

## A Volumetric Method of Moments for Integrated Lens Antennas

Ozzola, R.; Geng, J.; Llombart, N.; Freni, A.; Cavallo, D.; Neto, A.

**DOI**

[10.1109/IRMMW-THz50927.2022.9895976](https://doi.org/10.1109/IRMMW-THz50927.2022.9895976)

**Publication date**

2022

**Document Version**

Final published version

**Published in**

Proceedings of the 2022 47th International Conference on Infrared, Millimeter and Terahertz Waves (IRMMW-THz)

**Citation (APA)**

Ozzola, R., Geng, J., Llombart, N., Freni, A., Cavallo, D., & Neto, A. (2022). A Volumetric Method of Moments for Integrated Lens Antennas. In *Proceedings of the 2022 47th International Conference on Infrared, Millimeter and Terahertz Waves (IRMMW-THz)* (pp. 1-2). IEEE. <https://doi.org/10.1109/IRMMW-THz50927.2022.9895976>

**Important note**

To cite this publication, please use the final published version (if applicable). Please check the document version above.

**Copyright**

Other than for strictly personal use, it is not permitted to download, forward or distribute the text or part of it, without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license such as Creative Commons.

**Takedown policy**

Please contact us and provide details if you believe this document breaches copyrights. We will remove access to the work immediately and investigate your claim.

***Green Open Access added to TU Delft Institutional Repository***

***'You share, we take care!' - Taverne project***

**<https://www.openaccess.nl/en/you-share-we-take-care>**

Otherwise as indicated in the copyright section: the publisher is the copyright holder of this work and the author uses the Dutch legislation to make this work public.

# A Volumetric Method of Moments for Integrated Lens Antennas

R. Ozzola<sup>1</sup>, J. Geng<sup>1</sup>, N. Llombart<sup>1</sup>, A. Freni<sup>2</sup>, D. Cavallo<sup>1</sup>, A. Neto<sup>1</sup>

<sup>1</sup>Delft University of Technology, Delft, NL

<sup>2</sup>Università degli Studi di Firenze, Florence, IT

**Abstract**—A Volumetric Method of Moments accelerated by means of iterative solvers combined with FFT matrix-vector products has been presented. The method allows analyzing small lens antennas with their integrated feeds efficiently since different structures and geometries can be studied with the same discretization, just by adding the material characteristics in post-processing.

## I. INTRODUCTION

INTEGRATED lens antennas have gained popularity in the last decade in the field of wireless communications [1], [2], astronomical instrumentation [3], and security screening [4]. Small-size lenses, i.e., having a diameter  $\approx 1\lambda_0 - 2\lambda_0$ , have started being used as elements for coherent arrays [1], or as the core lens in core-shell lens structures [2]. However, techniques such as the PO/FO [5] are inaccurate when the lens diameter is not electrically large therefore computationally demanding full-wave simulations are the only strategy to simulate such structures. This contribution aims to develop a full-wave Volumetric Method of Moments (V-MoM) suitable for the analysis of lens antennas and their feeds.

## II. MATHEMATICAL FORMULATION

By resorting to the volume equivalence theorem [6], it is possible to define equivalent currents  $\vec{J}_{\text{eq}}$  as  $\vec{J}_{\text{eq}} \equiv j\omega\epsilon_0(\epsilon_r - 1)\vec{E}$  (with  $\vec{E}$  being the total electric field), which radiate in free space and replace the scatterer material. The integral volume equation can be written as

$$\vec{E}^i(\vec{r}) = \frac{\vec{J}_{\text{eq}}(\vec{r})}{j\omega\epsilon_0(\epsilon_r - 1)} - \iiint_{\mathcal{V}} G^{\text{fs}}(\vec{r}, \vec{r}') \cdot \vec{J}_{\text{eq}}(\vec{r}') d\vec{r}', \quad (1)$$

where  $\vec{E}^i$  and  $G^{\text{fs}}$  are the incident field and the free-space Green's function, respectively. By discretizing the problem with a hexahedral grid of edge  $\Delta$ , it is possible to expand the currents as

$$\vec{J}_{\text{eq}}(\vec{r}) \approx \sum_{n=1}^N i_n \vec{b}_n(\vec{r}), \quad (2)$$

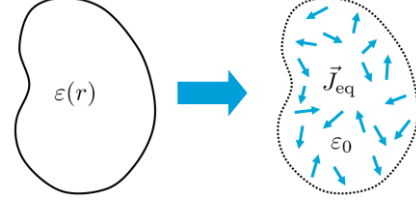
Where  $\vec{b}_n(\vec{r})$  are the basis functions defined as

$$\vec{b}_n(\vec{r}) = \frac{1}{\Delta^2} \text{rect}\left(\frac{\vec{r} - \vec{r}_n}{\Delta}\right) \hat{p}_n, \quad \text{with } \hat{p}_n \in \{\hat{x}, \hat{y}, \hat{z}\}. \quad (3)$$

By using the Galerkin's method on eq. (1), one obtains

$$\langle \vec{E}^i, \vec{b}_m \rangle = \sum_{n=1}^N \left[ \frac{\langle \vec{b}_n, \vec{b}_m \rangle}{j\omega\epsilon_0(\epsilon_r - 1)} - \left\langle \iiint_{\mathcal{V}} G^{\text{fs}} \cdot \vec{b}_n d\vec{r}', \vec{b}_m \right\rangle \right] i_n, \quad (4)$$

where the notation  $\langle \dots, \dots \rangle$  stands for the inner product operator.



**Fig. 1.** Depiction of the Volume Equivalence Theorem, with the original problem on the left and the equivalent problem on the right.

By defining the entries of the matrix impedance  $\mathbf{Z}^{\text{mat}}$  and  $\mathbf{Z}^{\text{rad}}$  as

$$[\mathbf{Z}^{\text{mat}}]_{m,n} = \frac{\langle \vec{b}_n, \vec{b}_m \rangle}{j\omega\epsilon_0(\epsilon_r - 1)},$$

$$[\mathbf{Z}^{\text{rad}}]_{m,n} = - \left\langle \iiint_{\mathcal{V}} G^{\text{fs}} \cdot \vec{b}_n d\vec{r}', \vec{b}_m \right\rangle,$$

And the forcing term  $\mathbf{v}$  as

$$[\mathbf{v}]_m = \langle \vec{E}^i, \vec{b}_m \rangle,$$

the resulting linear system is written as

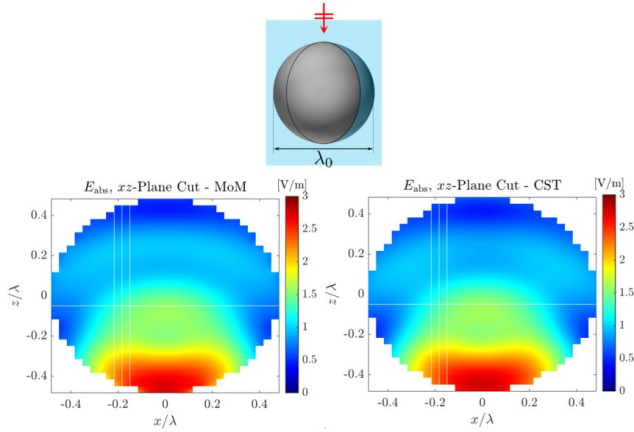
$$\mathbf{v} = (\mathbf{Z}^{\text{mat}} + \mathbf{Z}^{\text{rad}}) \cdot \mathbf{i}. \quad (5)$$

While  $\mathbf{Z}^{\text{mat}}$  is calculated analytically, thanks to the orthogonality of the basis functions, and considers the presence of the specific material and geometry, the evaluation of  $\mathbf{Z}^{\text{rad}}$  is computationally intensive but it happens in free space. The power of the volumetric formulation stands in the fact that different geometries can be studied with the same discretization for which  $\mathbf{Z}^{\text{rad}}$  is precomputed, and the specific material characteristics are added in post-processing with  $\mathbf{Z}^{\text{mat}}$ .

In the presented numerical solution, the linear system solution is obtained by means of the Conjugate Gradient (CG) method, for which  $n_s$  steps are required to reach the convergence. Acceleration techniques such as the ones proposed in [7] are applied for the solution. Thanks to the uniform sampling used for the discretization and to the space invariance of the free-space Green's function,  $\mathbf{Z}^{\text{rad}}$  is a Block Toeplitz-Toeplitz Block (BTTB) matrix. Due to the symmetries associated with this feature, the memory requirements for  $\mathbf{Z}^{\text{rad}}$  are scaled down from  $N^2$  (whole matrix storage) to  $2N - 1$ , allowing to handle large-scale problems. Moreover, the convolutional properties of BTTB matrices allow the calculation of fast matrix-vector products with the aid of the Fast Fourier Transform (FFT) in the CG solution. Combining the CG with the FFT improves the computational complexity from  $\mathcal{O}(N^3)$  of a direct solver to  $n_s \mathcal{O}(N \log_2 N)$ .

## III. RESULTS DISCUSSION

The method has been validated by studying the field scattered by a dielectric sphere having a relative permittivity  $\epsilon_r = 2.34$  and diameter  $D = \lambda_0$ , when illuminated by a plane wave



**Fig. 2.** Electric field inside a dielectric sphere of diameter  $D = \lambda_0$  and relative permittivity of  $\epsilon_r = 2.34$ , when a plane wave is impinging from broadside.

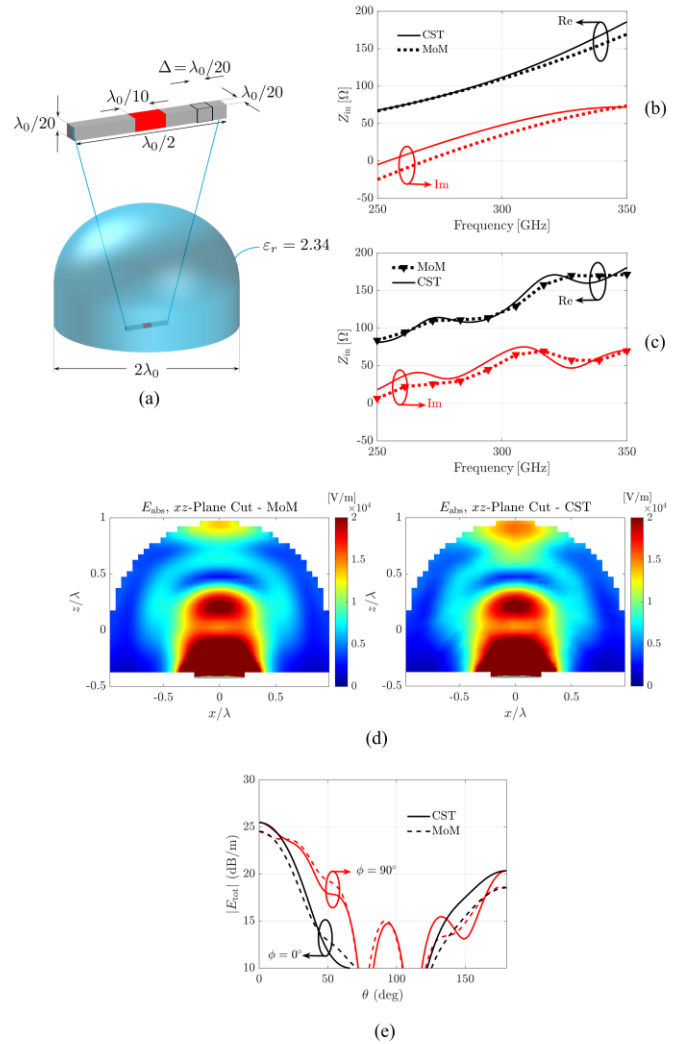
impinging from broadside. Basis functions having side  $\Delta = \lambda_0 / 30$  are used in the V-MoM discretization. The results are shown in Fig. 2, showing a good agreement between the V-MoM and a commercial full-wave solver [8].

The application scenario is shown in Fig. 3(a), where a semi-hemispherical lens with diameter  $D = 2\lambda_0$  made of dielectric with permittivity  $\epsilon_r = 2.34$  is fed by a half-wavelength dipole having a square cross-section of  $\lambda_0/20$  and feeding gap  $\lambda_0/10$ . The basis functions used to discretize the problem have edge  $\Delta = \lambda_0/20$ . The input impedance of the stand-alone dipole and the input impedance of the same dipole when used to illuminate the lens are shown in Figs. 3(b) and 3(c), respectively. Some differences between the V-MoM and [8] are present in modelling the lens and the dipole. In the V-MoM, the dipole is represented with only a basis function in the cross-section, and it is excited with a constant excitation on the entire gap volume, a feature that cannot be obtained with [8]. Moreover, the curved lens surface is not accurately represented with the voxel discretization of the V-MoM. Nevertheless, the agreement between the V-MoM and [8] is excellent for the input impedance of Figs. 3(b) and 3(c), and for the field distribution inside the lens when fed by the dipole at  $f = 294$  GHz.

The lens and the dipole need different levels of discretization: a sampling in the order of  $\lambda_d/10$  is sufficient to characterize the field inside the lens, a much finer discretization is needed to represent the thin metallization of a realistic dipole. Therefore, with the same mesh size the simulation of the lens-dipole ensemble would require an immense number of unknowns. It is our goal to decouple the simulation of the thin metallization from the one of the dielectric lens, by simulating the lens fed by an auxiliary dipole coarsely discretized and then to add in post-processing the analytical correction for the different metal.

## REFERENCES

- [1] H. Zhang, S. Bosma, A. Neto, and N. Llombart, "A Dual-Polarized 27 dBi Scanning Lens Phased Array Antenna for 5G Point-to-Point Communications," *IEEE Transactions on Antennas and Propagation*, vol. 69, no. 9, pp. 5640-5652, Sept. 2021.
- [2] N. van Rooijen, M. Alonso-delPino, M. Spirito, and N. Llombart, "Core-Shell Leaky-Wave Lens Antenna for 150GHz Fly's Eye Communication Systems," presented at the 16th European Conference on Antennas and Propagation (EuCAP), Madrid, Spain, Mar. 27-Apr. 1, 2022.



**Fig. 3.** (a) Hemispheric dielectric lens fed by a half-wave dipole, (b) input impedance of the dipole in free-space, (c) input impedance of the dipole when exciting the lens, (d) electric field distribution inside the lens, and (e) electric field pattern at 1 m when fed by the dipole at  $f = 294$  GHz.

- [3] L. Ferrari et al., "Antenna Coupled MKID Performance Verification at 850 GHz for Large Format Astrophysics Arrays," *IEEE Trans. Terahertz Sci Technol.*, vol. 8, no. 1, pp. 127-139, Jan. 2018.
- [4] E. Gandini, J. Svedin, T. Bryllert, and N. Llombart, "Optomechanical system design for dual-mode stand-off submillimeter wavelength imagers," *IEEE Trans. THz Sci. Technol.*, vol. 7, no. 4, pp. 393-403, Jul. 2017.
- [5] H. Zhang, S. O. Dabironezare, G. Carluccio, A. Neto, and N. Llombart, "A Fourier Optics Tool to Derive the Plane Wave Spectrum of Quasi-Optical Systems [EM Programmer's Notebook]," *IEEE Ant. and Propag. Mag.*, vol. 63, no. 1, pp. 103-116, Feb. 2021.
- [6] J. L. Volakis and K. Sertel, *Integral Methods for Electromagnetics*, SciTech Pub, 2012.
- [7] H. Gan and W.C. Chew, "A discrete BCG-FFT algorithm for solving 3D inhomogeneous scatterer problems," *J. Electromagn. Waves Appl.*, vol. 9, no. 10, pp. 1339-1357, 1995.
- [8] CST Microwave Studio. Dassault Systèmes Simulia Corp. [Online]. Available: <https://www.3ds.com/products-services/simulia/products/cst-studio-suite/>