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Abstract

We consider an ad-hoc network of wireless sensors that harvest energy from the environment and broadcasts measurements independently, at random, provided sufficient energy is available. Clients arriving at the network are interested in retrieving measurements from an arbitrary set of sensors of some fixed size s. We show that the sensors broadcast measurements according to a phase-type distribution. We determine the probability distribution of the time needed for a client to retrieve s sensor measurements. We provide a closed-form expression for the retrieval time of s sensor measurements for an asymptotically large capacity of the sensor battery or the rate at which energy is harvested. We also analyze numerically the retrieval time of s sensor measurements under various assumptions regarding the battery capacity of the sensors, the energy harvesting and consumption processes. The results provide a lower bound for the energy storage capacity of the sensors for which the retrieval time of measurements is below a targeted level. It is also shown that the ratio between the energy harvesting rate and the broadcasting rate significantly influences the retrieval time of measurements, whereas deploying sensors with large batteries does not significantly reduce the retrieval time of measurements. Numerical experiments also indicate that our theoretical

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results generalize to non-identical energy harvesting rates, various amount of energy consumed upon a broadcast and non-exponential distributions of the energy harvesting and broadcasting processes.

Keywords: wireless sensor networks, energy harvesting, data retrieval time, phase-type distribution, order statistics

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1. Introduction

This paper considers the problem of retrieving measurements from an ad-hoc wireless sensor network. The measurements should originate from an arbitrary set of sensors, where the size of the set is predefined and fixed. The sensors harvest energy from the environment independently of the other sensors and at random points in time. This reflects the stochastic nature of the availability of the energy source. We further assume that the sensors store their energy in batteries of limited capacity. When sensors have energy, they broadcasts measurements in a distributed manner. A broadcast implies energy consumption for the broadcasting sensor. Clients arrive at the network at random points in time and are interested in retrieving measurements from an arbitrary set of sensors. The size of the set is fixed and is considered to be the minimum number of measurements needed to compute an aggregate. Examples of applications are the case of sensors that estimate their position by combining several relative position measurements between themselves and the other sensors [1] or the case of users that obtain a reliable estimate of an attribute by combining noisy measurements from several sensors [2].

We determine the probability distribution of the time to retrieve measurements from an arbitrary set of sensors of fixed size. We also analyze the retrieval time of measurements when the capacity of the sensor battery or the rate at which energy is harvested are asymptotically large. These results show the impact of the energy availability, as well as the energy storage capabilities, on the process of measurement retrieval from an ad-hoc wireless sensor network with distributed data transmissions.

Energy harvesting for wireless communications has received significant attention in the last decade [3]. Energy harvesting brings new dimensions to the wireless communications problem in the form of intermittency and randomness of available energy [4]. Many authors have considered energy harvesting communication systems from the viewpoint of the communication channel of a single-transmitter to a single-receiver. For example, [5]

studies the minimization of the time to transmit a fixed number of bits using an Additive White Gaussian Noise (AWGN) broadcast channel. Here, a single transmitter harvests energy and has a finite-capacity rechargeable battery. In [6] optimal transmission policies are derived to specify whether to transmit incoming data packets or to drop them. The policies are derived based on a value attached to each packet and on the energy available at a single transmitter. The energy arrival process is assumed to be known in advance, in an offline manner. In [7] a general framework is provided to maximize the amount of transmitted data by a given deadline when the battery of the transmitter suffers from energy leakage, under similar conditions. In [8, 9, 10] dynamic programming is employed to determine an optimal energy allocation policy over a finite horizon so that the number of transmitted bits is maximized.

Significant research has been conducted in the area of information theory, with a focus on impairments in the communication channel such as white noise, fading and interference. In [11, 12] the minimization of the time to transmit a fixed number of bits using an Additive White Gaussian Noise (AWGN) broadcast channel is considered. However, the energy arrival process is assumed to be known in advance, in an offline manner. In [13, 14] the process of energy harvesting is stochastic. However, in these references *centralized* transmission policies that minimize the mean delay of data transmission are derived. In [14], the average delay of data packets arriving according to a Poisson process at a *single* transmitter is considered.

The problem of maximizing the amount of data transmitted within a fixed time window is considered in [7, 8, 9, 10]. In [15], the probability of successful reception of data packets and the energy cost per transmitted packet are determined for energy harvesting devices that broadcast using non-perfect transmission channels. The authors propose an erasure-based broadcast scheme to guarantee reliable transmissions. In [16] a game-theoretical approach is used to dynamically adjust the transmission power of sensors so that efficient use is made of the harvested energy. Such transmission policies require central coordination and, thus, may be difficult to implement for some ad-hoc wireless sensor networks. An interesting alternative viewpoint is taken in [17]. Power-neutral operations are proposed, where the instantaneous power consumption of the system must match the instantaneous harvested power (corresponding to very low energy storage capacity). Here, the focus is on processing and not on communications.

This paper contributes in the following way. Complementary to stud-

ies focusing on communication channel aspects, we further develop queueing theory in order to find analytical expressions describing fundamental performance trade-offs of an energy harvesting system, focusing on the impact of the energy harvesting process on the overall system (as opposed to only the communication channel). We analyze the time to retrieve measurements from a network of sensors (as opposed to a single source), with energy arriving according to a stochastic energy arrival process (not known in advance) to recharge the sensor batteries. Sensors transmit using a distributed (as opposed to centralized) protocol. Here a randomly arriving receiver needs to receive multiple distinct measurements (as opposed to a single measurement). We provide a formal analysis. Our viewpoint allows us to provide closed-form expressions for finite battery capacities, which, according to [4], is an important open research problem. We also conduct discrete event simulations for general energy harvesting and consumption models. The simulation results indicate that our theoretical results generalize to non-identical energy harvesting rates, various amount of energy consumed upon a broadcasting and non-exponential distributions for the energy harvesting and consumption processes. Overall, this work provides a formal theoretical support for the design of applications for ad-hoc sensor networks addressing the impact of energy arrival rate and storage capacity on the retrieval time of measurements under a distributed data transmission policy.

The remainder of this paper is organized as follows. In Section 2 we formulate the model and the problem statement. In Section 3 we determine the distribution of the time for a client to retrieve measurements from an arbitrary set of sensors of fixed size. We also determine the retrieval time of measurements when the rate at which energy is harvested and the maximum capacity of the sensor batteries are asymptotically large. We also conduct discrete event simulations to complement our numerical results and to investigate general energy harvesting and consumption models. In Section 4 we numerically compute the retrieval time of measurements from an arbitrary set of sensors of fixed size under various assumptions regarding the energy harvesting and consumption models. In Section 5 we discuss the results and provide conclusions.

2. Model and Problem Statement

We consider an ad-hoc network of N wireless sensors. Each sensor harvests one unit of energy from the environment at an exponential rate λ_e,

independently of the other sensors. Sensors have a maximum storage capacity of B energy units. When the harvested energy exceeds the storage capacity of the battery, the excessive energy is discarded.

Each sensor broadcasts a measurement at an exponential rate μ/N, independently of the other sensors. Clearly, a sensor broadcasts a measurement only if it has energy. Upon a broadcast, the energy of the broadcasting sensor decreases by one unit. The assumption that each sensor broadcasts at an exponential rate μ/N could be interpreted as the situation when the entire network of sensors broadcasts measurements at an exponential rate μ and this rate is shared uniformly among the N sensors of the network. Also, for simplicity, the energy of a sensor is assumed to decrease or increase by one unit upon a broadcast and an additional energy harvest, respectively. However, similar techniques as in this paper can be employed for the case of general rates at which the energy of a sensor varies due to broadcasts or additional energy harvests.

Clients arrive at the sensor network according to a Poisson process with rate λ_a. Each client waits until receiving $1 \leq s \leq N$ measurements from an arbitrary set of sensors. Each measurement should originate from a distinct sensor. Based on the retrieved set of measurements, each client computes an aggregate. Upon a sensor broadcast, all clients present in the system receive the broadcasted measurement simultaneously. The clients leave the system as soon as they acquire s measurements.

We are interested in the time, denoted by W_s, for a client to retrieve s measurements from an arbitrary set of sensors of size s.

Lastly, we introduce some notation that will be useful when working with phase-type distributions. Let $\mathbf{e}$ be a column vector with all unit entries for which the dimensions are determined by the context. Let I_k denote the $k \times k$ identity matrix. For $n \times n$ matrix M_1 and $m \times m$ matrix M_2, let $M_1 \otimes M_2$ denote the Kronecker product of matrices M_1 and M_2 and let $M_1 \oplus M_2$ denote their Kronecker sum, i.e., $M_1 \oplus M_2 = M_1 \otimes I_m + I_n \otimes M_2$. Finally, let $M^{\otimes n}$ and $M^{\oplus n}$ denote the n-fold Kronecker product and the n-fold Kronecker sum with itself, respectively.

3. Analysis

In this section we first determine the distribution of the time for a single sensor to broadcast, given that the system is in steady-state. We show that this is a phase-type distribution. Using these results, we then determine the

distribution of W_s. Lastly, we compute the $\mathbb{E}[W_s]$ for asymptotically large B, the maximum capacity of the sensor batteries, and λ_e, the rate at which a sensor harvests energy from the environment.

3.1. A single sensor

Firstly, we consider the steady-state probability that an arbitrary sensor has i units of energy, $0 \leq i \leq B$, which we denote by ν_i. The evolution of the units of energy at a sensor follows a Birth-and-Death model with a finite state space $\{0, 1, \ldots, B\}$ with births at rate λ_e and deaths at rate μ/N. For the special case $\lambda_e = \mu/N$, we denote the steady state probability that an arbitrary sensor has i units of energy by $\bar{\nu}_i$, $0 \leq i \leq B$. The steady-state distribution of this model is well known in literature (see, for instance, [18]) and is, thus, stated below without proof .

Lemma 1. *The steady-state probability for an arbitrary sensor to have i units of energy, $0 \leq i \leq B$, is:*

$$\nu_i = \begin{cases} \bar{\nu}_0, \textit{ if } \lambda_e = \mu/N \\ \nu_0 \left(\frac{\lambda_e N}{\mu}\right)^i, \textit{ otherwise,} \end{cases} \tag{1}$$

where $\nu_0 = (\lambda_e N/\mu - 1)/((\lambda_e N/\mu)^{B+1} - 1)$ and $\bar{\nu}_0 = 1/(B+1)$.

Note that in the above ν_0 ($\bar{\nu}_0$ if $\lambda_e = \mu/N$) is the probability that the battery of a sensor is depleted.

Next, we consider W, which denotes the time until an arbitrary sensor broadcasts, given that the system is in steady-state. Based on W, we compute the distribution of W_s by observing that, upon arrival, a client *sees* the steady-state energy available at the sensor [19]. This is valid since the sensors operate independently of the arrivals of the clients and since the clients arrive according to a Poisson process, and, thus, *see* the system in steady-state.

Observing that the evolution of the energy at an arbitrary sensor follows a continuous-time Markov process, the distribution of W can be modeled as a phase-type distribution as follows. Consider a continuous-time Markov chain with $B+2$ states. The state $i \in \{0, 1, \ldots, B\}$ is transient and corresponds to a sensor having i units of energy. The $(B+2)-$th state is an absorbing state. This state is reached when the sensor broadcasts a measurement. At an exponential rate λ_e, a jump occurs from state i to state $i+1$, $0 \leq i < B$. This corresponds to the sensor harvesting an additional unit of energy. At

an exponential rate μ/N, a transition occurs from state $1 \leq i \leq B$ to the absorbing state. This corresponds to a broadcast. Let the initial distribution over the transient states be ν_i ($\bar{\nu}_i$ if $\lambda_e = \mu/N$), as described in Lemma 1. Then, the time until absorption is W, as desired.

To this end, we simplify the model by observing that in the above description, the states 1 to B can be aggregated into a single transient state, which we denote by 1. There is a transition from state 0 to this aggregated state 1 at rate λ_e and there is a single outgoing transition from this aggregated state 1 to the absorbing state at rate μ/N. Below, we give the formal representation of this phase-type distribution as $(\mathbf{a}, T)$ and specify the row vector $\mathbf{a}$ and the matrix T. Given this representation as a phase-type distribution, we immediately obtain $\mathbb{P}(W \leq t) = 1 - \mathbf{a}e^{Tt}\mathbf{e}$. In this case, however, since T has a simple structure, we also obtain the distribution function in an explicit form. This yields the following result.

Lemma 2. *The distribution of W is phase-type $(\boldsymbol{a}, T)$, where*

$$\boldsymbol{a} = \begin{cases} \begin{bmatrix} \bar{\nu}_0 & 1-\bar{\nu}_0 \end{bmatrix}, \text{if } \lambda_e = \mu/N \\ \\ \begin{bmatrix} \nu_0 & 1-\nu_0 \end{bmatrix}, \text{otherwise.} \end{cases} \qquad T = \begin{bmatrix} -\lambda_e & \lambda_e \\ 0 & -\mu/N \end{bmatrix}. \tag{2}$$

The distribution function of W can be expressed as

$$\mathbb{P}(W \leq t) = \begin{cases} 1 - e^{-\lambda_e t} - \lambda_e t e^{-\lambda_e t}\bar{\nu}_0, \text{if } \lambda_e = \mu/N \\ 1 - e^{-\frac{\mu}{N}t} + \frac{\frac{\mu}{N}}{\frac{\mu}{N}-\lambda_e}\nu_0\left(e^{-\frac{\mu}{N}t} - e^{-\lambda_e t}\right), \text{otherwise.} \end{cases} \tag{3}$$

Proof. The representation in (2) follows from the discussion above and Lemma 1. Equation (3) is obtained by observing that, given that the system is in state 0, which happens with probability ν_0 (probability $\bar{\nu}_0$ if $\lambda_e = \mu/N$), the distribution of W is given by the sum of two exponentially distributed random variables with parameters μ/N and λ_e (if $\lambda_e = \mu/N$, then W is Erlang$(2, \lambda_e)$ distributed). Given that the system is in the aggregated state 1, which happens with probability $1 - \nu_0$ (if $\lambda_e = \mu/N$, $1 - \bar{\nu}_0$), the distribution of W is given by an exponentially distributed random variable with parameter μ/N (parameter λ_e if $\lambda_e = \mu/N$). Therefore,

$$\mathbb{P}(W \leq t) = \begin{cases} (1 - e^{-\lambda_e t} - \lambda_e t e^{-\lambda_e t})\bar{\nu}_0 + (1 - e^{-\lambda_e t})(1 - \bar{\nu}_0), \text{if } \lambda_e = \mu/N \\ \left(1 - \frac{\frac{\mu}{N}}{\frac{\mu}{N}-\lambda_e}e^{-\lambda_e t} + \frac{\lambda_e}{\frac{\mu}{N}-\lambda_e}e^{-\frac{\mu}{N}t}\right)\nu_0 + (1 - e^{-\frac{\mu}{N}t})(1-\nu_0), \text{otherwise.} \end{cases}$$

Equation (3) follows directly from the above expression. □

Observation 1: For the general case when $0 < l \leq B$ units of energy are harvested by an arbitrary sensor at an exponential rate λ_e, the same reasoning as above holds. It only remains to derive the steady state probability that an arbitrary sensor has i units of energy, which requires solving the balance equations of the continuous-time Markov chain which characterizes this system.

Observation 2: For the general case when $0 < k \leq B$ units of energy are consumed by an arbitrary sensor upon a broadcast, the same reasoning as above holds with the observation that, letting $\nu_i^{(k)}$ denotes the steady state probability that a sensor has i units of energy, the distribution of W is phase-type $(\mathbf{a}^{(k)}, T^{(k)})$, where

$$\mathbf{a}^{(k)} = \begin{bmatrix} \nu_0^{(k)} & \nu_1^{(k)} & \dots & \nu_{k-1}^{(k)} & 1 - \sum_{i=0}^{k-1} \nu_i \end{bmatrix}, \tag{4}$$

$$T^{(k)} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ \dots \\ k-1 \\ k \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & \dots & k-1 & k \end{array} \\ \begin{pmatrix} -\lambda_e & \lambda_e & 0 & 0 & 0 & 0 \\ 0 & \lambda_e & -\lambda_e & 0 & 0 & 0 \\ 0 & 0 & \lambda_e & -\lambda_e & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \lambda_e & -\lambda_e \\ 0 & 0 & 0 & 0 & 0 & -\mu/N \end{pmatrix} \end{array}. \tag{5}$$

For reasons of simplicity of notation and tractability of analytical results, in the following we will consider the case where $l = k = 1$.

3.2. Distribution of W_s

In this section, we determine the distribution of W_s. This result is general in the sense that the distribution of W can be expressed for a general energy harvesting and broadcasting process modeled as a phase-type distribution with a general representation $(\mathbf{a}, T)$.

Theorem 1. *The distribution of W_s is:*

$$\mathbb{P}(W_s \leq t) = 1 - \sum_{j=0}^{s-1} \binom{N}{j} \left(\sum_{k=0}^{j} \binom{j}{k} (-1)^{j-k} \boldsymbol{a}^{\otimes(N-K)} \exp\left(tT^{\oplus(N-k)}\right) \boldsymbol{e} \right).$$

Proof. Recall that a client leaves the system as soon as it retrieves s measurements. Thus, we need to compute the distribution of the time between the moment a client arrives at the network and the moment when the s-th broadcast occurs, all s broadcasts originating from distinct sensors. This can be seen as the distribution of the s-th order statistic of N phase-type distributed random variables with representation $(\mathbf{a}, T)$, as introduced above. The distribution of the s-th order statistic (see, for instance, [20]), for N variables, is

$$\mathbb{P}(W_s \leq t) = \sum_{j=s}^{N} \binom{N}{j} \mathbb{P}(W \leq t)^j (\mathbb{P}(W > t))^{N-j}. \tag{6}$$

$$= 1 - \sum_{j=0}^{s-1} \binom{N}{j} (1 - \mathbb{P}(W > t))^j (\mathbb{P}(W > t))^{N-j}$$

$$= 1 - \sum_{j=0}^{s-1} \binom{N}{j} \left(\sum_{k=0}^{j} \binom{j}{k} (-1)^{j-k} \mathbb{P}(W > t)^{j-k} \right) (\mathbb{P}(W > t))^{N-j} \tag{7}$$

$$= 1 - \sum_{j=0}^{s-1} \binom{N}{j} \left(\sum_{k=0}^{j} \binom{j}{k} (-1)^{j-k} \mathbb{P}(W > t)^{N-k} \right), \tag{8}$$

where in (7) we expanded the polynomial $(1 - \mathbb{P}(W > t))^j$.

Now, observe that the distribution of $\mathbb{P}(W > t)^{N-k}$ in (8) is:

$$\mathbb{P}(W > t)^{N-k} = \mathbb{P}(\min\{Y_1, Y_2, \ldots, Y_{N-k}\} > t), \tag{9}$$

where the $Y_i, 1 \leq i \leq N-k$ are i.i.d. phase-type distributed random variables with representation $(\mathbf{a}, T)$. Therefore, $\mathbb{P}(W > t)^{N-k}$ is the first order statistic of a phase-type distributed random variable for which it is well known (see, for instance, [21]) that it is phase-type distributed with representation $(\mathbf{a}^{\otimes(N-k)}, T^{\oplus(N-k)})$. The result follows directly by inserting the distribution function of this phase-type distribution into (8). □

We are next interested in determining $\mathbb{E}[W_s]$. In principle, $\mathbb{E}[W_s]$ can be obtained directly from Theorem 1. However, the moments of order statistics of phase-type distributed random variables are known in the literature [22]. Therefore, we will resort to the results from [22]. Let m_s^k denote the k-th moment of the s-th order statistic of N phase-type distributed random variables with representation $(\mathbf{a}, T)$.

Theorem 2. *[22, Thm 4.1]*

$$m_s^k = m_{s-1}^k + \sum_{j=1}^{s} (-1)^{j-1} \binom{N-s+j}{j-1} L_{N-s+j}^{(k)},$$

where $L_j^{(k)} = \binom{N}{j}(-1)^k k! \left(\boldsymbol{a}^{\otimes j}\right) \left(T^{\oplus j}\right)^{-k} \boldsymbol{e}$, $1 \leq j \leq s$, *and* $m_0^k = 0$.

Taking $k = 1$, $\mathbb{E}[W_s]$ can be computed from Theorem 2.

3.3. Expected retrieval time of s sensor measurements

Using Theorem 2 together with the results of Lemma 2 and Theorem 1, we can now compute $\mathbb{E}[W_s]$. However, this approach involves computing the matrices $T^{\otimes j}$, where j takes values up to N. The dimension of $T^{\otimes N}$ is $2^N \times 2^N$. Therefore, the complexity of these computations is exponentially increasing in N. Since we are interested in the behaviour of the system for arbitrary large values of N, in this section we derive an expression for $\mathbb{E}[W_s]$ that has at most polynomial complexity in all model parameters.

Theorem 3. *The expected time for a client to retrieve s measurements from arbitrary s different sensors is:*

$$\begin{aligned}\mathbb{E}[W_s] = & \sum_{j=0}^{s-1} \binom{N}{j} \sum_{k=0}^{j} \binom{j}{k} (-1)^{j-k} \sum_{v=0}^{N-k} \binom{N-k}{v} \\ & \cdot \left(\frac{\omega^v (1-\omega)^{N-k-v}}{\lambda_e (N-k-v) + \frac{\mu}{N} v} \boldsymbol{1}_{\lambda_e \neq \frac{\mu}{N}} + \frac{\bar{\nu}_0^{N-k-v} \Gamma(N-k-v+1)}{\lambda_e (N-k)^{N-k-v+1}} \boldsymbol{1}_{\lambda_e = \frac{\mu}{N}} \right),\end{aligned}$$

where $\omega = 1 - \nu_0 \frac{\frac{\mu}{N}}{\frac{\mu}{N} - \lambda_e}$.

Proof. The expected retrieval time for s measurements from distinct sensors can be expressed using Theorem 1 and Lemma 2 as follows.

$$\mathbb{E}[W_s] = \int_0^\infty \mathbb{P}(W_s > t) dt = \sum_{j=0}^{s-1} \binom{N}{j} \int_0^\infty \left(\sum_{k=0}^{j} \binom{j}{k} (-1)^{j-k} (\mathbb{P}(W > t))^{N-k} \right) dt, \tag{10}$$

where (10) follows from the derivations in (8).

i) Case $\lambda_e = \mu/N$.

$$\mathbb{E}[W_s] = \sum_{j=0}^{s-1} \binom{N}{j} \sum_{k=0}^{j} \binom{j}{k} (-1)^{j-k} \int_0^\infty (e^{-\lambda_e t} + \bar{\nu}_0 \lambda_e t e^{-\lambda_e t})^{N-k} dt \tag{11}$$

$$= \sum_{j=0}^{s-1} \binom{N}{j} \sum_{k=0}^{j} \binom{j}{k} (-1)^{j-k} \int_0^\infty \sum_{v=0}^{N-k} \binom{N-k}{v} (e^{-\lambda_e t})^v (\bar{\nu}_0 \lambda_e t e^{-\lambda_e t})^{N-k-v} dt$$

$$= \sum_{j=0}^{s-1} \binom{N}{j} \sum_{k=0}^{j} \binom{j}{k} (-1)^{j-k} \sum_{v=0}^{N-k} \binom{N-k}{v} (\lambda_e \bar{\nu}_0)^{N-k-v} \int_0^\infty e^{-\lambda_e t(N-k)} t^{N-k-v} dt$$

$$= \sum_{j=0}^{s-1} \binom{N}{j} \sum_{k=0}^{j} \binom{j}{k} (-1)^{j-k} \sum_{v=0}^{N-k} \binom{N-k}{v} (\lambda_e \bar{\nu}_0)^{N-k-v} \frac{\Gamma(N-k-v+1)}{(\lambda_e(N-k))^{N-k-v+1}}, \tag{12}$$

where (11) follows from Lemma 2 and (12) follows from multiplying by $\frac{\Gamma(N-k-v+1)(\lambda_e(N-k))^{N-k-v+1}}{\Gamma(N-k-v+1)(\lambda_e(N-k))^{N-k-v+1}}$ and from $\int_0^\infty \frac{(\lambda_e(N-k))^{N-k-v+1} t^{N-k-v} e^{-\lambda_e t(N-k)}}{\Gamma(N-k-v+1)} dt = 1$ since we integrate over the pdf of an Erlang$(N-k-v+1, \lambda_e(N-k))$ distributed random variable. The result follows.

ii) Case $\lambda_e \neq \mu/N$.

$$\mathbb{E}[W_s]$$

$$= \sum_{j=0}^{s-1} \binom{N}{j} \sum_{k=0}^{j} \binom{j}{k} (-1)^{j-k} \int_0^\infty \left(\omega e^{-\frac{\mu}{N} t} + (1-\omega) e^{-\lambda_e t}\right)^{N-k} dt \tag{13}$$

$$= \sum_{j=0}^{s-1} \binom{N}{j} \sum_{k=0}^{j} \binom{j}{k} (-1)^{j-k} \int_0^\infty \sum_{v=0}^{N-k} \binom{N-k}{v} \left(\omega e^{-\frac{\mu}{N} t}\right)^v \left((1-\omega) e^{-\lambda_e t}\right)^{N-k-v} dt$$

$$= \sum_{j=0}^{s-1} \binom{N}{j} \sum_{k=0}^{j} \binom{j}{k} (-1)^{j-k} \sum_{v=0}^{N-k} \binom{N-k}{v} \omega^v (1-\omega)^{N-k-v} \int_0^\infty e^{-\frac{\mu}{N} tv} \left(e^{-\lambda_e t}\right)^{N-k-v} dt$$

$$= \sum_{j=0}^{s-1} \binom{N}{j} \sum_{k=0}^{j} \binom{j}{k} (-1)^{j-k} \sum_{v=0}^{N-k} \binom{N-k}{v} \frac{\omega^v (1-\omega)^{N-k-v}}{\lambda_e(N-k-v) + \frac{\mu}{N} v}, \tag{14}$$

where (13) follows from Lemma 2, where we denoted by $\omega = 1 - \nu_0 \frac{\frac{\mu}{N}}{\frac{\mu}{N} - \lambda_e}$ and, thus, $\mathbb{P}(W > t) = \omega e^{-\frac{\mu}{N} t} + (1-\omega) e^{-\lambda_e t}$. □

3.4. Asymptotic analysis of $\mathbb{E}[W_s]$

In this section we determine $\mathbb{E}[W_s]$ for both asymptotically large rate of energy harvesting and capacity of the sensor battery. First, we introduce the following lemma.

Lemma 3. *For any* $0 \leq j \leq s$, $j \in \mathbb{N}$,

$$\binom{N}{j} \sum_{k=0}^{j} \binom{j}{k} (-1)^{j-k} \frac{N-j}{N-k} = 1. \tag{15}$$

Proof. This proof follows from induction on j. It is easy to see that (15) holds for $j = 0$. We assume that (15) holds for some $j > 0$. We next show that (15) holds for $j + 1$.

$$\begin{aligned}
&\binom{N}{j+1} \sum_{k=0}^{j+1} \binom{j+1}{k} (-1)^{j+1-k} \frac{N-(j+1)}{N-k} \\
&= \binom{N}{j+1} \sum_{k=0}^{j+1} \binom{j+1}{k} (-1)^{j+1-k} \frac{N-k+k-(j+1)}{N-k} \\
&= \binom{N}{j+1} \sum_{k=0}^{j+1} \binom{j+1}{k} (-1)^{j+1-k} 1^k + \binom{N}{j+1} \sum_{k=0}^{j+1} \binom{j+1}{k} (-1)^{j+1-k} \frac{k-j-1}{N-k} \\
&= 0 + \binom{N}{j+1} \sum_{k=0}^{j+1} \binom{j+1}{k} (-1)^{j+1-k-1} \frac{(j+1)-k}{N-k} \\
&= \binom{N}{j+1} \sum_{k=0}^{j} \binom{j+1}{k} (-1)^{j-k} \frac{(j+1)-k}{N-k} \\
&= \binom{N}{j+1} (j+1) \sum_{k=0}^{j} \frac{j!}{k!(j+1-k-1)!} (-1)^{j-k} \frac{1}{N-k} \\
&= \binom{N}{j} \sum_{k=0}^{j} \binom{j}{k} (-1)^{j-k} \frac{N-j}{N-k} = 1,
\end{aligned}$$

where the last equality follows from the induction hypothesis.

□

Theorem 4. *For* $1 \leq N < \infty$, $1 \leq s \leq N$ *and* $0 < B < \infty$,

$$\lim_{\lambda_e \to \infty} \mathbb{E}[W_s] = \sum_{j=0}^{s-1} \frac{1}{\frac{\mu}{N}(N-j)}.$$

Proof. Taking $\lambda_e \to \infty$ in Theorem 3, we have that

$$\lim_{\lambda_e \to \infty} \mathbb{E}[W_s] = \sum_{j=0}^{s-1} \binom{N}{j} \sum_{k=0}^{j} \binom{j}{k} (-1)^{j-k} \frac{1}{\frac{\mu}{N}(N-k)},$$

where we make the observation that $\nu_0 \to 0$, $\omega \to 1$ as $\lambda_e \to \infty$ and the terms in $\sum_{v=0}^{N-k} \binom{N-k}{v} \frac{\omega^v (1-\omega)^{N-k-v}}{\lambda_e (N-k-v) + \frac{\mu}{N} v}$ tend to zero except for the case when $v = N-k$. The result now follows by multiplying by $\frac{N-j}{N-k} \frac{N-k}{N-j}$ and from Lemma 3. □

We next consider the situation when the capacity of the sensors to store energy in the battery is asymptotically large.

For $\lambda_e < \mu/N$ and $B \to \infty$, the battery of a sensor is most of the time empty as the rate at which this sensor receives energy is lower than the rate at which this sensor broadcasts. As a consequence, in this case, the waiting time for a client to retrieve s measurements from distinct sensors largely depends on λ_e, which supports the broadcasting process. For $\lambda_e > \mu/N$ and $B \to \infty$, a sensor has most of the time energy for broadcasting since the rate at which it harvests energy is higher than the rate at which it broadcasts. In this case, the waiting time for a client to retrieve s measurements from distinct sensors depends on the broadcasting rate μ/N.

Theorem 5. *For* $1 \leq N < \infty$, $1 \leq s \leq N$ *and* $0 < \lambda_e < \infty$,

$$\lim_{B \to \infty} \mathbb{E}[W_s] = \begin{cases} \displaystyle\sum_{j=0}^{s-1} \frac{1}{\lambda_e (N-j)}, & \lambda_e < \frac{\mu}{N} \\ \displaystyle\sum_{j=0}^{s-1} \frac{1}{\frac{\mu}{N}(N-j)}, & \lambda_e \geq \frac{\mu}{N}. \end{cases}$$

Proof. We first consider the case $\lambda_e < \mu/N$. Then, from Lemma 1, it follows that $\lim_{B \to \infty} \nu_0 = 1 - \frac{\lambda_e}{\mu/N}$ and, thus, $\lim_{B \to \infty} \omega = 0$. Using this result in Theorem 3 we have that

$$\lim_{B \to \infty} \mathbb{E}[W_s] = \sum_{j=0}^{s-1} \binom{N}{j} \sum_{k=0}^{j} \binom{j}{k} (-1)^{j-k} \frac{1}{\lambda_e (N-k)}.$$

The result follows by multiplying by $\frac{N-j}{N-j}$ and from Lemma 3.

We next consider the case $\lambda_e \geq \mu/N$. Then $\lim_{B\to\infty} \nu_0 = 0$ and, thus, $\lim_{B\to\infty} \omega = 1$. Using this result in Theorem 3 we have that

$$\lim_{B\to\infty} \mathbb{E}[W_s] = \sum_{j=0}^{s-1} \binom{N}{j} \sum_{k=0}^{j} \binom{j}{k} (-1)^{j-k} \frac{1}{\frac{\mu}{N}(N-k)}.$$

Again, the result follows by multiplying by $\frac{N-j}{N-j}$ and from Lemma 3.

□

We next make the following simple observation regarding the waiting time of an arbitrary clients when the size of the network is arbitrarily large.

Lemma 4. *For any $s > 0$ and $\lambda_e \neq \mu/N$,*

$$\lim_{N\to\infty} E[W_s] = \frac{s}{\mu}.$$

Proof. From Lemma 1, $\lim_{N\to\infty} \nu_0 = 0$. Thus, the sensors always have energy for broadcasting. As a result, at an exponential rate μ, a broadcast occurs. Moreover, the probability that any s consecutive broadcasts are from distinct sensors, tends to 1 as $N \to \infty$. Thus, a client waits for an expected period of $s \cdot 1/\mu$ to retrieve s measurements. □

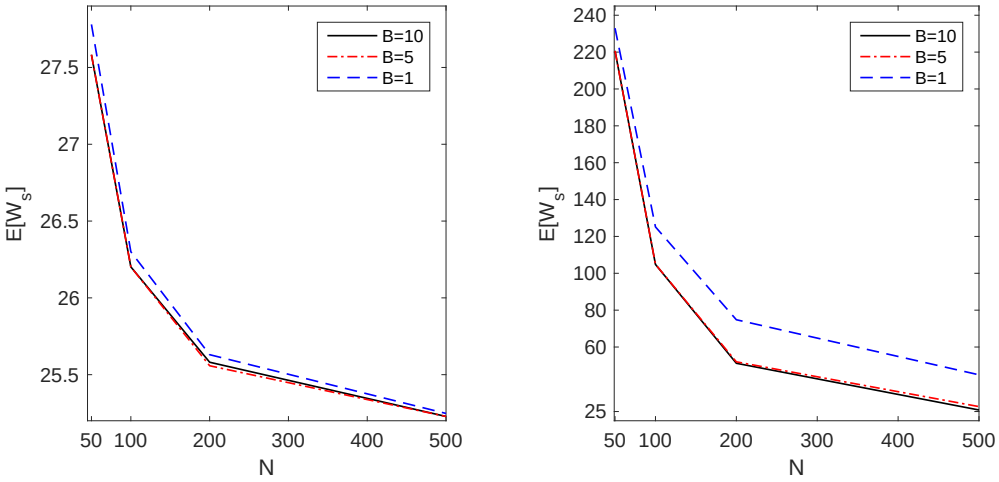
4. Numerical results

In this section we analyze numerically the expected time for a client to retrieve measurements from arbitrary s sensors under various assumptions concerning the size of the network, the capacity of the sensor battery, the energy harvesting and the broadcasting processes.

4.1. Size of the sensor network

Figures 1a and 1b show $\mathbb{E}[W_s]$ under various N, the size of the sensor network. The results are obtained analytically, following the derivations in Section 3.3. Figures 1a and 1b also show that the result in Lemma 4 is exhibited already for networks of size 500 sensors, where the expected time is well approximated by $s/\mu = 25$.

Figure 1a considers the case when $\lambda_e \geq \mu/N$, whereas Figure 1b considers the case when $\lambda_e < \mu/N$. When $\lambda_e > \mu/N$, it is expected that most of the



(a) $\lambda_e = 0.2$, $\mu = 0.4, s = 10$. (b) $\lambda_e = 0.001$, $\mu = 0.4, s = 10$.

Figure 1: Theoretical results: $\mathbb{E}[W_s]$ under various N, the size of the sensor network.

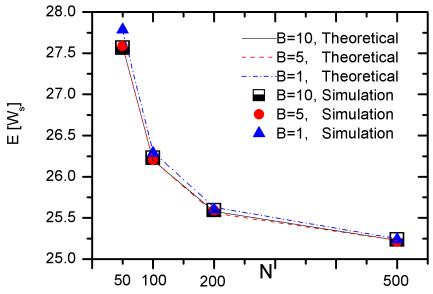
time the batteries of the sensors have energy. If $\lambda_e < \mu/N$, the batteries are expected to be empty most of the time. This explains the fact that $\mathbb{E}[W_s]$ takes lower values in Figure 1a than in Figure 1b.

Figures 1a and 1b also show as N increases, $\mathbb{E}[W_s]$ decreases. The reason is that, as N increases, the probability that at least one sensor has energy to broadcast a useful measurement, increases. Thus, it is expected that clients wait less to retrieve s measurements. Figures 1a and 1b show that, for a fixed λ_e, if B is increased, then $\mathbb{E}[W_s]$ decreases. This is because as B increases, more energy can be collected, which enables broadcasts. The results also indicate the minimum battery capacity of a sensor such that the retrieval time of s measurements remains below a targeted level, given a fixed N. Equally important, Figures 1a and 1b indicate that large battery sizes do not lead to a significant decrease in the expected time to retrieve measurements. We further investigate this observation in Figure 3.

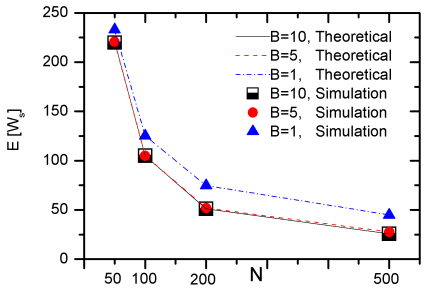
We also conducted discrete-event simulations to support the theoretical results for various sizes of sensor networks. Figure 2 shows that the simulation results coincide with the theoretical results.

4.2. Maximum battery capacities

Figure 3 shows $\mathbb{E}[W_s]$ for various B, the battery capacity of a sensor, and for various λ_e, the rate at which sensors harvest energy from the environment.



(a) $\lambda_e = 0.2$, $\mu = 0.4, s = 10$.



(b) $\lambda_e = 0.001$, $\mu = 0.4, s = 10$.

Figure 2: Theoretical results vs. Simulation results.

As expected, for a fixed B, $\mathbb{E}[W_s]$ decreases as λ_e increases. This is the case because the battery of the sensors are more frequently replenished and, thus, the sensors have energy to broadcast. Note that for $\lambda_e = 0.2$, $\lambda_e > \mu/N$, while for $\lambda_e \in \{0.002, 0.001\}$, $\lambda_e < \mu/N$.

Figure 3 also shows that, for a fixed λ_e, if B increases, then $\mathbb{E}[W_s]$ decreases. This decrease becomes less significant for large values of B. This can be explained as follows. If $\lambda_e \geq \mu/N$, then even though sensors are able to store large amounts of energy, the rate at which the sensors broadcast is low and thus, $\mathbb{E}[W_s]$ mostly depends on the broadcasting rate, rather than B. If $\lambda_e < \mu/N$, then even though B is large, the amount of energy in the batteries is expected to be low most of the times. Thus, in this case, the fact that B is very large does not result in a significant decrease in $\mathbb{E}[W_s]$.

4.3. Distributions governing the energy harvesting and measurement broadcasting processes

The model formulation in Section 2 assumes exponential distributions for the energy harvesting and broadcasting processes. By means of discrete event simulations, we investigated the influence of non-exponentiality on the retrieval time of measurements.

Figure 4 shows the expected time needed to retrieve $s = 10$ measurements when the energy harvesting and broadcast processes assume exponential distributions and uniform distributions. The uniform distributions were assumed to have the same mean as the exponential distribution assumed in Section 2. For example, the energy harvesting process was assumed to be $U(0, 2/\lambda_e)$, with mean $1/\lambda_e$.

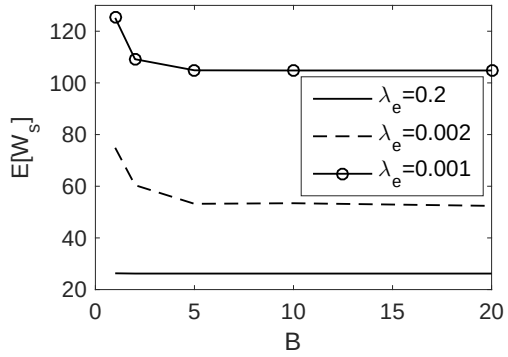


Figure 3: $\mathbb{E}[W_s]$ under various B, the maximum battery capacity of a sensor, $N = 100$, $\mu = 0.4, s = 10$.

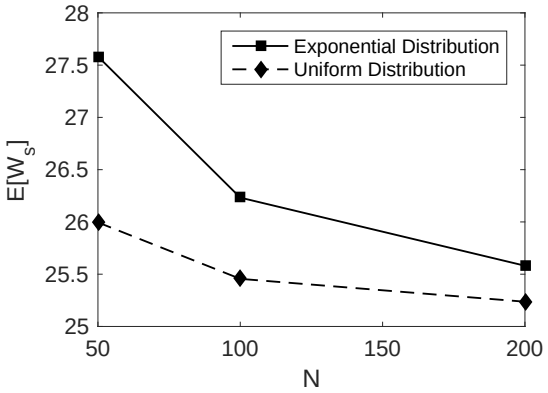


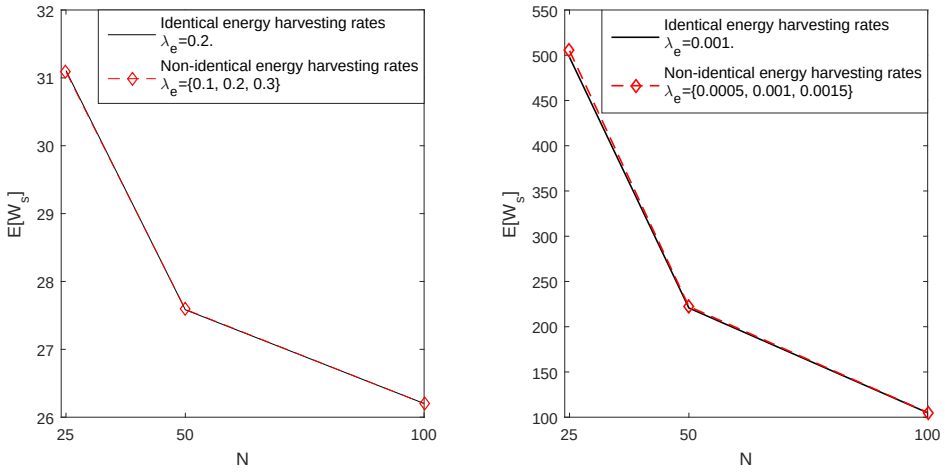
Figure 4: $\mathbb{E}[W_s]$ under various distributions governing the energy harvesting and broadcasting process, $B = 10$, $\mu = 0.4, \lambda_e = 0.2, s = 10$.

Figure 4 shows that the results obtained for exponential distributions are closely approximated by the results obtained for the uniform distributions.

4.4. Non-identical energy harvesting rates

The model formulation in Section 2 assumes identical energy harvesting rates λ_e. By means of discrete event simulations we investigated the influence of non-identical energy harvesting rates.

Figure 5 shows the expected time of retrieving $s = 10$ measurements from sensors that harvest energy at identical and non-identical exponential rates. In the case of non-identical rates, we have partitioned the network in



(a) $\mu = 0.4, s = 10, B = 10.$ (b) $\mu = 0.4, s = 10, B = 10.$

Figure 5: $\mathbb{E}[W_s]$ under identical vs. non-identical energy harvesting rates. For the case of non-identical energy harvesting rates, the N sensors are partitioned into 3 clusters of equal size, where sensors harvest energy according to the cluster's energy harvesting rate.

3 clusters of sensors of equal size $N/3$. Each cluster of sensors assumes a different exponential rate for the process of energy harvesting. To compare this case with the case of identical harvesting rates assumed in Section 2, we have maintained equal the mean harvesting rate of a sensor in the identical and non-identical case. Figure 5 shows that under these assumptions, the expected retrieval time obtained for the model assuming non-identical energy harvesting rates is closely approximated by the retrieval time obtained for the model assuming identical energy harvesting rates.

4.5. Amount of energy consumption upon broadcast

In this section we investigate by means of discrete event simulations the influence of general energy consumption rates on the retrieval time of measurements.

Figure 6 shows the expected time to retrieve $s = 10$ measurements when $k \in \{1, 2, 5\}$ units of energy are consumed upon a broadcast. As expected, increasing k , $k \leq B$, results in an increase in the expected retrieval time of measurements. However, for large sensor networks, the effect of amount of energy consumed upon transmission on the retrieval time is less significant

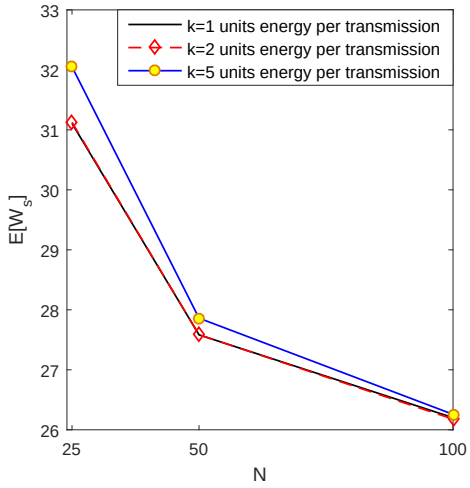


Figure 6: $\mathbb{E}[W_s]$ under various amounts of energy energy upon a transmission, $\mu = 0.4, \lambda_e = 0.2, s = 10, B = 10$.

since, as N increases, the probability of having at least one sensor with sufficient energy to broadcasts tends to 1. This is also supported by the result in Lemma 4.

5. Conclusions

In this paper, we considered the problem of retrieving a set of measurements from an ad-hoc wireless sensor network. We assumed that the sensors harvest energy from the environment and broadcast measurements in a distributed fashion.

We showed that the time until an arbitrary sensor broadcasts has a phase-type distribution. Based on this result, we determined the probability distribution of the time to retrieve measurements from an arbitrary set of sensors of fixed size. We provided a closed-form expression for the expected time to retrieve these measurements. We also determined the retrieval time of such a set of measurements when the energy available for harvesting or the storage capacity of the sensors are asymptotically large. The results show how the time to retrieve data from an ad-hoc wireless sensor network is influenced by a stochastic energy-harvesting process and a distributed data transmission policy. This provides a formal, theoretical support for the design and implementation of data retrieval applications for ad-hoc sensor networks where

the retrieval time of data is an important performance metric.

Lastly, we analyzed numerically the retrieval time of a set of measurements originating from distinct sensors for various network sizes, capacities of the sensor batteries, energy harvesting and broadcasting models. We show what is the minimum battery capacity such that the retrieval time of measurements is below a targeted threshold. We demonstrate that deploying sensors with very large batteries does not result in a significant decrease in the retrieval time of the measurements. Also, for large sensor network, the amount of energy consumed upon a broadcast, as well as the harvesting rate distribution, have a limited effect on the retrieval time of measurements. However, the ratio between the rate at which energy is harvested and the rate at which sensors broadcast, significantly influences the retrieval time of sensor measurements.

As future work, we will investigate more general settings for the sensor networks such as various sources of energy and corresponding energy harvesting rates, as well as general deployment of sensors in the plane and its impact on the amount of energy consumed for broadcasting.

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References

[1] P. Barooah, W. J. Russell, J. P. Hespanha, Approximate distributed Kalman filtering for cooperative multi-agent localization, Springer, 2010.

[2] M. Mitici, J. Goseling, M. de Graaf, R. J. Boucherie, Decentralized vs. centralized scheduling in wireless sensor networks for data fusion, in: Proceedings of IEEE International Conference on Acoustics, Speech and Signal Processing, 2014, pp. 5070–5074.

[3] S. Sudevalayam, P. Kulkarni, Energy harvesting sensor nodes: Survey and implications, IEEE Communications Surveys & Tutorials 13 (3) (2011) 443–461.

[4] S. Ulukus, A. Yener, E. Erkip, O. Simeone, M. Zorzi, P. Grover, K. Huang, Energy harvesting wireless communications: A review of recent advances, IEEE Journal on Selected Areas in Communications 33 (3) (2015) 360–381.

[5] J. Yang, S. Ulukus, Optimal packet scheduling in an energy harvesting communication system, IEEE Transactions on Communications 60 (1) (2012) 220–230.

[6] J. Lei, R. Yates, L. Greenstein, A generic Model for Optimizing Single-Hop transmission Policy of Replenishable Sensors, IEEE Transactions on Wireless Communications 8 (2) (2009) 547–551.

[7] B. Devillers, D. Gunduz, A general framework for the optimization of energy harvesting communication systems with battery imperfections, IEEE Communications and Networks 14 (2) (2012) 130–139.

[8] O. Ozel, K. Tutuncuoglu, J. Yang, S. Ulukus, A. Yener, Transmission with energy harvesting nodes in fading wireless channels: Optimal policies, IEEE Journal on Selected Areas in Communications 29 (8) (2011) 1732–1743.

[9] C. K. Ho, R. Zhang, Optimal energy allocation for wireless communications with energy harvesting constraints, IEEE Transactions on Signal Processing 60 (9) (2012) 4808–4818.

[10] K. Tutuncuoglu, A. Yener, Optimum transmission policies for battery limited energy harvesting nodes, IEEE Transactions on Wireless Communications, 11 (3) (2012) 1180–1189.

[11] M. A. Antepli, E. Uysal-Biyikoglu, H. Erkal, Optimal packet scheduling on an energy harvesting broadcast link, IEEE Selected Areas in Communications 29 (8) (2011) 1721–1731.

[12] J. Yang, O. Ozel, S. Ulukus, Broadcasting with an energy harvesting rechargeable transmitter, IEEE Transactions on Wireless Communications 11 (2) (2012) 571–583.

[13] V. Sharma, U. Mukherji, V. Joseph, S. Gupta, Optimal energy management policies for energy harvesting sensor nodes, Wireless Communications, IEEE Transactions on 9 (4) (2010) 1326–1336.

[14] A. Tandon, M. Motani, Has green energy arrived? delay analysis for energy harvesting communication systems, in: Proceedings of IEEE International Conference on Sensing, Communication, and Networking, 2014, pp. 582–590.

[15] C.-C. Kuan, G.-Y. Lin, H.-Y. Wei, R. Vannithamby, Reliable multicast and broadcast mechanisms for energy-harvesting devices, IEEE Transactions on Vehicular Technology 63 (4) (2014) 1813–1826.

[16] Q. Tan, W. An, Y. Han, Y. Liu, S. Ci, F.-M. Shao, H. Tang, Energy harvesting aware topology control with power adaptation in wireless sensor networks, Ad Hoc Networks 27 (2015) 44–56.

[17] D. Balsamo, A. Das, A. S. Weddell, D. Brunelli, B. M. Al-Hashimi, G. V. Merrett, L. Benini, Graceful performance modulation for power-neutral transient computing systems, IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems 35 (5) (2016) 738–749.

[18] S. Asmussen, Applied probability and queues, Vol. 51, Springer Science & Business Media, 2008.

[19] R. W. Wolff, Poisson arrivals see time averages, Operations Research 30 (2) (1982) 223–231.

[20] M. Ahsanullah, V. B. Nevzorov, M. Shakil, An introduction to order statistics, Springer, 2013.

[21] M. F. Neuts, Matrix-geometric solutions in stochastic models: an algorithmic approach, Courier Corporation, 1981.

[22] X. Zhang, Z. Hou, Computing the moments of order statistics from nonidentically distributed phase-type random variables, Journal of computational and applied mathematics 235 (9) (2011) 2897–2903.