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Reliability updating with survival information for dike slope stability using fragility curves

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ABSTRACT

Dikes and levees play a crucial role in flood protection. The main causes of levee failures are of geotechnical nature. Geotechnical failure modes are also the main contributors to the probability of failure of flood defenses such as levees due to the large uncertainties in ground conditions. Hence, information on ground conditions and soil properties is crucial in safety assessments and retrofitting designs of levees. The experience in practice with designs and risk assessments is that even with a substantial amount of site investigation and laboratory testing the uncertainties in soil properties often remain large. One way to further reduce uncertainties is to include past performance information.

The present papers shows how information of survived loads in the past can be incorporated in reliability assessments by means of Bayesian posterior analysis using the same (physics-based) performance models as the prior reliability assessments. The incorporation of survival information leads to reduced uncertainties and probabilities of failure. Furthermore, an approximate approach using fragility curves is proposed, mainly to make problems with computationally expensive performance functions tractable.

After briefly recapping Bayesian reliability updating, we provide examples for slope stability of (flood protection) dikes. The examples illustrate that posterior analysis enables us to reduce the large prior uncertainties in soil strength parameters by “eliminating” implausible samples or regions of the joint PDF of the strength properties using the evidence of survived load conditions (i.e. water levels). The results suggest that the effect of updating the probability of failure with survival evidence can be considerable, especially when the uncertainty in the strength properties dominates the reliability. Dominant strength uncertainties are rather typical in geotechnical engineering, as opposed to, for example, structural engineering, implying that using the method is promising also for other geotechnical applications.

INTRODUCTION

Dikes and levees play a crucial role in flood protection. The main causes of levee failures are of geotechnical nature; geotechnical failure modes are also the main contributors to the probability of failure of flood defenses such as levees due to the large uncertainties in ground conditions. Hence, information on ground conditions and soil properties is crucial in safety assessments and retrofitting designs of levees. The experience in practice with designs and risk assessments is that even with a substantial amount of site investigation and laboratory testing the uncertainties in soil properties often remain large. One way to further reduce uncertainties is to include past performance information.

Past performance refers to any kind of observation of the structural response to loading or ageing which can provide information to assess the future performance of the same structure, as also demonstrated by, for example, Baecher and Ladd (xx), Zhang et al. (2011) or Li et al. (2015). The present paper focuses on flood defenses and the survival of significant loadings in the past. We show how such information can be incorporated in reliability assessments using the same (physics-based) performance models as the prior reliability assessments and we demonstrate how this leads to reduction of uncertainties and updated probabilities of failure.

The key ingredient of the approach is Bayesian posterior analysis and the reliability updating method can be applied with virtually any standard reliability method. However, for some failure modes, the performance functions are computationally quite expensive to use these sampling methods directly, for which case we propose an approximate method using fragility curves. These can be determined a-priori using the original performance functions to generate an overall distribution of the resistance.

After briefly recapping Bayesian reliability updating, we introduce the approximate approach with fragility curves. Subsequently, we illustrate the approach with fragility curves using simplified examples before providing more realistic examples for slope stability of (flood protection) dikes.

BRIEF RECAP OF RELIABILITY UPDATING

Probability of failure (prior analysis). We define the probability of failure as $P(F) = P(g(\boldsymbol{X}) < 0)$, where F is the failure event, g is the performance (or limit state) function and $\boldsymbol{X}$ is the vector or random variables. The probability of failure is commonly also expressed in terms of the reliability index $\beta = \Phi^{-1}(1 - P(F))$, where Φ^{-1} in the inverse standard normal CDF.

Bayes' rule and reliability updating. Reliability updating as discussed in this paper is based on Bayes' rule (Bayes 1763) and often called Bayesian updating. In essence, there are two ways to apply Bayesian updating in a reliability context: (1) the 'indirect method', in which we update the probability distribution of the random variables first and then re-calculate the reliability estimate with the updated distribution; (2) the 'direct method', in which we apply Bayes' rule in the following form:

$$P(F|\varepsilon)=\frac{P(F\cap\varepsilon)}{P(\varepsilon)}=\frac{P\left(\{g(\boldsymbol{X})<0\}\cap\{h(\boldsymbol{X})<0\}\right)}{P\left(h(\boldsymbol{X})<0\right)} \tag{1}$$

where ε is the evidence expressed in terms of the observation function $h(\cdot)$. Observe that we deal with the evidence or observation similarly as we do with failure, by defining a function of the random variables that assumes negative values if the observation is true. That implies that we are dealing with inequality information, i.e. some observed quantity was greater or less than a certain value, which is typically the case for survival observations. Also equality information can be dealt with in reliability updating (i.e. ($h(\boldsymbol{X}) = 0$, see Straub 2011), yet this is beyond the scope of the current paper.

APPLICATION TO SLOPE INSTABILITY

Observed survival of water level w_{obs}. In this paper we focus on the instability of the inner slope of (river) dikes as failure mechanism, as can be seen in Figure 1

Figure 1. Breach in a river dike as a consequence of inner slope sliding at Breitenhagen, Germany, during the Elbe floods in June 2013

Generally, slope instability for dikes is assessed using two-dimensional (plane-strain) limit equilibrium methods (LEM) such as Bishop, Spencer or Uplift Van, which compute a safety factor *SF* based on moment equilibrium of potential sliding planes. Then, the corresponding performance function becomes $g = SF(\boldsymbol{X}, w) - 1$, in which w is the water level and $\boldsymbol{X}$ is the vector of the remaining random variables. Hence, the prior probability of failure can be estimated by assessing $P(F) = P(SF(\boldsymbol{X}, w) < 1)$.

Similarly, we can define the evidence in terms of a survived water level w_{obs} as $\varepsilon = \{\mathrm{SF}(\mathrm{X}_\varepsilon, \mathrm{w_{obs}}) > 1\}$, where X_ε is the vector of random variables (except w) at the time of the observed survival, i.e. for which we know that the safety factor was greater than 1. Consequently, we can re-write Eq. (1) in terms of safety factors for slope instability as follows:

$$P(F \mid \varepsilon) = P\left(SF(\mathbf{X}, w) < 1 \mid SF(\mathbf{X}_\varepsilon, w_{obs}) > 1\right) = \frac{P\left(\left\{SF(\mathbf{X}, w) < 1\right\} \cap \left\{SF(\mathbf{X}_\varepsilon, w_{obs}) > 1\right\}\right)}{P\left(SF(\mathbf{X}_\varepsilon, w_{obs}) > 1\right)} \qquad (2)$$

Note that in this study we incorporate model uncertainty by multiplying the safety factor obtained by an LEM analysis with a model uncertainty factor m (i.e. a random variable) to obtain SF. Furthermore, note that w is a random variable, whereas w_{obs} can be deterministic or a random variable, if measurement uncertainty in the observed water level is involved.

Equations 1 and 2 represent classical (structural) reliability problems, a parallel system reliability problem in the numerator and a standard component reliability problem in the denominator. Hence, they can be solved by virtually any standard reliability method (SRM, see Straub and Papaioannou 2015), except that first-order approximations may not work well for the numerator part. The implementation with sampling techniques such as Monte Carlo simulation, Importance Sampling or Subset Simulation is straightforward (Schweckendiek et al. 2014).

Epistemic and aleatory uncertainty. At this point, we need to elaborate on the difference between the (vector of) random variables representing the assessment conditions ($\boldsymbol{X}$) and representing the observation conditions ($\boldsymbol{X}_\varepsilon$). Though there is sometimes a grey zone, most random variables involved in the stability analysis can be clearly assigned to either representing epistemic or aleatory uncertainty. For example, the friction angle of a relevant sand layer may be uncertain due to limited site investigation. However, its true value can be assumed time-invariant and the uncertainty is purely of epistemic nature due to our lack of knowledge of what that true value is. On the other hand, an external load on a dike such as a traffic load on the crest typically represents an actual random process in time and hence, aleatory uncertainty.

While the difference between epistemic and aleatory does not matter for the prior reliability estimate in a Bayesian (subjective) probability interpretation, this difference is essential to reliability updating and posterior analysis, as epistemic uncertainty is reducible and aleatory uncertainty is not.

The pragmatic, and we believe often reasonable approach we propose to distinguish between aleatory and epistemic uncertainties in the reliability updating analyses introduced here is to literally assign each random variable to either category: epistemic (i.e. time-invariant, reducible uncertainty) or aleatory (i.e. inherently random process, irreducible uncertainty). Of course, if more accurate estimates can be made, the correlation can be modeled in more sophisticated ways. For the sake of illustration, in this paper all random variables except for the water level will be treated as epistemic.

Note that also known or best-guess differences between the assessment and observation conditions can be incorporated in the respective random vectors $\boldsymbol{X}$ and $\boldsymbol{X}_\varepsilon$. For example, settlements of the dike profile can be incorporated as well as differences in geo-hydraulic response etc. For more in-depth discussion on the relevance of modeling the differences and correlations between assessment and observation conditions refer to Schweckendiek et al. (2016).

Implementation with Monte Carlo simulation. While the approach can be easily adopted with virtually any standard reliability method, below we briefly explain how an implementation with Crude Monte Carlo simulation (MCS) would work in terms of the basic algorithm:

1. **Simulation of the event to be predicted:** Generate *n* realizations of the basic random variables according to their (prior) joint probability distribution. The *j*-th realization of the *i*-th random variable is denoted as $\boldsymbol{X}_{ij}$ and the *j*-th realization of the vector of basic random variables is denoted as $\boldsymbol{X}_j$.
2. **Prior probability of failure:** The prior probability of failure is the number of realizations in which the performance function assumes a negative value ($1[\cdot]$ is the indicator function), divided by *n*: $\hat{P}(F) = 1/n \cdot \sum_j 1[SF(\boldsymbol{X}_j) < 1]$.
3. **Simulation of the observed conditions:** The realizations of all variables with (fully) reducible, epistemic uncertainty obtain the same value as the event to be predicted (full auto-correlation in time or time-invariance): $\boldsymbol{X}_{\varepsilon,ij} = \boldsymbol{X}_{ij}$, i.e. for all *i* where the uncertainty is assumed reducible. The random variables assumed to be aleatory obtain new independent realizations according to their (joint) probability distribution.
4. **Posterior probability of failure:** The updating is achieved by conditioning on the observation and evaluating the following term:

$$\hat{P}(F \mid \varepsilon) = \frac{\sum_j \left(\mathbf{1}[SF(\mathbf{X_j} < 1)] \cdot \mathbf{1}[SF(\mathbf{X}_{\varepsilon,\mathbf{j}}) > 1] \right)}{\sum_j \mathbf{1}[SF(\mathbf{X}_{\varepsilon,\mathbf{j}}) > 1]} \quad (3)$$

The way the updating affects the reliability estimate is, practically speaking, by removing realizations from the failure sample which are implausible considering the survival of the observed load. The implementation with computationally more efficient reliability methods such as Importance Sampling, Directional Sampling or Subset Simulation is rather straightforward and essentially requires solving Equation (2).

APPROXIMATION WITH FRAGILITY CURVES

Since LEM-analyses take roughly one second on an ordinary PC, estimation of low probabilities of failure using sampling techniques as described above is hardly computationally tractable. Below we will describe an approximation using fragility curves, which avoids several computational difficulties in slope reliability analysis for dikes.

Definitions. Fragility curves are functions describing the conditional probability of failure given a (dominant) load variable. For dikes, typically the (water-side) water level w is used as the load of reference: $P(F|w) = P(SF(\boldsymbol{X}, w) < 1)$, in which case $\boldsymbol{X}$ becomes the vector of all random variables except for w. While the following elaboration focuses on the water level w to be used in fragility curves, any other load variable can be used instead.

The definition of fragility curves implies that the curve at the same time represents the cumulative distribution function (CDF) $\mathrm{F}_{\mathrm{w_c}}$ of the critical water level w_c, which is the water level at which the dike fails. This can be illustrated by defining the performance function $g = w_c - w$, for which case the probability of failure is given by:

$$P(F) = P(w_c < w) = \iint_{w_c < w} f(w_c) f(w)\, dw_c\, dw = \int F_{w_c}(w) f(w)\, dw = \int P(F \mid w) f(w)\, dw \qquad (4)$$

The fact that fragility curves represent the probability distribution of the overall resistance (quantified as the 'critical water level') is the key concept used in the proposed approximation.

Estimation. In reliability analysis for slope stability of dikes it is common practice in the Netherlands to first estimate the probability of failure conditional to several water levels in a relevant range using the First-Order Reliability Method (FORM). The results are represented as beta-h curves as depicted in Figure 2, which is just another representation of a fragility curve with the reliability index β on the vertical axis instead of the probability of failure. The "fragility points" are the result of the reliability analyses per water level. The red lines in Figure 2 indicate that we assume that the conditional reliability for other water levels than in the fragility points can be reasonably approximated by linear interpolation between the fragility points (in beta-w space). Note that the fragility points can in principle be determined using any other reliability method, not necessarily FORM.

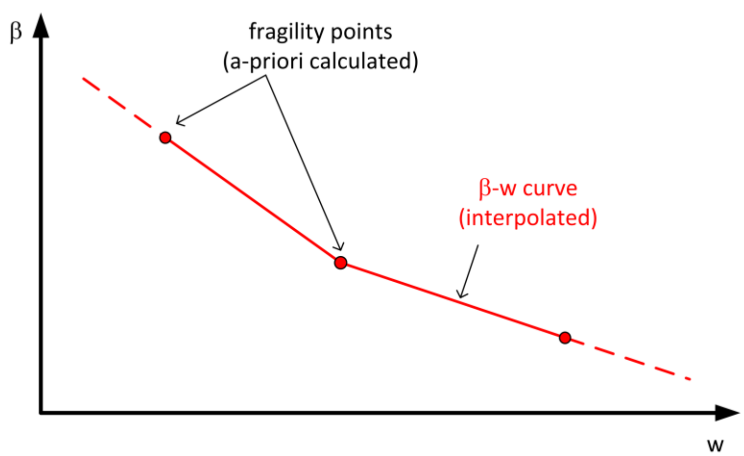

Figure 2. Illustration of a beta-w curve: The fragility points represent the reliability indices corresponding to the conditional probabilities of failure derived for discrete water levels. The conditional reliability for other water levels is obtained by linear interpolation.

Reliability updating with fragility curves. Being able to define the random variable of the critical water level based on beta-h curves as discussed in the previous section, we can directly apply the reliability updating approach using the direct method presented earlier. To that end we again define failure as the water level w exceeding the critical water level w_c: $F = \{w_c < w\}$. Furthermore, we define the observation or evidence (ε) as the critical water level at the observation, $w_{c,obs}$, exceeding the water level at the observation w_{obs} (which can also be a random variable due to measurement uncertainty): $\varepsilon = \{w_{c,obs} > w_{obs}\}$. As discussed, the conditions at the time of the observation may differ from the assessment conditions, in which case it is necessary to derive a separate beta-w curve for the observation. Having defined failure under assessment conditions (F) and the evidence in terms of survival of the observed conditions (ε), the basic formulation of reliability updating with fragility curves directly follows from Equation 1:

$$P(F|\varepsilon) = \frac{P(F \cap \varepsilon)}{P(\varepsilon)} = \frac{P\left(\{w_c < w\} \cap \{w_{c,obs} > w_{obs}\}\right)}{P\left(w_{c,obs} > w_{obs}\right)} \tag{5}$$

As Straub and Papaioannou (2014) have illustrated, Equation 5 can be solved by standard reliability methods. The numerator represents a parallel system reliability problem of two limit states, whereas the denominator is a classical component reliability problem.

Correlation between the resistance at assessment and observation. As discussed above, we can only reduce the epistemic (knowledge) uncertainty, since aleatory uncertainty is by definition irreducible. The proposed pragmatic approach is to divide the random variables in two categories: (1) epistemic, reducible uncertainty and (2) aleatory, irreducible uncertainty. For the present purposes considering a variable as epistemic implies considering it to be time-invariant and, hence, fully correlated between the assessment and the observation conditions. This information of auto-correlation in time of the individual basic random variables can then be used to estimate the correlation between the dike resistance in the assessment conditions (i.e. the critical water level w_c) and the observation conditions ($w_{c,obs}$) using the influence coefficients (α_i) from deriving the fragility curves. According to Vrouwenvelder (2006), the (linear) correlation coefficient ρ between the two resistance terms can be approximated by $\rho \approx \sum_i \alpha_i \alpha_i^{\varepsilon} \rho_i$ where α_i and α_i^{ε} are the FORM influence coefficients of variable i for the assessment and the observation conditions respectively. The correlation coefficient ρ_i describes the correlation of variable i between the observation and the assessment as discussed above (i.e. $\rho_i=1$ for epistemic and $\rho_i=0$ for aleatory uncertainty). Of course, better estimates can be used if available.

As discussed in Schweckendiek and Kanning (2016), there are several approaches to obtain the influence coefficients from the fragility points. In this paper, we use linear interpolation in the design point of the water level in the assessment conditions.

EXAMPLE DIKE SLOPE STABILITY

Example description. The example below illustrates the application and accuracy of the proposed approximate approach with fragility curves to a rather simple dike cross section with undrained shear strength parameters in the clay layers. Figure 3 illustrates the geometry and main characteristics of the example, consisting of a clay dike (“Clay”) on a clay blanket (“Claylayer”) on top of a sand aquifer (“Aquifer”). Table 1 defines all probability distributions and input parameters of the problem. The phreatic surface is assumed to assume a steady-state response to the water level, for which the phreatic level at the toe is assumed to be at surface level.

The stability analyses are made with D-GeoStability. For the sake of illustration, the sliding plane is fixed based on the lowest safety factor with mean values of all stochastic variables.

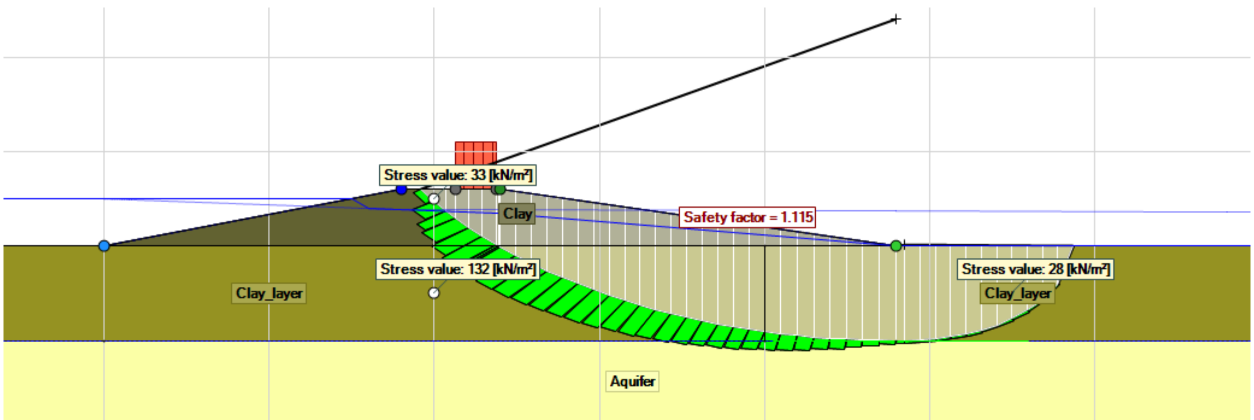

Figure 3. Simple slope geometry with clay dike on clay blanket on top of a sand aquifer.

Name	Unit	Description	Distribution	Parameters
Clay, *S*	[-]	Undrained shear strength ratio	Lognormal	μ=0.35 σ=0.10
Claylayer, *S*	[-]	Undrained shear strength ratio	Lognormal	μ=0.35 σ=0.10
Clay, *m*	[-]	Strength increase exponent	Lognormal	μ=0.90 σ=0.02
Claylayer, *m*	[-]	Strength increase exponent	Lognormal	μ=0.90 σ=0.02
Aquifer, *c*	[kN/m^2]	Cohesion	Deterministic	0
Aquifer, φ	[°]	Friction angle	Deterministic	35
Clay, yield	[kN/m^2]	Yield stress in the dike	Lognormal	μ=38 σ=6
Claylayer, yield	[kN/m^2]	Yield stress under the dike	Lognormal	μ=137 σ=6
Claylayer, yield	[kN/m^2]	Yield stress next to the dike	Lognormal	μ=28 σ=6
Water level	[m+REF]	Outside water level	Gumbel	*shift*=1.5 *scale*=0.4
m_d	[-]	Model undertainty	Lognormal	μ=0.995 σ=0.033

Table 1. Probability distributions of the random variables. The yield stress values belong to the white dots in the geometry in Figure 3, between which yield stresses are interpolated.

Prior reliability. For the prior reliability the full range of potential water levels with the corresponding probability distribution is taken into account. Table 2 shows that the prior reliability index as estimated using MCS and approximation using fragility curves (FC) are in good agreement, while the approximation with fragility curves requires substantially less model evaluations.

	β	D-GeoStability calculations
Monte Carlo simulation	2.15	20,000
Fragility Curve	2.07	250

Table 2: Prior reliability indices from Monte Carlo simulation and the approximation with fragility curves, including the number of model evaluations (with D-Geostability).

Posterior reliability. The observed survived water level is considered deterministic: $w_{obs} = 2\ m$. For both reliability updating approaches (MCS and FC), in general we would need to both the assessment conditions and the observed conditions. In the current example we assume all properties to be time-invariant and hence all uncertainty, except for the water level, epistemic.

Table 3 shows the posterior results of both methods, MCS and FC, which compare very well, while again the approximation with fragility curves requires substantially less stability analyses. Note that the approach actually involves two approximations, the linearization of the beta-h curves and the estimation of the fragility points by FORM (in this example). The probability of failure roughly decreases by one order of magnitude through incorporating the survival information.

	β	D-GeoStability calculations
Monte Carlo simulation	3.05	100,000
Fragility Curve approximation	3.05	500

Table 3: Posterior reliability indices from Monte Carlo simulation and the approximation with fragility curves, including the number of model evaluations (with D-Geostability).

CONCLUSION

Reliability updating using survived load conditions is a straightforward extension of prior reliability analysis and can be done with standard reliability methods. The effect on the estimated reliability can be considerable, especially when a problem is dominated by epistemic uncertainties in strength properties. The concept of Bayesian updating in geotechnical engineering is certainly not new and there are several displays of its potential in the literature. However, the authors feel that for actual practical application, simplified approaches are needed to close the gap between academics and practitioners. Though we have not been able to show this in detail in this paper due to space limitations, we have recently been in the lucky position to apply the presented approach to actual dikes in the Netherlands. The two related reports provide more in-depth background information on the proposed method and its accuracy

(Schweckendiek and Kanning 2016) and on two concrete case studies of lake dikes (Schweckendiek et al 2016). Additionally, at the time of writing there is ongoing work on three additional case studies with river dikes, in which also engineering firms are involved. With the accumulated experience from these cases, we strive to develop tools and best practices documents to enable practitioners to exploit the approach and the rich information contained in observations of survived loads.

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