



Delft University of Technology

## Robust flight-to-gate assignment using flight presence probabilities

van Schaijk, Oscar R.P.; Visser, Hendrikus G.

### DOI

[10.1080/03081060.2017.1355887](https://doi.org/10.1080/03081060.2017.1355887)

### Publication date

2017

### Document Version

Final published version

### Published in

Transportation Planning and Technology

### Citation (APA)

van Schaijk, O. R. P., & Visser, H. G. (2017). Robust flight-to-gate assignment using flight presence probabilities. *Transportation Planning and Technology*, 40(8), 928-945 .  
<https://doi.org/10.1080/03081060.2017.1355887>

### Important note

To cite this publication, please use the final published version (if applicable).  
Please check the document version above.

### Copyright

Other than for strictly personal use, it is not permitted to download, forward or distribute the text or part of it, without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license such as Creative Commons.

### Takedown policy

Please contact us and provide details if you believe this document breaches copyrights.  
We will remove access to the work immediately and investigate your claim.



## Robust flight-to-gate assignment using flight presence probabilities

Oscar R. P. van Schaijk & Hendrikus G. Visser

To cite this article: Oscar R. P. van Schaijk & Hendrikus G. Visser (2017) Robust flight-to-gate assignment using flight presence probabilities, *Transportation Planning and Technology*, 40:8, 928-945, DOI: [10.1080/03081060.2017.1355887](https://doi.org/10.1080/03081060.2017.1355887)

To link to this article: <http://dx.doi.org/10.1080/03081060.2017.1355887>



© 2017 The Author(s). Published by Informa UK Limited, trading as Taylor & Francis Group



Published online: 27 Jul 2017.



Submit your article to this journal [↗](#)



Article views: 39




View related articles [↗](#)



View Crossmark data [↗](#)

# Robust flight-to-gate assignment using flight presence probabilities

Oscar R. P. van Schaijk and Hendrikus G. Visser 

Faculty of Aerospace Engineering, Delft University of Technology, Delft, Netherlands

## ABSTRACT

In this paper we present a novel method to improve the robustness of solutions to the Flight-to-Gate Assignment Problem (FGAP), with the aim to reduce the need for gate re-planning due to unpredicted flight schedule disturbances in the daily operations at an airport. We propose an approach in which the deterministic gate constraints are replaced by stochastic gate constraints that incorporate the inherent stochastic flight delays in such a way so as to ensure that the expected gate conflict probability of two flights assigned to the same gate at the same time does not exceed a user-specified value. The novel approach is integrated into an existing multiple time slot FGAP model that relies on a binary integer programming formulation and is tested using real-life data pertaining to Amsterdam Airport Schiphol. The results confirm that the proposed approach holds out great promise to improve the robustness of the FGAP solutions.

## ARTICLE HISTORY

Received 12 February 2016  
Accepted 17 May 2017

## KEYWORDS

Gate assignment; gate conflict; stochastic gate constraints; robustness; mathematical programming; Amsterdam Airport Schiphol

## 1. Introduction

At busy large airports, the process of assigning flights to gates is a fairly complex task. The objective of this task is to assign each flight to a suitable gate while maximizing a combination of passenger conveniences, airline preferences, and airport operational efficiency elements. To deal with the Flight-to-Gate Assignment Problem (FGAP), a wide array of computer-based planning tools have been developed over the years, based on a variety of mathematical formulations and (exact optimal and heuristic) resolution methods, and that typically use a flight timetable with scheduled arrival and departure times as input data. While many of these tools produce efficient planning solutions under ideal conditions, the resulting gate assignment schedules often exhibit poor robustness when applied in a real-life environment. In daily airline operations, flights do not necessarily arrive or depart according to the planned flight schedule. An arriving aircraft may, for example, find its assigned gate still occupied by an aircraft that has suffered some departure delay. The arriving aircraft therefore has to either wait to be docked at its assigned gate or it needs to be re-assigned to a different gate. It is readily clear that an optimal

**CONTACT** Hendrikus G. Visser  [h.g.visser@tudelft.nl](mailto:h.g.visser@tudelft.nl)  Faculty of Aerospace Engineering, Delft University of Technology, Kluyverweg 1, Delft 2629HS, Netherlands

© 2017 The Author(s). Published by Informa UK Limited, trading as Taylor & Francis Group

This is an Open Access article distributed under the terms of the Creative Commons Attribution-NonCommercial-NoDerivatives License (<http://creativecommons.org/licenses/by-nc-nd/4.0/>), which permits non-commercial re-use, distribution, and reproduction in any medium, provided the original work is properly cited, and is not altered, transformed, or built upon in any way.

planning solution that cannot accommodate minor schedule disturbances will cause significant inherited delays and gate re-assignments in the daily operations, potentially resulting in major losses of revenues.

In the last two decades, a vast body of literature on robust gate assignment has emerged (Bouras et al. 2014). In the context of gate assignment, robustness relates to the ability of an assignment plan to remain sustainable under minor disturbances in the scheduled flight departure and arrival times. One of the earliest attempts aimed at improving the robustness of the flight-to-gate assignment plan is due to Mangoubi and Mathaisel (1985), who suggest the use of fixed time buffers time between two successive flights assigned to the same gate in order to hedge against flight schedule disturbances. Bolat (2000) proposes to distribute idle times uniformly over the gates. The notion of 'idle time' here refers to a time period between two successively assigned flights during which the gate is not used. By distributing the gate idle times uniformly over the gates, the probability of gate conflicts, which occur when two flights with overlapping apron times (gate occupancy times) are assigned to the same gate, is minimized. Diepen, van den Akker, and Hoogeveen (2012) present an integer linear programming formulation for the FGAP that features a robust objective function aimed at maximizing the idle time between each pair of consecutive flights. The study by Castaing et al. (2016) focused on incorporating the inherent stochasticity of the system into the planning process in order to reduce the prevalence and impact of gate blockage. More specifically, they formulated an optimization problem to assign flights to gates with the aim to minimize the expected impact of gate conflicts. Also in the work of Li (2009) a robust objective function is considered, aimed at minimizing the number of gate conflicts, using probability distribution functions on gate conflict between two aircraft as inputs. Şeker and Noyan (2012) consider the gate assignment problem under uncertainty in flight arrival and departure times and develop stochastic programming models incorporating robustness objective functions based on the number of conflicting flights, idle, and buffer times.

In this work we present a method to add robustness to the FGAP that differs from the traditional approaches reported in the literature cited above, which typically rely on the development of some robust 'gap cost' objective function based on the number of gate conflicts, idle, or buffer times. In contrast, the approach proposed here does not seek to modify the objective function, but rather one of the fundamental (strict) constraints that underlie most mathematical formulations of the FGAP, viz., the constraint that enforces that each gate can accommodate at most one aircraft at a time. We propose to replace this deterministic constraint with a stochastic gate conflict constraint that allows two flights with a predicted gate occupancy overlap time probability that remains below a specified threshold value to be assigned to the same gate concurrently. We refer to the probability of two flights destined to be docked at the same gate at the same time as 'gate overlap' or 'gate conflict' probability. The a priori specification of the value of the maximum allowed gate conflict probability in the model formulation proposed here can be regarded as a direct means to control the level of robustness of the FGAP solution.

The original deterministic FGAP gate conflict constraints are based on an input data set comprising the schedule arrival- and departure-time information provided by airlines. In contrast, the proposed stochastic gate conflict constraints rely on input data with uncertain arrival and departure times. More specifically, the coefficients of the gate conflict

constraints are determined based on the predicted ‘flight presence’ probabilities at the airport apron area. Flight presence probabilities are the probabilities that an aircraft is present at the apron  $x$  minutes before or after the planned arrival or departure time.

In this study, we use historic data on airport surface movements and gate assignments from a large hub airport to construct a regression model to predict flight presence probabilities distributions from a vector of selected explanatory variables (predictors). Using the developed regression model, a specific presence probability distribution can then be extracted for each individual flight that is scheduled to arrive or depart from the considered airport.

Our specific contributions to advancing the knowledge on robust flight-to-gate assignment are: (i) proposing an alternative FGAP formulation under uncertain conditions that permits direct control of the level of robustness of the assignment solutions by specifying upfront the permissible level of gate conflict probability, and (ii) incorporating a regression model to predict the flight presence probabilities distributions, based on extensive historic data pertaining to a large hub airport.

The paper is organized as follows. In Section 2, we introduce the regression model that has been conceived to predict the flight presence probability distributions at a major hub airport. Section 3 presents the problem definition and outlines the novel robust FGAP model concept. In Section 4, the usefulness of the proposed robust flight-to-gate assignment approach is demonstrated in a numerical example pertaining to Amsterdam Airport Schiphol (AAS in the Netherlands). Finally, in the conclusions in Section 5, the contributions of the present study are summarized, and the outlook for further research is provided.

## 2. Flight presence and gate conflict probability prediction

During the daily operations at airports disturbances of various kinds (e.g. aircraft or gate breakdown, extreme weather conditions, and turnaround delays) result in deviations from the planned schedule. In this study we use historical data to predict the variability in flight arrival and departure times at a major hub airport. More specifically, a regression model is developed to predict flight presence probability distributions based on real-time observations of airport surface movements and gate assignments, obtained during a measurement period of one week (i.e. from 8th to 14th April 2013) at AAS. At present, Schiphol airport features more than 1300 flight movements per day that need to be assigned to over 200 parking locations (gates and remote stands).

An extensive analysis of the historic surface movement data revealed that the presence probability distribution of different flights can vary significantly. Indeed, some flights are typically very punctual while others tend to be early or delayed quite often.

A regression analysis has been conducted to establish the correlations between a number of predictors and the response variable, which is taken as either the arrival delay cumulative probability distribution or the departure delay cumulative probability distribution. The arrival and departure time delays relate, respectively, to the difference between the Actual In Block Time and the Scheduled In Block Time, and the difference between the Actual Off Block Time and the Scheduled Off Block Time.

A number of potential predictors have been considered for inclusion in the regression model. Examples of predictors that were inferred to potentially influence the on-time

performance of a flight, include day of week, season, time of day, origin/destination region, and airline identity. However, due to the limited extent of the available historic data set, some of the identified potential predictors could not be explored (e.g. seasonality).

The two most influential predictors that were identified are the ‘airline identity’ and the ‘origin/destination region of flights’. [Figure 1\(a\)](#) shows the observed probability distributions for ‘arrivals per origin region’ for seven different (anonymized) regions for the origin airport. When inspecting the cumulative probability of a flight being present 10 minutes before the scheduled arrival time, it turns out to be 62% for Region 6, while for Region 5 this is just 6%. With respect to departures ([Figure 1\(b\)](#)), the cumulative probabilities for a flight being present 10 minutes after the planned departure time vary between 30% (Region 3) and 57% (Region 2).

Similar behaviour has been found for the predictor ‘airline identity’, where the variability between the cumulative probabilities per (anonymized) airline can be seen to vary significantly ([Figure 2](#)). Note that [Figure 2](#) only shows the results for the 10 largest airlines in terms of number of flight movements.

Although the two predictors ‘airline identity’ and ‘origin/destination region of flights’ proved to be correlated to some extent, the degree of collinearity remained sufficiently low to permit the construction of a linear regression model based on the combined use of these two predictors.

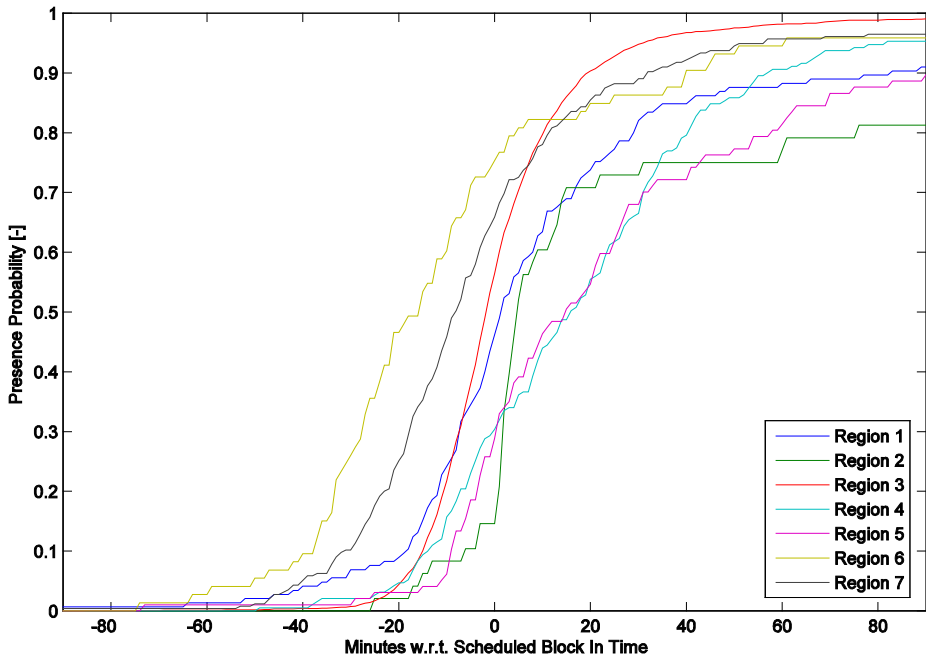
Using the developed regression model, the departure and arrival time delay cumulative probability distributions can be estimated for each flight ([Figure 3\(a\)](#)). By combining the arrival and departure delay cumulative probability distributions with the planned arrival and departure times, the probability distribution of flight presence at the apron can be constructed ([Figure 3\(b\)](#)).

The probability of a gate conflict, i.e. two flights being present at the same gate at the same time (gate conflict) can be readily obtained by multiplying the flight presence probabilities of two flights at the same gate at the same time. For instance, when a departing flight, which is scheduled to leave at 12:50 hours, has a 30% probability of being at least 10 minutes delayed and an arriving flight, which is scheduled to arrive at 13:10 hours, has a 20% probability of being at least 10 minutes early and both flights are assigned to the same gate, this would lead to an  $20\% \times 30\% = 6\%$  probability of the two flights having a gate conflict at 13:00 hours. [Figure 4](#) shows an actual example – pertaining to the Schiphol airport case – of a gate conflict probability history resulting from the overlap of two (predicted) flight presence probability histories. It is noted that in this particular example the peak in the overlap probability history has been slightly shifted in time (to the right) resulting from the use of discrete (sampling) time slots.

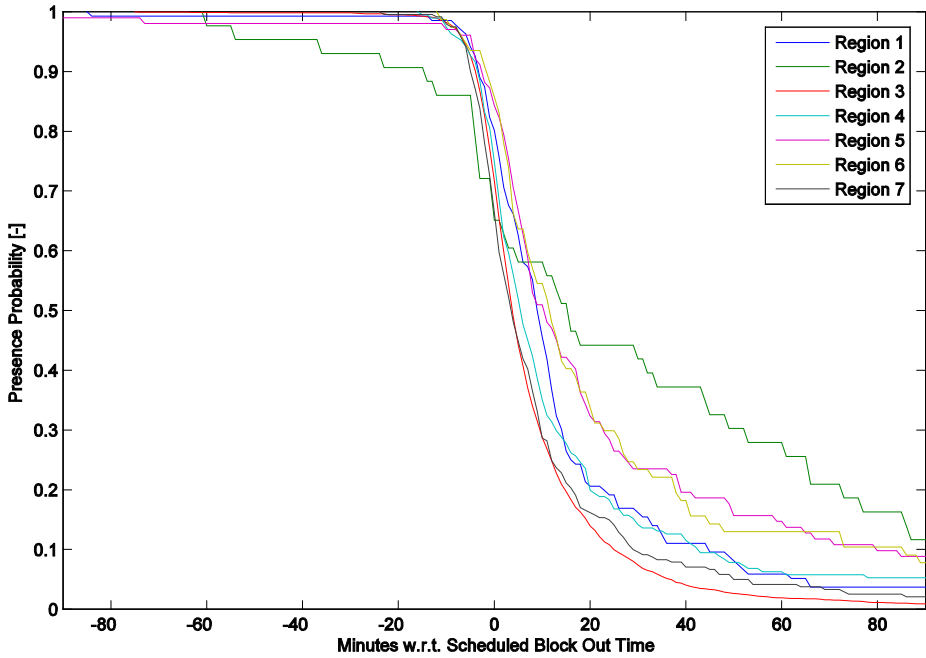
### 3. Gate assignment problem definition

#### 3.1. Basic multiple time slot binary integer programming model

The assignment of aircraft to gate positions at large hub airports is a fairly complicated problem due to the many assignment rules and criteria involved. Indeed, the imposition of strict constraints such as aircraft–gate size compatibility, security requirements, and availability of customs facilities, as well as soft constraints such as passenger comfort and preferences of the various stakeholders (airlines, airport, and ground services),

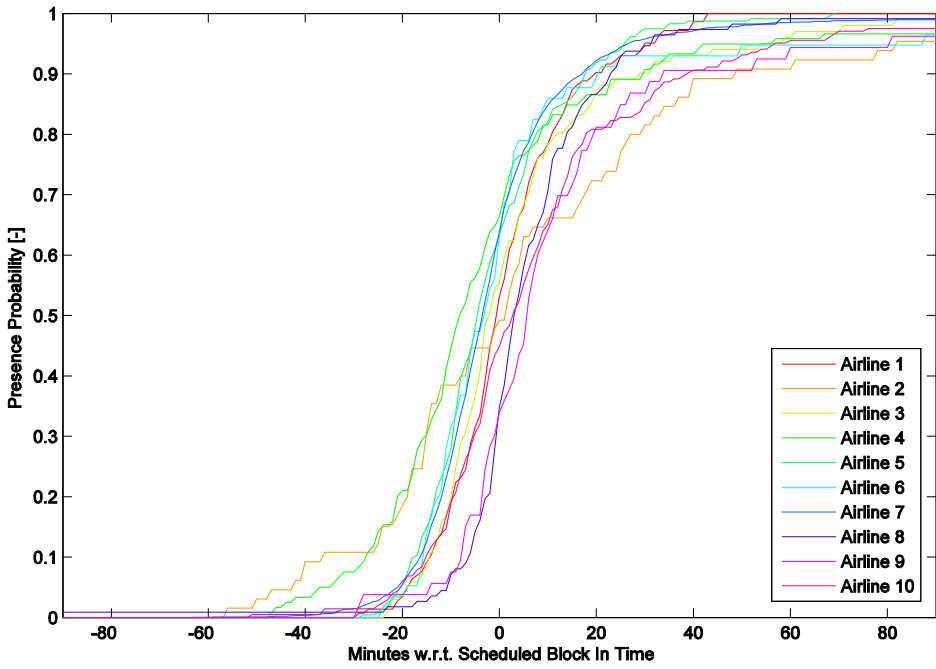


(a)

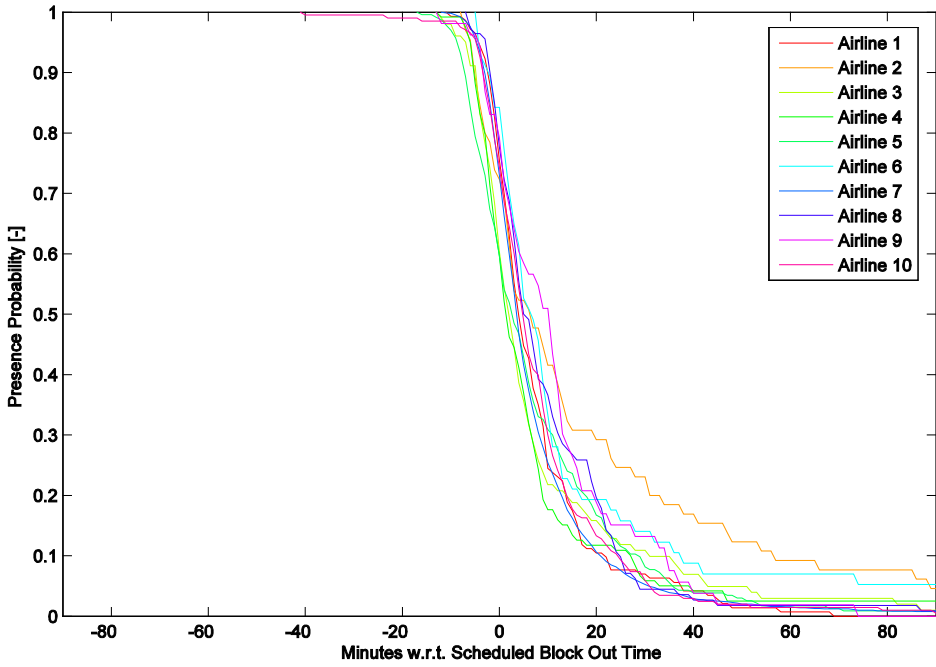


(b)

**Figure 1.** Arrival delay (a) and departure delay (b) cumulative probability distributions from seven different regions.



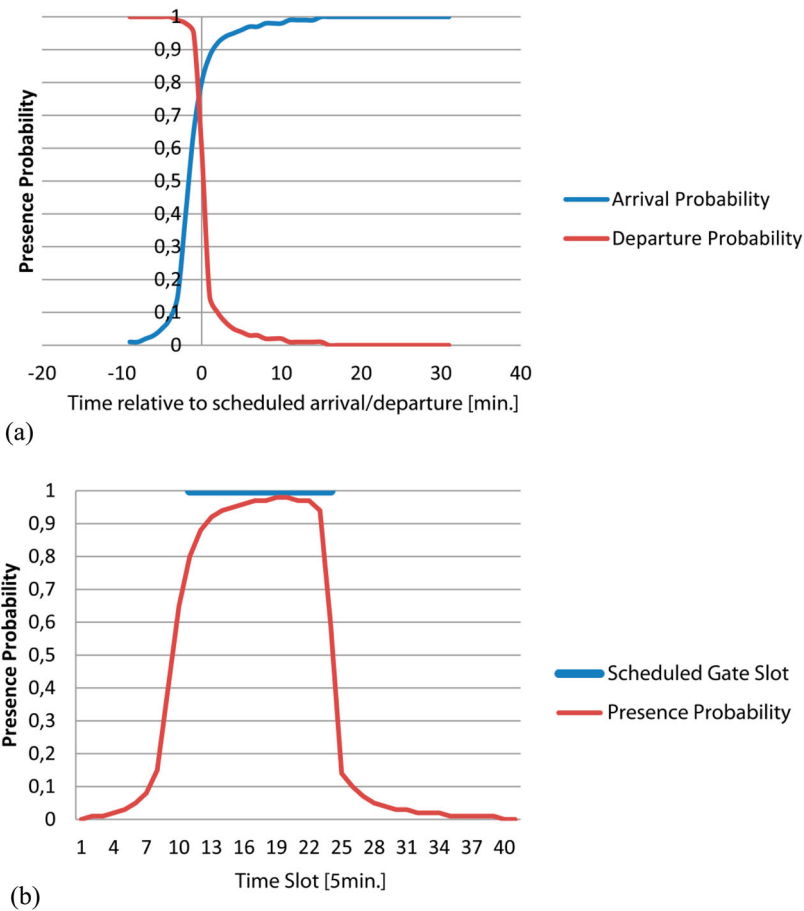
(a)



(b)

**Figure 2.** Arrival delay (a) and departure delay (b) cumulative probability distributions for 10 different airlines.

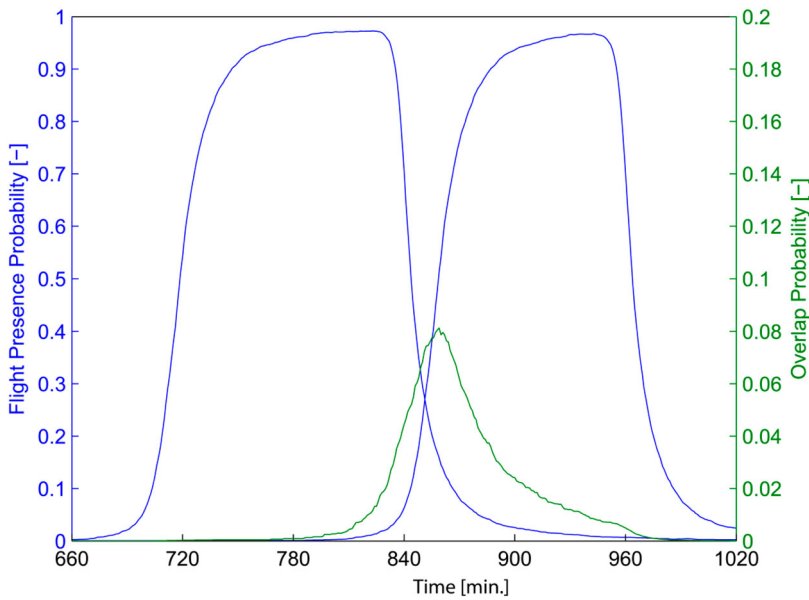




**Figure 3.** Example of assembly of flight presence probability distribution at the apron.

make gate assignment a very difficult problem in practice. To support airports in solving the gate assignment problem, a range of computer-based methods has been developed over the years. In van Rhee (1992) and Koot (2008), two mathematical models have been developed for the daily gate assignment that were specifically tailored to meet all the rules and criteria applicable at AAS. In both models the FGAP is formulated as a Mixed Integer Linear Programming (MILP) problem. While the formulation presented in van Rhee (1992) is a so-called single slot model, the model in Koot (2008) is based on a multiple time slot formulation. A single time slot model proposed by Dorndorf et al. (2007) considers the gate assignment of a batch of flights within a ‘significant’ time slot. A significant time slot begins with the arrival of a flight and ends with the departure of a flight. Within each significant time slot only one flight can be assigned to each gate. In contrast, in a multiple time slot model the overall time interval is just simply divided into a fixed number of time slots.

For a busy hub airport such as AAS, the use of the multiple time slot formulation proved to offer significant computational advantages over the single slot model. Yet, in order to make the multiple time slot approach computationally tractable for an airport



**Figure 4.** Example of assembly of a gate conflict probability distribution.

of the size of AAS, a range of simplifying measures still need to be taken in order to keep the size of the model and the associated computational cost in check. The most important measures introduced in Koot (2008) are the use of a rolling horizon approach for the daily assignment, restricting certain flights to select a gate out of a particular specified subset only (e.g. restricting the assignment of a flight of a low-cost carrier to a low-cost gate only) and aggregating remote stands with similar characteristics into a single remote gate area.

Although the models presented in van Rhee (1992) and Koot (2008) both produce efficient solutions under ideal conditions, the resulting schedules proved to be insufficiently robust in case disrupting events occur. The aim of the present study therefore is to develop an approach that renders the previously developed gate assignment algorithm in Koot (2008) more robust in the sense that the amount of re-planning needed due to schedule disturbances can be kept to a minimum.

To aid the development of the robust gate assignment algorithm, a simplified model has been conceived that considers only the most fundamental strict constraints that underlie the multiple time slot model reported in the Koot model, while implementing only a single component of the original composite multi-objective function defined. In contrast to the original (MILP) model, the simplified FGAP model relies on a binary integer programming (BIP) formulation. In the following, the details of this basic FGAP model will be described.

The basic multiple time slot FGAP model considers the assignments of flights to gates over an entire day in such a way as to minimize the overall cost. The 24-hour planning horizon is simply divided into a fixed number of time slots. In order to present the formulation of the basic FGAP, the following parameters are introduced first:

- $N$  is the set of scheduled flights to the airport during the planning horizon;

- $M$  is the set of gates available at the airport;
- $K$  is the set of time slots within the planning horizon;
- $n$  is the total number of scheduled flights;
- $m$  is the total number of available gates at the airport;
- $k$  is the total number of time slots;
- $c_{ij}$  is the cost of assigning flight  $i$  to gate  $j$  for a single time slot; and
- $s_{it}$  is a binary presence coefficient that indicates whether flight  $i$  is scheduled to be present at the airport apron in time slot  $t$  ( $s_{it} = 1$ ) or not ( $s_{it} = 0$ ).

The gate assignment plan is represented by the binary decision variables  $x_{ijt}$ , where  $x_{ijt}$  is equal to 1 if flight  $i \in N$  is assigned to gate  $j \in M$ , in time slot  $t \in K$ , and is 0 otherwise. The formulation of the basic multiple time slot FGAP can now be expressed as follows:

$$\min \left[ Z = \sum_{i=1}^n \sum_{j=1}^m \sum_{t=1}^k c_{ij} x_{ijt} \right], \quad (1)$$

taking into account all flights  $i = 1, 2, \dots, n$ , gates  $j = 1, 2, \dots, m$ , and time slots  $t = 1, 2, \dots, k$ . The minimization of the objective function (1) is subject to the constraint:

$$\sum_{j=1}^m s_{it} x_{ijt} = 1, \quad (2)$$

for all flights  $i = 1, 2, \dots, n$  and time slots  $t = 1, 2, \dots, k$ . The constraint (2) ensures that when a flight is present at the airport, it is assigned to a gate.

Another crucial element of the basic FGAP model is the set of constraints that allows only one flight at most to be assigned to a gate at a given time:

$$\sum_{i=1}^n s_{it} x_{ijt} \leq 1, \quad (3)$$

for all gates  $j = 1, 2, \dots, m$ ; time slots  $t = 1, 2, \dots, k$ .

Finally, it is important to ensure that a flight that is assigned to a particular gate at a particular time is not switched to a different gate in a subsequent time slot. This is achieved by introducing the constraint:

$$s_{it} \times x_{ijt+1} - s_{it+1} \times x_{ijt} = 0, \quad (4)$$

for all flights  $i = 1, 2, \dots, n$ ; gates  $j = 1, 2, \dots, m$ ; and time slots  $t = 1, 2, \dots, k-1$ . The constraint (4) ensures that when a flight is present at the apron in two consecutive time slots ( $s_{i,t} = s_{i,t+1} = 1$ ) and is assigned to gate  $j$  in time slot  $t$  (i.e.  $x_{ijt} = 1$ ), it will also be assigned to gate  $j$  in time slot  $t+1$  (i.e.  $x_{ijt+1} = 1$ ). Note that the presence coefficients  $s_{it}$  in Equations (2)–(4) are compiled from the input flight schedule for the day of operation considered, and possibly also incorporate (pre-assigned) buffer times.

The cost coefficients  $c_{ij}$  in the objective function (1) are compiled by aggregating several different cost elements, including costs associated with airline preferences, customs, and aircraft–gate size compatibility. An undesirable flight–gate combination is reflected by a very large value of the associated cost coefficients  $c_{ij}$ .

### 3.2. Adding robustness to the basic BIP model

To create a robust gate assignment model, the deterministic gate constraint (3) in the basic BIP model is modified into a stochastic constraint by incorporating the flight presence probabilities in such a way so as to ensure that the expected gate conflict (or overlap) probability of two flights assigned to the same gate at the same time does not exceed a user-specified value. The constraint modification entails replacing the original gate constraint equation (3) with the new stochastic constraint:

$$\sum_{i=1}^n f(p_{it}, r)p_{it}x_{ijt} \leq 1, \tag{5}$$

for all gates  $j = 1, 2, \dots, m$ , and time slots  $t = 1, 2, \dots, k$ . In Equation (5), the ‘scaling function’  $f(p_{it}, r)$  is given by:

$$f(p_{it}, r) = \frac{p_{it}}{r + p_{it}^2}, \tag{6}$$

where  $r$  is the maximum allowed overlap probability (input parameter) and  $p_{it}$  is the flight presence probability (at the apron) of flight  $i$  in time slot  $t$  (to be obtained from the regression model).

To explain the origin and nature of the gate constraints (5) and (6), a simple example will be given involving a single gate constraint. Let us assume that five flights (numbered I–V) are considered to be assigned to gate  $j$  at time  $t$ . The binary flight presence coefficients  $s_{it}$  (based on the original input schedule) along with the flight presence probabilities  $p_{it}$  (based on the regression model predictions) are listed in Table 1 for those five flights in time slot  $t$ .

The original gate constraint (3) considered for gate  $j$  in time slot  $t$  would read:

$$1 \cdot x_{Ijt} + 1 \cdot x_{IIjt} + 0 \cdot x_{IIIjt} + 1 \cdot x_{IVjt} + 1 \cdot x_{Vjt} \leq 1. \tag{7}$$

Constraint (7) permits the assignment of at most one flight (I, II, IV, or V) to gate  $j$  in time slot  $t$ . Now let us assume that in this example the constraint (3) is replaced by the stochastic constraint (5), however, with the scaling function  $f(p_{it}, r)$  uniformly set to 1 (rather than governed by Equation (6)). The new constraint would read:

$$0.85 \cdot x_{Ijt} + 0.45 \cdot x_{IIjt} + 0.20 \cdot x_{IIIjt} + 0.70 \cdot x_{IVjt} + 0.55 \cdot x_{Vjt} \leq 1. \tag{8}$$

In other words, in Equation (8) the binary presence coefficients have been replaced by the flight presence probabilities listed in Table 1. The fact that the constraint coefficients in Equation (8) are (sometimes substantially) smaller than 1 may result in the simultaneous assignment of more than one flight to gate  $j$  in time slot  $t$ . For example, constraint (8) permits flights II and V to be potentially assigned simultaneously, resulting in a gate overlap probability of  $0.45 \times 0.55 = 0.2475$ . In contrast, flights I and III cannot be assigned

**Table 1.** Example of flight presence in time slot  $t$  for five flights.

	Flight I	Flight II	Flight III	Flight IV	Flight V
Flight presence coefficient $s_{it}$	1	1	0	1	1
Flight presence probability $p_{it}$	0.85	0.45	0.20	0.70	0.55

simultaneously to gate  $j$  in time slot  $t$ , although the gate conflict probability for this pair (i.e.  $0.85 \times 0.2 = 0.17$ ) is actually much lower than for the previous pair. Clearly this is undesirable behaviour. To resolve this problem, the scaling factor given by Equation (6) is introduced in Equation (5) to ensure that flights that are assigned simultaneously do not exceed a user-specified value of the maximum permissible gate overlap probability  $r$ .

The required scaling function  $f(p_{it}, r)$  to meet this objective can be readily derived. First, the maximum allowed overlap probability  $r$  is defined as:

$$r = p_{it,max.allowed} \cdot p_{it}, \quad (9)$$

where  $p_{it}$  is a given probability of a flight being present at the apron in time slot  $t$ , and  $p_{it,max.allowed}$  is the maximum probability of a different flight to be assigned at the same gate at the same time without violating the maximum allowed overlap probability  $r$ . Equation (9) can be rewritten as:

$$p_{it,max.allowed} = \frac{r}{p_{it}}. \quad (10)$$

In order to incorporate the maximum allowed overlap probability into the BIP model, the probability  $p_{it}$  has to be scaled by the function  $f$ , as described above, leading to:

$$p_{it,scaled} = f \cdot p_{it} \quad (11)$$

and, similarly:

$$p_{it,max.allowed,scaled} = f \cdot p_{it,max.allowed}. \quad (12)$$

The scaling function  $f$  now needs to be selected in such a way that:

$$p_{it,scaled} + p_{it,max.allowed,scaled} = 1. \quad (13)$$

In words, the scaling function has to be selected such that the resulting scaled coefficients in the constraint equation (5) permit the simultaneous assignment of flight  $i$  in time slot  $j$  as well as a conflicting flight that has an overlap probability with flight  $i$  not exceeding the value  $r$ .

Substitution of Equations (11) and (12) into Equation (13) yields:

$$f \cdot p_{it} + f \cdot p_{it,max.allowed} = 1. \quad (14)$$

Finally, substitution of Equation (10) into Equation (14) yields the scaling function  $f$  as expressed by Equation (6). The coefficients in the constraint equation (5) are readily obtained by substituting Equation (6) into Equation (11):

$$p_{it,scaled} = f \cdot p_{it} = \frac{p_{it}^2}{r + p_{it}^2}. \quad (15)$$

Note that the scaled constraint coefficients in Equation (15) simply reduce to the value one in case the maximum overlap probability  $r$  is specified as zero.

Let us return to the example, and assess the constraint (5) while employing the scale factor given in Equation (6). Assuming a specified maximum permissible overlap probability  $r = .10$  and using the flight presence probabilities  $p_{it}$  listed in Table 1, the following

constraint is obtained for gate  $j$  in time slot  $t$ :

$$0.88 \cdot x_{Ijt} + 0.67 \cdot x_{IIjt} + 0.29 \cdot x_{IIIjt} + 0.83 \cdot x_{IVjt} + 0.75 \cdot x_{Vjt} \leq 1. \quad (16)$$

From the constraint equation (16) it is readily clear that only two flights can potentially be assigned simultaneously to gate  $j$  in time slot  $t$ , viz., flight II and flight III. Then, from Table 1 it follows that for this particular combination the predicted conflict probability is  $0.45 \times 0.20 = 0.09$ , which is indeed less than the specified maximum permissible value  $r = .10$ .

When we relax the specified value of the maximum permissible overlap probability to  $r = .15$ , the constraint equation (5) becomes:

$$0.83 \cdot x_{Ijt} + 0.57 \cdot x_{IIjt} + 0.21 \cdot x_{IIIjt} + 0.77 \cdot x_{IVjt} + 0.67 \cdot x_{Vjt} \leq 1. \quad (17)$$

In this case, in addition to the flight combination II, III, the combinations III, IV, and III, V are also possible. The latter two combinations feature a predicted conflict probability of, respectively, 0.14 and 0.11.

#### 4. Results

Adding robustness to the basic gate assignment problem model (i.e. replacing the gate constraint (3) by the stochastic constraint (5) in the basic FGAP model, given by Equations (1)–(3)) will only result in an actual improvement in the robustness of the planning solution if the predicted gate conflict probabilities vary significantly for a given idle time between each pair of consecutive flights according to the input schedule. Indeed, if this were not the case, using a fixed buffer time in the basic FGAP formulation would already lead to satisfactory robust solutions. From the historic flight presence probability histories presented in Figures 1 and 2, it can be readily inferred that the predicted gate conflict probabilities are likely to vary significantly between individual pairs of flights for the same amount of idle time. Where in the original basic gate assignment model all flights would essentially be treated the same, in the new robust model, the actual time gap between two consecutive assignments to the same gate will depend on the flight presence probabilities and on the specified value of the permissible maximum overlap probability. By varying the value of the permissible maximum overlap probability parameter, a direct trade-off can be made between (cost) efficiency and robustness requirements. By lowering the value the permissible overlap probability parameter  $r$ , the robustness of the planning solution is improved (i.e. the time gaps between consecutive assignments are increased), but this comes at the expense of a lower efficiency (i.e. a higher cost resulting from the fact that flights are assigned to less preferable gates).

The usefulness of the robust FGAP model is demonstrated here in a simple example, featuring only a subset of the flights and gates actually used during an operational day at AAS. Since the application of the robust planning approach is most crucial at highly utilized gates, only the assignment pertaining to the 22 most busy gates is considered in this example. Also, since these gates typically feature simple short-stay operations (i.e. an aircraft arrives at a gate, has a quick turnaround, and departs from the same gate), there is no need of having to deal with the added complexity of aircraft towing operations that are typically called for in the case of long-stay flights.

For the daily assignment of flights to these 22 gates according to the actual planning determined by the gate planners at AAS at a particular day (viz., 11 April 2013), the overall cost of assignment has been calculated using the cost coefficients implemented in the basic FGAP model. Additionally, the flight presence and gate overlap probabilities have been calculated using the regression model developed here. Figure 5 shows the largest recorded value of overlap probability throughout the day for each of the 22 selected gates. It is readily clear that there are a several gates that feature a relatively high value of conflict probability (up to 17%), while other gates exhibit a low maximum overlap probability. It needs to be noted here that in the planning at AAS typically a 20-minute buffer is applied between flights. However, based on experience, gate planners do occasionally apply smaller buffer sizes in practice.

All 133 flights that were assigned to the selected 22 gates in the actual planning were then collected in a 'reduced' flight schedule, which was subsequently used as the input schedule for the basic robust FGAP model. The robust FGAP model was run for a range of values of the permissible maximum overlap probability parameter  $r$ , while using a time slot of five minutes in all cases. In general, the size of the time slots must be judiciously chosen because it influences the problem size as well as possible gate utilization (Dorndorf et al. 2007). Since the flight plan used at AAS features a five-minute time resolution, and the current problem size is not prohibitively large, this particular value has been selected for the time slot size in this numerical example.

Figure 6 shows the results obtained using the robust FGAP model in terms of the maximum observed gate conflict probability at each of the 22 gates, where the target value of maximum permissible overlap probability  $r$  was specified as 7%. It needs to be noted that the value  $r = .07$  actually represents the smallest possible value for which a feasible solution can be found for the robust FGAP model. This relatively large minimum

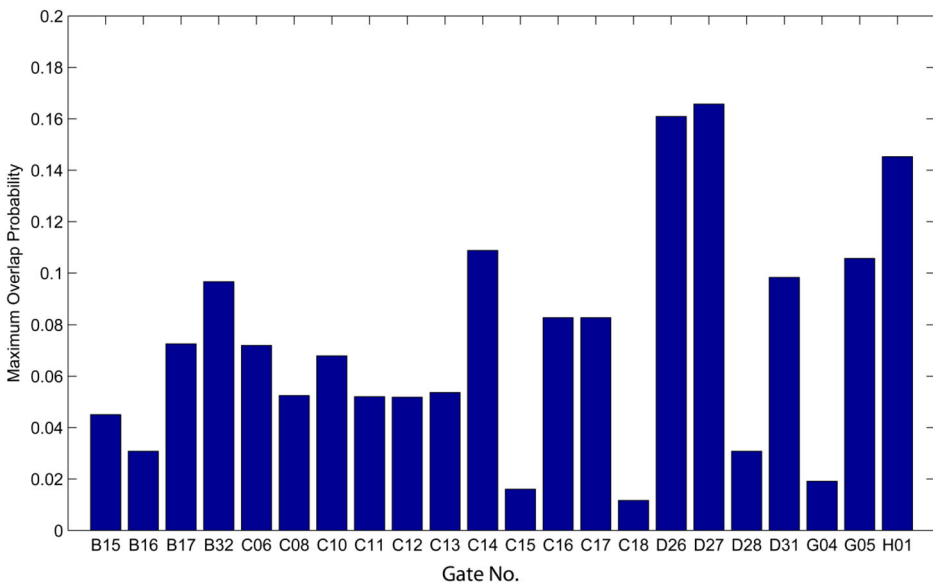
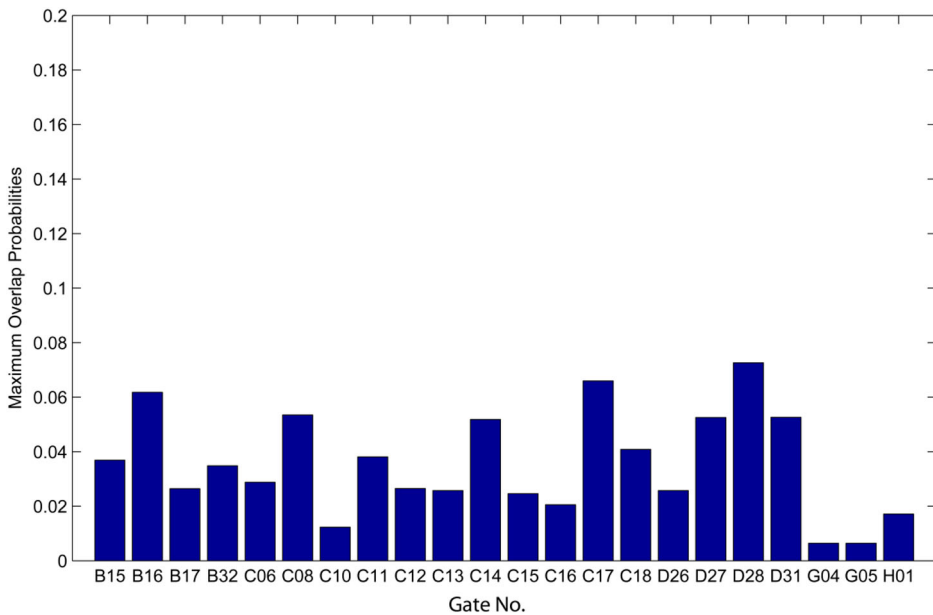


Figure 5. Predicted maximum overlap probabilities for 22 selected gates at AAS based on actual planning (11 April 2013).



**Figure 6.** Results of robust FGAP model for 22 selected gates at AAS (11 April 2013) for specified maximum overlap probability  $r = .07$ .

required value of  $r$  can be attributed to the fact that in this particular example the gates indeed proved to be highly utilized.

The results shown in [Figure 6](#) clearly bear out that for the considered subsets of flights and gates, the solution as produced by the robust basic FGAP model results in much lower gate conflict probabilities in comparison to the actual planning (see [Figure 5](#)). The results clearly demonstrate the value of the robust FGAP model: without compromising cost efficiency, a much more robust solution can be found just by making use of historical delay data. The flight-to-gate assignments resulting from the robust FGAP solution for this case are illustrated in [Figure 7](#).

In [Figure 8](#) the flight-to-gate assignments are shown for a case where the target value of maximum permissible overlap probability  $r$  is specified as 10%. The cost of assignment associated to this particular solution is significantly lower than for the baseline case with  $r = .07$ . More specifically, the 3% increase in accepted gate conflict probability  $r$  leads to an improvement in the cost of assignment of as much as 21%.

A further increase in the value of maximum permissible overlap probability parameter  $r$  to 15% results in a more modest additional reduction in the cost of assignment, i.e. the overall cost of assignment is reduced by about 24% relative to the baseline case. The flight-to-gate assignments for this case are illustrated in [Figure 9](#).

A close comparison of the graphs presented in [Figures 7–9](#) reveals that altering the value of the maximum permissible overlap probability parameter  $r$  has a profound impact on the resulting flight-to-gate assignments. In particular it can be observed that the time gaps between consecutive assignments at the same gate are indeed often much smaller when a large value for the maximum permissible overlap probability is specified.



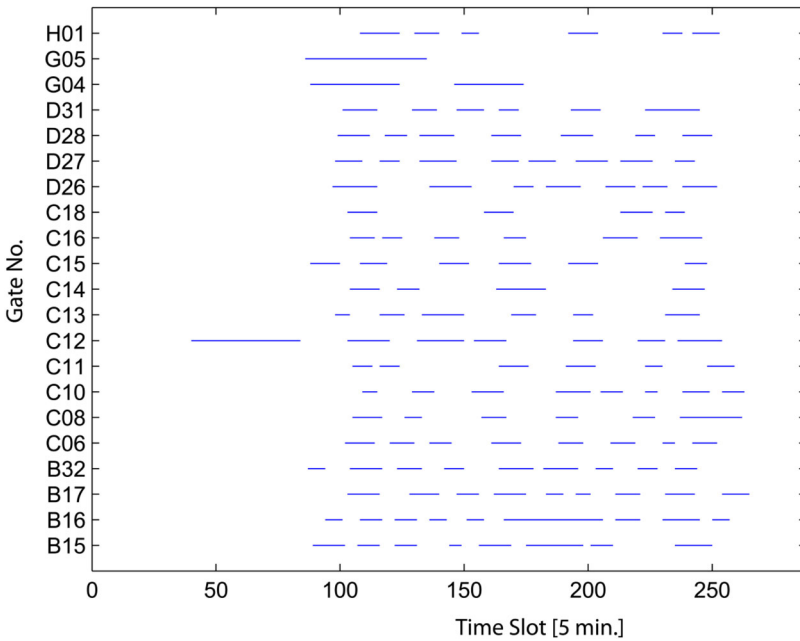


Figure 7. Flight-to-gate assignment resulting from robust FGAP solution for example case with  $r = .07$ .

Ultimately, it will be up to the gate planner to decide what level of robustness (i.e. what value of the maximum permissible overlap probability  $r$ ) needs to be specified for the flight-to gate planning at any given day. In other words, the gate planner will need to

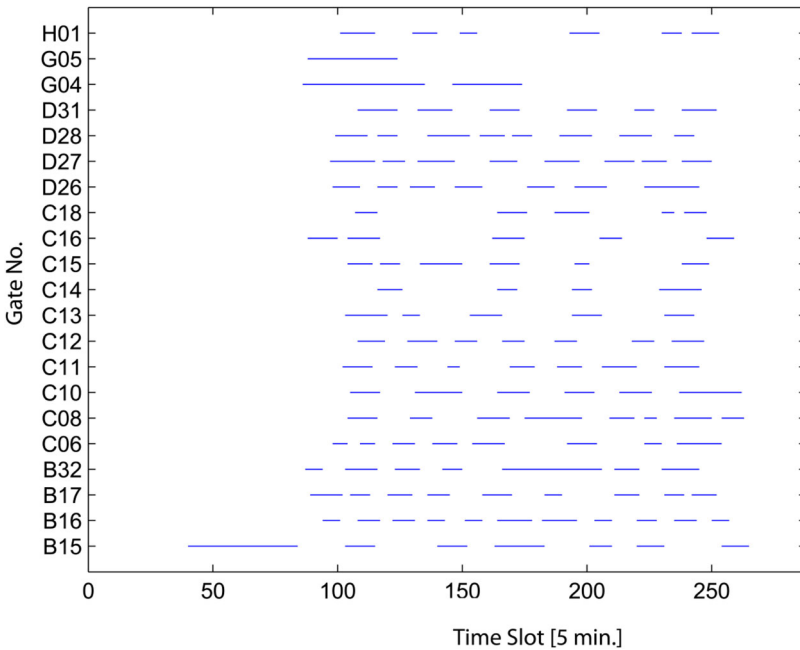
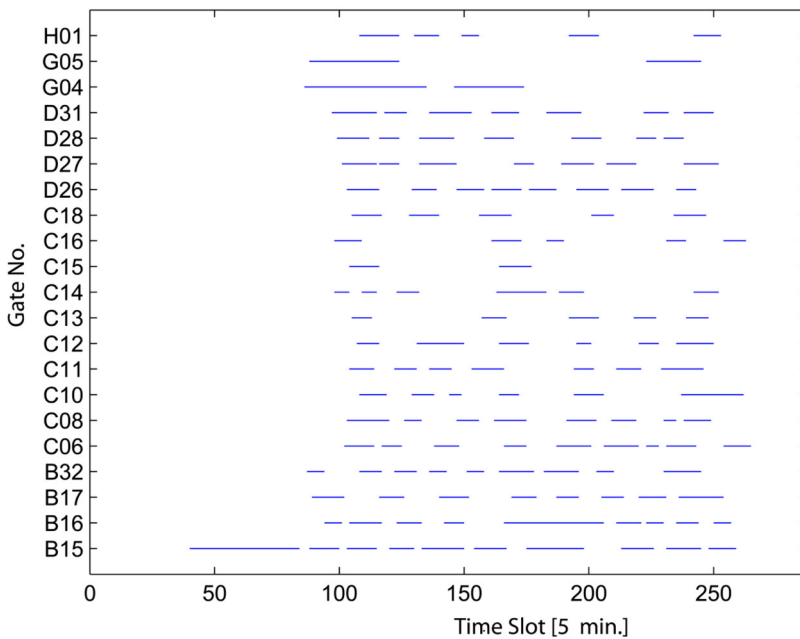


Figure 8. Flight-to-gate assignment resulting from robust FGAP solution for example case with  $r = .10$ .



**Figure 9.** Flight-to-gate assignment resulting from robust FGAP solution for example case with  $r = .15$ .

make a trade-off between efficiency and robustness against unpredicted delays occurring at the airport.

## 5. Conclusions

In this paper we have described an ongoing research effort pertaining to the development of a novel method for generating robust solutions to the multiple time slot FGAP, with the aim to reduce the need for gate re-planning due to unpredicted flight schedule disturbances. In contrast to the more common approaches to add robustness into the gate assignment schedules, such as the use of fixed buffer times, or the use of some stochastic objective function, we proposed a fundamentally different approach that is aimed at modifying the basic constraint underlying the multiple time slot FGAP that ensures that each gate can accommodate only a single aircraft at a time. More specifically, in the proposed formulation the deterministic gate constraints were modified into stochastic constraints by incorporating the inherent stochastic flight delays in such a way so as to ensure that the expected gate conflict probability of two flights assigned to the same gate at the same time does not exceed a user-specified threshold value. Setting this threshold parameter then provided a direct measure to regulate the robustness level in the proposed FGAP formulation. Indeed, the proposed model allowed the creation of a Pareto-front of cost efficiency versus robustness to aid gate planners in making their decision on how robust the solution should be.

Historical data of flight movements at AAS were used to predict probability distributions of flight presence at the apron, based on multiple identified predictors. Based on the expected flight presence probabilities, the gate conflict probabilities were readily

assessed. The novel approach was implemented in a prototype software tool that relies on a basic BIP formulation and was tested using real-life data. The results bear out that the proposed method to include uncertainty in the planning process holds out great promise to improve the robustness of the FGAP solutions. In particular, the proposed approach permits a direct trade-off evaluation to be made between (cost) efficiency and robustness in the flight-to-gate planning process.

The robust FGAP model proposed here features a computational complexity that is not significantly different from the basic multiple time slot FGAP formulation from which it originates. On the indication of the preliminary results, the concept therefore appears to hold out great promise for further development. An initial effort to implement the robust gate constraint formulation in the full-scale MILP-based multiple time slot FGAP model reported in Koot (2008) has already been undertaken, with very encouraging results (van Schaik 2013).

Future research will also need to focus on the development of an improved regression model to predict flight presence probabilities distributions. Having accurate flight presence probability estimations available is indeed of paramount importance for a successful implementation of the proposed robust FGAP model. The regression model developed and used here is rather rudimentary in nature, which is in part due to the fact that only a limited amount of historic surface movement data were available. Assuming that a significantly more extensive historic data set can be made available, future work will be aimed at developing a regression model based on a much wider set of (uncorrelated) predictors.

## Disclosure statement

No potential conflict of interest was reported by the authors.

## ORCID

Hendrikus G. Visser  <http://orcid.org/0000-0002-5644-0430>

## References

- Bolat, A. 2000. "Procedures for Providing Robust Gate Assignments for Arriving Aircraft." *European Journal of Operations Research* 120 (1): 63–80.
- Bouras, A., M. A. Ghaleb, U. S. Suryahatmaja, and A. M. Salem. 2014. "The Airport Gate Assignment Problem: A Survey." *The Scientific World Journal*. Article ID 923859, doi:10.1155/2014/923859.
- Castaing, J., I. Mukherjee, A. Cohn, L. Hurwitz, A. Nguyen, and J. J. Müller. 2016. "Reducing Airport Gate Blockage in Passenger Aviation: Models and Analysis." *Computers & Operations Research* 65 (1): 189–199.
- Diepen, G., J. M. van den Akker, and J. A. Hoogeveen. 2012. "Finding a Robust Assignment of Flights to Gates at Amsterdam Airport Schiphol." *Journal of Scheduling* 15 (6): 703–715.
- Dorndorf, U., A. Drexler, Y. Nikulin, and E. Pesch. 2007. "Flight Gate Scheduling: State-of-the-art and Recent Developments." *Omega* 35: 326–334.
- Koot, R. 2008. "Gate Assignment Using Linear and Binary Integer Programming Techniques in AMPL." Unpublished MSc thesis, Delft University of Technology, Delft.
- Li, C. 2009. "Airport Gate Assignment: A Hybrid Model and Implementation." <http://arxiv.org/abs/0903.2528>.

- Mangoubi, R., and D. Mathaisel. 1985. "Optimizing Gate Assignments at Airport Terminals." *Transportation Science* 19 (2): 173–188.
- Seker, M., and N. Noyan. 2012. "Stochastic Optimization Models for the Airport Gate Assignment Problem." *Transportation Research Part E* 48 (2): 438–459.
- van Rhee, G. 1992. "Gate Assignment Using Linear Programming." Unpublished MSc thesis, Delft University of Technology, Delft.
- van Schaik, O. R. P. 2013. "Finding Robust Solutions to the Flight to Gate Assignment Problem Using Flight Presence Probabilities." Unpublished MSc thesis, Delft University of Technology, Delft.