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Multi-GNSS PPP-RTK: Mixed-Receiver Network and User Scenarios

P.J.G. Teunissen, A. Khodabandeh, and B. Zhang

Abstract

In this contribution, we present full-rank observation equations of the network and user receivers, of mixed types, through an application of $\mathcal{S}$-system theory. We discuss the important roles played by the inter system biases (ISBs), and we show how the three-component structure of PPP-RTK is affected by the inclusion of the ISBs as extra parameters in the model.

Keywords

GNSS • Inter system bias (ISB) • ISB look-up table • PPP-RTK

1 Introduction

In recent years, we are witnessing rapid development in the satellite-based navigation and positioning, along with launching new global and regional satellite systems. This means that many more satellites will be visible to the GNSS users, tracking data on many more frequencies than the current GPS dual-frequency setup, thereby expecting considerable improvement in the performance of the positioning and non-positioning GNSS applications (Simsky et al. 2008; de Bakker et al. 2012; Teunissen et al. 2014; He et al. 2014; Odolinski et al. 2015; Nadarajah et al. 2015).

The stated improvement may not be realized however, would one not properly integrate the multi-system, multi-frequency data. Indeed, recent contributions have revealed the existence of non-zero inter system biases – experienced by receivers of different types – that, if ignored, result in a catastrophic failure of integer ambiguity resolution (IAR), thus deteriorating the corresponding ambiguity-resolved solutions (Odijk and Teunissen 2013a; Paziewski and Wielgosz 2014; Nadarajah et al. 2014; Torre and Caporali 2015). The availability of the new multi-system, multi-frequency data does therefore require proper functional models so as to enable one to correctly integrate such data, thus correctly linking the data to the estimable parameters of interest.

The present contribution is intended to provide such proper functional models through a careful application of $\mathcal{S}$-system theory (Baarda 1973; Teunissen 1985). The formulations are presented within the context of IAR-enabled precise point positioning, namely, PPP-RTK (Wubbena et al. 2005; Mervart et al. 2008; Teunissen et al. 2010). With the current single-system, dual-frequency PPP-RTK setup, the network-derived satellite orbit and clock corrections are further extended by the satellite phase bias corrections to recover the integerness of the user ambiguities, making single-receiver IAR feasible, see e.g., (Laurichesse and Mercier 2007; Collins 2008; Ge et al. 2008). Single-receiver IAR would then reduce the positioning convergence time as compared to that of the standard precise point positioning (Teunissen and Khodabandeh 2015).

In this contribution we show, with the multi-system, multi-frequency PPP-RTK setup, that additional estimable network-derived corrections are needed, in order to recover

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the integerness of the *entire* set of the single-receiver user's estimable ambiguities. A way to convey such additional corrections is also presented.

2 Single-System PPP-RTK

As our point of departure, we first briefly review the mechanism of single-system PPP-RTK. We will then be in a position to make a comparative analysis and identify all the subtle differences that are driven by the inclusion of additional systems.

The three components of PPP-RTK, together with their interactions, are visualized in Fig. 1 and discussed below.

2.1 Network-Component

The functionality of the network-component is to collect and process the GNSS observations. The outcomes of a least-squares network adjustment would then serve as the products well-suited to both the positioning and non-positioning users.

Consider the network receiver r ($r = 1, \ldots, n$) tracking satellite s ($s = 1, \ldots, m$) on frequency j ($j = 1, \ldots, f$). The corresponding observation equations read (Teunissen and Kleusberg 1998)

$$\begin{aligned} \Delta\phi^s_{r,j} &= \Delta\rho^s_r + dt_r - dt^s - \mu_j \iota^s_r + \lambda_j (z^s_{r,j} + \delta_{r,j} - \delta^s_{,j}) \\ \Delta p^s_{r,j} &= \Delta\rho^s_r + dt_r - dt^s + \mu_j \iota^s_r + d_{r,j} - d^s_{,j} \end{aligned} \tag{1}$$

where $\Delta\phi^s_{r,j}$ and $\Delta p^s_{r,j}$ denote the 'observed minus computed' phase and code observations, respectively. Here and in the following, the precise orbital corrections are assumed included in $\Delta\phi^s_{r,j}$ and $\Delta p^s_{r,j}$. The increment of the geometric range, lumped with that of the zenith tropospheric delay (ZTD), is denoted by $\Delta\rho^s_r$. This increment can be further parametrized by the position and ZTD increment Δx_r through $\Delta\rho^s_r = g^{sT}\Delta x_r$, with g^s containing the receiver-satellite direction vector and the tropospheric mapping function. The common receiver and satellite clock parameters are, respectively, denoted as dt_r and dt^s that are accompanied by the frequency-dependent code receiver and satellite biases $d_{r,j}$ and $d^s_{,j}$. Ambiguities, in units of cycles, are composed of the integer part $z^s_{r,j}$ and the receiver/satellite non-integer parts $\delta_{r,j}$ and $\delta^s_{,j}$, respectively. They show themselves through the wavelengths λ_j. The (first-order) slant ionospheric delay is denoted by ι^s_r that is experienced on the first frequency. Thus we have the scalars $\mu_j = (\lambda^2_j/\lambda^2_1)$ linking the ionospheric delays to the observations. Apart from $z^s_{r,j}$, $\delta_{r,j}$ and $\delta^s_{,j}$, the rest of the quantities are all expressed in units of range.

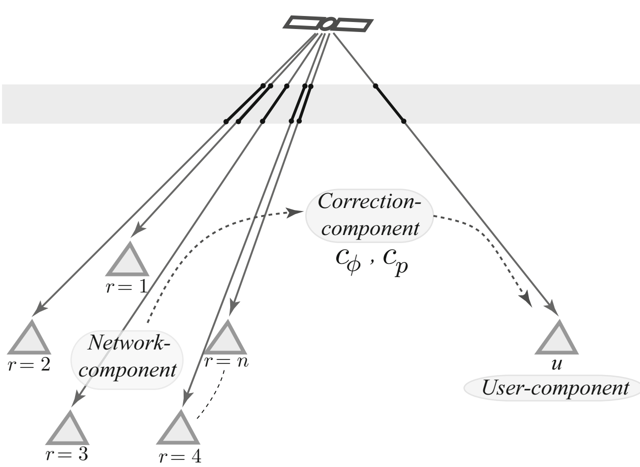

Fig. 1 Three components of PPP-RTK: (1) Network-component, (2) Correction-component and (3) User-component

The network's system of observation equations, as formulated in (1), is *not* yet in the form to enable one to perform the network adjustment. This is due to the fact that the information content in the observations (1) is not sufficient to determine the network's 'absolute' parameters. Only estimable combinations of the absolute parameters, the network observations are able to solve for. As shown in Teunissen and Khodabandeh (2015), a careful application of $\mathcal{S}$-system theory removes the underlying rank-deficiency of the model, thereby linking the observations to the stated estimable combinations. Through such rank-deficiency removal, a *minimum* set of parameters, the $\mathcal{S}$-basis, is chosen to make the system of equations (1) full-rank. A detailed explanation of the full-rank GNSS observation equations using the $\mathcal{S}$-system theory is presented in Odijk et al. (2015).

Given a choice of $\mathcal{S}$-basis parameters, a full-rank version of the network model (1) can be shown to be given as

$$\begin{aligned} \Delta\phi^s_{r,j} &= \Delta\tilde{\rho}^s_r + d\tilde{t}_r - d\tilde{t}^s - \mu_j \tilde{\iota}^s_r + \lambda_j (\tilde{z}^s_{r,j} + \tilde{\delta}_{r,j} - \tilde{\delta}^s_{,j}) \\ \Delta p^s_{r,j} &= \Delta\tilde{\rho}^s_r + d\tilde{t}_r - d\tilde{t}^s + \mu_j \tilde{\iota}^s_r + \tilde{d}_{r,j} - \tilde{d}^s_{,j} \end{aligned} \tag{2}$$

with $\Delta\tilde{\rho}^s_r = g^{sT}\Delta\tilde{x}_r$. Compare the full-rank model (2) with (1). Both look identical in structure. The absolute parameters are just replaced by the corresponding estimable combinations, highlighted by the $\tilde{.}$-symbol. For instance, the role of the absolute slant ionospheric delay ι^s_r is now taken by its estimable counterpart $\tilde{\iota}^s_r$. Despite their resemblance however, they differ in their interpretations, i.e. $\tilde{\iota}^s_r \neq \iota^s_r$. The estimable ionospheric delay $\tilde{\iota}^s_r$ is structured by the absolute ionospheric delay ι^s_r and the geometry-free (GF) combinations of the code biases, i.e. $d_{r,GF}$ and $d^s_{,GF}$ (cf. Table 1). We recall that these geometry-free combinations are nothing else, but scaled versions of the 'differential code biases' (Schaer 1999).

Table 1 presents the interpretation of the estimable parameters involved in the network model (2), together with the chosen $\mathcal{S}$-basis. As shown for instance, the estimable receiver and satellite clocks are, respectively, biased by the ionosphere-free (IF) combinations of the code biases, i.e. $d_{r,IF}$ and $d^s_{,IF}$, whereas the estimable ambiguities are the double-differenced (DD) integer-valued ambiguities.

2.2 Correction-Component

Not *all* of the network's parameters, given in Table 1, are of interest to the PPP-RTK users, of course. Apart from the orbital corrections, the PPP-RTK users need to be provided with the satellite clock, phase/code bias and (sometimes) the ionospheric corrections. Here we consider the case where no ionospheric correction is provided to the user. With this in mind, the PPP-RTK corrections here are referred to as the estimable satellite clocks $d\tilde{t}^s$, phase biases $\tilde{\delta}^s_{,j}$ and, in the multi-frequency scenario ($f > 2$), the code biases $\tilde{d}^s_{,j}$. One can therefore structure the following combined corrections,

$$\begin{aligned} c^s_{\phi,j} &= d\tilde{t}^s + \lambda_j \tilde{\delta}^s_{,j} \\ c^s_{p,j} &= \begin{cases} d\tilde{t}^s & j = 1,2 \\ d\tilde{t}^s + \tilde{d}^s_{,j} & j > 2 \end{cases} \end{aligned} \tag{3}$$

in which $c^s_{\phi,j}$ and $c^s_{p,j}$ are the combined corrections to be added to the user phase and code data, respectively.

As pointed out in the previous section, the estimable corrections do *not* represent the 'absolute' parameters, but instead *act* as such. Their interpretation changes, would the choice of the network's $\mathcal{S}$-basis change. With the aid of the interpretation given in Table 1, the combined corrections (3) can be characterized through the following *fivefold* expressions

$$\begin{bmatrix} c^s_{\phi,j} \\ c^s_{p,j} \end{bmatrix} = \mathrm{I} - \mathrm{II}_1 - \mathrm{III}_1 - \mathrm{IV}_1 - \mathrm{V}_{[1,2]} \tag{4}$$

Each of these five terms has its own insightful functionality (cf. Table 2). The first term I contains the 'absolute' parameters dt^s, $\delta^s_{,j}$ and $d^s_{,j}$. Its functionality is considered to be the most primary one, since it does what it is supposed to do, namely to remove the satellite clocks, phase and code biases from the user observation equations.

The second term II_1 contains the increment of the geometric/tropospheric range of the reference network receiver, i.e. $\Delta\rho^s_1$. Its functionality is therefore to establish a *positional* link between the user and the reference network receiver $r = 1$. That the first receiver is taken as the reference network is due to the choice of $\mathcal{S}$-basis by the network-component. Would one lump the geometric/tropospheric range of the second network receiver (i.e. $\Delta\rho^s_2$) with the satellite clocks, the interpretation of II_1 would then change to

$$\mathrm{II}_1 \mapsto \mathrm{II}_2 = \begin{bmatrix} \Delta\rho^s_2 \\ \Delta\rho^s_2 \end{bmatrix}, \tag{5}$$

which then establishes a positional link between the user and the reference network receiver $r = 2$. One can also consider a more general case, when the satellite clocks are lumped with an *average* of the geometric/tropospheric ranges over all the network stations, say $\Delta\rho^s_{\bar{r}} = (1/n)\sum_{r=1}^n \Delta\rho^s_r$. Given such $\mathcal{S}$-basis, the interpretation of II_1 changes to

$$\mathrm{II}_1 \mapsto \mathrm{II}_{\bar{r}} = \frac{1}{n}\sum_{r=1}^{n} \mathrm{II}_r, \tag{6}$$

Table 1 Estimable parameters formed by the chosen $\mathcal{S}$-basis of the single-system network model

Positions/ZTDs	$\Delta\tilde{x}_r = x_{1r};\ r \neq 1$
Ionospheric delays	$\tilde{\iota}^s_r = \iota^s_r + d_{r,GF} - d^s_{,GF}$
Receiver clocks	$d\tilde{t}_r = dt_{1r} + d_{1r,IF};\ r \neq 1$
Satellite clocks	$d\tilde{t}^s = (dt^s + d^s_{,IF}) - (dt_1 + d_{1,IF}) - g^{sT}\Delta x_1$
Ambiguities	$\tilde{z}^s_{r,j} = z^s_{1r,j} - z^1_{1r,j};\ r \neq 1,\ s \neq 1,$
Rec. phase biases	$\tilde{\delta}_{r,j} = \delta_{1r,j} + \frac{1}{\lambda_j}(\mu_j d_{1r,GF} - d_{1r,IF}) + z^1_{1r,j};\ r \neq 1$
Sat. phase biases	$\tilde{\delta}^s_{,j} = \delta^s_{,j} + \frac{1}{\lambda_j}(\mu_j[d^s_{,GF} - d_{1,GF}] - [d^s_{,IF} - d_{1,IF}]) - \delta_{1,j} - z^s_{1,j}$
Rec. code biases	$\tilde{d}_{r,j} = d_{1r,j} - (d_{1r,IF} + \mu_j d_{1r,GF});\ \ r \neq 1,\ j > 2$
Sat. code biases	$\tilde{d}^s_{,j} = [d^s_{,j} - (d^s_{,IF} + \mu_j d^s_{,GF})] - [d_{1,j} - (d_{1,IF} + \mu_j d_{1,GF})];\ j > 2$
$\mathcal{S}$-basis parameters	$\Delta x_1,\ dt_1, d_{1,j},\ \delta_{1,j},\ z^s_{1,j},\ z^1_{r,j},\ d_{r\neq 1,j=1,2},\ d^s_{,j=1,2}$

$(.)_{,IF} = \frac{1}{\mu_2-\mu_1}\{\mu_2(.)_{,1} - \mu_1(.)_{,2}\}$; $(.)_{,GF} = \frac{1}{\mu_2-\mu_1}\{(.)_{,2} - (.)_{,1}\}$

Table 2 The fivefold expression of the single-system PPP-RTK corrections, given the $\mathcal{S}$-basis in Table 1

		I		II_1		III_1		IV_1		$\mathrm{V}_{[1,2]}$
$\begin{bmatrix} c^s_{\phi,j} \\ c^s_{p,j} \end{bmatrix}$	=	$\begin{bmatrix} dt^s + \lambda_j\delta^s_{,j} \\ dt^s + d^s_{,j} \end{bmatrix}$	−	$\begin{bmatrix} \Delta\rho^s_1 \\ \Delta\rho^s_1 \end{bmatrix}$	−	$\begin{bmatrix} \lambda_j z^s_{1,j} \\ 0 \end{bmatrix}$	−	$\begin{bmatrix} \Delta dt_1 + \lambda_j\Delta\delta_{1,j} \\ \Delta dt_1 + \Delta d_{1,j} \end{bmatrix}$	−	$\begin{bmatrix} -\mu_j \\ +\mu_j \end{bmatrix} d^s_{,GF}$
		Absolute-term		Positional-link		Ambiguity-link		Receiver-specific link		Ionosphere-specific link

$\Delta dt_1 = dt_1 + d_{1,IF}$; $\Delta\delta_{1,j} = \delta_{1,j} + \frac{1}{\lambda_j}(\mu_j d_{1,GF} - d_{1,IF})$; $\Delta d_{1,j} = d_{1,j} - (\mu_j d_{1,GF} + d_{1,IF})$

making a positional link between the user and the average of the network receivers, i.e. $\bar{r}$.

The third term $\mathbb{III}_1$ contains the integer ambiguities of the reference network receiver $r = 1$, i.e. $z^s_{1,j}$. Thus it establishes an *ambiguity* link between the user and the reference network receiver $r = 1$. Similar to the second term, one can change its dependency on the first receiver to another by changing the network's $\mathcal{S}$-basis.

The fourth term $\mathbb{IV}_1$ contains the receiver-dependent parameters of the reference network receiver $r = 1$. Its functionality is to make the user receiver-dependent parameters estimable with respect to those of the reference receiver $r = 1$. Similar to the second and third terms $\mathbb{II}_1$ and $\mathbb{III}_1$, the interpretation of $\mathbb{IV}_1$ can change, for instance, to $\mathbb{IV}_2$ or $\mathbb{IV}_{\bar{r}}$, would the network's $\mathcal{S}$-basis change.

We finally consider the functionality of the last term $\mathbb{V}_{[1,2]}$. It contains the geometry-free components of the satellite code biases on the first two frequencies ($j = 1, 2$), i.e. $d^s_{,GF}$. As it is accompanied by the coefficients $[-\mu_j, \mu_j]^T$, it gets fully absorbed by the user ionospheric parameters. Due to its dependency on the network's $\mathcal{S}$-basis, the interpretation of $\mathbb{V}_{[1,2]}$ can change. One can form $d^s_{,GF}$ based on the first and *third* frequencies instead of the first and second frequencies (cf. Table 1). With such newly-defined geometry-free combinations, the last term $\mathbb{V}_{[1,2]}$ switches to $\mathbb{V}_{[1,3]}$, making a different estimable ionospheric parameter for the user.

2.3 User-Component

Replacing the subscript r by the user index u in (1), the single-receiver user observation equations follow as

$$\begin{aligned} \Delta\phi^s_{u,j} &= \Delta\rho^s_u + dt_u - dt^s - \mu_j \iota^s_u + \lambda_j (z^s_{u,j} + \delta_{u,j} - \delta^s_{,j}) \\ \Delta p^s_{u,j} &= \Delta\rho^s_u + dt_u - dt^s + \mu_j \iota^s_u + d_{u,j} - d^s_{,j} \end{aligned} \tag{7}$$

The above user system of observation equations is not solvable for an integer ambiguity resolved position. Applying the network-derived corrections (3), the user observation equations (7) can, however, be corrected as

$$\begin{aligned} \Delta\phi^s_{u,j} + c^s_{\phi,j} &= \Delta\tilde{\rho}^s_u + d\tilde{t}_u - \mu_j \tilde{\iota}^s_u + \lambda_j (\tilde{z}^s_{u,j} + \tilde{\delta}_{u,j}) \\ \Delta p^s_{u,j} + c^s_{p,j} &= \Delta\tilde{\rho}^s_u + d\tilde{t}_u + \mu_j \tilde{\iota}^s_u + \tilde{d}_{u,j} \end{aligned} \tag{8}$$

with $\Delta\tilde{\rho}^s_u = g^{sT} \Delta\tilde{x}_u$. The above user corrected observation equations are now solvable, but only for the estimable parameters (with the $\tilde{.}$-symbol) driven by the fivefold functionality of the corrections (4). Their interpretation follows from the user version of those in Table 1, i.e. with r replaced by u. We recall the *integer-recovery* role of the ambiguity link $\mathbb{III}_1$ in (4), making the integer-recovered user ambiguities the straightforward DD ambiguities, that is

$$\tilde{z}^s_{u,j} = z^{1s}_{1u,j} \in \mathbb{Z}, \quad s \neq 1 \tag{9}$$

3 Multi-System PPP-RTK

In the previous section, the three components of single-system PPP-RTK were discussed. In this section, we extend the concept to q satellite systems $\star = \text{G}, \text{J}, \ldots, \text{E}$. As one needs to discriminate between the satellites of different systems, our earlier satellite index 's' becomes obsolete. Instead, we make use of the satellite index $s_\star$ ($s_\star = 1_\star, \ldots, m_\star$) for the system $\star$. Although each system can broadcast signals on different frequency bands, in this study we restrict ourselves to those frequency bands that are in common with these q systems. With this in mind, our earlier frequency index 'j' now stands for the jth *overlapping* frequency of the systems. This restriction does, of course, not affect the generality of our discussion as one can apply the rank-deficiency removal to the multi-system models, of different frequencies, along similar lines as that of the single-system models.

3.1 Inter System Biases

In the multi-system case, the receiver bias delays are experienced in a *different* way from system to system, see e.g. (Hegarty et al. 2004; Montenbruck et al. 2011; Odijk and Teunissen 2013a). Under this assumption, the observation equations of the receiver r, tracking the system $\star \neq \text{G}$, follow as

$$\begin{aligned} \Delta\phi^{s_\star}_{r,j} &= \Delta\rho^{s_\star}_r + dt_r - dt^{s_\star} - \mu_j \iota^{s_\star}_r + \lambda_j [z^{s_\star}_{r,j} + \delta_{r,j} - \delta^{s_\star}_{,j} + \delta^{\text{G}\star}_{r,j}] \\ \Delta p^{s_\star}_{r,j} &= \Delta\rho^{s_\star}_r + dt_r - dt^{s_\star} + \mu_j \iota^{s_\star}_r + d_{r,j} - d^{s_\star}_{,j} + d^{\text{G}\star}_{r,j} \end{aligned} \tag{10}$$

Compare the above equations with their single-system counterparts (1). The additional parameters $\delta^{\text{G}\star}_{r,j}$ and $d^{\text{G}\star}_{r,j}$ are, respectively, referred to as the phase and code inter system biases (ISBs). They capture the difference between the receiver biases of two systems G and $\star \neq \text{G}$. They are therefore, by definition, *absent* in the observation equations of the first system $\star = \text{G}$. Note also that the data in (10) are registered in the 'time-system' of G, i.e. only one receiver clock dt_r is taken for all the systems. This is allowed as the difference between the time-systems of G and $\star \neq \text{G}$ is fully absorbed by the satellite clocks $dt^{s_\star}$ ($s_\star = 1_\star, \ldots, m_\star$).

3.2 Network-Component Affected by the ISBs

Inclusion of the additional unknowns $\delta_{r,j}^{\text{G}\star}$ and $d_{r,j}^{\text{G}\star}$ leads to extra rank-deficiencies in the network model (10). This means that the interpretation of some of the earlier estimable parameters, given in Table 1, would change. Indeed, after the rank-deficiency removal, a full-rank form of the network model can be shown to read

$$\begin{aligned} \Delta\phi_{r,j}^{s_\star} &= \Delta\tilde{\rho}_r^{s_\star} + d\tilde{t}_r - d\tilde{\tilde{t}}^{s_\star} - \mu_j \tilde{\tilde{\iota}}_r^{s_\star} + \lambda_j [\tilde{\tilde{z}}_{r,j}^{s_\star} + \tilde{\delta}_{r,j} - \tilde{\tilde{\delta}}_{,j}^{s_\star} + \tilde{\delta}_{r,j}^{\text{G}\star}] \\ \Delta p_{r,j}^{s_\star} &= \Delta\tilde{\rho}_r^{s_\star} + d\tilde{t}_r - d\tilde{\tilde{t}}^{s_\star} + \mu_j \tilde{\tilde{\iota}}_r^{s_\star} + \tilde{d}_{r,j} - \tilde{\tilde{d}}_{,j}^{s_\star} + \tilde{d}_{r,j}^{\text{G}\star} \end{aligned} \tag{11}$$

The two times fn number of absolute ISBs $\delta_{r,j}^{\text{G}\star}$ and $d_{r,j}^{\text{G}\star}$ are now, respectively, replaced by $f(n-1)$ number of *estimable* phase ISBs $\tilde{\delta}_{r,j}^{\text{G}\star}$ and $(f-1)(n-1)$ number of *estimable* code ISBs $\tilde{d}_{r,j}^{\text{G}\star}$ (per system $\star \neq \text{G}$). The ISBs also change the interpretation of the estimable parameters corresponding to the systems $\star \neq \text{G}$. They are highlighted by the $\tilde{\tilde{.}}$-symbol rather than the $\tilde{.}$-symbol. Their interpretations are given in Eqs. (12) and (17), but with the subscript r replaced by u.

3.3 Correction-Component Affected by the ISBs

As the ISB-affected network-derived estimable parameters, our earlier estimable satellite clocks $d\tilde{t}^{s_\star}$, phase biases $\tilde{\delta}_{,j}^{s_\star}$ and code biases $\tilde{d}_{,j}^{s_\star}$, of systems $\star \neq \text{G}$, are modified to

$$\begin{aligned} d\tilde{t}^{s_\star} &\mapsto d\tilde{\tilde{t}}^{s_\star} = d\tilde{t}^{s_\star} - d_{1,IF}^{\text{G}\star} \\ \tilde{\delta}_{,j}^{s_\star} &\mapsto \tilde{\tilde{\delta}}_{,j}^{s_\star} = \tilde{\delta}_{,j}^{s_\star} - \tfrac{1}{\lambda_j}(\mu_j d_{1,GF}^{\text{G}\star} - d_{1,IF}^{\text{G}\star}) - \delta_{1,j}^{\text{G}\star} \\ \tilde{d}_{,j}^{s_\star} &\mapsto \tilde{\tilde{d}}_{,j}^{s_\star} = \tilde{d}_{,j}^{s_\star} + (\mu_j d_{1,GF}^{\text{G}\star} + d_{1,IF}^{\text{G}\star}) - d_{1,j}^{\text{G}\star};\quad j > 2 \end{aligned} \tag{12}$$

These changes in the above PPP-RTK corrections are important, as they in turn change the estimability of the user parameters. To see this, we revisit the combined corrections (3), where the satellite index 's' is replaced by '$s_\star$'. With the link given in (12), our earlier fivefold expression (4) admits the extra term

$$\text{VI}_1^{\text{G}\star} = \begin{bmatrix} \lambda_j \Delta\delta_{1,j}^{\text{G}\star} \\ \Delta d_{1,j}^{\text{G}\star} \end{bmatrix} \tag{13}$$

as follows

$$\begin{bmatrix} c_{\phi,j}^{s_\star} \\ c_{p,j}^{s_\star} \end{bmatrix} = \text{I} - \text{II}_1 - \text{III}_1 - \text{IV}_1 - \text{V}_{[1,2]} - \text{VI}_1^{\text{G}\star} \tag{14}$$

where

$$\begin{aligned} \Delta\delta_{1,j}^{\text{G}\star} &= \delta_{1,j}^{\text{G}\star} + \tfrac{\mu_j}{\lambda_j} d_{1,GF}^{\text{G}\star} \\ \Delta d_{1,j}^{\text{G}\star} &= d_{1,j}^{\text{G}\star} - \mu_j d_{1,GF}^{\text{G}\star} \end{aligned} \tag{15}$$

The additional term $\text{VI}_1^{\text{G}\star}$ contains the ISB parameters of the network reference receiver $r = 1$. It is absent in the first system G, and present in the systems $\star \neq \text{G}$. Its functionality is to make the user ISB parameters estimable with respect to those of the reference receiver $r = 1$. The dependency on $r = 1$ stems from the fact that the ISBs of the first network receiver, i.e. $\delta_{1,j}^{\text{G}\star}$ and $d_{1,j}^{\text{G}\star}$, are chosen as the network's $\mathcal{S}$-basis.

3.4 User-Component Affected by the ISBs

Applying the network-derived corrections (14), the multi-system user corrected observation equations follow as

$$\begin{aligned} \Delta\phi_{u,j}^{s_\star} + c_{\phi,j}^{s_\star} &= \Delta\tilde{\rho}_u^{s_\star} + d\tilde{t}_u - \mu_j \tilde{\tilde{\iota}}_u^{s_\star} + \lambda_j [\tilde{\tilde{z}}_{u,j}^{s_\star} + \tilde{\delta}_{u,j} + \tilde{\delta}_{u,j}^{\text{G}\star}] \\ \Delta p_{u,j}^{s_\star} + c_{p,j}^{s_\star} &= \Delta\tilde{\rho}_u^{s_\star} + d\tilde{t}_u + \mu_j \tilde{\tilde{\iota}}_u^{s_\star} + \tilde{d}_{u,j} + \tilde{d}_{u,j}^{\text{G}\star} \end{aligned} \tag{16}$$

The above model is now solvable as it is linked to the network's $\mathcal{S}$-system. While the interpretation of the receiver specific parameters is the same as that of (8), the interpretation of the ionospheric delays and ambiguities changes, respectively, to

$$\begin{aligned} \tilde{\tilde{\iota}}_u^{s_\star} &= \tilde{\iota}_u^{s_\star} + d_{u,GF}^{\text{G}\star}, \\ \tilde{\tilde{z}}_{u,j}^{s_\star} &= \tilde{z}_{u,j}^{s_\star} - \tilde{z}_{u,j}^{1_\star} \in \mathbb{Z};\quad s_\star \neq 1_\star \end{aligned} \tag{17}$$

According to the user corrected model (16), the f number of *non-integer* phase ISBs $\tilde{\delta}_{u,j}^{\text{G}\star}$ take the role of the *integer* estimable ambiguities of the first satellite of each system $\star \neq \text{G}$, i.e. $\tilde{z}_{u,j}^{1_\star}$. Now the question is whether it is possible to bring back $\tilde{z}_{u,j}^{1_\star}$, thereby *maximizing* the number of integer estimable ambiguities. This would then result in an increase in the user model's redundancy. To address this question, recall that non-zero ISBs pop up, when the types of the network and user receivers are different. In case the type (i.e., make, type, firmware) of the user receiver u would be the *same* as that of a network receiver, say receiver q, their estimable code ISBs and the fractional part of their estimable phase ISBs are identical (Odijk and Teunissen 2013b). Such possibility can therefore be taken advantage of, would the network-component provide, next to the PPP-RTK corrections, a '*look-up*' table of its ISB solutions $\tilde{\delta}_{r,j}^{\text{G}\star}$ and $\tilde{d}_{r,j}^{\text{G}\star}$ for $r = 1, \ldots, n$. The user can then search the table

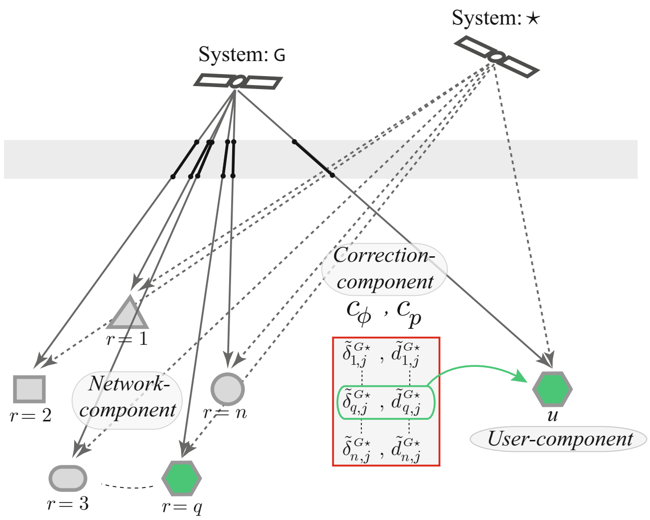

Fig. 2 Three components of multi-system PPP-RTK supported by the ISB look-up table (in *red*). Given a network of mixed-receiver types, the user 'u' has the possibility of finding the network-derived ISBs of the network receiver of the *same* type, say $r=q$ (in *green*)

for a network receiver of the same type (i.e. receiver $r=q$) and pick up the corresponding ISBs $\tilde{\delta}^{G\star}_{q,j}$ and $\tilde{d}^{G\star}_{q,j}$ (see Fig. 2). Such a user *ISB-corrected* model reads therefore

$$\begin{aligned} \Delta\phi^{s_\star}_{u,j} + c^{s_\star}_{\phi,j} - \lambda_j \tilde{\delta}^{G\star}_{q,j} &= \\ &\Delta\tilde{\rho}^{s_\star}_u + d\tilde{t}_u - \mu_j \tilde{\iota}^{s_\star}_u + \lambda_j [\tilde{z}^{s_\star}_{u,j} + \tilde{\delta}_{u,j}] \\ \Delta p^{s_\star}_{u,j} + c^{s_\star}_{p,j} - \tilde{d}^{G\star}_{q,j} &= \\ &\Delta\tilde{\rho}^{s_\star}_u + d\tilde{t}_u + \mu_j \tilde{\iota}^{s_\star}_u + \tilde{d}_{u,j} \end{aligned} \quad (18)$$

Compare the above model (18) with (16). The $(f-1)$ number of code ISBs $\tilde{d}^{G\star}_{u,j}$ are corrected. Thus the model is strengthened as the model's redundancy increases by $(f-1)$ per system $\star \neq$ G. Note also that the f number of integer ambiguities $\tilde{z}^{1_\star}_{u,j}$ are now recovered. Thus after integer ambiguity resolution, the redundancy even increases further by f per system $\star \neq$ G.

Also compare the user ISB-corrected model (18) with its single-system counterpart (8). Both are identical in structure. Therefore, the ISB-corrected model acts as if a single-system setup is considered, with a difference, that the number of visible satellites can then be much larger than that of the single-system setup.

4 Concluding Remarks

As the network observation equations are not capable of determining the absolute parameters, the $\mathcal{S}$-system theory must be applied to remove the rank-deficiencies underlying the model, thereby identifying the interpretation of the PPP-RTK corrections. Next to their most primary functionality, the corrections were shown to establish important links between the network and the users (cf. Table 2). In case of multi-system PPP-RTK, additional ISB parameters enter, affecting all the three components of PPP-RTK. The corrections are further biased by the ISBs (cf. (12)) and the estimability of the user ionospheric and ambiguity parameters would also change (cf. (17)). In this contribution, we proposed the network-derived ISB look-up table. Integrated with such additional information, the PPP-RTK corrections can then recover the integerness of the *entire* set of the single-receiver user's estimable ambiguities (cf. Fig. 2).

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