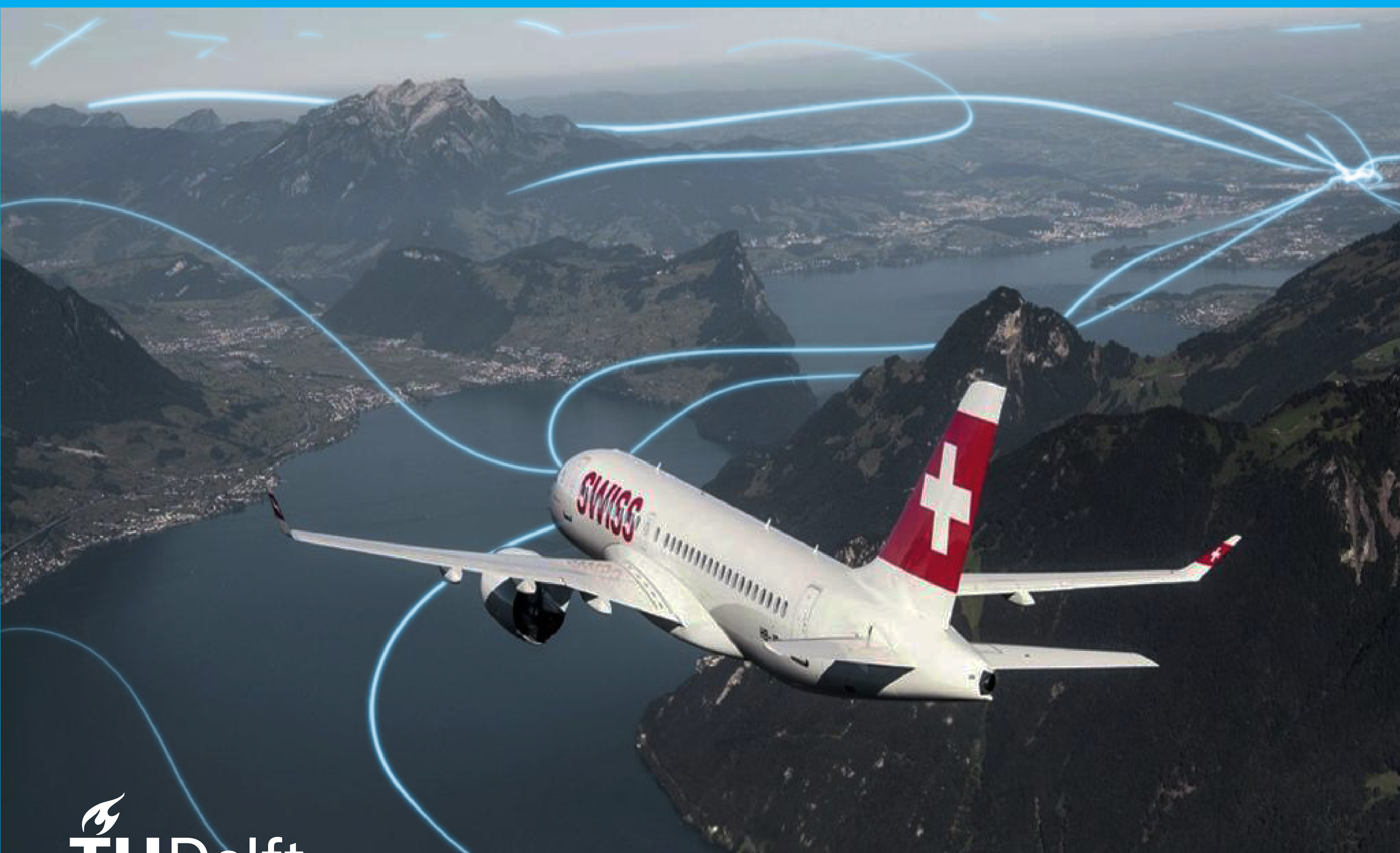
Flight planning quality assessment with predictive analysis using data clustering and reference trajectories

Koen Pieter Beukers



TU Delft

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by

Koen Pieter Beukers

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During my aerospace engineering bachelor there were some moments where I had doubts about my choice of study. However the masters profile of air transport operations reaffirmed my passion for aviation and airlines in particular. This was even further grown with the amazing opportunity I received from SWISS International Air Lines in the form of an internship at the operation research department. I am very grateful for this time even though global circumstances slightly complicating things. In particular I would like to thank Eric Nantier who backed my desire to do a thesis project in collaboration with SWISS and envisioned this project idea.

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Koen Beukers
Delft, September 2021

Contents

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List of Abbreviations

ABS-C	Automatic Dependent Surveillance Contract
AEDT	Aircraft Environmental Design Tool
AFT	Actual Flown Trajectory
ANSP	Air Navigation Service Provider
ASMA	Arrival Sequencing and Metering Area
ATC	Air Traffic Control
ATFM	Air Traffic Flow Management
ATM	Air Traffic Management
AU	Airspace User
CD	Continuous Decent
CH	Contraction Hierarchy
EPS	Ensemble Prediction System
FAB	Functional Airspace Block
FIFO	First In First Out
FIR	Flight Information Region
FMS	Flight Management System
FPP	Flight Planning Problem
FPT	Flight Plan Trajectory
GA	Genetic Algorithm
GCD	Great Circle Distance
HFPP	Horizontal Flight Planning Problem
ICAO	International Civil Aviation Organization
IF	Isolation Forest
IFR	Instrument Flight Rules
KPI	Key Performance Indicator
LOF	Local Outlier Factor
MIP	Mixed Integer Program
NMOC	Network Management Operations Centre
OCP	Optimum Control Problem
PI	Performance Indicator

QAR	Quick Access Recorder
RAD	Route Availability Document
RFC	Random Forest Classification
RNAV	Area Navigation
RPL	Repetitive Flight Plan
SES	Single European Sky
SESAR	Single European Sky initiative
SID	Standard Instrument Departure
SPP	Shortest Path Problem
SSPP	Stochastic Shortest Path Problem
STAR	Standard Terminal Arrival Route
TBO	Trajectory Based Operations
TDSPP	Time Dependent Shortest Path Problem
TO	Trajectory Optimisation
TOC	Top Of Climb

Introduction

Motivation

This thesis project was initiated in collaboration with SWISS International Air Lines after a very rewarding internship in their operation research department. The amount of data available to this team is fairly unique within the airline industry and therefore many exiting possibilities for research exist. At the start of this thesis, interest was expressed to analyse the efficiency of the used flight planning software. However, it was found that very little to none research about the topic was available. Therefore this research was proposed.

Scope

The scope of the project first included an implementation of the developed method in the operational systems, but as time passed this was found the be infeasible partly by the challenges remote working introduced. The project therefore should serve as a starting point from which could be continued to further provide value for SWISS. This also meant that the project had little to no agreed deliverables. It was tried to keep involved or interested SWISS team members up to date with the progress and any found phenomena. Found inefficiencies usually raised further questions and tested how much was able to be explained with the available data.

Objective

The purpose of this study is for flight planning software users to better understand the quality of their system. This mainly focuses on how quality can be expressed which in turn could then be used for performance monitoring or help decision making. The research objective is specified as:
"Create a framework to assess the quality of flight planning by predictive analysis using data clustering and reference trajectories."

Thesis Outline

This thesis report is organised as follows: In Part I, the scientific paper is presented. The Literature Study that was undertaken to support this work is given in Part II. Part III contains an additional remark about the available data for this study and a section about an investigated method that was decided to be excluded from the main scientific paper.

I

Scientific Paper

Flight planning quality assessment with predictive analysis using data clustering and reference trajectories

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Abstract

Flight planning is a process of optimisation that is commonly solved by complex software tools. Quantifying the quality of such a system is for the users an unsolved problem. This paper proposes a method that measures the inefficiencies in different domains to gain insights in the performance of flight planning tools. The established assessment framework includes relating actual operated performance to the flight plan instructions. Additionally, the framework is used to detect and predict possibly abnormal flight plans before they are operated.

1 Introduction

In commercial aviation, flight planning is an important process to ensure safe and efficient operations. Depending on the airlines operational criteria, the aircraft's trajectory is optimized for a balance between fuel cost, time cost and possibly more objectives like emissions. To solve this complex problem, airlines employ automated tools to generate efficient and feasible flight plans. These algorithms are build on certain assumptions so good solutions can be generated in a reasonable amount of computation time. The impact of these simplifications is to the best of my knowledge not extensively studied. As J. Rosenow states in [1] "An assessment of those tools is an unsolved challenge, because of a missing optimum trajectory which is hard to define and impossible to verify." However, there might be possibilities to assess such systems without defining the theoretical optimum by using historical operational data. Conventional statistical and machine learning techniques can give insight in if and where inefficiencies are present given the right set of reference data and representative performance measurements. Selectively introducing or omitting assumptions from reference simulations can be used to quantify their impacts. This impact is evaluated by performance indicators (PIs) which stipulates the need of creating a set that well reflects airlines objectives.

New PIs are continuously researched in the air traffic management (ATM) field, but to my knowledge no such public research exists in quantifying flight planning from an airline perspective. Providers of flight planning software do make claims to reduce fuel costs and increase productivity by some arbitrary percentage, but users i.e. airlines find this impossible to verify. Fuel savings as stated by these companies seemingly relates to the actual operated fuel costs. Therefore, associating the quality of a flight plan with its operated result. An 'optimal' flight plan has little value if it can not be operated as such.

Requiring the actual operated performance however limits the availability of the assessment framework. An envisioned use of the flight plan assessment framework is to identity abnormal instances. The flight planning process at an airline could benefit from a system that estimates the likelihood of a given plan to be operated inefficiently, so that suitable actions can be taken. This thus necessitates a predicting characteristic since operated data will not be available yet. With the recent popularity of machine learning methods, many different estimators are available to try and solve these challenges.

2 Literature Review

The flight planning problem (FPP) is an optimisation problem that can be considered as an optimum control problem or a shortest path problem. Traditionally this was solved by separating the FPP, first finding an optimum horizontal route and then optimising the altitudes and speeds. Recently, advancements in computational power and optimisation algorithms allow more and more of this separation to disappear and relax some of the frequently adopted assumptions to further improve the optimisation. Newly proposed methods include the wind component in the path finding phase and even allow for non linear and time dependent arc weights [2, 3, 4,

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5]. Other studies have focused on minimising the effect of uncertainties in the planning process by including heuristics [6, 7]. It is unclear if these newer methods are already used in commercial software.

In contrast, procedures and practices in the ATM field are very transparent. The ATM research community focuses a lot of its effort in reducing environmental impact by increasing airspace efficiency. This has resulted in new ways to quantify flight efficiency [8, 9, 10, 11, 12] which might be adapted and applied to reflect the airspace users (AU) objectives. Additionally, Jensen et al. in [13] propose a use of historical data to evaluate potential savings across an airspace. Here trajectories from all AUs are collectively analysed with the use of reconstructive trajectory modelling. This approach will be used in this research, but adapted and applied to flight plans rather that actually flown trajectories. An advantage in this project is the availability of actual aircraft data due to the collaboration with an AU. Therefore, trajectory reconstruction for actual flown flight paths will not be needed.

Today, many applications for machine learning are already envisioned and adopted in aviation. Anomaly detection is one of the sub-domains with a lot of potential benefit for airlines and is therefore extensively researched [14, 15, 16, 17]. Most studies however focus on safety analysis rather than efficiency. However, established methods designed to detect events that could harm safety can also be applied for detecting events that cause inefficiencies.

3 Methodology

The quality assessment framework should include performance indicators (PIs) so that all business values are covered. Additionally, PIs should be consistent regardless of the type of operation of the airline and objectively derivable from operational data. Since the optimum flight plan is not known in advance and as good as impossible to determine after operation, performance has to be quantified by comparing it to other consistent references.

3.1 Reference Trajectories

Multiple references are determined so that different factors of inefficiencies can be detected. Loses due to routing can be analysed by separating the horizontal component and do not need aircraft model simulations.

3.1.1 Horizontal trajectories

Horizontal flight efficiency is a well studied and often used metric in ATM analysis. The achieved distance is compared to the most direct path between airports. The great circle path therefore is the reference trajectory for this method. However, the routing near the source and target airports is determined the airport infrastructure, wind direction combined with fixed departure and arrival procedures. This thus describes the inefficiency in airspace structure rather than operation. Excluding terminal segments will eliminate this from the reference. A common boundary is the Arrival Sequencing and Metering Area (ASMA), which in Europe is often defined by a 40 nautical mile radius around an airport.

Where the planned or actual trajectory intersects this ASMA will serve as start and end points for the great circle reference geometries as shown in figure 1. One could either use the intersection of the Actual Flown Trajectory (AFT) or that of the Flight Plan Trajectory (FPT) denoted by the subscript p. In most cases these points are close to each other. However, if for instance the runway changed due to the wind direction, different standard arrival and departure routes might be used. Then, the difference could be considerable and should be noticeable when comparing both values.

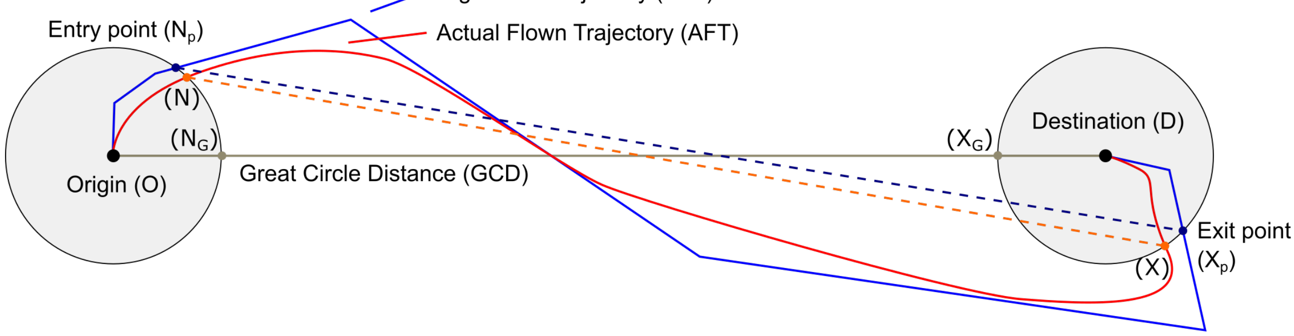


Figure 1: Horizontal reference geometries

The various reference track lengths as well as AFT and FPT length are calculated on the world geodetic system of 1984 ellipsoid. The relative change between the several combinations of AFT, FPT and the above mentioned reference geometries are used as the horizontal efficiency metrics as will be further described in section 3.2.

It is acknowledged that using only the track length of a given flight path can not capture all characteristics that could be valuable for assessing efficiency. Some inefficient flights can be explained by the aircraft needed to enter a holding pattern. This study therefore implements a holding pattern recognition algorithm and uses the presence of a holding as a boolean parameter on the AFT. The detection algorithm works by measuring the angle between track heading and the direct vector to the airport. This angle is calculated along with the distance to the destination airport for every available datapoint on the AFT. These observations are then ordered to find the minimum distance at which the relative angle is greater than 135 deg as depicted in figure 2.

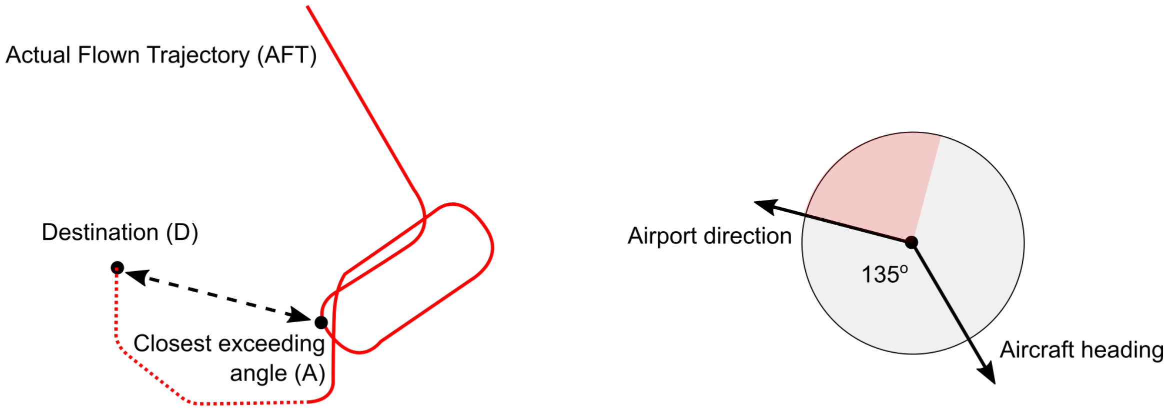


Figure 2: Holding pattern recognition method example

The threshold of 135 deg was chosen by trial and error, verified with flight records with known presence and absence of holding. In order to increase the methods robustness to noisy data, a minimum of 5 angle exceeding observations within ASMA is used before classifying a flight for having a holding.

3.1.2 Simulations

Similar to the different horizontal trajectories, selecting specific assumptions and conditions for a simulation can be used to determine what factor of inefficiency is analysed. For this research it is chosen to only select fuel optimised simulations. This is done because the alternative optimisation domain would be time, however the value of a flight to be on time is dependent on many factors and chosen to be out of scope for this research. To produce fuel optimal reference simulations, the aircraft database Piano X is used. The aircraft type, model and payload are conditions that are matched to the flight to compare with. The distance however is the parameter that is used to create the different references. In order to avoid having to run very similar simulations for every flight record, piece-wise linearity is assumed. This allows two-dimensional interpolation of the fuel burn given an payload and distance combination. The assumption is supported by figure 3 that shows little out of plane deformation.

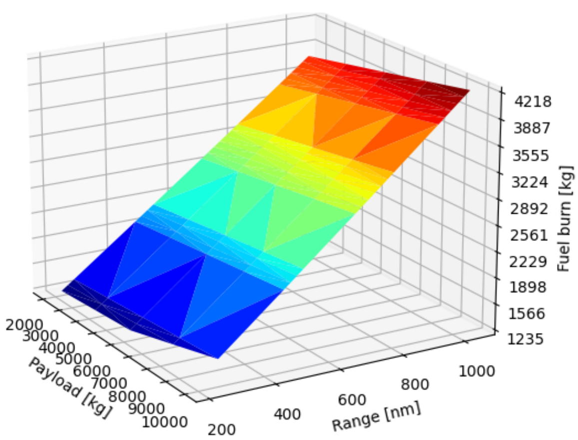


Figure 3: Piece-wise linear fuel burn approximation of Piano X model for Airbus 220-100

The simulation software provides besides the fuel burn also a detailed flight profile, with for instance the speeds and altitudes over the different segments of the flight. Since these are not single values and can therefore not be interpolated, the closest simulation is used in this regard for a given fuel and range. In total, 1970 simulations across 9 aircraft types were run to ensure that this nearest available point was reasonably close. The payload increments were set to 2000 kg and the step size for range was 20 and 50 nm for narrow and wide body aircraft respectively.

This research uses three approaches for selecting the range, resulting in three different simulation references as given below. They are denoted with a C to correspond to already existing research although in this case they are fuel optimised rather than cost optimised.

C1 - Great circle distance simulation This is the fuel optimised trajectory when free flight conditions are considered.

C2 - Planned distance simulation To capture the airspace structure, this trajectory is simulated using the horizontal distance of the planned track. Note that this solution will copy the shortest path solution for the flight planning software, but this will include avoiding adverse weather.

C3 - Weather component corrected distance Since wind is a major factor in the routing problem, this trajectory uses a distance that is extended or shortened depending on the forecasted wind speed and direction. The wind component is calculated using equation 1 where $\vec{w}$ is the wind vector and $\vec{v}$ the aircraft speed vector.

$$W/C = \sum_t \frac{\vec{w}_t \cdot \vec{v}_t}{||\vec{v}_t||} \tag{1}$$

3.2 Efficiency Domains

Airlines optimise their flight plans on a balance of different objectives. Therefore, to best analyse the quality of the optimisation, all objectives have to be reflected in the assessment framework. It can be argued that for an airlines operation, cost and time are the driving factors for flight plan optimisation. However, breaking down these objectives further does not hurt but allows identifying contributing factors of inefficiencies in the same way different references do. This approach was also taken in the research of López-Leonés et al. [18] and is largely replicated in this study.

Each of the domains will have a number of metrics that compare different trajectories and are named based on them. The domain specifies the first letter, with K for horizontal, V for vertical, T for time and F for fuel. This is followed by a generic E to depict efficiency. Than, the compared trajectories are noted, A for AFT, P for FPT and C for simulated references as in section 3.1.2. For horizontal metrics this is followed by an A if only the part outside ASMA is considered. The full list of metrics with description can be found in appendix A.

3.2.1 Horizontal

This domain quantifies the extra distance flown. Every metric will be normalised, therefore relative change is used. An example is shown in equation 2, where the extra distance is a measure of L_{AFT} as AFT track length and H as great circle distance between origin and destination.

$$KEA = \left(\frac{L_{AFT}}{H} - 1\right)\% \tag{2}$$

3.2.2 Vertical

For the vertical domain the average altitude is scored in the same way. A higher average often means a more fuel efficient flight. This is partly because a longer intermittent descent is more inefficient compared to a continues one and does result in a lower total altitude average. To conform the sign of this metric to the others, positive indicating inefficiency, this metric is inverted as given in equation 3. $\overline{h}$ denotes average altitude.

$$VEA\_P = -\left(\frac{\overline{h}_{AFT}}{\overline{h}_{FPT}} - 1\right)\% \tag{3}$$

3.2.3 Time

While ANSPs do not consider flight time as one of their operational objectives, for airlines this sometimes is the driving factor. It is common to allow a higher cost index in order to make up any existing delays that could cost the airline more than the additional fuel burn. Therefore, this study includes this domain with relative flight time calculated with equation 4 where t_{AFT} and t_{OCT1} are actual flown trajectory and Optimum cost trajectory 1 total flight time respectively.

$$TEA\_C1 = \left(\frac{t_{AFT}}{t_{OCT1}} - 1\right)\% \tag{4}$$

Relative delay recovery is also included in the assessment framework, but since only the AFT can have a delay, the arrival delay dt_{arr} is compared to the departure delay dt_{dep} as seen in equation 5. This is denoted by $TDEA\_P$ since the planned delay essentially is 0. Note the this metric only exists if there is departure delay.

$$TDEA\_P = \left(\frac{dt_{arr}}{dt_{dep}} - 1\right)\% \tag{5}$$

3.2.4 Fuel

The fuel metric is calculated with the total fuel burn of the airborne flight phase. As example FEA_C2 comparing the actual fuel burn with the simulated fuel burn over the planned distance is given by equation 6 where f denotes the fuel burn in kg.

$$FEA\_C2 = \left(\frac{f_{AFT}}{f_{OCT2}} - 1\right)\% \tag{6}$$

3.3 Anomaly Detection

The previously defined efficiency metrics are used to quantify flight plan quality in the different domains, but also to identify anomalies. Anomalies can be detected with methods that can be sub-classed in three different categories. These are; using statistical models, predicting models or machine learning detection models. [19] Statistical models use the behaviour of known observations to decide if the current subject is abnormal. This method is adapting to new data, but therefore will also change the interpretation with the new data. In this research, statistical outliers are determined with the z-score as defined in equation 7, where x is the observed value, μ and σ the mean and standard deviation of the sample respectively.

$$Z = \frac{x - \mu}{\sigma} \tag{7}$$

The second category in outlier detection is comparing the sample to a known prediction model. This is exactly what is done with the various reference trajectories. As example, horizontal distance can be classified as abnormal if a flight plan has a track length of 500km is the airports are only 300km apart. This case would result in a $KEP\_G = (\frac{500}{300} - 1)\% = +66.67\%$. Such samples could therefore be identified by means of a threshold on the absolute value of reference metrics.

Finally machine learning detection models have become more popular. This because they tend to have very capable fitting characteristics. However, selection of features and tuning the algorithms parameters are less well understood and often require much trial and error making them less explainable.

This research will apply 4 outlier detection models. One based on z-score, one by reference threshold and two machine learning algorithms; Local outlier factor (LOF) and Isolation forest (IF). LOF uses K-nearest neighbors clustering and the deviation of a sample to the local density to identify abnormal instances. This method should work well on datasets with multi dimensional correlated features. IF is a decision tree based algorithm. The method randomly selects a feature and value to split the dataset and repeats until the maximum group size is below a given value. One of these sets of splits is called a tree and many of these trees are then combined into a forest. Outlying instances will on average have a shorter path compared to normal samples, which is used as the detection characteristic in this method. IF performs well on high dimensional datasets that may include a lot of insignificant features. Validation of any of these models is a problem, because there is no labelled dataset available and only few samples can be checked manually. The methods will therefore be tested against each other.

3.4 Prediction

Part of the research objective is to detect anomalies before flight operation. This is predicting the outliers of previous section with only a subset of the efficiency metrics available, namely only the ones comparing the FPT with the reference trajectories. It can be regarded as a binary classification problem, where the two classes are normal and abnormal. The features are the metrics but additional information is added like aircraft type, origin and destination airports etc.

Random Forest Classification (RFC) was selected as the first predicting model, because it should work well with a high dimensional dataset possibly containing insignificant features. A benefit of the use of this estimator is that is also able to quantify feature importance. RFC is however sensitive to over-fitting, therefore test are always run on several different train test splits and the results averaged.

The second applied estimator is the local outlier factor, which is also used in the anomaly detection, but now for novelty detection. This model is trained with only the normal observations as determined from the completed metrics set. And any new observations with only pre-flight metrics are then compared to this to classify them as normal or abnormal.

Both methods have a number of parameters that can be tuned to give the best results. Since many of these are influenced by each other a simple grid search over the most common range of values is performed.

Performance scoring

Since any anomaly detection problem is unbalanced due to the number of normal samples compared to outliers, scoring on accuracy will not portray the wanted behaviour accurately. For the use-case in this research it is more important have a low number of abnormal records that are not detected. This is the type 2 error if outliers are classified as the positive result. Table 1 shows the errors for a binary classification problem. In machine learning this is also known as a confusion matrix. From this, several possible model quality scoring method can be derived. The most common one is given in equation 8, but in an unbalanced problem this is not that relevant. Equations 9 and 10 score the amount of true positives against both possible false classifications. Recall was chosen to be the leading scorer for model tuning.

$$accuracy = \frac{tn + tp}{tp + fp + tn + fn} \tag{8}$$

$$precision = \frac{tp}{tp + fp} \tag{9}$$

$$recall = \frac{tp}{tp + fn} \tag{10}$$

	Actual class (observation)	
Predicted clas (expectation)	tn (true negative) Correct absence of result	fn (false negative) Missing result
	fp (false positive) Unexpected result	tp (true positive) Correct result

Table 1: Binary classification confusion matrix

4 Description of the Case Studies

The scenario to study the proposed use of efficiency metrics is based on a by an airline provided dataset including several short- and longhaul destinations for the year 2019.

4.1 Event Measuring System

The airline makes use of an event measuring system datastore where multiple sources are combined. This includes data from the flight planning software as well as operational data from the quick access recorders. The system allows for extensive preset computed variables. Time series data for instance can already be matched to the closest point on the flight plan.

4.2 Data Anonymisation

In agreement with the airline and their pilot union it was decided to anonymise the provided flight records. All actually operated time-series data below 10000ft was removed as well as reducing the date information to weekday and month.

Removing the terminal sections of the flight records requires some adjustments for the metrics in the horizontal domain. Some have the ASMA region already excluded, but for the others a projection of the available actual trajectory is made onto the original reference and used instead. This closest point projection is denoted in the metric naming scheme with an additional C as listed in appendix A.

4.3 Code Structure

The flight record extraction and anonymisation is performed on the airlines hardware before transferred externally. A local postgres database is used for the storing of records, because of the build-in geometry handling capabilities with postGIS and convenient array type for time series data. A schematic overview of the dataflow is given in figure 4.

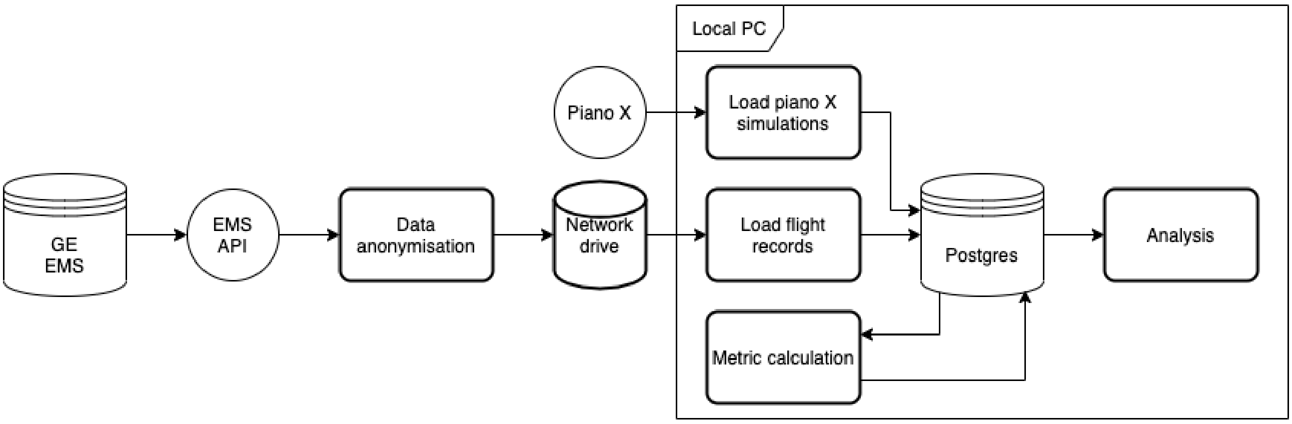


Figure 4: Schematic data flow

Python with the object relational mapper package sqlalchemly is used to manage flight record instances. These can be handled individually for metric calculations, keeping a low memory usage. The metrics are then stored in a separate table making retrieving the complete dataset easy with the pandas library for analysis. The machine learning package scikit learn is used for anomaly detection and prediction.

5 Results

To evaluate the proposed assessment framework, first the results of the different metrics regarding the case study are presented. Afterwards, the effectivity of using this framework for both anomaly detection and performance prediction is analysed.

5.1 Operating Performance of Flight Plans

Table 14 gives the mean and standard deviation for the AFT compared to FPT metrics of all assessed flight records. Note that positive values denote higher inefficiency. KEA_PA shows that on average, the horizontal flown distance outside AMSA is 2.8% shorter than planned. This replicates findings in similar research in the ATM field. AUs often have to plan longer routes, but get shortcuts assigned. [12] The average distance saving is lessened by the flights that have to enter a holding pattern. With the method described in section 3.1.1 it is found that 17.1% of analysed records include holding within ASMA. Further investigation showed that two of the studied destinations do have holding stacks located outside of this 40nm range to the airport. However, there were just 15 records that would have been picked up by the detection algorithm is the range was increased to 70nm. It is worth mentioning that with this method setting a maximum range is required to prevent detecting standard instrument departures to the opposite direction for shorter routes.

Metric	Description	Mean	Std
KEA_PA	Extra distance AFT in comparison with FPT excluding ASMA	-2.8141%	4.9168%
VEA_P	Difference average altitude AFT in comparison with FPT	3.5532%	4.9901%
TEA_P	Extra flight time AFT in comparison with FPT	-3.0199%	20.1196%
TDEA_P	Arrival delay in comparison with departure delay	-32.1224%	206.4666%
FEA_P	Extra fuel AFT in comparison with FPT	+1.5644%	5.9901%

Table 2: Averages of metrics comparing AFT with FPT

Despite the shorter flown distance, the fuel burn is on average higher than planned. In fact, all metrics of table 2 are found to be uncorrelated as can be seen in figure 5. This supports the use of all the domains, because they do describe different behaviour.

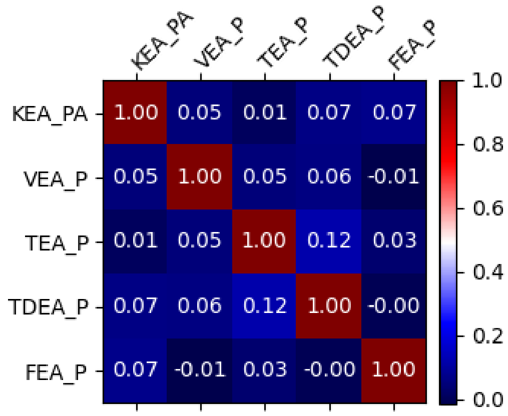


Figure 5: Linear correlation between metrics comparing AFT with FPT

5.2 Flight Plan Performance Compared to Reference

Table 3 shows the average of the horizontal domain metrics comparing both AFT and FPT to the GCD outside ASMA. GCD1 refers to the geometry linking actual flown track entry and exit of ASMA whereas GCD2 is from and to the flight plan trajectory intersections with ASMA. These values present the inefficiency of the airspace structure and therefore ca not be influenced by an AU, but can be used to attribute inefficiencies in other domains to their simulated reference trajectories.

Domain		GCD1	GCD2
Horizontal	AFT	+5.926%	+1.462%
	FPT	+9.426%	+4.412%

Table 3: Average metrics comparing to great circle distance outside AMSA

The other domain averages are given in table 4, but can also be found in appendix B where all metrics are listed. Whether these reference metrics are representative as quality metrics depends on how well simulations relate to the actual conditions. This is checked by analysing the consistency of the given metrics using a kernel density estimation. The result of the fuel domain for a longhaul flight separated by direction is given in figure 6. The difference between C1 and C2 shows the omission of the routing problem. Subsequently C3 should remove the wind effect, however is can be seen that the different directions do not respond comparable. This indicates that there is a phenomena that is not included in the simulations.

Domain		C1	C2	C3
Vertical	AFT	+6.029%	+8.640%	+8.681%
	FPT	+2.444%	+5.335%	+5.367%
Time	AFT	-0.559%	-9.656%	-7.406%
	FPT	+1.151%	-8.288%	-5.841%
Fuel	AFT	+16.498%	+6.388%	+8.059%
	FPT	+14.851%	+4.736%	+6.465%

Table 4: Averages of metrics comparing to simulated reference trajectories

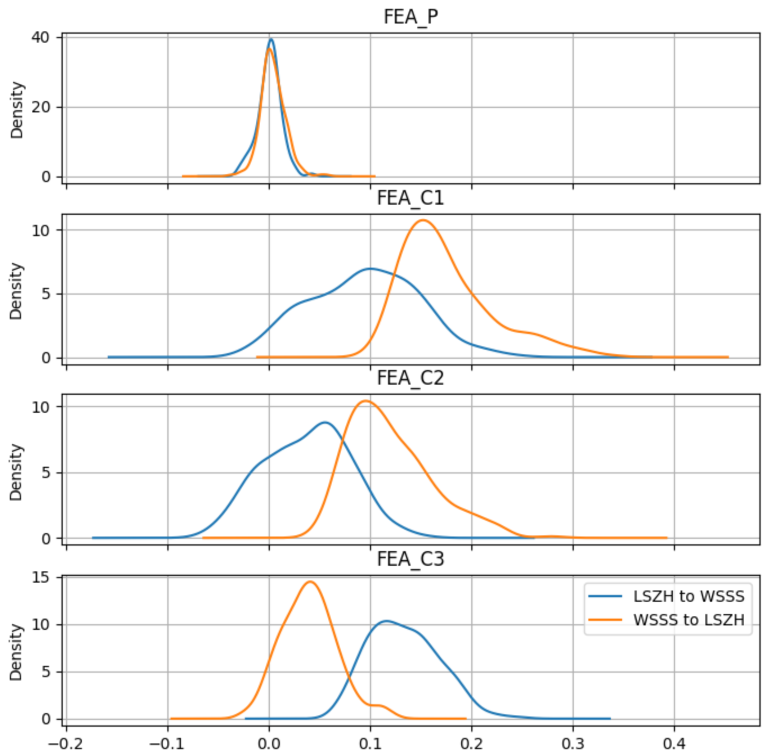


Figure 6: Directional kernel density estimators for different references in the fuel domain for longhaul route.

5.3 Anomaly Investigation

Anomaly detection is performed on the unlabelled dataset created with the use of efficiency metrics. To validate the methods, some identified samples were checked manually and the behaviour explained. One case is shown here whereas two more are presented in appendix C. It is recognised that sampling will only give a very limited validation and will not be able to give optimisation of tuning the methods parameters.

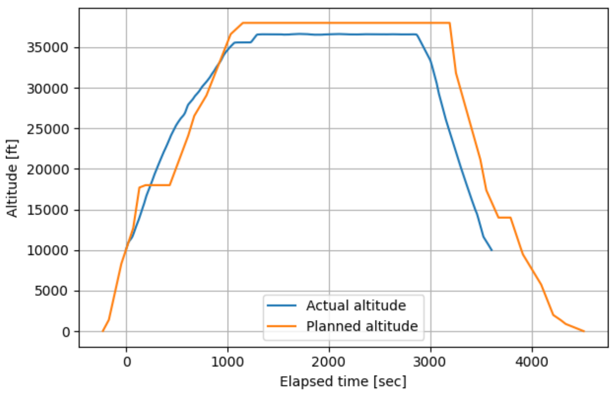
Flight record 2517904 was detected as an outlier by the z-score method. Table 5 shows a selection of the efficiency metrics comparing AFT with FPT. The value of $FEA\_P = 0.339$ was the cause of the detection, because this is more than 5 times higher than the standard deviation of that metrics giving a z-score of 5.09.

Metric	Value	Parameter	Meaning
FEA_P	0.339	Fuel burn	Lot more than planned
KEA_PA	-0.018	Horizontal distance	More direct than planned
VEA_P	+0.045	Altitude	Lower than planned
TDEA_P	-0.2	Delay recovery	Reduced delay
TEA_P	-0.05	Flight time	Faster than planned

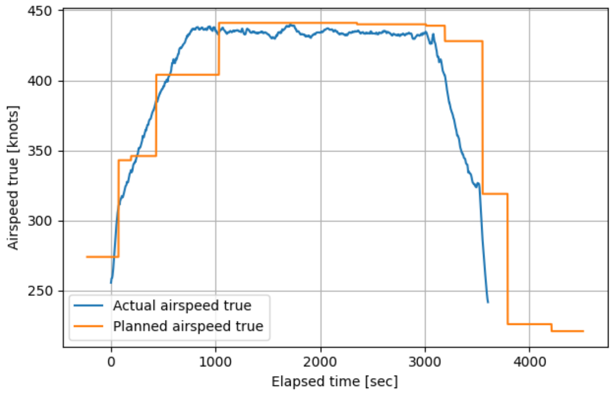
Table 5: Subset of efficiency metrics of flight record 2517904

The origin of the values of KEA_PA and VEA_P can be seen in figures 7 and 8(a) respectively. The flight time savings were a result of the shorter horizontal track since, in figure 8(b), it can be seen that the true airspeed did not exceed planned. The cause of the fuel inefficiency can in this case be attributed to inaccurate weight estimations shown in table 6.

Figure 7: Trajectories of flight 2517904; Flown trajectory red, flight plan trajectory blue, great circle distance gray.



(a) Altitude profile



(b) Speed profile

Figure 8: Time series plot for flight record 2517904

	Zero fuel weight	Fuel burn	Take-off fuel weight
Actual	64555	4032.5	7529.9
Flight plan	52490	3011.0	4770.0
Difference	+12065	+1021.5	+2759.9

Table 6: Weight differences between AFT and FPT for flight record 2517904

Simulating the flight with additional weight results in an additional fuel burn of 470kg and 110kg for differences in ZFW and TOFW respectively.

In order to analyse the sensitivity of the different detection methods, they were all compared to the results the other methods. The default parameters of sklearn were used for both the isolation forest and local outlier factor methods. The absolute value method threshold was set at 0.4 for metrics comparing to a simulated trajectory. And z-score identification absolute value of 3. Table 7 shows the number of records that are classified as normal (negative) or abnormal (positive) for every comparison between methods. Note that the IF method is stochastic so values may differ between runs. Two observations can be made from table 7. There are almost no records detected by IF that are not picked up by statistical methods. And very little overlap exists in abnormal classified records between LOF and any of the other methods.

		Z-score		Absolute value		Isolation forest	
		neg	pos	neg	pos	neg	pos
Absolute value	neg	3810	355				
	pos	72	616				
Isolation forest	neg	3876	388	4158	124		
	pos	6	583	7	564		
Local outlier factor	neg	3657	797	3860	594	3947	507
	pos	225	174	305	94	314	85

Table 7: Anomaly detection matching matrices comparing all methods

For the Z-score and absolute value methods, it is possible to attribute the classification of an outlier to a specific metric or domain. Table 8 shows what part of the total dataset is classified as outlier by each method separated by domain. Note that Z-score is also applied to non metric parameters which are grouped into the other column. A full list with the percentages per single metric is given in appendix D. For the absolute value method, most of the outliers are exceeding the threshold in either the time domain, or specifically in the fuel domain when compared to C1. For Z-score, time is the main contributor too, but followed by the horizontal metrics. From table 8 it is also clear that the different domains overlap considerably. The total percentage of outliers is not the sum of the value in each domain, because a single record can exceed normal limits in multiple domains, but is only recognised as one outlier.

Domain		Z-score	Absolute value
Horizontal	K	6.84	10.44
Vertical	V	5.42	0.32
Time	T	7.89	11.77
Fuel	F	2.56	6.00
Other		7.67	-
Total		20.01	19.10

Table 8: Percentage of outliers detected by each method separated by domain

5.4 Predictive Anomaly Detection

Predictive anomaly detection looks to find any records by only using metrics that are available before operation. These therefore are just the metrics comparing FPT with the references. However, as figure 9 shows for fuel domain metrics, some FPT metrics are linear correlated to AFT metrics, which does hint to a prediction possibility. But it has to be considered that the reference simulations are not very consistent, shown by the width of the peeks in figure 6. Thus it is likely that the linear correlation comes from the offset itself rather that the individual performance.

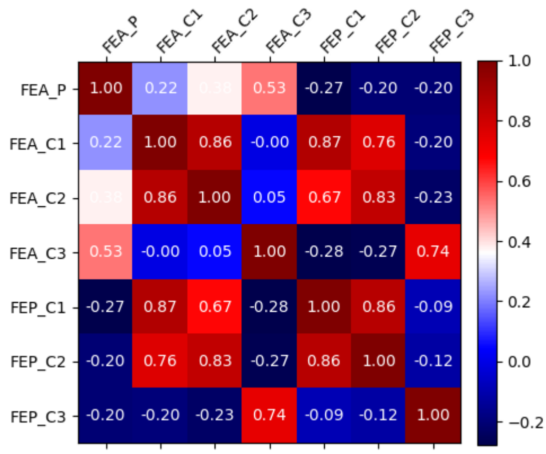


Figure 9: Linear correlation between fuel domain metrics

Nevertheless, both random forest and local outlier factor classifications models tested on labelled datasets created with the methods described in section 3.3. A 0.75, 0.25 train test split is maintained and both models are tested with the full set of metrics and pre-flight set and averaged over 10 individual runs.

Random forest classifier

The first step was to optimise the algorithm parameter by a grid search to maximise the recall score. This found the values in table 9.

Parameter	Value
max depth	15
max features	10
min sample split	10
n estimators	300

Table 9: Random forest classifier parameters optimised for recall score on isolation forest labelled anomaly dataset.

When applied to the dataset with all available metrics it gave an accuracy of 0.978 and a recall score of 0.911. These come from the averaged confusion matrix in table 10.

	pred_neg	pred_pos
neg	1145.5	16.5
pos	12.2	124.8

Table 10: Confusion matrix for random forest classifier on isolation forest labelled anomaly dataset

However, more interesting is the same algorithm applied on the subset of metrics given in table 11. This equates to an accuracy of 0.960 and a recall score of 0.790.

	pred_neg	pred_pos
neg	1139.4	22.6
pos	28.8	108.2

Table 11: Confusion matrix for random forest classifier on isolation forest labelled anomaly dataset limited to pre-flight available metrics

With random forest classification it is possible to quantify the importance of each of the features. It is found that for both the all metric and the pre-flight scenario, all time domain reference metrics ranked as most important. The full list of features with their importance can be found in appendix E.

Local outlier factor

For this method the same procedure is followed and tested on an anomaly labelled dataset by the local outlier factor method itself. The resulting accuracy for all metrics was found to be 0.910 with a recall score of 0.989 as calculated from table 12.

	pred_neg	pred_pos
neg	1092.0	117.0
pos	1.0	89.0

Table 12: Confusion matrix for local outlier factor on LOF labelled anomaly dataset

And for the pre-flight available only metrics subset the accuracy is 0.891 with a recall score of 0.700. The confusion matrix of this is shown in table 13.

	pred_neg	pred_pos
neg	1094.0	115.0
pos	27.0	63.0

Table 13: Confusion matrix for local outlier factor on LOF labelled anomaly dataset limited to pre-flight available metrics

It is found that the performance of both of these models is very dependent on what outlier labelling method is used and with which settings. This also prevents any useful tuning to be done to the predictive algorithms since this too will be specific on the labelling method. However, a general observation could be made that with the increase in sensitivity of the outlier labelling method, the prediction performance went down.

Another observation was that grouping the dataset by either aircraft type or origin destination pair resulted in worse predicting scores. This is likely due to the reduced size of training data, but has not been tested intensively.

6 Conclusions

Efficiency metrics as used in air traffic management are assets that can be used by an airline to monitor trajectory related inefficiencies. This work proposed a framework to assess the quality of flight planning that also takes into account how well a flight plan is operated. The developed method was applied on historical data and its results analysed.

On average a slight over-consumption compared to the flight plan was observed. However, the proposed framework was found to be inadequate to attribute this aspect to a specific cause. This because the established references failed to deliver consistent results. Compared to the flight plan predicted error, the error of reference metrics was much larger and differences between the reference trajectories could not be fully explained.

Noteworthy is the average actual flown track length, which is shorter than planned despite the presence of some records of flights with holding. However, it can be argued that this does not has relevance in improving the flight planning process. A flight plan should not be filed expecting less distance than given by the planned route anticipating directer in flight routing. Besides, even if the flight plan advises a reduced fuel load, this is often disregarded as the captain has the final say on this. The average takeoff fuel weight difference between planned and actual operating already shows this. Overall, the assessment framework presented is considered to be good for monitoring and detecting phenomena that reduce the efficiency. However, this knowledge should then be further substantiated in order to make decisions about improving the process.

Using the proposed framework for anomaly detection is shown to be feasible. Abnormal flight records can be detected using operating efficiency metrics and helped explained by looking at reference metrics. It however is not yet shown that detecting outliers from reference metrics results in meaningful insights. The amount of pollution from uninteresting record using these methods is hard to quantify due to the lack of a existing labelled dataset. All proposed methods require tuning to fit their use-case, which is not yet defined and therefore sufficient validation is still needed.

This work also outlined a method to detect abnormal flight plans before operation. It showed promising results, but also considerable sensitivity to how outlier identification is specified. Performing more adequate validation on the anomaly detection algorithms is needed before substantial conclusions can be drawn on this proposed method.

7 Recommendations

This research was initiated due to the lack of comparable studies examining the quality of flight planning from an airlines perspective. The framework described in this paper makes a start with exploring the possibilities in this field but as can be read in previous section it is acknowledged that in many ways further improvements are needed to increase the validity.

One envisioned improvement is to extend the framework by increasing the number of metrics measuring smaller subsections within each efficiency domain. Rather than focusing on flight spanning metrics like average altitude, it might be beneficial to create separate metrics for different flight phases. For instance, time to cruise height, average cruising altitude and length of decent to name a few possibilities in the vertical domain.

Also, since time was found to be an important domain, it will be worth looking into how much schedules are padded. If the planned times do already include this or if removing it would improve the metrics accuracy.

Further analysis could also be applied to the accuracy of weather forecasting and its effects on the achieved efficiency. A fourth reference trajectories could be created using actual measured wind direction and speed when computing the correcting wind component.

Additionally, the analysis of averaged metrics could be expanded. This research did try to find understanding in correlation between the different proposed metrics, but partitioning the data is different subsets could also lead to better insights. This might include comparing behaviour for different seasons, aircraft types or route lengths. Dividing the records like this would require more data to start with, because it will reduce the size of the data subsets. In the setup of this case study this was not achievable, but better integration of the assessment framework in the airlines systems is possible and would solve this.

If fact, this research would benefit from actual implementation. If the framework can be used to monitor flight planning and operation for a longer period of time on recent flights, specialists will be able to validate the relevance of the difference inefficiency qualifiers. With this, a solid use-case specification can be built to tune anomaly detection for. In turn allowing for analysis about the feasibility of predicting these outliers.

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Appendices

A Complete Efficiency Metrics Table

Metric	Domain	Reference	Description
KEA_GCA	Horizontal	GCD	Extra distance AFT in comparison with AFT projection on GCD.
KEA_PCA		GCD	Extra distance AFT in comparison with FPT projection on GCD.
KEA_GA		GCD	Extra distance AFT in comparison with GCD from and to AFT intersection with ASMA.
KEA_PA		FPT	Extra distance AFT in comparison with FPT excluding ASMA.
KEA_GAP		GCD	Extra distance AFT in comparison with GCD from and to FPT intersection with ASMA.
KEP_GCA		GCD	Extra distance FPT in comparison with FPT projection on GCD.
KEP_GA		GCD	Extra distance FPT in comparison with GCD from and to FPT intersection with ASMA.
KEP_GAP		GCD	Extra distance FPT in comparison with GCD excluding ASMA.
VEA_P	Vertical	FPT	Difference average altitude AFT in comparison with FPT.
VEA_C1		OCT1	Difference average altitude AFT in comparison with OCT1.
VEA_C2		OCT2	Difference average altitude AFT in comparison with OCT2.
VEA_C3		OCT3	Difference average altitude AFT in comparison with OCT3.
VEP_C1		OCT1	Difference average altitude FPT in comparison with OCT1.
VEP_C2		OCT2	Difference average altitude FPT in comparison with OCT2.
VEP_C3		OCT3	Difference average altitude FPT in comparison with OCT3.
TEA_P	Time	FPT	Extra flight time AFT in comparison with FPT.
TEA_C1		OCT1	Extra flight time AFT in comparison with OCT1.
TEA_C2		OCT2	Extra flight time AFT in comparison with OCT2.
TEA_C3		OCT3	Extra flight time AFT in comparison with OCT3.
TEP_C1		OCT1	Extra flight time FPT in comparison with OCT1.
TEP_C2		OCT2	Extra flight time FPT in comparison with OCT2.
TEP_C3		OCT3	Extra flight time FPT in comparison with OCT3.
TDEA_P		FPT	Arrival delay in comparison with departure delay.
FEA_P	Fuel	FPT	Extra fuel AFT in comparison with FPT.
FEA_C1		OCT1	Extra fuel AFT in comparison with OCT1.
FEA_C2		OCT2	Extra fuel AFT in comparison with OCT2.
FEA_C3		OCT3	Extra fuel AFT in comparison with OCT3.
FEP_C1		OCT1	Extra fuel FPT in comparison with OCT1.
FEP_C2		OCT2	Extra fuel FPT in comparison with OCT2.
FEP_C3		OCT3	Extra fuel FPT in comparison with OCT3.

Table 14: List of efficiency metrics with description

B Metrics averages

metric	all	with holding	without holding
KEA_GA	0.059258	0.051859	0.060837
KEA_GAP	0.014616	0.029453	0.011452
KEA_GCA	0.053951	0.079722	0.048626
KEA_PA	-0.028141	-0.012987	-0.031374
KEA_PCA	-0.023244	0.008512	-0.029497
KEP_GA	0.094257	0.06618	0.100059
KEP_GAP	0.044123	0.043337	0.044285
KEP_GCA	0.043606	-0.007715	0.054212
VEA_C1	-0.060287	-0.080571	-0.05613
VEA_C2	-0.086398	-0.08896	-0.085871
VEA_C3	-0.086806	-0.090817	-0.085982
VEA_P	-0.035532	-0.025925	-0.037513
VEP_C1	-0.024444	-0.054789	-0.018225
VEP_C2	-0.053345	-0.063623	-0.051232
VEP_C3	-0.053664	-0.065477	-0.051236
TEA_C1	-0.005593	-0.021475	-0.002337
TEA_C2	-0.096563	-0.100779	-0.095696
TEA_C3	-0.074064	-0.078462	-0.07316
TEA_P	-0.030199	-0.005044	-0.035383
TEP_C1	0.011507	-0.041183	0.022307
TEP_C2	-0.082877	-0.118204	-0.075616
TEP_C3	-0.058413	-0.091735	-0.051564
TDEA_P	-0.321224	0.620354	-0.521832
FEA_C1	0.164982	0.166668	0.164634
FEA_C2	0.063881	0.073264	0.061902
FEA_C3	0.080658	0.10155	0.076459
FEA_P	0.015644	0.026554	0.013395
FEP_C1	0.148511	0.135983	0.151093
FEP_C2	0.047357	0.045485	0.047752
FEP_C3	0.064652	0.07492	0.062588

Table 15: Average metrics of all records and split by presence of holding.

parameter	all	with holding	without holding
takeoff_fuel_weight_difference	1372.374068	1453.532324	1355.658281
touchdown_fuel_weight_difference	1167.769183	1234.216619	1154.083315
fuel_burn_difference	204.604885	219.315705	201.574966
zero_fuel_weight_difference	-166.153279	-106.148874	-178.54998
holding	0.170691	1	0

Table 16: Average absolute parameters of all records and split by presence of holding.

C Anomaly Investigation Examples

Under-consumption due to direct flight path

This case was identified by a z-score for FEA_P of -4.6. This can be explained by looking at the horizontal trajectories in figure 10.

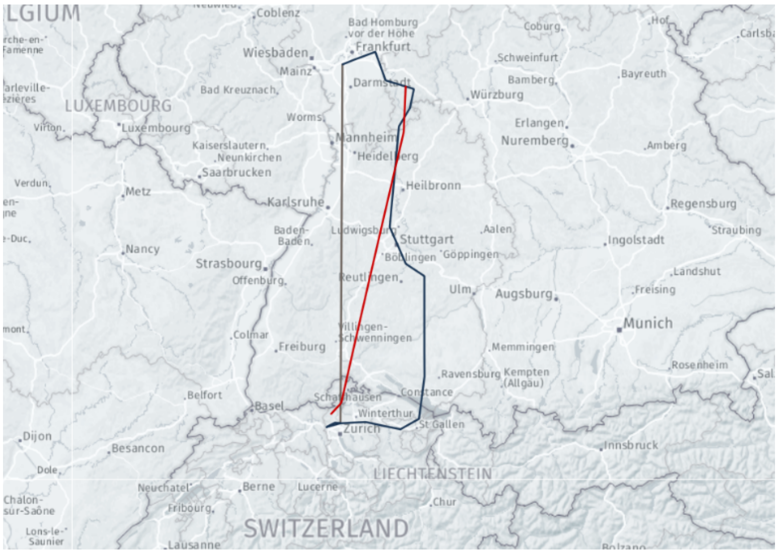


Figure 10: Trajectories of flight 2560981; Flown trajectory red, flight plan trajectory blue, great circle distance gray.

Flight 2560981 was operated on a much directer route, which resulted in the efficiency metrics of table 17. It however is much harder for this flight to check the fuel savings with a simulation which better matches the actual condition. Although the weights are known and shown in table 18, the actual flown track length can not be accurately estimated due to the significant segment below 10000ft for such a short flight.

Metric	Value	Parameter	Meaning
FEA_P	-0.282	Fuel burn	Lot less than planned
KEA_PA	-0.227	Horizontal distance	More direct than planned
TDEA_P	-	Delay recovery	No departure delay
TEA_P	-0.024	Flight time	Faster than planned

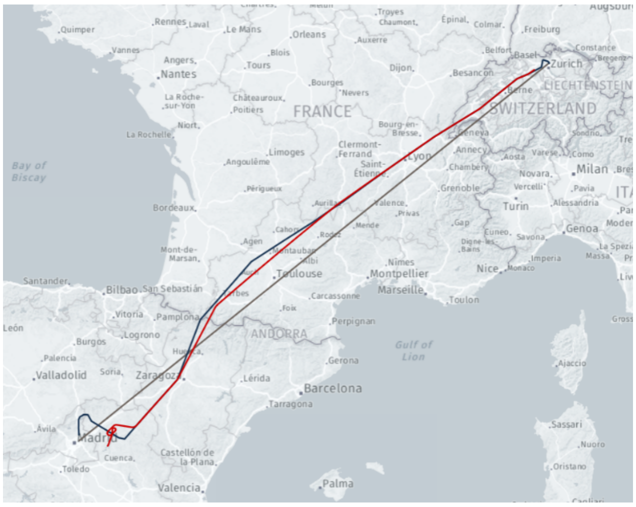
Table 17: Subset of efficiency metrics of flight record 2560981

	Zero fuel weight	Fuel burn	Take-off fuel weight
Actual	51744	1730.6	5075.6
Flight plan	57884	2409.2	4480.0
Difference	-6140	-678.6	+595.6

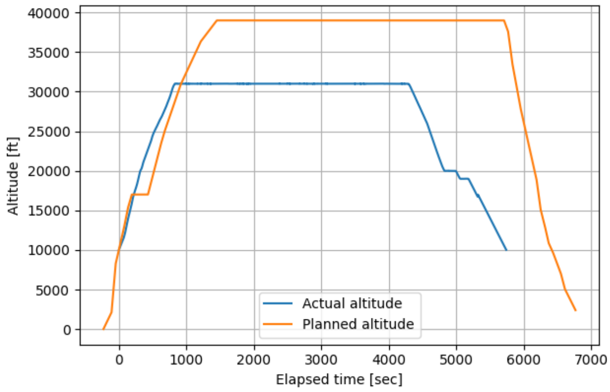
Table 18: Weight differences between AFT and FPT for flight record 2560981

Over-consumption due to delay recovery

Flight 2533673 was again identified as anomaly due to its z-score. In this case for VEA_P with 4.0. With the available data, a guess can be made why this flight was operated as such. With the times listed in table 20 it can be seen that flight departed with a considerable delay. However, the arrival delay is not reduced by much. This despite what, based on the lower cruising altitude, appears to be operation with a higher cost index, figure 11(b). But figure 11(a) shows a holding before arrival, which likely diminished any achieved delay reduction.



(a) Horizontal trajectories



(b) Altitude profile

Figure 11: Trajectories of flight record 2533673

Metric	Value	Parameter	Meaning
VEA_P	+0.2467	Altitude	Lower than planned
FEA_P	0.1726	Fuel burn	More less than planned
TDEA_P	-0.231	Delay recovery	Reduced delay
TEA_P	-0.050	Flight time	Faster than planned

Table 19: Subset of efficiency metrics of flight record 2533673

	Take-off	Touchdown
Scheduled	20:09	22:08
Planned	20:35	22:28
Delay	+0:26	+0:20

Table 20: Departure and arrival times of flight record 2533673

D Outlier percentage by metric

Note that outliers can overlap, therefore percentages of tables 21 and 22 should not be added. Also, the absolute value method is only applied on reference metrics and therefore excludes the non metric parameters of table 22.

metric	Z score	Absolute value
KEA_GA	3.21	0.10
KEA_GAP	4.08	0.00
KEA_GCA	1.32	0.05
KEA_PA	4.33	-
KEA_PCA	2.04	-
KEP_GA	5.40	6.61
KEP_GAP	0.56	0.02
KEP_GCA	2.66	5.63
VEA_C1	1.26	0.00
VEA_C2	3.11	0.20
VEA_C3	3.34	0.25
VEA_P	0.35	-
VEP_C1	0.84	0.05
VEP_C2	2.14	0.05
VEP_C3	2.14	0.05
TEA_C1	5.46	7.11
TEA_C2	5.46	5.15
TEA_C3	5.46	5.35
TEA_P	0.97	-
TEP_C1	5.32	9.34
TEP_C2	5.32	4.73
TEP_C3	5.32	5.28
TDEA_P	2.08	-
FEA_C1	0.60	3.51
FEA_C2	0.35	0.22
FEA_C3	0.37	0.20
FEA_P	1.09	-
FEP_C1	0.89	4.30
FEP_C2	0.31	0.08
FEP_C3	0.08	0.03

Table 21: Percentage of outliers detected by each method separated by metric

parameter	Z score
flight plan cruise mach	2.49
fuel burn difference	2.12
planned cost index	0.12
planned fuel burn	0.35
planned takeoff fuel weight	0.12
planned touchdown fuel weight	0.35
scheduled liftoff time	0.00
scheduled touchdown time	0.00
takeoff fuel weight difference	1.48
touchdown fuel weight difference	1.63
zero fuel weight difference	1.38

Table 22: Percentage of outliers detected using Z score separated by parameter

E Random Forest Classifier Feature Importance

Metric	All	Pre-flight
TEA_C2	0.125106	
TEA_C3	0.100976	
TEA_C1	0.080476	
TEP_C1	0.076863	0.171253
TEP_C3	0.06912	0.168968
TEP_C2	0.066631	0.158424
VEA_C3	0.050919	
VEP_C3	0.050879	0.122301
VEA_C2	0.035625	
VEP_C2	0.031306	0.060549
KEP_GA	0.026871	0.059563
scheduled touchdown time	0.024604	0.039756
FEP_C1	0.021113	0.030339
KEP_GCA	0.020742	0.038399
KEA_PCA	0.016334	
planned cost index	0.015189	0.010119
planned fuel burn	0.014979	0.023812
TEA_P	0.014709	
planned takeoff fuel weight	0.013909	0.01514
KEA_PA	0.01289	
KEA_GAP	0.012527	
FEP_C2	0.01006	0.020157
fp cruise mach	0.009752	0.020355
FEA_C1	0.008641	
FEA_C2	0.008294	
KEA_GA	0.007323	
VEP_C1	0.007038	0.019502
FEA_C3	0.00694	
touchdown fuel weight difference	0.00652	
planned touchdown fuel weight	0.006474	0.013601
VEA_C1	0.006186	
zero fuel weight difference	0.006079	
takeoff fuel weight difference	0.005851	
TDEA_P	0.004801	
fuel burn difference	0.004569	
FEP_C3	0.004506	0.016223
VEA_P	0.003767	
FEA_P	0.00346	
KEA_GCA	0.003202	
KEP_GAP	0.002677	0.00791
scheduled liftoff time	0.002093	0.003627

Table 23: Feature importance of random forest classifier on dataset labelled by isolation forest.

II

Literature Study

1
Introduction

Airlines are always looking to reduce their operating costs. With the unprecedented challenges of the 2020 pandemic this might even be more than ever, since almost all airlines world wide are under immense financial pressure. In 2019, fuel costs amounted to 23.7% of the airlines operating costs. [44] This will likely be different for 2020 since the global passenger capacity is estimated to be down 40.4% relative to previous year. [45] Nevertheless, with the also increasing global focus on aviation emissions, reduction of fuel use will stay a major priority for airlines.

Generalized, there are three ways to reduce fuel burn and emissions. Operational changes, technological improvements and alternative fuels.[51] The first one is the shortest term solution and therefore significant effort is put into this by not only the airlines, but all commercial aviation stakeholders. Since 2007, Europe has a coordinated effort to develop the new generation of Air Traffic Management (ATM) in the Single European Sky initiative (SESAR). [31] The United States have a similar program with NextGEN. [33] Between the two, there is a vast amount of topics being researched.

The SESAR program invented Trajectory Based Operations (TBO). This is a 4 dimensional trajectory management system that is envisioned to replace the traditional way-point based navigation. It would allow airline operators to freely route between airports and therefore increase the efficiency of the airspace and operation. One of the challenges that remain is the management of air traffic flows by air traffic control (ATC). Improvements in 4D trajectory predictions will have to be made in order to accurately agree upon overfly times of specific points, that should allow ATC to maintain separation between aircraft. Research in this area also allows improvement in the flight planning, since accurately modelling flight trajectories is the first step in optimizing these trajectories.

Traditionally the flight planning problem (FPP) is regarded as a shortest path problem (SPP). Depending on the airlines operational criteria, this is optimized for a balance between fuel cost, time cost and possibly more objective like emissions. To solve this complex problem, airlines employ automated tools to generate efficient and feasible flight plans. These algorithms are build on certain assumptions and simplifications to the FPP, so good solutions can be generated in a reasonable amount of computation time. The impact of these assumptions and simplifications is to the best of my knowledge not extensively studied. As J. Rosenow states in [79] “An assessment of those tools is an unsolved challenge, because of a missing optimum trajectory which is hard to define and impossible to verify.” However, with the advancements in trajectory optimization for TBO this might not be impossible anymore.

The aim of this study is to develop an assessment framework for flight planning. Both the quality of a flight planning system should be analysed based on historic results and newly generated flight plans should be able to be evaluated before operation. The first is important because like the well known business quote: “If you can not measure it, you can not improve it". And the second is relevant since with the ever increasing amount of automation, it can be vary valuable to identify possible problematic solutions and have them checked by experts as a human in the loop system.

In order to create a assessment framework for flight planning, first the current day's flight planning is analysed in section 2. Section 3 describes the flight planning problem in detail and discusses the methods that are used to solve it. This is then followed by section 4 that discusses the state-of-the-art trajectory optimization approaches. Next the major components of trajectory optimization will be addressed in sections 5 and 6 about the aircraft and weather model respectively. Section 7 reviews the metrics that are currently used or proposed by researchers to measure the efficiency of an air network regarding different stakeholders. Thereafter, novel methods to cluster aircraft data in order to detect abnormal flight plans are reviewed in section 8. This is relevant to the topic since it might allow identification of inefficient flight plans without knowing the optimal trajectory, but using big data and machine learning.

2

Today's Flight Operation Procedures

This section reviews the procedures that are currently in place to operate a commercial flight. Although the topic of this paper is flight plan analysis, in the end we are interested in the efficiency of the operated flight. Therefore not only the planning, but also the procedures while operating the flight are discussed in this section.

2.1. Planning

In Europe, every operated flight is required to submit a flight plan in advance. The basis of this regulation is given in Doc 4444 of the International Civil Aviation Organization (ICAO) [47]. In order to manage the traffic flow for high demand airports with limited capacity, regularly operated IFR flights are submitted as a Repetitive Flight Plan (RPL). These should be received at least two weeks before operation, but usually airlines plan the next seasons timetable 6 month ahead and already submit the RPLs then. The RPL contains information like; days of operation, aircraft type, departure and arrival airport, off-block and estimated elapsed time, route, cruising altitude(s) and speed(s). Since the dispatchers can not know many of the conditions that influence the planning months in advance, they have to work with estimated averages for things like weather and fuel costs.

The RPL is refined on the day of operation. For flights that are subject to air traffic flow regulations, changes to the RPL have to be submitted to the Network Management Operations Centre (NMOC) 3 hours before departure. The NMOC will than calculate a 4D trajectory and checks if any airspace sector capacity limits are exceeded. [79] Airspaces usually have a constant capacity over time, but in the case of strong weather or unusual circumstances like understaffed control centers, this might be reduced. If airspaces are overcrowded, the NMOC has two options. It can propose re-routings to the airspace users to divert traffic. Or it can delay departure times. In this case the NMOC will distribute departures slots that determine the time at which aircraft allowed to take-off.

In contrast to the RPL, a daily flight plan is optimized for the forecasted weather conditions. The route should avoid severe weather, but might also be routed in a way such that it will benefit from favorable winds. This is a very complex optimization, since not only the route has to be determined laterally, but altitude and speed play an important role too. In literature this is referred to as the flight planning problem and will be further discussed in section 3. Airlines have several software tools available to them to generate efficient routes. Programs like Jeppesen's JetPlan.com, Lufthansa System's LIDO or Sabre's Airline Solution use advanced algorithms to optimize the FPP for an airlines specific target function. [79]

These target function are a balance between different objectives. Airlines want to minimize operating cost, but a flight should also fit within the schedule therefore flight time is important too. Furthermore, environmental and maintenance considerations could influence how a flight is planned. Lower thrust setting climb for instance could reduce noise and engine wear. Since the introduction of the Flight Management System (FMS) in commercial aircraft, operating costs are managed by the cost index. This is a ratio between the time related cost over the cost of fuel. [78] Time cost are the direct operating cost of an aircraft without fuel and include things like crew wages, leasing or owning cost and maintenance cost expressed in hours. Setting a cost index to zero would result in the fuel optimal trajectory whereas a maximum cost index minimizes the flight time. Airlines usually operate their aircraft above the minimum fuel setting. This might be further increased if a flight is sped up to reduce delay.

2.2. Operation

Once an aircraft is airborne, it has to follow the direction given by air traffic control (ATC). This causes the actual flown trajectory to almost always deviate from the filed flight plan. Some Flight Information Regions (FIR) apply area navigation (RNAV). This allows for routes with freely selectable way-points, instead of the traditional fixed ground navigation infrastructure. Flights could therefore be vectored between FIR entry points, allowing more efficient use of the airspace and being more fuel efficient. RNAV is seen as the first step towards TBO. As mentioned, today it is used for direct lateral deviations. It the future this is expected to be extended to also include vertical and lateral components. Lateral meaning along the flight path, so time dependent, 4D. [46] Currently however, airline specific routes or even wind optimal routes are not yet manageable due to the highly dynamic characteristics of weather forecasts. [79]

To the best of my knowledge no literature exist about applying results of flown trajectories to improve flight planning. Likewise there is no literature found of using flown trajectories to assess the quality of flight planning. Air Navigation Service Providers (ANSP) do use flown trajectories in a measurement of achieved efficiency as will be further reviewed in section 7.

3

Flight Planning Problem

The Flight Planning Problem (FPP) is the network representation of the aircraft trajectory planning, where optimization can be seen as solving a shortest path problem (SPP). This is in contrast to what is often revert to as trajectory optimization, which models the aircraft as a dynamic system. Trajectory optimization is therefore an Optimum Control Problem (OCP), which will be addressed in section 4. The FPP is 4D, 3 spatial dimensions and one time. Normally the networks nodes are given by way-points with a latitude, longitude, flight-level and associated time. The arcs connecting the nodes are the airway segments. The size of the global air network graph varies between studies. Some researchers construct their own, and others cooperate with the industry to get this data. Lufthansa Systems has a database containing 109314 nodes and 838114 arcs per level, on 43 flight levels [82]. The average node degree of an air network in a 2-dimensional case is around 6. This is already much higher than the 2 to 3 of road networks, without adding the vertical and time dimensions. The number of possible start and end nodes of any path through the network is however much smaller. These can only be airports which are at a single altitude. Currently only about 1300 airports exist with regular commercial services. [9] One can imagine that solving such a large shortest path problem is very computationally intensive. And with weather changing dynamically, this has to be solved for every flight individually. Therefore, the industry standard is to decompose the FPP into a 2D horizontal routing problem, which is then followed by a speed and altitude optimization. The impact of this to the optimality of the solutions is not well studied in literature. Recent literature does focus on ways to omit the decomposition whilst maintaining reasonable computing times. This is done by introducing various assumptions and simplifications, which are reviewed in the subsections following the decomposed form.

3.1. Decomposed FPP

The decomposed FPP starts with the horizontal routing problem, which in respect to the flight planning is called the horizontal flight planning problem (HFPP). This is then followed by speed and altitude optimization, described in the following subsections.

3.1.1. HFPP

Some of the factors that have to be taken into account in the HFPP are the overfly cost, restriction to the airspace, weight-dependent fuel consumption, wind and avoidance of severe weather [9].

Overfly costs Overfly costs are the air traffic control charges that authorities use to finance themselves. These are different for each airspace, usually divided by country. There exist a several different schemes to determine the amount of overfly costs, but these can be generalized into three function groups with 2 different metrics. The fee functions are either linear, constant or piecewise linear and the metric being either the flown distance or the great circle distance (GCD) between airspace point of entry and exit. [8] European countries charge linearly by GCD and maximum take-off weight [29], but the rates per mile can differ considerable between countries. As example, in October 2020, Switzerland's and Ireland's unit rate are 91.95 and 24.48 euro respectively. [30] How overfly cost are considered during conventional flight planning is not exactly reported, literature does indicates the importance, but only recently methods where published by Blanco et al. in [8] that address this in the SPP.

Airspace restrictions Airspace restrictions introduce constraints into the HFPP. Within the European airspace managed by EUROCONTROL, there is the route availability document (RAD). This contains rules that can be generalized into two types of constraints, mandatory and forbidden pairs. Forbidden pair type shortest path problems are proven to be NP-hard. Although some specific graph structures can make this problems complexity P-hard [58], these structures are not guaranteed in a airspace network. In total there are approximately 16.000 constraints in the RAD which might even be updated multiple times a day.

Weight-dependent fuel consumption Weight-dependent fuel consumption makes the cost a flight path segment dependent on the path up to that segment. Additionally take-off fuel weight is one of the variables that needs to be determined in the FPP. For the HFPP, take-off weight is often chosen as a constant determined by historical data. This is then refined in the second step, speed and altitude optimization.

Weather Weather plays an important role in flight planning and is my the problem is so dynamic. Commercial flight planning tools are based on two weather aspects, wind optimization and avoidance of deep convective storms. There are more hazardous conditions involved like icing and turbulence, but these usually have to be manually accounted for by the dispatchers. [81]. Dutta et al. in [27] do propose a algorithm for real-time weather avoidance including turbulence, but this is yet to be applied in planning tools. Wind has a big impact on the flight time between two points and therefore also on the total fuel burn. Winds can be different for different altitudes, so in the HFPP usually the altitude is assumed to be at the aircraft manufacturer's recommend cruise. Convective storm activity effects the entire vertical air column and can thus only be avoided by routing around it. The Federal Aviation Association recommends aircraft to avoid deep convective storms by at least 20 miles. [1] Weather conditions are given as a function of time, making the FPP a time dependent shortest path problem (TDSPP). [9] For the HFPP, time dependence is a problem since speed is not yet optimized. Speed determines the time of traversing an arc, and therefore the arc weight can not have a time dependent function. This is solved in the same way as altitude, by setting the speed as a constant.

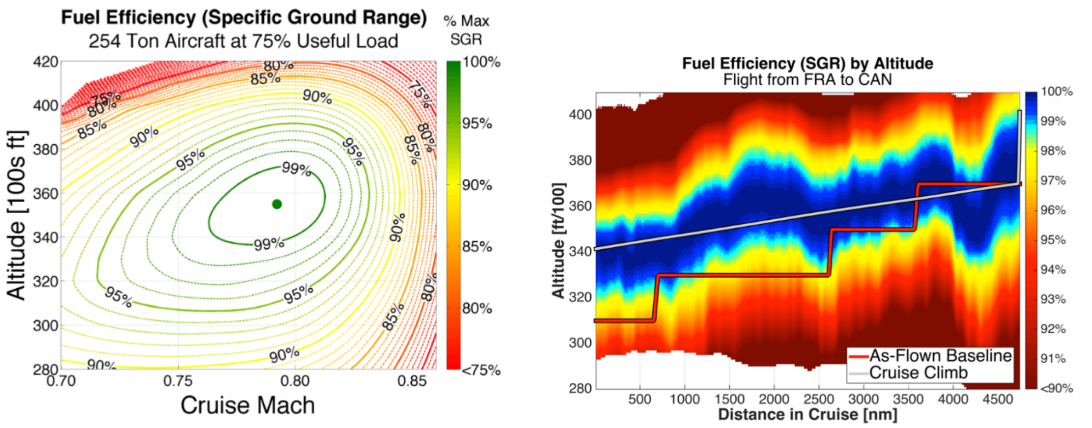
Path finding algorithms Both the classic SPP and TDSPP have been researched extensively. In the past decades preprocessing techniques have been developed to speed up the classic Dijkstra's path finding algorithm. A* is the most well known and was already introduced in 1968. [42] It is based on underestimating a potential function for every node. The natural metric for this potential in FPP is GCD. [82] Contraction Hierarchies (CHs) [37] is the leading technique in shortest path calculation for road networks. [9] The basic idea is to remove unimportant nodes and replace them by shortcut arcs so that all shortest paths remain. This algorithm is also suitable for the time dependent case. [6] However for air networks in particular, Blanco et al in [9] show that A* outperforms CHs for all their studied models. They do note that there might be adaptations of CHs that are not developed yet, which could attain better performance.

3.1.2. Speed and altitude optimization

With a given horizontal trajectory from the HFPP, the vertical and speed profile can be optimized. Flight planning tools use two factors for their optimum speed and altitude generation, namely aircraft performance models and estimates of key environmental factors. [87] Both will be extensively discussed in sections 5 and 6 respectively. Part of the speed and altitude optimization is the determining of the take-off weight. This is dependent on the payload, which has to be estimated, but also on the initial fuel weight. This fuel weight however has to correspond with the amount of fuel that is needed for the route that is calculated, therefore not known in advance. For long-haul flights, the fuel loading is approximately 1 to 3, meaning to carry 3000 kg of redundant fuel will cause the aircraft to burn 1000 kg more. [88] Speed and altitude optimization algorithms usually do not consider initial fuel weight as optimization problem, but perform a enclosing line-search for this value. [57] The optimization itself is usually divided into climb, cruise and decent phases. In literature, most emphasis is put on the cruise phase.

Climb The climb phase is the trajectory from the altitude at take-off to the Top of Climb (TOC). The first section is often dictated by a Standard Instrument Departure (SID) and both the horizontal and vertical route are constraint. However, many studies found in literature model this differently or not at all. Jensen et al. in [48–50] estimate TOC weight from a regression model and Dancila and Botez in [20] assume unconstrained continuous climb.

Cruise Fuel efficiency, in this case fuel burn over distance, is closely related to both speed and altitude. This can be seen in figure 3.1(a). For the minimum fuel path, one would want to fly at the optimum point. Current air navigation structure however does not allow this, since flight have to adhere to fixed flight levels for separation purposes. Additionally, the optimum fuel efficiency point is not static and changes for aircraft weight and weather conditions. A illustration of this is given in figure 3.1(b). In order to take advantage of the shifting optimum while adhering to the flight levels, step climb was introduced. A newer development is cruise climb, were the altitude rate change is set constant. It is thought to outperform step climb, but Jensen et al. in [49] note that their observed negligible difference would not be worth the complexity increase for air traffic management.



(a) Instantaneous fuel efficiency of a typical long-haul aircraft at a fixed weight (calm winds, standard atmosphere) [50]

(b) Side view of a typical long-haul altitude optimization profile and optimal cruise climb solution [50]

Figure 3.1: Typical fuel efficiency of a long-hail aircraft as expressed in ground range per pound of fuel consumption (SGR)

There are several methods to solve the speed and altitude optimization. It is not publicly known what methods are used by commercial flight planning tools, but literature proposes multiple approaches. Knudsen et al. in [57] represent the speed and altitude optimization as a network and by introducing some of the techniques discussed in section 3.2. Jensen et al. in [48] approach it as a Mixed Interger Program (MIP), but state that the number of combinations make this problem computationally infeasible to solve with a linear solver and therefore use a "greedy" heuristic algorithm. Recent research also focuses on genetic algorithm, like Murrieta-Mendoze et al. in [69]. Essentially, all of the algorithms used in trajectory optimization in section 4 could also be used to solve the speed and altitude optimization.

Decent Similar to climb, decent is often subject to a standard terminal arrival route (STAR). Although continuous decent (CD) approaches are a proven procedure developed in the SESAR program, they are not yet widely implemented. A stepped decent is still the norm. In literature however, most speed and altitude optimization algorithms do assume CD.

3.2. Full 4D methods

The 4 dimensional flight planning problem is the horizontal, vertical and speed optimization combined, but still expressed in a network representation. Due to the weather being a function of time, this also is a TDSPP. Orda and Rom showed in [74] that solving such a problem does not scale polynomially and is therefore referred to as NP-hard. This is a problem for flight planning where the generation of a route should take seconds rather than hours. Therefore a number of assumptions and simplifications have been studied, which can reduce the computational intensity of an 4D FPP.

3.2.1. FIFO

First In First Out (FIFO) is an assumption to TDSPP that state that any path P' with a greater cost than some other path P to any node, can not be part of the optimal solution. Kaufman et al. proved in [52] that with FIFO, TDSPPs are solvable in polynominal time. In real flight planning this theoretically does not hold. Knudsen et al. in [57] note that it is unclear how much this assumption is violated in real data and if so, how much worse the solution is. In their study, they tested 3600 scenarios and found only one instance of an non FIFO algorithm finding a better solution than the much faster algorithms. In the following paper of the same authors [56], none of the optimal solutions were found to violate the FIFO assumption although relaxing this caused not all scenarios to even come to a optimal solution in reasonable time.

3.2.2. Constraint handling

As mentioned before, part of the complexity of the FPP is the airspace network structure with all of its constraints. To keep track of all the constraints while the route is constructed in a search algorithm is computationally expense. Knudsen et al. in [56] develop an A* algorithm where partly dominated labels are only expanded when a constraint violated occurs on the then cheapest label, rather than always expanding partly dominated labels. In the same paper, they evaluate “lazy constraints". This is a method where all constraints are ignore at first and the problem solver normally. Then the solution is checked for feasibility and if it violates one or more constraint, only these constraints are added to the problem. This is then iteratively done until a valid solution is reached. This approach is also used in the study of Schienle et al. [82], where they claim that this is also the method used in practice. When a route is submitted to ATM it is either approved or rejected, and if rejected the route is recomputed. The lazy constraint approach showed to outperform both the normal and lazy expansion methods in real-life instances of the FPP. [56]

3.2.3. Pruning

Pruning is a way to leverage constraints in order to reduce the problem size. When setting lower and upper bounds to the various variables, part of the network might be discarded. Schienle et al. in [82] discuss four strategies. The first is dead-end elimination. Nodes that can not be reached from both the start or end point can be eliminated. Since the graph is directional, this does includes for instance approached to airports that are not the target airport. The second strategy is Tank-Capacity pruning. Nodes that can not be reached with the amount of fuel that the aircraft can hold do not have to be considered. This however does depend on fuel burn. In order to not have to solve this time dependent problem, underestimation of fuel burn is performed for all arcs. This would involve finding the most favourable conditions for a given arc, referred to as Super-Optimal-Wind in the study of Blanco et al. [9]. The third pruning strategy is also using underestimation, but for both fuel and time costs combined. This can be done in the same computational step as the previous one. In final strategy is the pruning of crossing cost. These are the cost related to overfly charges. A minimum crossing cost path can be found when regarding only these charges and then set as the lower bound. These strategies are shown to yield a noticeable reduction of search space and therefore computational time. [82]

3.3. Stochastic shortest path problem

Any modelling of real world problem would require some factor of uncertainty. Aircraft models, but especially weather models have some unpredictability. Some weather forecast providers do recognize this are publishing ensemble forecasts as will be explained in 6. Modelling flight planning under uncertainty is a form of an Stochastic Shortest Path Problem (SSPP) and is a very new field of research. SSPP research does exist for quite some time but the constraint version, which is needed for flight planning, is more recent. [86] In SSPP the cost of an traversing an arc is given by the required action to move from the start to end node for all possible states. Geißer et al. in [38] develop an algorithm to solve constraint SSPP for the FPP. Their method is based on column generation from linear programming. First, the SPP is solved for one state. Then, other state are searched for lower cost paths. These “reduces negative cost" states are then added to the problem and it is solved again. This method allows for the uses of non optimal path finding algorithms since it is enough to find any reduced negative cost. Only to prove optimality, would one require a optimal path finding algorithm. For now this method is only proven on experimental networks that are much smaller that real-world problems.

4
Optimum Control Problem

Trajectory optimization (TO) refers to the finding the control input of a dynamic system so that a certain optimum is achieved. This is more generally known are the Optimal Control Problem (OCP). In regards of TO, the aircraft is typically modelled as a six-dimension system derived around the center of mass. [55] A possible system of differential equation is given in equation 4.1

$$
\begin{aligned}
\dot{V} &= g\left(\frac{T\cos\alpha - D}{mg}\right)\\
\dot{\gamma} &= \frac{1}{mg}((T\sin\alpha + L)\cos\mu - mg\cos\gamma)\\
\dot{\chi} &= \frac{(T\sin\alpha + L)\sin\mu}{mV\cos\gamma}\\
\dot{x} &= V\cos\gamma\cos\chi\\
\dot{y} &= V\cos\gamma\sin\chi\\
\dot{h} &= V\sin\gamma
\end{aligned}
\tag{4.1}
$$

where V, γ, χ, α and μ are the speed, the angle of descent, yaw angle, angle of attack and roll angle respectively. x, y and h is the position of the aircraft. T, D, L, m and g are thrust, drag, lift, mass and gravitational acceleration respectively. The position of the aircraft can also be expressed in spherical coordinates and then using latitude, longitude and altitude and note that this system does not have wind components included. Generalized the differential equations can be written in a explicit for

$$\dot{\boldsymbol{y}} = [\boldsymbol{y}(t), \boldsymbol{u}(t), \boldsymbol{p}, t]$$

Where $\boldsymbol{y}(t)$ are the state variables, $\boldsymbol{u}(t)$ the control variables, $\boldsymbol{p}$ any parameters that are not dependent on time. Initial conditions can be defined by

$$\psi_{0l} \leq \psi[\boldsymbol{y}(t_0), \boldsymbol{u}(t_0), \boldsymbol{p}, t_0] \leq \psi_{0u}$$

And the same hold for final conditions where t_0 is replaced by t_f. Additionally any constraints can be represented by

$$\boldsymbol{g}_l \leq \psi[\boldsymbol{y}(t_0), \boldsymbol{u}(t_0), \boldsymbol{p}, t_0] \leq \boldsymbol{g}_u$$

The basic OCP is to determine the vectors $\boldsymbol{u}$ and $\boldsymbol{p}$ to minimize a performance index [7]

$$J = \phi[\boldsymbol{y}(t), \boldsymbol{p}, t]$$

ϕ could for instance be fuel flow [65], but also a more complex objective function. [83]

4.1. Numerical methods

Over the years, different numerical algorithms for TO have been studied. The most common ones will be briefly reviewed here.

Direct Direct methods include two steps. First the OCP is transformed in an non-linear programming problem. And secondly, this non-linear programming problem is solved numerically. [91] This can be done by for instance choosing a limited number of time points for the control variables. These will then need to be integrated to get the control values for the intermediate points. With this numerical state of the differential equations an non-linear programming algorithm can be used. Depending on the acceptable computational time and error, different algorithms can be used.

In-direct In-direct methods transform the OCP into a Hamiltonial boundary value problem with the use of Pontryagin's maximum principle. However it is very hard in regards to the equality and inequality constraints. Therefore indirect methods are not suitable for real engineering problems. [83]

Dynamic programming Dynamic programming subdivides the problem into smaller sub-problems and makes use of the idea that a sub-strategy of the optimal strategy is always optimal. [43] Dynamic programming defines a grid in the search space of state variables and computes the solution for every intersection. With this method, inequality constraints are easy to implement, since it just limits the search space. However, computational time and memory do not scale well with increasing dimensionality. [66]

Genetic algorithms Genetic algorithms (GA) are a search techniques that simulates natural selection to find optimal solutions. It the TO literature review Betts did in 1998 he remarks that there is no reason to use GAs in this field. [7] However, in recent literature GAs are often used to solve non-linear problems as will be reviewed in the next section.

4.2. Application to commercial flight planning

Trajectory optimization is widely studied for several applications. Most notable are; space missions, military missions and unmanned air vehicles. For commercial flight planning, this is less. [83] However, in the scope of TBO this recently gained some more attention.

Solet et al. in [83] develop a direct method that uses database modelled aircraft and a nonlinear solver to compute full 4D trajectories. Key to this technique is the differentiation between flight phases, climb, cruise and decent, and make the switching times part of the state. Although the work is still far from a full optimization as the trajectories are computed with preset cost indexes and initial weight estimations, it does show the use of nonlinear solvers in flight planning.

Similarly Miyazawa et al. in [66] show that with the improvements of computers for numerical computation, dynamic programming is a feasible method for 4D trajectory optimization. This is used in the study of Wickramasinghe et al. [90] where they use dynamic programming in the 3D and 4D OCP to assess the fuel efficiency of conventionally routed flights compared to the freely routed optimum.

Lastly, genetic algorithms have seen a interesting development for flight planning. Several studies discretize the trajectory optimization problem to be solved using this method. Partón et al. in [34] use a genetic algorithm to optimize lateral routing including wind. In their continued works [75, 76] they make adaptations to expand the algorithm for the vertical dimension. GAs with dynamic discritization are also studied. Entz et al. in [28] develop a lateral routing optimizer, using the so called ant colony optimization. Murrieta-Mendoza et al. in [68] show promising results for 4D flight planning using the artificial bee colony algorithm. Due to the low computational intensity of GAs, these algorithms are seen as good candidates for on-board trajectory optimization in the FMS. [34]

5
Aircraft Model

Trajectory optimization needs trajectory prediction. In order to to this, the aircraft has to be modelled. In the real world, the pilot give flight commends to the aircraft systems. These interact with the environment and produce a trajectory. When modelling this the input instructions are referred to as aircraft intent. [89] These combined with initial conditions are fed into a trajectory computation infrastructure. This comprises of two models that interact with each other. The first being the aircraft model discussed in this section and the second being the environment model, which in case of TO is the earth/weather model reviewed in section 6. Together these can predict a trajectory, but it is important to note that this predicted trajectory is not perfect and will differ from the real trajectory.

5.1. Current models

The aircraft performance model is an essential tool for flight planners. Currently, ground-based technologies work with performance tables that are determined by the aircraft manufacturer by wind tunnel testing and flight tests. [88] Actually, according to Dancila et al. in [21] also the on-board FMS does utilize these performance tables. Fuel burn is determined by the fuel burn rate which is a function of aircraft parameters and flying conditions. These parameters are the aircraft's speed, weight, air temperature and altitude and the function is not linear. [22] In both the FMS and ground-based flight planning tools, the furn burn is estimated by interpolation of the performance tables. In the FMS this is commonly linear interpolation and this linearization might even consider only a subset of parameters. [21] Adding to the inaccuracy is the fact that every aircraft is slightly different. Factors such as aging, maintenance and different configurations causes a variation in performance. This problem is usually handled by the airlines multiplying the performance models by a tail specific performance/aging factor. In literature studies, this is often not the case, since these research does not take individual aircraft into account. Research usually makes use of aircraft performance databases from various sources.

5.2. Aircraft performance databases

There are multiple different aircraft performance database models. Eurocontrol has its Base of Aircraft Data (BADA) [36], FAA the Aircraft Environmental Design Tool (AEDT), ICAO the Engine Emissions Databank and there is also Lissys Piano-X a industry aircraft performance and design tool. Analysis of these tools on actual flight data shows various results. Chati and Balakrishnan found in [14] that the ICAO databank overestimates the fuel burn in almost all studied cases. Poles et al. showed in [77] that the latest version of BADA is already a large improvement over the previous edition, but since these are generalized aircraft models, there will always be a difference when compared to individual reference trajectories. There is however research in using machine learning techniques to further improve the models for individual tails.

5.3. Machine learning models

Improving the aircraft models using machine learning models work by training algorithms on actual quick access recorder (QAR) data. The studies of Khadilkar, Chati and Balakrishnan in [15–17, 54] use Gaussian regression to improve the fuel estimates for the taxi, climb, cruise and descent phases. They are shown to reduce the mean square error considerably with a 5% significance level. Reduction of up to 50% compared to BADA can be achieved. Trani et al. in [85] and Uzun et al. in [88] both use neural networks to improve the fuel burn estimation. Both report promising results that outperform the BADA model. It would have to be investigated what the influence of an more accurate fuel estimation would be for the results of a FPP.

6 Weather Model

Weather is a very important factor in the optimization of a flight trajectory. There are several aspects that influence the efficiency. Aircraft have to be routed around convective storms, air temperature affects engine performance and wind has a major impact on ground speed. During the planning of a flight only forecasts are available and these are an inherent source of inaccuracy. Firstly, a forecast does not describe every possible point exactly and therefore has to be interpolated to get flight relevant data. Betts in [7] noted that interpolation was the number one impediment in finding efficient trajectories. Secondly the accuracy of a forecast model in perfect either. Meteorological predicting is an entire research field on its own and the complex models that are the basis of current day's weather forecasts will not be discussed in this review. However, there are some specific challenges that come with weather predicting for aviation and where the aviation research community is involved in.

6.1. Improving forecasts

Obtaining a accurate 3 dimensional wind field is one of these issues, since there are very few available weather sensors apart from weather balloons. [84] Various studies are dedicated to increasing this number of sensors by retrieving wind data from aircraft. De Haan in [23] developed a method to capture wind speed from mode-S radar responses. He combined the ground track with the aircraft's measured flight level, speed and direction. This is similar to the workings of weather data generated for Automatic Dependent Surveillance Contract (ADS-C) messages, but there the ground track is also determined on-board the aircraft. [24] These airborne wind estimation models are not perfect and with future operations like (TBO) requiring accurate wind speeds for trajectory predictions, efforts are directed into further improving these. [25] Since flight planning also depends on accurate trajectory predictions, any advancements in this field will also increase the efficiency of trajectory optimization. Besides improving the data sources for the weather models, a different deep/machine learning techniques are also being studied for forecast improvement purposes. Uzun et al. in [87] use the same technique they employed to improve aircraft models and show a similar reduction in standard deviation for wind components. Cabos et al. in [12] use historical forecast data to predict errors in future forecasts. This methods involves training of machine learning algorithms on extremely large dataset and requires dedicated computing-clusters but does show promising results.

6.2. Uncertainty

Today, flight planning tools use deterministic weather models. The weather however is inherently uncertain and to deal with this numerical weather prediction centers created the Ensemble Prediction System (EPS). This system attempts to capture the uncertainty by producing sets of forecast. Sets typically have 10 to 50 members [41] and give ome probability to a forecast occurring. Figure 6.1 gives a schematic representation of EPS compared to a deterministic model.

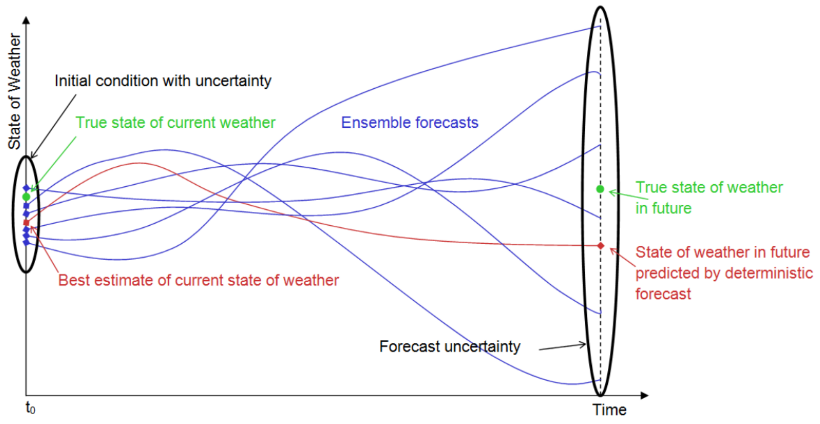


Figure 6.1: Schematic showing uncertainty captured in an Ensemble Weather Forecast. [19]

SESAR noticed the value of EPS and supported work to analyse the affects of uncertainty for TBO. [18, 19] Here is was found that EPS could also be of interest for flight planning proposes, although it was acknowledged that real flight planning was far more complex than the scenarios then studied. There since been several studies that use EPS, like Geißer in [38] for the FPP, and Gonzales-Arribas et al. in [2, 40] for the OPC.

7

Efficiency Metrics

Flight efficiency is a collective term that depending who you ask means something different. Airlines are most interested in cost effective operations and schedule adherence, while Air Navigation Service Providers (ANSP) value factors like capacity, ATC interventions, emissions and noise. Although the efficiency metrics are different for both parties, they often do require comparison with a optimal reference. The cost efficiency of a flight for instance, can be expressed as a ratio between that flight and the optimal flight. The same holds for noise or emissions. As discussed in previous sections, the optimum flight is hard to predict and therefore the current approaches to evaluate flight efficiency is reviewed in this section.

7.1. Air Navigation Service Providers

In order to manage effectively, Air Navigation Service Providers (ANSP) need to measure their performance. This needs quantifiable Key Performance Indicators (KPIs) that properly represent the Key Performance Areas (KPAs). ANSPs are continuously developing these to advance the knowledge and awareness of the overall air transport system. Some of the KPIs used are legally bound by the governing authorities. In Europe there are 37 different ANSPs that are organized by the Single European Sky (SES) performance scheme. [32] This also divines targets, which are in Europe mainly focused towards horizontal flight efficiency and Air Traffic Flow Management (ATFM) delays. In the United States, the FAA is more focused on capacity and capacity efficiency. [13] Both parties however do yearly publish combined work where they assess the effectiveness of both approaches with to goal to learn for each other and advance the combined knowledge. One of the incentives is to relate ANSP performance more to the airlines objectives. [35]

The current method of analysis horizontal efficiency is often about expressing the actual trajectory length with the great circle distance (GCD). This however often excludes the areas close to the origin and target airports known as the Arrival Sequencing and Metering Area (ASMA). In Europe the ASMA circles the airport by 40 NM, whereas the FAA uses 100 NM. The GCD for en-route efficiency calculations is taken between the intersection points of the actual trajectory and the ASMA. Another reference can be taken when using the closest points between ASMAs, this is called the achieved distance as shown in figure 7.1.

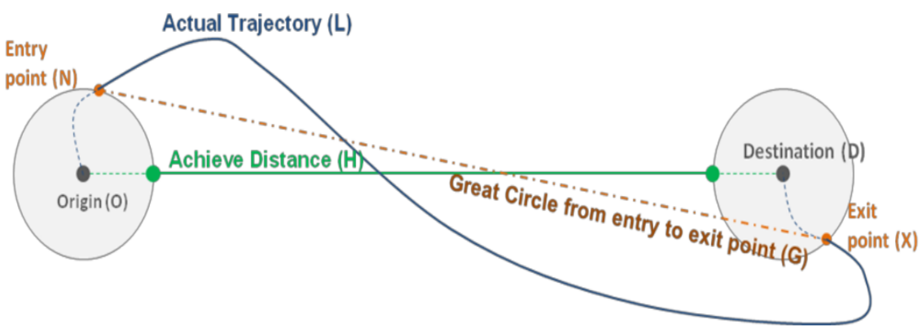


Figure 7.1: Reference distances [13]

The achieved distance somewhat better captures the objectives from the airlines, but as discussed in previous sections, the optimal trajectory is often not a straight line between origin and destination. Besides, especially in Europe the deviation of a route compared to this achieved distance could be due to inefficient routing in different functional airspace blocks (FAB) managed by other ANSPs. Therefore recent developments created partitioned KPIs on a FAB level. [59] These however are still not using wind optimal trajectories as reference although is an idea that already exists for numerous years. But because of the complexity in obtaining this reference, not much headway is made. [53] A independent improvement would be to include the vertical dimension in the efficiency evaluation. The ANSP of the UK, NATS, leaded this effort and developed the 3Di metric. [70] The goal was to better capture the fuel efficiency, which is more directly related to emissions and cost than the usual horizontal distance. This however does need the generation of optimal 3 dimensional reference trajectories. Any of the methods reviewed in sections 3 and 4 can be used for this. Wickramasinghe et al. in [90] use dynamic programming for reference trajectory calculation, but NATS keep it relatively simple and assume International standard atmosphere condition with still winds for their 3Di reference. Furthermore, aircraft performance was modelled using the BADA performance tables, using nominal take-off weights. The 3Di score is believed to provide a good measure, but it is not yet widely adopted. One of the reasons for this is that the 3Di score still does not capture the performance compared to the user preferred operational objectives. The business strategy of an airline calls for a balance between cost and time, so time should be somehow added to the evaluations. Calvo et al. in [13] propose a method where the efficiency is related to flight plan adherence, which they argue should include the airlines incentives regarding cost and time, since the airline has optimized to route with their preferred cost index. This assumption however is not fully true, since as discussed in this review, airlines are not aware of how close to the optimum their flight planning tools are.

7.2. Air Network Users

Flight efficiency to airlines is mainly about cost and schedule adherence. This is important for the entire operation of a flight and therefore one of the bigger criticisms about the ANSP efficiency KPIs is the exclusion of the ASMA. Additionally, with future TBO in mind, airlines are also interested in equity metrics that as of now do not exists. To ensure fair competition between airlines, it is clear that ANSPs should handle airline request similarly. One airline should not always get directer routing than another airline. However for the scope of this review, this is less important.

One of the proposed methods to evaluate flight planning quality is to analyse the performance of the execution of a flight plan and relate this to the flight plan itself. To do this, user-centric flight efficiency is reviewed as is suggested in the recent studies of López-Leonés et al. [60, 61]. A prominent part is their method is the construction of several different reference trajectories that are outlined below.

Optimal distance trajectory This is the GCD that is use in the SES perfomance scheme. It does not consider airspace structure.

Optimal cost trajectory 1 Here free flight conditions are assumed. It is a form of the OCP where cost and time are minimized. López-Leonés et al. do not use wind optimal routing and omit overfly charges, but these can essentially be added if the complexity can be coped with.

Optimal cost trajectory 2 This differs from the optimal cost trajectory 1 only by adhering to horizontal airspace structure. It is a decomposed FPP, where speed and altitude are freely optimized.

Flight plan trajectory This is the trajectory that corresponds to the filed flight plan. The difference between this and the actual flown route lies in inaccuracies of the operation, but also interventions by ATC or weather diversions.

All of these trajectories are compared to the actual flown trajectory which is retrieved from surveillance data or QAR data. The comparison involves four categories namely; horizontal, vertical, fuel and cost. This results in 13 indicators since the optimal distance trajectory in not 3D and can only be compared horizontally. Equation 7.1 gives an example of an indicator calculation.

$$CEA\_CW1 = \left(\frac{C_{AFT}}{C_{OCT1}} - 1 \right) \% \tag{7.1}$$

Where $CEACW1$ is the cost efficiency indicator of the Actual Flown Trajectory (AFT) versus the Optimum Cost Trajectory 1 (OCT1) considering Weather. [60] The other indicator are defined in a analog way. Results shown that the cost indicators do not align with the horizontal only and therefore emphasizes the importance of including the vertical dimension for flight efficiency evaluation. For flight plan assessment this should therefore definitely be included too.

8

Clustering

As the assessment of flight plans will likely involve grouping them into subsets with similar features, this section will review the research on clustering in the aviation domain. Generally, clustering is applied for one of two reasons. To either uncover underlying trends or to detect outliers. In the scope of trajectories, most research is done to detect traffic flows and this thus falls in the first category. Outlier detection however could also be of interest, since a use of the flight plan assessment framework would be to flag potential problematic flight plans that are generated automatically and hand them over to expects for manual checking.

8.1. Trajectory clustering

Trajectory clustering is extensively studied for several aviation applications like; performance assessment, airspace design and monitoring, and traffic flow management. [67] Clustering is often an multi-step process first a coarse grouping is made after with this is further refined. This is done because finer clustering methods often require comparison with all other items and this can be very computationally intensive. Therefore most often the first grouping is made based on origin destination pairs. DeArmon et al. in [26] developed a interesting alternative for this though, by clustering the airports themselves into spatially coherent groups and defining the first trajectory grouping by which airport clusters the trajectory connects. After this first coarse grouping trajectories further refining requires some measure of distance between the flight paths. Bombelli et al. in [10, 11] use Fréchet distance for this. Fréchet distance is a measure of separation between two paths and is often explained with the dog on a leash metaphor. Consider someone walking a dog on a leash. Both the person and the dog move along a different trajectory but the distance between them is bounded by the length of the leash. One can compute the maximum difference between trajectories and cluster them according to this distance. Additional features can be added to this like heading angles or speed over ground. [64] To point of trajectory clustering is often to get the means of the clusters and use these as flow patterns. The difference in features can also be used to detect irregularities as explained in the next subsection.

Recent studies have employed the techniques of trajectory clustering to improve the trajectory prediction models. As mentioned before, one of the challenges in TBO is the accurate prediction of aircraft's locations. In current operations the deviation between the reference waypoints and actual position can be up to 15 km. [39]. Ayhan et al in [3, 4] make use of clustered historical trajectories to train hidden markov models to better predict this. Newer work by Liu et al. [63] and Georgiou et al. [39] use a similar approach but the do include the flight plan into their models to enrich the prediction model. This is a very interesting method also for flight plan assessment, because results show that these models are closer to the actual flown trajectory than trajectories generated with only the flight plan.

8.2. Anomaly detection

Very recently Barosa et al. reviewed all the advances in anomaly detection for the aviation domain in [5]. This contain a very extensive list of both applications and methods, and mainly focuses on unsupervised technique suited for time series data. They distinguish anomalies in three different groups. The first is point anomalies, this is when a single point stands out compared to others. The most traditional detection systems in the industry are based on parameters exceeding a certain threshold and therefore are capable in detecting these. Then there are contextual anomalies, which are only outliers in a certain context for a given time of location. Finally, there are collective anomalies. This is when a group of data within a set combined is a anomaly, but an individual in itself is not.

Generalized, anomaly detection programs follow three steps. First continuous or discrete data is transformed into high dimensional vectors, often with normalization of the continuous parameters. Then the number of dimensions is reduced with for instance the use of pricipal component analysis. This is to decrease the problem size, without losing to much relevant behaviour. Lastly, clustering method can be applied to the reduced vectors. Some of the methods are k-mean, DBSCAN or HDBSCAN. These themselves come with parameters that can be tuned by doing a sensitivity analysis. [62]

Regarding anomaly detection in trajectories, interesting advancements have been made to identify thing like bad weather rerouting and ATC interventions for for instance deconfliction or sequencing purposes. [71–73] These method could be useful when analysing flight plan performance, since ATC interventions for instance could have a profound effect of the efficiency of a flight, but they might not be documented in the historical data.

9 Conclusion

Airlines are always looking to reduce their operating costs. And with the global focus on reducing emission, lots of effort is put into decreasing aviation fuel burn. The most short term solution for this would be to develop more efficient operating procedures and therefore this is a well studied field in aviation.

Flight operation will likely change over the next few decades, with free flight procedures replacing the current ground navigation infrastructure based one. However, until then, advances made in this field could already be used to evaluate and improve the performance of the current system.

Due to the complexity of the flight planning problem, simplification had to be introduced to generate solutions. The impact of this in regards of the resulting performance if a unsolved question, because evaluation techniques require the determination of the optimum trajectory, which is hard to define. However new methods developed for trajectory based operations are capable of modelling the problem much closer to reality than before, therefore reducing the gap with the theoretical optimum.

These newer techniques are being employed to evaluate the performance of airspace infrastructure, but no literature is found of this being used to evaluate current flight planning systems. In fact, no research is found that relates the results of flown trajectories to the planning stage of the flight. To explore this gap in knowledge, current flight efficiency metrics were reviewed. Most of the existing work there focuses on flight efficiency in the eyes of the air navigation service providers. However, their key performance indicators shift more and more to the objectives of the users, which is useful for this study.

Advancements in deep learning on aviation data allows to discover patterns in historical trajectories that should be able to create an assessment framework for flight planning tools. Although some researchers have commented that this is an unfeasible challenge, it seems like it is not thoroughly studied. The aim of this thesis project is to change this an explore the uses of new techniques in this context.

III

Supporting work

A

Available Records

Table A.1 gives the total count of records that were available for this project. Unfortunately did the circumstances not allow for direct access in the data system of SWISS international air lines. An alternative data flow was setup, but it had some limitations that resulted in the incompleteness of the released dataset. The extracting method of time series data in particular was very computational extensive, because some parameters were sampled with a 1 hz interval producing very large files. The attained dataset is in my opinion still a good representation to apply this research to. It is fairly balanced in regards of to the different destination distances.

Origin	Destination	Jan	Feb	Mär	Apr	Mai	Jun	Jul	Aug	Sep	Okt	Nov	Dez	Total
EDDF	LSZH		19											19
EGLL	LSZH	189	190	208	205	188	107							1087
KJFK	LSZH	54	48	60	60	62	58	61	62	58	60	57	61	701
LEMD	LSZH		21											21
LGAV	LSZH	56	56	83	65	86	86	87	91					610
LSZH	EDDF	94	74	73		76	84							401
LSZH	EGLL	196	189	198		199	204							986
LSZH	KJFK	50	49	56	58	59	58	23	57	24	62	56	61	613
LSZH	LEMD	101	81	89		85	82							438
LSZH	LGAV	56	56	83	65	86	86	87	89					608
LSZH	WSSS	25	25	29	29	28	27	28	26	16	27	5		265
WSSS	LSZH	16	22	30	28	29	27	29	26	18	25	6		256
Total		837	830	909	510	898	819	315	351	116	174	124	122	6005

Table A.1: Available record count by route and month

B
Dynamic Time Warping

This researched used relative change in all efficiency domains to map a flight into a series of floating-point values that were subsequently used for statistic and machine learning methods. However, another method was also tried, which will be described in this appendix. Dynamic time warping is a method that quantifies the difference between two time series. It is much used in the field of voice recognition, since the speed of change is not necessarily directly relevant to the similarity. This is also the case when comparing the altitude profiles of an actual flight and a planned trajectory. If for instance the actual flight in operated at a higher speed, one would expect characteristics linked to position to shift in time. DTW allows for this by calculating the minimum possible shift/warping regardless of the dimension it is measured in.

B.1. Method

Using DTW usually requires calculation of all distances between all points of the trajectories. This however is very computationally intensive for very large time series data like an 1hz altitude measurement for an intercontinental flight. Therefore, the Fast DTW method from S. Salvador and P. Chan [80] is used and the search window is limited to 30 minutes. The measured variables are filtered for high frequency noise by applying a running average over 30 seconds. And for irregular sparse variables, i.e. simulation results, and linear interpolation is used match the data sampling rates.

DTW can be used with various distance definitions. For this research it is important that all metrics are generalised for meaningful comparison between different flights. To normalise the minimum warping distance this value is divided by the maximum possible of that flight. To be able to calculate this it is chosen to use the manhattan distance as defined in equation B.3 as distance metric for DTW in equation B.2 with n being the amount of dimensions, k number of data points and i and j the compared trajectories. The minimum distance found with equation B.1 is than normalised using equation B.4.

$$D_{\min}(i_k, j_k) = \min_{i_{k-1}, j_{k-1}} D_{\min}(i_{k-1}, j_{k-1}) + d(i_k, j_k | i_{k-1}, j_{k-1}) \tag{B.1}$$

$$D = \sum_k d(i_k, j_k) \tag{B.2}$$

$$d(i,j) = \sum_n |x_{i_n} - x_{j_n}| \tag{B.3}$$

$$D_{norm} = \frac{D_{\min}}{x_k \sum_k (\max(y_{i_k}, y_{j_k}))} \tag{B.4}$$

B.2. Results

Analysing typical DTW values for different length routes showed a problem with the proposed method. As example, average values for warping distance between AFT and FPT are given In table B.1 for the fuel and speed profiles, FEA_P and VEA_P respectively. Here it is clear that all values scale by route distance, pointing to the normalisation being flawed.

Distance	FEA_P	VEA_P
EDDF-LSZH	0.35167	0.07832
LGAV-LSZH	0.05010	0.02879
KJFK-LSZH	0.02837	0.01454

Table B.1: Selection of average dynamic time warping distances for routes with different distances

B.3. Conclusion

With the distance values not being normalised, it will not allow appropriate comparison. This is a big problem since this is the sole purpose of the proposed assessment framework. The only value for the method in this research is that on an individual flight record basis is can be determined what trajectory is closest within the set of AFT, FPT and different reference trajectories.

Finally, DTW in the form used in this research is not practical due to the computational intensive nature of the calculations. Even though the fast DTW implementation was selected, calculation took considerable time and due to the lack of usable results it was opted to withhold calculation on the second half of the provided data.

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