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Summary

Rigid boom electromagnetic surveys that use coil-coil configurations are often used to obtain information about the subsurface conductivity. Semi-analytic solutions help to simulate electromagnetic induction measurements for a large number of horizontally layered models, which can then be stored and used as a lookup table. This procedure is performed once and then used to find the corresponding model that produces the best data fit, eliminating the need for running numerous simulations in every minimization step of an inversion scheme for large field datasets. We apply this methodology to a numerical example and field data acquired in The Netherlands. Our results from both cases using the global search demonstrate its ability to estimate electrical conductivity distributions in two-layered models in a fast and accurate manner. Furthermore, we apply the workflow using a lookup table based on low induction number approximation-derived measurements. The outcome of implementing this methodology using the low induction number lookup table shows poor accuracy in the electrical conductivity estimations for both the numerical example and the field data in comparison to the semi-analytical approach.

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Introduction

Electromagnetic induction (EMI) instruments are commonly used to estimate the electrical conductivity of the shallow subsurface for hydrogeological, environmental, agricultural, and archeological purposes. McNeill (1980) proposed a way to derive linearly an approximated value of the apparent electrical conductivity σ_a of a horizontally layered subsurface from EMI measurements, which is called the low induction number (LIN) approximation. This methodology assumes two main conditions: 1) the low induction number (LIN) condition $\beta << 1$, where the induction number is defined as $\beta = s/\delta$, s is the coil-coil separation and $\delta = \sqrt{\omega\mu_0\sigma/2}$ is the skin depth (ω is the angular frequency of the source, μ_0 is the magnetic permeability of free space), 2) the instrument lays on the ground surface. From the LIN approach McNeill (1980) also defined based on Wait (1962) sensitivity functions that calculate for horizontally layered models the contributions of each layer thickness and electrical conductivity to the apparent electrical conductivity. Subsequently, due to its simplicity and linearity, numerous inversion schemes to estimate electrical conductivity vertical distribution profiles using McNeill (1980) sensitivity functions have been developed (e.g. Hendrickx et al. (2002); Monteiro Santos et al. (2011)). However, previous studies suggest that using the LIN approximation approach, even under the LIN conditions provides inaccurate estimations of electrical conductivity in the subsurface. For example, Callegary et al. (2007) compared the sensitivity of the apparent electrical conductivity with respect to the vertical distribution using LIN approximation and Maxwell's equations for two-layered models and observed clear differences. Additionally, von Hebel et al. (2019) performed a multi-layered inversion using both the LIN approximation and Maxwell's exact solutions as forward models, finding erroneous electrical conductivity estimations in the LIN approximation case. Using the full non-linear solution from Maxwell's equations for horizontally layered models can be computationally expensive to perform in an inversion scheme that needs to calculate many forward models in each minimization step. In this study, we use the semi-analytical modeler *empymod* (from Werthmüller (2017)) that provides the exact magnetic response due to a three-dimensional magnetic source in a layered-earth model with vertical transverse isotropic (VTI) electrical conductivity. This modeler provides us with the opportunity to calculate the measurements for a large range of models and create a lookup table that can be used in a global search to find the electrical conductivity model that better explains the EMI measurements. For completeness, we also use the LIN approximation function to create a lookup table and perform the global search methodology. The workflow using both semi-analytical and LIN lookup tables is applied on a numerical example and in data acquired for hydrogeophysical evaluation in Ackerdijkse Plassen, The Netherlands showing satisfactory results for a precise estimation of the electrical conductivity in the shallow subsurface using the semi-analytical approach.

Methodology

The basic EMI instrument consists of a pair of coils: a transmitter coil (T_x) and a receiver coil (R_x). The system uses as a source an alternating current flowing in the transmitter coil that generates a primary magnetic field H_p. This time-varying flux produces an electromotive force that induces eddy currents in conductive materials of the subsurface. The eddy currents then generate a secondary magnetic H_s. Both H_p and H_s fluxes go through the receiver coil. Consequently, a voltage is induced in the receiver coil (measuring then the time-varying magnetic flux). Since the primary field H_p is known, EMI instruments can provide the coupling ratio between the secondary magnetic field induced in the ground and the primary magnetic field from the direct wave propagating from the transmitter through the air to the receiver coil, which is a complex ratio ($H_s/H_p = IP + iQ$), where IP represents the in-phase part and Q represents the out-of-phase or quadrature part.

Semi-analytic (SA) forward model

From Ward and Hohmann (1987), for EMI sensors where the distance between the transmitter and receiver coils is at least five times larger than the radius of the coils, the coils are considered magnetic dipoles. Ward and Hohmann (1987) derived the formulations for the axial component of the magnetic field generated by a magnetic dipole assuming a horizontally layered subsurface. The derived equations for the horizontal coplanar (H), vertical coplanar (V), and perpendicular (P) coil orientations placed at an elevation h are shown in the following equations:

$$\begin{aligned} Z^H &= \frac{H_z^s}{H_z^p} = -s^3 \int_0^\infty r_{\mathrm{TE}} e^{-2\lambda h} \lambda^2 J_0(\lambda s) d\lambda, \\ Z^V &= \frac{H_x^s}{H_x^p} = -s^2 \int_0^\infty r_{\mathrm{TE}} e^{-2\lambda h} \lambda J_1(\lambda s) d\lambda, \\ Z^P &= \frac{H_x^s}{H_z^p} = s^3 \int_0^\infty r_{\mathrm{TE}} e^{-2\lambda h} \lambda^2 J_1(\lambda s) d\lambda, \end{aligned} \tag{1}$$

where λ is the Hankel transform integral parameter, r_{TE} is the reflection coefficient, H_s and H_p represent the primary and secondary fields, J_0 and J_1 are the Bessel functions of zeroth and first order respectively. With the SA forward model we compute for H_p and H_s and calculate the ratios to obtain Z^H,Z^V and Z^P, obtaining Q and IP values. The equations were solved for the following coil configurations: 3 coil geometries (H, V, and P) with three different offsets each (2, 4, 8 m for H and V coils; 2.1, 4.1, 8.1 m for the P coil) at a frequency of 9000 Hz. EMI data was modeled for $n = 226981$ electrical conductivity 2 layered models with a maximum thickness of the first layer h_1 of 10 m. The electrical conductivity for each layer ranged from 1 mS/m up to 100 mS/m, where the values for the proposed coil geometry fulfill the LIN conditions. Therefore, the models are formulated by two layers where the parameters are the logarithms of the conductivity and the thicknesses $m = [log(\sigma_1), log(\sigma_2), h_1]$. The data generated for each model contains the values of Q and IP for each coil geometry in ppt $d = [Q_H, Q_V, Q_P, IP_H, IP_V, IP_P]$. The EMI data d was computed and stored in the lookup table $D_{SA} = [d_1, d_2, ..., d_n]$, which is then used in the global search. The workflow for the methodology is shown in Figure 1 where this stage is shown as step 1: Forward operation.

Low induction number (LIN) forward model

McNeill (1980) proposed that equations in 1 can be modified to a linear relationship between the apparent conductivity of the subsurface σ_a and the imaginary part of the ratio $\Im(H_s/H_p) = Q$ for low induction numbers as follow:

$$\sigma_a^{LIN} = \frac{4}{\mu_0 \omega s^2} Q. \tag{2}$$

This approximation is defined for coils directly laying on the ground surface. From the LIN approximations the vertical distribution of the electrical conductivity is linearly related to cumulative sensitivity functions derived in McNeill (1980) from Wait (1962). Taking into account these functions, then the response for 2 layered electrical conductivity models is calculated by adding the contribution from each layer, weighting the sensitivity according to the cumulative sensitivity functions CS:

$$\sigma_a^{LIN} = \sigma_1 [CS(r_1)] + \sigma_2 [1 - CS(r_1)]. \tag{3}$$

We used this equation to repeat step 1 Forward operation and obtain for $n = 226981$ possible models the lookup table $D_{LIN} = [d_1, d_2, ..., d_n]$.

Global search

After storing the solutions computed in step 1 of the workflow (see Figure 1) we perform for each position i in a survey a global search that minimizes the misfit $min(d - d_i)$ between the measurement d_i and all the measurements d stored in lookup table D. The minimum misfit yields the best electrical conductivity model σ_i.

Numerical example

A two-layered numerical earth model was created in step 2 of the workflow to test the applicability of the global search. The global search in step 3 (Figure 1) was applied for each position in the 2D section of stitched 1D earth models using both lookup tables D_{SA} and D_{LIN} separately. In Figure 2 top left the

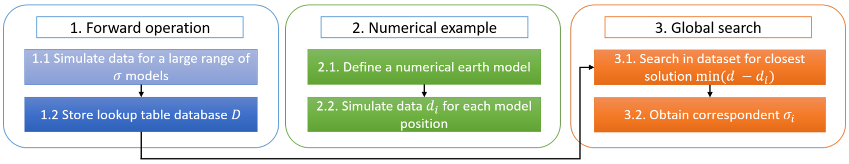

***Figure 1** Global search workflow*

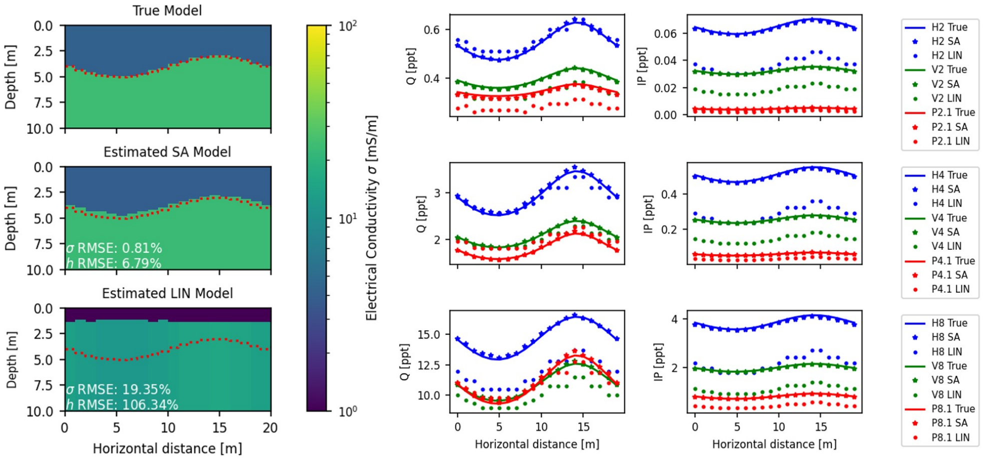

***Figure 2** Electrical conductivity estimated models for a numerical example. Top left: True model. Middle left: model estimated using D_{SA}. Bottom left: model estimated using D_{LIN}. Right: Data (Q and IP) simulated using SA forward for each model and coil configuration*

true numerical model used to test the methodology is shown, below the resulting estimations using both lookup tables D_{SA} and D_{LIN} are presented. The model estimation returned by the global search using D_{SA} has a better fit compared to the model estimation returned when using D_{LIN}. In the right of Figure 2 we show the data simulated with the SA forward function for the true model and the estimated models. The model estimated using D_{LIN} cannot accurately reproduce the data measurements in Q or IP.

Case study: Ackerdijkse Plassen

We applied the methodology for EMI data acquired in the natural monument Ackerdijkse Plassen, The Netherlands. In this case, we used only the H and V coil geometries in the methodology. The global search was performed using both lookup tables D_{SA} and D_{LIN} separately. The estimated electrical conductivity models are shown in Figure 3. On the right side of Figure 3 we show the comparison of the

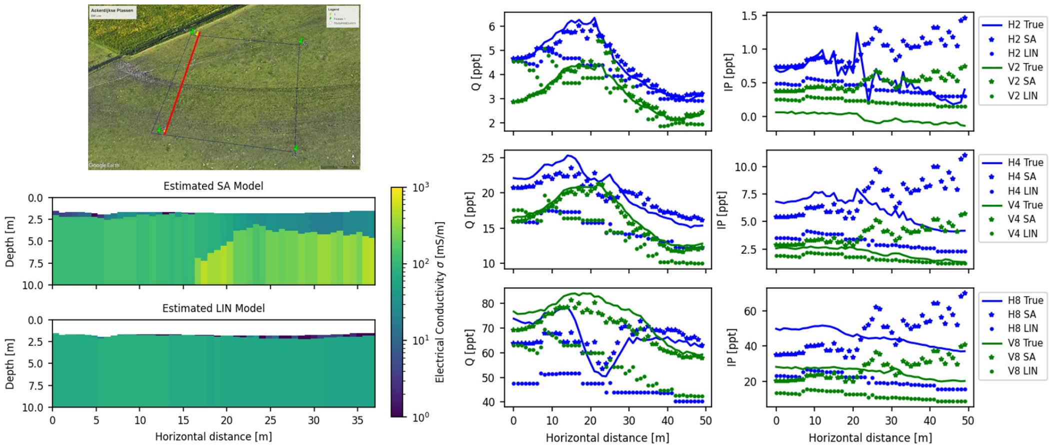

***Figure 3** Case study: Ackerdijkse Plassen. Top Left: Satellite image of location (EMI line acquired in red). Bottom Left: Models estimated with D_{SA} and D_{LIN}. Right: Field data and simulated data*

acquired data and the simulated data using the SA forward approach for each estimated model. The Q data simulated for the estimated model using D_{SA} has a better fitting with the measured Q data, with respect to the simulated Q data for the estimated model using D_{LIN}. However, the IP data for both model estimations do not have a good fit with the field data. For offsets up to 4 m the IP values are considered less accurate according to the EMI instrument manufacturer (Taylor (2023)). Additionally, there could be 3D effects in the field data that are not accounted for in equations 1 and 2. It is also important to consider that the electrical conductivities measured in the field showed values of σ_a where the LIN conditions are not satisfied for the $H8$ coil configuration. Generating and storing the lookup table for the examples shown only took minutes. Afterward, finding the best fitting data for one position in the lookup table can be accomplished in just 7.68 milliseconds. In our numerical example, we estimated the electrical conductivity model using a dataset with 20 horizontal distance positions, which only took 70.78 milliseconds. For the field example, we utilized only the H and V coil geometries in the data measurements. Estimating the 50 positions presented in this case only took 118.90 milliseconds. Our computations were performed using an Intel(R) Core(TM) i7-8665U CPU with 8 cores.

Conclusions

We used a semi-analytical modeler to simulate the exact response of EMI measurements in a layered earth and obtain a lookup table to be used in a global search workflow. After computing the lookup table to find the best fitting measurement for one position can be done in milliseconds. This method yields a fast estimation for horizontal two-layered models of the subsurface which can be easily applied in field datasets. Furthermore, the application of the semi-analytical approach to the numerical example reproduced accurately the electrical conductivity model. We compared the results of using the semi-analytical simulated measurements with respect to using LIN approximation functions to generate the lookup table. The global search methodology using the LIN approximation lookup table showed a poor estimation in the case of two-layered models even when the LIN conditions are satisfied.

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References

Callegary, J.B., Ferré, T.P.A. and Groom, R.W. [2007] Vertical Spatial Sensitivity and Exploration Depth of Low-Induction-Number Electromagnetic-Induction Instruments. *Vadose Zone Journal*, **6**(1), 158–167.

Hendrickx, J., Borchers, B., Corwin, D., Lesch, S., Hilgendorf, A. and Schlue, J. [2002] Inversion of soil conductivity profiles from electromagnetic induction measurements: Theory and experimental verification. *Soil Science Society of America Journal*, **66**(3), 673 – 685. Type: Article.

McNeill, J.D. [1980] Electromagnetic terrain conductivity measurement at low induction numbers. Publisher: Geonics Limited Mississauga, ON, Canada.

Monteiro Santos, F.A., Triantafilis, J. and Bruzgulis, K. [2011] A spatially constrained 1D inversion algorithm for quasi-3D conductivity imaging: Application to DUALEM-421 data collected in a riverine plain. *GEOPHYSICS*, **76**(2), B43–B53.

Taylor, R. [2023] EM Induction https://dualem.com/documents/em-induction/.

von Hebel, van der Kruk, Huisman, Mester, Altdorff, Endres, Zimmermann, Garré and Vereecken [2019] Calibration, Conversion, and Quantitative Multi-Layer Inversion of Multi-Coil Rigid-Boom Electromagnetic Induction Data. *Sensors*, **19**(21), 4753.

Wait, J.R. [1962] A note on the electromagnetic response of a stratified earth. *GEOPHYSICS*, **27**(3), 382–385.

Ward, S.H. and Hohmann, G.W. [1987] 4. Electromagnetic Theory for Geophysical Applications.

Werthmüller, D. [2017] An open-source full 3D electromagnetic modeler for 1D VTI media in Python: empymod. *GEOPHYSICS*, **82**(6), WB9–WB19.