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1 **Dynamic analysis of layered systems under a moving harmonic**
2 **rectangular load based on the spectral element method**

3
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10
11 **Abstract:** In order to design high-performance roadways, a robust tool which can
12 compute the structural response caused by moving vehicles is necessary. Therefore,
13 this paper proposes a spectral element method-based model to accurately and
14 effectively predict the 3D dynamic response of layered systems under a moving load.
15 A layer spectral element and a semi-infinite spectral element are developed to
16 respectively model a layer and a half-space, and the combinations of these two
17 elements can simulate layered systems. The detailed mathematical derivation and
18 numerical validation of the proposed model are included. Addition- ally, this model
19 is used to investigate the dynamic characteristics of a pavement structure under a
20 moving harmonic rectangular load. The results show that the proposed model can
21 accurately predict the dynamic response of layered systems caused by a moving load.
22 It is also found that the vertical displacement amplitude curves of surface points
23 caused by a moving harmonic load are asymmetric along the moving direction, and
24 this property is more dominant at higher velocities. In addition, the amplitudes of
25 these vertical displacements are smaller if the loading frequency is higher or the loss
26 factor is bigger. Finally, the loading area and Poisson's ratio only have effect on the
27 displacement amplitudes of points in the close vicinity of the loading area. The
28 proposed model is beneficial to the development of engineering methods for
29 pavement design and is a promising parameter back-calculation engine for pavement

30 quality evaluation.

31

32 **Keywords:** Dynamic response; Layered systems; Moving load; Spectral element

33 method; Doppler effect

34 **1. Introduction**

35 Roadways are important infrastructures and should be well- designed. In order to
36 ensure the performance, a clear understanding of the response of roadways caused by
37 moving vehicles is necessary. Theoretically, this problem can be regarded as the
38 dynamic analysis of semi-infinite or layered media caused by a moving load, which is
39 generally solved by using either analytical or numerical methods.

40 Analytical methods generally give exact solutions to dynamic problems, and
41 these methods are usually efficient. For example, Eason (1965) investigated the
42 stresses in a semi-infinite elastic solid caused by moving surface forces with different
43 loading conditions using integral transforms. Vostroukhov and Metrikine (2003)
44 proposed a theoretical model to analyse the steady-state dynamic response of a
45 railway track caused by moving trains, through which an analytical expression of the
46 steady-state deflection of the rails was obtained. However, the analytical solutions are
47 generally only valid for specific structural and loading configurations, and these
48 solutions are usually difficult to calculate because they often contain complicated
49 integrals with singular points.

50 Numerical methods, such as the finite element method (FEM) and the boundary
51 element method (BEM), are powerful tools for the dynamic analysis of solid media
52 with different structural combinations and loading conditions. For instance, Zaghoul
53 and White (1993) developed a three-dimensional dynamic finite element program to
54 analyse the behaviour of flexible pavements caused by loads moving at different
55 velocities. Andersen and Nielsen (2003) conducted boundary element analysis of the
56 steady-state response of an elastic half-space caused by a surface moving load.
57 However, numerical methods are usually time and resource intensive, and numerical
58 distortions may occur in some cases.

59 The limitations of analytical and numerical methods may hinder their application
60 in engineering, especially for the dynamic analysis of layered systems. Hence, a
61 semi-analytical method called the spectral element method (SEM) (Doyle, 1997;
62 Al-Khoury et al., 2002; Lee, 2009) is used in this paper to analyse the 3D dynamic
63 response of layered systems caused by a moving load. The SEM is promising for
64 efficient dynamic analysis because it has the advantages of both spectral analysis and
65 finite element method. In the SEM, one element is sufficient to represent a whole
66 layer because of the exact description of mass distribution, which reduces the size of

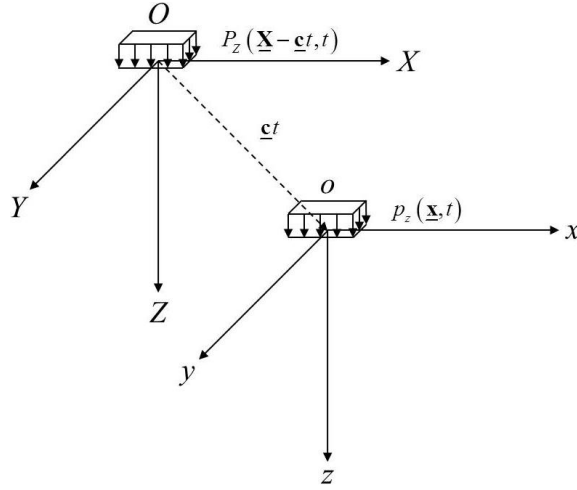
67 the system of dynamic equations and further increases the computational efficiency.
68 Moreover, this method discretises the continuous integrals into series summations,
69 which is more convenient for numerical calculation. The SEM has been successfully
70 used for analysing the 2D dynamic response of layered systems. For example, You et
71 al. (2018) investigated dynamic response of transversely isotropic pavement structure
72 under axisymmetric impact load in cylindrical coordinate system based on the SEM.
73 Yan et al. (2018) applied the SEM to predict the dynamic response of a 2D layered
74 system subject to a moving harmonic strip load. However, the SEM has rarely been
75 applied for the 3D dynamic analysis of layered systems under a moving harmonic
76 rectangular load, which is the main focus of this study.

77 This paper includes the detailed mathematical formulation of a 3D dynamic
78 model for layered systems under a moving harmonic rectangular load based on the
79 SEM. The accuracy of this model has been verified both numerically and
80 experimentally. The proposed model can be used to analyse the 3D dynamic response
81 of pavement structures caused by a moving harmonic rectangular load, which
82 contributes to the development of engineering methods for pavement design.
83 Furthermore, this model could be combined with proper optimisation algorithms to
84 back-calculate the parameters of pavement structures by analysing the response,
85 which is useful for pavement quality evaluation.

86 **2. Model formulation**

87 In this section, the detailed formulation of a model which can predict the 3D
88 dynamic response of elastic layered systems subjected to a uniformly moving,
89 harmonically varying, evenly distributed, rectangular surface load is presented. With
90 considering the loading conditions caused by moving vehicles and structural
91 parameters of pavement systems, this model can be used as a tool for structural
92 design to ensure the durability.

93 *2.1. Introduction of moving coordinate system*



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Figure 1. Schematic representation of coordinate system transformation.

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As shown in Figure 1, in order to deal with the moving load problem, it is convenient to introduce a stationary Cartesian coordinate system ($OXYZ$) and a moving Cartesian coordinate system ($oxyz$) (Jones et al., 1998; Lefeuve-Mesgouez et al., 2000; Metrikine, 2004). The stationary coordinate system does not move and its origin is located at the centre of the loading area when time is zero. The moving coordinate system follows the load and its origin is located at the centre of the moving loading area. The moving velocity is assumed to be constant and is described by a vector $\underline{c} = [c_x \ c_y \ c_z]^T$. The stationary coordinate vector is notated as $\underline{X} = [X \ Y \ Z]^T$, and the moving coordinate vector is notated as $\underline{x} = [x \ y \ z]^T$. The relationship between these two coordinate vectors can be expressed as follows:

$$\underline{x} = \underline{X} - \underline{c}t \quad (1)$$

in which t is time. These two coordinate systems are coincident when $t = 0$.

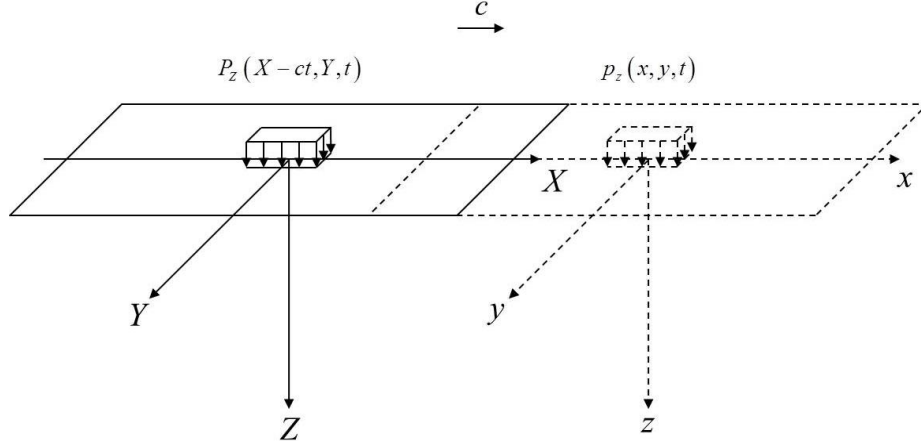
Additionally, the partial derivatives in the two coordinate systems have the following relationships for nonnegative integer n :

$$\frac{\partial^n}{\partial \underline{X}^n} = \frac{\partial^n}{\partial \underline{x}^n} \quad (2)$$

$$\frac{\partial^n}{\partial t^n} \Big|_{\underline{x}} = \left(\frac{\partial}{\partial t} - \underline{c} \cdot \frac{\partial}{\partial \underline{x}} \right)^n \Big|_{\underline{x}} \quad (3)$$

113 where $\underline{\mathbf{c}} \cdot \frac{\partial}{\partial \underline{\mathbf{x}}} = c_x \frac{\partial}{\partial x} + c_y \frac{\partial}{\partial y} + c_z \frac{\partial}{\partial z}$.

114 2.2. Wave motion in a semi-infinite medium under a surface moving load



115
116 Figure 2. Schematic representation of a semi-infinite medium under a surface
117 moving load.

118 As shown in Figure 2, a homogeneous, isotropic, and linear-elastic semi-infinite
119 medium is subjected to a surface load which moves along X-axis with a constant
120 speed c . The corresponding wave motion in this medium is considered first. In the
121 stationary coordinate system ($OXYZ$), the equations of motion for the medium can be
122 expressed by Navier's equation in the absence of body forces:

123
$$(\lambda + \mu) \nabla_0 \nabla_0 \cdot \underline{\mathbf{U}} + \mu \nabla_0^2 \underline{\mathbf{U}} = \rho \frac{\partial^2 \underline{\mathbf{U}}}{\partial t^2} \quad (4)$$

124 in which $\nabla_0 = \left[\frac{\partial}{\partial X} \quad \frac{\partial}{\partial Y} \quad \frac{\partial}{\partial Z} \right]^T$ is the Del operator, $\nabla_0^2 = \frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2} + \frac{\partial^2}{\partial Z^2}$ is

125 the Laplacian operator, $\underline{\mathbf{U}}(\underline{\mathbf{X}}, t) = [U_x \quad U_y \quad U_z]^T$ is the displacement vector, ρ

126 is the mass density, and λ and μ are Lamé constants defined by Young's

127 modulus E and Poisson's ratio ν .

128 An elegant approach to solve the Navier's equation is using the Helmholtz
129 decomposition, which expresses a displacement field in the following form:

130
$$\underline{\mathbf{U}} = \nabla_0 \Phi + \nabla_0 \times \underline{\Psi} \quad (5)$$

131 where $\Phi(\underline{\mathbf{X}}, t)$ is a scalar potential related to the P-wave, and

132 $\underline{\Psi}(\underline{\mathbf{X}}, t) = [\Psi_x \quad \Psi_y \quad \Psi_z]^T$ is a vector potential related to the S-wave. It can be
133 seen that the three components of the displacement vector are related to four other
134 functions, the scalar potential and the three components of the vector potential,
135 which indicates that an additional constraint condition is needed (Achenbach, 1999).
136 The additional constraint condition can have different forms (Vostroukhov and
137 Metrikine, 2003; Hung and Yang, 2001), but the solution is uniquely determined by
138 the governing equations and boundary conditions by virtue of the uniqueness
139 theorem. In this paper, the Gauge condition $\nabla_0 \cdot \underline{\Psi}(\underline{\mathbf{X}}, t) = 0$ is taken as the
140 additional constraint condition.

141 The velocity vector of the load is $\underline{\mathbf{c}} = [c \quad 0 \quad 0]^T$, which means movement with
142 constant velocity c along the x -axis. According to the relationship between the two
143 coordinate systems, equation (4) has the following form in the moving coordinate
144 system:

$$145 \quad (\lambda + \mu) \nabla \nabla \cdot \underline{\mathbf{u}} + \mu \nabla^2 \underline{\mathbf{u}} = \rho \left(\frac{\partial}{\partial t} - c \frac{\partial}{\partial x} \right)^2 \underline{\mathbf{u}} \quad (6)$$

146 in which $\nabla = \left[\frac{\partial}{\partial x} \quad \frac{\partial}{\partial y} \quad \frac{\partial}{\partial z} \right]^T$ is the Del operator in the moving coordinate system,

147 $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the Laplacian operator in the moving coordinate system,

148 $\underline{\mathbf{u}}(\underline{\mathbf{x}}, t) = [u_x \quad u_y \quad u_z]^T$ is the displacement vector in the moving coordinate
149 system.

150 In the moving coordinate system, equation (5) has the following form:

$$151 \quad \underline{\mathbf{u}} = \nabla \phi + \nabla \times \underline{\Psi} \quad (7)$$

152 where $\phi(\underline{\mathbf{x}}, t)$ and $\underline{\Psi}(\underline{\mathbf{x}}, t) = [\psi_x \quad \psi_y \quad \psi_z]^T$ are the scalar potential and the
153 vector potential in the moving coordinate system, respectively. The Gauge condition
154 in the moving coordinate system reads $\nabla \cdot \underline{\Psi}(\underline{\mathbf{x}}, t) = 0$.

155 By substituting equation (7) into equation (6), the following uncoupled wave

156 equations in the moving coordinate system are obtained (for more details see
 157 Appendix A):

$$158 \quad \nabla^2 \phi - \frac{1}{c_p^2} \left(\frac{\partial}{\partial t} - c \frac{\partial}{\partial x} \right)^2 \phi = 0 \quad (8)$$

$$159 \quad \nabla^2 \underline{\Psi} - \frac{1}{c_s^2} \left(\frac{\partial}{\partial t} - c \frac{\partial}{\partial x} \right)^2 \underline{\Psi} = \underline{\mathbf{0}} \quad (9)$$

160 in which $c_p = \sqrt{(\lambda + 2\mu) / \rho}$ is the velocity of the P-wave, and $c_s = \sqrt{\mu / \rho}$ is the
 161 velocity of the S-wave.

162 In order to solve equations (8) and (9), the following Fourier transform pair with
 163 respect to time is used:

$$164 \quad \hat{q}(\underline{\mathbf{x}}, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} q(\underline{\mathbf{x}}, t) e^{-i\omega t} dt \quad (10)$$

$$165 \quad q(\underline{\mathbf{x}}, t) = \int_{-\infty}^{\infty} \hat{q}(\underline{\mathbf{x}}, \omega) e^{i\omega t} d\omega \quad (11)$$

166 where i is the imaginary unit satisfying $i^2 = -1$, ω is angular frequency, $q(\underline{\mathbf{x}}, t)$ is
 167 an arbitrary quantity in time domain, and $\hat{q}(\underline{\mathbf{x}}, \omega)$ is the corresponding quantity in
 168 frequency domain. After applying the above Fourier transform, equations (7) to (9)
 169 become:

$$170 \quad \hat{\underline{\mathbf{u}}} = \nabla \hat{\phi} + \nabla \times \hat{\underline{\Psi}} \quad (12)$$

$$171 \quad \nabla^2 \hat{\phi} - \frac{1}{c_p^2} \left(i\omega - c \frac{\partial}{\partial x} \right)^2 \hat{\phi} = 0 \quad (13)$$

$$172 \quad \nabla^2 \hat{\underline{\Psi}} - \frac{1}{c_s^2} \left(i\omega - c \frac{\partial}{\partial x} \right)^2 \hat{\underline{\Psi}} = \underline{\mathbf{0}} \quad (14)$$

173 in which the ‘‘hat’’ means that these quantities are expressed in the frequency
 174 domain.

175 In the Cartesian coordinate system, the solutions of equations (13) and (14) can
 176 be retrieved in exponential forms. According to the phase matching principle (Zhao
 177 et al., 2016), different waves should have the same phase at the boundary (e.g. the

178 surface $z = 0$). Consequently, the P-wave and S-wave have the same wavenumbers
 179 not only in x -direction, but also in y -direction. Therefore, the general expressions of
 180 $\hat{\phi}(\underline{\mathbf{x}}, \omega)$ and $\hat{\underline{\psi}}(\underline{\mathbf{x}}, \omega)$ are:

$$181 \quad \hat{\phi}(\underline{\mathbf{x}}, \omega) = A e^{-ik_x x} e^{-ik_y y} e^{-ik_{Pz} z} \quad (15)$$

$$182 \quad \hat{\underline{\psi}}(\underline{\mathbf{x}}, \omega) = [B \quad C \quad D]^T e^{-ik_x x} e^{-ik_y y} e^{-ik_{S_z} z} \quad (16)$$

183 where A , B , C , D are unknown coefficients to be determined by the boundary
 184 conditions, k_x is the wavenumber in the x -direction, k_y is the wavenumber in the
 185 y -direction, and k_{Pz} and k_{S_z} are respectively the wavenumbers in the z -direction for
 186 the P-wave and S-wave. Note that the signs of k_{Pz} and k_{S_z} should be chosen
 187 carefully to ensure that the waves propagate and/or attenuate in the positive
 188 z -direction. After substituting equations (15) and (16) into equations (13) and (14),
 189 the expressions for k_{Pz} and k_{S_z} can be obtained:

$$190 \quad k_{Pz}^2 = \frac{(\omega + ck_x)^2}{c_p^2} - k_x^2 - k_y^2 \quad (17)$$

$$191 \quad k_{S_z}^2 = \frac{(\omega + ck_x)^2}{c_s^2} - k_x^2 - k_y^2 \quad (18)$$

192 By substituting equation (12) into the expressions of the constitutive equations
 193 in frequency domain, the following relationships between the stresses and the
 194 potentials in frequency domain are obtained:

$$195 \quad \hat{\sigma}_{xx}(\underline{\mathbf{x}}, \omega) = \lambda \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \hat{\phi} + 2\mu \left(\frac{\partial^2 \hat{\phi}}{\partial x^2} + \frac{\partial^2 \hat{\psi}_z}{\partial x \partial y} - \frac{\partial^2 \hat{\psi}_y}{\partial z \partial x} \right) \quad (19)$$

$$196 \quad \hat{\sigma}_{yy}(\underline{\mathbf{x}}, \omega) = \lambda \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \hat{\phi} + 2\mu \left(\frac{\partial^2 \hat{\phi}}{\partial y^2} + \frac{\partial^2 \hat{\psi}_x}{\partial y \partial z} - \frac{\partial^2 \hat{\psi}_z}{\partial x \partial y} \right) \quad (20)$$

$$197 \quad \hat{\sigma}_{zz}(\underline{\mathbf{x}}, \omega) = \lambda \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \hat{\phi} + 2\mu \left(\frac{\partial^2 \hat{\phi}}{\partial z^2} + \frac{\partial^2 \hat{\psi}_y}{\partial z \partial x} - \frac{\partial^2 \hat{\psi}_x}{\partial y \partial z} \right) \quad (21)$$

$$198 \quad \hat{\sigma}_{xy}(\underline{\mathbf{x}}, \omega) = \mu \left[2 \frac{\partial^2 \hat{\phi}}{\partial x \partial y} + \frac{\partial^2 \hat{\psi}_x}{\partial z \partial x} - \frac{\partial^2 \hat{\psi}_y}{\partial y \partial z} - \left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} \right) \hat{\psi}_z \right] \quad (22)$$

199
$$\hat{\sigma}_{yz}(\underline{\mathbf{x}}, \omega) = \mu \left[2 \frac{\partial^2 \hat{\phi}}{\partial y \partial z} + \frac{\partial^2 \hat{\psi}_y}{\partial x \partial y} - \frac{\partial^2 \hat{\psi}_z}{\partial z \partial x} - \left(\frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} \right) \hat{\psi}_x \right] \quad (23)$$

200
$$\hat{\sigma}_{zx}(\underline{\mathbf{x}}, \omega) = \mu \left[2 \frac{\partial^2 \hat{\phi}}{\partial z \partial x} + \frac{\partial^2 \hat{\psi}_z}{\partial y \partial z} - \frac{\partial^2 \hat{\psi}_x}{\partial x \partial y} - \left(\frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial x^2} \right) \hat{\psi}_y \right] \quad (24)$$

201 in which $\hat{\psi}_x$, $\hat{\psi}_y$, and $\hat{\psi}_z$ are three components of $\hat{\underline{\Psi}}(\underline{\mathbf{x}}, \omega)$.

202 In addition, by combining the expressions of c_p and c_s with equations (17)

203 and (18), the relationship between Lamé constants can be obtained:

204
$$\lambda = - \frac{k_x^2 + k_y^2 + 2k_{pz}^2 - k_{sz}^2}{k_x^2 + k_y^2 + k_{pz}^2} \mu \quad (25)$$

205 2.3. Spectral element formulation

206 In the SEM, the response of an element is determined by its total wave field,
 207 which is the superposition of wave fields originating from different boundaries
 208 (Al-Khoury et al., 2001). In this method, the number of elements needed for a
 209 simulation is the same as the number of layers because one element is sufficient to
 210 simulate a whole layer, which makes it efficient for dynamic analysis of layered
 211 systems. In this section, a layer spectral element and a semi-infinite spectral element
 212 are formulated to simulate a layer and a half-space, respectively. The combinations
 213 of these two spectral elements are capable of modelling different layered systems.

214 2.3.1. Layer spectral element

215 As shown in Figure 3(a), the layer spectral element consists of two parallel
 216 horizontal rectangular surfaces, which constrain the waves to propagate within the
 217 element. The element vertically covers the whole simulated layer, and it horizontally
 218 extends to a certain distance after which the response caused by waves is negligible.
 219 In addition, the spectral element of a layer with thickness h is physically defined by
 220 two nodes located at $(0, 0, 0)$ and $(0, 0, h)$, each of which has three degrees of
 221 freedom. In the layer spectral element, the total potentials can be expressed as
 222 follows (Al-Khoury et al., 2001; van Dalen et al., 2015):

223
$$\hat{\phi}(\underline{\mathbf{x}}, \omega) = e^{-ik_x x} e^{-ik_y y} \left[A_1 e^{-ik_{pz} z} + A_2 e^{ik_{pz}(z-h)} \right] \quad (26)$$

$$224 \quad \underline{\hat{\Psi}}(\underline{\mathbf{x}}, \omega) = e^{-ik_x x} e^{-ik_y y} \begin{bmatrix} B_1 e^{-ik_{S_z} z} + B_2 e^{ik_{S_z}(z-h)} \\ C_1 e^{-ik_{S_z} z} + C_2 e^{ik_{S_z}(z-h)} \\ D_1 e^{-ik_{S_z} z} + D_2 e^{ik_{S_z}(z-h)} \end{bmatrix} \quad (27)$$

225 where A_1 , A_2 , B_1 , B_2 , C_1 , C_2 , D_1 , and D_2 are the unknown coefficients to be
 226 determined by boundary conditions. The first terms are the potentials of the wave
 227 fields originated from the top surface, while the second terms are the potentials of
 228 the wave fields reflected from the bottom surface.

229 The Fourier transform is applied to the Gauge condition in the moving
 230 coordinate system to obtain its spectral form, which is expressed as follows:

$$231 \quad \nabla \cdot \underline{\hat{\Psi}}(\underline{\mathbf{x}}, \omega) = 0 \quad (28)$$

232 After substituting equation (27) into equation (28), the following relationships are
 233 obtained:

$$234 \quad D_1 = -\frac{k_x B_1 + k_y C_1}{k_{S_z}} \quad (29)$$

$$235 \quad D_2 = \frac{k_x B_2 + k_y C_2}{k_{S_z}} \quad (30)$$

236 Equations (29) and (30) are substituted into equation (27) first to decrease the
 237 number of unknown coefficients. Then equations (26) and (27) are substituted into
 238 equation (12) to obtain the expressions for the displacements in frequency domain,
 239 which have the following forms:

$$240 \quad \hat{u}_x(\underline{\mathbf{x}}, \omega) = -ie^{-ik_x x} e^{-ik_y y} \begin{bmatrix} k_x \left(A_1 e^{-ik_{P_z} z} + A_2 e^{ik_{P_z}(z-h)} \right) - \frac{k_x k_y}{k_{S_z}} \left(B_1 e^{-ik_{S_z} z} - B_2 e^{ik_{S_z}(z-h)} \right) \\ -\frac{k_y^2 + k_{S_z}^2}{k_{S_z}} \left(C_1 e^{-ik_{S_z} z} - C_2 e^{ik_{S_z}(z-h)} \right) \end{bmatrix} \quad (31)$$

$$241 \quad \hat{u}_y(\underline{\mathbf{x}}, \omega) = -ie^{-ik_x x} e^{-ik_y y} \begin{bmatrix} k_y \left(A_1 e^{-ik_{P_z} z} + A_2 e^{ik_{P_z}(z-h)} \right) + \frac{k_x^2 + k_{S_z}^2}{k_{S_z}} \left(B_1 e^{-ik_{S_z} z} - B_2 e^{ik_{S_z}(z-h)} \right) \\ + \frac{k_x k_y}{k_{S_z}} \left(C_1 e^{-ik_{S_z} z} - C_2 e^{ik_{S_z}(z-h)} \right) \end{bmatrix} \quad (32)$$

$$242 \quad \hat{u}_z(\underline{\mathbf{x}}, \omega) = -i e^{-ik_x x} e^{-ik_y y} \left[\begin{array}{l} k_{pz} (A_1 e^{-ik_{pz} z} - A_2 e^{ik_{pz}(z-h)}) - k_y (B_1 e^{-ik_{sz} z} + B_2 e^{ik_{sz}(z-h)}) \\ + k_x (C_1 e^{-ik_{sz} z} + C_2 e^{ik_{sz}(z-h)}) \end{array} \right] \quad (33)$$

243 The displacements of the top node are notated as \hat{u}_x^1 , \hat{u}_y^1 , and \hat{u}_z^1 , and the
 244 displacements of the bottom node are notated as \hat{u}_x^2 , \hat{u}_y^2 , and \hat{u}_z^2 . Then, the
 245 coordinates of the nodes are substituted into equations (31) to (33) to obtain the
 246 nodal displacements, which can be expressed as follows:

$$247 \quad \hat{\underline{\mathbf{u}}}_0^e = \hat{\underline{\mathbf{L}}}_0^e \cdot \hat{\underline{\mathbf{a}}} \quad (34)$$

248 in which the superscript “e” means the corresponding quantities are expressed for an
 249 element, $\hat{\underline{\mathbf{u}}}_0^e = [\hat{u}_x^1 \quad \hat{u}_y^1 \quad \hat{u}_z^1 \quad \hat{u}_x^2 \quad \hat{u}_y^2 \quad \hat{u}_z^2]^T$ is the nodal displacement vector of the
 250 element, $\hat{\underline{\mathbf{a}}} = [A_1 \quad A_2 \quad B_1 \quad B_2 \quad C_1 \quad C_2]^T$ is the unknown coefficient vector, and
 251 $\hat{\underline{\mathbf{L}}}_0^e$ is a frequency and wavenumber dependent matrix which has the following
 252 expression:

$$253 \quad \hat{\underline{\mathbf{L}}}_0^e = i \left[\begin{array}{cccccc} -k_x & -k_x e^{-ik_{pz} h} & \frac{k_x k_y}{k_{sz}} & -\frac{k_x k_y}{k_{sz}} e^{-ik_{sz} h} & \frac{k_y^2 + k_{sz}^2}{k_{sz}} & -\frac{k_y^2 + k_{sz}^2}{k_{sz}} e^{-ik_{sz} h} \\ -k_y & -k_y e^{-ik_{pz} h} & -\frac{k_x^2 + k_{sz}^2}{k_{sz}} & \frac{k_x^2 + k_{sz}^2}{k_{sz}} e^{-ik_{sz} h} & -\frac{k_x k_y}{k_{sz}} & \frac{k_x k_y}{k_{sz}} e^{-ik_{sz} h} \\ -k_{pz} & k_{pz} e^{-ik_{pz} h} & k_y & k_y e^{-ik_{sz} h} & -k_x & -k_x e^{-ik_{sz} h} \\ -k_x e^{-ik_{pz} h} & -k_x & \frac{k_x k_y}{k_{sz}} e^{-ik_{sz} h} & -\frac{k_x k_y}{k_{sz}} & \frac{k_y^2 + k_{sz}^2}{k_{sz}} e^{-ik_{sz} h} & -\frac{k_y^2 + k_{sz}^2}{k_{sz}} \\ -k_y e^{-ik_{pz} h} & -k_y & -\frac{k_x^2 + k_{sz}^2}{k_{sz}} e^{-ik_{sz} h} & \frac{k_x^2 + k_{sz}^2}{k_{sz}} & -\frac{k_x k_y}{k_{sz}} e^{-ik_{sz} h} & \frac{k_x k_y}{k_{sz}} \\ -k_{pz} e^{-ik_{pz} h} & k_{pz} & k_y e^{-ik_{sz} h} & k_y & -k_x e^{-ik_{sz} h} & -k_x \end{array} \right]$$

254 By substituting equations (26) and (27) into equations (19) to (24) and
 255 considering equation (25), the transformed expressions of the stresses can be
 256 obtained:

$$257 \quad \hat{\sigma}_{xx}(\underline{\mathbf{x}}, \omega) = -\mu e^{-ik_x x} e^{-ik_y y} \left[\begin{array}{l} (k_x^2 - k_y^2 - 2k_{pz}^2 + k_{sz}^2) (A_1 e^{-ik_{pz} z} + A_2 e^{ik_{pz}(z-h)}) \\ - \frac{2k_x^2 k_y}{k_{sz}} (B_1 e^{-ik_{sz} z} - B_2 e^{ik_{sz}(z-h)}) - \frac{2k_x (k_y^2 + k_{sz}^2)}{k_{sz}} (C_1 e^{-ik_{sz} z} - C_2 e^{ik_{sz}(z-h)}) \end{array} \right] \quad (35)$$

$$258 \quad \hat{\sigma}_{yy}(\underline{\mathbf{x}}, \omega) = \mu e^{-ik_x x} e^{-ik_y y} \left[\begin{aligned} & (k_x^2 - k_y^2 + 2k_{pz}^2 - k_{sz}^2) (A_1 e^{-ik_{pz} z} + A_2 e^{ik_{pz}(z-h)}) \\ & - \frac{2k_y (k_x^2 + k_{sz}^2)}{k_{sz}} (B_1 e^{-ik_{sz} z} - B_2 e^{ik_{sz}(z-h)}) - \frac{2k_x k_y^2}{k_{sz}} (C_1 e^{-ik_{sz} z} - C_2 e^{ik_{sz}(z-h)}) \end{aligned} \right] \quad (36)$$

$$259 \quad \hat{\sigma}_{zz}(\underline{\mathbf{x}}, \omega) = \mu e^{-ik_x x} e^{-ik_y y} \left[\begin{aligned} & (k_x^2 + k_y^2 - k_{sz}^2) (A_1 e^{-ik_{pz} z} + A_2 e^{ik_{pz}(z-h)}) \\ & + 2k_y k_{sz} (B_1 e^{-ik_{sz} z} - B_2 e^{ik_{sz}(z-h)}) - 2k_x k_{sz} (C_1 e^{-ik_{sz} z} - C_2 e^{ik_{sz}(z-h)}) \end{aligned} \right] \quad (37)$$

$$260 \quad \hat{\sigma}_{xy}(\underline{\mathbf{x}}, \omega) = -\mu e^{-ik_x x} e^{-ik_y y} \left[\begin{aligned} & 2k_x k_y (A_1 e^{-ik_{pz} z} + A_2 e^{ik_{pz}(z-h)}) + \frac{k_x (k_x^2 - k_y^2 + k_{sz}^2)}{k_{sz}} (B_1 e^{-ik_{sz} z} - B_2 e^{ik_{sz}(z-h)}) \\ & + \frac{k_y (k_x^2 - k_y^2 - k_{sz}^2)}{k_{sz}} (C_1 e^{-ik_{sz} z} - C_2 e^{ik_{sz}(z-h)}) \end{aligned} \right] \quad (38)$$

$$261 \quad \hat{\sigma}_{yz}(\underline{\mathbf{x}}, \omega) = -\mu e^{-ik_x x} e^{-ik_y y} \left[\begin{aligned} & 2k_y k_{pz} (A_1 e^{-ik_{pz} z} - A_2 e^{ik_{pz}(z-h)}) + (k_x^2 - k_y^2 + k_{sz}^2) (B_1 e^{-ik_{sz} z} + B_2 e^{ik_{sz}(z-h)}) \\ & + 2k_x k_y (C_1 e^{-ik_{sz} z} + C_2 e^{ik_{sz}(z-h)}) \end{aligned} \right] \quad (39)$$

$$262 \quad \hat{\sigma}_{zx}(\underline{\mathbf{x}}, \omega) = -\mu e^{-ik_x x} e^{-ik_y y} \left[\begin{aligned} & 2k_x k_{pz} (A_1 e^{-ik_{pz} z} - A_2 e^{ik_{pz}(z-h)}) - 2k_x k_y (B_1 e^{-ik_{sz} z} + B_2 e^{ik_{sz}(z-h)}) \\ & + (k_x^2 - k_y^2 - k_{sz}^2) (C_1 e^{-ik_{sz} z} + C_2 e^{ik_{sz}(z-h)}) \end{aligned} \right] \quad (40)$$

263 Based on the Cauchy stress principle, for a certain surface, the relationship
264 between the surface traction vector $\underline{\mathbf{t}}$ and the Cauchy stress matrix $\underline{\underline{\sigma}}$ can be
265 expressed as follows:

$$266 \quad \underline{\mathbf{t}} = \underline{\underline{\sigma}} \cdot \underline{\mathbf{n}} \quad (41)$$

267 where $\underline{\mathbf{n}}$ is the unit outward normal vector of the surface. The tractions of the top

268 node are denoted as \hat{t}_x^1 , \hat{t}_y^1 , and \hat{t}_z^1 , and the tractions of the bottom node are

269 denoted as \hat{t}_x^2 , \hat{t}_y^2 , and \hat{t}_z^2 . On the basis of equation (41), the nodal tractions have

270 the following relationships with the nodal Cauchy stresses:

$$271 \quad \hat{t}_x^1 = -\hat{\sigma}_{zx}^1, \quad \hat{t}_y^1 = -\hat{\sigma}_{zy}^1, \quad \hat{t}_z^1 = -\hat{\sigma}_{zz}^1 \quad (42)$$

$$272 \quad \hat{t}_x^2 = \hat{\sigma}_{zx}^2, \quad \hat{t}_y^2 = \hat{\sigma}_{zy}^2, \quad \hat{t}_z^2 = \hat{\sigma}_{zz}^2 \quad (43)$$

273 in which $\hat{\sigma}_{zx}^1$, $\hat{\sigma}_{zy}^1$, and $\hat{\sigma}_{zz}^1$ are the Cauchy stresses of the top node, and $\hat{\sigma}_{zx}^2$, $\hat{\sigma}_{zy}^2$,

274 and $\hat{\sigma}_{zz}^2$ are the Cauchy stresses of the bottom node.

275 The nodal coordinates are substituted into equations (35) to (40) to derive the

276 nodal stresses, which are then incorporated into equations (42) and (43) to obtain the

277 expressions of nodal tractions, which are expressed as:

$$278 \quad \underline{\hat{\mathbf{t}}}_0^e = \underline{\hat{\mathbf{H}}}^e \cdot \underline{\hat{\mathbf{a}}} \quad (44)$$

279 where $\underline{\hat{\mathbf{t}}}_0^e = [\hat{t}_x^1 \quad \hat{t}_y^1 \quad \hat{t}_z^1 \quad \hat{t}_x^2 \quad \hat{t}_y^2 \quad \hat{t}_z^2]^T$ is the nodal traction vector of the element,

280 $\underline{\hat{\mathbf{H}}}^e$ is a frequency and wavenumber dependent matrix which has the following

281 form:

$$282 \quad \underline{\hat{\mathbf{H}}}^e = \mu \begin{bmatrix} 2k_x k_{pz} & -2k_x k_{pz} e^{-ik_{pz}h} & -2k_x k_y & -2k_x k_y e^{-ik_{sz}h} & k_3^2 & k_3^2 e^{-ik_{sz}h} \\ 2k_y k_{pz} & -2k_y k_{pz} e^{-ik_{pz}h} & k_2^2 & k_2^2 e^{-ik_{sz}h} & 2k_x k_y & 2k_x k_y e^{-ik_{sz}h} \\ -k_1^2 & -k_1^2 e^{-ik_{pz}h} & -2k_y k_{sz} & 2k_y k_{sz} e^{-ik_{sz}h} & 2k_x k_{sz} & -2k_x k_{sz} e^{-ik_{sz}h} \\ -2k_x k_{pz} e^{-ik_{pz}h} & 2k_x k_{pz} & 2k_x k_y e^{-ik_{sz}h} & 2k_x k_y & -k_3^2 e^{-ik_{sz}h} & -k_3^2 \\ -2k_y k_{pz} e^{-ik_{pz}h} & 2k_y k_{pz} & -k_2^2 e^{-ik_{sz}h} & -k_2^2 & -2k_x k_y e^{-ik_{sz}h} & -2k_x k_y \\ k_1^2 e^{-ik_{pz}h} & k_1^2 & 2k_y k_{sz} e^{-ik_{sz}h} & -2k_y k_{sz} & -2k_x k_{sz} e^{-ik_{sz}h} & 2k_x k_{sz} \end{bmatrix}$$

283 with $k_1^2 = k_x^2 + k_y^2 - k_{sz}^2$, $k_2^2 = k_x^2 - k_y^2 + k_{sz}^2$, and $k_3^2 = k_x^2 - k_y^2 - k_{sz}^2$.

284 By combining equations (34) and (44), the relationship between the nodal
285 traction vector and the nodal displacement vector is obtained, which can be
286 expressed as:

$$287 \quad \underline{\hat{\mathbf{t}}}_0^e = \underline{\hat{\mathbf{k}}}^e \cdot \underline{\hat{\mathbf{u}}}_0^e \quad (45)$$

288 in which $\underline{\hat{\mathbf{k}}}^e = \underline{\hat{\mathbf{H}}}^e \cdot (\underline{\hat{\mathbf{L}}}^e)^{-1}$ can be regarded as the element stiffness matrix, and the
289 detailed expressions of its components are shown in Appendix B.

290 2.3.2. Semi-infinite spectral element

291 As shown in Figure 3(b), the semi-infinite spectral element is composed of a
292 horizontal rectangular surface, and physically defined by a node located at (0, 0, 0)
293 with three degrees of freedom. In the semi-infinite spectral element, the waves
294 originated from the surface travel in the positive z -direction and no reflection occurs,
295 which physically means that the energy is radiated away. Actually, the semi-infinite
296 spectral element can be regarded as a special case of the layer spectral element that
297 only contains the top surface, which requires the coefficients of A_2 , B_2 , C_2 , and
298 D_2 in equations (26) and (27) to be zero. Accordingly, the transformed
299 displacements for the semi-infinite spectral element can be expressed as follows:

$$300 \quad \hat{u}_x(\underline{\mathbf{x}}, \omega) = -ie^{-ik_x x} e^{-ik_y y} \left(k_x A_1 e^{-ik_{pz} z} - \frac{k_x k_y}{k_{S_z}} B_1 e^{-ik_{S_z} z} - \frac{k_y^2 + k_{S_z}^2}{k_{S_z}} C_1 e^{-ik_{S_z} z} \right) \quad (46)$$

$$301 \quad \hat{u}_y(\underline{\mathbf{x}}, \omega) = -ie^{-ik_x x} e^{-ik_y y} \left(k_y A_1 e^{-ik_{pz} z} + \frac{k_x^2 + k_{S_z}^2}{k_{S_z}} B_1 e^{-ik_{S_z} z} + \frac{k_x k_y}{k_{S_z}} C_1 e^{-ik_{S_z} z} \right) \quad (47)$$

$$302 \quad \hat{u}_z(\underline{\mathbf{x}}, \omega) = -ie^{-ik_x x} e^{-ik_y y} \left(k_{pz} A_1 e^{-ik_{pz} z} - k_y B_1 e^{-ik_{S_z} z} + k_x C_1 e^{-ik_{S_z} z} \right) \quad (48)$$

303 After substituting the coordinates of the node, the nodal displacements can be
304 expressed as:

$$305 \quad \begin{bmatrix} \hat{u}_x^1 \\ \hat{u}_y^1 \\ \hat{u}_z^1 \end{bmatrix} = \mathbf{i} \begin{bmatrix} -k_x & \frac{k_x k_y}{k_{S_z}} & \frac{k_y^2 + k_{S_z}^2}{k_{S_z}} \\ -k_y & -\frac{k_x^2 + k_{S_z}^2}{k_{S_z}} & -\frac{k_x k_y}{k_{S_z}} \\ -k_{pz} & k_y & -k_x \end{bmatrix} \begin{bmatrix} A_1 \\ B_1 \\ C_1 \end{bmatrix} \quad (49)$$

306 The stresses in frequency domain become:

$$307 \quad \hat{\sigma}_{xx}(\underline{\mathbf{x}}, \omega) = -\mu e^{-ik_x x} e^{-ik_y y} \left[\begin{array}{l} (k_x^2 - k_y^2 - 2k_{pz}^2 + k_{S_z}^2) A_1 e^{-ik_{pz} z} \\ -\frac{2k_x^2 k_y}{k_{S_z}} B_1 e^{-ik_{S_z} z} - \frac{2k_x (k_y^2 + k_{S_z}^2)}{k_{S_z}} C_1 e^{-ik_{S_z} z} \end{array} \right] \quad (50)$$

$$308 \quad \hat{\sigma}_{yy}(\underline{\mathbf{x}}, \omega) = \mu e^{-ik_x x} e^{-ik_y y} \left[\begin{array}{l} (k_x^2 - k_y^2 + 2k_{pz}^2 - k_{S_z}^2) A_1 e^{-ik_{pz} z} \\ -\frac{2k_y (k_x^2 + k_{S_z}^2)}{k_{S_z}} B_1 e^{-ik_{S_z} z} - \frac{2k_x k_y^2}{k_{S_z}} C_1 e^{-ik_{S_z} z} \end{array} \right] \quad (51)$$

$$309 \quad \hat{\sigma}_{zz}(\underline{\mathbf{x}}, \omega) = \mu e^{-ik_x x} e^{-ik_y y} \left[(k_x^2 + k_y^2 - k_{S_z}^2) A_1 e^{-ik_{pz} z} + 2k_y k_{S_z} B_1 e^{-ik_{S_z} z} - 2k_x k_{S_z} C_1 e^{-ik_{S_z} z} \right] \quad (52)$$

$$310 \quad \hat{\sigma}_{xy}(\underline{\mathbf{x}}, \omega) = -\mu e^{-ik_x x} e^{-ik_y y} \left[\begin{array}{l} 2k_x k_y A_1 e^{-ik_{pz} z} + \frac{k_x (k_x^2 - k_y^2 + k_{S_z}^2)}{k_{S_z}} B_1 e^{-ik_{S_z} z} \\ + \frac{k_y (k_x^2 - k_y^2 - k_{S_z}^2)}{k_{S_z}} C_1 e^{-ik_{S_z} z} \end{array} \right] \quad (53)$$

$$311 \quad \hat{\sigma}_{yz}(\underline{\mathbf{x}}, \omega) = -\mu e^{-ik_x x} e^{-ik_y y} \left[2k_y k_{pz} A_1 e^{-ik_{pz} z} + (k_x^2 - k_y^2 + k_{S_z}^2) B_1 e^{-ik_{S_z} z} + 2k_x k_y C_1 e^{-ik_{S_z} z} \right] \quad (54)$$

$$312 \quad \hat{\sigma}_{zx}(\underline{\mathbf{x}}, \omega) = -\mu e^{-ik_x x} e^{-ik_y y} \left[2k_x k_{pz} A_1 e^{-ik_{pz} z} - 2k_x k_y B_1 e^{-ik_{S_z} z} + (k_x^2 - k_y^2 - k_{S_z}^2) C_1 e^{-ik_{S_z} z} \right] \quad (55)$$

313 After substituting the nodal coordinates and considering equation (42), the nodal

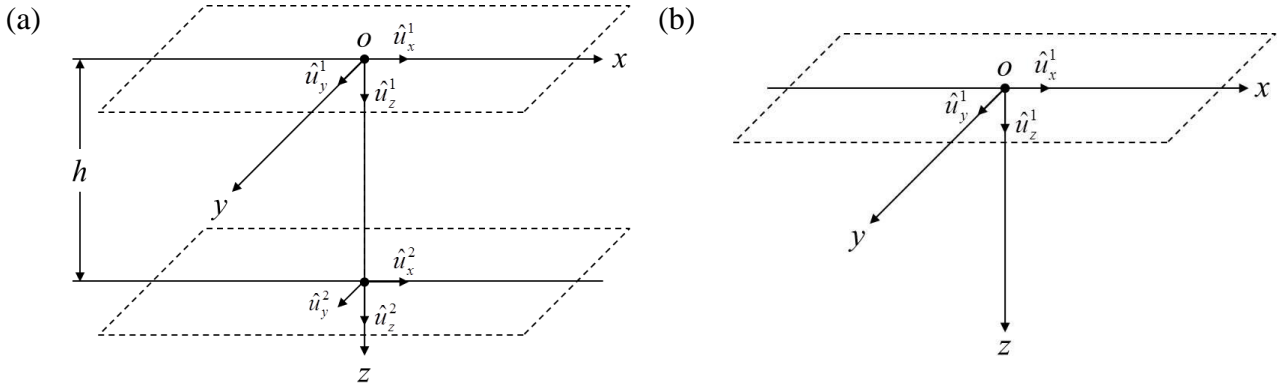
314 traction vector is expressed as:

$$315 \begin{bmatrix} \hat{t}_x^1 \\ \hat{t}_y^1 \\ \hat{t}_z^1 \end{bmatrix} = \mu \begin{bmatrix} 2k_x k_{Pz} & -2k_x k_y & k_x^2 - k_y^2 - k_{S_z}^2 \\ 2k_y k_{Pz} & k_x^2 - k_y^2 + k_{S_z}^2 & 2k_x k_y \\ -k_x^2 - k_y^2 + k_{S_z}^2 & -2k_y k_{S_z} & 2k_x k_{S_z} \end{bmatrix} \begin{bmatrix} A_1 \\ B_1 \\ C_1 \end{bmatrix} \quad (56)$$

316 By combining equations (49) and (56), the relationship between the nodal traction
317 vector and the nodal displacement vector is obtained:

$$318 \begin{bmatrix} \hat{t}_x^1 \\ \hat{t}_y^1 \\ \hat{t}_z^1 \end{bmatrix} = i\mu \begin{bmatrix} \frac{(k_x^2 + k_{S_z}^2)k_{Pz} + k_y^2 k_{S_z}}{k_x^2 + k_y^2 + k_{Pz} k_{S_z}} & \frac{k_x k_y (k_{Pz} - k_{S_z})}{k_x^2 + k_y^2 + k_{Pz} k_{S_z}} & \frac{k_x k_0^2}{k_x^2 + k_y^2 + k_{Pz} k_{S_z}} \\ \frac{k_x k_y (k_{Pz} - k_{S_z})}{k_x^2 + k_y^2 + k_{Pz} k_{S_z}} & \frac{(k_y^2 + k_{S_z}^2)k_{Pz} + k_x^2 k_{S_z}}{k_x^2 + k_y^2 + k_{Pz} k_{S_z}} & \frac{k_y k_0^2}{k_x^2 + k_y^2 + k_{Pz} k_{S_z}} \\ -\frac{k_x k_0^2}{k_x^2 + k_y^2 + k_{Pz} k_{S_z}} & -\frac{k_y k_0^2}{k_x^2 + k_y^2 + k_{Pz} k_{S_z}} & \frac{(k_x^2 + k_y^2 + k_{S_z}^2)k_{S_z}}{k_x^2 + k_y^2 + k_{Pz} k_{S_z}} \end{bmatrix} \begin{bmatrix} \hat{u}_x^1 \\ \hat{u}_y^1 \\ \hat{u}_z^1 \end{bmatrix} \quad (57)$$

319 with $k_0^2 = k_x^2 + k_y^2 + 2k_{Pz} k_{S_z} - k_{S_z}^2$.



320 Figure 3. Schematic representation of spectral elements: (a) Layer spectral element
321 and (b) Semi-infinite spectral element.

322 2.4. Boundary conditions

323 As shown in Figure 2, the external load is applied on the surface of the layered
324 system in the positive Z-direction. The load is assumed to be a uniformly distributed
325 traction over a rectangular area, the amplitude of the load varies with time. In the
326 moving coordinate system, the loading area is fixed and it can be expressed as
327 follows:

$$328 p_z(x, y, t) = h_0(x, y) p(t) \quad (58)$$

329 where $p_z(x, y, t)$ is the traction applied in the positive z-direction, $h_0(x, y)$ is the

330 spatial distribution function of the traction without dimension, $p(t)$ is the loading
 331 history function of the traction with dimension of force/area.

332 The spatial distribution function $h_0(x, y)$ can be expressed as follows:

$$333 \quad h_0(x, y) = H(x_0 - |x|)H(y_0 - |y|) \quad (59)$$

334 in which $H(\cdot)$ is the Heaviside function, $2x_0$ is the length of the loading area in
 335 the x -direction, and $2y_0$ is the width of the loading area in the y -direction.

336 According to equations (35) to (41), the distribution of tractions on the
 337 horizontal surfaces is in the form of $e^{-ik_x x} e^{-ik_y y}$, so $h_0(x, y)$ should be expressed in
 338 the same form to match the traction conditions. This can be achieved by using the
 339 Fourier series representation:

$$340 \quad h_0(x, y) = \sum_m \sum_n \tilde{h}_{xm} \tilde{h}_{yn} e^{-ik_{xm} x} e^{-ik_{yn} y} \quad (60)$$

341 where m is an integer that ranges from $-M$ to M and n is an integer that ranges from
 342 $-N$ to N , where M and N should be large enough to ensure the accuracy of the
 343 representation. In addition, $k_{xm} = m\pi/X_0$ and $k_{yn} = n\pi/Y_0$, where $2X_0$ is the
 344 length of the space window of interest in the x -direction, and $2Y_0$ is the
 345 corresponding width in the y -direction. The dimensions of the space window should
 346 be large enough to cover the influencing area of the applied load. The \tilde{h}_{xm} and \tilde{h}_{yn}
 347 are the Fourier coefficients defined as follows:

$$348 \quad \tilde{h}_{xm} = \frac{1}{2X_0} \int_{-X_0}^{X_0} H(x_0 - |x|) e^{ik_{xm} x} dx \quad (61)$$

$$349 \quad \tilde{h}_{yn} = \frac{1}{2Y_0} \int_{-Y_0}^{Y_0} H(y_0 - |y|) e^{ik_{yn} y} dy \quad (62)$$

350 The moving load considered in this paper is harmonically varying, hence the
 351 loading history function $p(t)$ can be expressed as follows:

$$352 \quad p(t) = p_0 e^{i\omega_b t} \quad (63)$$

353 in which p_0 is the amplitude of the traction, $\omega_0 = 2\pi f_0$ with ω_0 being the
 354 loading angular frequency and f_0 being the loading frequency. By applying the
 355 forward Fourier transform based on equation (10), the expression of $p(t)$ in the
 356 frequency domain is obtained:

$$357 \quad \hat{p}(\omega) = p_0 \delta(\omega - \omega_0) \quad (64)$$

358 where $\delta(\cdot)$ is the Dirac delta function.

359 Hence, the Fourier transformed expression of the applied surface traction is:

$$360 \quad \hat{p}_z(x, y, \omega) = \hat{p}(\omega) \sum_m \sum_n \tilde{h}_{xm} \tilde{h}_{yn} e^{-ik_{xm}x} e^{-ik_{yn}y} \quad (65)$$

361 In addition, it can be concluded from equation (65) that the expressions of the
 362 potentials should be represented as summations over all k_{xm} and k_{yn} to match the
 363 traction conditions.

364 2.5. Solution scheme

365 According to the SEM, the combination of several layer spectral elements on top
 366 of a semi-infinite spectral element is capable of simulating a layered system. The
 367 numbering and assembling of these elements follow the same procedure as in the
 368 traditional FEM. However, because of the wavenumber dependence of the element
 369 stiffness matrix in the SEM, the whole assembly process is done for each
 370 wavenumber combination. The total number of the nodes in the spectral element
 371 model of a layered system is notated as l , and the global system of equations for a
 372 certain wavenumber combination can be expressed as:

$$373 \quad \hat{\underline{\underline{T}}}_0^{mn}(\omega) = \hat{\underline{\underline{K}}}_0^{mn}(\omega) \cdot \hat{\underline{\underline{U}}}_0^{mn}(\omega) \quad (66)$$

374 in which the superscript “ mn ” indicates that the quantities correspond to a certain
 375 wavenumber combination of k_{xm} and k_{yn} , $\hat{\underline{\underline{T}}}_0^{mn}(\omega)$ is the global nodal traction
 376 vector with dimensions $3l$ by 1, $\hat{\underline{\underline{K}}}_0^{mn}(\omega)$ is the global stiffness matrix with
 377 dimensions $3l$ by $3l$, and $\hat{\underline{\underline{U}}}_0^{mn}(\omega)$ is the global nodal displacement vector with
 378 dimensions $3l$ by 1.

379 According to equation (65), the traction of the top node can be expressed as
 380 follows:

$$381 \quad \hat{p}_z(0,0,\omega) = \hat{p}(\omega) \sum_m \sum_n \tilde{h}_{xm} \tilde{h}_{yn} \quad (67)$$

382 Therefore, the global nodal traction vector for a certain wavenumber combination
 383 can be expressed as follows:

$$384 \quad \hat{\mathbf{T}}_0^{mn}(\omega) = \hat{p}(\omega) \tilde{h}_{xm} \tilde{h}_{yn} \mathbf{e}_3 \quad (68)$$

385 where \mathbf{e}_3 is a $3l$ by 1 unit vector with the third component being 1.

386 According to equation (66), the global nodal displacement vector for a certain
 387 wavenumber combination is calculated by:

$$388 \quad \hat{\mathbf{U}}_0^{mn}(\omega) = \hat{p}(\omega) \tilde{h}_{xm} \tilde{h}_{yn} \hat{\mathbf{G}}^{mn}(\omega) \cdot \mathbf{e}_3 \quad (69)$$

389 in which $\hat{\mathbf{G}}^{mn}(\omega)$, the inverse of $\hat{\mathbf{K}}^{mn}(\omega)$, can be regarded as the transfer matrix.

390 The nodal displacement vectors for different wavenumber combinations are summed
 391 to obtain the total nodal displacement vector caused by the applied load, such that:

$$392 \quad \hat{\mathbf{U}}_0(\omega) = \hat{p}(\omega) \sum_m \sum_n \tilde{h}_{xm} \tilde{h}_{yn} \hat{\mathbf{G}}^{mn}(\omega) \cdot \mathbf{e}_3 \quad (70)$$

393 where $\hat{\mathbf{U}}_0(\omega)$ is the total nodal displacement vector. Then, the inverse Fourier
 394 transform is used to obtain the nodal displacements in time domain, which can be
 395 expressed as:

$$396 \quad \mathbf{U}_0(t) = p_0 e^{i\omega_0 t} \sum_m \sum_n \tilde{h}_{xm} \tilde{h}_{yn} \hat{\mathbf{G}}^{mn}(\omega_0) \cdot \mathbf{e}_3 \quad (71)$$

397 According to equations (31) to (33), for a certain wavenumber combination, the
 398 displacement vector of the horizontal plane where a node is located equals the
 399 product of the nodal displacement vector and the term $e^{-ik_{xm}x} e^{-ik_{yn}y}$. Therefore, the
 400 displacements of points on the nodal horizontal planes can be calculated as:

$$401 \quad \mathbf{U}_0^{\text{plane}}(x,y,t) = p_0 e^{i\omega_0 t} \sum_m \sum_n \tilde{h}_{xm} \tilde{h}_{yn} e^{-ik_{xm}x} e^{-ik_{yn}y} \hat{\mathbf{G}}^{mn}(\omega_0) \cdot \mathbf{e}_3 \quad (72)$$

402 in which $\mathbf{U}_0^{\text{plane}}(x,y,t)$ is a vector with dimensions $3l$ by 1, which contains the
 403 displacement components of all the horizontal planes where nodes are located.

404 If the displacement field in a specific layer is desired, the following steps can be

405 followed. Firstly, one obtains the nodal displacement vector of this layer for a certain
 406 wavenumber combination from equation (69). Secondly, one calculates the
 407 corresponding coefficient vector via equation (34). Then, one substitutes these
 408 coefficients into equations (31) to (33) and sums over all the wavenumber
 409 combinations to compute the total displacement field in frequency domain. Finally,
 410 one applies the inverse Fourier transform via equation (11) to obtain the total
 411 displacement field within this layer in time domain. Corresponding stress and strain
 412 fields can also be calculated using the constitutive equations. The procedure to
 413 determine the response fields in a half-space is the same as that for a layer. It should
 414 be highlighted that all the calculated response fields are steady-state solutions, so
 415 they are changing over time with the same frequency as the applied load. For a
 416 certain response field (displacement field, stress field, or strain field), it can be
 417 expressed as follows:

$$418 \quad \underline{\mathbf{f}}(\underline{\mathbf{x}}, t) = \underline{\mathbf{F}}(\underline{\mathbf{x}}, \omega_0) e^{i\omega_0 t} \quad (73)$$

419 where $\underline{\mathbf{f}}(\underline{\mathbf{x}}, t)$ is a certain response field, $\underline{\mathbf{F}}(\underline{\mathbf{x}}, \omega_0)$ is the corresponding
 420 time-independent quantity which is normally complex-valued.

421 For different loading history functions, a certain component of the response field
 422 vector has different forms:

$$423 \quad f_k(\underline{\mathbf{x}}, t) = \begin{cases} \text{Re}[F_k(\underline{\mathbf{x}}, \omega_0) e^{i\omega_0 t}], & p(t) = p_0 \cos(\omega_0 t) \\ \text{Im}[F_k(\underline{\mathbf{x}}, \omega_0) e^{i\omega_0 t}], & p(t) = p_0 \sin(\omega_0 t) \end{cases} \quad (74)$$

424 in which the subscript “ k ” represents the considered component of the corresponding
 425 vector, $\text{Re}(\cdot)$ denotes the real part of a complex term, and $\text{Im}(\cdot)$ denotes the
 426 imaginary part of a complex term.

427 Assuming the loading history is in cosine form, equation (74) can be rewritten as
 428 follows accordingly:

$$429 \quad f_k(\underline{\mathbf{x}}, t) = \text{Re}[F_k(\underline{\mathbf{x}}, \omega_0)] \cos(\omega_0 t) - \text{Im}[F_k(\underline{\mathbf{x}}, \omega_0)] \sin(\omega_0 t) \quad (75)$$

430 Equation (75) indicates that a response field component equals to the real part (or
 431 imaginary part) of corresponding time-independent quantity at a specific time. In
 432 addition, equation (75) can also be written as:

433
$$f_k(\mathbf{x}, t) = |F_k(\mathbf{x}, \omega_0)| \cos[\omega_0 t + \theta_k(\mathbf{x}, \omega_0)] \quad (76)$$

434 where $|F_k(\mathbf{x}, \omega_0)|$ is the amplitude of vibration, and $\theta_k(\mathbf{x}, \omega_0)$ is the corresponding

435 phase angle which satisfies $\tan[\theta_k(\mathbf{x}, \omega_0)] = \frac{\text{Im}[F_k(\mathbf{x}, \omega_0)]}{\text{Re}[F_k(\mathbf{x}, \omega_0)]}$.

436 It can be concluded from equation (76) that, in the moving coordinate system, any
 437 response quantity of a point is harmonically varying with the same frequency as the
 438 applied load, but different points have different amplitudes and phase angles, which
 439 consequently forms a periodically varying profile over time. In this paper, all the
 440 results are presented in the moving coordinate system; corresponding results in the
 441 stationary coordinate system can be obtained based on the relationship between the
 442 coordinates. Working in the moving or stationary coordinate system should give
 443 equivalent solutions, because the physical nature of the problem is coordinate system
 444 independent (Louhghalam et al., 2013).

445 Although the presented model is formulated for elastic layered systems, it can be
 446 combined with different damping models to simulate layered systems with damping.
 447 Note that the damping models should be transformed to the moving coordinate system.
 448 Additionally, the presented model can handle different types of surface moving loads
 449 by changing the spatial distribution function and the loading history function of the
 450 applied load. In this paper, a hysteretic damping model defined in the
 451 frequency-wavenumber domain related to the moving coordinate system is used to
 452 simulate the damping effect in the system by replacing Young's modulus E with
 453 $E[1+i\eta\text{sgn}(\omega+ck_x)]$, in which η is the loss factor and $\text{sgn}(\cdot)$ is the signum function. In
 454 addition, in view of the practical speeds of vehicles on roadways, all the considered
 455 velocities of the load are taken smaller than the Rayleigh wave speed in layered
 456 systems.

457 **3. Model validation**

458 The accuracy of the presented model is validated in this section. At first, this
 459 model is implemented in a computer program to compute the response of a layered
 460 system by executing the following steps:

- 461 (1) For every wavenumber combination, it calculates the element stiffness
- 462 matrices and assembles them to the global stiffness matrix;
- 463 (2) It applies the boundary conditions and computes the global nodal

464 displacement vector by solving the corresponding global system of equations;

465 (3) It calculates the response field within a certain layer on the basis of the nodal
466 displacements and obtains the total response field by summing all the contributions at
467 different wavenumbers.

468 Then, two cases are used to compare the simulated results with corresponding
469 boundary element solutions given by Andersen and Nielsen (2003). These two cases
470 consider the surface deflections of a homogeneous half-space and a layered system
471 caused by a moving harmonic rectangular load. The points along the x -axis on the
472 surface are considered in the result comparison, where specifically the corresponding
473 amplitudes and phase angles of displacements in z -direction $u_z(\underline{\mathbf{x}}, t)$ are analysed.
474 Note that the loading amplitude used in the current paper is 10^6 times that in the
475 reference literature to make the results comparable with realistic pavement response.

476 Finally, the proposed model is validated by comparing simulated results with field
477 measurements. A pavement testing facility called LINTRACK (for more details see
478 Appendix C) was used to measure the strains of a pavement structure. The measured
479 maximum longitudinal strains (in moving direction) of the pavement structure are
480 used for comparison with corresponding simulated results.

481 *3.1. Response of a homogeneous half-space under a moving harmonic load*

482 This case considers the dynamic response of a homogeneous half-space caused by
483 a harmonically varying load moving on its surface. The load is uniformly distributed
484 over an area of 3 by 3 m², and the amplitude is 1/9 MPa (instead of 1/9 Pa in the
485 literature). The load varies at frequency of 40 Hz and moves in the positive direction
486 of the x -axis with velocities of 0, 50, 100 and 150 m/s. The structural parameter
487 values of the half-space are shown in Table 1, these parameter values are
488 corresponding to some unsaturated sandy soil with moderate stiffness. With
489 considering the practical speeds of vehicles on roadways, all the moving velocities of
490 the load considered in this paper are smaller than the Rayleigh wave speed in the
491 layered systems.

492 Table 1 Structural parameter values of the half-space

Layer	ρ kg/m ³	E MPa	ν -	η -	h m
1	1550	369	0.257	0.1	Infinite

493

494 The amplitudes and phase angles of the displacements in z -direction $u_z(\mathbf{x}, t)$ for
495 points along the x -axis on the surface of the half-space are calculated by the presented
496 SEM-based model. In order to obtain converged solutions, 4096×4096
497 wavenumbers are used, and this holds for all the results shown in this paper. The
498 simulated results are compared with those given in the reference literature (Andersen
499 and Nielsen, 2003) in Figure 4. The comparison shows that the results calculated by
500 these two methods are almost identical for different moving velocities, which proves
501 the accuracy of the proposed semi-infinite spectral element. In addition, some
502 observations can be made:

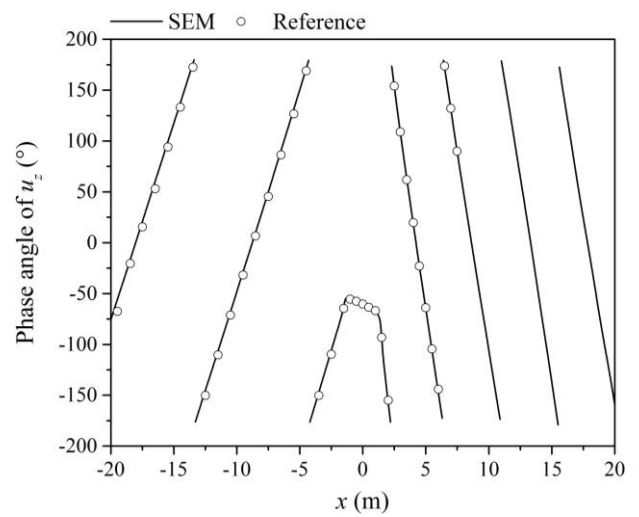
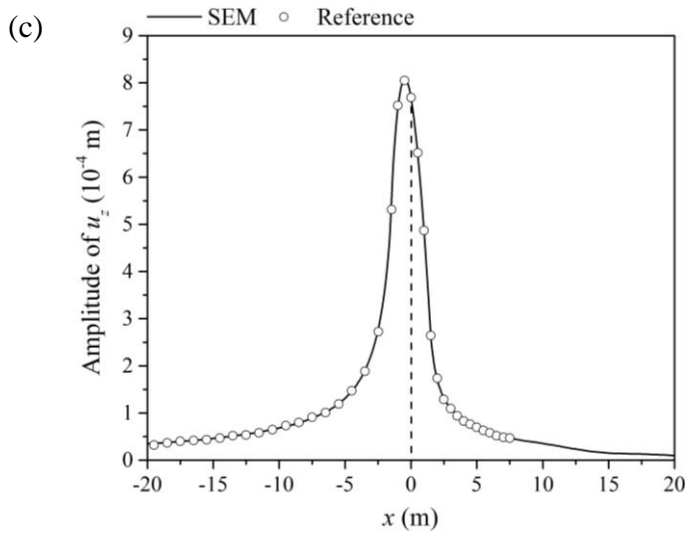
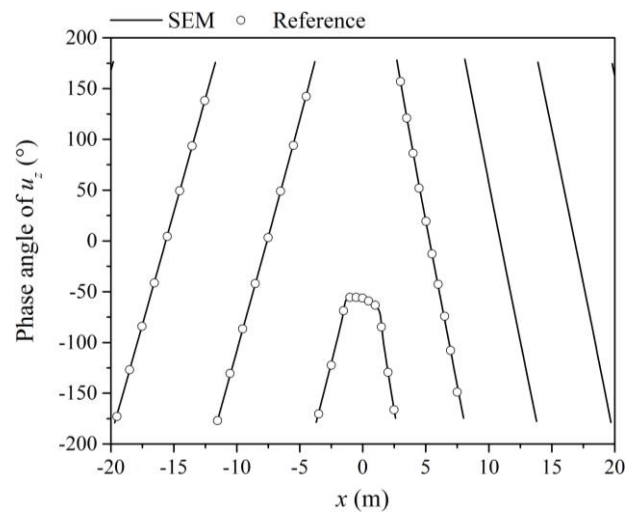
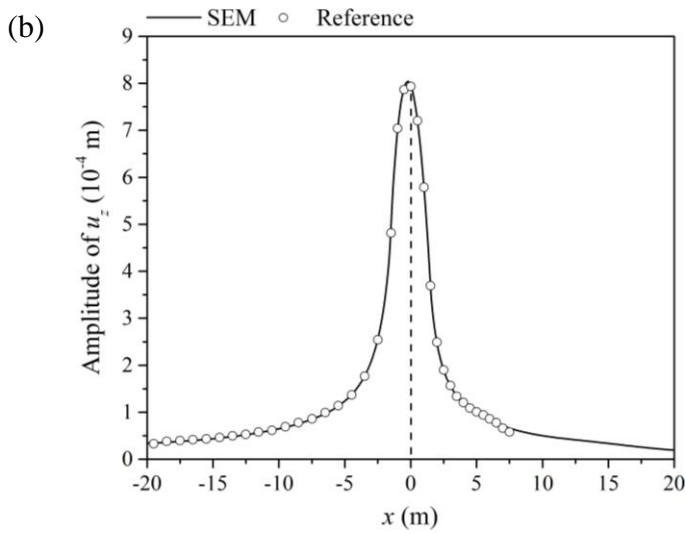
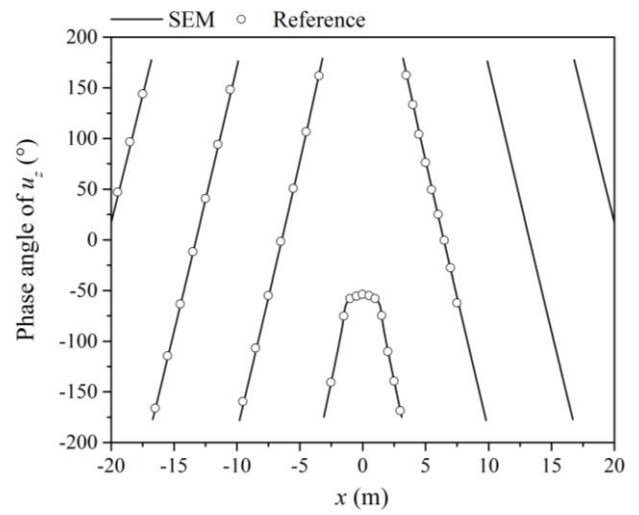
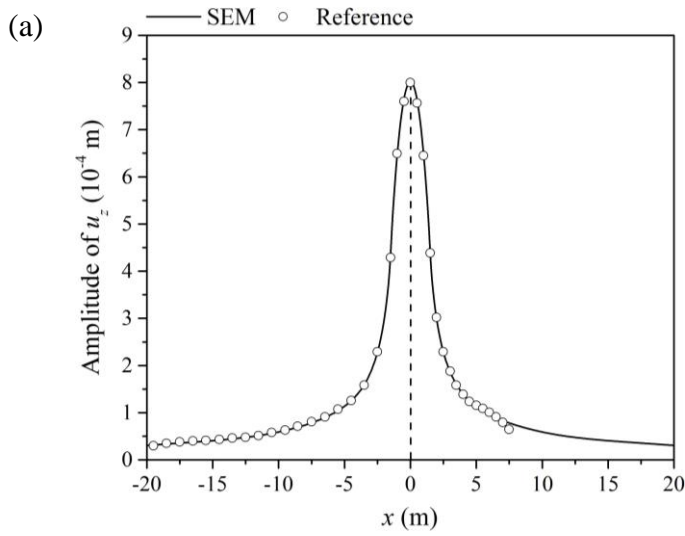
503 (1) When the load does not move, the displacement amplitude curve along the
504 x -axis is symmetric with respect to $x = 0$ and the displacement amplitude is maximum
505 at $x = 0$.

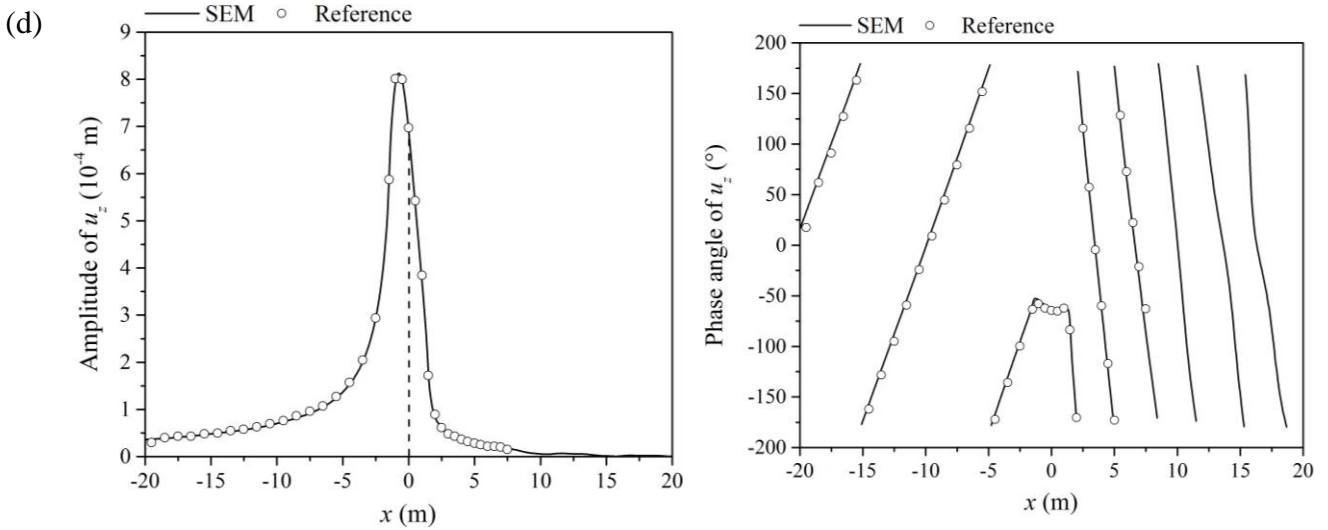
506 (2) When the load moves, the displacement amplitude curve along x -axis is
507 asymmetric with respect to $x = 0$. The displacement amplitudes at the points in front
508 of the load decrease more rapidly than on the other side, and this trend is more
509 obvious if the moving velocity is higher.

510 (3) When the moving velocity is increased, the position of the peak of the
511 displacement amplitude curve along the x -axis shifts to the left, and the maximum
512 value is slightly higher.

513 (4) When the moving velocity is zero, the phase angle curve along the x -axis is
514 symmetric with respect to $x = 0$. However, with increasing moving velocity, the phase
515 angles of u_z at points in front of the loading area change more rapidly, and
516 consequently the phase angle curve is denser on this side.

517





518 Figure 4. Comparison of u_z for points along the x -axis on the half-space surface
 519 calculated by different methods at different moving velocities:
 520 (a) $c = 0$ m/s, (b) $c = 50$ m/s, (c) $c = 100$ m/s, and (d) $c = 150$ m/s.

521 3.2. Response of a layered system under a moving harmonic load

522 This case considers the dynamic response of a layered system caused by a
 523 uniformly distributed harmonic load moving on its surface. The loading area and
 524 amplitude are the same as those in the case of the half-space, while the loading
 525 frequency is 20 Hz and the moving velocities are 0, 25, 50, and 75 m/s in the positive
 526 direction of the x -axis. The layered system is composed of a horizontal layer with a
 527 certain thickness and a homogeneous half-space. The structural parameter values of
 528 the layered system are shown in Table 2. The parameter values of this layered system
 529 correspond to two kinds of soil, and the soil in the layer is softer than that in the
 530 half-space.

531 Table 2 Structural parameter values of the layered system

Layers	ρ kg/m ³	E MPa	ν -	η -	h m
1	1500	100	0.40	0.1	2.0
2	2000	300	0.45	0.1	Infinite

532

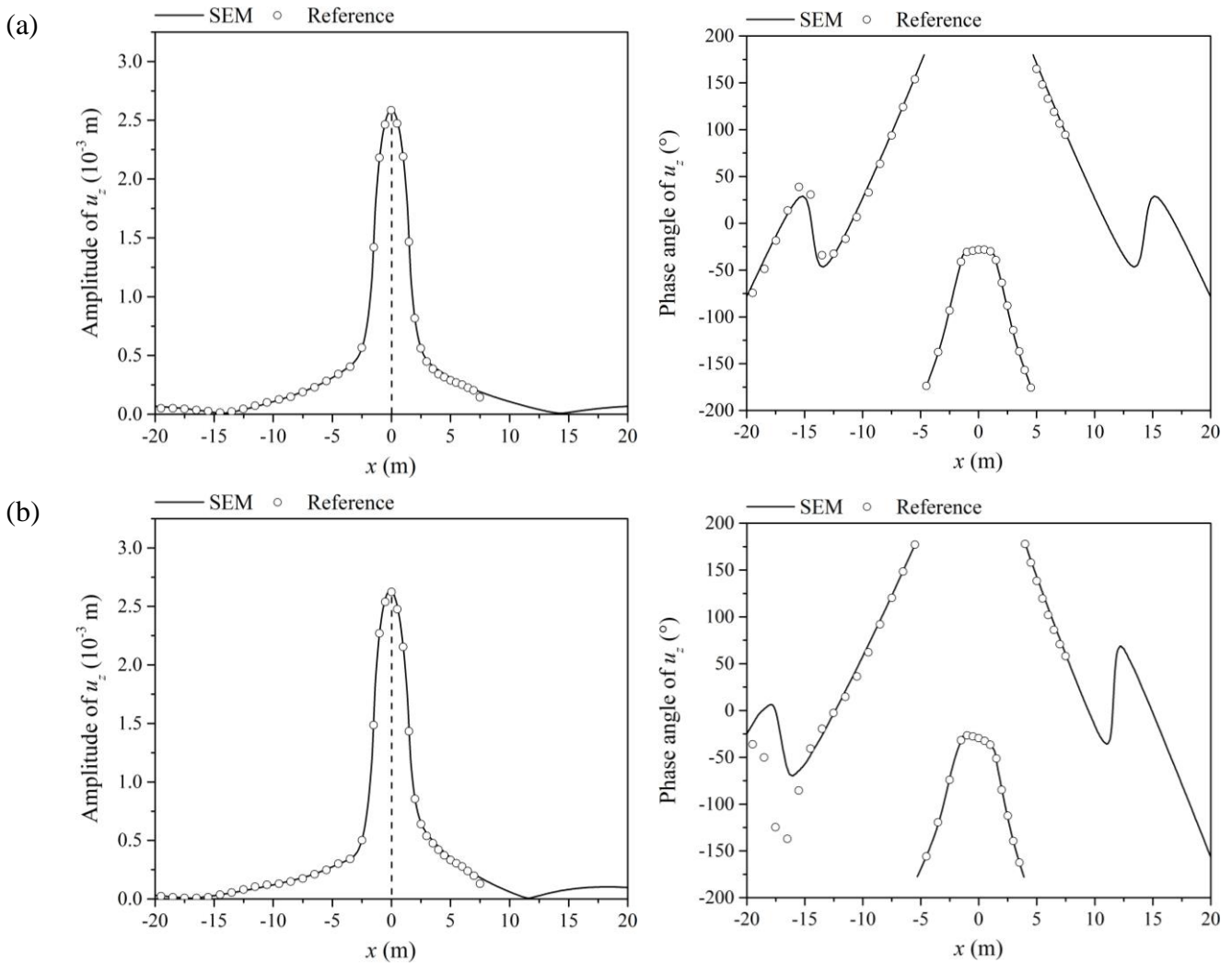
533 The displacements in the z -direction $u_z(\underline{x}, t)$ at points along the x -axis on the
 534 surface of the layered system are computed by the presented SEM-based model, and
 535 the corresponding amplitudes and phase angles are compared with those given in the
 536 reference literature (Andersen and Nielsen, 2003) in Figure 5. The comparison
 537 indicates that the results calculated by the different methods have good agreement for

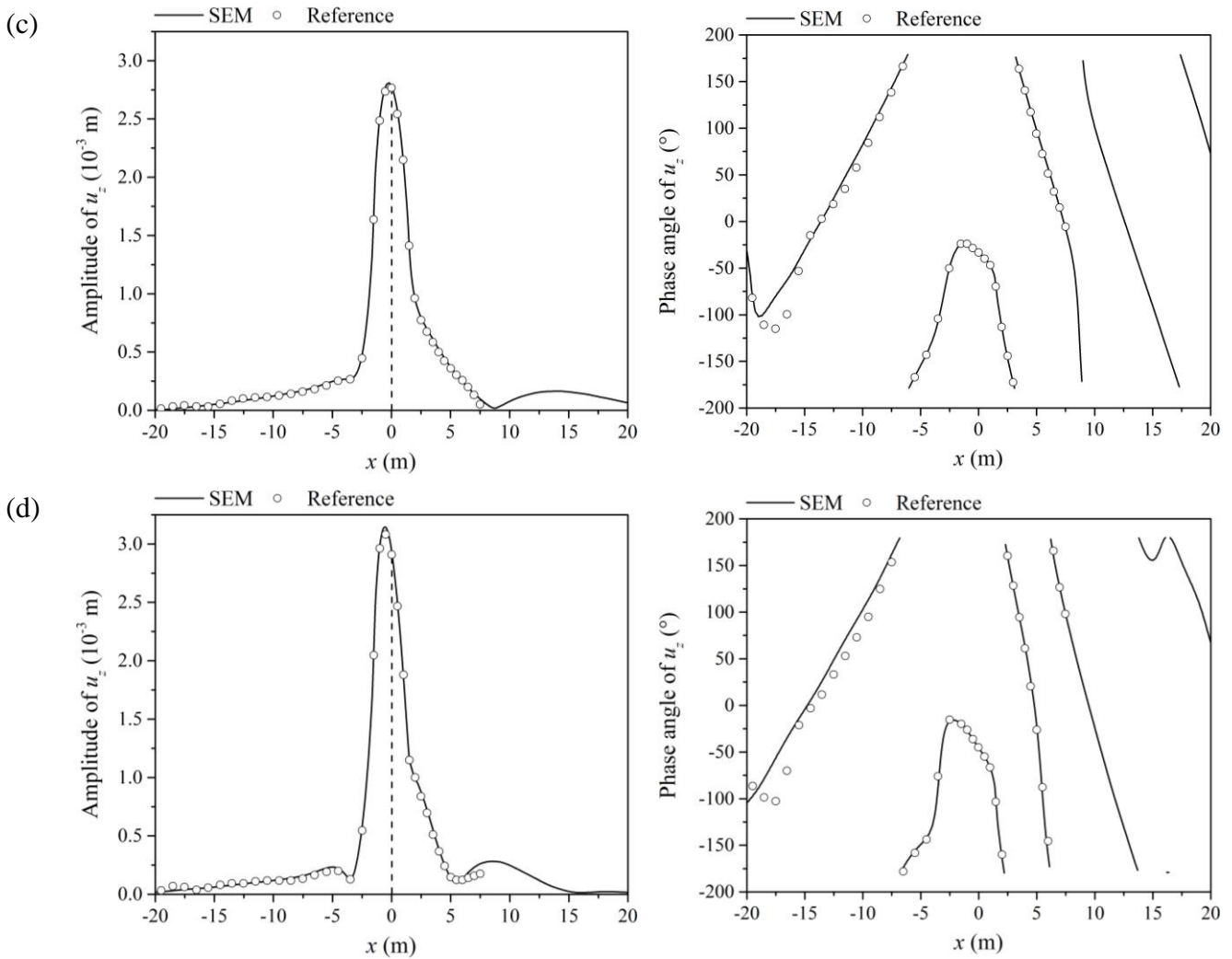
538 different moving velocities, which confirms the accuracy of the proposed layer
 539 spectral element and its combination with semi-infinite spectral element. Additionally,
 540 some observations can be made:

541 (1) The displacement amplitude curves along the x -axis on the layered system
 542 surface have similar changing trends as in the case of homogeneous half-space if the
 543 moving velocity is increased. However, the curves have some fluctuations for the
 544 layered system, which might be attributed to the complicated wave field in the layer
 545 spectral element. The half-space has higher stiffness than the layer above it, so the
 546 contribution of the reflected waves is pronounced.

547 (2) The phase angle curves along the x -axis on the layered system surface are
 548 more complicated, but the changing trends are similar to the case of homogeneous
 549 half-space when the moving velocity is increased.

550





551 Figure 5. Comparison of u_z for points along the x -axis on the layered system surface
 552 calculated by different methods at different moving velocities:
 553 (a) $c = 0$ m/s, (b) $c = 25$ m/s, (c) $c = 50$ m/s, and (d) $c = 75$ m/s.

554 The results shown in this section indicate that the displacement amplitude curve
 555 decreases more rapidly in front of the loading area, which is more obvious at higher
 556 velocities. The reason of this phenomenon is the uneven wave field distribution in
 557 the vicinity of the loading area caused by the Doppler effect (Lefeuvre-Mesgouez et
 558 al., 2002). The wavelengths of the waves in front of the loading area are shorter
 559 while the wavelengths of the waves behind the loading area are longer. Hence, the
 560 moving load has a smaller influencing area in front of the load than behind it.

561 3.3. Comparison with field measurements

562 LINTRACK was used to measure the strains of an asphalt pavement structure
 563 which was designed for a heavily loaded motorway. The first layer is porous asphalt

564 concrete (PAC), the second layer is newly applied stone asphalt concrete (New
565 STAC), the third layer is old STAC, the fourth layer is asphalt granulate cement
566 (AGRAC), and the foundation is a thick and well-compacted sand subgrade. The
567 parameter values of the tested pavement structure are shown in Table 3. Strain
568 gauges were installed at the bottom of the first layer in the longitudinal direction
569 (direction of movement). During the measurements, the LINTRACK belt moved
570 straight over the built-in strain gauges at a constant speed of 2.5 m/s. A constant
571 force was applied on the tire, while the tire pressure was maintained to be 900 kPa.

572 Table 3 Parameter values of the tested pavement structure

Layers	ρ kg/m ³	E MPa	ν -	η -	h m
PAC	2090	5525	0.25	0.1	0.05
New STAC	2395	7225	0.25	0.1	0.06
STAC	2395	8500	0.25	0.1	0.17
AGRAC	2141	5400	0.25	0.2	0.25
Subgrade	1733	126	0.4	0.4	Infinite

573

574 The maximum longitudinal strains of the pavement structure calculated by the
575 presented model are compared with those measured by the strain gauges. The results
576 are shown in Table 4, which indicates a good match between the simulated and
577 measured data, and thus further proves the accuracy of the presented model.

578 Table 4 Comparison between the simulated and measured maximum longitudinal
579 strains

Cases	Forces	Maximum longitudinal strains (10 ⁻⁶)	
	kN	Simulated	Measured
1	20	19	19
2	25	21	21
3	30	22	22
4	35	24	23
5	40	25	24
6	45	27	26

580

581 **4. Response analysis of a pavement structure**

582 This section focuses on a specific pavement structure subjected to a surface
583 moving load, and the parameter sensitivity analysis and stress analysis are conducted.

584 The reference loading conditions are described as follows:

- 585 • A uniformly distributed harmonically varying load moves in the positive
586 direction of the x -axis on the surface of a pavement structure;
- 587 • The moving velocity is $c = 25$ m/s (90 km/h);
- 588 • The loading frequency is $f_0 = 20$ Hz ;
- 589 • The loading amplitude is $p_0 = 550$ kPa ;
- 590 • The dimensions of loading area are $2x_0 = 2y_0 = 0.2683$ m ;
- 591 • The dimensions of the space window are $2X_0 = 2Y_0 = 400$ m .

592 The total force applied on the surface is about 39.6 kN, which is comparable to
593 the actual traffic load. The detailed reference parameter values of a pavement
594 structure are shown in Table 5.

595 Table 5 Reference parameter values of a pavement structure

Layers	ρ kg/m ³	E MPa	ν -	η -	h m
1	2400	1000	0.35	0.1	0.1
2	2000	500	0.35	0.1	0.3
3	1600	60	0.35	0.1	Infinite

596

597 *4.1. Parameter sensitivity analysis*

598 By using single factor analysis, the sensitivity of the displacement amplitude
599 curve along the x -axis on the pavement surface to different parameters is
600 investigated. The results are shown in Figure 6, in which the response of the
601 reference structural configuration to the reference loading is shown in solid line. It is
602 assumed that all the layers in the pavement structure have the same Poisson's ratio
603 and loss factor.

604 4.1.1. Sensitivity to moving velocity

605 The displacement amplitude curves along the x -axis on the pavement surface
606 caused by a load moving at different velocities ($c = 5, 25, \text{ and } 45 \text{ m/s}$) are shown in
607 Figure 6(a). The effect of the moving velocity is similar to that observed in the
608 previous section. However, for a realistic pavement structure, the curves are very
609 smooth because of the relatively high structural stiffness, and the Doppler effect is
610 not as significant as that observed for the layered soil systems. In addition, within the
611 range of analyses, the maximum of the displacement amplitude curve is slightly
612 affected by the moving velocity.

613 4.1.2. Sensitivity to loading frequency

614 The displacement amplitude curves along the x -axis on the pavement surface
615 caused by a moving load with different loading frequencies ($f_0 = 10, 20, \text{ and } 30 \text{ Hz}$)
616 are shown in Figure 6(b). The vertical displacement amplitudes of the surface points
617 along the moving direction are smaller if the loading frequency is higher, which
618 might be the result of the damping mechanism playing a more pronounced role.

619 4.1.3. Sensitivity to loading area

620 The displacement amplitude curves along the x -axis on the pavement surface
621 caused by a moving load with different loading areas ($s_0 = 0.036, 0.072, \text{ and } 0.108$
622 m^2) but the same amplitude of the total force (39.6 kN) are shown in Figure 6(c). It
623 can be seen that the maximum of the curve is higher if the loading area is smaller,
624 which is caused by the increase of the loading pressure. However, the differences
625 appear only in the close vicinity of the loading area, the displacement amplitudes of
626 points outside are almost identical. Therefore, if the applied force is the same, the
627 effect of the loading area is localised in the close vicinity of the load.

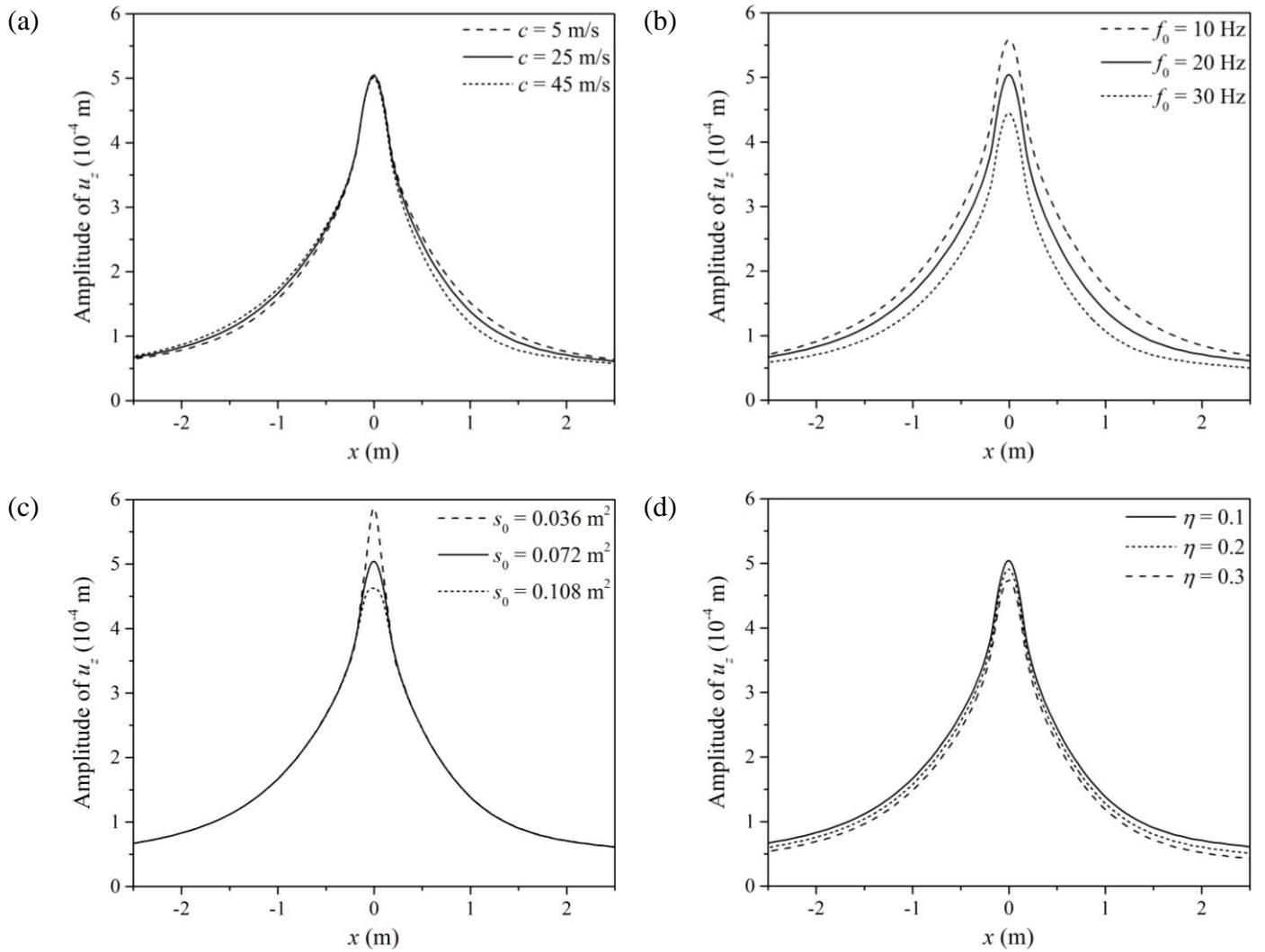
628 4.1.4. Sensitivity to loss factor

629 The surface displacement amplitude curves along the x -axis for pavement
630 structures with different loss factors ($\eta = 0.1, 0.2, \text{ and } 0.3$) under reference loading
631 conditions are shown in Figure 6(d). It can be seen that the curve is slightly lower if
632 the loss factor is higher. More energy is dissipated for a system with higher loss
633 factor, which results in smaller displacements.

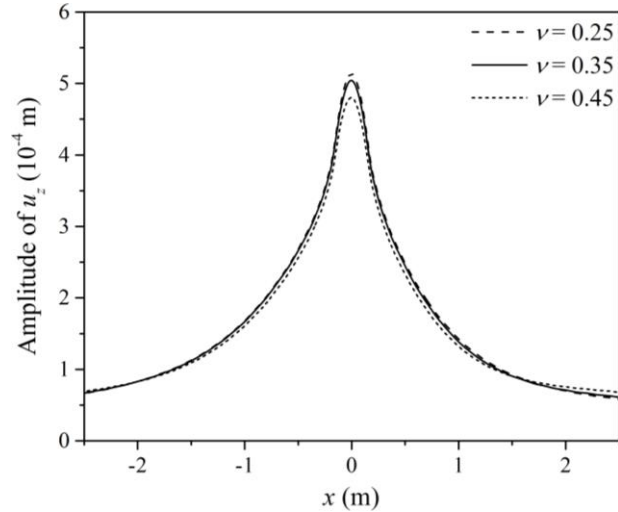
634 4.1.5. Sensitivity to Poisson's ratio

635 The surface displacement amplitude curves along the x -axis for pavement
 636 structures with different Poisson's ratios ($\nu = 0.25, 0.35, \text{ and } 0.45$) under reference
 637 loading conditions are shown in Figure 6(e). The maximum of the curve is slightly
 638 smaller if the Poisson's ratio is larger, while the displacement amplitudes of points
 639 outside the loading area are almost unaffected.

640

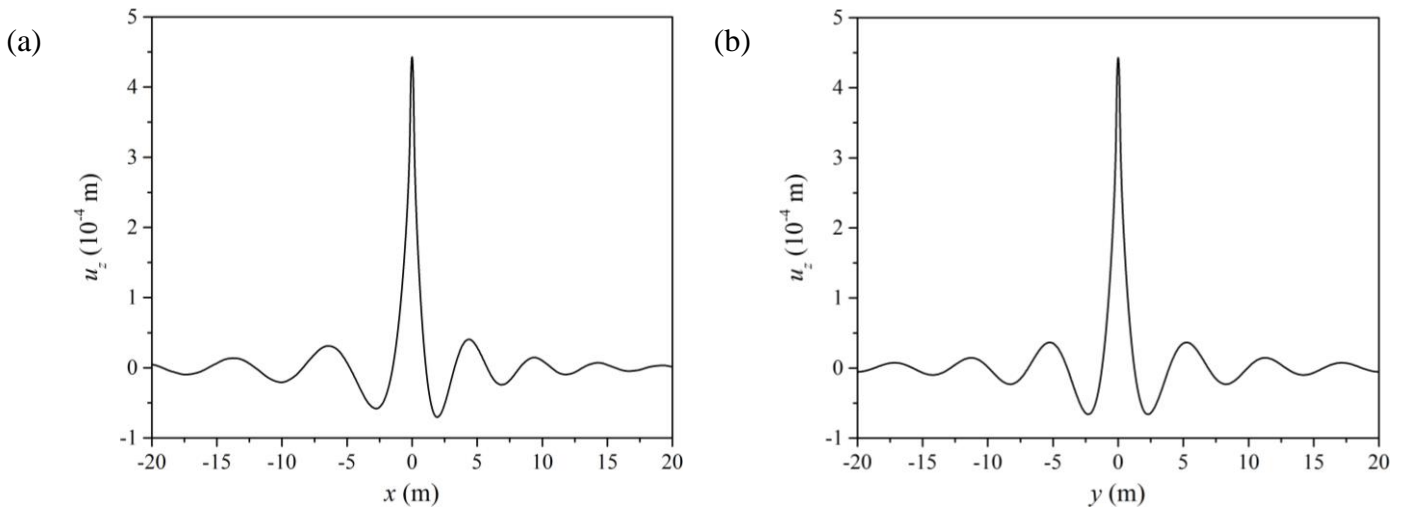


(e)



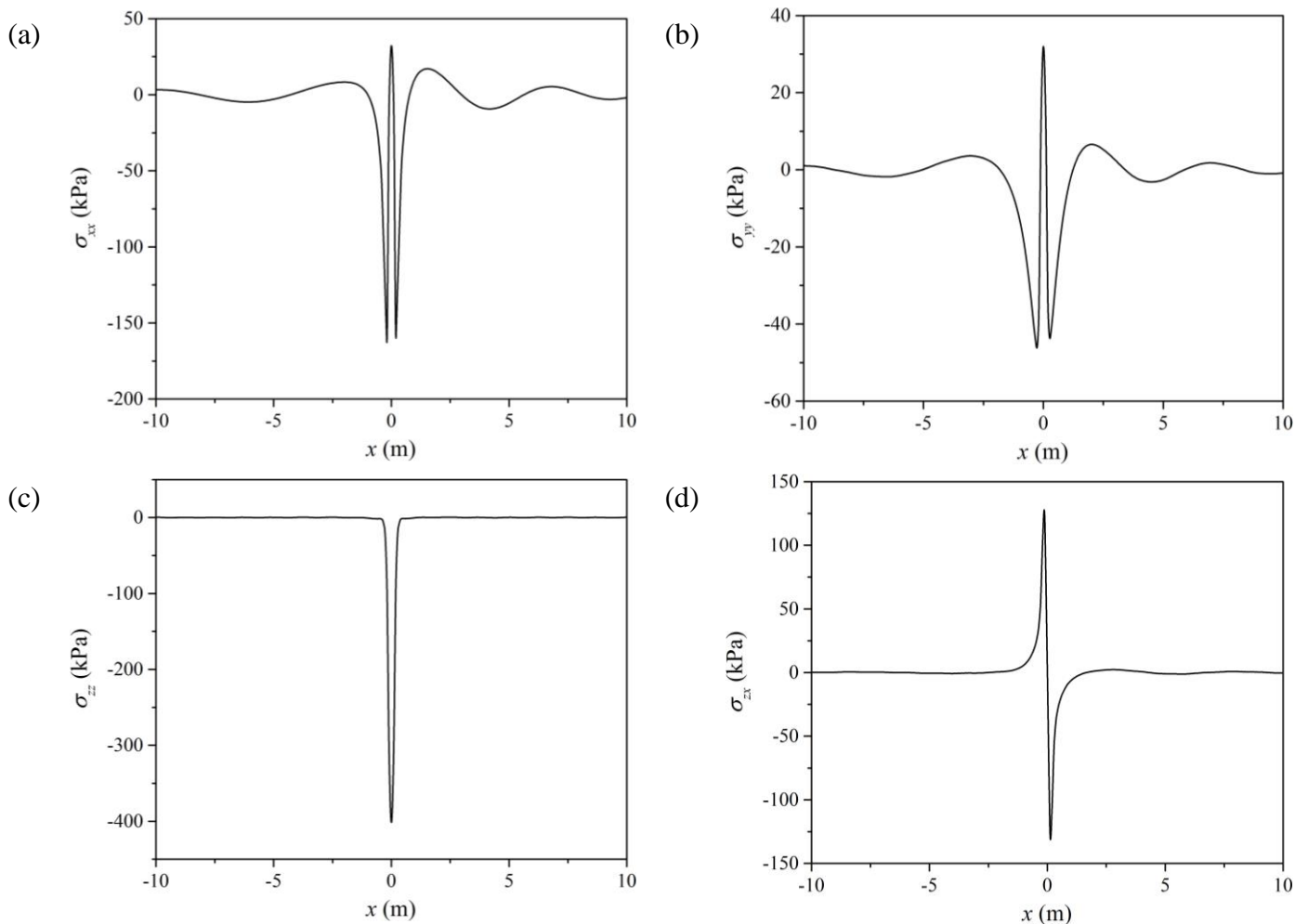
641 Figure 6. Sensitivity of the amplitudes of u_z for points along the x -axis on the
642 pavement surface to different parameters: (a) Moving velocity, (b) Loading
643 frequency, (c) Loading area, (d) Loss factor, and (e) Poisson's ratio.

644 It should be highlighted that Figure 6 only shows the amplitudes of u_z for points
645 along the x -axis on the surface. In reality, all quantities at all points are harmonically
646 varying, as shown in equation (76). Furthermore, for a pavement structure with the
647 reference structural configuration subjected to the reference loading condition, the
648 profiles of u_z for points along different axes on the pavement surface for $t = 0$ are
649 shown in Figure 7. The results show that the profile of u_z is asymmetric along the
650 x -axis while symmetric along the y -axis, which means the Doppler effect appears
651 only in the moving direction.



652 Figure 7. Profiles of u_z for points along different axes on the pavement surface for $t =$
653 0: (a) x -axis and (b) y -axis.

655 For a pavement structure with the reference loading and structural configuration,
 656 the stresses of points along the x -axis at depth 0.1 m are simulated by the presented
 657 model. The results for $t = 0$ are shown in Figure 8, which indicates that the points
 658 under the loading area are most critical. For these points, the maximum stress
 659 component is σ_{zz} , which is followed by σ_{xx} , σ_{zx} , and σ_{yy} . In addition, the stress
 660 components of σ_{xy} and σ_{yz} are negligibly small. The stresses calculated by the
 661 presented model could be used for pavement structural design to ensure its
 662 durability.



663 Figure 8. Stresses of points along the x -axis at depth 0.1 m for $t = 0$:

664 (a) σ_{xx} , (b) σ_{yy} , (c) σ_{zz} , and (d) σ_{zx} .

665 5. Conclusions and recommendations

666 This paper proposes a SEM-based model which can be used to analyse the

667 dynamic response of layered systems caused by a moving load. Based on the
668 discussion shown in this paper, the following conclusions can be drawn:

669 (1) The proposed model is robust for the dynamic analysis of layered systems
670 under a moving load, and this model is a potential tool for pavement structural
671 design.

672 (2) The displacement amplitude curves and phase angle curves along the
673 moving direction are asymmetric when the load moves, and this asymmetry is more
674 dominant if the moving velocity is higher. The reason of this phenomenon is the
675 inhomogeneous wave field distribution caused by the Doppler effect. However, the
676 moving velocity only has slight effect on the maximum of the surface displacement
677 amplitude curve within the scope of analysis.

678 (3) The surface displacement amplitude curve will be lower if the loading
679 frequency is higher or the loss factor is bigger, and the effect of the former is more
680 dominant.

681 (4) If the amplitude of the applied total force is constant, the loading area only
682 has influence on the displacement amplitudes of points in the close vicinity of the
683 load.

684 (5) The Poisson's ratio has slight effect on the maximum of the displacement
685 amplitude curve, and it almost does not affect the displacement amplitudes of points
686 outside the loading area.

687 (6) The presented model is a promising parameter back-calculation engine for
688 pavement quality evaluation.

689 This paper proposed a 3D dynamic model for elastic layered systems under a
690 moving load, which is combined with a hysteretic damping model to analyse the
691 response of a pavement structure caused by a moving harmonic load. In order to
692 consider the frequency-dependent viscous effect in pavement structures, it is
693 recommended to use more suitable damping models.

694

695 **Conflict of interest**

696 None.

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701 Civil Engineering and Geosciences, Delft University of Technology.

702 **Appendix A**

703 In the moving coordinate system, the Navier's equation has the following form:

$$704 \quad (\lambda + \mu) \nabla \nabla \cdot \underline{\mathbf{u}} + \mu \nabla^2 \underline{\mathbf{u}} = \rho \left(\frac{\partial}{\partial t} - \underline{\mathbf{c}} \cdot \frac{\partial}{\partial \underline{\mathbf{x}}} \right)^2 \underline{\mathbf{u}} \quad (\text{A1.1})$$

705 The Helmholtz decomposition of the displacement field is expressed as:

$$706 \quad \underline{\mathbf{u}} = \nabla \phi + \nabla \times \underline{\boldsymbol{\psi}} \quad (\text{A1.2})$$

707 By substituting equation (A1.2) into equation (A1.1), considering the identities of

708 $\nabla \cdot \nabla \phi = \nabla^2 \phi$ and $\nabla \cdot \nabla \times \underline{\boldsymbol{\psi}} = 0$, and interchanging the order of the operators gives:

$$709 \quad \nabla \left[(\lambda + 2\mu) \nabla^2 \phi - \rho \left(\frac{\partial}{\partial t} - \underline{\mathbf{c}} \cdot \frac{\partial}{\partial \underline{\mathbf{x}}} \right)^2 \phi \right] + \nabla \times \left[\mu \nabla^2 \underline{\boldsymbol{\psi}} - \rho \left(\frac{\partial}{\partial t} - \underline{\mathbf{c}} \cdot \frac{\partial}{\partial \underline{\mathbf{x}}} \right)^2 \underline{\boldsymbol{\psi}} \right] = \underline{\mathbf{0}} \quad (\text{A1.3})$$

710 This equation will be satisfied if the terms in the square brackets vanish, hence:

$$711 \quad \nabla^2 \phi - \frac{1}{c_p^2} \left(\frac{\partial}{\partial t} - \underline{\mathbf{c}} \cdot \frac{\partial}{\partial \underline{\mathbf{x}}} \right)^2 \phi = 0 \quad (\text{A1.4})$$

$$712 \quad \nabla^2 \underline{\boldsymbol{\psi}} - \frac{1}{c_s^2} \left(\frac{\partial}{\partial t} - \underline{\mathbf{c}} \cdot \frac{\partial}{\partial \underline{\mathbf{x}}} \right)^2 \underline{\boldsymbol{\psi}} = \underline{\mathbf{0}} \quad (\text{A1.5})$$

713 with $c_p = \sqrt{(\lambda + 2\mu) / \rho}$ and $c_s = \sqrt{\mu / \rho}$.

714 If the velocity vector has the form of $\underline{\mathbf{c}} = [c \ 0 \ 0]^T$, then the equations (A1.4)

715 and (A1.5) become:

$$716 \quad \nabla^2 \phi - \frac{1}{c_p^2} \left(\frac{\partial}{\partial t} - c \frac{\partial}{\partial x} \right)^2 \phi = 0 \quad (\text{A1.6})$$

$$717 \quad \nabla^2 \underline{\boldsymbol{\psi}} - \frac{1}{c_s^2} \left(\frac{\partial}{\partial t} - c \frac{\partial}{\partial x} \right)^2 \underline{\boldsymbol{\psi}} = \underline{\mathbf{0}} \quad (\text{A1.7})$$

718

719 **Appendix B**

720 The element stiffness matrix $\hat{\underline{\mathbf{k}}}^e$ of the layer spectral element can be expressed

721 as follows:

$$722 \quad \hat{\underline{\mathbf{k}}}^e = \begin{bmatrix} \hat{k}_{11}^e & \hat{k}_{12}^e & \hat{k}_{13}^e & \hat{k}_{14}^e & \hat{k}_{15}^e & \hat{k}_{16}^e \\ \hat{k}_{12}^e & \hat{k}_{22}^e & \hat{k}_{23}^e & \hat{k}_{15}^e & \hat{k}_{25}^e & \hat{k}_{26}^e \\ -\hat{k}_{13}^e & -\hat{k}_{23}^e & \hat{k}_{33}^e & \hat{k}_{16}^e & \hat{k}_{26}^e & \hat{k}_{36}^e \\ \hat{k}_{14}^e & \hat{k}_{15}^e & -\hat{k}_{16}^e & \hat{k}_{11}^e & \hat{k}_{12}^e & -\hat{k}_{13}^e \\ \hat{k}_{15}^e & \hat{k}_{25}^e & -\hat{k}_{26}^e & \hat{k}_{12}^e & \hat{k}_{22}^e & -\hat{k}_{23}^e \\ -\hat{k}_{16}^e & -\hat{k}_{26}^e & \hat{k}_{36}^e & \hat{k}_{13}^e & \hat{k}_{23}^e & \hat{k}_{33}^e \end{bmatrix}$$

723 in which

$$724 \quad \hat{k}_{11}^e = \frac{i\mu}{\Delta} \left\{ 2k_{Pz} \left[k_x^4 + k_x^2 k_y^2 + (k_x^2 - 2k_y^2) k_{S_z}^2 \right] \left[e^{-2i(k_{Pz} + k_{S_z})h} + e^{-2ik_{S_z}h} \right] - (k_x^2 + k_y^2 + k_{Pz} k_{S_z}) \left[(k_x^2 + k_{S_z}^2) k_{Pz} + k_y^2 k_{S_z} \right] \left[e^{-2i(k_{Pz} + 2k_{S_z})h} + 1 \right] \right. \\ \left. + 8k_y^2 k_{Pz} k_{S_z}^2 \left[e^{-i(k_{Pz} + k_{S_z})h} + e^{-i(k_{Pz} + 3k_{S_z})h} \right] - (k_x^2 + k_y^2 - k_{Pz} k_{S_z}) \left[(k_x^2 + k_{S_z}^2) k_{Pz} - k_y^2 k_{S_z} \right] \left(e^{-2ik_{Pz}h} + e^{-4ik_{S_z}h} \right) \right\}$$

$$725 \quad \hat{k}_{12}^e = \frac{i\mu k_x k_y}{\Delta} \left\{ 2k_{Pz} (k_x^2 + k_y^2 + 3k_{S_z}^2) \left[e^{-2i(k_{Pz} + k_{S_z})h} + e^{-2ik_{S_z}h} \right] - (k_x^2 + k_y^2 + k_{Pz} k_{S_z}) (k_{Pz} - k_{S_z}) \left[e^{-2i(k_{Pz} + 2k_{S_z})h} + 1 \right] \right. \\ \left. - 8k_{Pz} k_{S_z}^2 \left[e^{-i(k_{Pz} + k_{S_z})h} + e^{-i(k_{Pz} + 3k_{S_z})h} \right] - (k_x^2 + k_y^2 - k_{Pz} k_{S_z}) (k_{Pz} + k_{S_z}) \left(e^{-2ik_{Pz}h} + e^{-4ik_{S_z}h} \right) \right\}$$

$$726 \quad \hat{k}_{13}^e = -\frac{i\mu k_x}{\Delta} \left\{ 2 \left[(k_x^2 + k_y^2)^2 - (k_x^2 + k_y^2 - 2k_{Pz}^2) k_{S_z}^2 \right] \left[e^{-2i(k_{Pz} + k_{S_z})h} - e^{-2ik_{S_z}h} \right] - (k_x^2 + k_y^2 + k_{Pz} k_{S_z}) (k_x^2 + k_y^2 + 2k_{Pz} k_{S_z} - k_{S_z}^2) \left[e^{-2i(k_{Pz} + 2k_{S_z})h} - 1 \right] \right. \\ \left. + 4k_{Pz} k_{S_z} \left[3(k_x^2 + k_y^2) - k_{S_z}^2 \right] \left[e^{-i(k_{Pz} + 3k_{S_z})h} - e^{-i(k_{Pz} + k_{S_z})h} \right] - (k_x^2 + k_y^2 - k_{Pz} k_{S_z}) (k_x^2 + k_y^2 - 2k_{Pz} k_{S_z} - k_{S_z}^2) \left(e^{-2ik_{Pz}h} - e^{-4ik_{S_z}h} \right) \right\}$$

$$727 \quad \hat{k}_{14}^e = -\frac{2i\mu}{\Delta} \left\{ 2k_{Pz} \left[k_x^4 + k_x^2 k_y^2 + (k_x^2 + 4k_y^2) k_{S_z}^2 \right] e^{-i(k_{Pz} + 2k_{S_z})h} - k_{S_z} (k_y^4 + k_x^2 k_y^2 + k_x^2 k_{Pz}^2 + 2k_y^2 k_{Pz} k_{S_z} + k_{Pz}^2 k_{S_z}^2) \left[e^{-i(2k_{Pz} + 3k_{S_z})h} + e^{-ik_{S_z}h} \right] \right. \\ \left. - k_x^2 k_{Pz} (k_x^2 + k_y^2 + k_{S_z}^2) \left[e^{-ik_{Pz}h} + e^{-i(k_{Pz} + 4k_{S_z})h} \right] + k_{S_z} (k_y^4 + k_x^2 k_y^2 + k_x^2 k_{Pz}^2 - 2k_y^2 k_{Pz} k_{S_z} + k_{Pz}^2 k_{S_z}^2) \left[e^{-i(2k_{Pz} + k_{S_z})h} + e^{-3ik_{S_z}h} \right] \right\}$$

$$728 \quad \hat{k}_{15}^e = -\frac{2i\mu k_x k_y}{\Delta} \left\{ k_{S_z} (k_x^2 + k_y^2 - k_{Pz}^2 + 2k_{Pz} k_{S_z}) \left[e^{-i(2k_{Pz} + 3k_{S_z})h} + e^{-ik_{S_z}h} \right] - k_{Pz} (k_x^2 + k_y^2 + k_{S_z}^2) \left[e^{-ik_{Pz}h} + e^{-i(k_{Pz} + 4k_{S_z})h} \right] \right. \\ \left. - k_{S_z} (k_x^2 + k_y^2 - k_{Pz}^2 - 2k_{Pz} k_{S_z}) \left[e^{-i(2k_{Pz} + k_{S_z})h} + e^{-3ik_{S_z}h} \right] + 2k_{Pz} (k_x^2 + k_y^2 - 3k_{S_z}^2) e^{-i(k_{Pz} + 2k_{S_z})h} \right\}$$

$$729 \quad \hat{k}_{16}^e = -\frac{2i\mu k_x k_{Pz} k_{S_z}}{\Delta} (k_x^2 + k_y^2 + k_{S_z}^2) \left[e^{-ik_{Pz}h} - e^{-i(k_{Pz} + 4k_{S_z})h} - e^{-i(2k_{Pz} + k_{S_z})h} + e^{-i(2k_{Pz} + 3k_{S_z})h} - e^{-ik_{S_z}h} + e^{-3ik_{S_z}h} \right]$$

$$730 \quad \hat{k}_{22}^e = \frac{i\mu}{\Delta} \left\{ 2k_{Pz} \left[k_y^4 + k_x^2 k_y^2 - (2k_x^2 - k_y^2) k_{S_z}^2 \right] \left[e^{-2i(k_{Pz} + k_{S_z})h} + e^{-2ik_{S_z}h} \right] - (k_x^2 + k_y^2 + k_{Pz} k_{S_z}) \left[(k_y^2 + k_{S_z}^2) k_{Pz} + k_x^2 k_{S_z} \right] \left[e^{-2i(k_{Pz} + 2k_{S_z})h} + 1 \right] \right. \\ \left. + 8k_x^2 k_{Pz} k_{S_z}^2 \left[e^{-i(k_{Pz} + k_{S_z})h} + e^{-i(k_{Pz} + 3k_{S_z})h} \right] - (k_x^2 + k_y^2 - k_{Pz} k_{S_z}) \left[(k_y^2 + k_{S_z}^2) k_{Pz} - k_x^2 k_{S_z} \right] \left(e^{-2ik_{Pz}h} + e^{-4ik_{S_z}h} \right) \right\}$$

$$731 \quad \hat{k}_{23}^e = -\frac{i\mu k_y}{\Delta} \left\{ 2 \left[(k_x^2 + k_y^2)^2 - (k_x^2 + k_y^2 - 2k_{Pz}^2) k_{S_z}^2 \right] \left[e^{-2i(k_{Pz} + k_{S_z})h} - e^{-2ik_{S_z}h} \right] - (k_x^2 + k_y^2 + k_{Pz} k_{S_z}) (k_x^2 + k_y^2 + 2k_{Pz} k_{S_z} - k_{S_z}^2) \left[e^{-2i(k_{Pz} + 2k_{S_z})h} - 1 \right] \right. \\ \left. - 4k_{Pz} k_{S_z} \left[3(k_x^2 + k_y^2) - k_{S_z}^2 \right] \left[e^{-i(k_{Pz} + 3k_{S_z})h} - e^{-i(k_{Pz} + k_{S_z})h} \right] - (k_x^2 + k_y^2 - k_{Pz} k_{S_z}) (k_x^2 + k_y^2 - 2k_{Pz} k_{S_z} - k_{S_z}^2) \left(e^{-2ik_{Pz}h} - e^{-4ik_{S_z}h} \right) \right\}$$

$$732 \quad \hat{k}_{25}^e = -\frac{2i\mu}{\Delta} \left\{ 2k_{Pz} \left[k_y^4 + k_x^2 k_y^2 + (4k_x^2 + k_y^2) k_{S_z}^2 \right] e^{-i(k_{Pz} + 2k_{S_z})h} - k_{S_z} (k_x^4 + k_x^2 k_y^2 + 2k_x^2 k_{Pz} k_{S_z} + k_y^2 k_{Pz}^2 + k_{Pz}^2 k_{S_z}^2) \left[e^{-i(2k_{Pz} + 3k_{S_z})h} + e^{-ik_{S_z}h} \right] \right. \\ \left. - k_y^2 k_{Pz} (k_x^2 + k_y^2 + k_{S_z}^2) \left[e^{-ik_{Pz}h} + e^{-i(k_{Pz} + 4k_{S_z})h} \right] + k_{S_z} (k_x^4 + k_x^2 k_y^2 - 2k_x^2 k_{Pz} k_{S_z} + k_y^2 k_{Pz}^2 + k_{Pz}^2 k_{S_z}^2) \left[e^{-i(2k_{Pz} + k_{S_z})h} + e^{-3ik_{S_z}h} \right] \right\}$$

$$733 \quad \hat{k}_{26}^e = -\frac{2i\mu k_y k_{p_z} k_{s_z}}{\Delta} (k_x^2 + k_y^2 + k_{s_z}^2) \left[e^{-ik_{p_z}h} - e^{-i(k_{p_z}+4k_{s_z})h} - e^{-i(2k_{p_z}+k_{s_z})h} + e^{-i(2k_{p_z}+3k_{s_z})h} - e^{-ik_{s_z}h} + e^{-3ik_{s_z}h} \right]$$

$$734 \quad \hat{k}_{33}^e = \frac{i\mu k_{s_z}}{\Delta} (k_x^2 + k_y^2 + k_{s_z}^2) \left\{ \begin{aligned} & (k_x^2 + k_y^2) \left[e^{-2ik_{p_z}h} - e^{-2i(k_{p_z}+2k_{s_z})h} + e^{-4ik_{s_z}h} - 1 \right] \\ & - k_{p_z} k_{s_z} \left[e^{-2ik_{p_z}h} - 2e^{-2i(k_{p_z}+k_{s_z})h} + e^{-2i(k_{p_z}+2k_{s_z})h} - 2e^{-2ik_{s_z}h} + e^{-4ik_{s_z}h} + 1 \right] \end{aligned} \right\}$$

$$735 \quad \hat{k}_{36}^e = -\frac{2i\mu k_{s_z}}{\Delta} (k_x^2 + k_y^2 + k_{s_z}^2) \left\{ \begin{aligned} & (k_x^2 + k_y^2) \left[e^{-i(2k_{p_z}+k_{s_z})h} - e^{-i(2k_{p_z}+3k_{s_z})h} - e^{-ik_{s_z}h} + e^{-3ik_{s_z}h} \right] \\ & - k_{p_z} k_{s_z} \left[e^{-ik_{p_z}h} - 2e^{-i(k_{p_z}+2k_{s_z})h} + e^{-i(k_{p_z}+4k_{s_z})h} \right] \end{aligned} \right\}$$

736 where Δ is defined as follows:

$$737 \quad \Delta = (k_x^2 + k_y^2 + k_{p_z} k_{s_z})^2 \left[e^{-2i(k_{p_z}+2k_{s_z})h} - 1 \right] + 8(k_x^2 + k_y^2) k_{p_z} k_{s_z} \left[e^{-i(k_{p_z}+k_{s_z})h} - e^{-i(k_{p_z}+3k_{s_z})h} \right] \\ + (k_x^2 + k_y^2 - k_{p_z} k_{s_z})^2 (e^{-2ik_{p_z}h} - e^{-4ik_{s_z}h}) - 2 \left[(k_x^2 + k_y^2)^2 + k_{p_z}^2 k_{s_z}^2 \right] \left[e^{-2i(k_{p_z}+k_{s_z})h} - e^{-2ik_{s_z}h} \right]$$

738

739 **Appendix C**

740 LINTRACK is a pavement tester in the Faculty of Civil Engineering and
741 Geosciences, Delft University of Technology. As shown in Figure C1, it consists of a
742 free-rolling wheel that moves forward and backward with a guidance system. The
743 force applied on the wheel can be varied between 15 and 100 kN and the moving
744 speed can be changed between 0 and 20 km/h. A fully automatic electronic control
745 system makes it possible to run LINTRACK continuously with automatic data
746 collection. Various measuring instruments (e.g. strain gauges) can be built into test
747 sections to collect necessary information about the response of a pavement structure.

748



749
750

Figure C1. LINTRACK device with wide base tire.

751

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