Complexity of Scheduling Charging in the Smart Grid

Extended Abstract

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ABSTRACT

The problem of optimally scheduling the charging demand of electric vehicles within the constraints of the electricity infrastructure is called the *charge scheduling problem*. The models of the charging speed, horizon, and charging demand determine the computational complexity of the charge scheduling problem. For about 20 variants the problem is either in P or weakly NP-hard and dynamic programs exist to compute optimal solutions. About 10 other variants of the problem are strongly NP-hard, presenting a potentially significant obstacle to their use in practical situations of scale.

KEYWORDS

Complexity Theory; Scheduling; Charging Electric Vehicles

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1 INTRODUCTION

The problem of deciding when to charge electric vehicles [8] under a shared network constraint gives rise to a new class of scheduling problems. The defining difference from the traditional scheduling literature [5, 15] is that such charging jobs are more flexible: not only can they be shifted in time, but the charging speed can also be controlled. Additionally, the charging resources ("the machines" in ordinary scheduling) may vary over time [6, 14]. Further, providers that control flexible demand will need to solve such scheduling problems repeatedly. Therefore, it is important to understand when such problems can be solved optimally within the time limits required, and what aspects of the model make the problem intractable. We refer to this class of problems as the *charge scheduling problem*.

While the existing scheduling literature is extensive [5, 9, 15], the unique setting of charge scheduling gives rise to a number of novel variants of the general scheduling problem. The charge scheduling problem can be seen as a special case of the so-called resource-constrained project scheduling problem (RCPSP) [9] if additionally the problem is extended to deal with continuously divisible resources [3, Ch.12.3], a varying availability of resources with time [10], and the possibility to schedule subactivities of the same activity in parallel, called fast tracking [17]. An initial investigation shows that minimizing the make-span for this extension, but for one resource and without preemption is strongly NP-hard even if there are only three processors available [4]. This NP-hardness result, however, does not apply to the charge scheduling problem, because of the difference in the objective and because in charge scheduling preemption is allowed. However, there are some directly applicable results for fixed supply: then the charge scheduling problem is equivalent to a multi-machine problem where all agents have an identical charging speed and the supply is a multiple of the charging speed. From this we immediately obtain a dynamic program and that this variant is weakly NP-hard [12, 13]. However, when supply (i.e., the number of machines) varies over time, or when charging speed limits (i.e., the maximum number of machines allowed for a single job) differ per agent, the existing literature does not readily provide an answer to the question of the charge scheduling problem complexity.

We identify over 30 variants, and in our full (IJCAI-ECAI-18) publication their computational complexity is proven, and for the easy problems a polynomial dynamic programming algorithm is provided [7].

2 THE CHARGE SCHEDULING PROBLEM

The availability of the supply at time t is represented by a value $m_t \in \mathbb{R}$, modeling for example remaining network capacity at a congestion point. This supply is allocated to a set of n agents, and the allocation to agent i is denoted by a function $a_i : T \to \mathbb{R}$. The value of an agent i for such an allocation is denoted by $v_i : [T \to \mathbb{R}] \to \mathbb{R}$. This value function for an allocation can represent both dynamic prices of charging in certain time slots as well as user preferences for when their vehicle is charged. In this paper, we focus on problems where the valuation function of agent i can be represented by triples of a value $v_{i,k}$, a deadline $d_{i,k}$ and a resource demand $w_{i,k}$, such that the value $v_{i,k}$ is obtained if and only if the demand $w_{i,k}$ is met by the deadline $d_{i,k}$. This allows the agent (app) to represent user preferences such as: "I value being able to go to work at $100, I must leave for work at 8am, and it requires 25 kWh to complete the trip." By adding a second deadline, the user could express: "I may suddenly fall ill, so I value having at least the option to take my car to urgent hospital care at 10pm at $20, and it would require 10 kWh to complete that trip."

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single deadline

$\lvert T \rvert$	*gaps* demand constant	*gaps* demand polynomial	*gaps* demand unbounded	*fixed charging speed* demand constant	*fixed charging speed* demand polynomial	*fixed charging speed* demand unbounded	*unbounded charging speed* demand constant	*unbounded charging speed* demand polynomial	*unbounded charging speed* demand unbounded
$O(1)$	P	P	weak NP-c	P	P	weak NP-c	P	P	weak NP-c
$O(n^c)$	strong NP-c	strong NP-c	strong NP-c	?	?	? NP-c	P	P	weak NP-c

multiple deadlines

$\lvert T \rvert$	*gaps* demand constant	*gaps* demand polynomial	*gaps* demand unbounded	*fixed charging speed* demand constant	*fixed charging speed* demand polynomial	*fixed charging speed* demand unbounded	*unbounded charging speed* demand constant	*unbounded charging speed* demand polynomial	*unbounded charging speed* demand unbounded
$O(1)$	P	P	weak NP-c	P	P	weak NP-c	P	P	weak NP-c
$O(n^c)$	strong NP-c	strong NP-c	strong NP-c	strong NP-c	strong NP-c	strong NP-c	strong NP-c	strong NP-c	strong NP-c

Table 1: Problem complexity of variants with a single deadline per agent (top) and multiple deadlines (bottom), where '? NP-c' means that the problem variant is NP-complete, but it is an open problem whether this is strong or weak.

To be able to express such a valuation function concisely, we denote the total amount of resources allocated to an agent i up to and including interval t by $\bar{a}_i(t) = \sum_{t'=1}^{t} a_i(t')$. We then write

$$v_i(a_i, v_{i,k}, d_{i,k}, w_{i,k}) = \begin{cases} v_{i,k} & \text{if } \bar{a}_i\left(d_{i,k}\right) \geq w_{i,k} \\ 0 & \text{otherwise} \end{cases}$$

and $v_i(a_i) = \sum_{k \in K(i)} v_i(a_i, v_{i,k}, d_{i,k}, w_{i,k})$, where $|K(i)|$ is the number of deadlines for i.

We aim to find an allocation that maximizes social welfare subject to the resource constraints, i.e.,

$$\begin{array}{ll} \max & \sum_i v_i(a_i) \\ \text{subject to} & \sum_i a_i(t) \leq m_t \text{ for every } t \end{array}$$

We then consider the following dimensions:

- Each agent i has a maximum *charging speed* s_i and for all t and i, $a_i(t) \leq s_i$. We consider three variants of such a constraint: *fixed* means that the maximum charging speed is the same at all times, *unbounded* means that there is no bound on the charging speed for each individual agent, and *gaps* means that the maximum charging speed may be 0 for some time steps and unbounded for others.
- The number of *periods* T is *constant* (denoted by $O(1)$) when there is an a-priori known number of periods for all instances of the problem, while we say it is *polynomially bounded* ($O(n^c)$) when the number of periods may be large, but is bounded by a polynomial function of the input size.
- The model of the *demand* $d_{i,k}$ is *constant* when $d_{i,k} \leq D$ for all i, k and this D is an a-priori known constant, it is *polynomial* when each $d_{i,k}$ is bounded by a polynomial function of the input size, and it is *unbounded* when there is no bound on the demand size.
- We can have either a *single deadline* per agent, $k = 1$, or *multiple* deadlines where there may be more than one value-demand-deadline triple.

The overview of the the complexity of the charge scheduling problem along these dimensions can be found in Table 1.

3 DISCUSSION AND FUTURE WORK

These complexity results provide an important step towards practical applicability. However, a number of questions are still open.

First, the hardness results for instances with three deadlines extend to any constant number of deadlines at least three, and we have separate results for most of the single-deadline cases. However, it is open exactly what happens with two deadlines (except that it is at least as hard as with one deadline).

Second, if the number of time periods is constant but too large, the dynamic programs we use to prove weak NP-hardness do not scale to realistic problem instances [7]. Therefore, a relevant direction for future work is to develop fast heuristic algorithms.

Third, the results presented are realistic if good predictions for future supply and demand are available (such as based on weather predictions and historical charging patterns, which can be quite accurate for larger groups). If these predictions are only somewhat good, it becomes important to explicitly think about charge scheduling as an online problem [1], where we will want to re-solve the offline problem at each point in time. Although our hardness results still apply, this opens new questions, such as the competitive performance of on-line algorithms (compared to off-line). For deterministic algorithms we can conclude from [2] that if realized charging speeds needs to be either the maximum or 0, no constant competitive algorithm exists. However, algorithms exist for other variants, e.g., without the supply constraint [16] or with a weak supply constraint [18], and there exist randomized algorithms for other variants of on-line scheduling [11]. It remains an open question which of these can be effectively applied to an on-line variant of the charge scheduling problem and how they would perform against simpler heuristics.

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