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A controlled sewer system should be treated as a sampled data system with events

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Abstract: Arguments are presented in favour of modelling sewer systems and in particular Dutch sewer systems as a sampled data system with events. Basic limitations on controlling these systems when ignoring their hybrid nature are stated. The traditional control scheme for the Dutch systems is shown to be event driven. The control schemes under discussion are: local event driven control for a group of pump stations, sampled data control for a group of pump stations, hierarchical control with sampled data control for the group and event driven control for the individual stations.

Keywords: hybrid system, sewer system, water management

1. INTRODUCTION

Climate change, increasing urbanization, and stricter environmental standards result in higher demands being placed on sewer systems. At the same time it is not economically feasible to make drastic changes to the system because of the high associated costs. In Maurer et al. (2005, Table 1) the value of existing sewer infrastructure is estimated to be between 1700 and 5300 US dollars per capita. The existing systems were mostly designed before the age of affordable computers and means of electronic communication. They were designed to operate under local control. For some systems the original design dates from the 19th century and has been extended many times since then.

The introduction of computers in manufacturing led to attempts to use them in the context of sewer systems, see for example Anderson (1972); Bell (1974); Brandstetter et al. (1973). Much work has been done since then, see for example Marinaki and Papageorgiou (2005); Ocampo-Martinez (2010); García et al. (2015). However, the work tended to concentrate on models that treated the sewer system as a sampled data system. As a further simplification it was often assumed that a hierarchical control system was in place. A high level optimal control scheme, for instance Model Predictive control (MPC), to determine flow rates and valve settings and local controllers that would take care of implementing a flow rate, moving a valve, and starting or stopping a pump.

During a pilot project in the Hoeksche Waard, an island in the Dutch Rhine-Meuse Delta for two municipalities, a problem with this approach surfaced. In this project the decisions to switch individual pump stations on or off were based on calculations done centrally at the water board, see van Nooijen et al. (2011a,b, 2012); van Loenen et al. (2012). As this was a pilot project it was decided to add the following rule: if certain bounds on local levels were exceeded then the whole system would switch back to the old local control scheme. This was done to avoid air ingestion by the pump. If a pump ingested air then it would be out of operation until a worker visited the pump station to bleed the air from the pump. It turned out that due to this rule the central control scheme had less freedom to operate the pumps than the local control scheme. In this paper we analyse the problem and examine whether it may be solved by using a hierarchical control scheme where a local event driven controller can switch the pump station on or off.

2. DESCRIPTION OF THE SYSTEM

Combined sewer systems in low lying areas of the Netherlands consist of sub-networks connected to each other and to the Waste Water Treatment Plant (WWTP) by pump stations. Within the sub-networks the flow is gravity driven. The sub-networks collect sewage and run-off from connected surfaces. Some sub-networks also function as a link in the transport chain between another sub-network and the WWTP. At a pump station there is a wet well to accommodate the traditional local control method used, which consists of triggering pump state changes based on the water level in the wet well. During dry weather the wet well collects water until a certain level h_{on} is reached, then the pump switches on and runs until the level drops to a lower level h_{off}. The wet well is designed to be the lowest point in the sub-network and the level h_{on} is usually chosen to be at or below the lowest point in the sub-network pipe system, this guarantees that during dry weather the sewage flows freely into the wet well. The presence of the wet well assures that there is enough volume available to run the pump for a reasonable time. While starting and stopping the pump could in principle be done electromechanically, modern sewer pump stations usually have a specialized computer. This local computer also takes care of alternating the use of pumps when there are multiple pumps, specific requirements for pump start-up, and it allows for different start and stop levels for different pumps. The wet well mentioned earlier is dimensioned for local control and its volume for small systems tends to be about 5 minutes worth of pump capacity. To limit the costs of the pilot project a coupling was created between the existing municipal SCADA system and the system used to calculate the control actions at the waterboard. This meant transmission of commands and measurements incurred an additional delay.

A final restriction that should be mentioned is the following. Sewer pumps are designed to start and stop quite often, but there is an upper limit imposed by the manufacturer. This limit needs to be respected because every start causes some additional wear and tear on the pump motor. Please note that even in small Dutch systems the pumps may vary in capacity from $13.7\text{m}^3/\text{h}$ to $291\text{m}^3/\text{h}$, the wet wells vary in area from 1.3m^2 to 5.3m^2, and the wet well may extend to more than 4m below ground level, while the local shallow ground water level may be at 2m below ground level or less. In a small system pumps may routinely run for 5 minutes or less at a time.

For all storm or combined sewer networks there are limits on how much precipitation the network can transport. If these limits are exceeded then sewage will spill into open water at locations with emergency spillways or it will flow back into the streets. At the same time these systems are designed for the high flow rates expected during precipitation, so the pipes tend to have relatively large diameters and during dry weather they are only partially full. If a heavy precipitation event occurs then in a complex network under local control a local spill may happen when there is still room to store sewage temporarily in the pipes in other parts of the network. Some form of coordination between local controllers or a hierarchical control scheme can remedy this. Such a control scheme could also optimize use of storage tanks for temporary storage of sewage during heavy precipitation and activation of certain spillways to avoid spills in more sensitive locations. Many proposed sewer control schemes calculate a specific trajectory for the volume to be stored in each sub-network, see Marinaki and Papageorgiou (2005); Ocampo-Martinez (2010); García et al. (2015) and references therein. This implies that the controller subsystem at the top level of the hierarchy will be implemented in a digital computer. This in itself may not force a sampled data system approach, but practical considerations tend to limit the frequency with which communication takes place to times steps whose length cannot be neglected. So in practice the system is usually analysed as a sampled data system.

3. SEWER SYSTEM MODEL DESCRIPTION

During dry weather the sewer pipes are partially filled and the inflow rate into the wet wells is only a fraction of the pump capacity. During heavy rain the inflow rate into the wet wells exceeds the pump capacity. For the moment we do not model all the individual pipes. Instead we model sub-networks, wet wells and pumping stations. We use a graph structure to do so with sub-networks as nodes and pumping stations as edges, see for example van Nooijen and Kolechkina (2013). We suppose that the resulting graph is a directed tree with the WWTP at its root to simplify notation. Each sub-network receives sewage, run-off, and outflow from pumps discharging into the subnetwork.

When level measurements are taken only in the wet well, the distinction between wet well and subnetwork is difficult to incorporate in the control scheme. If we only consider high level goals, such as optimal use of in system storage during heavy rain events, ignoring this distinction can perhaps be justified. However, for control during dry weather, low volume precipitation events, and transitions between wet and dry weather the distinction may be important to the correct functioning of the control system.

3.1 Signals and subsystem models

We will use the signals: the wet well inflow rate $q_{\text{in}}(t)$, the flow rate through the pump $q_{\text{out}}(t)$, the power supply to the pump $p(t)$ with values in $\{0,1\}$, and the level in the wet well $h(t)$. The dry weather part of the inflow tends to be rather predictable, has a more or less periodic character with a daily and weekly cycle, and it has a clear upper bound that is rarely if ever exceeded. Even though dry weather inflow will rarely be zero, it may come very close to zero. Inflow due to precipitation is harder to model, because hydrologists are uncomfortable with upper bounds on precipitation intensity. However, most will agree that it is never infinite, so when considered over a finite time interval, there will be an upper limit to the mean intensity over the interval. Based on this we assume that $q_{\text{in}}(t)$ is non-negative, essentially bounded and Lebesgue measurable on its domain ($q_{\text{in}} \in L^{\infty}(\mathbb{R})$), and $q_{\text{in}} \geq 0$. Moreover, the mean intensity tends to decrease as the duration over which the mean is taken increases, see for instance Langousis and Veneziano (2007). The uncertainty in intensity also seems to decrease with increasing duration, see Zhang and Singh (2007, Figure 1). When taking the mean over an area uncertainty decreases with increasing area, see Sivapalan and Blöschl (1998). Based on this we will also make the following assumption.

A1 For a given region containing a sewer system there is a non-increasing function $\bar{q}_{\max}(T')$ and a $T > 0$ such that for all $T' \geq T$ and for all t we have

$$\frac{1}{T'} \int_{t'=t}^{t+T'} q_{\text{in}}(t')\,dt' \leq \bar{q}_{\max}(T') \tag{1}$$

Model for the wet well We model the wet well as a reservoir with a fixed cross section a with the bottom at level h_{b}, the lowest point of the pipe opening into the wet well at h_{in}, and a spillway to serve as a Combined Sewer Overflow (CSO) at h_{sp}. It has two continuous-time inputs: the wet well inflow rate $q_{\text{in}}(t)$, and the flow rate through the pump $q_{\text{out}}(t)$; two continuous time outputs: the wet well level $h(t)$, and the flow over the spillway $q_{\text{sp}}(t)$; and one continuous-time state: the stored volume $v(t)$. Its time evolution is given by

$$\dot{v}(t) = q_{\text{in}}(t) - q_{\text{sp}}(t) - q_{\text{out}}(t) \tag{2}$$

The flow over the spill way is given by

$$q_{\text{sp}}(t) = c_{\text{sp}}\left(\max\left(0, h(t) - h_{\text{sp}}\right)\right)^{\frac{3}{2}} \tag{3}$$

and the wet well level is given by

$$h(t) = h_b + \frac{v(t)}{a} \tag{4}$$

Model for the pump All pumps need some time to start up and shut down. For sewer pumps it may be quite important to reach full speed before being shut down again as debris in the sewer might otherwise get stuck in the pump. While sewer pumps are designed for frequent starts and stops, there are still limits to be observed. We assume the pump has a fixed flow rate q_{cap}. We will ignore the dependence of the flow rate realized by the pump on inlet and outlet pressure. The pump can be in several discrete different states: “off”, “starting up”, “shutting down”, “running”, and “out of order”. Actions of repair crews will not be modelled, so once the pump enters the “out of order” state it will stay there. It has two continuous time inputs, one is the level $h(t)$ in the wet well, the other is a binary signal $p(t)$ with 0 corresponding to request to stop or stay off and 1 to a request to start or keep running. It has one continuous time state: $q_{\text{out}}(t)$ which is also its output.

Starting up the pump takes a given amount of time τ_{su} and during that time the flow rate will increase as a function of time up to full capacity, we approximate this by a linear function. Shutting down the pump takes a given amount of time τ_{sd} and during that time the flow rate will decrease as a function of time down to zero flow, we approximate this by a linear function. We will denote the level below which the pump will ingest air and enter the “out of order” state by h_{oo}. We assume this is higher than h_b. If the pump receives a start command while stopping or a stop command while starting it will also enter the “out of order” state.

The pump has a soft upper limit of n_{ul} on the number of starts per time unit. We assume that $n_{\text{ul}}(\tau_{\text{su}} + \tau_{\text{sd}})$ is less than one, else this restriction would be superfluous.

For safe pump operation with near zero inflow it is necessary that

$$a\frac{h_{\text{sp}} - h_{\text{oo}}}{q_{\text{cap}}} > \frac{\tau_{\text{su}} + \tau_{\text{sd}}}{2} \tag{5}$$

If we assume that (5) holds then for an inflow that is zero except for impulses that fill up the wet well just as the pump stops the condition

$$a\frac{h_{\text{sp}} - h_{\text{oo}}}{q_{\text{cap}}} > \frac{1}{n_{\text{ul}}} \tag{6}$$

is necessary to avoid switching the pump too often.

Model for the local controller We will use the following model for the local controller. It will have one input: the level in the wet well; two states: “off” and “on”, and one continuous time binary output: the signal to the pump $p(t)$. When in the state “off” $p(t)$ will be 0 and when in the state “on” $p(t)$ will be one, switching levels are h_{off} and h_{on}.

It is essential to keep the pump away from the “out of order” state and keep the average number of starts per unit of time below n_{ul}. The design parameters of the controller are h_{off} and h_{on}. It is also important to keep the level in the wet well below h_{sp} whenever possible.

4. LIMITATIONS ON THE EFFECTIVENESS OF DIFFERENT CONTROL SCHEMES

4.1 Local event driven control

No redistribution of storage use in the system is possible. The switching levels for local controller are constrained by the design of the pump station and the choice of pump. The constraints are captured in the following two Lemmas.

Lemma 1. The following conditions are necessary and sufficient to keep the locally controlled pump station from entering the “out of order” state:

$$h_{\text{off}} > h_{\text{oo}} + \frac{q_{\text{cap}}\tau_{\text{sd}}}{2a} \tag{7}$$

$$h_{\text{on}} - h_{\text{off}} > \frac{q_{\text{cap}}\tau_{\text{su}}}{2a} \tag{8}$$

To prevent a spill for inflows with $\|q_{\text{in}}\|_\infty < q_{\text{cap}}$ we need

$$h_{\text{on}} < h_{\text{sp}} - \frac{q_{\text{cap}}\tau_{\text{su}}}{2a} \tag{9}$$

Proof. Condition (7) is necessary and sufficient to avoid dropping below h_{oo}. Condition (8) is necessary and sufficient to avoid the pump being switched off before reaching full speed. Condition (9) is needed to avoid a spill due to the lower pump flow rate during start-up.

Lemma 2. The following condition is sufficient to limit the number of times a pump starts to less than n_{ul} times per time interval τ_u

$$a\frac{h_{\text{on}} - h_{\text{off}}}{q_{\text{cap}}} > \frac{\tau_u}{n_{\text{ul}}} \tag{10}$$

Proof. As long as condition (10) holds the pump will not have more than n_{ul} starts per unit of time τ_u. Even if the inflow is such that it maximizes the number of starts, for instance by consisting of a series of impulses that lift the level from h_{off} to h_{on}, the pump will run for longer than τ_u/n_{ul} after each impulse. Now n_{ul} starts take at least

$$n_{\text{ul}}a\frac{h_{\text{on}} - h_{\text{off}}}{q_{\text{cap}}} > n_{\text{ul}}\frac{\tau_u}{n_{\text{ul}}} \geq \tau_u$$

We can say the following about the level in the wet well. We assume we start the system at $t = 0$. In the next two Lemmas we consider controller performance.

Lemma 3. Assume that (8) and (7) hold. For all inflows $q_{\text{in}} \in L^\infty(\mathbb{R}_{\geq 0})$ with $q_{\text{in}}(t) \geq 0$ such that

$$\|q_{\text{in}}\|_\infty < q_{\text{cap}} \tag{11}$$

and all starting conditions for the wet well with $h_{\text{off}} \leq h(0) \leq h_{\text{on}}$ we have

$$h(t) \in \left[h_{\text{off}} - \frac{q_{\text{cap}}\tau_{\text{sd}}}{2a}, h_{\text{on}} + \frac{q_{\text{cap}}\tau_{\text{su}}}{2a}\right]$$

Proof. Follows from (11).

Lemma 4. Assume that (8) and (7) hold. Furthermore assume that we consider only inflows $q_{\text{in}} \in L^\infty(\mathbb{R}_{\geq 0})$ for which assumption A1 holds for T and $\bar{q}_{\max}$ with $\bar{q}_{\max}(T) \leq q_{\text{cap}} - \epsilon$. We see that

$$\frac{1}{T'}\int_{t'=t}^{t+T'} q_{\text{in}}(t')\,dt' \leq q_{\text{cap}} - \epsilon \tag{12}$$

and all starting conditions for the wet well with $h_{\text{off}} \leq h(0) \leq h_{\text{on}}$ we have

$$h(t) \in \Big[h_{\text{off}} - \frac{q_{\text{cap}}\tau_{\text{sd}}}{2a}, \quad h_{\text{on}} + \frac{q_{\text{cap}}\tau_{\text{su}}}{2a} + \frac{T}{a}(q_{\text{cap}} - \epsilon)\left(1 - \frac{q_{\text{cap}}}{\|q_{\text{in}}\|_\infty}\right)\Big] \tag{13}$$

and, if $h(t) > h_{\text{on}}$ then the level in the wet well will drop below h_{off} somewhere in the interval

$$\left[t, t + T + \frac{1}{\epsilon}\left(a\frac{h_{\text{on}} - h_{\text{off}}}{q_{\text{cap}}} + \tau_{\text{su}} + T\right)\right] \tag{14}$$

Proof. The lower bound of (13) follows immediately. To derive the upper bound in (13) we use that for all $\tau \leq T$ we may write

$$\int_{t'=t}^{t+\tau} q_{\text{in}}(t')\,dt' + \int_{t'=t+\tau}^{t+T} q_{\text{in}}(t')\,dt' \leq T(q_{\text{cap}} - \epsilon) \tag{15}$$

In the worst case scenario with near zero inflow until we reach h_{on} and then a rapid increase in inflow rate, we get an inflow volume of

$$T(q_{\text{cap}} - \epsilon)$$

over a period τ. The length of the period $\tau_{\min}$ is limited by the upper bound on the particular inflow given by $\|q_{\text{in}}\|_\infty$,

$$\tau_{\min} = \frac{T(q_{\text{cap}} - \epsilon)}{\|q_{\text{in}}\|_\infty}$$

If $\tau_{\min} < \tau_{\text{su}}$ then the net inflow volume is limited by

$$\begin{aligned}
\int_{t'=t}^{t+\tau_{\min}} \left(\|q_{\text{in}}\|_\infty - \frac{q_{\text{cap}}t'}{\tau_{\text{su}}}\right) dt' &\leq \|q_{\text{in}}\|_\infty \tau_{\min} - \frac{q_{\text{cap}}\tau_{\min}^2}{2\tau_{\text{su}}} \leq \\
& T(q_{\text{cap}} - \epsilon)\left(1 - \frac{q_{\text{cap}}}{2\|q_{\text{in}}\|_\infty}\right) \leq \\
T(q_{\text{cap}} - \epsilon)\left(1 - \frac{q_{\text{cap}}}{\|q_{\text{in}}\|_\infty}\right) &+ T(q_{\text{cap}} - \epsilon)\frac{q_{\text{cap}}}{2\|q_{\text{in}}\|_\infty} \leq \\
& T(q_{\text{cap}} - \epsilon)\left(1 - \frac{q_{\text{cap}}}{\|q_{\text{in}}\|_\infty}\right) + \frac{q_{\text{cap}}\tau_{\text{su}}}{2}
\end{aligned}$$

else the net inflow volume is limited by

$$\begin{aligned}
&\int_{t'=t}^{t+\tau_{\text{su}}} \left(\|q_{\text{in}}\|_\infty - \frac{q_{\text{cap}}t'}{\tau_{\text{su}}}\right) dt' \\
&+ \int_{t'=t+\tau_{\text{su}}}^{t+\tau_{\min}} \left(\|q_{\text{in}}\|_\infty - \frac{q_{\text{cap}}t'}{\tau_{\text{su}}}\right) dt' \leq \\
&\left(\|q_{\text{in}}\|_\infty - \frac{q_{\text{cap}}}{2}\right)\tau_{\text{su}} + (\|q_{\text{in}}\|_\infty - q_{\text{cap}})(\tau_{\min} - \tau_{\text{su}}) \leq \\
&\left(\|q_{\text{in}}\|_\infty - \frac{q_{\text{cap}}}{2}\right)\tau_{\text{su}} \\
&+ (\|q_{\text{in}}\|_\infty - q_{\text{cap}})\left(\frac{T(q_{\text{cap}} - \epsilon)}{\|q_{\text{in}}\|_\infty} - \tau_{\text{su}}\right) \leq \\
&\left(\|q_{\text{in}}\|_\infty - \frac{q_{\text{cap}}}{2}\right)\tau_{\text{su}} \\
&+ \left(\|q_{\text{in}}\|_\infty - \frac{1}{2}q_{\text{cap}} - \frac{1}{2}q_{\text{cap}}\right)\left(\frac{T(q_{\text{cap}} - \epsilon)}{\|q_{\text{in}}\|_\infty} - \tau_{\text{su}}\right) = \\
&(\|q_{\text{in}}\|_\infty - q_{\text{cap}})\frac{T(q_{\text{cap}} - \epsilon)}{\|q_{\text{in}}\|_\infty} + \frac{q_{\text{cap}}\tau_{\text{su}}}{2}
\end{aligned}$$

which completes the proof of the first assertion. Next take

$$T'' = T + \frac{1}{\epsilon}\left(\frac{a(h_{\text{on}} - h_{\text{off}})}{q_{\text{cap}}} + \tau_{\text{su}} + T\right)$$

and suppose that $h(t) \geq h_{\text{on}}$ and for all $t' \in [t, t + T'']$ the level $h(t + T'')$ is above h_{off}. We see that for all $t'' \in [t + T, t + T'']$

$$\begin{aligned}
\int_{t'=t}^{t''} q_{\text{in}}(t')\,dt' - \left(T' - \frac{\tau_{\text{su}}}{2}\right)q_{\text{cap}} &\leq \\
(t'' - t)(q_{\text{cap}} - \epsilon) - \left(t'' - t - \frac{\tau_{\text{su}}}{2}\right)q_{\text{cap}} &= \\
\left(\frac{\tau_{\text{su}}}{2} - \epsilon(t'' - t)\right)q_{\text{cap}}&
\end{aligned}$$

We know that

$$h(t) \leq h_{\text{on}} + \frac{q_{\text{cap}}\tau_{\text{su}}}{2a} + \frac{T}{a}(q_{\text{cap}} - \epsilon)\left(1 - \frac{q_{\text{cap}}}{\|q_{\text{in}}\|_\infty}\right)$$

so

$$\begin{aligned}
h(t + T'') = h_{\text{on}} &+ \frac{q_{\text{cap}}\tau_{\text{su}}}{2a} + \frac{T}{a}(q_{\text{cap}} - \epsilon)\left(1 - \frac{q_{\text{cap}}}{\|q_{\text{in}}\|_\infty}\right) \\
&+ \frac{1}{a}\left(\frac{1}{2}\tau_{\text{su}} - \epsilon T''\right)q_{\text{cap}} \\
&\leq h_{\text{on}} - \left(\frac{\epsilon T q_{\text{cap}}}{a} + (h_{\text{on}} - h_{\text{off}})\right) < h_{\text{off}}
\end{aligned}$$

and we have a contradiction.

For low inflow rates the system will spend a lot of time slowly moving from h_{off} to h_{on} so a definition of stability that is in accordance with the standard operation of the system would need to be in terms of a set, not a point, for example using invariant sets as in Michel et al. (2015).

4.2 Emulating local control using a remote controller

We consider the effect of not allowing events to trigger control actions. Suppose a measurement is taken with a time step τ_{stp} and the time needed for transmission of the measurement, the calculation to decide whether or not to switch on the pump (possibly at a central location and involving many pumping stations), and the transmission of the commands to the pumping stations involves a delay of τ_{del}.

Lemma 5. The following conditions are necessary to keep the pump station from entering the "out of order" state. The pump must always be switched off when the measured level is at or below a predetermined level h_{off} with

$$h_{\text{off}} > h_{\text{oo}} + \left(\tau_{\text{stp}} + \tau_{\text{del}} + \frac{\tau_{\text{sd}}}{2}\right)\frac{q_{\text{cap}}}{a} \tag{16}$$

The pump may only only be switched on when the level is above a predetermined level h_{on} with

$$h_{\text{on}} - h_{\text{off}} > \frac{q_{\text{cap}}\tau_{\text{su}}}{2a} \tag{17}$$

To prevent a spill for inflows with $\|q_{\text{in}}\|_\infty < q_{\text{cap}}$ we need

$$h_{\text{on}} < h_{\text{sp}} - \left(\tau_{\text{stp}} + \tau_{\text{del}} + \frac{\tau_{\text{su}}}{2}\right)\frac{q_{\text{cap}}}{a} \tag{18}$$

Proof. Condition (16) is necessary to avoid dropping below h_{oo}. Condition (17) is necessary and sufficient to avoid the pump being switched off before reaching full speed. Condition (18) is needed in case a period with a flow rate of nearly q_{cap} starts when the pump is off, the level reaches h_{on} and t is just past the moment at which the measurement is taken.

The controller should keep track of the number of pump starts to avoid exceeding the number of starts per hour. We see that a discrete controller without events may need to switch off a pump $\tau_{\text{stp}} + \tau_{\text{del}}$ earlier than a local event driven controller. It also needs to switch on $\tau_{\text{stp}} + \tau_{\text{del}}$ earlier than a local event driven controller.

5. SET-POINT TRACKING

Most sewer control schemes try to plan ahead to avoid the need to spill untreated sewage into open water. To this end storage in pipes and in purpose built basins is used. One way to implement optimal use of available storage is to calculate time varying set-points for local storage centrally and adjust flows in different locations to track those set-points. In general the local set-points are related to the total amount of sewage in the system and possibly the expected inflows for the different sub-networks. In a simple, but reasonably popular, scheme where the percentage of total storage used is taken as target of the percentage of storage used in the different districts.

Another form of set-point tracking may occur when multiple pumping stations discharge to the same WWTP. In that case it can be advantageous to keep the total flow between given lower and upper bounds. In this case it may be necessary to temporarily store sewage in a sub-network.

At low flows into the subnetwork the wet well acts as a buffer between the subnetwork as a whole and the pump. In effect we have a large reservoir (the subnetwork) with area a_{sn} and a small reservoir with area $a << a_{\text{sn}}$ (the wet well) connected by a pipe and the flow rate in the pipe q_{in} will more or less match the flow rate into the subnetwork. At high inflows into the subnetwork the flow rate into the wet well will depend on the levels in the system upstream and downstream of the pipe.

We formalize this as follows. From a model of the subnetwork we may determine a function f_{v} that gives the total volume of sewage that would be present in the system for a given level in the wet well if we assume equal water pressure in all parts of the subnetwork (so zero flow rate in all pipes). For simplicity we assume there is no "dead volume", that is there are no locations in the system form where sewage does not flow to the wet well. We assume that f_{v} is invertible. We can now define

$$a_{\text{v}}(h) = \begin{cases} 0 & h < h_{\text{b}} \\ a & h_{\text{b}} < h \leq h_{\text{in}} \\ \dfrac{df_{\text{v}}(h)}{dh} & h > h_{\text{in}} \end{cases}$$

Recall that h_{in} is lowest point of the pipe opening into the wet well. In practice $a_{\text{v}}(h)$ varies from a near $h = h_{\text{in}}$ to $20a$ or even $100a$ once all pipes in the subnetwork start to contribute. We wish to track a set-point $v_{\text{trk}}(t)$ for the volume $v(t)$ by using the pump. This set-point translates into a hypothetical level $h_{\text{trk}}(t) = f_{\text{v}}^{-1}(v_{\text{trk}}(t))$ in the wet well and a volume change $\Delta v_{\text{trk}}(t) = v(t) - v_{\text{trk}}(t)$ to be removed from the system to arrive at the set-point. For that level the available in-system storage is maximal. A theoretical upper limit for the in system storage that can be used is given by

$$v_{\max} = f_{\text{vh}}(h_{\text{sp}}) - f_{\text{vh}}(h_{\text{oo}})$$

this limit assumes zero start-up and shutdown times for the pump station. To illustrate the advantages of including a local event driven subordinate controller we consider a situation where the inflow rate into a subnetwork is below the q_{cap} for that subnetwork and there are other sub-networks that would benefit when a volume $v_1 > f_{\text{vh}}(h_{\text{oo}})$ of in-system storage in this network is used.

5.1 Simple set-point tracking with events

The simplest scheme to use v_1 in-system storage in a subnetwork in use is to keep the level in the wet well near $h_1 = f_{\text{vh}}^{-1}(v_1)$. If we assume that $a_{\text{v}}(h_1) >> a$ then the simplest way to achieve this is to start the pump at time step k_0 if $h(k_0\tau_{\text{stp}})$ is at or above h_1 and the pump is off and stop it either locally when $h(t)$ reaches h_{off} or when $h(kt) \leq f_{\text{vh}}^{-1}(v_1)$ and $(k - k_0)\tau_{\text{stp}} \geq \tau_{\text{su}}$ and $(k - k_0)\tau_{\text{stp}} q_{\text{cap}} \geq f_{\text{vh}}(h(k_0\tau_{\text{stp}})) - f_{\text{vh}}^{-1}(v_1)$. Clearly if we are to avoid spillage then we cannot wait to reach h_1 when it is above the limit set by (8).

Where theoretically h_{off} can be chosen to be

$$h_{\text{oo}} + \frac{q_{\text{cap}}\tau_{\text{sd}}}{2a}$$

5.2 Set-point tracking without events

Here we have the problem that for h_1 above the limit set by (17) we cannot wait to reach h_1 so compared to set-point tracking with events we lose a volume of potential storage given by

$$f_{\text{vh}}\left(h_{\text{sp}} - \frac{q_{\text{cap}}\tau_{\text{su}}}{2a}\right) - f_{\text{vh}}\left(h_{\text{sp}} - \left(\tau_{\text{stp}} + \tau_{\text{del}} + \frac{\tau_{\text{su}}}{2}\right)\frac{q_{\text{cap}}}{a}\right)$$

Moreover, we need to stop pumping at the limit set by (16), so we cannot properly empty the wet well. If $\tau_{\text{stp}} + \tau_{\text{del}}$ is long then we might not even be able to lower the level in the wet well to h_{in}.

6. CONCLUSIONS

We showed that use of a hierarchical control scheme that combines a discrete controller with time step τ_{stp} at the top level combined with a local event driven controller will outperform a discrete controller with time step τ_{stp} without allowance for local events. Moreover, the local controller cannot be seen as a black box that implements commands, the central controller needs to take into account the behaviour of the local controller to avoid giving commands that would result in a conflict between local and central control, for instance by violating constraints on pump station operations.

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