Improvement of the Richardson-Zaki liquid-solid fluidisation model on the basis of hydraulics (PPT)

Kramer, Onno

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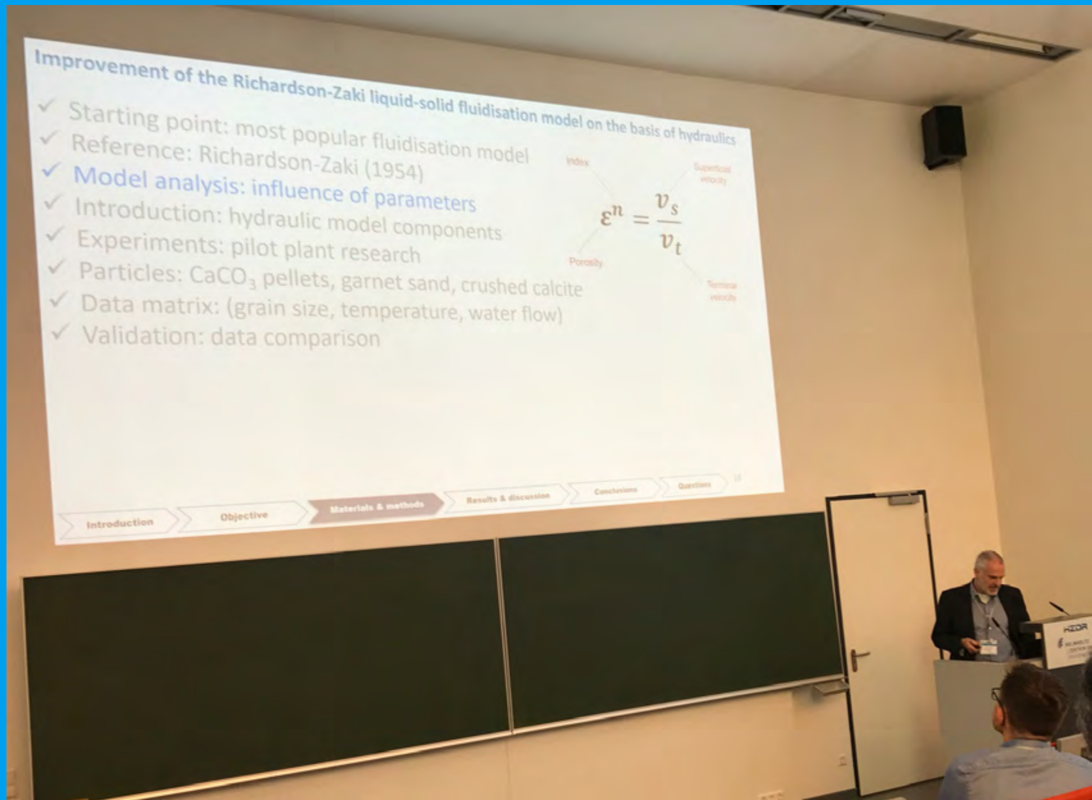
15 November 2018

16th Multiphase Flows conference & shore course – Simulation, Experiment and Application



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Onno Kramer

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16th Multiphase Flows conference & shore course – Simulation, Experiment and Application

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- Introduction
- Objectives
- Materials and methods
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- Questions

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- **Introduction**
- Objectives
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Hydraulic modelling of liquid-solid fluidisation in drinking water treatment processes

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Eric Baars[1]
Peter de Moel[3, 5]
Wim van Vugt[2]
Johan Padding[4]
Jan Peter van der Hoek[1, 3]

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[3] **TUD** Delft University of Technology,
Faculty of Civil Engineering and Geosciences

[4] **TUD** Delft University of Technology,
Faculty of Mechanical, Maritime and Materials Engineering

[5] **Omnisys**
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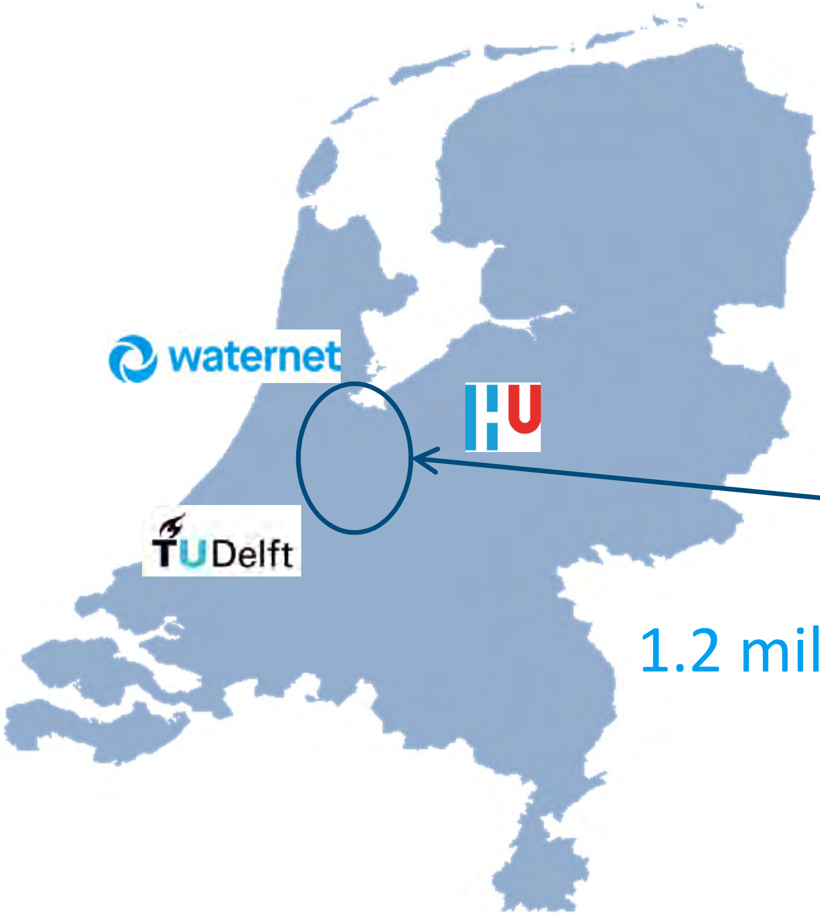
Improvement of the Richardson-Zaki liquid-solid fluidisation model on the basis of hydraulics



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Introduction | Objective | Materials & methods | Results & discussion | Conclusions | Questions

Improvement of the Richardson-Zaki liquid-solid fluidisation model on the basis of hydraulics

- ✓ Background: (water cycle)
- ✓ Field: (drinking water treatment processes)
- ✓ System: (multiphase flows)
- ✓ Process: (softening)
- ✓ Fluidisation: (liquid-solid = water-calcite pellets)

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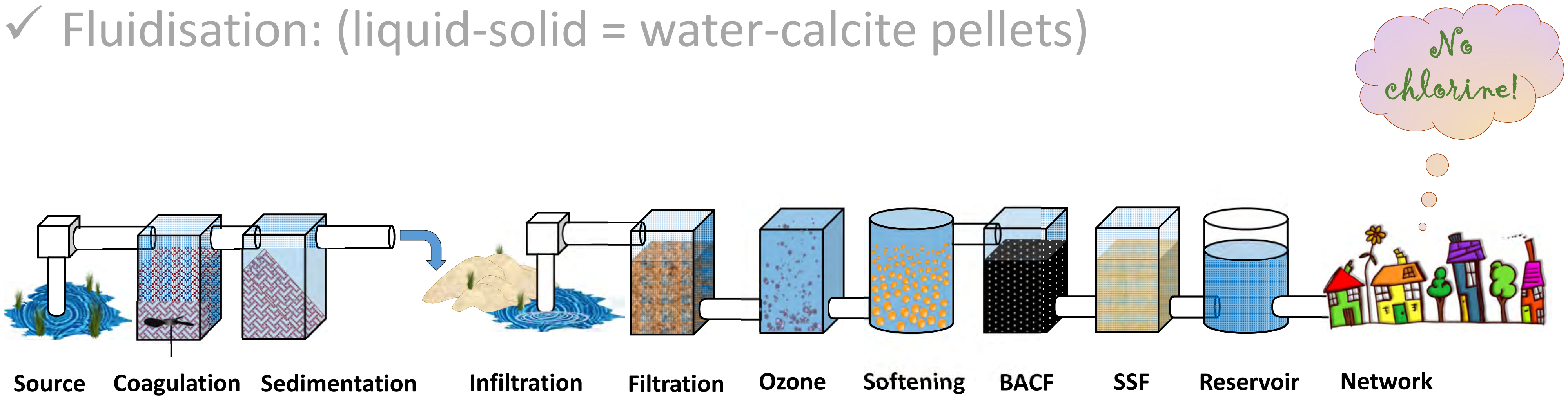

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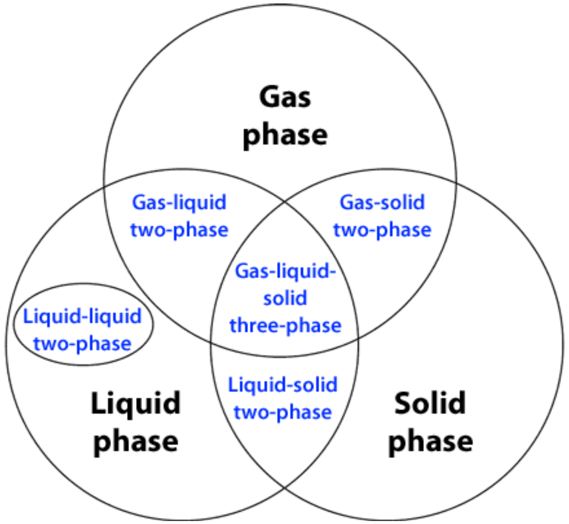

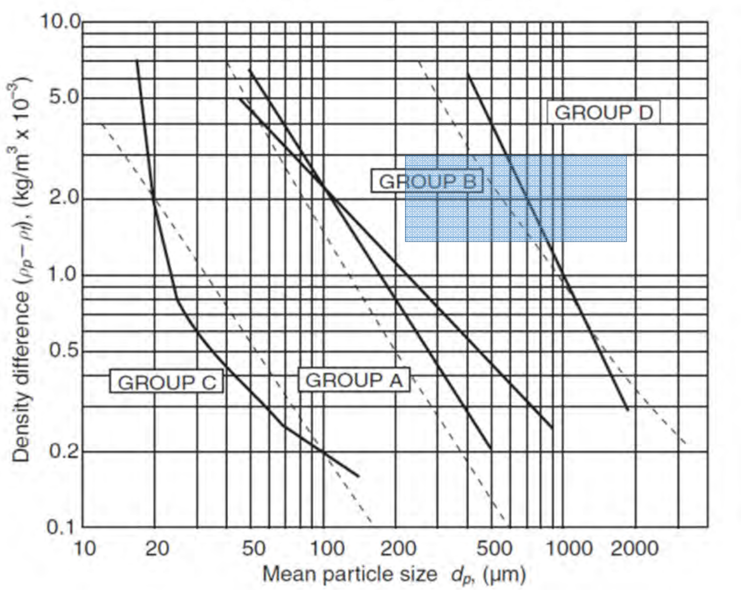

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$OH^- + HCO_3^- \leftrightarrow CO_3^{2-} + H_2O$

$CO_3^{2-} + Ca^{2+} \rightarrow CaCO_3\downarrow$

- Hardness reduction to 1.4 mmol/L
- Reduces solubility of lead (public health) and copper (environment)
- Economic benefits and comfort
    - Reduction of washing powder
    - Increase life time hot water equipment
    - Cleaner laundry, tasteful tea

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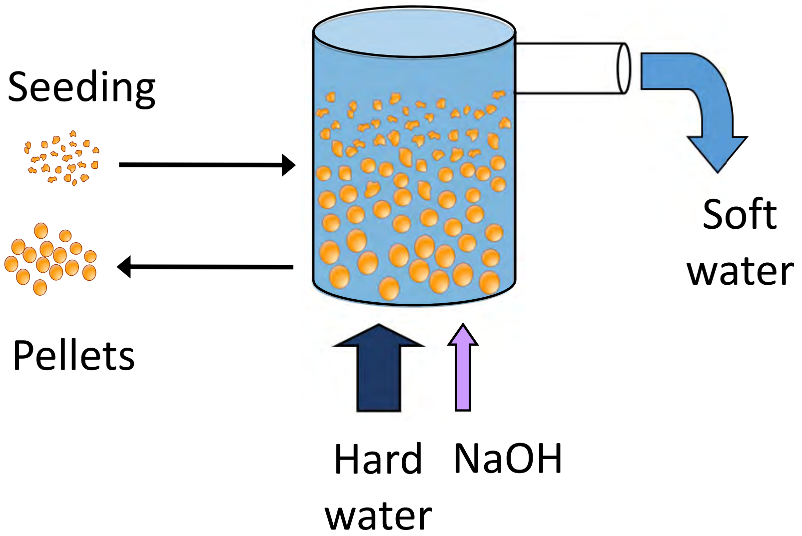

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Improvement of the Richardson-Zaki liquid-solid fluidisation model on the basis of hydraulics

- ✓ Objectives:
  - Increasing sustainability
  - Reducing chemical use
  - Improving water quality
- ✓ Method: improved model based on hydraulics (porosity)
- ✓ Focus: crystallisation on specific surface area

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- ✓ Reference: Richardson-Zaki (1954)
- ✓ Model analysis: influence of parameters
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$$\varepsilon^n = \frac{v_s}{v_t}$$

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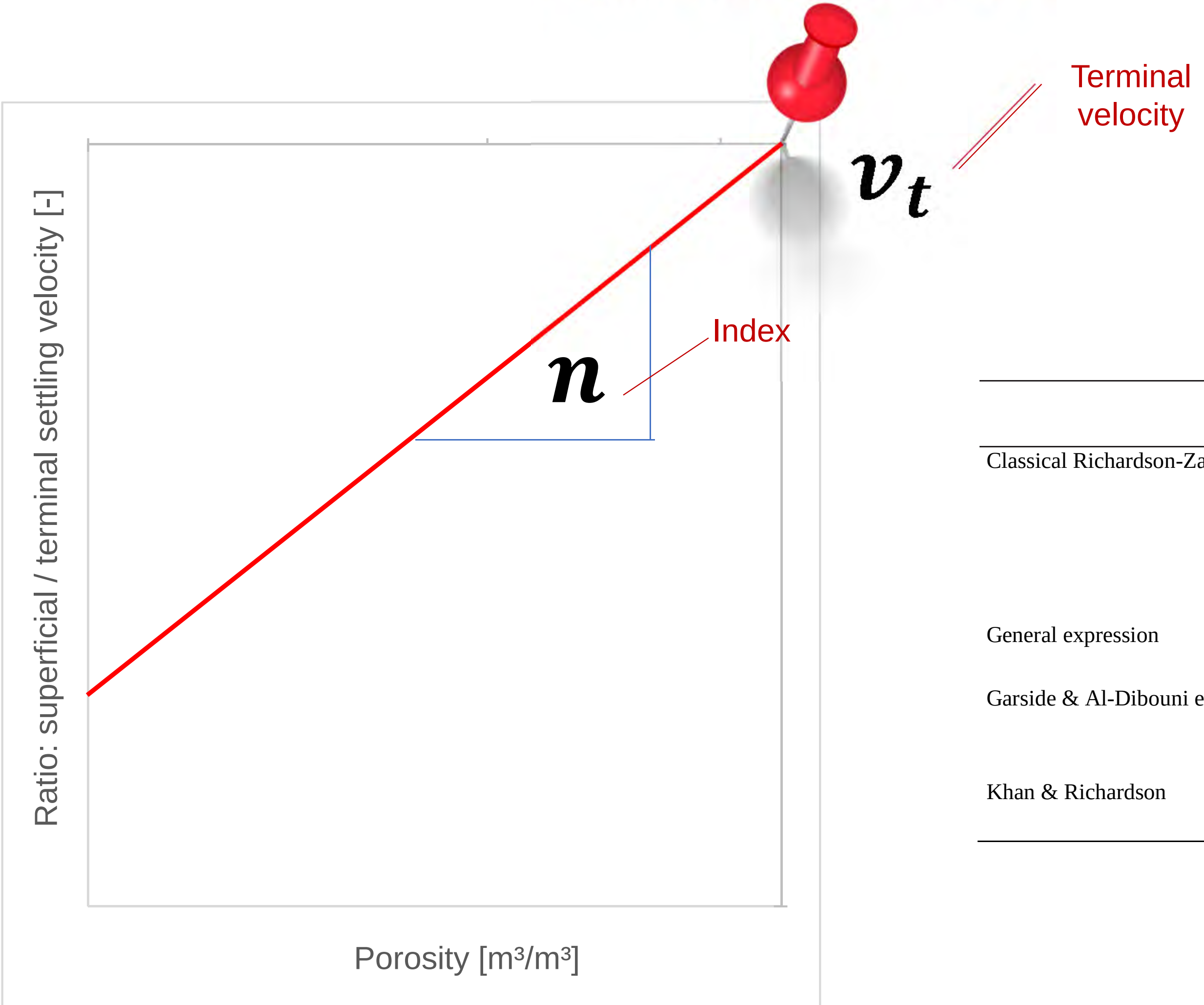
Index
Superficial velocity
Porosity
Terminal velocity

Improvement of the Richardson-Zaki liquid-solid fluidisation model on the basis of hydraulics



	Formula
Classical Richardson-Zaki equation	$n = \begin{cases} Re_t < 0.2, & n = 4.65 \\ 0.2 \le Re_t < 1, & n = 4.4\, Re_t^{-0.03} \\ 1 \le Re_t < 500, & n = 4.4\, Re_t^{-0.1} \\ Re_t \ge 500, & n = 2.4 \end{cases}$
General expression	$n = c_1 Re_t^{c_2}$
Garside & Al-Dibouni equation	$\frac{n_L - n}{n - n_T} = \alpha\, Re_t^{\beta}$
Khan & Richardson	$\frac{n_L - n}{n - n_T} = \alpha\, Ar^{\beta}$

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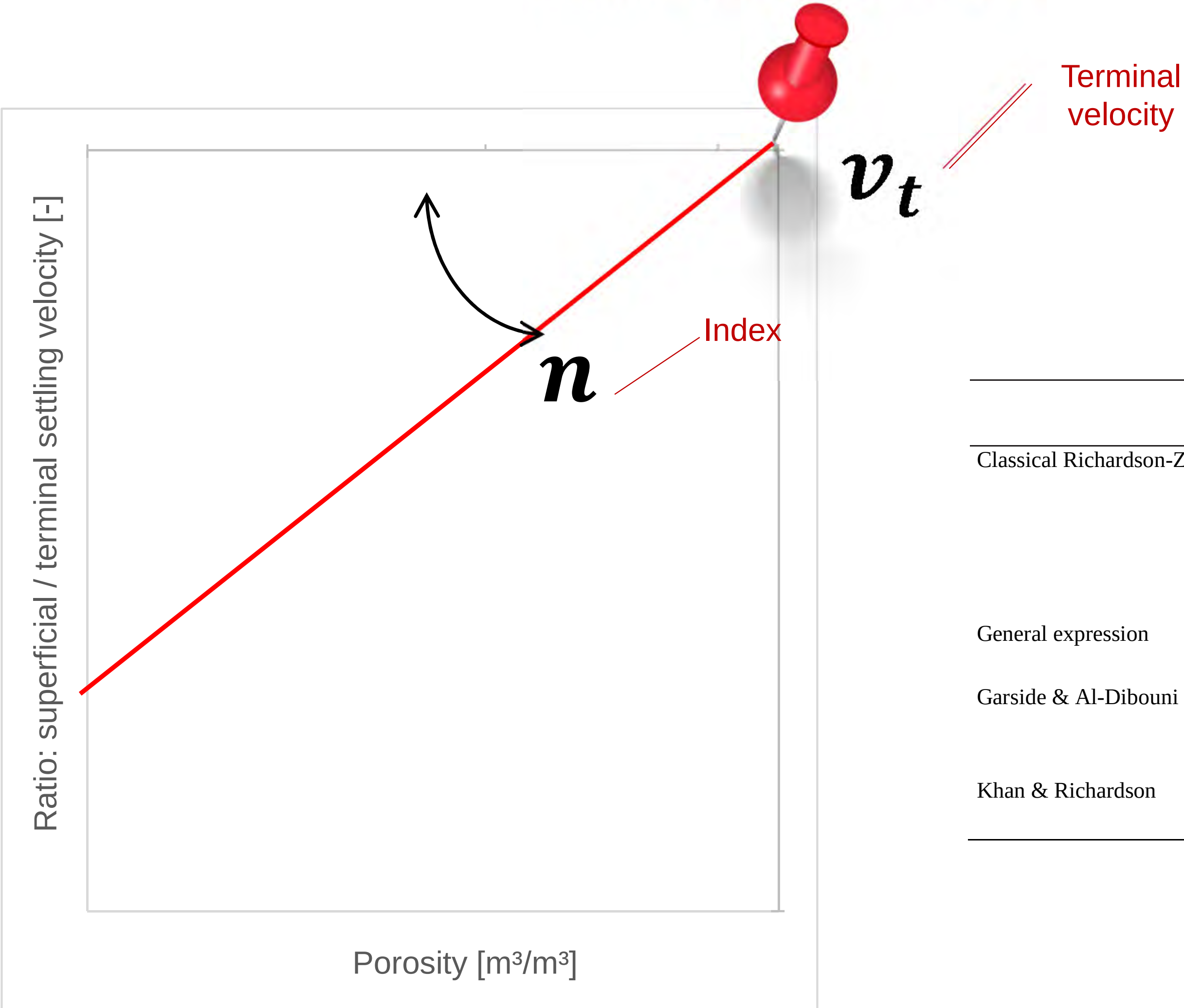

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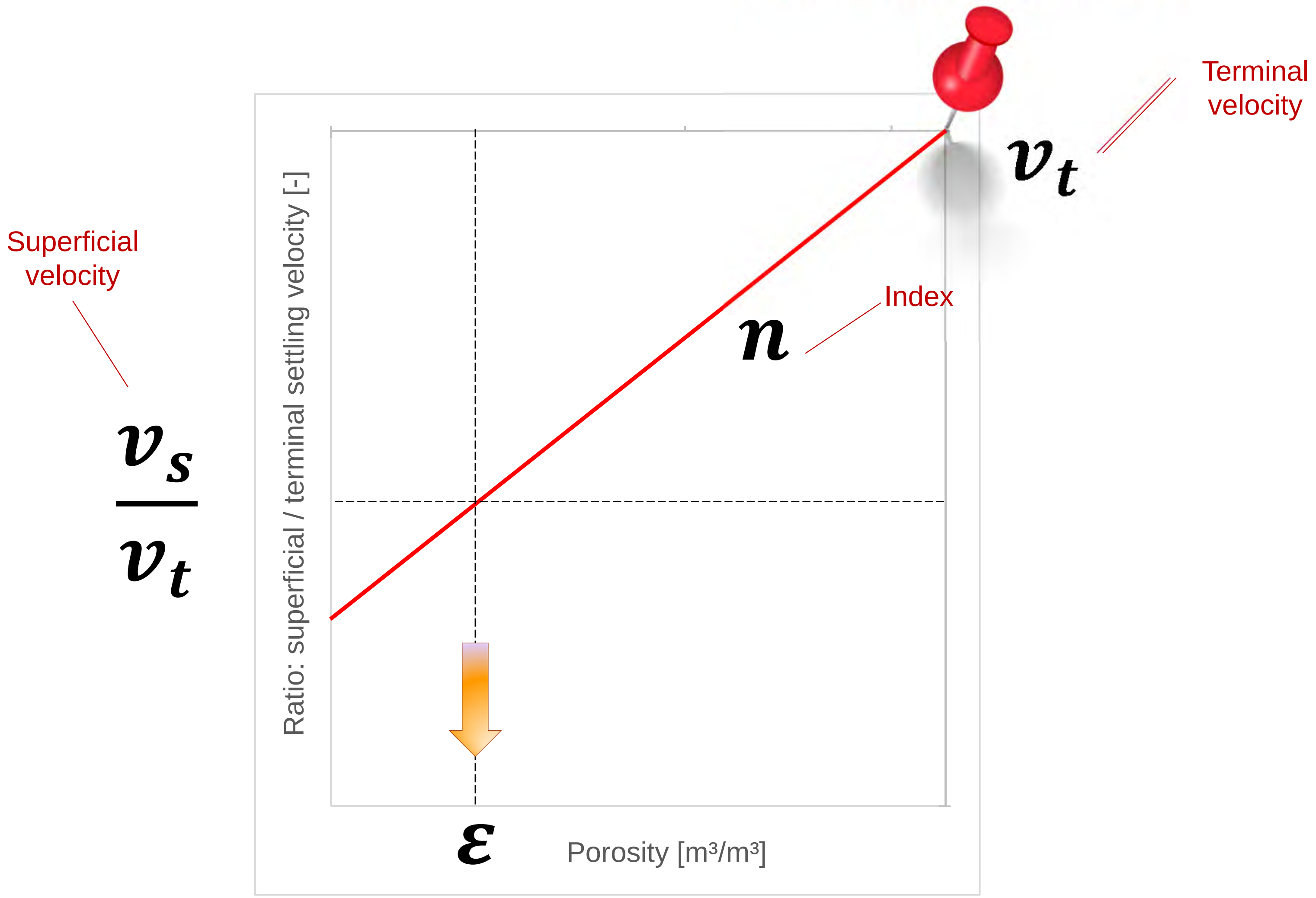

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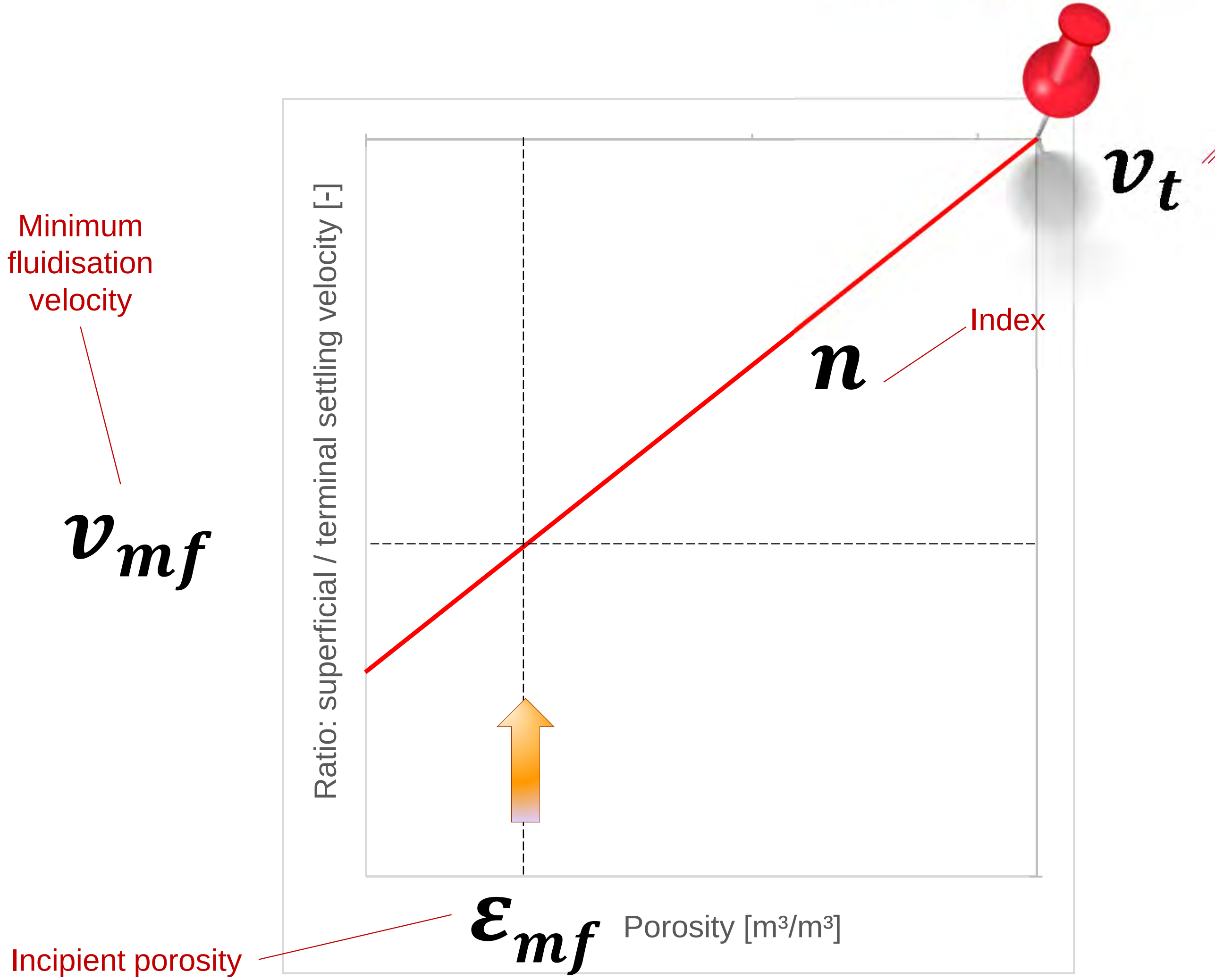

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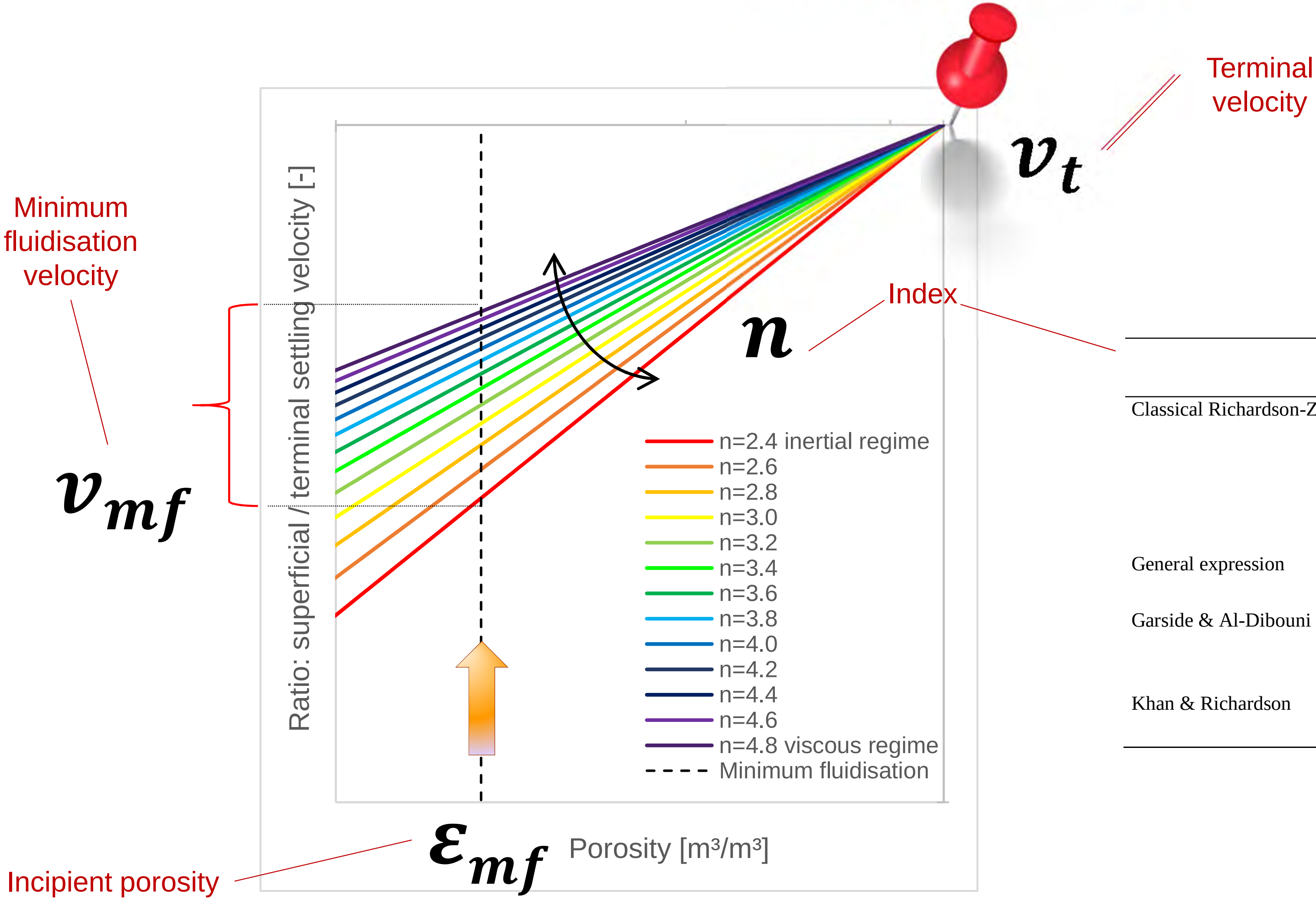

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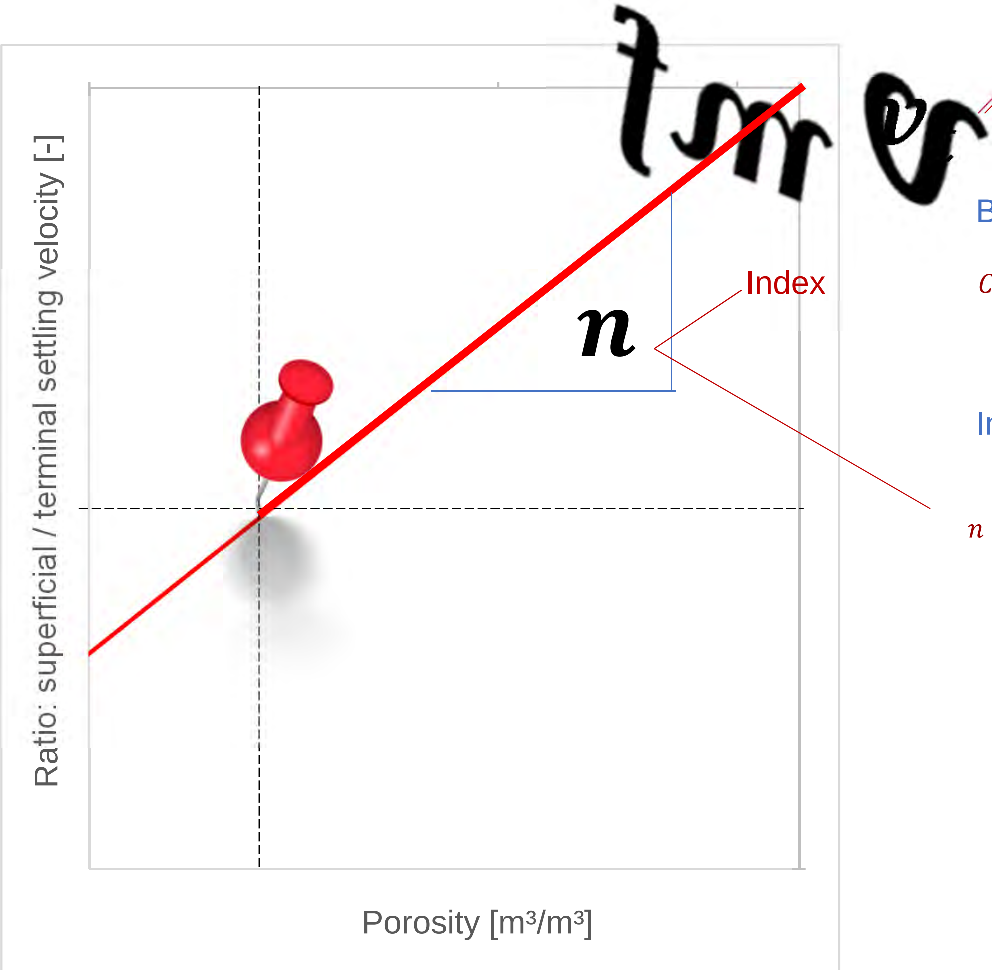
Improvement of the Richardson-Zaki liquid-solid fluidisation model on the basis of hydraulics

Minimum fluidisation velocity

v_{mf}

Carman-Kozeny

$$C_D = \frac{180}{Re_\varepsilon} + \frac{2.87}{{Re_\varepsilon}^{0.1}}$$



Terminal velocity

Brown-Lawler (improved Schiller-Naumann)

$$C_D = \frac{24}{Re_t}\left(1 + 0.15{Re_t}^{0.681}\right) + \frac{0.407}{1 + \frac{8710}{Re_t}}$$

Interpolation

$$n = \frac{\log\left(\frac{Re_{\varepsilon,mf}}{Re_t}\left(1 - \varepsilon_{mf}\right)\right)}{\log \varepsilon_{mf}}$$

$$Re_t = \frac{\rho_f d_p v_t}{\eta}$$

$$Re_{\varepsilon,mf} = \frac{\rho_f d_p v_{mf}}{\eta} \frac{1}{1 - \varepsilon_{mf}}$$

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Superficial velocity

$$\varepsilon^n = \frac{v_s}{v_t}$$

Porosity

Terminal velocity

Carman-Kozeny (at minimum fluidisation)

$$C_D = \frac{180}{Re_\varepsilon} + \frac{2.87}{{Re_\varepsilon}^{0.1}}$$

Brown-Lawler (at terminal settling settling)

$$C_D = \frac{24}{Re_t}\left(1 + 0.15{Re_t}^{0.681}\right) + \frac{0.407}{1 + \frac{8710}{Re_t}}$$

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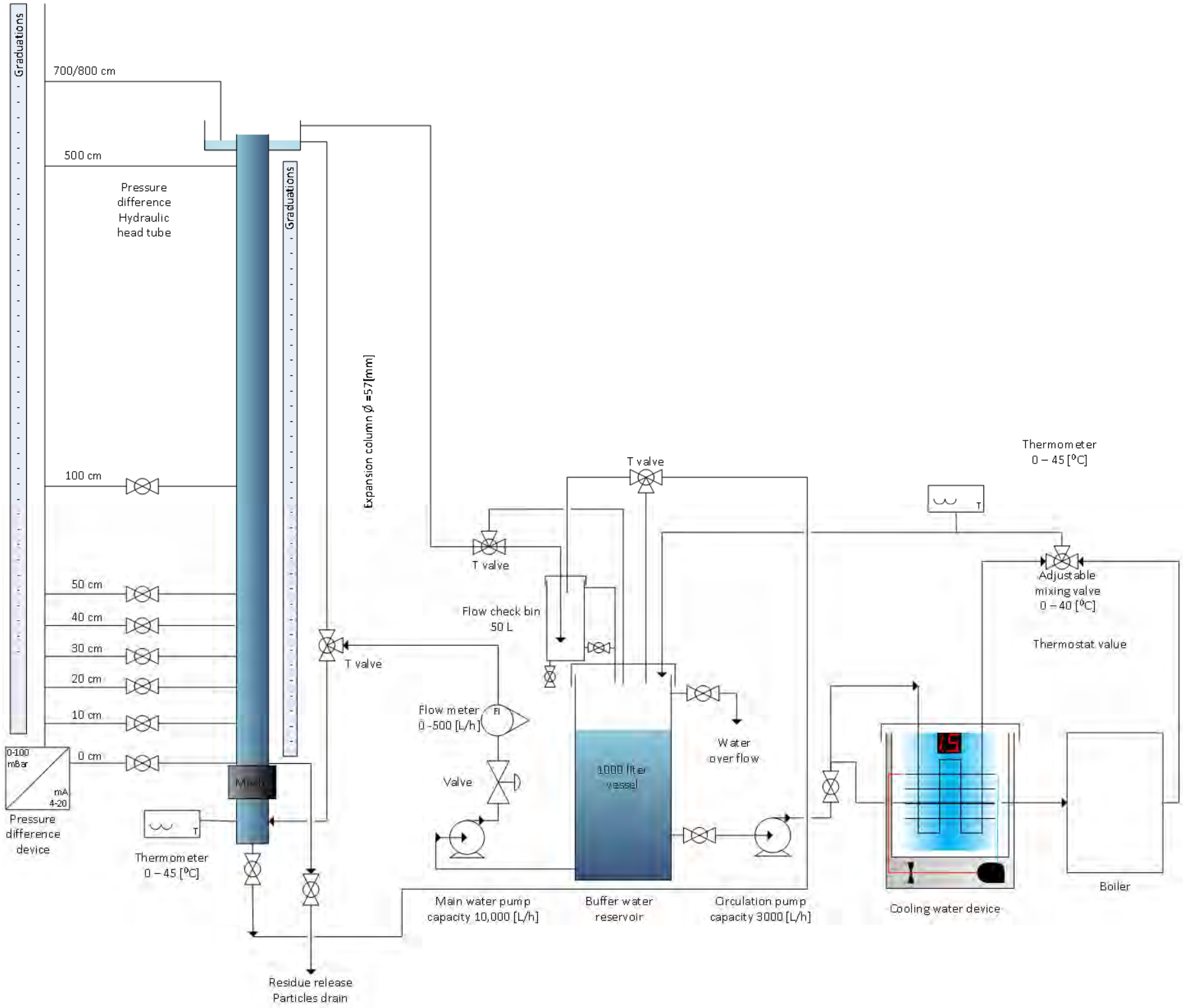

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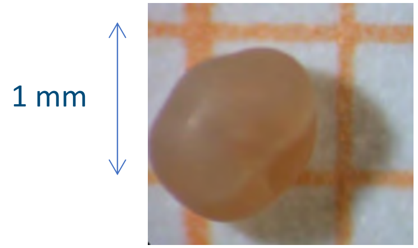
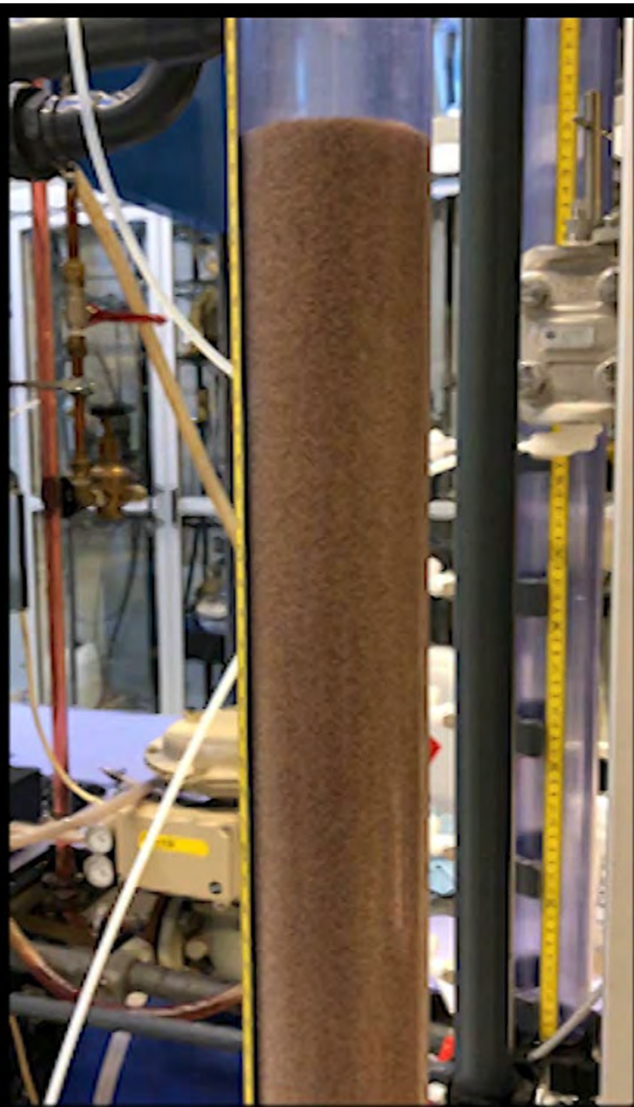


1.4 - 1.7 mm calcite pellets



0.425 - 0.5 crushed calcite

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- ✓ 10 sieved fractions
  ($0.4 < d_z < 2.0$ mm)
- ✓ 4 temperatures
  (5, 15, 25, 35 °C)
- ✓ 25 ascending water flows
  (0-180 m/h)

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Improvement of the Richardson-Zaki liquid-solid fluidisation model on the basis of hydraulics

- ✓ Experiments: 76 fluidisation characteristics
- ✓ Results: model (implicit) and simplified model (explicit)
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- ✓ Model accuracy improvement

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$$\frac{4.8 - n}{n - 2.4} = 0.015\, Ar^{0.5}$$

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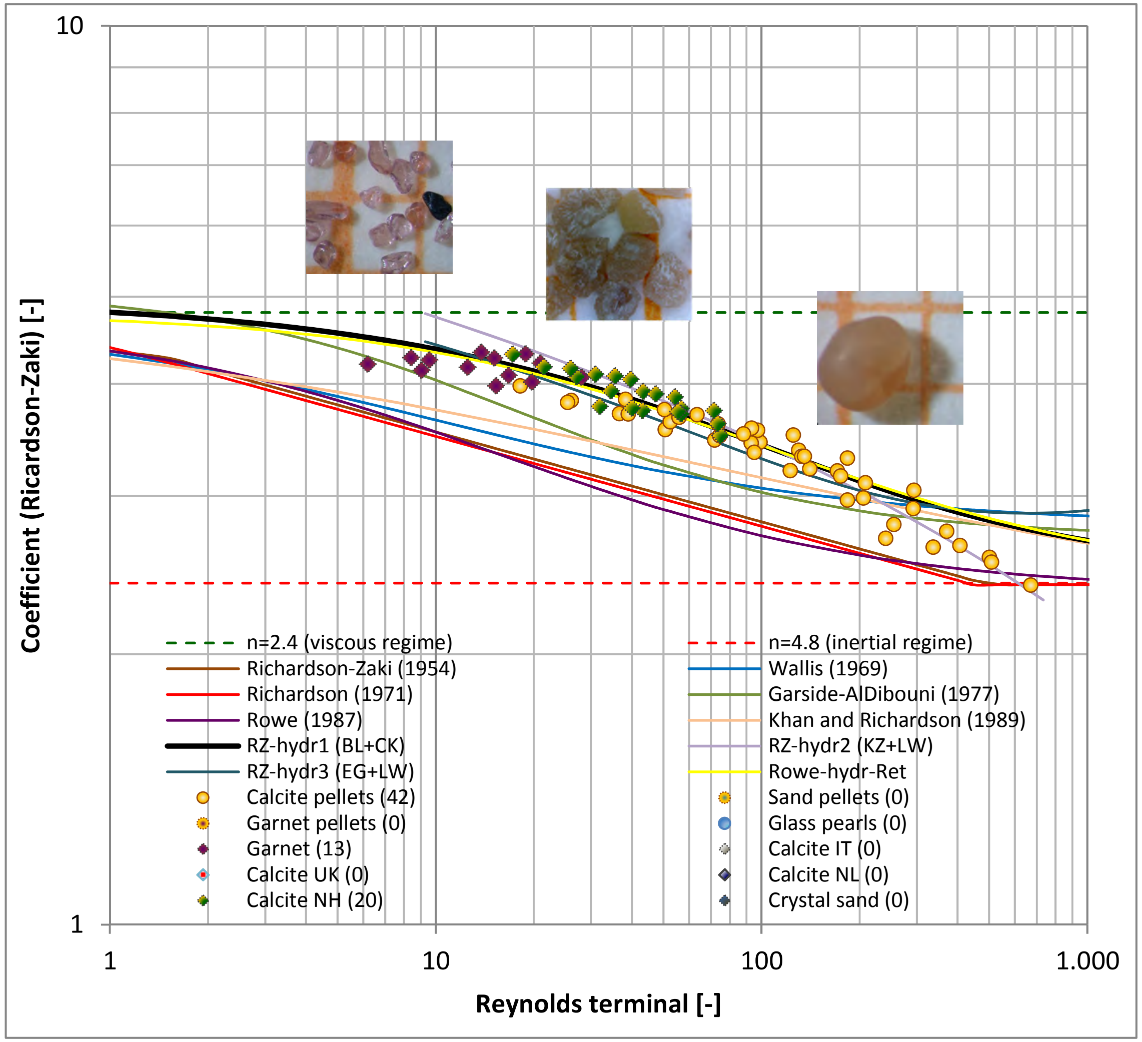
$$\varepsilon^n = \frac{v_s}{v_t}$$

$$Re_t = \frac{\rho_f d_p v_t}{\eta}$$



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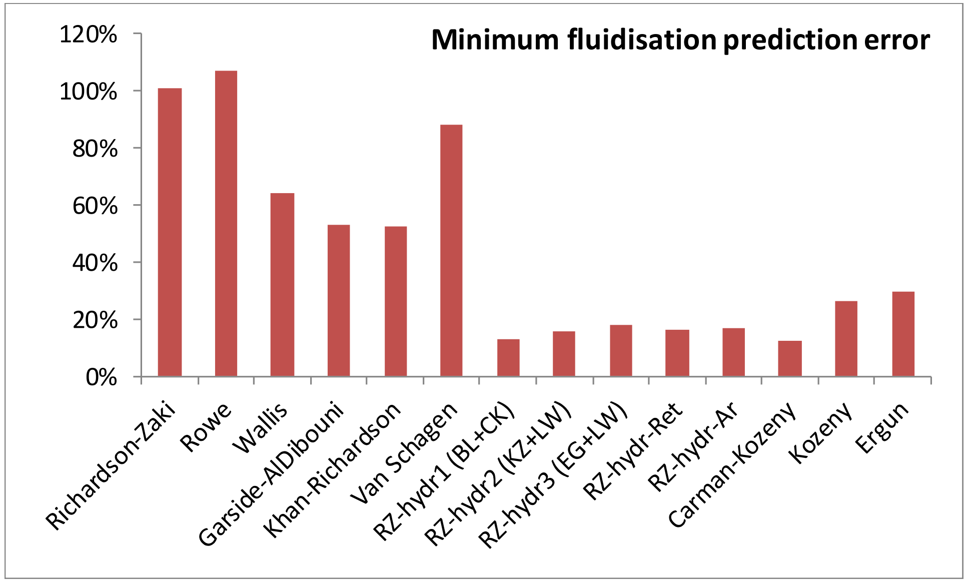

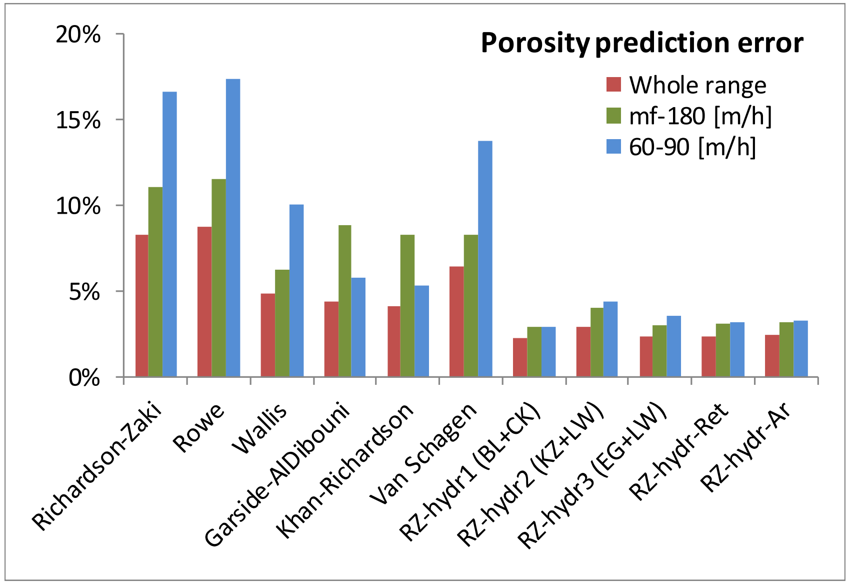

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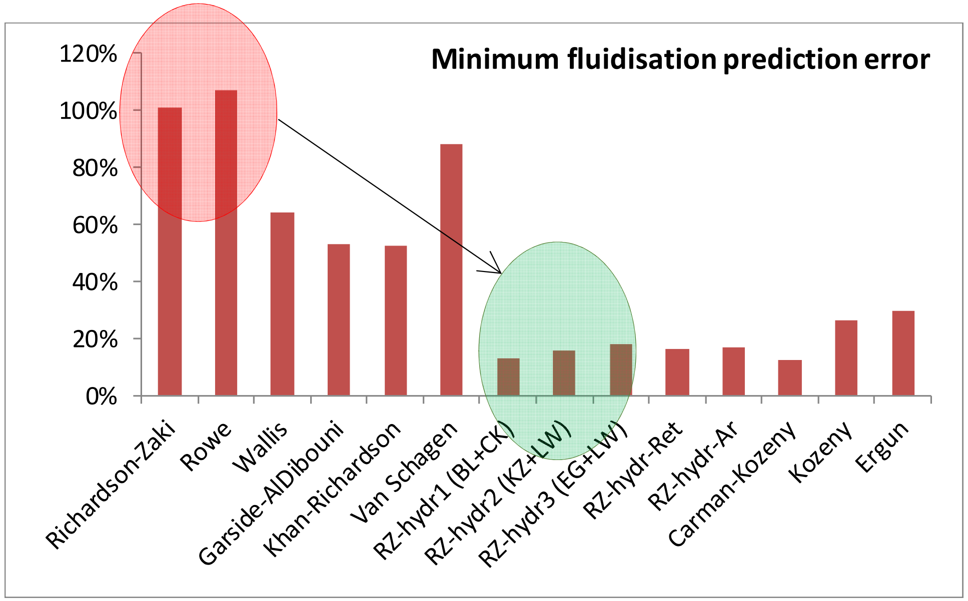
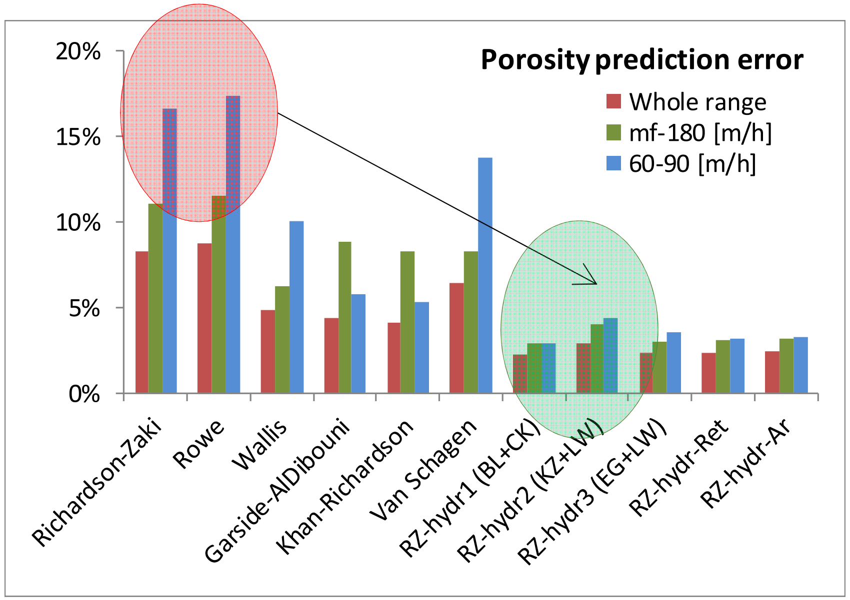
minimum fluidisation >100%→12%   porosity >15%→3%





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- ✓ RZ models can be improved based on hydraulics principles i.e. 3 points (ε,v) (0,0) (ε_{mf}, v_{mf}) ($\varepsilon \rightarrow 1$, v_t)
- ✓ Porosity can be predicted more accurately
- ✓ Recommendations:
  - Model enhancement (more general)
  - Identification of irregularly shaped particles
  - Implications for specific surface area (Interfacial Area Density)

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O.J.I. Kramer [a, b, c, d], P.J. de Moel [a, e], E.T. Baars [c], W.H. van Vugt [d], J.T. Padding [b], J.P. van der Hoek [a, c]

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Faculty of Mechanical, Maritime and Materials Engineering, Department Process and Energy, Section Intensified Reaction and Separation Systems

TU Delft

Optional for questions

A.R. Khan, J.F. Richardson, Fluid-particle interactions and flow characteristics of fluidized beds and settling suspensions of spherical particles, Chem. Eng. Commun. 78 (1989) 111–130. doi:10.1080/00986448908940189.

Chem. Eng. Comm. 1989, Vol. 78, pp. 111–130
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FLUID-PARTICLE INTERACTIONS AND FLOW CHARACTERISTICS OF FLUIDIZED BEDS AND SETTLING SUSPENSIONS OF SPHERICAL PARTICLES

A.R. KHAN* and J.F. RICHARDSON

Department of Chemical Engineering
University College of Swansea
Singleton Park
Swansea SA2 8PP, UK

(Received October, 1987; in final form August 4, 1988)

The published correlations for the velocity-voidage relationship observed during fluidization and sedimentation of uniformly sized spherical particles in solid–liquid systems are compared with published experimental results. It is found that the expression suggested by Richardson and Zaki represents the experimental data well over a wide range of values of Galileo number ($10^{-2} < Ga < 10^{20}$) and voidage ($0.4 < \varepsilon < 1$). Methods are given for predicting the constants in the expression as a function of the properties of the system, including wall effects.
KEYWORDS Fluidized beds Fluid–particle interactions Spherical particles.

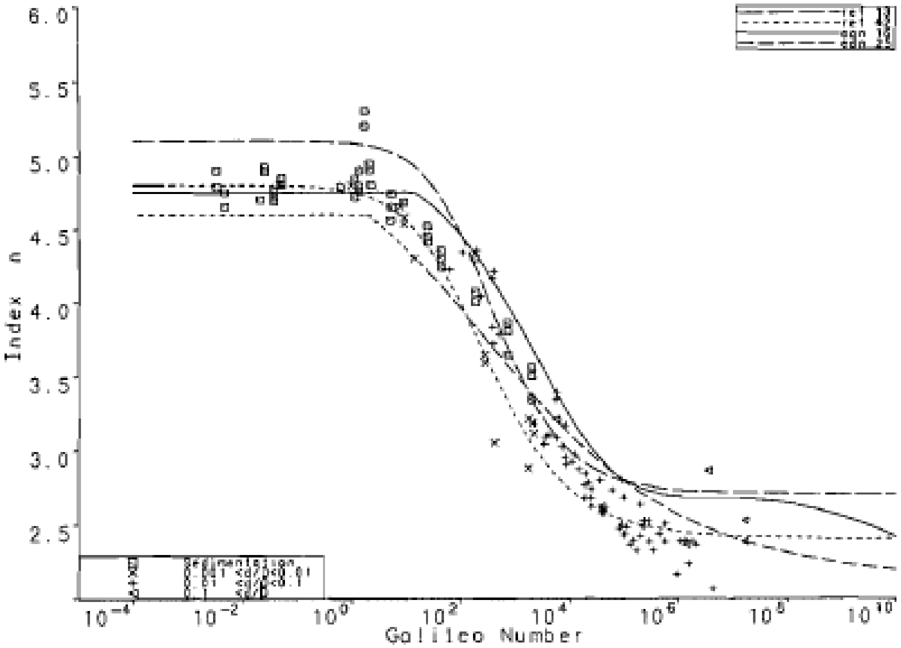

FIGURE 5 Comparison of published correlations with experimental values of index *n*.

J. Garside, M.R. Al-Dibouni, Velocity-voidage relationships for fluidization and sedimentation in solid-liquid systems, Ind. Eng. Chem. Process Des. Dev. 16 (1977) 206–214. doi:10.1021/i260062a008.

Velocity–Voidage Relationships for Fluidization and Sedimentation in Solid–Liquid Systems

John Garside* and Maan R. Al-Dibouni

Department of Chemical Engineering, University College London, London WC1E 7JE

Published experimental results for the velocity–voidage relationship observed during fluidization and sedimentation of uniformly sized spheres in solid–liquid systems are compared and new experimental results are presented. Predictions of various published correlations that are available to describe this relation are compared with these experimental results and the inadequacy of most of the correlations is demonstrated. New correlations are suggested. The most reliable of these is based on a logistic curve and can be represented by the equation $(U_R - A)/(B - U_R) = 0.06\mathrm{Re}_t^{\epsilon+0.2}$, where $A = \epsilon^{4.14}$ and $B = 0.8\epsilon^{1.28}$ for $\epsilon \leqslant 0.85$ or $B = \epsilon^{2.65}$ for $\epsilon > 0.85$. $U_R = U_\epsilon/U_t$ where U_ϵ is the relative average velocity between particles and fluid, U_t is the terminal velocity of a single particle, Re_t is the particle Reynolds number based on U_t, and ϵ is the bed voidage.

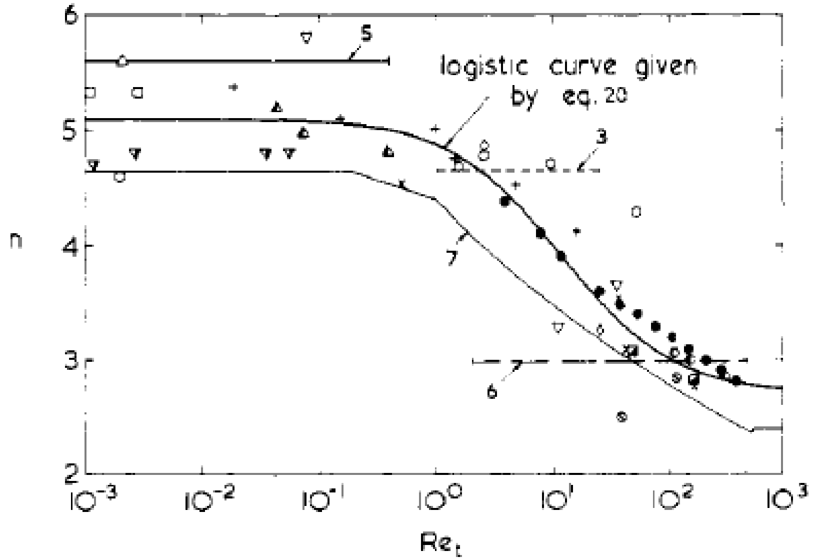

Figure 8. Variation of exponent n with Reynolds number. (See Table I for explanation of symbols; See Table III for key to different curves.)

A.D. Maude, R.L. Whitmore, A generalized theory of sedimentation, Br. J. Appl. Phys. 9 (1958) 477–482. doi:10.1103/RevModPhys.20.35.

A generalized theory of sedimentation

By A. D. Maude, B.Sc., Ph.D., A.Inst.P.,* and R. L. Whitmore, B.Sc., Ph.D., A.Inst.P., Department of Mining and Fuels, University of Nottingham

[*Paper first received* 3 *January, and in final form* 25 *June*, 1958]

A theoretical relationship between the concentration and the sedimentation velocity of non-flocculated suspensions of particles is derived. It is shown that the settling velocity relative to that of a single particle in the suspension is $(1 - c)^{\beta}$ where β is a function of particle shape, size distribution and Reynolds number and c is the volume of solid per unit volume of suspension. The expression is shown to satisfy the experimental results of other workers. An empirical relationship between β and the Reynolds number is suggested.

1. INTRODUCTION

When a body falls through a fluid, it accelerates until it reaches a constant terminal velocity. This velocity is determined by the density of the fluid ρ_l, the density of the body ρ_s, the viscosity of the fluid η, the shape and orientation of the body and by some length characterizing the size of the body d. The velocity may also depend, to some extent, upon the size and shape of the containing vessel, but if this is large its influence may be neglected.

The problem becomes more complicated if many bodies are present and the system becomes a sedimenting suspension. When the bodies are more or less evenly dispersed throughout the fluid, their rate of fall is decreased, and it is of considerable practical interest to know the relation between the concentration of bodies and the magnitude of this decrease.

Many theoretical and empirical relations[1–8] have been proposed to solve this problem, but they suffer from various defects, in particular they lack generality. It would be of considerable practical value if an equation could be derived which would cover a wide range of particle shapes, size ranges and rates of fall, and in the following paper an attempt is made to do this.

2. FALL OF SPHERES AT LOW REYNOLDS NUMBER

Consider a mass of equi-velocity particles falling through a pure fluid. If v is the average volume of one particle, and g is the acceleration due to gravity, then the average force on one particle is $F = (\rho_s - \rho_l)gv$. If c is the volume of solid in unit form of suspension, then it may be shown from

* Now at the City of Liverpool College of Technology.

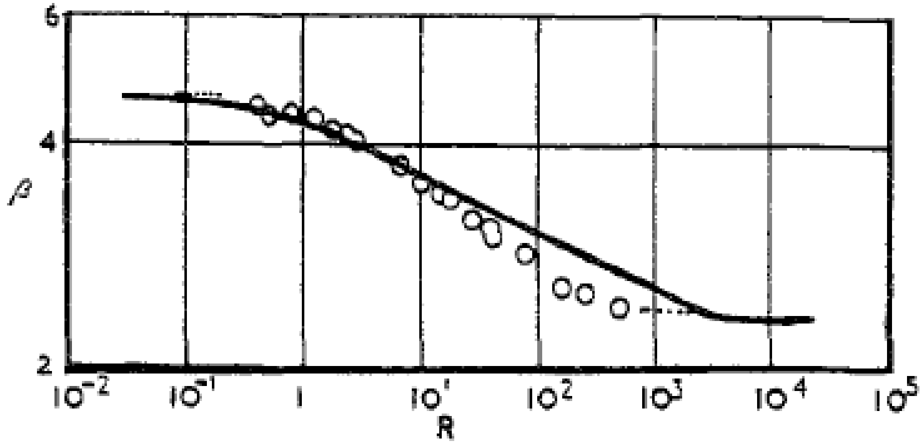

Fig. 2. Variation of the exponent of $(1 - c)$ with Reynolds number

J.F. Richardson, M.A. da S. Jerónimo, Velocity-voidage relations for sedimentation and fluidisation, Chem. Eng. Sci. 34 (1979) 1419–1422. doi:10.1016/0009-2509(79)85167-2.

Chemical Engineering Science Vol 34 pp 1419–1422
Pergamon Press Ltd 1979 Printed in Great Britain

Velocity–voidage relations for sedimentation and fluidisation

(*Received 26 February 1979, accepted 30 April 1979*)

A concentrated suspension of uniform particles settles at a lower rate than one of the particles in isolation in a large expanse of fluid This phenomenon arises from a combination of factors Thus in a suspension there is a significant upflow of displaced fluid, there are changed buoyancy effects and steeper velocity gradients at a given particle velocity relative to the fluid The relation between sedimentation velocity and concentration in a suspension is similar to that between fluidisation velocity and concentration in a liquid solid system, and various empirical relations have been suggested It is now proposed to examine how the constant in one of these relations can be calculated from the slope of the curve of drag coefficient against particle Reynolds number, and to show how calculated and experimental values compare

THE EFFECT OF CONCENTRATION ON THE DRAG FORCE–VELOCITY RELATION FOR A SPHERE

For a single isolated particle settling in a fluid

$$\frac{\pi}{6} d^3(\rho_s - \rho)g = \left(\frac{R'}{\rho u_0^2}\right) \rho u_0^2 \frac{\pi}{4} d^2 \quad (1)$$

from which

$$\left(\frac{R'}{\rho u_0^2}\right)\frac{u_0^2 d^2 \rho^2}{\mu^2} = \frac{2}{3}\frac{d^3 g(\rho_s - \rho)\rho}{\mu^2} \quad (2)$$

i e

$$\frac{R'}{\rho u_0^2} \mathrm{Re}_0^2 = \frac{2}{3}\mathrm{Ga} \quad (3)$$

Suppose that in a suspension of voidage ϵ, the force on a particle at a given relative velocity is increased by some factor $f(\epsilon)$ Then

$$\frac{\pi}{6} d^3(\rho_s - \rho)g = \left(\frac{R'}{\rho u^2}\right)\rho u^2 \frac{\pi}{4} d^2 f(\epsilon) \quad (4)$$

where $f(\epsilon)$ takes account of all interparticle effects including the increase in buoyancy force, and $(R'/\rho u^2)$ is still the friction factor for the isolated particle Combining eqns (3) and (4)

$$f(\epsilon) = \frac{2}{3}\frac{\mathrm{Ga}}{(R'/\rho u^2)\,\mathrm{Re}^2} \quad (5)$$

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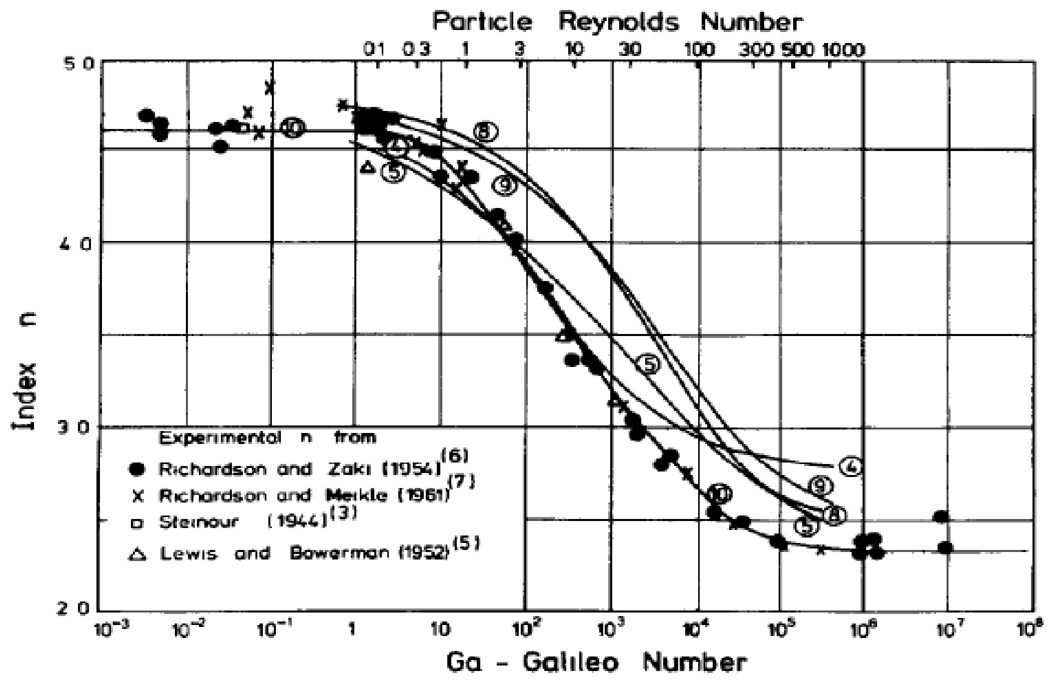


Fig 1 Experimental and calculated values of n as a function of Galileo and particle Reynolds numbers

P.N. Rowe, A convenient empirical equation for estimation of the Richardson-Zaki exponent, Chem. Eng. Sci. 43 (1987) 2795–2796. doi:https://doi.org/10.1016/0009-2509(87)87035-5.

Chemical Engineering Science, Vol. 42, No. 11, pp. 2795–2796, 1987.
Printed in Great Britain.

0009–2509/87 $3.00 + 0.00

A convenient empirical equation for estimation of the Richardson–Zaki exponent

(*Received* 1 *April* 1987; *accepted* 11 *May* 1987)

Wilhelm and Kwauk (1948) were the first to publish studies of the variation of voidage with fluid velocity for fluidized particles and to show that their results using water as the fluid (described as particulate fluidization) were correlated by an equation of the form

$$\mathrm{Re} = K\varepsilon^n \quad (1)$$

Richardson and Zaki (1954) showed some of the logic behind this choice which can be conveniently written

$$u = u_t\varepsilon^n \quad (2)$$

and made a systematic experimental study of how the exponent, n (commonly referred to now as the Richardson–Zaki exponent) varies with Re_t. Their results, conveniently presented in Richardson (1971), for cases where particle size is small compared with vessel diameter, were described by four empirical equations covering different Reynolds number ranges. These equations are a little awkward to use particularly in the regions where they overlap and a continuous functions covering all values of Re_t is more useful especially when embodied in a general theory such as that of Foscolo and Gibilaro (1984).

Inspection of their data suggests it can be fitted by a logistic curve which is symmetrical and asymptotes to limiting values

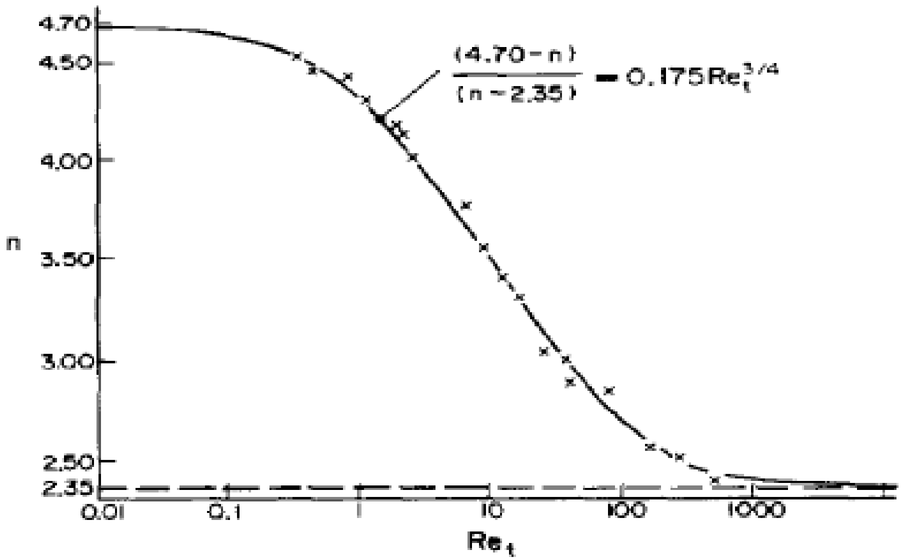
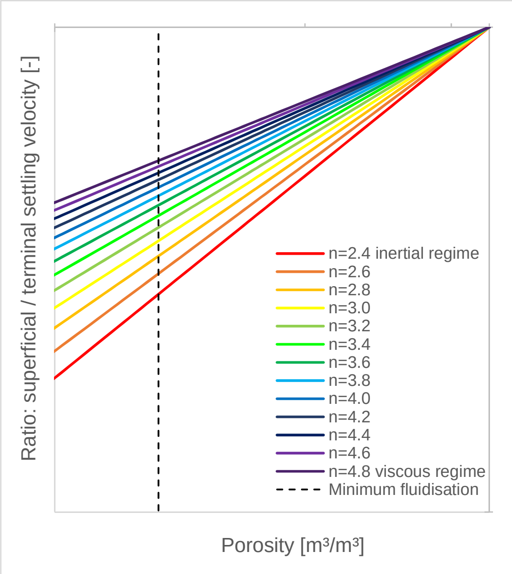


Fig. 1.

Linear relationship log v_s/v_t versus log ε like Richardson-Zaki



J. Fluid Mech. (1972), *vol.* 52, *part* 2, *pp.* 245–268 245
Printed in Great Britain

Sedimentation in a dilute dispersion of spheres

By G. K. BATCHELOR
Department of Applied Mathematics and Theoretical Physics, University of Cambridge

(Received 26 August 1971)

The dispersion considered c… spheres with random position… gravity. The volume fraction… dispersion is statistically hom… that the mean volume flux a… is to determine the mean val… scribed a systematic and rig… ficulty presented by the occ… choice of a quantity V whose… same long-range dependence… mean of U – V can be expres…
The result is that, correct to order c, the mean value of U is $U_0(1-6{\cdot}55c)$, where U_0 is the velocity of a single sphere in unbounded fluid. The only assumption made in the calculation is that the centres of spheres in the dispersion take with equal probability all positions such that no two spheres overlap; arguments are

The results (5.3), (5.4) and (5·8), together with (3.14), show that the mean translational velocity of a sphere in a dispersion of identical rigid spheres with (small) volume concentration c is

$$\bar{U} = U_0 + cU_0(-5{\cdot}50 + 0{\cdot}50 - 1{\cdot}55) = U_0(1 - 6{\cdot}55c), \quad (5.9)$$

…rsion over … is difficult, and many observers have sought other kinds of average …icle velocity, such as the speed of fall of the relatively sharp 'top' to the cloud of particles in a vessel. The available data of different kinds have been examined by Maude & Whitmore (1958) who concluded that the observed mean settling velocity of identical spheres moving with small Reynolds number is best represented, over a wide range of values of c including small values, by a relation of the form

$$\bar{U} = U_0(1-c)^{\beta},$$

G. K. Batchelor, 1972, Sedimentation in a dilute dispersion of spheres, Journal of Fluid Mechanics, Volume 52, Issue 2, pp. 245-268, https://doi.org/10.1017/S0022112072001399

A generalized theory of sedimentation

By A. D. Maude, B.Sc., Ph.D., A.Inst.P.,* and R. L. Whitmore, B.Sc., Ph.D., A.Inst.P., Department of Mining and Fuels, University of Nottingham

[*Paper first received* 3 *January, and in final form* 25 *June*, 1958]

A theoretical relationship between the concentration and the sedimentation velocity of non-flocculated suspensions of particles is derived. It is shown that the settling velocity relative to that of a single particle in the suspension is $(1-c)^{\beta}$ where β is a function of particle shape, size distribution … sion. The expres…

Thus equation (16) becomes

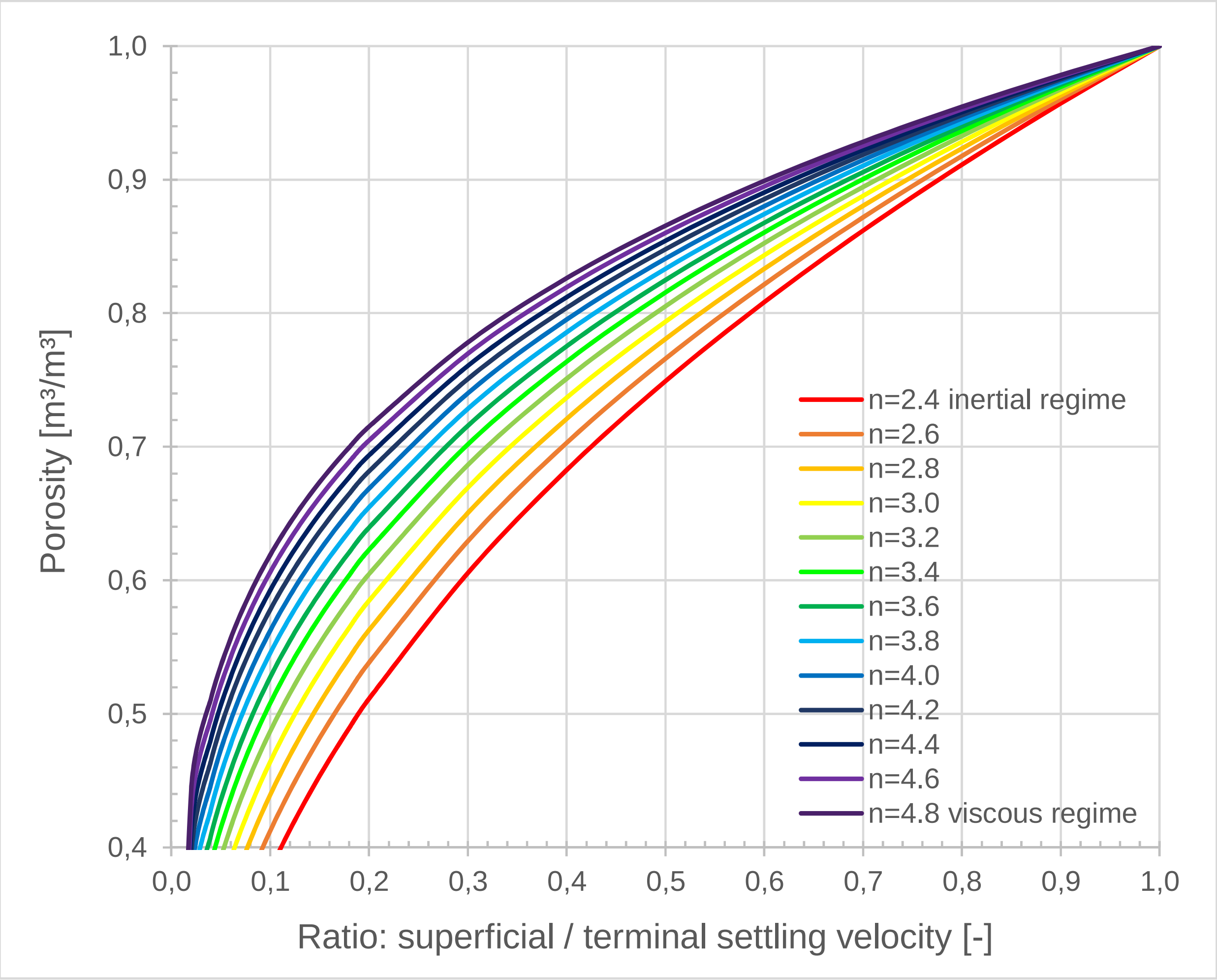
$$\frac{F\rho_l}{\eta^2} \propto \left(\frac{U\rho_l d}{\eta}\right)^m (1-c)^{\alpha}$$

which gives

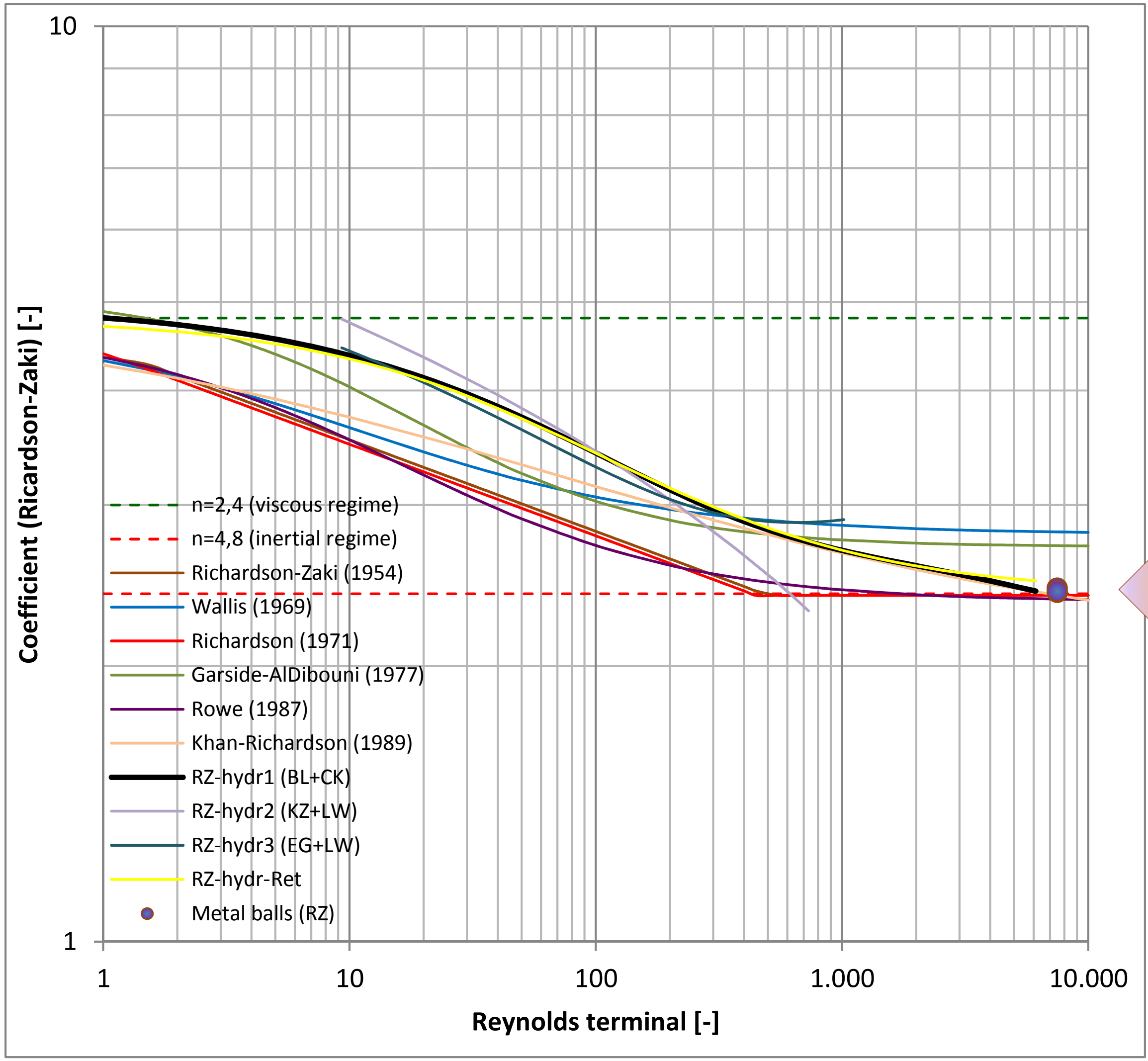
$$U = U_0(1-c)^{\alpha/m}$$

A.D. Maude, R.L. Whitmore, A generalized theory of sedimentation, Br. J. Appl. Phys. 9 (1958) 477–482. doi:10.1103/RevModPhys.20.35.

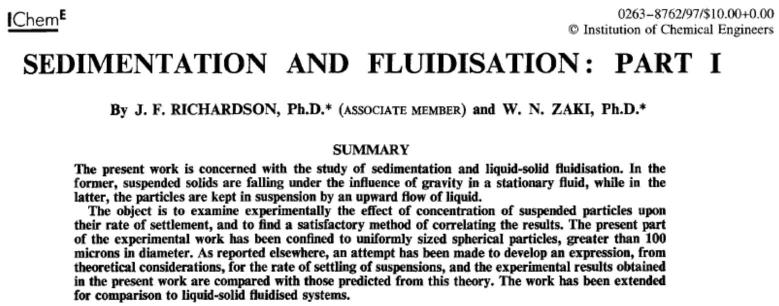
Improvement of the Richardson-Zaki liquid-solid fluidisation model on the basis of hydraulics



Richardson-Zaki (1954) experimental data



RZ data



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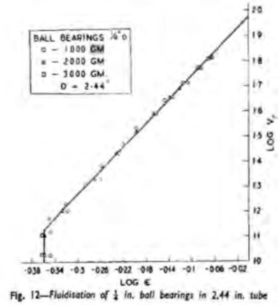
SEDIMENTATION AND FLUIDISATION: PART I

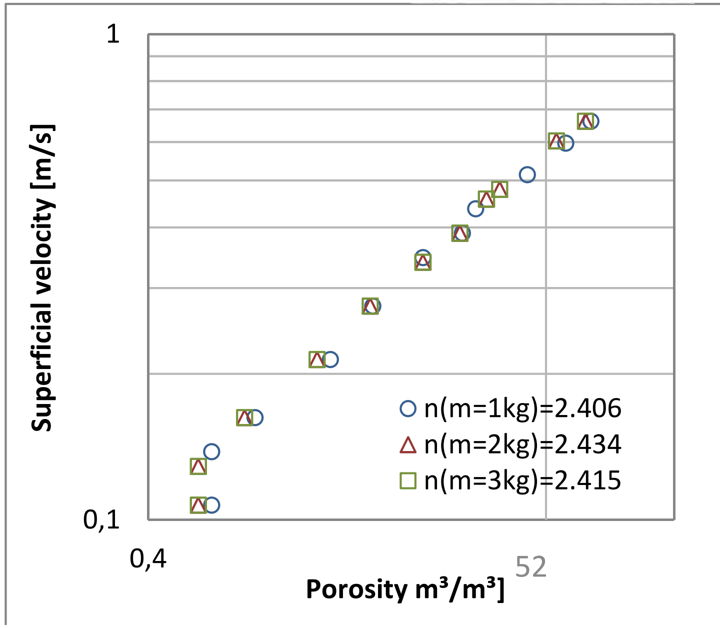
By J. F. RICHARDSON, Ph.D.* (ASSOCIATE MEMBER) and W. N. ZAKI, Ph.D.*

SUMMARY

The present work is concerned with the study of sedimentation and liquid-solid fluidisation. In the former, suspended solids are falling under the influence of gravity in a stationary fluid, while in the latter, the particles are kept in suspension by an upward flow of liquid.

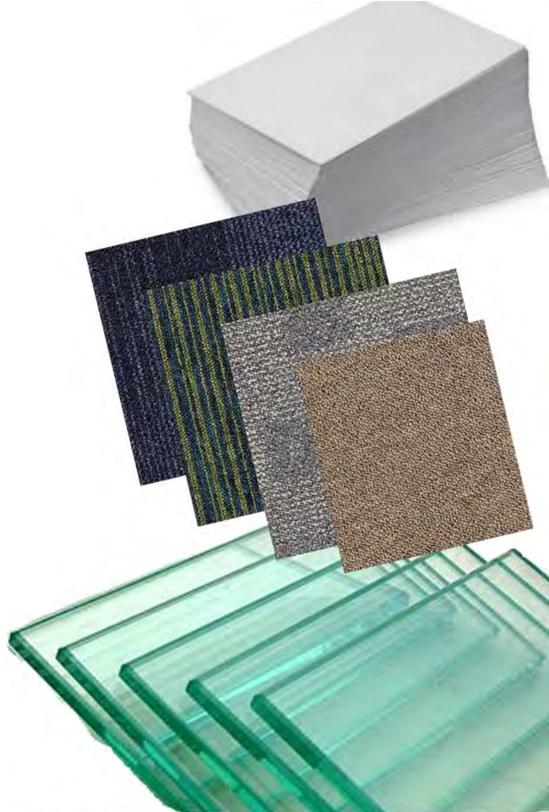
The object is to examine experimentally the effect of concentration of suspended particles upon their rate of settlement, and to find a satisfactory method of correlating the results. The present part of the experimental work has been confined to uniformly sized spherical particles, greater than 100 microns in diameter. As reported elsewhere, an attempt has been made to develop an expression, from theoretical considerations, for the rate of settling of suspensions, and the experimental results obtained in the present work are compared with those predicted from this theory. The work has been extended for comparison to liquid-solid fluidised systems.





Drinking water softening (circular economy)

✓ Profit: re-use calcite as a seeding material
  - Cost reduction: 100.000 €/year (0,4%)
  - Sustainability: 40.000 eco-points/year (5%)
  - Valorisation: high market segments: glass/paper/capet…
  - Vision: possibilities introduction of process cycles in industry
  - So much to learn…
    - Legislation
    - Hydraulic
    - LCA calculation



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Circular economy in drinking water treatment: reuse of ground pellets as seeding material in the pellet softening process

M. J. A. Schetters, J. P. van der Hoek, O. J. I. Kramer, L. J. Kors,
L. J. Palmen, B. Hofs and H. Koppers

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Overview

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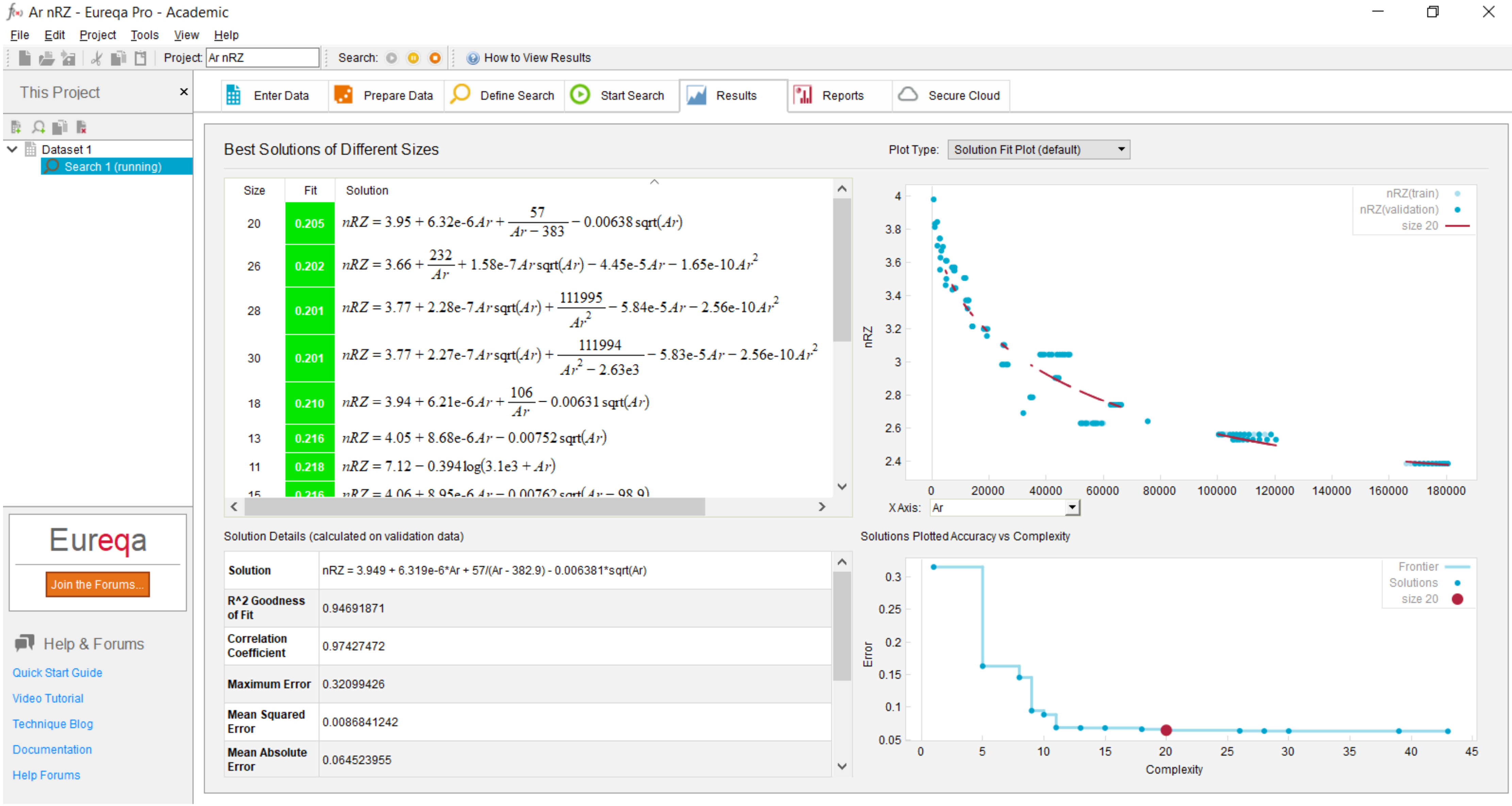
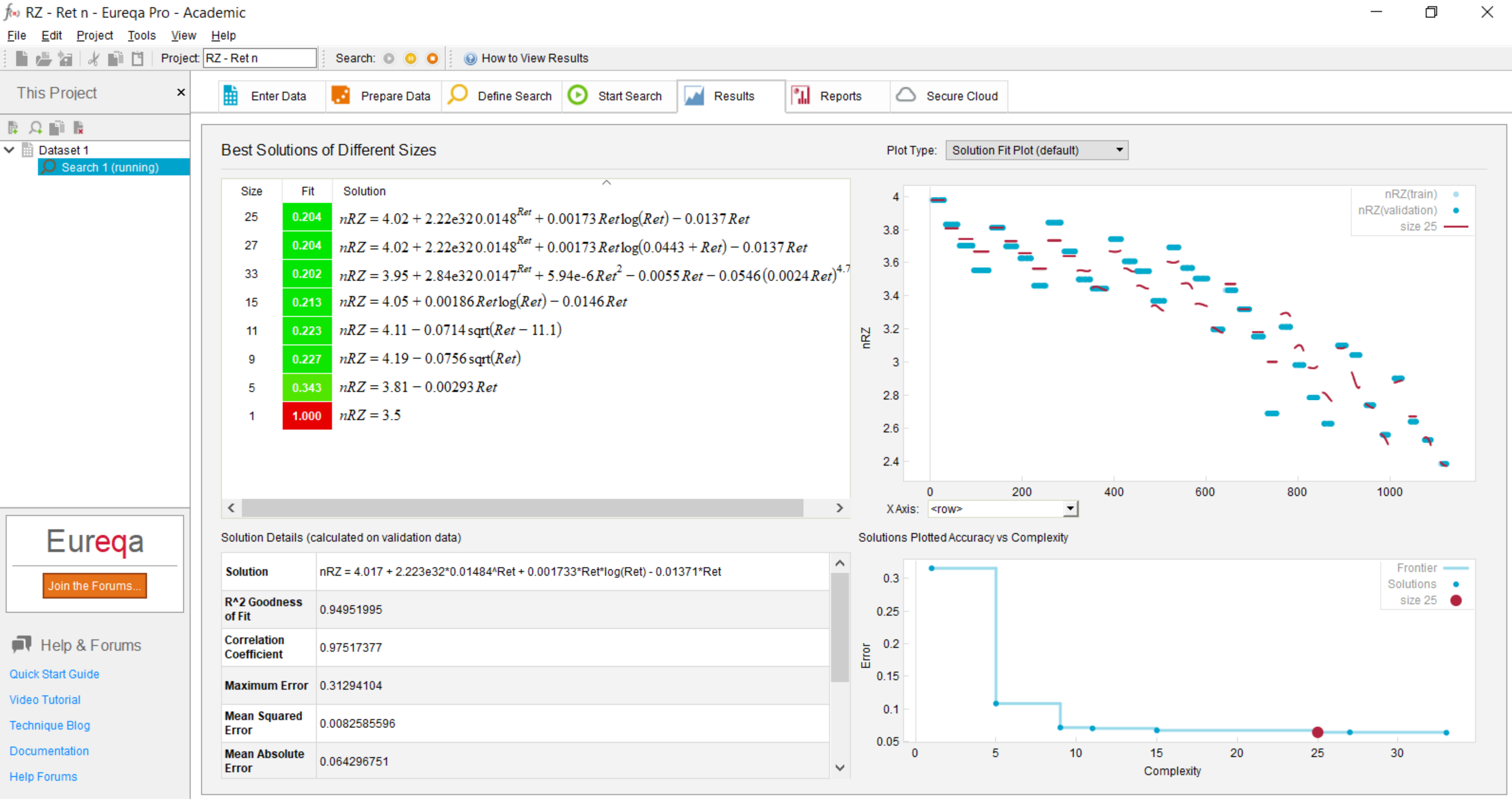
Symbolic regression: Archimedes number $\longrightarrow n_{RZ}$



Symbolic regression: Reynolds terminal number ⟶ n_{RZ}



CFD oppertunities

- Interstitial velocity versus terminal settling velocity
- Tortuosity versus ratio terminal and interstitial velocity
- Influence of the geometric representation (shape) on the specific surface area
- Particle interactions and collisions versus drag
- Relevant forces buoyancy, gravity and friction
- Surface roughness impact
- …

Any suggestions are welcome.
Please mail me at: o.j.i.kramer@tudelft.nl