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**DOI**

[10.29333/ejmste/93398](https://doi.org/10.29333/ejmste/93398)

**Publication date**

2018

**Document Version**

Final published version

**Published in**

EURASIA Journal of Mathematics, Science and Technology Education

**Citation (APA)**

Tursucu, S., Spandaw, J. G., Flipse, S., & de Vries, M. (2018). Teachers' core beliefs about improving transfer of algebraic skills from mathematics into physics in senior pre-university education. *EURASIA Journal of Mathematics, Science and Technology Education*, 14(10). <https://doi.org/10.29333/ejmste/93398>

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# Teachers' Core Beliefs about Improving Students' Transfer of Algebraic Skills from Mathematics into Physics in Senior Pre-university Education

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Received 29 September 2017 • Revised 19 March 2018 • Accepted 12 April 2018

## ABSTRACT

Students in senior pre-university education face difficulties in the application of mathematics in physics. This paper presents the results of a qualitative study on teachers' core beliefs about improving the transfer of algebraic skills to physics. Teachers were interviewed about their beliefs regarding a transfer problem from mathematics to physics for which solution algebraic skills were needed. We obtained large amount of data which were reduced to sixteen core beliefs including constraints and affordances influencing students' demonstration of coherent mathematics education (CME) and transfer of algebraic skills from mathematics into physics. These core beliefs were grouped into the five main categories 'Collaboration', 'Curricula', 'Students', 'Teachers' and 'Textbooks'. We think that our approach to pattern coding is both elegant and generally applicable to reduce code trees including large amount of data. Four core beliefs were identified as naïve beliefs, which may impede transfer. We provided a powerful remedy against such unproductive beliefs: through professional development programs teachers with such beliefs should be made aware, reflect and reconcile their naïve beliefs with those required for transfer. These core beliefs contain data to extract teachers' belief systems. Quantitative research could investigate to which extent this is the case and which beliefs these contain.

**Keywords:** coherent mathematics education, qualitative research, senior pre-university education, teachers' core beliefs, transfer of learning

## INTRODUCTION

Teachers have the experience that students encounter difficulties when applying mathematics in physics class (Ivanjek et al., 2016; Karam, 2014; Nilsen, Angell & Grønmo, 2013; Quinn, 2013). Such transfer problems can be intractable and concern students of all ages, including those in pre-university education (Awodun, Omotade & Adeniyi, 2013; Basson, 2002; Molefe, 2006; Roorda, 2012).

van Hiele (1974) was among the first authors to explain scientifically why pre-university students lacked transfer, especially algebraic skills to physics. He points out two main causes: mathematics and physics which are taught as two separate school disciplines, and the difference in pedagogical approaches between mathematics and physics teachers. For instance, for the lens formula in geometrical optics a physics teacher writes  $b^{-1} + v^{-1} = f^{-1}$  ( $b \neq 0, v \neq 0$ ), whereas a mathematics teacher writes  $(b - f)(v - f) = f^2$ .

**Contribution of this paper to the literature**

- Characterization of 16 new and relevant teachers' beliefs about constraints and affordances influencing transfer of algebraic skills in physics. Five of them were harmful for transfer.
- When used properly, these 16 beliefs may help enhance effectivity of physics education.
- Development of an elegant, generally applicable method to reduce large amount of data.

**Table 1.** Semi-structured questionnaire, which was based on the transfer problem in the case

Question number	Rationale
	Case: during a physics lesson a student does not recognize that the physical formula (formula in short) for displacement, $s = \frac{1}{2}at^2$ , has a similar algebraic structure as the mathematical equation (equation in short), $y = bx^2$ . This student is also unable to express $t$ in terms of $s$ . However, earlier that day during mathematics, the student managed to express $x$ in terms of $y$ , implying that besides a lack of recognition, the student is not able to apply algebraic skills from mathematics to physics successfully. Now we want the same student to <i>recognize</i> that in both cases a similar algebraic structure is used.
1.	<i>Is this a familiar problem?</i>
2.	<i>Do you consider it an important problem?</i>
3.	<i>What may be the reason?</i>
4.	<i>As a physics (mathematics) teacher, what would you do about it?</i>
5.	<i>What may the mathematics (physics) teacher do about it?</i>
6.	<i>What does it mean for the formal physics (mathematics) curriculum?</i>
7.	<i>What does it mean for the math- and physics textbooks?</i> <i>We also want this student to be competent in the application of algebraic skills from mathematics into physics. In this case, to express <math>t</math> in terms of <math>s</math>: <math>t = \sqrt{\frac{2s}{a}}</math></i>
8.	<i>Do you consider it an important problem?</i>
9.	<i>What may be the reason?</i>
10.	<i>As a physics (mathematics) teacher, what would you do about it?</i>
11.	<i>What may the mathematics (physics) teacher do about it?</i>
12.	<i>What does it mean for the formal physics (mathematics) curriculum?</i>
13.	<i>What does it mean for the math- and physics textbooks?</i>
14.	<i>To what extent do you follow textbooks during teaching?</i> In the above-mentioned case, it can be seen that math- and physics are closely related to one another. Teachers appear to have different ideas about their relation.
15.	<i>How do you see the relation between math- and physics?</i>
16.	<i>Do you have any cooperation with your mathematics colleagues?</i>
17.	<i>How do you see the optimal cooperation with your mathematics colleagues?</i> Our pre-university physics education is permeated with algebraic problems from mathematics, such as the case above.
18.	<i>How can the application of algebraic skills from mathematics to physics be improved for solving algebraic problems that occur in our pre-university physics education?</i>

The lack of transfer may also related to the mismatch between teachers' beliefs and teaching practice. Indeed, beliefs have a major impact on teacher behavior (Ernest, 1991; Pajares, 1993). For example, when physics teachers *naïvely* (Schoenfeld, 2014) believe that extensive practice in math class automatically leads to transfer in physics. They completely ignore mathematics in physics class, but soon find themselves re-teaching elementary mathematics. This may be both frustrating and time-consuming, overshadowing the physics content that needs to be taught (Roorda, Goedhart & Vos, 2014; Turşucu, Spandaw, Flipse, & de Vries, 2017).

Although teachers' beliefs are relevant for transfer and coherence across these subjects, they are not studied extensively. This makes it worthwhile to further investigate them.

### Article Aim and Central Research Question

This paper reports the findings of a qualitative study on teachers' core beliefs about improving transfer of algebraic skills to physics. The central research question is "What are the core beliefs of mathematics and physics teachers about improving students' transfer of algebraic skills from mathematics into physics for solving algebraic problems that occur in senior pre-university education?" The word 'core' in core belief should not be confused with the stable and unchangeable beliefs (Haney & McArthur, 2002). Instead, they are the final product of data reduction of the common code tree including the continuum of teachers' individual beliefs from the previous qualitative exploratory study (Table 1) (Turşucu, Spandaw, Flipse, & de Vries, 2017). This code tree concerns the whole set of mathematics and physics teachers' individual beliefs about transfer of algebraic skills into physics, and coherence across both subjects. Hence, these core beliefs provide further insight into these teachers' individual beliefs.

## BACKGROUND

### Coherent Mathematics Education and Transfer

Mathematics and science are intertwined subjects (Atiyah, Dijkgraaf, & Hitchin, 2010). Mathematics offers science a formal language in which quantitative relationships can be described, evaluated and predicted. On the other hand, science offers meaning to mathematics through contexts. Education aiming at enhancement of connection between both subjects lies at the heart of coherent mathematics education (CME), and is of major importance to students (Alink, Asselt, & Braber, 2012; Berlin & White, 2012, 2014).

Connecting mathematics and science, especially physics is possible through alignment such as using compatible notations, concept descriptions, pedagogy of equivalent mathematical methods, and organization of the learning process across time (Alink et al., 2012). The latter implies that the required mathematics had already been explained *before* it was used in physics lessons (Roorda, 2012).

Traditionally, the application of mathematics (initial learning) (Bransford, Brown, & Cocking, 2000; Larsen-Freeman, 2013; Singley & Anderson, 1989) into physics (new learning situation) forms the foundation of CME. Educational researchers view this approach to transfer as one of the main goals of education (Haskell, 2001). In this framework, the existence of transfer is determined by the expert (teacher) in advance, and measured by comparing the learners (student) test answers with that of the experts' correction scheme (Cui, 2006; Lobato, 2003; Rebello et al., 2007). However, "little agreement in the scholarly community about the nature of transfer, the extent to which it occurs, and the nature of its underlying mechanisms" (Barnett & Ceci, 2002, p. 612) led to a shift from traditional to alternative models, such as actor-oriented transfer (AOT in short). As to AOT, transfer is defined as the students' construction of similarities between the initial and new learning situation (Lobato, 2003). The expert tries to understand *how* they are constructed. Thus, the extent to which transfer occurs moves from the teachers' to the students' point of view.

### The Three Classroom Actors

Dutch students start with senior pre-university education (SPE) in Grade-10 after they finished three years of junior pre-university education (JPE). In that year, they should choose between mathematics A and mathematics B. The former puts less attention on algebra than the latter. The content of these subjects is determined by curricula ("Netherlands institute for curriculum development", 2017) and tested in national final examinations. These curricula shape the textbooks which are closely followed by the teachers and their students (Stein & Smith, 2010; van Zanten & van den Heuvel - Panhuizen, 2014). Hence, curricula, teachers and textbooks are the three main actors in Dutch pre-university education.

Teachers in the Netherlands are not sufficiently aware of the content of curricula (Turşucu, Spandaw, Flipse, & de Vries, 2017). For them textbooks *are* the curricula. Thus, their beliefs about CME and transfer are influenced by the content of textbooks, e.g. a physics teacher who discovers that the method of *how* the algebraic technique substitution (Drijvers, 2011) is explained in the physics textbook is different from that in the mathematics textbook.

### Teachers' Individual Beliefs about CME and Transfer

In the previous study we examined mathematics and physics teachers' individual beliefs about CME and transfer (Turşucu, Spandaw, Flipse, & de Vries, 2017), and not the organized teachers' belief systems (Ernest, 1991) containing a set of mutually supporting beliefs. We answered the two research questions: (A) 'How do mathematics and physics teachers characterise the transfer problem in the case?', and (B) 'What sort of beliefs do mathematics and physics teachers' beliefs have about improving students' transfer of algebraic skills from mathematics into physics for solving algebraic problems that occur in senior pre-university education (SPE)?'.

We used convenience sampling to gather data from 10 mathematics and 10 physics teachers who were qualified to teach in SPE and had at least 5 years of teaching experience. The interviews were conducted by means of a semi-structured questionnaire that was based on one specific *case* containing a transfer problem from mathematics to physics (Table 1). Afterwards, each interview was transcribed ad verbatim for analysis.

We used open coding (Bryman, 2012) to label each fragment of the transcripts, which provided a short description of teachers' individual beliefs regarding research questions (A) and (B). For each of the twenty transcripts this led to a set of labels identifying teachers' beliefs. Then, we used axial coding including two steps. First, labels with the same content were put together, resulting in a grouping of the labels. Each group of labels was summarized as a subtheme and included at least three different beliefs uttered by at least three different teachers. If not, it was considered as an outlier. In the subsequent step, we organised 28 subthemes into 9 core themes (coherence, curriculum, education, pedagogy of algebra, relation between scientific subjects, school subjects, teacher, the use of textbooks and transfer), see Table 2. Hence, we obtained one hierarchical structured common code tree for all 20 teachers, with the core themes as main branches. The latter branches out into subthemes, the

**Table 2.** Common code tree including the continuum of teachers' beliefs

Core theme/ subtheme	Mathematics teachers	Physics teachers
<b>1. Coherence</b>	<b>126</b>	<b>135</b>
1.1 Alignment	2 <sup>a</sup>	10/6
1.2 Collaboration and cooperation	85/10	75/10
1.3 Ideal collaboration and cooperation	39/10	50/10
<b>2. Curriculum</b>	<b>65</b>	<b>86</b>
2.1 Curriculum (general)	25/9	10/7
2.2 Mathematics curriculum	23/10	31/10
2.3 Physics curriculum	17/10	45/10
<b>3. Education</b>	<b>7</b>	<b>26</b>
3.1 Junior pre-university education (JPE)	7/5	26/7
<b>4. Pedagogy of algebra</b>	<b>82</b>	<b>72</b>
4.1 Algebraic skills	40/10	26/7
4.2 Algebraic methods	7/4	8/5
4.3 Practice (general)	21/9	30/9
4.4 Practice within mathematics	9/5	3/3
4.5 Practice within physics	5/3	5/3
<b>5. Relation between scientific subjects</b>	<b>87</b>	<b>52</b>
5.1 Mathematics and physics	27/10	15/10
5.2 Mathematics within physics	35/10	23/10
5.3 Physics within mathematics	25/10	14/10
<b>6. School subjects</b>	<b>30</b>	<b>20</b>
6.1 Mathematics	19/7	13/6
6.2 Physics	11/6	7/4
<b>7. Teacher</b>	<b>193</b>	<b>112</b>
7.1 Mathematics teacher	97/10	48/10
7.2 Physics teacher	96/10	64/10
<b>8. The use of textbooks</b>	<b>143</b>	<b>139</b>
8.1 Following textbooks	31/10	43/10
8.2 Mathematics textbook	66/10	31/10
8.3 Physics textbook	37/10	45/10
8.4 Textbook general	9/5	20/7
<b>9. Transfer</b>	<b>144</b>	<b>89</b>
9.1 Activating prior knowledge	8/5	10/4
9.2 Affordances (specific)	34/10	8/5
9.3 Constructing relations (general constraints)	27/10	23/9
9.4 Constructing relations (specific constraints)	75/10	48/10
9.5 Focus on students	1/1 <sup>a</sup>	1/1 <sup>a</sup>

smaller branches. The leaves of the tree are the last and finest level of the hierarchy and represent the underlying continuum of approximately 1.300 individual teachers' beliefs about aspects influencing students' demonstration of CME and aspects influencing transfer.

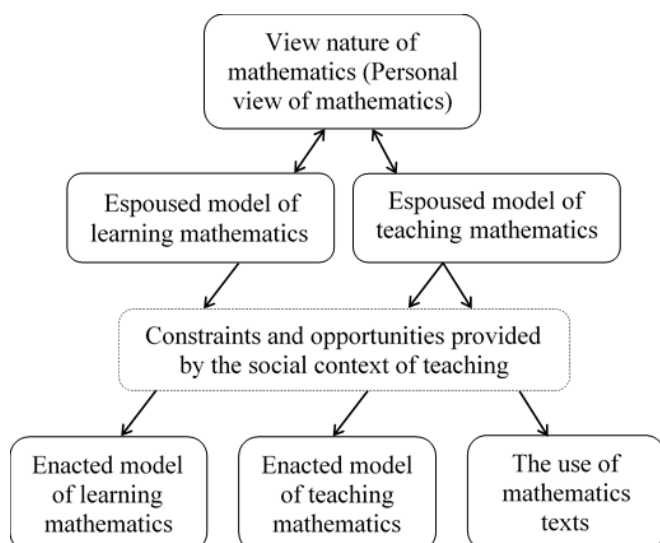
### Findings in a Nutshell

Almost all teachers acknowledged the case and considered it important that students are competent at applying algebraic skills in physics.

In line with literature (Dierdorff, Bakker, van Maanen, & Eijkelhof, 2014; Karakok, 2009; Nashon & Nielsen, 2007; Roorda et al., 2014) most teachers believe that transfer did not occur, because of *compartmentalized thinking* in which students see math and science as separate disciplines.

Most of the mathematics teachers mentioned that they did not feel the need to collaborate with physics teachers. In contrast, physics teachers mentioned that they were willing to work together with mathematics teachers. This result makes developing common teaching strategies to tackle transfer problems analogous to the case more difficult.

Data indicated the existence of two extreme, opposite beliefs about how transfer may be realized. The first one is related to prioritizing basic skills (Wu, 1999) in mathematics class, and the second to reinventing the same mathematical wheel in different physics contexts. An intermediate group thinks that only an integrated approach can solve the transfer problem.



**Figure 1.** Scheme describing how a teachers' belief system is influenced by the social context of teaching (adopted from Ernest, 1991)

### Espoused Versus Enacted Model

The upper rectangle in [Figure 1](#) (Ernest, 1991) represents a teachers' belief system (view) about the nature of mathematics. This offers a foundation for their mental (espoused) models about learning and teaching mathematics. As to this study, the espoused model concerns naïve beliefs (Schoenfeld, 2014) about CME and transfer. As indicated by the arrows pointing down, transformation of such beliefs into classroom practice (enacted model) can be affected by constraints and opportunities provided by the social context in school. The distinction between teacher's espoused and enacted models is necessary and follows from case studies showing a great disparity between both models (Cooney, 1985). Indeed, we have seen that for naïve beliefs which did not match with those required for transfer in the classroom (Turşucu, Spandaw, Flipse, & de Vries, 2017). Hence, as a working definition of naïve beliefs we use unproductive beliefs (espoused models) that do not match those needed for students' transfer of algebraic skills from mathematics into physics (enacted models). This implies that naïve beliefs are harmful for transfer. Through professional development programs (Guskey, 2002) teachers should be made aware of such unproductive beliefs, reflect on the mismatches between both models and reconcile these to enhance transfer.

### From Individual Beliefs to Core Beliefs

The common code tree ([Table 2](#)) is too large to extract belief systems in one single data reduction step. Hence, we further reduced this tree by using the second cycle coding technique pattern coding (Saldaña, 2013). This is explained in the next section.

## METHODOLOGY

Pattern coding grasps the essence of the common code tree and leaves out less important details. In this section we explain how we used this method in three consecutive data reduction steps D1, D2 and D3 (D = data reduction step).

Different from e.g. Gibson and Brown (2009) and Saldaña (2013) who offer general directions and explanations on how to *further* reduce coded data, we worked out their method in detail to reduce the common code tree including the continuum of about 1300 beliefs. We think that our approach to pattern coding is elegant since we used refined and systematic data reduction steps (see [Figure 2, 3 and 4](#)), and offers a generally applicable second cycle coding tool to further reduce data of (common) code trees containing large amounts of data.

### D1: Data Reduction Step 1

After the common code tree ([Table 2](#)) was split into the code tree for mathematics and physics teachers, we followed D1 including two sub steps. *Firstly*, we reduced the collected individual beliefs of each subtheme to zero up to seven summarizing beliefs for each teaching group. These summarizing beliefs contain the essence of the

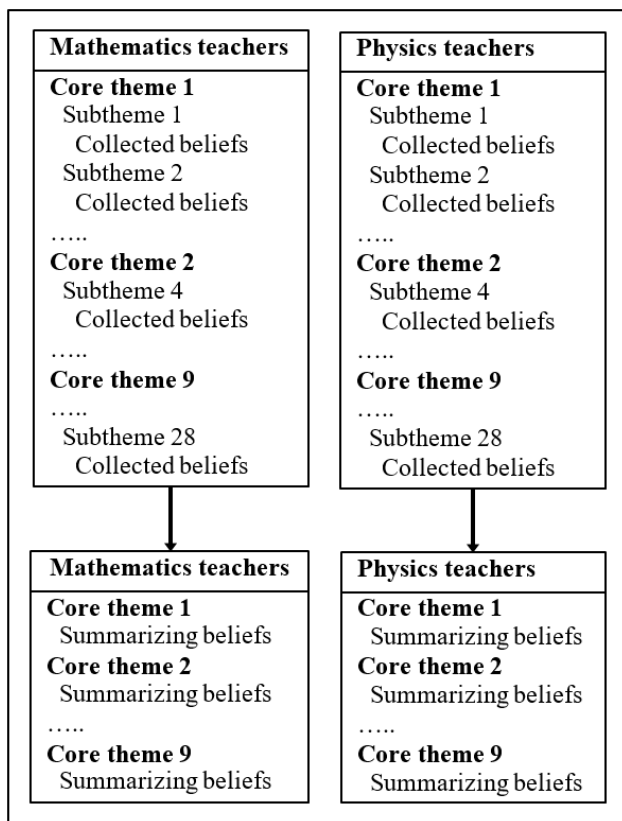


Figure 2. Data reduction step 1: the forming of summarizing beliefs

individual beliefs for each subtheme. *Secondly*, we grouped the summarizing beliefs of different subthemes belonging to the same core theme. This led to summarizing beliefs for each teacher group and is shown in Figure 2.

Herein, the collected beliefs refer to the collected individual teachers' beliefs of Table 2.

### D2: Data Reduction Step 2

As shown in Figure 3, in the second data reduction step we combined both previous datasets including summarizing beliefs into one single dataset. Each core theme of the mathematics group was compared to the same core theme of the physics group. Summarizing beliefs that had the same content were grouped to form main beliefs.



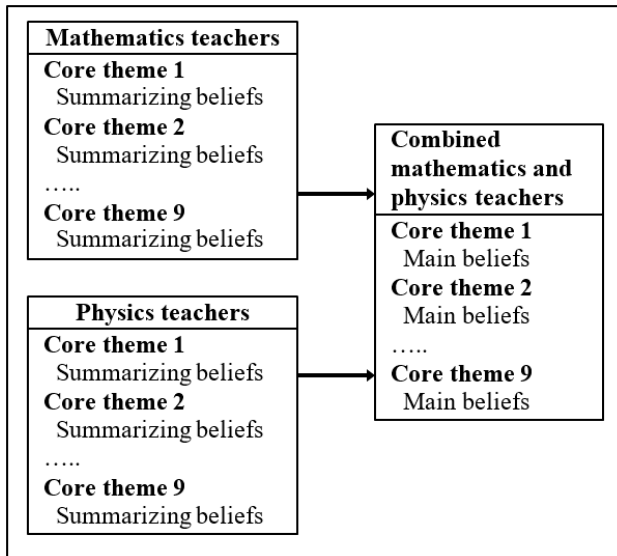


Figure 3. Data reduction step 2: the forming of main beliefs

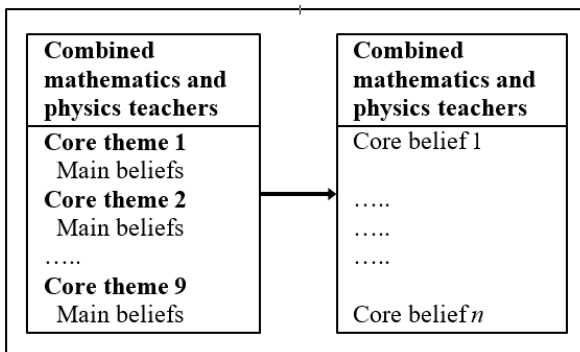


Figure 4. Data reduction step 3: the forming of core beliefs

### D3: Data Reduction Step 3

The last step of pattern coding is shown in Figure 4 and concerned the reduction of core themes together with main beliefs into core beliefs. This process consisted of two smaller steps. Firstly, we replaced the nine core themes including main beliefs by three categories. The new categorization differed from the structure of core themes, and described

(1) the causes for the lack of transfer, (2) the effects of the lack of transfer, and (3) the potential solutions from different perspectives. In this regard, the main beliefs of each core theme were attached to one of these categories. Secondly, the set of differently categorized main beliefs were reduced into one single set of core beliefs. To this extent, main beliefs with the same content were removed. This led to the remaining beliefs, which are the core beliefs.

During the triangulation process, the steps D1, D2 and D3 were independently carried out by an independent researcher. After each round the first author and the independent researcher crosschecked the results. Each dataset was thoroughly discussed and led to 100% agreement among both researchers.

## RESULTS

The results of this study are described in the subsequent sub sections 'D1: Forming of Summarizing Beliefs', 'D2: Forming of Main Beliefs' and 'D3: Forming of Core Beliefs'.



**Table 3.** Core themes including summarizing beliefs

Core theme	Summarizing beliefs
	<b>76/64</b>
1. Coherence	11/12
2. Curriculum	5/7
3. Education	2/3
4. Pedagogy of algebra	11/10
5. Scientific subjects	9/6
6. School subjects	6/5
7. Teacher	11/11
8. The use of textbooks	12/8
9. Transfer	14/10

**Table 4.** Core themes including main beliefs

Core theme	Main beliefs
	<b>31</b>
1. Coherence	5
2. Curriculum	2
3. Education	1
4. Pedagogy of algebra	3
5. Scientific subjects	4
6. School subjects	2
7. Teacher	3
8. The use of textbooks	4
9. Transfer	7

### D1: Forming of Summarizing Beliefs

The results of the first step of pattern coding (D1) are presented in **Table 3**. The first column 'Core theme' represents the nine core themes that we found in the previous study (**Table 2**). The second column 'Summarizing beliefs' including the bold numbers.

'76/64' refers to the total number of summarizing beliefs. The first number deals with mathematics teachers and the second with physics teachers. This also holds for the other numbers. For instance, the second core theme 'Curriculum' consists of five summarizing beliefs for mathematics and seven summarizing beliefs for physics teachers.

### D2: Forming of Main Beliefs

The results of the second step of pattern coding (D2) are presented in **Table 4**. The second column 'Main beliefs' with bold number '31' combines mathematics and physics teachers' summarizing beliefs and refers to the total number of main beliefs. Similarly, e.g. number '5' corresponds to the core theme 'Coherence' and includes five main beliefs.

### D3: Forming of Core Beliefs

The result of the third step of pattern coding (D3) is presented in **Table 5** and contains the set of sixteen core beliefs. The latter concerns beliefs about CME and beliefs influencing students' transfer of algebraic skills from mathematics into physics. This dataset was further organized into the five main categories 'Collaboration', 'Curricula', 'Students', 'Teachers' and 'Textbooks'. Each of these contains core beliefs which are related to each other.

**Table 5.** Sixteen core beliefs

Core belief	Rationale
1.	Mathematics teachers often lack time for cooperation
2.	There is a lack of collaboration between mathematics and physics teachers
3.	Algebraic skills taught in mathematics A do not match sufficiently with physics
4.	Mathematics contains less algebra
5.	Mathematics should incorporate more physics contexts
6.	The physics curriculum should contain manipulation of formulas
7.	Transfer can be stimulated if students practice in different physics contexts
8.	Transfer is being hindered because students regard mathematics and physics as separate subjects
9.	Transfer often will occur spontaneously if students recognize the contexts
10.	Both mathematics and physics teachers can stimulate transfer
11.	There is no consensus whether mathematics and physics teachers should be able to teach basic mathematics that is needed for transfer
12.	Transfer can be stimulated if mathematics and physics teachers agree on the used notations for formulas
13.	Transfer can be stimulated if prior knowledge is activated in physics class
14.	Transfer can be stimulated if students are taught to see connections between contexts
15.	Mathematics and physics teachers stick to the lesson book
16.	There is no consensus whether mathematics and physics textbooks should be adapted

The first two core beliefs are part of 'Collaboration', core beliefs number '3' up to '6' of 'Curricula', number '7' up to '9' of 'Students', number '10' up to '14' of 'Teachers' and the last two core beliefs belong to the main category 'Textbooks'.

## RESULTS INTERPRETATION

In this section we will first interpret the core beliefs. Subsequently, the five main categories are discussed. We finalize with the limitations of this study and make some recommendations for further research.

### Core Themes Versus Main Categories

In short, through pattern coding (Saldaña, 2013) the nine core themes in **Table 2** were further reduced into the five main categories in **Table 5**. This means that the main categories somehow are related to the nine core themes. Indeed, this was the case. The main category 'Collaboration' corresponds to the core theme 'Coherence (core theme 1, or 1 in short)' in **Table 2**. In an analogous manner, 'Curricula' corresponds to 'Curriculum (2)', 'Education (3)' and 'School subjects (6)'; 'Students' to 'Pedagogy of algebra (4)' and 'Transfer (9)'; 'Teachers' to 'Relation between scientific subjects (5)' and 'Teacher (7)'; 'Textbooks' to 'The use of textbooks (8)' (Turşucu, Spandaw, Flipse, & de Vries, 2017). These correspondences may seem a bit rough, since some of the main categories slightly overlap with the same core theme. Nevertheless, we conclude that these matches are reasonable. For instance, core belief number seven of the main category 'Students' corresponds to individual teachers' beliefs such as more practice with different physics problems improves transfer (subtheme 'Practice within physics' of the core theme 'Pedagogy of algebra' (4)). Core belief number '8' matches with, e.g. the individual teachers' belief that mathematics class ends when the student enters the physics class (subtheme 'Constructing relations (general constraints)' of 'Transfer' (9)).

### Loss of Information

The main difference between **Table 2** and **Table 5** concerns the information density: the continuum of circa 1300 individual teachers' beliefs were condensed into sixteen core beliefs. Therefore, some of the detailed information in **Table 2** about CME and transfer of algebraic skills in physics were lost. For instance, core belief number 6 'The physics curriculum should contain manipulation of formulas' does not offer any information about the sort manipulation of formulas. In contrast, **Table 2** includes detailed information such as practice with transfer problems analogous to the case. Another example is core belief number 10 'Both mathematics and physics teachers can stimulate transfer'. While the latter does not explicitly describe *how* to stimulate transfer, **Table 2** includes detailed information, e.g. physics teachers who should write down the mathematics equation  $y = bx^2$  next to the corresponding physics formula  $s = \frac{1}{2}at^2$  in physics class.

### Main category '1': Collaboration

The first two core beliefs refer to a lack of collaboration between mathematics and physics teachers. The majority of mathematics teachers said (as opposed to physics teachers) that they did not feel the need to collaborate with

physics teachers (Turşucu, Spandaw, Flipse, & de Vries, 2017). They believe that physics teachers encounter problems, and should contact them [mathematics teachers] for solutions to transfer problems (espoused models). Accordingly, there are no meetings between both departments (enacted models). See **Figure 1** for both models. The only interactions between them concern individual efforts on a small scale during informal meetings. The lack of time and a huge workload among mathematics teachers were regarded as impeding factors, which supports the first core belief.

On the other hand, a small number of mathematics teachers already is working together with physics teachers. They emphasized the importance of aligning both subjects across time. In this way, certain mathematical concepts are not introduced in mathematics class *before* they were used in physics class (Alink et al., 2012; Roorda, 2012).

We conclude that for teaching practice aiming at CME (Berlin & White, 2012, 2014; Davison, Miller, & Metheny, 1995), and improving students' transfer of algebraic skills from mathematics in physics, a shift in mathematics teachers' espoused beliefs in **Figure 1** (Ernest, 1991) towards more openness to collaboration is needed (enacted beliefs). This involves the development of common pedagogical strategies such as alignment of notations, formulas and the application of algebraic skills in both subjects.

### Main category '2': Curricula

Core beliefs number '3' up to '6' refer to beliefs about the content of mathematics and physics subjects and thus belong to the main category 'Curricula'. Number '3' follows from the belief that the algebra involved in mathematics A is insufficient for that needed in physics class. Indeed, we had already seen this in the previous study (Turşucu, Spandaw, Flipse, & de Vries, 2017). Extensive quantitative research is required to determine whether this belief is widely shared among Dutch mathematics and physics teachers. Even if the latter would be the case, it does not imply that mathematics A is insufficient for physics. Moreover, both mathematics A and mathematics B curricula are designed in such a way that the algebra involved in these subjects should be sufficient for physics ("Netherlands institute for curriculum development", 2017).

As to number '4', both teacher groups mention that mathematics contains less algebra. So, besides mathematics A, mathematics B also seems to lack sufficient algebra for physics students. This result seems to contradict the belief above that *only* mathematics A does not contain algebra. Nevertheless, from the previous study we know that most of the mathematics and physics teachers wish to see that the content standards of both subjects should include the application of algebraic skills (e.g. manipulating formulas and solving for a variable) in physics contexts. This desire is close to core belief number '5'. Furthermore, the mismatch between number '3' and '4' may be related to teachers' contradicting opinions between the first and the second half of the interviews.

We note that too much focus on physics contexts on algebra lessons may be harmful for transfer Bransford, Brown, and Cocking (2000). Curriculum designers aiming at CME and transfer need to take this issue into consideration. They should focus on an 'integrated' approach in which algebraic skills are mainly taught algebraically with some applications in physics context. Such an approach may be visible through explicit descriptions of different aspects of algebraic skills in the content standards, e.g. analogous explanation of an algebraic method in both curricula.

Both groups think that the actual physics curriculum should contain descriptions about manipulation of formulas (number '6'). Nevertheless, the current physics curriculum already includes explicit descriptions about manipulation of formulas. This emphasizes our earlier findings on physics teachers who were unaware of the actual physics curriculum (Turşucu, Spandaw, Flipse, & de Vries, 2017). Although mathematics teachers are not directly involved in physics class, they share this belief. For Dutch teachers, who usually quite strictly follow their textbooks (van Zanten & van den Heuvel - Panhuizen, 2014) and faithfully think that textbooks reflect the intended curriculum (Valverde, Bianchi, Wolfe, Schmidt & Houang, 2002), textbooks *are* the curriculum. Therefore, we regard core belief number '6' as naïve (Schoenfeld, 2014).

We think that transfer can be improved when designers of mathematics curricula and those engaged in physics curricula put effort into collaboration aiming at pedagogy of equivalent mathematical methods regarding the application of algebraic skills. For instance, using algebraic techniques (Drijvers, 2012) in the same way. Design principles should also focus on organization of the learning process across time (Alink et al., 2012).

Furthermore, for two main reasons the alignment above should not lead to attempts to integrate both subjects. Firstly, mathematics has a serving role which is not restricted to physics, but also includes subjects such biology and chemistry. Secondly, it has an intrinsic unifying role: similar expressions and formulas used in different contexts outside mathematics can be reduced to the same abstract mathematics (Atiyah et al., 2010).

### Main category '3': Students

Core belief number '7' is related to basic algebraic skills first: thoroughgoing practice in physics class should automatically lead to transfer. However, earlier studies have shown that successful execution of basic skills in school mathematics also involves conceptual understanding (e.g., Kilpatrick, Swafford & Findell, 2001; Wu, 1999). Not considering the latter may lead to routine based on tricks, thereby impeding transfer (Drijvers, 2011; Roorda, 2012; Turşucu, Spandaw, Flipse, & de Vries, 2017). Thus, this belief is identified as naïve. The same argument holds for extensive practice in mathematic class with algebraic skills. To improve transfer, both basic skills and conceptual understanding should be taught in an integrated manner: *"Advocates of insightful learning are often accused of being soft on training. Rather than against training, my objection to drill is that it endangers retention of insight. There is, however a way of training - including memorisation - where every little step adds something to the treasure of insight: training integrated with insightful learning."* (Freudenthal, 1991).

We think that practicing algebraic skills should happen in both classes with emphasis on conceptual understanding of the underlying mathematics behind these skills. Concrete indications are provided in [Table 1](#). In physics class students should practice with formulas analogous to  $s = (1/2)at^2$ , and in mathematics class with equations analogous to  $y = bx^2$ .

Core belief number '8' is in line with earlier literature on compartmentalized thinking in which students see math and physics as separated subjects (e.g., Dierdorff et al., 2014; Osborne, 2013). The quote "Math is for math class" (Nashon & Nielsen, 2007, p. 97) summarizes our finding. This is reinforced by the fact that in the Netherlands (and in many other countries) both subjects are taught as separate disciplines (Roorda, 2012). Reducing this mental wall could be possible through coherent mathematics education (CME) (Berlin & White, 2014), which aims at fostering connections between mathematics and science education.

As to core belief number '9', recognition of the same algebraic structure in a mathematics equation and a physics formula does not necessarily lead to transfer, but could be an essential precondition for a strategy in which algebraic skills are applied successfully. Therefore, we regard core belief number '9' as naïve. As stated earlier, naïve beliefs (espoused models) are harmful for transfer, since they do not match with beliefs needed for productive classroom practice (enacted models) (Schoenfeld, 2014). Teacher educators who are well-informed about both models could use [Figure 1](#) to make teachers aware of their unproductive beliefs and make them reflect on the mismatches between both models and reconcile them to improve transfer. Otherwise, because of the socialization effect of teaching, teachers are often observed to stick to unproductive teaching practice (Cooney, 1985).

### Main category '4': Teachers

Core belief numbers '10', '12', '13' and '14' confirm earlier research on the crucial role of teachers regarding transfer (Alink et al., 2012; Quinn, 2013). In addition to the aforementioned collaboration between both departments (mesoscopic level in school), individual efforts of mathematics and physics teachers (microscopic level in school) in respectively mathematics and physics classes could enhance students seeing connections between both subjects (number '14'). As to physics teachers, activation of prior mathematical knowledge in physics class should concern mathematics involved in physics problems (number '13'). For instance, physics teachers could write  $y = bx^2$  next to  $s = (1/2)at^2$ , and solving for  $x$  in  $y = bx^2$  next to solving for  $t$  in  $s = (1/2)at^2$ . Physics teachers could also develop their own teaching materials in which such examples are elucidated in detail. Similar arguments hold for mathematics teachers, e.g. mentioning that the equation of a parabola  $y = bx^2$  has the same mathematical structure as the distance formula  $s = (1/2)at^2$  in physics. Such interventions are small enough to be easily adopted by mathematics teachers, and provide context and meaning for the formal language of mathematics (Dierdorff et al., 2014).

The belief that transfer is stimulated when there is agreement on the used notations for formulas (number '12') is in line with earlier literature (e.g. Roorda, 2012; Quinn, 2013). A key example concerns that formulas used in physics class should be equivalent to those used in mathematics class.

Remarkably, only half of both mathematics and physics teachers agree that both teacher groups should be able to teach basic mathematics (number '11'). This belief seems to indicate that half of the mathematics and physics teachers think that students' transfer is independent of whether mathematics and physics teachers possess a solid basis of basic mathematics. This belief is quite astonishing. Indeed, if teachers have not mastered basic algebraic skills, then probably many of their students also lack these skills. This makes transfer impossible. It is very likely that teachers lacking basic mathematics, also lack sufficient basic algebraic skills. Therefore, core belief number '11' simply overlooks the fact that both mathematics and physics teachers should be sufficiently knowledgeable in explaining basic mathematics. Therefore, we regard core belief number '11' as naïve. We also think that this belief is more harmful to transfer than the former naïve beliefs.

Furthermore, the belief above follows from core theme '5' (Table 2). The relationship between mathematics and physics is extremely strong and goes back thousands of years (Atiyah et al., 2010); e.g. Galileo mentioned that the book of nature is written in the language of mathematics. We think that such historical facts brings responsibilities for both mathematics and physics teachers: they should be able to teach basic mathematics.

We conclude that there are many concrete things that individual mathematics and physics teachers could do to connect both subjects. In most cases these concern small interventions, feasible for teachers. Even mentioning that math and physics are not separate subjects, but closely related to each other could contribute to students' transfer and experiencing CME.

### Main category '5': Textbooks

In line with earlier research (Stein & Smith, 2010; van den Heuvel – Panhuizen & Wijers, 2005; van Zanten & van den Heuvel – Panhuizen, 2014) core belief number '15' confirms that Dutch teachers are highly textbook-driven and teach these them to their students. In short, the content of mathematics and physics textbooks shape what students learn. But what if these textbooks contain pedagogical mismatches on how algebraic skills are learned? Algebraic techniques are part of the machinery of algebraic skills and pivotal in algebraic manipulation of formulas. Hence, the question "*To what extent do differences in pedagogical methods to how algebraic techniques are treated in mathematics and physics textbook series affect students solving algebraic physics problems where these techniques are needed?*" is worthwhile to investigate in a new study. It could give insight in the underlying mechanisms that affect students' application of these techniques in such physics problems, and provide design principles about how pedagogical methods should be used in curricula and textbooks to improve transfer.

Although there is hardly any scientific research examining alignment in textbooks between these subjects, there have been talks between mathematics and physics textbook publishers on this matter (Alink et al., 2012). Textbook publishers mention that Dutch teachers are highly textbook-driven and teach them to their students (confirming earlier studies above). They think that textbooks focusing on alignment could strengthen students experiencing CME. We think that curriculum designers should take this matter into account, since curricula determine the content of textbooks.

In practice, however, even if alignment should have been explicitly described in these curricula (ideal scenario), the idea to develop textbook series for this purpose remains difficult. The main reasons are twofold: mathematics and physics textbook publishers working separately, and the absence of learning lines aiming at coherence across both subjects.

Furthermore, among the respondents these teachers there is no consensus whether the content of mathematics and physics textbooks should be adapted (number '16'). This implies that teachers believe that with respect to improving transfer, there is no need to adapt the content of mathematics and physics textbooks. However, Alink et al. (2012) mention that during various consultations with teachers, the teachers frequently asked themselves why actual textbooks did not pay attention to alignment. Therefore, we regard core belief number '16' as naïve. This result also implies that our 'what if' question above about potential pedagogical mismatches in these textbooks was legitimate.

We conclude that alignment of both subjects is crucial for CME and transfer (Konijnenberg, Paus, Pieters, Rijke & Sonneveld, 2015; Roorda, 2012). This includes using compatible pedagogical strategies to teach algebraic methods, using compatible notations, compatible concept descriptions, and especially the organization of the learning process across time in which mathematics had already been explained *before* it was used in physics class. Textbook publishers should take these issues into account.

### Limitations of This Study and Recommendations

The core beliefs obtained in this study were extracted from the common code tree (Table 2). The latter resulted from open and axial coding (Bryman, 2012) of transcripts of twenty interviews with ten mathematics and ten physics teachers who were qualified to teach in SPE (Turşucu, Spandaw, Flipse, & de Vries, 2017). They were selected from different regular Dutch schools in urban, rural and sub-urban areas within a radius of  $\pm 50$  kilometres. Each teaching group consisted of eight male and two female teachers, being in good agreement with the gender ratio in Dutch SPE (Mullis, Martin, Kennedy, Trong, & Sainsbury, 2009). They had varying teaching experience ranging from five to forty years. Therefore, we think that our sample may be representative for Dutch teachers in SPE.

The common code tree in Table 2 contained saturated beliefs, because we did not see much change in the diversity of teachers' individual beliefs after a total of eight interviews including four mathematics and four physics teachers. Since core beliefs follow from this common code tree, they can be regarded as saturated too.



Based on the sample properties together with saturation of core beliefs, the results of this study may be generalizable for the major part of mathematics and physics teachers teaching in SPE in the Netherlands. This also holds for teachers in general secondary education (GSE), but not for preparatory vocational secondary education (PVSE). In PVSE, the mathematics needed in physics are different from those than in SPE and GSE (“Netherlands institute for curriculum development”, 2017). This may lead to different beliefs.

In the Netherlands the content of subjects in SPE is centralized through curricula, shaping to a large extent textbooks and also teachers who strictly follow and teach these to their students (Stein & Smith, 2010; van Zanten & van den Heuvel – Panhuizen, 2014). Consequently, textbooks influence teachers’ beliefs about CME and transfer. In many countries however, such centralized curricula do not exist (Valverde et al., 2002). Thus, we do not expect that our results are generalizable to other countries outside the Netherlands. Although (to our knowledge) there are no other studies examining *teachers’ beliefs* about transfer of mathematics in physics, the latter (non-generalizable) result does not trivialize the fact that teachers *do* observe students experiencing difficulties in physics class (e.g. Basson, 2002; Ivanjek et al., 2016; Karam, 2014; Quinn, 2013; Roorda, 2012). In this sense, our study shares the finding that teachers acknowledge students encountering algebraic difficulties in physics, and even mention the importance of being competent at it. Other studies in which algebraic skills play a more profound role confirm these findings (e.g. Awodun et al., 2013; Bolton & Ross, 1997; Hameed, Metwally, Al Shaya, & Abdo, 2015; Hudson & McIntire, 1977). For instance, in a study examining the mathematics performance among 120 senior pre-university physics students in physics class (four different schools in the North-West Province of South Africa), the results show a very poor level of application of algebraic skills (Molefe, 2006). These students tend to treat mathematics and physics as two separate subjects. Moreover, the individual studies above are in line with large scale international studies in which there is a clear decline in students’ achievements in physics related to insufficient mathematical competence in a number of different countries (e.g. Mullis et al., 2016; Nilsen et al., 2013).

Within the triangulation process, some of the core beliefs have been removed because these were regarded as outliers. This was the case when a summarizing belief was mentioned less than three times by less than three teachers. To some degree this measure is arbitrary. But what if outliers contain important information about missing core beliefs such as the integration of the mathematics and physics curriculum or the textbooks. We recommend to further investigate this matter.

Furthermore, we recommend to identify teachers who think that both mathematics and physics teachers are not required to be sufficiently knowledgeable to teach basic mathematics (number ‘11’). Among all naïve beliefs, this turns out to be the most harmful for transfer. Conducting in depth-interviews (Bryman, 2012) with those teachers may provide insight in why they have such harmful unproductive beliefs. Similar to aforementioned cases, we recommend these teachers to take part in professional teaching programs, since basic mathematics lies at the heart of transfer. Teacher educators could pay attention on their espoused and enacted models (see [Figure 1](#)) to change mathematics and physics teachers’ unproductive belief in one that is productive for teaching practice.

## CONCLUSION

We used pattern coding to answer the central research question “*What are the core beliefs of mathematics and physics teachers about students’ transfer of algebraic skills from mathematics into physics for solving algebraic problems that occur in senior pre-university education?*”. Our approach to this coding technique turned out to be very useful. The common code tree ([Table 2](#)) including the continuum of individual teachers’ beliefs was further reduced into sixteen core beliefs containing constraints and affordances influencing both students’ demonstration of CME and transfer. Different from e.g. Saldaña (2013) who provided general methods to *further* reduce coded data, we used refined and systematic data reduction steps (see [Figure 2](#), [3](#) and [4](#)). Moreover, we think that our approach is generally applicable to further reduce data of code trees containing large amounts of data.

These core beliefs provided new insight into the individual teachers’ beliefs, not found in the previous study. They were grouped into five main categories, i.e. ‘Collaboration’ (number ‘1’ and ‘2’), ‘Curricula’ (number ‘3’ up to ‘6’), ‘Students’ (number ‘7’, ‘8’ and ‘9’), ‘Teachers’ (number ‘10’, ‘11’ and ‘12’) and ‘Textbooks’ (number ‘15’ and ‘16’). So, to enhance students’ demonstration of CME and solve the transfer problem in the *case*, one needs to focus on these categories.

Core belief numbers ‘6’, ‘7’, ‘9’, ‘11’ and ‘16’ concerned naïve (unproductive) beliefs (espoused model) which may stand in the way of both CME and transfer (enacted model). Through professional development programs teachers with such beliefs should be made aware of their unproductive beliefs, reflect on them and reconcile their espoused and enacted models, see [Figure 1](#). This may enhance students’ demonstration of CME and transfer.

‘Collaboration’ between both departments is of major importance to tackle transfer problems, thereby confirming earlier studies.

As to ‘Curricula’, teachers believe that both mathematics A and B lack sufficient algebra for physics. This result seems to contradict the belief that only mathematics A is insufficient for physics.

With respect to 'Students', teachers think that the lack of transfer is due to compartmentalized thinking in which students see mathematics and physics as separate subjects. Again, this finding confirms earlier research.

To improve transfer both basic skills and insightful learning should be taught in an integrated manner.

'Textbooks' should be designed in such a way that last-mentioned integration is taken into consideration. This may enhance both CME and transfer.

Contrary to the common code tree, these core beliefs provide data small enough to extract belief systems including an organized set of mutually supporting core beliefs about CME and transfer in one single data reduction step. Quantitative research could investigate to which extent this is the case and which core beliefs these belief systems contain.

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