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Chapter 10
Computationally Tractable Reserve Scheduling for AC Power Systems with Wind Power Generation

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Abstract This work presents a solution method for a day-ahead stochastic reserve scheduling (RS) problem using an AC optimal power flow (OPF) formulation. Such a problem is known to be non-convex and in general hard to solve. Existing approaches follow either linearized (DC) power flow or iterative approximation of nonlinearities, which may lead to either infeasibility or computational intractability. In this work we present two new ideas to address this problem. We first develop an algorithm to determine the level of reserve requirements using vertex enumeration (VE) on the deviation of wind power scenarios from its forecasted value. We provide a theoretical result on the level of reliability of a solution obtained using VE. Such a solution is then incorporated in OPF-RS problem to determine up- and down-spinning reserves by distributing among generators, and relying on the structure of constraint functions with respect to the uncertain parameters. As a second contribution, we use the sparsity pattern of the power system to reduce computational time complexity. We then provide a novel recovery algorithm to find a feasible solution for the OPF-RS problem from the partial solution which is guaranteed to be rank-one. The IEEE 30 bus system is used to verify our theoretical developments together with a comparison with the DC counterpart using Monte Carlo simulations.

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10.1 Introduction

The reserve scheduling (RS) problem deals with day-ahead scheduling of the reserve power to accommodate possible mismatches between forecasted and actual wind power. Stochastic variants of the RS problem, where violations of operational constraints, e.g. power generations, bus voltage and line flow constraints, are allowed with a small probability to achieve better performance, have received a lot of attention in the past few years, see [1–4] and the references therein. A stochastic RS problem is typically formulated using a lossless DC model based on the assumption of constant voltage magnitudes and small voltage angles, while ignoring the active power losses [5]. These assumptions do not hold in general and may lead to sub-optimality or even infeasibility when implemented on real world systems, especially for networks under a high degree of stress [6].

Using an AC model of the power network enables the stochastic RS formulation to accurately model the effect of large deviations of wind power from its forecasted value, and to offer a-priori suitable reserves such that both active and reactive power, and complex-valued voltage are globally optimal. Due to the non-convexity of the OPF problem, identifying such an optimal operating point of a power system may not be straightforward. In [7], different reformulations and relaxations of the AC OPF problem were presented and their connections were discussed. By means of semidefinite programming (SDP), in [7] a convex relaxation was provided under the existence of a rank-one SDP solution to guarantee the recovery of an optimal solution of the power network.

An RS problem that incorporates an OPF formulation has been introduced in [8] where a chance-constrained OPF problem was formulated. With some modifications, the authors in [8] provided a theoretical guarantee that the OPF-RS problem yields a rank-one feasible solution. Using a heuristic sampling approach, they showed that the resulting optimization problem involves an OPF problem for each wind power profile. Our work here is motivated by [8] to provide some results in a more systematical approach.

While preparing the final version of this work, [9, 10] independently gave an approach to solve an OPF-RS problem in each hour separately, based on the results in [4]. The OPF-RS formulation in [9] is similar to [8] with some modifications, whereas in [10] the formulation is weaker compared to [8], since the condition to distribute reserves among generators is relaxed. Even though the authors in [8] presented a complete day-ahead OPF-RS formulation with up- and down spinning reserves, the results in the aforementioned references are limited either to be heuristic or to a single hourly-based RS with the relaxed conditions. The major barrier of representing an OPF-RS problem as a SDP is the necessity of defining a square SDP matrix variable, which makes the cardinality of scalar variables of the OPF-RS problem quadratic with respect to the number of buses in the power network. This may yield a very large-scale SDP problem for realistic large-scale power networks of interest.

Our work here differs from the aforesaid references in two important aspects. We propose an algorithm to determine a worst-case reserve requirement in each hour by vertex enumeration (VE) of all possible deviations of wind power scenarios from the forecast value. The outcome of VE determines the up- and down-spinning reserves in a probabilistic sense. Using the OPF-RS problem formulation, similarly to [8] with some modifications, we distribute the up- and down-spinning reserves among generators together with the generator dispatch planning for day-ahead schedules. To address the resulting high-dimensional SDP problem, we leverage the sparsity pattern in power networks to break down the large-scale positive-semidefinite (PSD) constraints into small-size constraints, similarly to [11, 12]. We then propose a novel recovery algorithm to obtain a rank-one solution based on the results in [13]. It is important to highlight that this work is based on the same authors' conference paper published in [14] and thesis report in [15].

The layout of this work is as follows. Section 10.2 formulates the RS problem using AC OPF model of power systems by including the uncertain wind power generation, whereas in Sect. 10.3, we provide a computationally tractable reformulation to solve the resulting large-scale SDP in infinite dimensional spaces. Section 10.4 provides a simulation result using IEEE benchmark, whereas Sect. 10.5 concludes this work with remarks and future work.

Notations

$\mathbb{R}$, $\mathbb{R}_+$ denote the set of real and positive real numbers, $\mathbb{S}$, $\mathbb{S}_+$ denote the set of symmetric matrices and positive-semidefinite matrices, respectively. $\mathbb{C}$ denotes the set of complex numbers. Vectors are denoted by lowercase-bold letters $\boldsymbol{a} \in \mathbb{R}^n$, and uppercase letters are reserved for matrices $A \in \mathbb{R}^{n\times n}$. The symbols $A^\top$, A^*, and A^H are used for the transpose, complex conjugate and conjugate transpose of a matrix, respectively. The notations $\underline{a}$ and $\overline{a}$ are used to denote the minimum and maximum allowed values, respectively. The cardinality of a set $\mathcal{A}$ is denoted by $|\mathcal{A}|$.

10.2 Problem Formulation

AC OPF Problem

Consider a power system with a set of buses $\mathcal{N}$, a set of lines $\mathcal{L} \subseteq \mathcal{N} \times \mathcal{N}$ and a set of generator buses $\mathcal{G} \subseteq \mathcal{N}$ such that $|\mathcal{N}| = N_b$ and $|\mathcal{G}| = N_G$. The set of wind power generation buses is denoted by $\mathcal{F} \subseteq \mathcal{N}$ such that $|\mathcal{F}| = N_w$. A set of hours $\mathcal{T}$ forms the scheduling horizon of the hourly-based RS optimization problem and in this work $|\mathcal{T}| = 24$. The vectors $\boldsymbol{p} \in \mathbb{R}^{N_b}$, $\boldsymbol{q} \in \mathbb{R}^{N_b}$ and $\boldsymbol{s} \in \mathbb{C}^{N_b}$ denote real, reactive and apparent power, respectively.

Define the decision variables to be the generator dispatch $\boldsymbol{p}_t^G, \boldsymbol{q}_t^G \in \mathbb{R}^{N_G}$ and the complex bus voltages $\boldsymbol{v}_t \in \mathbb{C}^{N_b}$ for each time step $t \in \mathcal{T}$. Using the rectangular voltage notation: $\boldsymbol{x}_t := [\Re\boldsymbol{v}_t{}^\top \Im\boldsymbol{v}_t{}^\top]^\top \in \mathbb{R}^{2N_b}$, we follow [7, Lemma 1] to determine the data-matrices Y_k, Y_k^*, Y_{lm}, Y_{lm}^*, M_k. The cost function is the cost of real power generation, expressed as a second order polynomial [16], where the coefficient vectors $\boldsymbol{c}^{\mathrm{qu}}, \boldsymbol{c}^{\mathrm{li}} \in \mathbb{R}_+^{N_G}$ correspond to the quadratic and linear cost coefficients, respectively, and $[\boldsymbol{c}^{\text{qu}}]$ represents a diagonal matrix with entries $\boldsymbol{c}^{\text{qu}}$. We now formulate the AC OPF problem by taking into account the effect of wind power generations as follows:

$$\underset{\{\boldsymbol{x}_t, \boldsymbol{p}_t^G, \boldsymbol{q}_t^G\}_{t\in\mathcal{T}}}{\text{minimize}} \quad \sum_{t\in\mathcal{T}} (\boldsymbol{c}^{\text{li}})^\top \boldsymbol{p}_t^G + (\boldsymbol{p}_t^G)^\top [\boldsymbol{c}^{\text{qu}}] \boldsymbol{p}_t^G \tag{10.1a}$$

subject to:

1. Power generation limits $\forall k \in \mathcal{G}, \forall t \in \mathcal{T}$:

$$\begin{aligned} \underline{p_k^G} &\le p_{k,t}^G \le \overline{p_k^G}\,, \\ \underline{q_k^G} &\le q_{k,t}^G \le \overline{q_k^G}\,. \end{aligned} \tag{10.1b}$$

2. Power balance at every bus $\forall k \in \mathcal{G}, \forall t \in \mathcal{T}$:

$$\begin{aligned} \boldsymbol{x}_t^\top Y_k \boldsymbol{x}_t &= p_{k,t}^G - p_{k,t}^D + p_{k,t}^w\,, \\ \boldsymbol{x}_t^\top Y_k^* \boldsymbol{x}_t &= q_{k,t}^G - q_{k,t}^D\,. \end{aligned} \tag{10.1c}$$

   where $\boldsymbol{p}_t^w := \{p_{k,t}^w\}_{k\in\mathcal{F}}$ is the wind power, and $\boldsymbol{s}_t^D := \{s_{k,t}^D\}_{k\in\mathcal{N}}$ is the demanded power such that $s_{k,t}^D = p_{k,t}^D + q_{k,t}^D$. Note that it is assumed[1] $\mathcal{G} \bigcap \mathcal{F} = \emptyset$ which means there is no wind power at generator buses.
3. Bus voltage limits $\forall k \in \mathcal{G}, \forall t \in \mathcal{T}$:

$$|\underline{v_k}|^2 \le \boldsymbol{x}_t^\top M_k \boldsymbol{x}_t \le |\overline{v_k}|^2\,. \tag{10.1d}$$

4. Lineflow limits $\forall (l, m) \in \mathcal{L}, \forall t \in \mathcal{T}$:

$$\left(\boldsymbol{x}_t^\top Y_{lm} \boldsymbol{x}_t\right)^2 + \left(\boldsymbol{x}_t^\top Y_{lm}^* \boldsymbol{x}_t\right)^2 \le |\overline{s_{lm}}|^2\,,$$

   which can be reformulated using the Schur Complement [17] to form a linear matrix inequality constraint, such that the fourth order dependence on the voltage vector is reduced to quadratic terms:

$$\begin{bmatrix} -|\overline{s_{lm}}|^2 & \boldsymbol{x}_t^\top Y_{lm} \boldsymbol{x}_t & \boldsymbol{x}_t^\top Y_{lm}^* \boldsymbol{x}_t \\ \boldsymbol{x}_t^\top Y_{lm} \boldsymbol{x}_t & -1 & 0 \\ \boldsymbol{x}_t^\top Y_{lm}^* \boldsymbol{x}_t & 0 & -1 \end{bmatrix} \preceq 0\,. \tag{10.1e}$$

5. Reference bus constraint $\forall t \in \mathcal{T}$:

$$\boldsymbol{x}_t^\top E_{\text{ref}} \boldsymbol{x}_t = 0\,, \tag{10.1f}$$

[1] This assumption is considered to streamline the presentation and it is not restrictive for our proposed framework.

where E_{ref} is a diagonal matrix from the standard basis vector $e_{N_b+i_{\text{ref}}}$, and i_{ref} denotes the reference bus.

Remark 10.1 The power balance constraints (10.1c) can be used to reformulate the real and reactive generator dispatch in terms of the voltage vector as follows $\forall k \in \mathcal{N}, \forall t \in \mathcal{T}$:

$$p_{k,t}^G = \boldsymbol{x}_t^\top Y_k \boldsymbol{x}_t + p_{k,t}^D - p_{k,t}^w \,, \tag{10.2a}$$
$$q_{k,t}^G = \boldsymbol{x}_t^\top Y_k^* \boldsymbol{x}_t + q_{k,t}^D \,. \tag{10.2b}$$

Using this reformulation, one can substitute for $p_{k,t}^G$ and $q_{k,t}^G$ in (10.1b) to have $\forall k \in \mathcal{N}, \forall t \in \mathcal{T}$:

$$\underline{p_k^G} \leq \boldsymbol{x}_t^\top Y_k \boldsymbol{x}_t + p_{k,t}^D - p_{k,t}^w \leq \overline{p_k^G} \,, \tag{10.3a}$$
$$\underline{q_k^G} \leq \boldsymbol{x}_t^\top Y_k^* \boldsymbol{x}_t + q_{k,t}^D \leq \overline{q_k^G} \,, \tag{10.3b}$$

where the lower and upper generation limits have also been extended to $\mathcal{N}$ using $\underline{p_k^G} = \overline{p_k^G} = 0 \, \forall k \in \{\mathcal{N} \setminus \mathcal{G}\}$.

Remark 10.2 Following Remark 10.1, one can reformulate the cost function (10.1a) using the voltage vector $\boldsymbol{x}_t$:

$$f_G^x(\boldsymbol{x}_t, \boldsymbol{p}_t^w, \boldsymbol{p}_t^D) := \sum_{k \in \mathcal{G}} c_k^{\text{li}} \big(\boldsymbol{x}_t^\top Y_k \boldsymbol{x}_t + p_{k,t}^D - p_{k,t}^w\big) + c_k^{\text{qu}} \left(\left(\boldsymbol{x}_t^\top Y_k \boldsymbol{x}_t + p_{k,t}^D - p_{k,t}^w\right)\right)^2 \,. \tag{10.4}$$

It is important to note that this function is of order four with respect to $\boldsymbol{x}$, but it can be also made quadratic.[2] To streamline the presentation, these steps are skipped.

Using $\{\boldsymbol{x}_t\}_{t \in \mathcal{T}}$, we reformulate the problem (10.1) in a more compact form:

$$\text{OPF}(\{\boldsymbol{p}_t^w\}) : \begin{cases} \underset{\{\boldsymbol{x}_t\}_{t \in \mathcal{T}}}{\text{minimize}} & \sum_{t \in \mathcal{T}} f_G^x(\boldsymbol{x}_t, \boldsymbol{p}_t^w, \boldsymbol{p}_t^D) \\ \text{subject to} & (10.1\text{d}), (10.1\text{e}), (10.1\text{f}), (10.3) \end{cases} ,$$

where the time dependency of $\text{OPF}(\{\boldsymbol{p}_t^w\}_{t \in \mathcal{T}})$ is dropped for clarity of the notation.

$\text{OPF}(\{\boldsymbol{p}_t^w\})$ is a quadratically constrained quadratic program (QCQP) in $\{\boldsymbol{x}_t\}_{t \in \mathcal{T}}$, and a non-convex optimization problem, since the data matrices Y_k, Y_k^*, Y_{lm}, Y_{lm}^* are indefinite [7], which is in fact an NP-hard problem [18] and very hard to solve, in general.

---

[2] The cost function can be made linear with the use of the epigraph notation (see also [17, Chapter 4.1.3]). The resulting inequality constraint can be converted to a LMI using the Schur Complement (see also [17, Chapter A.5.5]), which yields a quadratic function of $\boldsymbol{x}$.

Convexified AC OPF Problem

Using a semi-definite reformulation (SDR) technique (see [7, 19] and the references therein), one can reformulate OPF($\{\boldsymbol{p}_t^w\}$) as an equivalent problem in a matrix variable $W_t := \boldsymbol{x}_t \boldsymbol{x}_t^\top \in \mathbb{S}^{2N_b}$. W_t represents the operating state of the network, and is therefore called the state matrix. We define $\mathcal{W} \subset \mathbb{S}^{2N_b}$ as the set of feasible operating states, such that $W_t \in \mathcal{W}$, using the following characteristics:

$$
\begin{aligned}
\mathcal{W}(\boldsymbol{p}^w, \boldsymbol{s}^D) := \Big\{ W \in \mathbb{S}^{2N_b} \ \Big|\ & \mathrm{Tr}\,(E_{\mathrm{ref}} W) = 0\,, \\
& \underline{p_k^G} \le \mathrm{Tr}\,(Y_k W) + p_k^D - p_k^w \le \overline{p_k^G}, \forall k \in \mathcal{N}, \\
& \underline{q_k^G} \le \mathrm{Tr}\left(Y_k^* W\right) + q_k^D \le \overline{q_k^G}, \forall k \in \mathcal{N}, \\
& |\underline{v_k}|^2 \le \mathrm{Tr}\,(M_k W) \le |\overline{v_k}|^2, \forall k \in \mathcal{N},\ \forall (l,m) \in \mathcal{L}, \\
& \begin{bmatrix} -|\overline{s_{lm}}|^2 & \mathrm{Tr}\,(Y_{lm} W) & \mathrm{Tr}\left(Y_{lm}^* W\right) \\ \mathrm{Tr}\,(Y_{lm} W) & -1 & 0 \\ \mathrm{Tr}\left(Y_{lm}^* W\right) & 0 & -1 \end{bmatrix} \preceq 0 \Big\}\,,
\end{aligned}
\tag{10.5}
$$

where $\boldsymbol{p}^w$ is the wind power, and $\boldsymbol{s}^D = \boldsymbol{p}^D + i\boldsymbol{q}^D$ is the demanded power. Consider now the following formulation as an equivalent optimization problem to OPF($\{\boldsymbol{p}^w\}$):

$$
\underset{\{W_t\}_{t\in\mathcal{T}}}{\text{minimize}} \quad \sum_{t\in\mathcal{T}} f_G(W_t, \boldsymbol{p}_t^w, \boldsymbol{p}_t^D) \tag{10.6a}
$$

$$
\text{subject to} \quad W_t \in \mathcal{W}(\boldsymbol{p}_t^w, \boldsymbol{s}_t^D), \qquad \forall t \in \mathcal{T}, \tag{10.6b}
$$

$$
W_t \succeq 0, \qquad \forall t \in \mathcal{T}, \tag{10.6c}
$$

$$
\mathrm{rank}\,(W_t) = 1, \qquad \forall t \in \mathcal{T}, \tag{10.6d}
$$

where f_G is defined in (10.4), using $W_t = \boldsymbol{x}_t \boldsymbol{x}_t^\top$. Constraints (10.6c) and (10.6d) are introduced to guarantee the exactness of SDR and consequently, OPF($\{\boldsymbol{p}_t^w\}$) and (10.6) to be equivalent.

The optimization problem (10.6) is non-convex, due to the presence of rank-one constraint (10.6d). Removing (10.6d) relaxes the problem to a semi-definite program (SDP). It has been shown in [7] and later in [20] that the rank-one constraint can be dropped without affecting the solution for most power networks. In [8, Proposition 1], the authors showed that when the convex relaxation of the AC OPF problem has solutions with rank at most two, then, forcing any arbitrary selected entry of the diagonal of the matrix W_t to be zero results in a rank-one solution W_t^{opt}. This condition is motivated by the fact that in practice the voltage angle of one of the buses (the reference bus) is often fixed at zero. We denote by C-OPF($\{\boldsymbol{p}_t^w\}$) the convexified version of OPF($\{\boldsymbol{p}_t^w\}$), i.e. Problem (10.6) with the rank-one constraint (10.6d) removed.

OPF-RS Problem Formulation

Consider a power network where a TSO aims to solve a day-ahead AC OPF problem to determine an optimal generator dispatch for the forecasted wind power trajectory such that: (1) the equipments of power system remain safe and (2) the power balance (10.1c) in the power network is achieved. As a novel feature in our proposed formulation C-OPF($\{\boldsymbol{p}_t^w\}$) has a dependency on $\{\boldsymbol{p}_t^w\}_{t\in\mathcal{T}}$, and thus, it solves the OPF problem by taking into account the actual wind power trajectory $\{\boldsymbol{p}_t^w\}_{t\in\mathcal{T}}$. We here define the difference between a generic actual wind power realization and the forecasted wind power, as the mismatch wind power at each time step, e.g. $\boldsymbol{p}_t^m = \boldsymbol{p}_t^w - \boldsymbol{p}_t^{w,f}$. Due to the fact that $\{\boldsymbol{p}_t^m\}_{t\in\mathcal{T}}$ is a random variable, the following technical assumption is necessary in order to proceed to the next steps.

Assumption 1 $\{\boldsymbol{p}_t^m\}_{t\in\mathcal{T}}$ are defined on some probability space $(\mathcal{P}, \mathfrak{B}(\mathcal{P}), \mathbb{P})$, where $\mathfrak{B}(\cdot)$ denotes a Borel σ-algebra, and $\mathbb{P}$ is a probability measure defined over $\mathcal{P}$.

In order to ensure the feasibility and validity of the power network (top TSO priority), we formulate the following problem:

$$\underset{\left\{W_t^f\right\}_{t\in\mathcal{T}}}{\text{minimize}} \quad \sum_{t\in\mathcal{T}} f_G(W_t^f, \boldsymbol{p}_t^{w,f}, \boldsymbol{p}_t^D) \tag{10.7a}$$

$$\text{subject to} \quad W_t^f \in \mathcal{W}(\boldsymbol{p}_t^{w,f}, \boldsymbol{s}_t^D), \qquad \forall t \in \mathcal{T}, \tag{10.7b}$$

$$W_t \in \mathcal{W}(\boldsymbol{p}_t^w, \boldsymbol{s}_t^D),\ \forall \boldsymbol{p}_t^m \in \mathcal{P}, \qquad \forall t \in \mathcal{T}, \tag{10.7c}$$

$$W_t^f \succeq 0,\ W_t \succeq 0 \qquad \forall t \in \mathcal{T}, \tag{10.7d}$$

where $\{\boldsymbol{p}_t^{w,f}\}_{t\in\mathcal{T}}$ denotes the forecasted wind power trajectory, $\{\boldsymbol{p}_t^w\}_{t\in\mathcal{T}}$ is a generic wind power trajectory, $\{W_t^f\}_{t\in\mathcal{T}}$ is related to the state of the network in the case of forecasted wind power, and $\{W_t\}_{t\in\mathcal{T}}$ is a generic network state for a generic wind power trajectory. Constraints (10.7b) and (10.7c) ensure feasibility for every network state, while constraints (10.7d) enforce positive semidefiniteness of all network states.

As a second task of the TSO, the power balance of the power network has to be achieved to ensure demand satisfaction even in the presence of uncertain wind power generation. To address this issue, the TSO employs *reserve power scheduling*, using the fact that a mismatch between actual wind power and forecasted wind power can be mitigated by the controllable generators [1]. We can thus express

$$r_{k,t} := p_{k,t}^G - p_{k,t}^{G,f}, \tag{10.8}$$

where $\boldsymbol{r}_t = \{r_{k,t}\}_{k\in\mathcal{G}} \in \mathbb{R}^{N_G}$ denotes the amount of reserve requirement in the power network. Following Remark 10.1, we have:

$$p_{k,t}^G = \text{Tr}\left(Y_k W_t\right) + p_{k,t}^D - p_{k,t}^w,$$
$$p_{k,t}^{G,f} = \text{Tr}\left(Y_k W_t^f\right) + p_{k,t}^D - p_{k,t}^{w,f},$$

and one can substitute them in (10.8) to obtain the reserve power in terms of the network states W_t and W_t^f as follows:

$$\begin{aligned} r_{k,t} &:= \mathrm{Tr}\left(Y_k\left(W_t - W_t^f\right)\right) - (p_{k,t}^w - p_{k,t}^{w,f}) \\ &= \mathrm{Tr}\left(Y_k\left(W_t - W_t^f\right)\right) - p_{k,t}^m \,, \\ &= \mathrm{Tr}\left(Y_k\left(W_t - W_t^f\right)\right) \,, \end{aligned} \tag{10.9}$$

where the term $p_{k,t}^m$ is dropped, since it is assumed that there is no wind power at generator buses. The elements of $\boldsymbol{r}_t = \{r_{k,t}\}_{k \in \mathcal{N}_\mathcal{G}}$ can be positive and negative (the upspinning and downspinning reserve power, respectively) such that they are deployed for a power deficit and surplus to bring balance to the network and satisfy the demanded power [16]. Following the automatic generator regulation (AGR) mechanism [4], we also define two vectors $\boldsymbol{d}_t^{\mathrm{us}}, \boldsymbol{d}_t^{\mathrm{ds}} \in \mathbb{R}^{N_G}$ to distribute the amount of up- or downspinning reserve power among the available generators for each hour $t \in \mathcal{T}$. To obtain the optimal control strategies for AGR, we consider the following equality constraint $\forall \boldsymbol{p}_t^m \in \mathcal{P}$, $\forall k \in \mathcal{G}$ and $\forall t \in \mathcal{T}$:

$$\begin{aligned} r_{k,t} &= \mathrm{Tr}\left(Y_k\left(W_t - W_t^f\right)\right) \\ &= -\, d_{k,t}^{\mathrm{us}} \min\left(0, \boldsymbol{1}^\top \boldsymbol{p}_t^m\right) - d_{k,t}^{\mathrm{ds}} \max\left(0, \boldsymbol{1}^\top \boldsymbol{p}_t^m\right). \end{aligned} \tag{10.10}$$

In order to always negate the mismatch wind power using the reserve power and bring balance to the power network, we enforce the sum of the distribution vectors to be equal to one using the following constraint $\forall t \in \mathcal{T}$:

$$\boldsymbol{1}^\top \boldsymbol{d}_t^{\mathrm{us}} = 1 \,, \; \boldsymbol{1}^\top \boldsymbol{d}_t^{\mathrm{ds}} = 1 \,, \tag{10.11}$$

where $\boldsymbol{1}$ is a vector of appropriate dimensions with all entries equal to one. Define $\boldsymbol{r}_t^{\mathrm{ds}}, \boldsymbol{r}_t^{\mathrm{us}} \in \mathbb{R}^{N_G}$ such that $\forall t \in \mathcal{T}$:

$$-\boldsymbol{r}_t^{\mathrm{ds}} \leq \boldsymbol{r}_t \leq \boldsymbol{r}_t^{\mathrm{us}} \,, \tag{10.12a}$$

$$0 \leq \boldsymbol{r}_t^{\mathrm{us}} \,, \; 0 \leq \boldsymbol{r}_t^{\mathrm{ds}} \,, \tag{10.12b}$$

and consider corresponding linear up- and downspinning cost coefficients $\boldsymbol{c}^{\mathrm{us}}, \boldsymbol{c}^{\mathrm{ds}} \in \mathbb{R}_+^{N_G}$ yielding the total reserve cost:

$$f_R(\boldsymbol{r}_t^{\mathrm{us}}, \boldsymbol{r}_t^{\mathrm{ds}}) := (c^{\mathrm{us}})^\top \boldsymbol{r}_t^{\mathrm{us}} + (c^{\mathrm{ds}})^\top \boldsymbol{r}_t^{\mathrm{ds}}.$$

Using $\Xi := \left\{W_t^f,\; W_t, \boldsymbol{d}_t^{\mathrm{us}}, \boldsymbol{d}_t^{\mathrm{ds}}, \boldsymbol{r}_t^{\mathrm{us}}, \boldsymbol{r}_t^{\mathrm{ds}}\right\}_{t \in \mathcal{T}}$ as the set of decision variables, and combining our previous discussions with the optimization problem (10.7), we are now in the position to formulate the OPF($\{\boldsymbol{p}_t^w\}$) problem with RS in a more compact form:

$$\text{C-OPF-RS}:\begin{cases}\min\limits_{\Xi} & \sum_{t\in\mathcal{T}}\left(f_G(W_t^f,\boldsymbol{p}_t^{w,f})+f_R(\boldsymbol{r}_t^{\text{us}},\boldsymbol{r}_t^{\text{ds}})\right)\\ \text{s.t.} & (10.7\text{b}),(10.7\text{c}),(10.7\text{d}),(10.10),(10.11),(10.12)\end{cases}.$$

Notice that one needs to substitute $\boldsymbol{r}_t$ in (10.12a) with (10.9).

C-OPF-RS is an uncertain infinite SDP program, due to the unknown and unbounded set $\mathcal{P}$. It is therefore computationally intractable and in general difficult to solve. In the next section, we propose a technique to approximate $\mathcal{P}$ such that it contains the probability mass distribution of $\mathcal{P}$ almost surely with a high level of confidence.

10.3 Tractable Reformulations

In this section, we first present a tractable approach to approximately solve C-OPF-RS, and then, we leverage the sparsity in the problem data to decompose the computationally expensive PSD constraints.

Vertex Enumeration Scheme

The constraint function of C-OPF-RS is a linear function with respect to the uncertainty $\boldsymbol{p}_t^m$, if we exclude (10.10). However, due to the nonlinear operators, max-min, it is not straightforward to reformulate (10.10) as a linear constraint. In fact such operators lead to a hybrid operation, and especially in (10.10), the two terms on the right-hand side cannot be non-zero simultaneously. Following this observation, one can approximate the uncertainty set $\mathcal{P}$ using two sets $\overline{\mathcal{B}}$, $\underline{\mathcal{B}}$, and reformulate (10.10) as follows $\forall k\in\mathcal{G}$, $\forall t\in\mathcal{T}$:

$$\begin{aligned}\text{Tr}\left(Y_k\left(W_t-W_t^f\right)\right)&=-d_{k,t}^{\text{ds}}(\mathbf{1}^\top\boldsymbol{p}_t^m)\,,\forall\boldsymbol{p}_t^m\in\overline{\mathcal{B}}\,,\\ \text{Tr}\left(Y_k\left(W_t-W_t^f\right)\right)&=-d_{k,t}^{\text{us}}(\mathbf{1}^\top\boldsymbol{p}_t^m)\,,\forall\boldsymbol{p}_t^m\in\underline{\mathcal{B}}\,.\end{aligned}\tag{10.13}$$

Remark 10.3 It is important to notice that all other uncertain constraints in C-OPF-RS have to be satisfied for all $\boldsymbol{p}_t^m\in\overline{\mathcal{B}}$ and $\boldsymbol{p}_t^m\in\underline{\mathcal{B}}$, separately, for all $k\in\mathcal{G}$ and for all $t\in\mathcal{T}$.

Our goal here is to approximate the uncertainty set $\mathcal{P}$ by employing recent results in randomized optimization, the so-called scenario approach [21], to characterize $\overline{\mathcal{B}}$, $\underline{\mathcal{B}}$ and provide feasibility certificates. A similar technique has been also used in [3, 22, 23] based on [24]. It is now of interest to characterize $\overline{\mathcal{B}}$, $\underline{\mathcal{B}}$ such that $\overline{\mathcal{B}}\bigcup\underline{\mathcal{B}}$ approximates $\mathcal{P}$. We assume for simplicity that $\overline{\mathcal{B}}$ and $\underline{\mathcal{B}}$ are two axis-aligned hyper-rectangular sets. This is not a restrictive assumption and any convex set could have been chosen instead as described in [22]. We define $\overline{\mathcal{B}}:=[\mathbf{0},\overline{\boldsymbol{p}}_t^m]$, and $\underline{\mathcal{B}}:=[\underline{\boldsymbol{p}}_t^m,\mathbf{0}]$ as two intervals, where the vectors $\overline{\boldsymbol{p}}_t^m\in\mathbb{R}^{N_w}$ and $\underline{\boldsymbol{p}}_t^m\in\mathbb{R}^{N_w}$ define the bounds of hyper-rectangular sets. We now propose Algorithm 6 that aims to determine both sets $\overline{\mathcal{B}}$ and $\underline{\mathcal{B}}$ with minimal volume such that $\overline{\mathcal{B}}\bigcap\underline{\mathcal{B}}=\emptyset$. Consider $\overline{\boldsymbol{p}^*}_t^m$ and $\underline{\boldsymbol{p}^*}_t^m$

to be the outcome of Algorithm 6 that determine $\overline{\mathcal{B}^*}$ and $\underline{\mathcal{B}^*}$, respectively. Defining $\mathcal{B}^* = \overline{\mathcal{B}^*} \bigcup \underline{\mathcal{B}^*}$, we next provide the following theorem that establishes a theoretical connection between $\mathcal{B}^*$ and $\mathcal{P}$ by means of the level of approximation.

Theorem 10.1 *Fix* $\varepsilon \in (0, 1)$, $\beta \in (0, 1)$,

$$N_s \geq \frac{2}{\varepsilon}\left(2N_w + \ln\frac{1}{\beta}\right), \tag{10.14}$$

and construct the set $\mathcal{S} = \{\boldsymbol{p}_t^{m,1}, \boldsymbol{p}_t^{m,2}, \cdots, \boldsymbol{p}_t^{m,N_s}\}$. *Then,*

$$\mathbb{P}^{N_s}\left[\mathcal{S} \in \mathcal{P}^{N_s} : \mathbb{P}[\, \boldsymbol{p}_t^m \in \mathcal{P} : \boldsymbol{p}_t^m \notin \mathcal{B}^*] \leq \varepsilon\right] \geq 1 - \beta,$$

where $\mathbb{P}^{N_s}$ *denotes an* N_s*-fold product probability.*

Proof The proof is a direct result of [25, Theorem 1].

The interpretation of Theorem 10.1 is as follows. Given a generic sample $\boldsymbol{p}_t^m \in \mathcal{P}$, the probability of $\boldsymbol{p}_t^m \in \mathcal{B}^*$ is greater than $1 - \varepsilon$ with high confidence level $1 - \beta$.

Algorithm 6 Vertex Enumeration (VE) Algorithm

Input: ε, β
$N_s \leftarrow \lceil \frac{2}{\varepsilon}\left(2N_w + \ln\frac{1}{\beta}\right)\rceil$
Extract $\{\boldsymbol{p}_t^{m,1}, \boldsymbol{p}_t^{m,2}, \cdots, \boldsymbol{p}_t^{m,N_s}\} \in \mathcal{P}^{N_s}$
for $t \in \mathcal{T}$ **do**
  $\mathcal{I} \leftarrow \emptyset$, $\{\overline{\boldsymbol{p}^*}_t^m\} \leftarrow \emptyset$, $\{\underline{\boldsymbol{p}^*}_t^m\} \leftarrow \emptyset$
  for $k \in \mathcal{F}$ **do**
    $\mathcal{I} \leftarrow \mathcal{I} \cup \arg\max_i\{p_{k,t}^{m,i}\}$
    $\mathcal{I} \leftarrow \mathcal{I} \cup \arg\min_i\{p_{k,t}^{m,i}\}$
  end for
  $\mathcal{I} \leftarrow \mathcal{I} \cup \arg\max_i\{\mathbf{1}^\top \boldsymbol{p}_t^{m,i}\}$
  $\mathcal{I} \leftarrow \mathcal{I} \cup \arg\min_i\{\mathbf{1}^\top \boldsymbol{p}_t^{m,i}\}$
  for $i \in \mathcal{I}$ **do**
    if $\mathbf{1}^\top \boldsymbol{p}_t^{m,i} > 0$ **then**
      $\{\overline{\boldsymbol{p}^*}_t^m\} \leftarrow \{\overline{\boldsymbol{p}^*}_t^m\} \cup \boldsymbol{p}_t^{m,i}$
      $\{\underline{\boldsymbol{p}^*}_t^m\} \leftarrow \{\underline{\boldsymbol{p}^*}_t^m\} \cup \mathbf{0}$
    else if $\mathbf{1}^\top \boldsymbol{p}_t^{m,i} < 0$ **then**
      $\{\overline{\boldsymbol{p}^*}_t^m\} \leftarrow \{\overline{\boldsymbol{p}^*}_t^m\} \cup \mathbf{0}$
      $\{\underline{\boldsymbol{p}^*}_t^m\} \leftarrow \{\underline{\boldsymbol{p}^*}_t^m\} \cup \boldsymbol{p}_t^{m,i}$
    end if
  end for
end for
Output: $\{\overline{\boldsymbol{p}^*}_t^m, \underline{\boldsymbol{p}^*}_t^m\}_{t \in \mathcal{T}}$

A complete description of Algorithm 6 can be found in [15, Section 4.1.2]. Combining the previous discussion with Remark 10.3, the resulting problem is tractable by means of the robust SDP program which is a finite dimensional optimization problem. We consider the generic network state $\{W_t\}_{t\in\mathcal{T}}$ to be the sum of forecasted network state $\{W_t^f\}_{t\in\mathcal{T}}$ and a corrective control action. In this way, it emulates the way the set-points of the AGR unit can be adjusted as a function of the wind power. Due to the fact that the constraint functions of C-OPF-RS, by replacing (10.10) with (10.13), will be convex functions with respect to the uncertainty $\boldsymbol{p}_t^m \in \mathcal{B}^*$, it suffices to enforce the constraints only at the values that correspond to the vertices $\overline{\boldsymbol{p}^*}_t^m$ and $\underline{\boldsymbol{p}^*}_t^m$ of $\mathcal{B}^* = \overline{\mathcal{B}^*} \bigcup \underline{\mathcal{B}^*}$. To solve numerically the resulting robust SDP problem, we use a similar approach as in [3, 22] based on [26] together with enforcing the PSD constraints $\overline{W_t} \succeq 0$ and $\underline{W_t} \succeq 0$ in each hour $t \in \mathcal{T}$, separately, for the worst-case uncertainties $\overline{\boldsymbol{p}^*}_t^m$ and $\underline{\boldsymbol{p}^*}_t^m$, respectively.

It is worth mentioning that compared to the result of [9], the number of samples needed from $\mathcal{P}$ is much lower, since the dimension of the decision variable is much smaller. We formulate a robust variant of the OPF-RS problem that uses far less samples of the uncertainty compared to the approach in [9], whilst having the same theoretical probabilistic guarantees.

Sparsity Pattern Decomposition

SDPs with matrix variables subject to PSD constraints are computationally complex. One can reduce the size of the computationally expensive PSD constraints by selecting certain submatrices of the original matrix variables and only imposing PSD property on those matrices. The solution will be a partially filled matrix, with only those entries filled that correspond to at least one of the submatrices. All other entries will be undetermined. Various algorithms are available for *matrix completion*, the a-posteriori filling of the undetermined entries. In [27], Grone et al. provided the *chordal theorem*, that guarantees the completed matrix will be PSD if and only if specific submatrices are PSD. Consider a symmetric matrix $X \in \mathbb{S}^d$, and let G be a graph with nodes $\{1, \ldots, d\}$. The chordal theorem states that one can reconstruct the PSD Hermitian[3] matrix X using only the entries of X that correspond to the nodes in the maximal cliques[4] of G, if and only if G is a chordal graph.[5] The chordal theorem can thus be used to prove the equivalence between the PSD property of a matrix and the PSD property of its submatrices, thereby reducing the size of the PSD constraints with the overall computational complexity.

Consider a graph over all the buses of the power network such that the edges correspond to the non-zeros in all the data-matrices Y_k, Y_k^*, Y_{lm}, Y_{lm}^*, M_k, where the

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[3] Note that a symmetric matrix is a Hermitian matrix with all its imaginary values equal to zero, i.e. $\mathbb{S} \subset \mathbb{H}$, so the chordal theorem also holds for symmetric matrices.

[4] A *clique* is a subset of nodes that together form a complete graph, i.e. the number of edges between any two nodes in a clique is equal to one. A clique is *maximal* if it is not a subset of any other cliques in the graph [28].

[5] A graph is *chordal* if every cycle of length greater than three has a chord (an edge between non-consecutive vertices in the cycle) [28].

aggregate sparsity pattern can be found. Due to the definition of the nodal admittance matrix, the sparsity pattern is identical to the network topology.

Using a greedy decomposition algorithm [29], we decompose the network in K subsets of buses, corresponding to the maximal cliques of the chordal graph. Denote every clique with $C_k \subset \mathcal{N}$, and collect all cliques in $C = \{C_1, \ldots, C_K\}$, and let N_C be the number of buses in the largest maximal clique: $N_C := \max_k |C_k|$. Every subset C_k induces a submatrix from the original matrix by selecting the columns and rows corresponding to the buses in it. Note that the decomposed problem has K matrix variables of dimension N_C at most.

We now decompose the PSD constraints (10.7d) on every matrix variable in C-OPF-RS $\forall t \in \mathcal{T}$ using the following constraints:

$$W_t^f(C_k, C_k) \succeq 0, \qquad \forall C_k \in C \tag{10.15a}$$

$$\overline{W_t}(C_k, C_k) \succeq 0, \qquad \forall C_k \in C \tag{10.15b}$$

$$\underline{W_t}(C_k, C_k) \succeq 0, \qquad \forall C_k \in C \tag{10.15c}$$

We call the proposed decomposed formulation as CD-OPF-RS. The following proposition is the direct result of [13, Theorem 1].

Proposition 10.1 *The optimal objective value of CD-OPF-RS is equivalent to the optimal objective value of C-OPF-RS.*

The obtained solution using CD-OPF-RS is a partially filled matrix, denoted by $\tilde{W}_t^f$. From this matrix, we wish to reconstruct a PSD matrix which is rank-one and an optimal solution for the proposed original OPF-RS. Although the chordal theorem proves the possibility of completing a PSD matrix, it does not provide a PSD matrix with the desired rank. We therefore aim to develop a matrix recovery algorithm such that the resulting solution is a PSD matrix with rank one.

Inspired by the voltage vector recovery algorithm in [13], we propose a matrix recovery algorithm which is guaranteed to complete a partially filled state matrix to a rank-one PSD matrix. We modify their algorithm for the rectangular voltage notation, and extract a complex voltage vector from the partially filled solution. We then recover the full state matrix from the complex voltage vector. Algorithm 7 summarizes our proposed recovery procedure.

The magnitude of the entries of $\boldsymbol{v}$ is determined by summing the entries on the diagonal that correspond to the real and imaginary part of the same bus, and taking the square root. After this, the angle difference between buses is calculated based on the filled entries in $\tilde{W}_t^f$. Since the sparsity pattern coincides with the network topology, the filled entries will correspond to the lines of the network. The convex program in Algorithm 7 of extracts the globally optimal voltage vector if $\tilde{W}_t^f$ is rank-one such that $\sum_{(l,m)\in\mathcal{L}} |\angle \tilde{W}_{lm} - \angle v_l + \angle v_m| = 0$. If this is not the case, the program aims to find a voltage vector for which the corresponding angle differences are as close to those suggested by the matrix $\tilde{W}_t^f$ as in [13]. The determined solution is used to build $\boldsymbol{x}$, which is then used to form W_t^f.

Algorithm 7 Matrix Completion

Given: partially filled state matrix $\tilde{W} \in \mathbb{S}^{2N_b}$
Initialize: $\boldsymbol{v} \in \mathbb{C}^{N_b}$
for $k \in \mathcal{N}$ **do**
  $|v_k| \leftarrow \sqrt{\tilde{W}(k,k) + \tilde{W}(k+N_b, k+N_b)}$
end for
for $(l, m) \in \mathcal{L}$ **do**
  $\angle \tilde{W}_{lm} := \tan^{-1}\left(\frac{\tilde{W}(l+N_b,m)-\tilde{W}(l,m+N_b)}{\tilde{W}(l,m)+\tilde{W}(l+N_b,m+N_b)}\right)$
end for
$\angle \boldsymbol{v} \leftarrow \underset{-\pi \leq \angle \boldsymbol{v} \leq \pi}{\arg\min} \sum_{(l,m)\in\mathcal{L}} |\angle \tilde{W}_{lm} - \angle v_l + \angle v_m|$
$\boldsymbol{x} \leftarrow \left[\left(|\boldsymbol{v}| \cos \angle \boldsymbol{v}\right)^\top, \left(|\boldsymbol{v}| \sin \angle \boldsymbol{v}\right)^\top\right]^\top$
Output: $W \leftarrow \boldsymbol{x}\boldsymbol{x}^\top$

Remark 10.4 It is worth mentioning that the proposed approach in [13] first completes $\tilde{W}$ and then extracts the optimal voltage vector from this completed matrix. We however skip the completion step compared to [13], since our proposed formulation allows us to directly use $\tilde{W}_t^f$ to extract a voltage vector and then reconstruct a completely filled state matrix.

Remark 10.5 Comparing the computational time complexity of CD-OPF-RS with C-OPF-RS by considering that $N_C \ll N_b$, one can clearly see the impact of decomposition on the computational complexity. For realistic networks, N_C is still of reasonable dimensions (see [13] for a list of power systems and their corresponding treewidth, i.e. the size of the largest maximal cliques plus one). CD-OPF-RS has K matrix variables of dimension N_C at most, and the worst-case overall dimension of the matrix variable is therefore $K N_C (N_s + 1) T$.

10.4 Numerical Study

Simulation Setup

We carried out a simulation study using the 30-bus IEEE benchmark power system [30] assuming only a single wind-bus infeed at bus 10. We follow the approach of [31] to generate trajectories for the wind power, with a data-set corresponding to the hourly aggregated wind power production of Germany over the period 2006–2011. The load profile is assumed to be known (see [15, Fig. 6.1]) and the nominal load from MATPOWER[6] [32] is multiplied with this profile to get a time-varying load.

Following Theorem 10.1, we fix $\varepsilon = 10^{-2}$, $\beta = 10^{-5}$, and $N_w = 1$ to obtain the number of required wind power samples at each hour $N_s = 541$. We use Matlab

[6] MATPOWER is a commercial software for solving power flow problems using successive quadratic programming.

together with YALMIP [33] as an interface and MOSEK [34] as a solver. All optimizations are run on a MacBook Pro with a 2.4 GHz Intel Core i5 processor and 8 GB of RAM.

After obtaining a solution, the scheduled generator power (the generator power based on the forecasted wind trajectory) and the voltage magnitudes are extracted from $\{W_t^f\}$ for all time steps using the following relations $\forall k \in \mathcal{G}, \forall t \in \mathcal{T}$:

$$p_{k,t}^G = \text{Tr}\left(Y_k W_t^f\right) + p_{k,t}^D - p_{k,t}^w, \tag{10.16a}$$

$$q_{k,t}^G = \text{Tr}\left(Y_k^* W_t^f\right) + q_{k,t}^D, \tag{10.16b}$$

$$|v_{k,t}| = \sqrt{W_t^f(k,k) + W_t^f(N_b + k, N_b + k)}. \tag{10.16c}$$

A comparison using the DC model of power network to solve the OPF-RS problem is delivered as a benchmark approach. A detailed description of DC model can be obtained from [3, 4]. The solution of the benchmark program is the real generator power and distribution vectors for every hour, $\left\{\boldsymbol{p}_t^{G,\text{dc}}, \boldsymbol{d}_t^{\text{us,dc}}, \boldsymbol{d}_t^{\text{ds,dc}}\right\}$. One also needs the reactive generator power and generator voltage magnitudes in order to have a more realistic comparison. In [8], the nominal value of such variables was extracted from the MATPOWER test case for all time steps and scenarios. This will result in large violations, since the reactive generator power is not adapted to the time-varying demand.

We here develop a novel benchmark approach, namely converted DC (CDC), to have a more sophisticated comparison by solving the following program:

$$\begin{aligned} \min_{\{W_t\}_{t\in\mathcal{T}}} \quad & \sum_{t\in\mathcal{T}} \sum_{k\in\mathcal{G}} \left(p_{k,t}^{G,\text{dc}} - \left(\text{Tr}\,(Y_k W_t) + p_{k,t}^D - p_{k,t}^{w,f}\right)\right)^2 \\ \text{s.t.} \quad & W_t \in \mathcal{W}(\boldsymbol{p}_t^{w,f}, \boldsymbol{s}_t^D), \qquad \forall t \in \mathcal{T}, \\ & W_t \succeq 0, \qquad \forall t \in \mathcal{T}. \end{aligned}$$

The solution to this program is a feasible (AC) network state $\{W_t\}_{t\in\mathcal{T}}$ where the real generator power is as close as possible to the obtained generator power from the DC solution. The distribution vectors used in simulation will be equal to those obtained from the original solution of the DC framework. A schematic overview of the optimization and simulation process to obtain and validate both the benchmark and proposed formulations is given in Figs. 10.1 and 10.2.

After retrieving a solution, we simulate the network power flow using MATPOWER such that the power and voltage magnitude of generators and all the loads are fixed without imposing any constraints for 10,000 different wind power scenarios. The wind power is implemented as a negative load on the wind-bus. Afterward, the resulting power flows and voltage magnitudes are evaluated by means of counting the number of violated constraints.

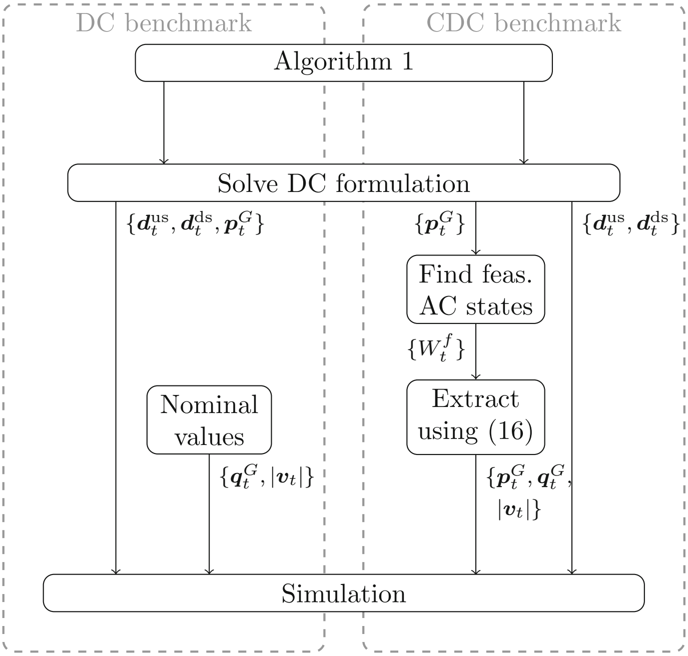
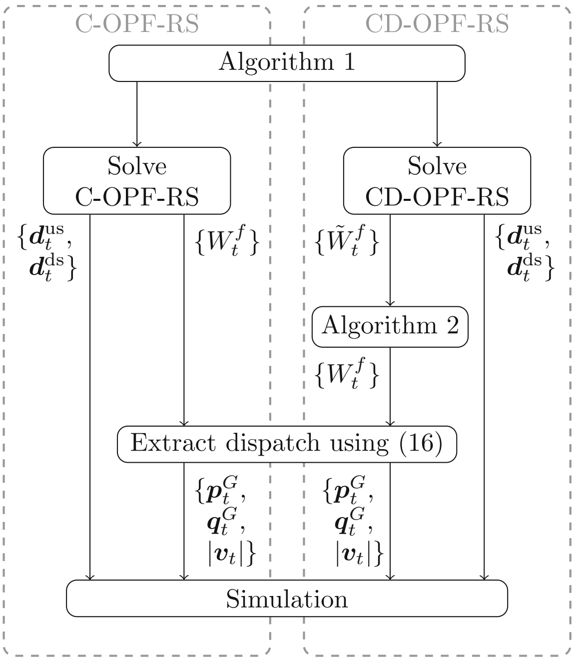


Fig. 10.1 Schematic overview of optimization and simulation process for the DC benchmarks

Fig. 10.2 Schematic overview of optimization and simulation process for the proposed formulations



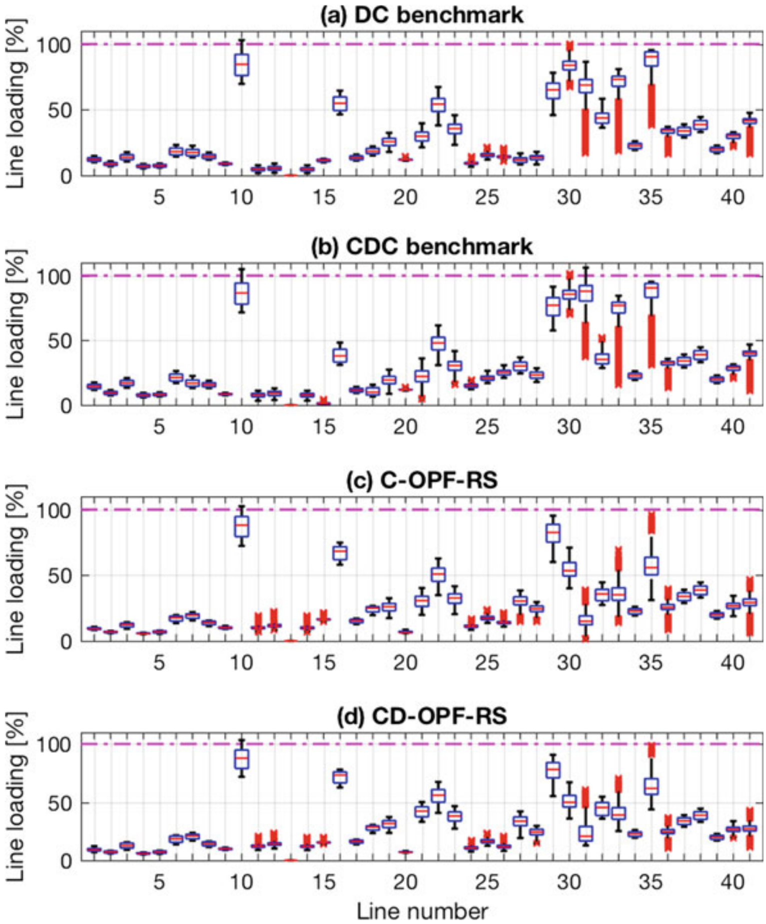


Fig. 10.3 Relative line loading for all hours and scenarios per line. The red line represents the median value, edges of each box correspond to the 25th and 75th percentiles, the whiskers extend to 99% coverage, and the red marks denote the data outliers. The upper plots **a** and **b** show the Benchmark results, and the lower plots **c** and **d** show the proposed approaches

Simulation Results

Figure 10.3 depicts the line loadings[7] as boxplots for DC, CDC, C-OPF-RS, and CD-OPF-RS formulations, for all hours and scenarios. Such a boxplot has been also used in [9] to show line flow violations. It can be seen that all formulations have some violations of the lineflow limits for line 10. To further assess the performance

[7] Line loadings are defined as the apparent power flow over a line divided by the maximum apparent power flow for that line, such that a line loading higher than 100% corresponds to the violation of the lineflow limit.

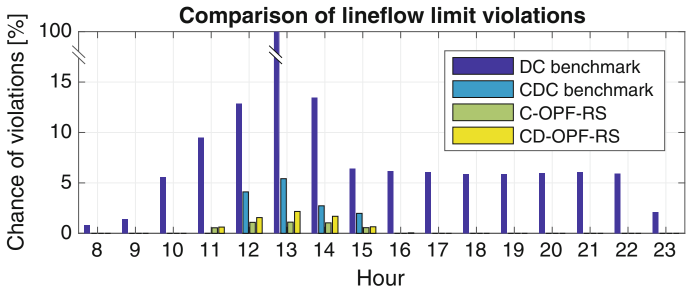


Fig. 10.4 Empirical violation level of lineflow limit for different formulations

of these results, we calculate the level of violation for lineflow limit empirically using the different formulations at each hour (see Fig. 10.4).

In Fig. 10.4 the results for the peak hours are shown. For all other hours, the chance of constraint violation is very close to zero for all formulations. As expected, the DC solution shows a very high level of violation during the peak hours. Although the CDC solution improves the chance of lineflow limit violation, the theoretical limit at the peak hours is still not respected. It is important to notice that the empirical chance of constraint violation for the C-OPF-RS and CD-OPF-RS results are well below the theoretical limit (5%), and they are at most 1.1 and 2.2%, respectively. The proposed decomposition and reconstruction process caused the solution to be slightly less conservative compared to the results of C-OPF-RS.

We next examine the bus voltage magnitudes for all formulations. It is observed that the DC, C-OPF-RS and CD-OPF-RS solutions are always within the limits for all hours and scenarios. However, using the CDC formulation the bus voltage limits show a violation of 100% for all hours. This can be explained by the fact that in the DC framework, the bus voltages are assumed to be constant at nominal value. When we implement the obtained solution in the AC framework, it can be seen that this assumption does not hold. We can thus conclude that for both the DC formulations, the empirical chance of constraint violation is well above the theoretical limits once the solution is implemented in the AC power flow simulations. For both C-OPF-RS and CD-OPF-RS, the a-priori probabilistic guarantees are confirmed to be valid.

10.5 Conclusions

We developed a framework to solve the reserve scheduling (RS) problem using AC optimal power flow (AC OPF) formulation. We first integrated the effect of wind power generation in power networks into an AC OPF problem formulation. Using this new formulation, we unified the RS problem with the AC OPF problem. The final optimization problem leads to an uncertain infinite semi-definite program (SDP) formulation, since the uncertainty set is unknown and unbounded. We approximated the uncertainty set with a-priori probabilistic guarantee using a set-based characterization technique, and then solved a robust finite SDP problem at each hour. A decomposition technique is employed to reduce computational time complexity of the resulting problem. As a final contribution, we proposed a new recovery algorithm to determine a rank-one solution from the decomposed problem. The resulting solutions are validated using Monte Carlo simulations and a commercial power flow simulator, and found to perform as expected. The obtained solutions via our proposed formulation perform better than the solution obtained from the DC framework, which is currently used in industry. Future work will focus on extending the current results to multi-area power systems.

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