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Towards understanding the impact of the evanescent elastodynamic mode coupling in Marchenko equation based de-multiple methods

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SUMMARY

Marchenko equation-based methods promise data-driven, true-amplitude internal multiple elimination. The method is exact in 1-D acoustic media, however it needs to be expanded to account for the presence of 2- and 3-D elastodynamic wavefield phenomena, such as compressional (P) to shear (S) mode conversions, total reflections or evanescent waves. Mastering high waveform-fidelity methods such as this, could further advance amplitude vs offset analysis and lead to improved reservoir characterization. This method-expansion may comprise of re-evaluating the underlying assumptions and/or appending the scheme with additional constraints (e.g. minimum phase). To do that, one may need to better understand the construction of the Marchenko equation solutions, the so-called focusing functions, in a mathematically simple and numerically stable fashion. The latter could be a challenge at large angles of incidence where the elastodynamic effects and evanescent waves start playing a dominant role. We demonstrate that the elastodynamic focusing functions are the bridge between the Marchenko equation theory and the transfer matrix formalism. Using the latter, we show how we can try to gain further insights into how time-reversal (correlations) behaves when either of the elastic modes becomes evanescent. We also show how this construction allows us to shed light on into the mathematical properties of elastodynamic inverse transmissions, which takes us a step closer towards understanding the elastodynamic minimum phase reconstruction.

INTRODUCTION

Advances in seismic data processing provide greater certainty about the subsurface, and at the same time continue blurring the lines between time processing, imaging and reservoir characterization. This blurring is particularly true for the inverse scattering methodologies such as those based on the Marchenko equation. This suite of methods (Wapenaar et al., 2021a) amounts to mapping the reflection response due to the entire medium, to an inverse transmission response due to just a part of it, a so-called focusing function. Subsequently, the latter or the de-reverberation operators (forward-propagated focusing functions) can be used to suppress multiples in a variety of ways (e.g. Dukalski and de Vos, 2022), without damaging primaries by e.g. adaptive subtraction. One could hope that this process, in particular attenuating the short-period multiples (Dukalski et al., 2019; Elison et al., 2020; Peng et al., 2021), could significantly contribute to improvements in the amplitude-versus-offset (AVO) analysis and in seismic inversion. This in turn could lead to an improved reservoir characterization for resource exploration or CO2 sequestration. In order to accomplish that however, one would need a robust elastodynamic Marchenko equation method (da Costa Filho et al., 2014; Wapenaar, 2014; Reinicke et al., 2020).

The Marchenko equation consists of three main ingredients: (1) a temporal mute, (2) an initial condition as well as (3) a time-reversal. These three try converting a single scattering relation featuring four wavefields, into a consistent and determined set of equations. In the 1-D acoustic case, with well separated reflectors, assumptions behind the conversion are met relatively easily. However, in elastic media they are not. For instance, implementing (1) can become difficult due to non-trivial, angle of incidence dependent, temporal overlaps between the focusing and the Green's functions (Reinicke et al., 2020). One of these overlaps defines the initial condition (2) and can be circumvented by exploiting the minimum-phase property of the de-reverberation operator. Despite recent advances on exploiting the minimum-phase property beyond single-mode wavefields (Elison et al., 2020; Peng et al., 2021), further research is needed to handle elastodynamic wavefields (Dukalski, 2020; Reinicke, 2020). At large incidence angles, in high velocity layers, one can observe increased P- and S-wave coupling as well as totally reflected and evanescent waves. Such waves are not universally time-reversed (ingredient 3) by complex conjugating the signal in the frequency domain. At large angles of incidence, the scheme is also prone to generating linear artefacts in time offset domain. This might be related to how the method handles evanescent waves, however whether or not these are handled correctly remains a debatable point in the literature between Diekmann and Vasconcelos (2021), and Kiraz et al. (2021), Wapenaar et al. (2021b) and Dukalski et al. (2022). In media without strongly dipping reflectors, this problem was addressed by reducing the maximum angle of incidence, e.g. via a wavenumber-frequency filter both in the acoustic (Elison et al., 2020; Reinicke and Dukalski, 2020) and the elastic media Reinicke et al. (2021), however that might impact one's ability to perform AVO analysis.

Bringing this technology another step closer to field data applications requires understanding the impact of these challenges. Here we show, how the elastodynamic extension of the work by Dukalski et al. (2022), could be used (a) to better understand the evanescent P- and S-waves and (b) to help explore the elastodynamic minimum-phase property. We hope that these insights would lead to similar impact as the work of Elison et al. (2021) had on the 2-D Augmented Marchenko presented by Peng et al. (2021).

THEORY

This section has four parts: (1) the one-way wave equation, (2) the scattering and transfer matrices and the focusing functions, (3) the so-called path reversal and its relation to time-reversals, and (4) the focusing functions modeling and their properties.

Elastodynamic one-way wave-equation

We consider lossless elastic horizontally-layered media where horizontal ray-parameters $p=(p_x^2+p_y^2)^{1/2}$ are decoupled. Here, the subscripts are associated with the spatial coordinates $\mathbf{x}=(x,y,z)$. According to the unified representation notation from Wapenaar et al. (2016), the (first order) wave equation can be written in a simple form

$$\partial_z \mathbf{q} - \mathscr{A}\mathbf{q} = \mathbf{d} \tag{1}$$

in the space-frequency domain ($\mathbf{x}-\omega$). The quantities $\mathbf{q}$ and $\mathbf{d}$, contain the two-way wavefield and source components, respectively. Choosing the z-axis as the preferential direction, we define the composition operator $\mathscr{L}$, which relates two-way fields ($\mathbf{q}$ and $\mathbf{d}$) to their up (-) and down (+) -going components, e.g.

$$\mathbf{q} = \mathscr{L}\mathbf{p} = \mathscr{L}\begin{pmatrix}\mathbf{p}^+\\ \mathbf{p}^-\end{pmatrix}, \quad \text{and} \quad \mathscr{A} = \mathscr{L}\mathscr{H}\mathscr{L}^{-1}. \tag{2}$$

The operator $\mathscr{H}$ is a 2×2 block matrix associated with the up and down-going fields which are the eigenbasis of the operator $\mathscr{A}$. Each block is composed of 2×2 matrices where the two columns (rows) represent P- and S-wave sources (receivers). Since these operators depend on the local medium parameters, they are defined for each layer, indicated by the subscript j ($\mathscr{H}_j$ and $\mathscr{L}_j$). The operator

$$\begin{aligned}\mathscr{W}_j &= \exp\left[i\mathscr{H}_j\left(z_j - z_{j-1}\right)\right],\\ &= \exp\left[i\,\mathrm{diag}\left[\mathfrak{h}_\mathbf{j}, -\mathfrak{h}_\mathbf{j}\right]\left(z_j - z_{j-1}\right)\right],\\ &\equiv \mathrm{diag}\left[\mathbf{w_j}, \mathbf{w_j}^{-1}\right],\end{aligned} \tag{3}$$

extrapolates the up- and downgoing wavefield components through the j[th] layer. Here $\mathfrak{h}_j = \mathrm{diag}\left[k_{z,P,j}, k_{z,S,j}\right]$ is a 2×2 diagonal matrix with the vertical wavenumbers for the P/S wave potentials $k_{z,P/S,j} = \omega\left(c_{P/S,j}^{-2} - p^2\right)^{1/2}$, which depend on the P/S wave propagation velocity in the j[th] layer $c_{P/S,j}$. Defining 4×4 matrices

$$\begin{aligned}\mathbf{Q}_1 &= \mathfrak{e}_{13}+\mathfrak{e}_{31}-\mathfrak{e}_{24}-\mathfrak{e}_{42},\\ \mathbf{Q}_2 &= \mathfrak{e}_{11}+\mathfrak{e}_{33}-\mathfrak{e}_{24}-\mathfrak{e}_{42},\\ \mathbf{Q}_3 &= \mathfrak{e}_{11}+\mathfrak{e}_{33}-\mathfrak{e}_{22}-\mathfrak{e}_{44},\end{aligned} \tag{4}$$

where $\mathfrak{e}_{mn}$ is 1 in mth row and nth column, we have

$$\begin{aligned}\mathscr{W}_j^* &= \mathbf{Q}_{1,j}\mathscr{W}_j\mathbf{Q}_{1,j}, &&\text{for} \quad \mathrm{Im}(k_{z,P/S,j}) = 0,\\ \mathscr{W}_j^* &= \mathbf{Q}_{2,j}\mathscr{W}_j\mathbf{Q}_{2,j}, &&\text{for} \quad \mathrm{Im}(k_{z,S,j}) = 0 = \mathrm{Re}(k_{z,P,j}),\\ \mathscr{W}_j^* &= \mathbf{Q}_{3,j}\mathscr{W}_j\mathbf{Q}_{3,j}, &&\text{for} \quad \mathrm{Re}(k_{z,P/S,j}) = 0.\end{aligned}$$

The case where $\mathrm{Re}(k_{z,S,j}) = 0 = \mathrm{Im}(k_{z,P,j})$, i.e. travelling P-wave and an evanescent S-wave, is of little interest since $c_{S,j} < c_{P,j}$. The matrices $\mathbf{Q}_{n,j}$ are also labeled per layer j. Whether or not $\mathscr{W}_j^{-1} = \mathscr{W}_j^*$ and what that means for the scattering transfer matrix elements will be the focus of the following sections.

Scattering/transfer matrices and focusing functions

The scattering and transfer (Born and Wolf, 1965) matrices (S and $\mathscr{T}$) relate the one-way fields in the top $\left(\mathbf{p}_1^\pm\right)$ and bottom $\left(\mathbf{p}_N^\pm\right)$ layers according to

$$\begin{pmatrix}\mathbf{p}_N^+\\ \mathbf{p}_1^-\end{pmatrix} = \begin{pmatrix}\mathbf{T}^\downarrow & \mathbf{R}^\cap\\ \mathbf{R}^\cup & \mathbf{T}^\uparrow\end{pmatrix}\begin{pmatrix}\mathbf{p}_1^+\\ \mathbf{p}_N^-\end{pmatrix} \equiv S\begin{pmatrix}\mathbf{p}_1^+\\ \mathbf{p}_N^-\end{pmatrix}, \tag{5}$$

and

$$\begin{pmatrix}\mathbf{p}_N^+\\ \mathbf{p}_N^-\end{pmatrix} = \begin{pmatrix}\mathbf{A} & \mathbf{B}\\ \mathbf{C} & \mathbf{D}\end{pmatrix}\begin{pmatrix}\mathbf{p}_1^+\\ \mathbf{p}_1^-\end{pmatrix} \equiv \mathscr{T}\begin{pmatrix}\mathbf{p}_1^+\\ \mathbf{p}_1^-\end{pmatrix}. \tag{6}$$

In equation 5 we have labeled the reflection and transmission responses due to sources above (below) the medium with $\mathbf{R}^\cup$ ($\mathbf{R}^\cap$) and $\mathbf{T}^\downarrow$ ($\mathbf{T}^\uparrow$) respectively. To study the form of operators $\mathbf{A}$ to $\mathbf{D}$ (see equation 6), we exploit the continuity condition of the two-way fields $\mathscr{L}_{j+1}\mathbf{p}_{j+1} = \mathscr{L}_j\mathbf{p}_j$ and use the extrapolation operator $\mathscr{W}_j$ to propagate the fields between the interfaces of the layers. This allows us to write the transfer matrix and its inverse as

$$\mathscr{T} = \mathscr{K}_{N-1}\mathscr{K}_{N-2}\cdots\mathscr{K}_2\mathscr{K}_1, \tag{7}$$

and

$$\mathscr{T}^{-1} = \mathscr{K}_1^{-1}\mathscr{K}_2^{-1}\cdots\mathscr{K}_{N-2}^{-1}\mathscr{K}_{N-1}^{-1}, \tag{8}$$

where $\mathscr{K}_j = \mathscr{L}_{j+1}^{-1}\mathscr{L}_j\mathscr{W}_j \equiv \mathscr{M}_j\mathscr{W}_j$. The operators $\mathscr{T}$ as well as $\mathscr{T}^{-1}$ exist provided that the eigenvector matrices $\mathscr{L}_k$ are non-singular, which holds except for horizontally-travelling waves. We can also write

$$\mathscr{T}^{-1} = \begin{pmatrix}\mathbf{X}^{-1} & -\mathbf{Y}\\ -\mathbf{D}^{-1}\mathbf{C}\mathbf{X}^{-1} & \mathbf{D}^{-1}+\mathbf{D}^{-1}\mathbf{C}\mathbf{Y}\end{pmatrix}, \tag{9}$$

where $\mathbf{X} = \mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C}$ and $\mathbf{Y} = \mathbf{X}^{-1}\mathbf{B}\mathbf{D}^{-1}$. Since $\mathscr{T}^{-1}$ exists, then it will take the above form given $\mathbf{D}^{-1}$ exists, in which case $\mathbf{X}^{-1}$ exist. The alternative formula for the inverse of a block matrix, asserts the existence of $\mathbf{A}^{-1}$.

We can now make the connection between the scattering matrix, the (inverse of the) transfer matrix, and the focusing functions. Rearranging equations 5 and 6 we obtain

$$\begin{pmatrix}\mathbf{T}^\downarrow & \mathbf{R}^\cap\\ \mathbf{R}^\cup & \mathbf{T}^\uparrow\end{pmatrix} = \begin{pmatrix}\mathbf{X} & \mathbf{B}\mathbf{D}^{-1}\\ -\mathbf{D}^{-1}\mathbf{C} & \mathbf{D}^{-1}\end{pmatrix}, \tag{10}$$

which means that the inverses of the transmissions $\mathbf{T}^\downarrow$ and $\mathbf{T}^\uparrow$ exist (waves travelling horizontally anywhere in the medium were excluded in the derivation). Moreover, we can identify that

$$\mathbf{D} = \mathbf{T}^{\uparrow -1} \equiv \mathbf{f}_2^- \quad \text{and} \quad \mathbf{B} = \mathbf{R}^\cap\mathbf{T}^{\uparrow -1} \equiv \mathbf{f}_2^+, \tag{11}$$

are the up- and down- going components of the Marchenko focusing function $\mathbf{f_2}$, i.e., the focusing function that focuses at $z=0$.

Using the form of $\mathbf{B}$ and $\mathbf{D}$ for a single reflector, and the form $\mathscr{L}_j$ from Wapenaar and Berkhout (1989) to identify operators $\mathbf{A}$ and $\mathbf{C}$, one can show that

$$\mathscr{K}_j = \begin{pmatrix}\sigma_\mathbf{z}\mathbf{t}_\mathbf{j}^{\uparrow -1}\sigma_\mathbf{z}\mathbf{w_j} & \mathbf{r}_\mathbf{j}^\cap\mathbf{t}_\mathbf{j}^{\uparrow -1}\mathbf{w_j}\\ \sigma_\mathbf{z}\mathbf{r}_\mathbf{j}^\cap\mathbf{t}_\mathbf{j}^{\uparrow -1}\sigma_\mathbf{z}\mathbf{w_j} & \mathbf{t}_\mathbf{j}^{\uparrow -1}\mathbf{w_j}\end{pmatrix}, \tag{12}$$

where $\mathbf{t}_\mathbf{j}^\uparrow$ and $\mathbf{r}_\mathbf{j}^\cap$ are the matrices of transmission and reflection coefficients in the PS space and where $\sigma_\mathbf{z} = \mathrm{diag}[1,-1]$. This form remains unchanged whether the fields are pressure or flux normalized. We can define now the path reversal $\mathscr{P}$ as

$$\mathscr{P}\left[\mathbf{w_j}\right] = \mathbf{w_j}^{-1}, \quad \text{and} \quad \mathscr{P}\left[\mathfrak{m}\right] = \sigma_\mathbf{z}\mathfrak{m}\sigma_\mathbf{z}, \tag{13}$$

with $\mathfrak{m} = \mathbf{t_j^{\uparrow}}$, or $\mathbf{r_j^{\cap}}$, or their products and inverses (if such exist). The path reversal is equivalent to the sign-reversal of the ray-parameter p and inverting the extrapolation operator $\mathbf{w_j}$. Naturally $\mathscr{P}[\mathscr{P}[X]] = X$ holds. Path reversal introduced in equation 13 is a generalization of time reversal. The path reversal, is a context-dependent operation which reverses the phase shift for the traveling waves and exponential decay for the evanescent ones. In comparison, the time reversal, only reverses the phase shift, which is typically implemented using complex conjugation in the frequency domain or when combining fields, a convolution is replaced with a correlation. This is why (without undertaking additional steps – e.g. Wapenaar, 2020) as shown by Dukalski et al. (2022), when dealing with evanescent waves, correlations no longer appear to be the correct approach. The latter as well as the companion paper by Wapenaar et al. (2022), seems to be however concerned mainly with whether or not the waves are travelling or evanescent in the top and bottom layers. We will address that in the next section. Before that, we notice that using the form of $\mathscr{K}_j$ in equations 7, and 12, we can show that,

$$\mathscr{T} = \begin{pmatrix} \mathscr{P}\left[\mathbf{f_2^-}\right] & \mathbf{f_2^+} \\ \mathscr{P}\left[\mathbf{f_2^+}\right] & \mathbf{f_2^-} \end{pmatrix}, \quad \text{and} \quad \mathscr{T}^{-1} = \begin{pmatrix} \mathbf{f_1^+} & \mathscr{P}\left[\mathbf{f_1^-}\right] \\ \mathbf{f_1^-} & \mathscr{P}\left[\mathbf{f_1^+}\right] \end{pmatrix}, \tag{14}$$

with

$$\mathbf{f_1^-} = \mathbf{R^{\cup}T^{\downarrow-1}}, \quad \text{and} \quad \mathbf{f_1^+} = \mathbf{T^{\downarrow-1}}, \tag{15}$$

(the focusing function that focuses in the Nth layer), such that $\mathscr{P}[\mathscr{T}] = \mathbf{Q}_1 \mathscr{T} \mathbf{Q}_1$. This result is consistent with the result of Wapenaar et al. (2022), who choose to describe the path reversal as a combination of complex conjugation and a wavefield modeling with $\mathscr{H}^*$.

Path reversal vs time reversal – in bulk and at the boundaries

In the following derivation, we show that the apparent need for path reversal in bulk (e.g. with tunneling through a high velocity layer as an evanescent wave) is compensated by scattering coefficients, such that only the type of wave in the top or bottom layer is of importance. We observe that elements of the simplest, single-reflector transfer matrix $\mathscr{T} = \mathscr{K}_j$ from equations 7 and 12, observe

$$\mathscr{M}_j = \mathbf{Q}_{1,j} \mathscr{M}_j \mathbf{Q}_{1,j}\,, \tag{16}$$

and

$$\mathscr{P}[\mathscr{W}_j] = \mathscr{W}_j^{-1} = \mathbf{Q}_{1,j} \mathscr{W}_j \mathbf{Q}_{1,j}\,. \tag{17}$$

The equivalence between path and time reversal will be established if we can show that for a large product of links $\mathscr{K}_j$, i.e. for the full transfer matrix we have

$$\mathscr{T}^* = \mathbf{Q}_{1,j=N-1} \mathscr{T} \mathbf{Q}_{1,j=1} = \mathscr{P}[\mathscr{T}]\,. \tag{18}$$

Each link $\mathscr{K}_j$, when complex conjugated will be compensated by its own set of operators $\mathbf{Q}_{n,j}$ with $n = 1,2,3$ and $j = 1,,N-1$, which will be reabsorbed either by cancelation with its neighbors in the product, or using the properties above. It is easy to check that pressure normalized $\mathscr{M}_j$ obeys

$$\mathscr{M}_j^* = \mathbf{Q}_{b,j} \mathscr{M}_j \mathbf{Q}_{a,j}\,, \tag{19}$$

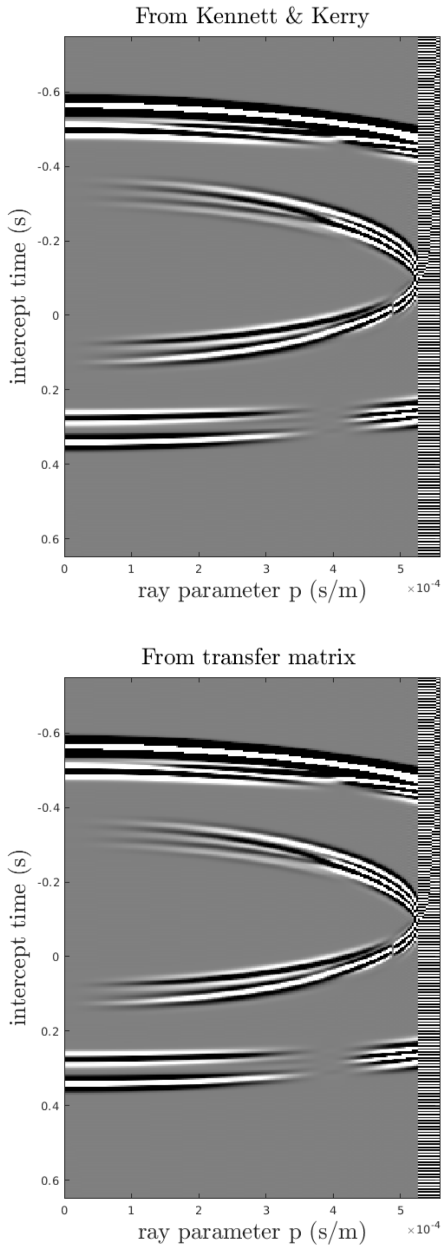

Figure 1: Results of modeling $\mathbf{f_1^+}$ using the Kennett & Kerry method (top) and the transfer matrix (bottom), in a four layer medium with $c_P = [1.9, 2.5, 1.9, 2.1]$km/s, $c_S = [0.9, 1.2, 1.0, 1.2]$km/s, densities $\rho = [2.1, 3.2, 2.2, 2.5]$Mg/m^3 and layer depts $z = [100, 40, 420, 720]$m. SS component is shown, with expected $4^{3-1} = 16$ events, with the figure showing 8 double events, as the 2nd layer (40m) makes for unresolvable pairs of events using a 30Hz Ricker wavelet. For $p > 1/1900$ s/m (evanescent in the top layer), amplitudes grow exponentially large while favouring high frequencies.

where we have a separate label $a(b)$ to denote the layer above (below), with action from right (left) such that

$$\begin{array}{ll} a=1 & \mathrm{Im}(k_{z,S,j}) = 0 = \mathrm{Im}(k_{z,P,j})\,, \\ a=2 & \mathrm{Im}(k_{z,S,j}) = 0 = \mathrm{Re}(k_{z,P,j})\,, \\ a=3 & \mathrm{Re}(k_{z,S,j}) = 0 = \mathrm{Re}(k_{z,P,j})\,, \end{array} \tag{20}$$

and the same holds for b if we replace j with $j+1$. This means that $\mathscr{K}_j^* = \mathbf{Q}_{b,j}\mathscr{K}_j\mathbf{Q}_{a,j}$, under the same conditions. Furthermore, the b type in $\mathbf{Q}_{b,j}$, must coincide with the a type in $\mathbf{Q}_{a,j+1}$ on $\mathscr{K}_{j+1}^*$, because they are both dependent on whether $k_{z,P,j+1}$ or $k_{z,S,j+1}$, are real or imaginary. This means that the $\mathbf{Q}_{b,m}$ and $\mathbf{Q}_{a,m+1}$ at any location in the chain of products of matrices will have to cancel out (their product is an identity), leaving

$$\mathscr{T}^* = \mathbf{Q}_{b,N-1}\mathscr{T}\mathbf{Q}_{a,1}\,. \tag{21}$$

This is equivalent to the path reversal if both $a = b = 1$, i.e. if the waves in the top and bottom layer are propagating. This means that it does not matter if either of the waves is evanescent anywhere inside the medium, but if they are at the boundaries, then that suggest that time reversal has to be replaced with path-reversal in the scattering relations and hence inside the Marchenko equation (Dukalski et al., 2022). This suggests that a direct evanescent modes measurement cannot be recovered with the convolve-correlate type Marchenko method.

Focusing functions modeling and their properties

Transfer matrix formalism can be used to gain further insight into the mathematical properties of solutions to the Marchenko equation – an underconstrained inverse problem we wish to solve, before we use its solutions to suppress internal multiples. A common modeling strategy inverts the Kennett and Kerry (1979) relations to find

$$\mathbf{f_1^+} = \mathbf{T_j^{\downarrow -1}} = \mathbf{T_{j-1}^{\downarrow -1}}\left(1 - \mathbf{R_{j-1}^{\cap -1}}\mathbf{w_j}\mathbf{r_j^{\cup}}\mathbf{w_j}\right)\mathbf{w_j}^{-1}\mathbf{t_j^{\downarrow -1}}\,, \tag{22}$$

which, on addition of a reflector requires updating

$$\mathbf{R_j^{\cap}} = \mathbf{r_j^{\cap}} + \mathbf{t_j^{\downarrow}}\mathbf{w_j}\left(1 - \mathbf{R_{j-1}^{\cap}}\mathbf{w_j}\mathbf{r_j^{\cup}}\mathbf{w_j}\right)^{-1}\mathbf{R_{j-1}^{\cap}}\mathbf{w_j}\mathbf{t_j^{\uparrow}}\,. \tag{23}$$

This requires computing the term $\left(1 - \mathbf{R_{j-1}^{\cap}}\mathbf{w_j}\mathbf{r_j^{\cup}}\mathbf{w_j}\right)^{-1}$, which amounts to inverting a potentially close-to-singular operator, especially in acoustic media when at large p we deal with total reflections and evanescent wave tunneling through thin beds. Calculating (the inverse of) the transfer matrix is not only more convenient, but also faster and more stable (no such inverses need to be calculated). In Figure 1 we show a numerical experiment for elastic media, where the problem is less severe at large p and both the transfer matrix and Kennett methods give close-to-identical results. The transfer matrix method, however, gives an easy access to some properties of the focusing functions, e.g. equations 7, 12 and 14 show that the number of events in each component of the focusing function is given by the dimension of $\mathscr{K}_j$ to the power of the number of reflectors minus one.

The transfer matrix construction could allow extending the analysis presented by Sherwood and Trorey (1965) to elastodynamic waves, however the elastodynamic minimum-phase property is actually observed by the de-reverberation operator $\mathbf{v}^+ = \mathbf{T}^{\downarrow -1}\mathbf{T_{dir}^{\downarrow -1}}$, (Dukalski, 2020; Reinicke, 2020). This requires calculating the early part of the transmission $\mathbf{T_{dir}^{\downarrow -1}}$, which can be found by setting off-diagonal blocks (or even all off-diagonal elements) of $\mathscr{M}_j$ to zero and then analytically inverting the result.

CONCLUSIONS

We have shown the relationship between the scattering and transfer matrices as well as focusing functions in the context of elastodynamic and evanescent waves. We expect that this work will enable studying the elastodynamic minimum-phase property to advance the Marchenko equation de-multiple methods, and understand the performance of the latter in elastic media including the evanescent mode spectrum, and pave the way towards improved AVO and reservoir characterization.

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