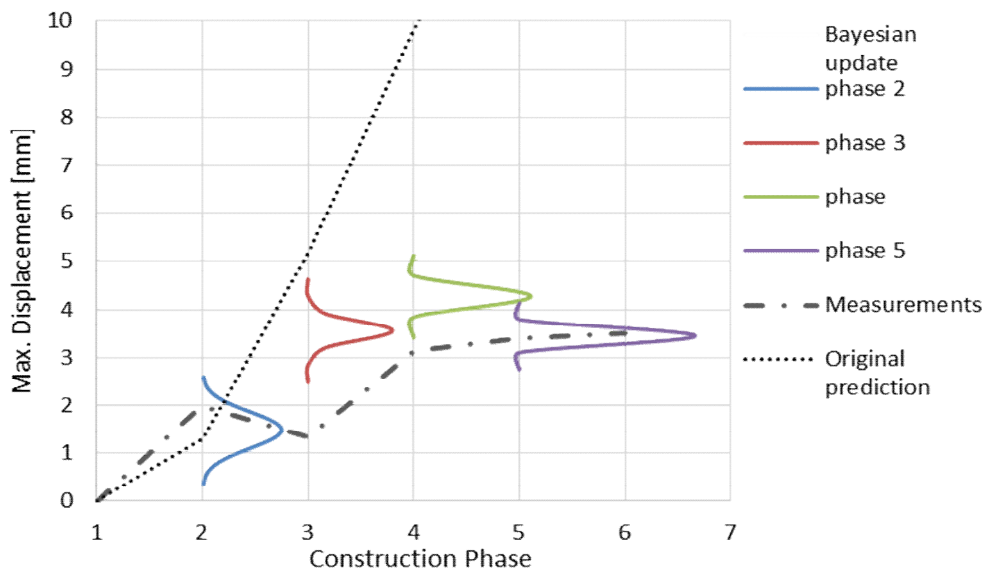


Figure 6. Bayesian update for construction phases 2-5.

input is confirmed as the Bayesian update shifts more towards the measurements. At phase 5 the Bayesian update has fully converged. This means that at that phase the updated prediction coincides with the field observations. The parameters found by the calibration in phase 5 should thus be representative for the overall observed structural deflections. Figure 7 presents the results of a forward simulation of the Spring model, performed with these calibrated input parameters (Table 3).

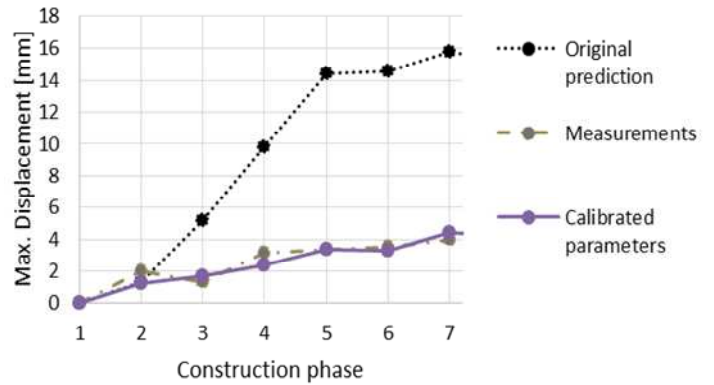


Figure 7. Forward simulation with calibrated parameters of phase 5 shows a fit towards the overall observed structural displacements.

### 3.4 Calibrated results

Figure 8 illustrates the change of the probability density function (pdf) of the friction angle of layer 2B for different phases. The calibrated results follow the outcome of the sensitivity analysis. It can be seen in Figure 5 that the biggest change in the mean  $\mu$  is at phase 4 as its sensitivity score strongly increased relative to phase 3. This update in  $\mu$  can be

noticed by the shift of the pdf of phase 4 towards the right. Consequential calibration of phase 5 confirmed this updated  $\mu$ , leading to a significant decrease of standard deviation  $\sigma$ .

Such results could not be found for every input parameter. For both the stiffness  $EI$  and the modulus of subgrade reaction  $k_l$  of layer 2B the coefficients of variations remained around 10%. As noticed during the first calibration phases, many mutual combinations were possible between the two parameters leading to the same calculated retaining wall displacement (Fig. 9). This means that a slightly higher modulus of subgrade reaction, with a lower stiffness value, leads to the same outcome. This result makes sense as both parameters have a comparative sensitivity score with a comparable effect on the outcome of the Spring model as can be derived from the sensitivity analysis.

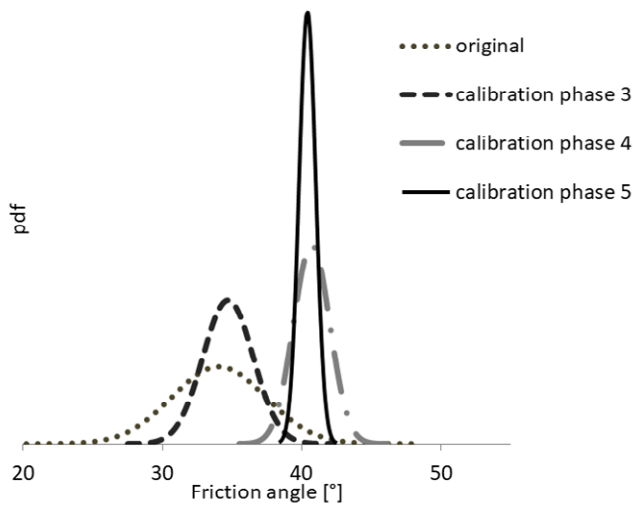


Figure 8. Calibrated friction angle of layer 2B.

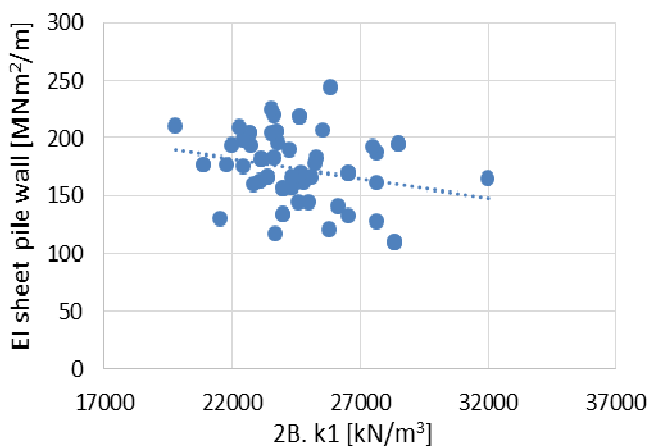


Figure 9. Possible parameter input combinations with same displacement as outcome as found for calibration of phase 3.

Table 3. Mean parameter values for MIP wall and soil layer 2B assumed in design phase (original) and calibrated via Bayesian update in phase 5.

	Mean parameter values: $\mu$	
	Original	Calibrated phase 5
2B. $\varphi$ [°]	33.5	40.4
2B. $OCR$ [-]	4.0	3.6
2B. $k_l$ [kN/m <sup>3</sup> ]	9.0E+03	50.5E+03
EI [kNm <sup>2</sup> /m]	8.75E+04	19.19E+04

## 4 DISCUSSION

### 4.1 Interpretation of the calibrated solution

The calibrated parameters of phase 5, presented in Table 3, fit the overall measured wall deflection as shown in Figure 7. However, these values are not truly representative for the actual soil conditions: Both the calibrated values for wall stiffness  $EI$  and the modulus of subgrade reaction  $2B.k_l$  are unrealistically high. Instead, it should be realized that the results are simply a way to fit the computational model to the observations. Unrealistic calibrated parameters could thus indicate shortcomings in the computational model.

The process of the Bayesian update allows to assess the structure's safety in real-time if, and only if, these results hold till the end of construction. Therefore, the type of computational model is an important choice as it should accurately simulate soil behaviour and soil-structural behaviour in time. For soft soil conditions, this means that it is especially important that the model is able to recognize the temporary effects of undrained soil behaviour. In the case study however, the long timespan of the overall measurement set indicated that undrained soil behaviour was not relevant during the staged excavation (De Wolf, 2019). Instead, the observed limited MIP wall displacements are most likely a result of more favourable force distributions between the structural elements and the soil than previously assumed. This might have been caused by the following explanations:

(1) The cement-soil mixture of the MIP wall has been modelled incorrectly by assuming a too low modulus of subgrade reaction.

(2) The stiffness due to the asymmetric shape of the deep excavation might have been more favourable than assessed by the 2D Spring model.

The contribution of each explanation could not be

found due to a lack of additional data. In the project, no strut forces were measured alongside the considered cross-section. It would have been valuable to have this extra data to be more decisive on the actual force distributions between the structural elements. Additionally, there is little guidance for the selection of input parameters for modelling a MIP wall in existing literature. All in all, it should be realized that limitations of the model and observational data affect the possibilities of engineers to fully explain the site-conditions.

#### *4.2 Features affecting the performance of the Bayesian update*

A valuable feature of the Bayesian update is that uncertainties can be taken into account for both the model and the observations. With this method the prediction on the structure's performance can be updated anytime during the construction process based on the weight of each uncertainty. According to equations (2) and (3) enough  $n$  measurements should be performed in order to converge. Before convergence, the structure's performance is carefully assessed by not fully rejecting the original prediction. Depending on the certainty of each Bayesian update with regards to this original prediction, the structure's performance can be reassessed. This can be beneficial in both safety and economic points of view as this methodology can timely reveal flaws in the set-up of the original model prediction. For instance, based on the growing set of measurements in the case study, the decision could have been made to adjust the structure in construction phase 4, as the Bayesian update indicated the incorrectness of the original prediction. Therefore, the Bayesian update and corresponding calibrated parameters can be used to justify the decision to continue with a more economical design alternative with two struts instead of three.

Typically, the necessary number of measurements in order to converge depends on the specification of the model and measurement errors. However, determining the magnitude of these errors can still be subjective and case dependent. As demonstrated in this case study, it might be desired to adjust the measurement error as the number of observations grows. However, such choices are not generally specified and might not be easily made in real-time.

## 5 CONCLUSIONS

The case study showed a successful application of the Bayesian update. The value of the proposed methodology is primarily in the ability to determine a set of soil parameters to fit the computational model to the observational data. Consequently, this can be used to justify decisions to adjust the original design. It needs to be emphasized that the calibrated results are not truly representative soil parameters if the constitutive model is not suitable for the site-conditions. Also, additional sources of information might be needed to fully interpret the measured data. The presented methodology with the Bayesian update works with different computational models. In order to use this method for real-time measurement processing in the Observational method it is necessary to investigate some aspects further.

The use of different computational models could be tested in order to take into account undrained soil behaviour as relevant for soft soil conditions. It would be valuable to extend this method for an excavation with adjacent buildings and to combine multiple observed quantities. Therefore, the methodology could be extended to add, for example, surface settlements as an additional information source. Finally, the quantification of the uncertainties in both the model and observations play an important role in the outcome of the Bayesian update. Therefore, more case studies should be performed to come up with a more ambiguous formulation of these uncertainties.

To conclude, it is believed that the presented methodology with the Bayesian update has the potential to enrich the Observational method in the applications of deep excavations. It could be a starting point for a more active use of real-time measurement interpretation.

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