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Summary

A revised Marchenko scheme that avoids the need to compute the Green's function is presented for artefact-free image of the subsurface with single-sided reflection response as input. The initial downgoing Green's function which can be modelled from a macro model is needed for solving the revised Marchenko equations instead of its inverse. The retrieved upgoing focusing function can be correlated with the modelled initial downgoing Green's function to image the medium without artefacts. The numerical example shows the effectiveness of the revised scheme in a 2D layered case.

Introduction

It has been shown that the Green's function for a virtual receiver inside a 3D heterogeneous medium and a physical source at the acquisition plane can be obtained using the single-sided reflection response. This requires two steps. In the first step the wavefield that focuses at the virtual receiver location is computed using the reflection response. In the second step the Green's function is computed from the focusing wavefield and the reflection response. This Green's function can be used for imaging or other application of interest. The development of the 1D single-sided Marchenko scheme has been inspired by Rose (2002). Broggini and Snieder (2012) introduce this to the geophysical field as an approach to compute the 1D Green's function. Wapenaar et al. (2013) derive the theory for 3D media. Slob et al. (2014) use the reciprocity relations to create coupled Marchenko equations that can be solved for the up- and downgoing parts of the focusing function simultaneously. The expansion to 3D is given by Wapenaar et al. (2014).

In this abstract we present a revised scheme that avoids the need to compute the Green's function. By applying a new truncation operator, we show that the initial downgoing focusing function can be computed from the scheme whereas it is required as input of the standard Marchenko-type scheme. In the revised Marchneko-type scheme we need an estimate of the time-reversed version of the initial downgoing Green's function. We show how the new truncation operator shifts the reflection response of the reflector that is located at a vertical distance above or below the focal point within half the size of the wavelet to the upgoing part of the focusing function. We demonstrate that the upgoing part of the focusing function can be used to image the medium by correlating it with the modelled initial estimate of the downgoing Green's function. We theoretically compare the revised scheme with the standard scheme. We give a 2D layered example to illustrate the performance of the standard and revised schemes and draw conclusions.

Theory

We indicate time as t and the position vector of a spatial coordinate as $\mathbf{x} = (x, y, z)$, where z denotes depth and (x, y) denote the horizontal coordinates. The acoustically transparent acquisition boundary $\partial \mathbf{D}_0$ is defined as $z_0 = 0$. For convenience, the coordinates at $\partial \mathbf{D}_0$ are denoted as $\mathbf{x}_0 = (\mathbf{x}_H, z_0)$, with $\mathbf{x}_H = (x, y)$. Similarly, the position vector of a point at an arbitrary depth level $\partial \mathbf{D}_i$ is denoted as $\mathbf{x}_i = (\mathbf{x}_H, z_i)$, where z_i denotes the depth of $\partial \mathbf{D}_i$. The acoustic reflection response is denoted as $R(\mathbf{x}'_0, \mathbf{x}_0, t)$, where $\mathbf{x}_0$ denotes the source position and $\mathbf{x}'_0$ the receiver position at the acquisition surface $\partial \mathbf{D}_0$. The focusing function $f_1(\mathbf{x}_0, \mathbf{x}_i, t)$ is the solution of the homogeneous wave equation in a truncated medium and focuses at the focal point $\mathbf{x}_i$. We define the truncated domain as $\partial \mathbf{D}_0 < z < \partial \mathbf{D}_i$ with $z_0 < z < z_i$. Inside the truncated domain, the properties of the medium are equal to the properties of the physical medium. Outside the truncated domain, the truncated medium is reflection free. The Green's function $G(\mathbf{x}_i, \mathbf{x}_0, t)$ is defined for an impulsive source that is excited at $\mathbf{x}_0$ and a receiver positioned at the focal point $\mathbf{x}_i$. The Green's function is defined in the same physical medium as the measured data. The focusing and Green's functions can be partitioned into up- and downgoing parts and for this we use power-flux normalized quantities (Wapenaar et al. 2014).

Revised Marchenko scheme

We start with the 3D versions of one-way reciprocity theorems for flux-normalized wave fields and use them for the depth levels z_0 and z_i. When the medium above the acquisition level z_0 is reflection-free, the Green's function representations are given by (Wapenaar et al. 2014),

$$G^-(\mathbf{x}_i, \mathbf{x}'_0, t) = \int_{\partial \mathbf{D}_0} d\mathbf{x}_0 \int_0^{+\infty} R(\mathbf{x}'_0, \mathbf{x}_0, t') f_1^+(\mathbf{x}_0, \mathbf{x}_i, t - t') dt' - f_1^-(\mathbf{x}'_0, \mathbf{x}_i, t), \qquad (1)$$

$$G^+(\mathbf{x}_i,\mathbf{x}'_0,-t) = -\int_{\partial\mathbf{D}_0} d\mathbf{x}_0 \int_{-\infty}^{0} R(\mathbf{x}'_0,\mathbf{x}_0,-t')f_1^-(\mathbf{x}_0,\mathbf{x}_i,t-t')dt' + f_1^+(\mathbf{x}'_0,\mathbf{x}_i,t). \tag{2}$$

Superscripts $+$ and $-$ stand for downgoing and upgoing parts, respectively. We write the downgoing focusing and Green's functions as the sum of a direct part and a coda

$$f_1^+(\mathbf{x}_0,\mathbf{x}_i,t) = f_{1d}^+(\mathbf{x}_0,\mathbf{x}_i,t) + f_{1m}^+(\mathbf{x}_0,\mathbf{x}_i,t), \tag{3}$$

$$G^+(\mathbf{x}_i,\mathbf{x}_0,t) = G_d^+(\mathbf{x}_i,\mathbf{x}_0,t) + G_m^+(\mathbf{x}_i,\mathbf{x}_0,t), \tag{4}$$

where f_{1d}^+ and G_d^+ indicate the direct part, whereas f_{1m}^+ and G_m^+ indicate the following coda. We rewrite equations (1) and (2) with the help of equation (4) as

$$f_1^-(\mathbf{x}'_0,\mathbf{x}_i,t) = \int_{\partial\mathbf{D}_0} d\mathbf{x}_0 \int_0^{+\infty} R(\mathbf{x}'_0,\mathbf{x}_0,t')f_1^+(\mathbf{x}_0,\mathbf{x}_i,t-t')dt', \qquad \text{for } -t_d-\varepsilon < t < t_d+\varepsilon \tag{5}$$

$$f_1^+(\mathbf{x}'_0,\mathbf{x}_i,t) - G_d^+(\mathbf{x}_i,\mathbf{x}'_0,-t) = \int_{\partial\mathbf{D}_0} d\mathbf{x}_0 \int_{-\infty}^{0} R(\mathbf{x}'_0,\mathbf{x}_0,-t')f_1^-(\mathbf{x}_0,\mathbf{x}_i,t-t')dt', \quad \text{for } -t_d-\varepsilon < t < t_d+\varepsilon \tag{6}$$

where t_d denotes the one-way travel time from a surface point $\mathbf{x}'_0$ to the focusing point $\mathbf{x}_i$, and ε is a positive value to account for the finite bandwidth. Note that the truncation interval is longer in equations (5) and (6) than it is in the conventional Marchenko scheme. When the focal point is located at a vertical distance above or below a reflector within half the size of the wavelet, the earlier truncation in equation (6) would keep f_{1d}^+ in f_1^+ such that it will be updated during the computing, then the G_d^+ (it is present in equation (6) because the time-reversed version of it is overlapped with f_{1d}^+ in time) in equation (6) can be the initial estimate and input for solving the revised Marchenko scheme, the later truncation in equation (5) would keep the reflection of that reflector in f_1^-, which would be the first event in G^- in standard Marchenko scheme, such that f_1^- can be correlated with the modelled G_d^+ to image that reflector. This can be written as

$$\mathbf{I}(\mathbf{x}_i,\mathbf{x}_i,t) = \int_{\partial\mathbf{D}_0} d\mathbf{x}'_0 \int_{-\infty}^{+\infty} G_d^+(\mathbf{x}_i,\mathbf{x}'_0,-t')f_1^-(\mathbf{x}'_0,\mathbf{x}_i,t-t')dt', \tag{7}$$

Equation (7) shows that $\mathbf{I}(\mathbf{x}_i,\mathbf{x}_i,t)$ can be used for estimating the image of $\mathbf{x}_i$ with $t=0$.

Standard Marchenko scheme

The standard Marchenko scheme is obtained from equations (1) and (2) with the help of equation (3)

$$f_1^-(\mathbf{x}'_0,\mathbf{x}_i,t) = \int_{\partial\mathbf{D}_0} d\mathbf{x}_0 \int_0^{+\infty} R(\mathbf{x}'_0,\mathbf{x}_0,t')f_1^+(\mathbf{x}_0,\mathbf{x}_i,t-t')dt', \qquad \text{for } -t_d+\varepsilon < t < t_d-\varepsilon \tag{8}$$

$$f_{1m}^+(\mathbf{x}'_0,\mathbf{x}_i,t) = \int_{\partial\mathbf{D}_0} d\mathbf{x}_0 \int_{-\infty}^{0} R(\mathbf{x}'_0,\mathbf{x}_0,-t')f_1^-(\mathbf{x}_0,\mathbf{x}_i,t-t')dt', \qquad \text{for } -t_d+\varepsilon < t < t_d-\varepsilon \tag{9}$$

These two equations are the known coupled Marchenko equations that can be solved by the iterative scheme for f_1^- and f_1^+. Note that the truncation interval is shorter in equations (8) and (9) than it is in the revised Marchenko scheme. When the focal point is located at a vertical distance above or below a reflector within half the size of the wavelet, the earlier truncation in equation (9) would exclude f_{1d}^+ from f_1^+ such that it will not be updated during the computing and the estimation of it is required as input for solving the standard scheme, the later truncation in equation (8) would keep the reflection of that reflector in G^-, such that G^- can be used to image that reflector. Once f_1^- and f_1^+ are found the Green's functions G^- and G^+ can be computed from equations (1) and (2). The retrieved Green's function can be used to image the medium by

$$\mathbf{I}'(\mathbf{x}_i,\mathbf{x}_i,t) = \int_{\partial\mathbf{D}_0} d\mathbf{x}'_0 \int_{-\infty}^{+\infty} G_d^+(\mathbf{x}_i,\mathbf{x}'_0,t')G^-(\mathbf{x}_i,\mathbf{x}'_0,t+t')dt' \tag{10}$$

Equation (10) shows that $\mathbf{I}'(\mathbf{x}_i,\mathbf{x}_i,t)$ can be estimated for the artefact-free image of $\mathbf{x}_i$ with $t=0$.

As shown in equations (5) and (6), the revised Marchenko scheme has a new truncation operator. This leads to using $G_d^+(\mathbf{x}_i,\mathbf{x}'_0,-t)$ as the exact initial estimate to start the scheme. In practice, we need to

estimate it using velocity analysis and modelling. The imaging scheme given by equation (7) is derived from the upgoing focusing function and can image the medium in the same way as the scheme shown in equation (10) without need to solve equations (1) and (2) for Green's functions. Moreover, the derivation of the revised scheme is similar to the standard one, the assumptions for the standard scheme have been inherited by the revised scheme.

Example

To illustrate the method we give a layered numerical example. Figures 1(a) and (b) show the values for the velocity and density of the model. The source emits a $20\,Hz$ Ricker wavelet. We compute single-sided reflection responses with 401 sources and 401 receivers on a fixed spread with 10m spacing at the top of the model. The initial downgoing Green's functions are computed with sources at focal points and receivers at acquisition level. Absorbing boundary conditions are applied around the model and the direct wave is removed. The computed single-sided reflection responses and the time reversed version of the initial downgoing Green's function are used as inputs to solve the standard and revised Marchenko schemes for focusing and Green's functions. Figure 2 gives the retrieved focusing functions with the focal point shown by the red star in Figure 1(a) and Figure 3 gives the retrieved Green's functions. The truncation in the revised scheme moves the reflection response of the third reflector from the first event in the upgoing Green's function (pointed at by the red arrow in Figure 3(b)) in the standard scheme to the last event of the upgoing focusing function (pointed at by the red arrow in Figure 2(d)) in the revised scheme. The changed truncation leads to extra events visible in Figure 2(c) compared to Figure 2(a) because the last reflection is now present in the upgoing focusing function that must be generated using more downgoing parts in the focusing function as well. For this reason there are also more events in the upgoing part of the focusing function. The procedures described in equations (7) and (10) lead to the images as shown in Figures 4(a) and (b). Note that both imaging results are nearly perfect without ghost images due to internal multiple reflections, but the image retrieved by the revised scheme has better amplitude fidelity.

(a)   (b)
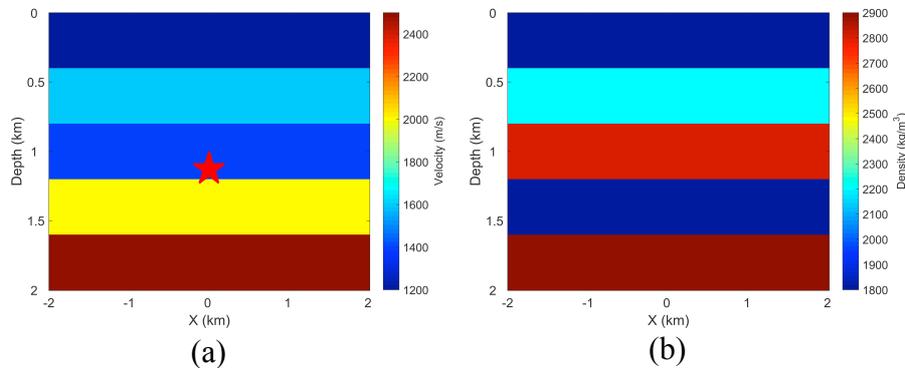
Figure 1 *(a) The velocity model and (b) the density model.*

(a)   (b)   (c)   (d)
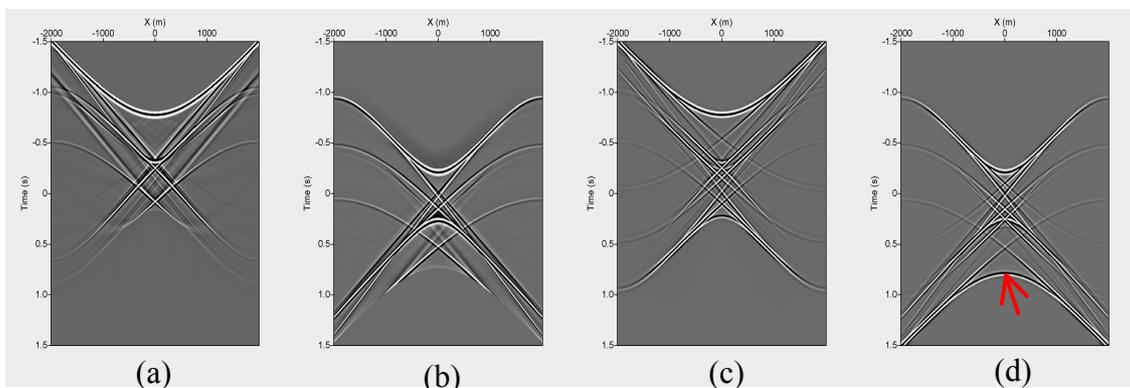
Figure 2 *Down- (a) and upgoing (b) parts of the focusing functions retrieved with the standard scheme and down- (c) and upgoing (d) parts of the focusing functions retrieved with the revised scheme, all with the focal point indicated by the red star in Figure 1(a).*
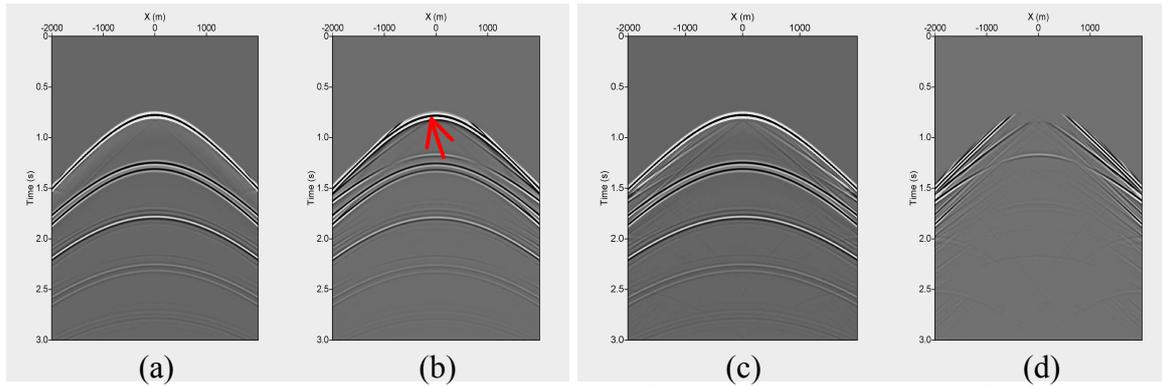
Figure 3 *(a) and (b) indicate the down- and upgoing Green's functions retrieved from the standard Marchenko scheme, (c) and (d) indicate the down- and upgoing Green's functions retrieved from the revised Marchenko scheme; all with a virtual receiver indicated by the red star in Figure 1(a).*
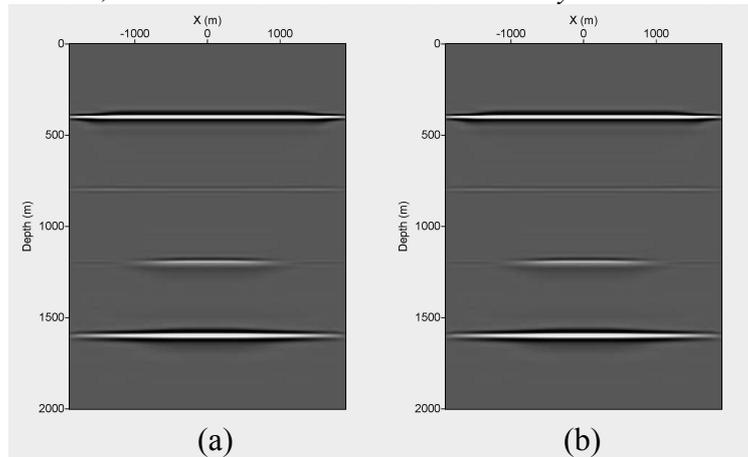
Figure 4 *(a) and (b) indicate images retrieved from equation (7) and equation (10).*

Conclusions

Two aspects have been improved by intruding new truncation in the Marchenko scheme. We can use an estimate of the initial downgoing Green's function instead of its inverse. We capture a reflection event in the upgoing focusing function when the focal point is within half the wavelet of that reflector. We can therefore create an artefact-free image by correlating the initial estimate with the retrieved upgoing focusing function. Applicability to field data requires properly sampled data, which condition can be fulfilled in 2D. In 3D it is not fulfilled and modifications will be necessary before the method can work.

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