

A life cycle costing approach for discounting in age and interval replacement optimisation models for civil infrastructure assets

van den Boomen, Martine; Schoenmaker, Rob; Wolfert, Rogier

DOI

[10.1080/15732479.2017.1329843](https://doi.org/10.1080/15732479.2017.1329843)

Publication date

2017

Document Version

Final published version

Published in

Structure & Infrastructure Engineering: maintenance, management, life-cycle design & performance

Citation (APA)

van den Boomen, M., Schoenmaker, R., & Wolfert, R. (2017). A life cycle costing approach for discounting in age and interval replacement optimisation models for civil infrastructure assets. *Structure & Infrastructure Engineering: maintenance, management, life-cycle design & performance*.
<https://doi.org/10.1080/15732479.2017.1329843>

Important note

To cite this publication, please use the final published version (if applicable).
Please check the document version above.

Copyright

Other than for strictly personal use, it is not permitted to download, forward or distribute the text or part of it, without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license such as Creative Commons.

Takedown policy

Please contact us and provide details if you believe this document breaches copyrights.
We will remove access to the work immediately and investigate your claim.



Structure and Infrastructure Engineering

Maintenance, Management, Life-Cycle Design and Performance

ISSN: 1573-2479 (Print) 1744-8980 (Online) Journal homepage: <http://www.tandfonline.com/loi/nsie20>

A life cycle costing approach for discounting in age and interval replacement optimisation models for civil infrastructure assets

M. van den Boomen, R. Schoenmaker & A.R.M. Wolfert

To cite this article: M. van den Boomen, R. Schoenmaker & A.R.M. Wolfert (2017): A life cycle costing approach for discounting in age and interval replacement optimisation models for civil infrastructure assets, Structure and Infrastructure Engineering, DOI: [10.1080/15732479.2017.1329843](https://doi.org/10.1080/15732479.2017.1329843)

To link to this article: <http://dx.doi.org/10.1080/15732479.2017.1329843>



© 2017 The Author(s). Published by Informa UK Limited, trading as Taylor & Francis Group



Published online: 05 Jun 2017.



Submit your article to this journal [↗](#)



Article views: 147



View related articles [↗](#)



View Crossmark data [↗](#)

Full Terms & Conditions of access and use can be found at
<http://www.tandfonline.com/action/journalInformation?journalCode=nsie20>

A life cycle costing approach for discounting in age and interval replacement optimisation models for civil infrastructure assets

M. van den Boomen, R. Schoenmaker and A.R.M. Wolfert

Civil Engineering and Geosciences, Delft University of Technology, Delft, The Netherlands

ABSTRACT

Civil infrastructure assets, such as roads, locks, bridges, treatment plants and storm surge barriers, are often characterised by long service lives and corresponding technical life cycles. When life cycles are long, the time value of money plays a role in asset management decision-making on capital investments and operation and maintenance expenditures. In this paper, a new life cycle costing (LCC) approach for discounting in two classes of maintenance optimisation models is developed. These models are the age replacement model and the interval replacement model. Three well-known LCC techniques, which are the present worth, the capital recovery and the capitalised equivalent worth, are combined and used to develop a stepwise methodology. This methodology is validated with the few case-specific mathematical equations that exist in the literature. The advantage of using this alternative LCC approach is its applicability and flexibility for reliability and maintenance engineers. The resulting LCC method builds on well-known LCC formula and enhances the understanding of the inclusion of discounting principles in reliability models. Understanding these principles makes the method flexible. Practitioners can extend or adapt the method to changing circumstances, such as additional cash flows and altering reliability modelling.

ARTICLE HISTORY

Received 24 January 2017
Revised 30 March 2017
Accepted 4 May 2017

KEYWORDS

Civil infrastructure assets; discounting; life cycle costing; age replacement; present value; equivalent annual cost; capitalised equivalent worth

1. Introduction

The international standard on infrastructure asset management (ISO, 2014) and the British Institute of Asset Management (IAM, 2015) both stress the importance of life cycle cost optimisation at a desired service level. The application of infrastructure life cycle costing (LCC) in practice is supported worldwide by several standards and guidelines. Good examples hereof are provided by the U.S. National Highway Federation (FHWA, 2017), who presents a whole range of reports and case studies, including supporting software. Other examples are given by the U.S. Department of Energy (DOE, 2014), the U.S. Transportation and Research Board (NCHRP, 2003), the World Road Association PIARC (PIARC, 2017) and the International Standards Organisation (ISO, 2008). The guidelines stress the importance of a probabilistic approach and dealing with uncertainty in LCC analyses, an area that is in development.

Probabilistic LCC for maintenance optimisation is of importance for asset owners, asset managers and service providers. In general, fundamental probabilistic cost optimisation in maintenance strategies is widely covered in the literature on reliability engineering but often lacks discounting of costs. The cost of failure is set against the cost of preventive maintenance to find optimised preventive or corrective maintenance strategies. An overview of fundamental probabilistic maintenance and replacement cost optimisation models is provided by Jardine

and Tsang (2013) and Campbell, Jardine, and McGlynn (2011). In our paper, the focus is on two of these optimisation models: the age replacement and interval replacement models.

The fundamental probabilistic models provide a quick estimate for optimised preventive replacement (or major overhaul) intervals considering a trade-off between corrective and preventive replacement costs. The value of these generic optimisation models is their ease and broad applicability for practitioners to establish a long-term asset planning for similar types of assets, in addition to more case-specific and advanced probabilistic LCC analyses. However, the fundamental probabilistic maintenance cost optimisation models hardly include discounting of costs. Discounting accounts for the time value of money. The time value of money gains in importance when maintenance or replacement intervals cover more than a few years, which is often the case for civil infrastructure assets. To allow for fair comparison of life cycle costs of different optimisation strategies, future costs are to be converted to their present values.

Although, LCC concepts are well-known, LCC analyses are still far from satisfactory in many fields in practice. Korpi and Ala-Risku (2008) only found 55 international LCC cases studies suitable for analysis out of a total of 205 potential articles. The authors concluded an overall unsatisfactory level of the execution of LCC analyses and specifically addressed the deterministic nature of most LCC case studies. Similar conclusions were drawn in a small-scale study on the quality of LCC analyses in the Dutch

public water sector (van den Boomen, Schoemaker, Verlaan, & Wolfert, 2016). Herein, only 10 suitable case studies for analysis were identified. The study primarily addressed common mistakes found in the execution of the investigated LCC case studies, which were all deterministic in nature.

Fundamental probabilistic maintenance optimisation models deal with uncertainty, however, hardly with the discounting of costs. The inclusion of the time value of money complicates the calculations. Mathematical solutions for discounting in specific fundamental maintenance optimisation models have been provided by only a few authors. Fox (1966) demonstrated a mathematical relationship for discounting in age replacement models. Chen and Savits (1988) established mathematical discounted cost relationships for both age and block replacement policies and the relation between them. Rackwitz (2001) incorporated discounting in a renewal model for structural failures with systematic reconstruction. van Noortwijk (2003) derived a formula for calculating the present value over an unbounded time horizon in age replacement optimisation models as input for a condition-based lifetime extension model. Practical implications were shown in several papers, for example, in an article by van Noortwijk and Frangopol (2004). Van der Weide, Suyono, and van Noortwijk (2008) extended these results to other types of discounting such as hyperbolic and generalised hyperbolic discounting in renewal processes. Mazzuchi, van Noortwijk, and Kallen (2007) reviewed mathematical decision models to optimise time-based and condition-based maintenance intervals. The results were later extended to the derivation of formulas for calculating the discounted costs in combined condition-based and age-based optimisation models (Van der Weide, Pandey, & van Noortwijk, 2010).

These papers all have in common the derivation of mathematical formulas for discounting of costs for explicit and case-specific types of maintenance optimisation models. Other case-specific literature combines advanced probabilistic deterioration models with discounted life cycle costs for structure and infrastructure assets. For example, Frangopol, Lin, and Estes (1997) developed an approach for optimising inspection and repair intervals based on discounted costs, related to the maximum allowable service life of a bridge. Furuta, Frangopol, and Nakatsu (2011) extended the work of Frangopol et al. (1997) to allow for the inclusion of more variables, like different combinations of inspection techniques, by developing multi-objective mathematical algorithms to find the minimum discounted life cycle costs. Almeida, Teixeira, and Delgado (2015) developed degradation algorithms using Markov matrices for bridges and discounted the costs over medium- and long-term finite time horizons. An extension to a discounted renewal approach is provided by Kumar and Gardoni (2014). These authors developed mathematical equations to calculate model variables such as repair time and age, as a function of the system's reliability and, discounted expenditures over a finite time horizon.

All these studies have a strong focus on developing case-specific probabilistic deterioration models. The total discounted costs of preventive and corrective measures over the allowable service life are hereafter minimised to arrive at optimised intervention intervals (inspection, preventive maintenance, repair, partial replacement). The focus of current study is not on developing advanced probabilistic deterioration models for specific

types of infrastructure assets to predict and optimise life cycle costs. Instead, focus is put on discounting life cycle costs in existing and fundamental maintenance optimisation models, using the concepts of equivalent annual cost (EAC) and the capital equivalent worth, which will be explained in Section 2. This alternative LCC approach for discounting in age and interval replacement models has not yet been elaborated on in the literature.

From an engineering asset management point of view, there is an interest in a more generalised, rather quick and flexible approach that allows for discounting in different types of fundamental maintenance optimisation models. Instead of the derivation of a unique set of mathematical formulas for a specific maintenance optimisation problem, three LCC techniques are used in combination and in a specific order to arrive at the required results. This stepwise LCC approach is demonstrated in two fundamental maintenance optimisation models: the age replacement model and the interval (block) replacement model. In the age replacement model, an asset is replaced upon failure or at a preventive replacement interval, whichever comes first. In the interval replacement model, an asset is replaced upon failure and at a preventive replacement interval.

The reason for selecting the age and interval replacement models for developing this alternative LCC method is twofold. First, the existence of mathematical formulas for discounting in age and interval replacement models allows for validation of the alternative LCC approach. A second reason is their ease and quick applicability in practice for infrastructure assets with long life cycles and periodic major overhauls. An inventory of different maintenance policies over the last 50 years still denotes the popularity of these models (Asis, Subhash Chandra, & Bijan, 2011). The models are used in practice by organisations that own and/or maintain infrastructure assets with long life cycles, for instance, for the interval estimation of the conservation of steel lock gates (age replacement), the block replacement of street lighting luminaires (interval replacement) and the revisions, major overhauls or replacements of hydraulic cylinders (both age and interval replacement).

One may argue that the age and interval replacement optimisation models are based on oversimplifications on the failure behaviour of assets and forecasts of future expenditures. An additional argument is that preventive age and interval replacement models ignore the benefits to be gained by measures directed at life time extension, risk reduction and postponement of the actual replacement. Periodic age and interval replacement optimisation is just one of the alternatives for optimised life cycle management of infrastructure assets. The value of age and interval replacement optimisation models is not the actual decision for a preventive or corrective replacement. These short-term and mid-term decisions are made based on actual condition monitoring and technical state combined with detailed LCC analyses which are commonly referred to as a defender (the existing asset) and challenger (the alternative option) analyses. In these types of LCC analyses is investigated whether the postponement of a replacement justifies the cost of measures like major overhauls or renovations to keep an asset some additional years in service. (Blank & Tarquin, 2012; Newnan, Lavelle, & Eschenbach, 2016; Park, 2011; Sullivan, Wicks, & Koeling, 2012).

The discounted age and interval replacement models are also not a substitute for the more advanced probabilistic life cycle optimisation models as referred to in the aforementioned

literature. Within their field of application, the value of using age and interval replacement strategies is that the models provide quick and easy long-term costs and interval estimates as input for the overall long-term asset and maintenance planning. The models also support the decision where successive detailed probabilistic LCC analyses are most effective. Even with simplifications, the probabilistic generic age and interval replacement models provide accurate results for the objectives they are used and meant for.

In this paper, three areas of expertise are merged: reliability engineering, engineering economics (LCC analysis) and infrastructure asset management. Terminology differs between and within these fields. The terminology used in this paper follows the best common denominator of the herein stated literature. The focus in this paper is on cost optimisation models. In the maintenance optimisation models covered in this paper, yearly benefits are considered to be non-differential for different scenarios and are therefore left out of the equations. Salvage values are also left out in the method development. The reason is that salvage values are cash flows that become available when an asset is sold at the end of its life (Brealey, Myers, & Allen, 2011, p. 131). In general, civil infrastructure assets have long service lives, even longer functionalities and are often replaced or renewed at the end of their service lives. Most infrastructure assets cannot be sold.

Occasionally, infrastructure assets may have some scrap or recycling values but these are frequently negligible compared to the renewal costs. Another common situation is that worn-out parts (without salvage values) are periodically renewed. In that case, an infrastructure asset will never be fully replaced. Infrastructure assets do have demolition costs which are often included in the renewal costs. Demolition costs are mostly not differential in a sequence of continuous renewals for age and interval replacement strategies. For these reasons, salvage values are left out in development of the alternative LCC approach and demolition costs are considered to be part of the new investment costs. However, the LCC approach developed in this paper is flexible, and allows for easy separate inclusion of salvage values or demolition costs.

The outline of this article is as follows: first, three generic LCC techniques, which are the present worth, the capital recovery and the capitalised equivalent worth, will be explained. This is followed by a stepwise approach on how to use these three LCC techniques in combination for discounting in age and interval replacement optimisation models. After this, the article is divided into two parts: one for age replacement modelling and one for interval (block) replacement modelling. For each part, the fundamental optimisation model without discounting will be shortly reviewed. Hereafter, the LCC techniques will be used to include discounting in the fundamental maintenance optimisation models. The results will be validated using the mathematical discounted cost relationships found in the literature on an example. The paper ends with overall conclusions on using an LCC approach for discounting in fundamental maintenance optimisation models.

2. LCC techniques and method development

For the inclusion of the time value of money in both the age replacement model and the interval (block) replacement model,

three LCC techniques are of immediate interest: the so-called single payment present worth factor, the equal payment series capital recovery factor and the capitalised equivalent worth. These will be explained briefly. The terminology used follows the stated literature.

The single payment present worth factor ($P/F, i, t$) transforms a future value F to its present value P and is given by (Park, 2011, p. 105; Sullivan et al., 2012, p. 114):

$$(P/F, i, t) = \frac{1}{(1+i)^t}, \quad (1)$$

where i is the real interest or discount rate [-] and t is the time of occurrence [time].

The functional notation ($P/F, i, t$) reads as follows: find the present value P , given the future value F , the discount factor i and the time of occurrence t . Both t and i are generally expressed (but not necessarily), respectively, in years and discount rate per year.

The present worth factor is used for standard discrete discounting and is, for commonly used interest rates, comparable with and close to continuous exponential discounting. Continuous exponential discounting is frequently used in the literature that demonstrates mathematical derivations for the inclusion of the time value of money in maintenance optimisation models. In the latter case, a continuous discount function e^{-it} is used instead of the present worth factor. General inflation is implicitly incorporated using an inflation-free discount rate (real discount rate). Many considerations can be made on discount rate estimations and fluctuations in time. In general, the discount rate in cost models should at minimum cover the long-term weighted average cost of capital of an organisation. The methodology described in this paper allows for a flexible handling of discount rates if required.

The second factor of interest is the equal payment series capital recovery factor or annuity factor ($A/P, i, t$). This factor transforms a present value into the EAC over a chosen number of time units t , generally years. The EAC is analogous to A . The capital recovery factor is given by (Park, 2011, p. 121; Sullivan et al., 2012, p. 124):

$$(A/P, i, t) = \frac{i(1+i)^t}{(1+i)^t - 1}. \quad (2)$$

Here, ($A/P, i, t$) reads as: find A (analogous to EAC) given a present value P , a discount rate i and a number of time units t . An interesting, important and often forgotten feature of the EAC is that the EAC of one life cycle equals the EAC of any number of repeating life cycles assuming identical replacements and identical life cycle costs (Blank & Tarquin, 2012, p. 151; Newnan et al., 2016, p. 181). Therefore, the EAC of one life cycle is the same as the EAC of an infinite number of replacement cycles, under the given assumptions.

The third expression of interest is the capitalised equivalent worth (CW). The capitalised equivalent worth equation converts the EAC of one life cycle to the present value of an infinite number of replacement cycles (Park, 2011, p. 255; Sullivan et al., 2012, p. 187):

$$CW = \frac{EAC}{i}. \quad (3)$$

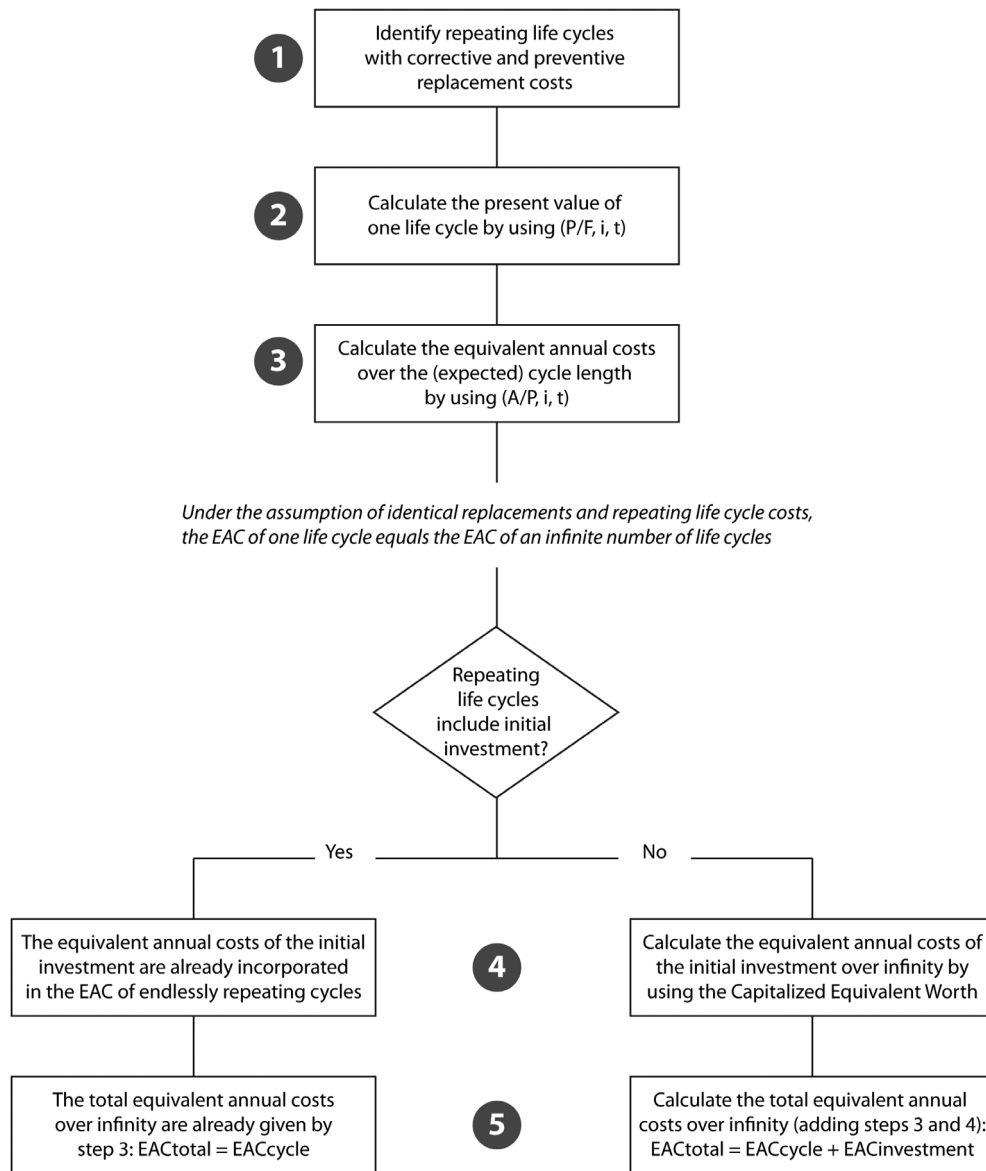


Figure 1. LCC approach to include the time value of money in the age and interval replacement models.

The capitalised equivalent worth factor i^{-1} is found by letting t approach infinity in the recursive formula of the capital recovery factor (Park, 2011, p. 255; Sullivan et al., 2012, p. 187):

$$\lim_{t \rightarrow \infty} (P/A, i, t) = \lim_{t \rightarrow \infty} \frac{(1+i)^t - 1}{i(1+i)^t} = \frac{1}{i}. \quad (4)$$

The basic concept of these LCC techniques is not new but the application of them in a specific order and in combination to arrive at discounted maintenance optimisation models has not been elaborated on before in the literature or textbooks on LCC, maintenance optimisation, engineering economy and infrastructure asset management. Uncertainty is hardly covered in the textbooks on LCC (engineering economy), and discounting of costs is hardly covered in the textbooks on reliability engineering. The framework in Figure 1 depicts how these three LCC techniques, are used in combination for the inclusion of the time value of money in age and interval replacement models. In Sections 3 and 4, this approach will be expressed in formulas, demonstrated

with a practical example and validated with the dedicated mathematical equations found in the literature. Step 3 considers the (expected) cycle length. As will be explained in Section 3, the expected cycle length of an age replacement interval is a weighted average of the probability of a corrective cycle length in the case of failure and the probability of a preventive cycle length in the case of no failure. The cycle length in an interval replacement model is per definition given by the length of a preventive cycle, which will be explained in Section 4.

The total EAC of step 5 gives the basis for comparison of an age or interval replacement strategy with another age or interval replacement strategy. The optimum is found at the minimum total EAC. Following the same principles as shown in Figure 1, one can add time-dependent operation and maintenance expenditures without searching for another dedicated mathematical formula.

There is one important limitation that is hardly mentioned in the literature and textbooks. The age and interval replacement models, with or without discounting, assume a repeatability of

the costs of a replacement cycle. If this repeatability assumption does not hold, neither approach can be used. The repeatability assumption will not hold if, for example, an asset is replaced by another alternative with a different cost and/or failure probability density profile. This may be the case when replacement options are prone to technology development.

These limitations do not automatically refute (discounted) age and interval replacement models for civil infrastructure assets. There are many situations where the age and interval replacement models provide good estimates for an initial investment decision and the long-term asset planning. Changing cash flow patterns of replacement cycles due to technology developments, for instance, a new type of asphalt, are often not that deviant for civil infrastructure assets with long life cycles. Furthermore, deviations frequently occur after decennia and the discounting process mutes the deviations. The argument here is that the applicability of discounted age and interval replacement models should be checked on the presence of an approximated repeatability assumption of replacement cycles, something that is not well stated in the literature.

3. Age replacement model

This section addresses the age replacement model. After a short review of the fundamental age replacement model without discounting, the LCC approach as described in Section 2 is demonstrated. Two situations are dealt with: ending and starting a cycle with a preventive replacement. Hereafter, a dedicated mathematical formula that includes discounting of costs over an infinite time horizon is presented. A practical example is used to compare the mathematical equation with the LCC techniques. After this, the results will be discussed and conclusions formed.

3.1. Fundamental age replacement model without the time value of money

In an age replacement model an asset is replaced correctively upon failure or preventively at a certain replacement interval, whichever comes first. As an example, Figure 2 depicts the cash flow development of an age replacement model with a preventive replacement interval of three years. Here, it is assumed that the initial investment I_0 equals a preventive replacement costs C_p . The cost of a corrective replacement is given by C_f . The failure probability density function is designated with $f(t)$. The reliability function $R(t)$ is defined by $1 - \int f(t)dt$.

An age replacement model⁰ searches for the optimum of a preventive replacement interval, given a certain failure probability density function and corrective and preventive replacement costs. Age replacement models are well described in the

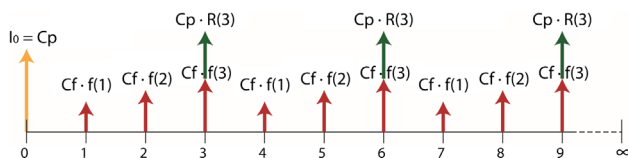


Figure 2. Cash flow diagram of an age replacement policy for a preventive replacement interval of three years.

Note: Three full cycles and an initial investment are shown.

literature, early by Barlow and Proschan (1965, p. 95), chapter 3) and, more recently, for example, by Jardine and Tsang (2013, pp. 48–55). In its basic form, the economic optimum is found by minimising the expected total costs per unit of time. In formula (Jardine & Tsang, 2013, p. 51):

$$c(t) = \frac{C_f \cdot (1 - R(t)) + C_p \cdot R(t)}{M(t) \cdot (1 - R(t)) + t \cdot R(t)}, \quad (5)$$

where:

$c(t)$	expected total costs per unit of time for interval $[0, t]$ [currency/unit of time]
t	time [unit of time]
C_f	corrective replacement costs or failure costs [currency]
$R(t)$	reliability [-]
C_p	preventive replacement costs [currency]
$M(t)$	the mean of the failure probability density function from $t = [0, t]$ [unit of time]

The numerator of this equation expresses the expected total costs per cycle length, which is given by the probability of a failure multiplied by the corrective replacement costs and the probability of no failure multiplied by the preventive replacement costs. The denominator expresses the expected cycle length $E(L)$, which is a weighted average of the probability of a corrective cycle length in the case of failure and the probability of a preventive cycle length in the case of no failure:

$$E(L) = M(t) \cdot (1 - R(t)) + t \cdot R(t). \quad (6)$$

$M(t)$ is, according to usual statistics, defined as follows (Jardine & Tsang, 2013, p. 51):

$$M(t) = \int_0^t \frac{t \cdot f(t)dt}{1 - R(t)}, \quad (7)$$

where:

$M(t)$	the mean of the failure probability density function from $t = [0, t]$ [unit of time]
$f(t)$	failure probability density function [-]
$R(t)$	reliability function [-]

For practical reasons, the failure probability density function is assumed to be 0 at $t = 0$.

3.2. Discounted age replacement optimisation model with use of the LCC techniques

In this section, the time value of money is included in the fundamental age replacement model (Equation (5)). Hereby, the LCC techniques and approach described in Section 2 are used. Two situations are dealt with: ending a repeating cycle with a preventive replacement and starting a repeating cycle with a preventive replacement. The reason for doing so is that the mathematical equations in the literature all end a repeating cycle with a preventive replacement, while in practice a maintenance engineer would like to start a cycle with a preventive replacement.

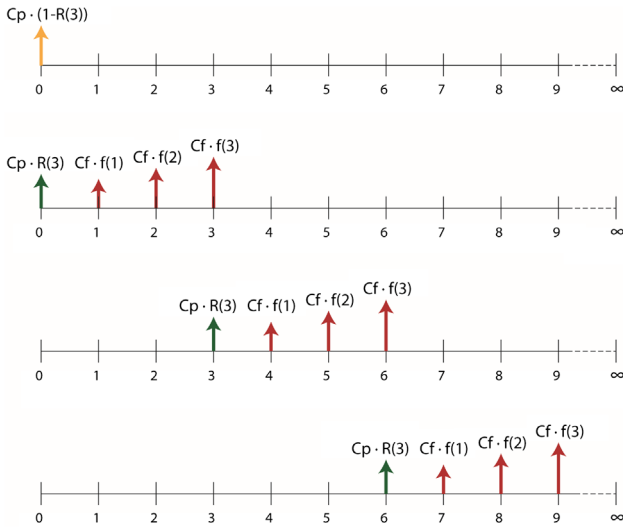


Figure 3. Cash flow diagram of an age replacement policy for a preventive replacement interval of three years, ending with a preventive replacement.

Note: Three full cycles are shown. The initial investment is fully excluded from the repeating life cycle costs.

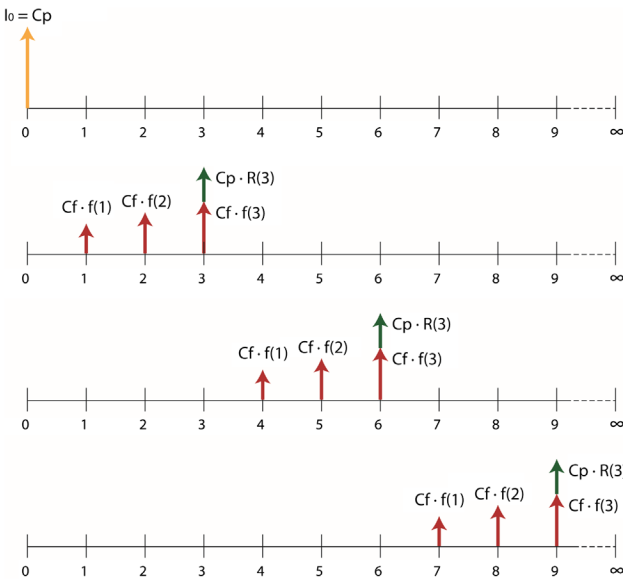


Figure 4. Cash flow diagram of an age replacement policy for a preventive replacement interval of three years, starting with a preventive replacement.

Note: Three full cycles are shown. The initial investment is partly excluded from the repeating life cycle costs.

3.2.1. Alternative 1: ending a repeating cycle with a preventive replacement C_p

Step 1: Identify repeating life cycle costs.

A repeating pattern of cash flows is identified in Figure 2 by taking the initial investment $I_0 = C_p$ with probability 1 out of the cash flow development, as shown in Figure 3. If the asset fails in this example at $t = 1, 2$ or 3 , it will be replaced correctively. If the asset has not failed at the end of $t = 3$, it will be replaced preventively. The total probability of a replacement cycle is 1. Because of the repeatability assumption, only the present value and EAC of one cycle needs to be calculated for derivation of the present value and EAC for repeating cycles up to infinity (Blank & Tarquin, 2012, p. 151; Newnan et al., 2016, p. 181).

Step 2: Calculate the present value of one life cycle.

Using the present worth factor $(P/F, i, t)$, the present value of the expected total replacement costs of the first cycle are given by:

$$P_{cycle} = C_f \cdot \sum_{t=1}^T (P/F, i, t) f(t) + C_p \cdot (P/F, i, T) \cdot \left(1 - \sum_{t=1}^T f(t) \right). \quad (8)$$

Equation (8) can also be written as follows:

$$P_{cycle} = \frac{C_f \cdot f(1)}{(1+i)^1} + \frac{C_f \cdot f(2)}{(1+i)^2} + \dots + \frac{C_f \cdot f(T)}{(1+i)^T} + \frac{C_p \cdot R(T)}{(1+i)^T}. \quad (9)$$

Step 3: Calculate the EAC over the expected cycle length.

The expected cycle length $E(L)$ is calculated according to Equation (6). The EAC of a cycle is found by using the capital recovery factor $(A/P, i, t)$ (Equation (2)) where t is equal to the expected cycle length $E(L)$:

$$EAC_{E(L)} = (A/P, i, E(L)) \cdot P_{cycle}, \quad (10)$$

Under the assumption of identical replacements and repeating life cycle costs, the EAC of one life cycle equals the EAC of an infinite number of life cycles.

Step 4: Calculate the EAC of the initial investment over infinity.

The initial investment costs $I_0 = C_p$ are equally distributed over an infinite time horizon by using the capitalised equivalent worth (Equation (3)).

$$EAC_{I_0} = I_0 \cdot i. \quad (11)$$

Step 5: Calculate the total EAC over infinity.

The total EAC of the age replacement strategy concerned is given by $EAC_{total} = EAC_{E(L)} + EAC_{I_0}$. The optimum is found by minimising the EAC_{total} of different age replacement strategies.

3.2.2. Alternative 2: beginning a repeating cycle with a preventive replacement C_p

In subsection 3.2.1, a repeating cycle of cash flows was identified after instalment of a new asset with investment costs I_0 . The investment costs were converted to EAC over an infinite time horizon and added to the EAC of the cycles. In practice, a maintenance engineer would prefer to start an asset's life cycle with a preventive maintenance or initial investment. To show the deviations, the discounted age replacement model that starts with a preventive replacement will be presented.

Step 1: Identify repeating life cycle costs.

A repeating pattern that starts with a preventive replacement can also be derived from Figure 2 by dividing the initial investment (or preventive replacement) $I_0 = C_p$ with probability 1 into a part $C_p R(t)$ and a part $C_p (1 - R(T))$. This is shown in Figure 4.

Step 2: Calculate the present value of one life cycle.

The present value of the replacement costs of the first cycle are now given by:

$$P_{cycle} = C_f \cdot \sum_{t=1}^T (P/F, i, t) f(t) + C_p \cdot \left(1 - \sum_{t=1}^T f(t) \right). \quad (12)$$

There is no need for discounting C_p as in this approach C_p of the first cycle occurs at $t = 0$. Equation (12) can also be written as:

$$P_{cycle} = \frac{C_f \cdot f(1)}{(1+i)^1} + \frac{C_f \cdot f(2)}{(1+i)^2} + \dots + \frac{C_f \cdot f(T)}{(1+i)^T} + C_p \cdot R(T). \quad (13)$$

Step 3: Calculate the EAC over the expected cycle length.

The expected cycle length is unchanged, as the probabilities and interval times of a preventive cycle and corrective cycle are unchanged. Again, the expected cycle length is calculated according to Equation (6). The EAC of a cycle is found using the capital recovery factor ($A/P, i, t$) (Equation (2)), where t is equal to the expected cycle length $E(L)$:

$$EAC_{E(L)} = (A/P, i, E(L)) \cdot P_{cycle}, \quad (14)$$

Step 4: Calculate the EAC of the initial investment over infinity.

The rest of the term of the initial investment costs $C_p (1 - R(T))$ is equally distributed over an infinite time horizon using the capitalised equivalent worth (Equation (3)):

$$EAC_{rest_term_I_0} = C_p \cdot (1 - R(T)) \cdot i. \quad (15)$$

Step 5: Calculate the total EAC over infinity.

The total EAC of the age replacement strategy concerned is given by $EAC_{total} = EAC_{E(L)} + EAC_{rest_term_I_0}$. The optimum is found by minimising the EAC_{total} of different age replacement strategies.

Summarising: The differences between the LCC approaches with a preventive replacement at the end or beginning of a cycle are, respectively:

- Discounting or no discounting of C_p for the first cycle;
- Distributing the entire initial investment I_0 or the part $C_p (1 - R(T))$ over an infinite time horizon.

Both approaches will be demonstrated with an example in Section 3.4.

3.3. Mathematical equation for discounted age replacement optimisation found in the literature

Chen and Savits (1988); Fox (1966) and van Noortwijk (2003) established mathematical relationships for discounting in a fundamental age replacement model. Apart from differences in mathematical expressions, these relationships do not differ from each other. The expression of van Noortwijk (2003) will be used in this article, slightly adapted for reasons of uniform notations.

The expected total discounted costs of an age replacement interval, assuming identical replacements over an infinite time horizon $E(K(n, \alpha))$, are written as follows (van Noortwijk, 2003):

$$\lim_{n \rightarrow \infty} E(K(n, \alpha)) = \frac{C_f \cdot \left(\sum_{t=1}^T \alpha^t \cdot f(t) \right) + C_p \cdot \alpha^T \left(1 - \sum_{t=1}^T f(t) \right)}{1 - \left[\left(\sum_{t=1}^T \alpha^t \cdot f(t) \right) + \alpha^T \left(1 - \sum_{t=1}^T f(t) \right) \right]}, \quad (16)$$

where:

- α discount factor defined as $\frac{1}{(1+i)}$ with i as inflation free discount rate
- t time, often expressed in years [unit of time]
- T preventive replacement time [unit of time]
- C_p preventive replacement costs [currency]
- C_f corrective replacement costs or failure costs [currency]
- $f(t)$ failure probability density function [-]

Note that α^t in Equation (16) is equal to the present worth factor in Equation (1). The numerator of Equation (16) expresses the discounted costs of one cycle length and is equal to Equation (8). It is further observed that the preventive replacement costs C_p are discounted with α^T at the end of a replacement interval and not at the beginning.

The denominator of Equation (16) transfers the discounted costs of one cycle length to the total discounted costs over an infinite time horizon by assuming continuous repeatability of the first cycle. The denominator of Equation (8) follows from a mathematical derivation where van Noortwijk (2003) uses Feller (Feller, 1950, chapter 13). Transformation to an infinite time horizon is practical for reasons of comparison. It is, for example, not justified to compare the total discounted costs of a cycle of 10 years with the total discounted costs of a cycle of 15 years. However, if both cycles are repeated to infinity, the same time basis of comparison is created. The expected total discounted costs over an infinite time horizon in Equation (16) excludes the initial investment costs. These can be added. In that case, Equation (16) is extended to:

$$\lim_{n \rightarrow \infty} E(K(n, \alpha)) = I_0 + \frac{C_f \cdot \left(\sum_{t=1}^T \alpha^t \cdot f(t) \right) + C_p \cdot \alpha^T \left(1 - \sum_{t=1}^T f(t) \right)}{1 - \left[\left(\sum_{t=1}^T \alpha^t \cdot f(t) \right) + \alpha^T \left(1 - \sum_{t=1}^T f(t) \right) \right]}, \quad (17)$$

where I_0 represents the initial investment costs at $t = 0$.

Table 1. Discounted age replacement model: results calculated on a yearly basis (discrete).

Approaches to discounted age replacement models	Optimum [y]	EAC [€]	CW = P_{∞} [€]	$R(T)$ (%)
Discounting with LCC techniques alternative 1	12	€ 3,586	€ 71,716	96
Discounting with LCC techniques alternative 2	12	€ 3,587	€ 71,734	96
Discounting with mathematical Equation (17)	12	€ 3,586	€ 71,717	96

3.4. Practical example comparing discounted age replacement calculations

For comparison, a slightly adapted example of van Noortwijk (2003) is used. It concerns the maintenance of a cylinder on an existing swing bridge. The cost of preventive and corrective replacements are, respectively, € 30,000 and € 100,000. The initial investment is equal to the cost of a preventive replacement. The failure of the cylinder is modelled with a normal probability distribution with a mean of 15 years and a standard deviation of 1.5 years. The inflation-free discount rate is 5% per year.

A discrete approach on a yearly basis is used to perform the calculations. These were checked with more accurate discrete computations on a monthly basis. The differences were marginal from a practical point of view. The computations are made for the situations: (1) discounting with use of the LCC techniques and ending an interval with a preventive replacement, (2) discounting with use of the LCC techniques and starting an interval with a preventive replacement and (3) discounting with use of mathematical Equation (17). The results of the calculations on a yearly basis are given in Table 1. The discrete failure probability density function is presented in Figure 5. The graphs that contain the age replacement interval calculations on a yearly basis are shown in Figures 6–8. The EAC of the mathematical equation of van Noortwijk (2003) is obtained using the capitalised equivalent approach (Equation (3)) on Equation (17); that is, the EAC of the present value $E(K(\alpha, n))$ is obtained by multiplying this present value by the interest rate i .

3.5. Discussion of results

The first observation is the marginal differences between the mathematical Equation (17) and the use of the LCC techniques. The three calculations give nearly identical outcomes. All calculations arrive at the same economic optimum for the preventive replacement interval. The EAC only slightly differs. The slight difference between calculations 1 and 3 (Figures 6 and 8) is explained by the mathematical transform that is used in Equation (17).

The difference between LCC alternatives 1 and 2 (Figures 6 and 7) is more difficult to explain. From a LCC perspective, scenarios 1 and 2 should arrive at the same results for the total EAC because the total cash flows in Figures 3 and 4 do not differ from each other. The difference is explained by the influence of the expected cycle length (the denominator of Equation (5)) when calculating the EAC of a cycle. Instead of distributing the present value of one cycle over a preventive replacement interval, the present value is distributed over the expected cycle length, which is a weighted average of the probability of a preventive cycle length and the probability of a corrective cycle length.

The initial investment, however, is converted to EAC over an infinite time horizon using the capitalised equivalent worth. The capitalised equivalent worth does not consider expected cycle lengths. Thus, there is a distortion that disappears when the expected cycle length is replaced by the length of a preventive replacement cycle. From a reliability point of view, this would not be acceptable, as the expected cycle length is bounded by the failure probability function. The total probability of a replacement is always 1 for a cycle. There is no probability of survival after

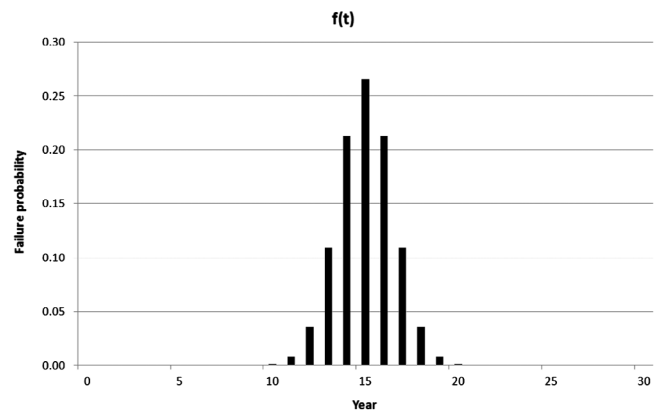


Figure 5. Failure probability density function $f(t)$.

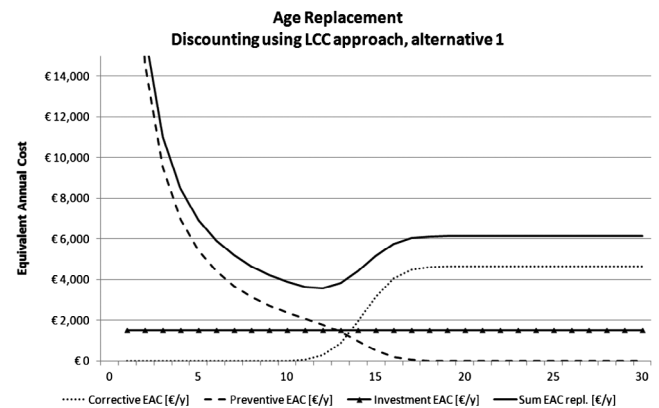


Figure 6. Age replacement with discounting using LCC techniques and ending an interval with a preventive replacement (subsection 3.2.1).

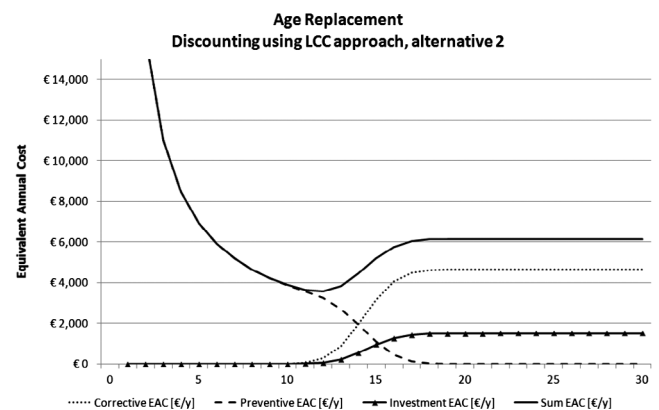


Figure 7. Age replacement with discounting using LCC techniques and beginning an interval with a preventive replacement (subsection 3.2.2).

a certain time, which is the reason for a horizontal asymptote in graphs 5, 6 and 7. However, as the capital recovery factor $(A/P_i E(L))$ for the time span considered is close to $(A/P_i t)$, the distortion is small and from a practical point of view hardly significant. A sensitivity analysis supports this statement in the previous example. Increasing the standard deviation, decreasing and increasing the C_p/C_f -ratio and increasing the interest rate would not lead to differences in optimised preventive replacement intervals. Slight differences in EAC may occur.

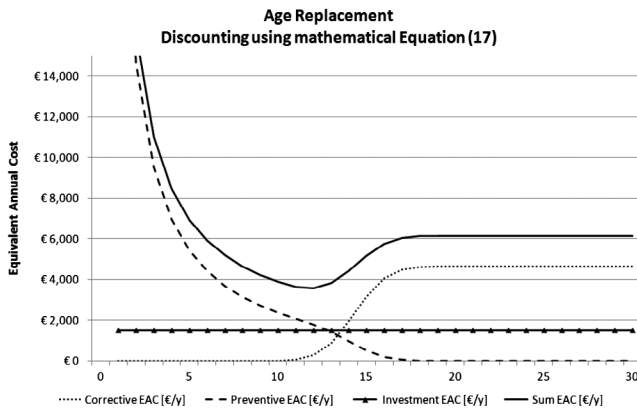


Figure 8. Age replacement with discounting using mathematical Equation (17). An interval ends with a preventive replacement. Transformation to EAC conform Equation (3).

A second observation concerns the reliability at the economic optimum. In this example, the economic optimum is found at 12 years. The reliability at that point is approximately 96%. One could argue whether an organisation or maintenance department would accept 96% reliability, for example, for critical assets. Optimised replacement costs are not the only replacement criterion and should always be viewed in a broader context.

4. Fundamental interval or block replacement model

A second fundamental model in the field of maintenance optimisation is the interval (block) replacement model. In this case, an asset is correctively replaced upon failure and preventively at a certain interval. This type of maintenance optimisation is often found in combination with asset groups. The entire group (block) is preventively replaced at a certain interval. In between, corrective replacements of individual assets are carried out when assets fail. First, the interval (block) replacement model without discounting is reviewed. Then, the LCC techniques presented in Section 2 are used to include the time value of money into the interval (block) replacement model. Hereafter, two mathematical equations for discounting of costs in interval replacement models are shown. Finally, the LCC approach and mathematical approach are demonstrated with an example, compared and discussed.

4.1. Fundamental interval replacement model without the time value of money

The interval replacement model searches for the optimum preventive replacement interval given preventive replacement costs C_p , corrective replacement costs C_f and a renewal function $H(t)$. The renewal function expresses the total number of failures in an interval given a failure probability density function $f(t)$ and constant renewal at failure with identical assets. The interval replacement model is described by Barlow and Proschan (1965, p. 95). In this paper, the expression of Jardine and Tsang (2013, p. 41) is used to describe the model:

$$c(t) = \frac{C_p + C_f \cdot H(t)}{t}, \quad (18)$$

where:

- $c(t)$ expected total costs in interval $[0, t]$ /length of interval [currency/unit of time]
- t time [unit of time]
- C_p preventive replacement costs [currency]
- C_f corrective replacement costs or failure costs [currency]
- $H(t)$ the expected number of failures between $t = [0, t]$ [-]

For block replacement, the number of assets m is added:

$$c(t) = m \cdot \left[\frac{C_p + C_f \cdot H(t)}{t} \right]. \quad (19)$$

The number of assets is not relevant for the optimisation question. For practical reasons, m is assumed to be 1 in this paper.

The difficulty in the interval replacement model is the determination of the renewal function $H(t)$ and its derivate $h(t)$, which expresses the expected number of failures per unit of time, often year. The renewal density function $h(t)$ is needed for discounting on a yearly (or other time unit) basis, which will be demonstrated in subsection 4.2. The renewal density function $h(t)$ is given by Barlow and Proschan (1965, p. 50):

$$h(t) = \sum_{k=1}^{\infty} f^{(k)}(t), \quad (20)$$

where $f^{(k)}(t)$ is the k -fold convolution of the probability density function $f(t)$ with itself. Suppose that an asset fails according to a certain probability density function $f(t)$. An asset can only fail once. A failure will lead to a full replacement by an identical asset with the same probability density function $f(t)$ that will start at the time of replacement. The probability functions move along the time axis and are combined to find the k -fold convolution. The expected number of failures in time $[0, t]$ is given by:

$$H(t) = \int_0^t h(t)dt, \quad (21)$$

4.2. Discounted interval replacement optimisation model with use of the LCC techniques

Including the time value of money in the fundamental interval replacement model is done with the LCC techniques explained in Section 2. Again, two alternatives will be demonstrated, ending a repeating cycle with a preventive replacement (subsection 4.2.1) and beginning a repeating cycle with a preventive replacement (subsection 4.2.2). The reason for demonstrating two alternatives is that the mathematical equations found in the literature (see Section 4.3) all end repeating cycles with a preventive replacement. For comparison between using the LCC techniques and mathematical equations, the same cash flow pattern should be used. From a practical point of view, a maintenance engineer would prefer to consider the first instalment as the start of a cycle. Therefore, two alternatives are demonstrated with the use of the stepwise LCC approach.

4.2.1. Alternative 1: ending a repeating cycle with a preventive replacement C_p

Step 1: Identify repeating life cycle costs.

The initial investment and repeating cycles are presented in Figure 9. The renewal density function for the expected number of failures per year is represented as $h(t)$ (Equation (20)).

Step 2: Calculate the present value of one life cycle.

The present value of the expected total costs of a cycle is calculated using the present worth factor $(P/F, i, t)$ according to Equation (1):

$$P_{cycle} = C_f \cdot \sum_{t=1}^T (P/F, i, t) \cdot h(t) + C_p \cdot (P/F, i, T). \quad (22)$$

This could also be written as follows:

$$P_{cycle} = \frac{C_f \cdot h(1)}{(1+i)^1} + \frac{C_f \cdot h(2)}{(1+i)^2} + \dots + \frac{C_f \cdot h(T)}{(1+i)^T} + \frac{C_p}{(1+i)^T}. \quad (23)$$

Step 3: Calculate the EAC of one life cycle.

The present value of a cycle is now distributed over the cycle length $L = [0, T]$ by using the capital recovery factor $(A/P, i, t)$ with $t = T$:

$$EAC_L = (A/P, i, T) \cdot P_{cycle}. \quad (24)$$

Under the repeatability assumption, the EAC of a cycle is equal to the EAC of repeating cycles up to infinity.

Step 4: Calculate the EAC of the initial investment over infinity.

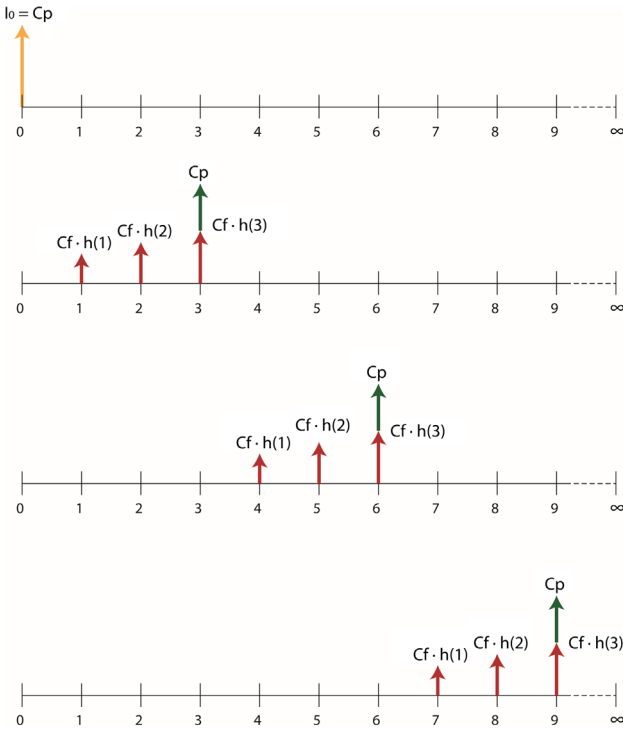


Figure 9. Cash flow diagram of an interval replacement policy for a preventive replacement interval of three years, ending with a preventive replacement. Note: Three full cycles are shown. The initial investment is fully excluded from the repeating life cycle costs.

The initial investment costs $I_0 = C_p$ are equally distributed over an infinite time horizon using the capitalised equivalent worth (Equation (3)):

$$EAC_{I_0} = C_p \cdot i. \quad (25)$$

Step 5: Combining the EAC's of step 3 and 4 gives the total EAC of the strategy concerned.

The total EAC of the interval replacement strategy concerned is given by $EAC_{total} = EAC_L + EAC_{I_0}$. The optimum is found by minimising EAC_{total} of different interval replacement strategies.

4.2.2. Alternative 2: beginning a repeating cycle with a preventive replacement C_p

Instead of ending an interval with a preventive replacement, one could start an interval with a preventive replacement. This is considered to be more realistic from a maintenance perspective.

Step 1: Identify repeating life cycle costs.

An even faster result is obtained by starting each repeating cycle with a preventive replacement C_p . In that case, there is no need to distribute the initial investment $I_0 = C_p$. The repeating cash flows are illustrated in Figure 10.

Step 2: Calculate the present value of one life cycle.

The present value of the expected total costs of a cycle is now calculated as follows:

$$P_{cycle} = C_p + C_f \cdot \sum_{t=1}^T (P/F, i, t) \cdot h(t). \quad (26)$$

Step 3: Calculate the EAC of one life cycle.

The present value of a cycle is again distributed over the cycle length $L = [0, T]$ using the capital recovery factor $(A/P, i, t)$ with $t = T$:

$$EAC_L = (A/P, i, T) \cdot P_{cycle} \quad (27)$$

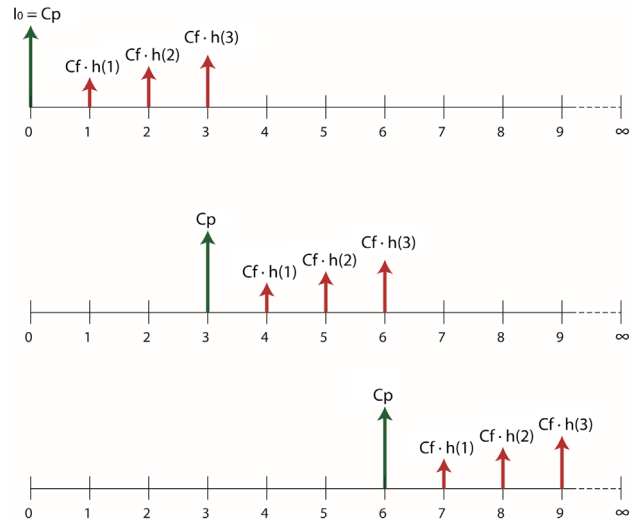


Figure 10. Cash flow diagram of an interval replacement policy for a preventive replacement interval of three years, beginning with a preventive replacement. Note: Three full cycles are shown. The initial investment is fully included in the repeating life cycle costs.

Because of the validity of the repeatability assumption, the EAC_L already provides a basis for comparison between various preventive replacement intervals and gives the required result. Steps 4 and 5 are redundant, as the initial investment $I_0 = C_p$ is already taken into account in the EAC of a cycle.

4.3. Mathematical equations for discounted interval replacement optimisation found in the literature

Chen and Savits (1988) developed a mathematical relationship for the expected total discounted costs over an infinite time horizon for an interval (block) replacement model that is given by:

$$J_B = \frac{\int_0^T e^{-it} dQ(t)}{1 - e^{-iT}}, \quad (28)$$

where J_B represents the expected total discounted costs from $t = [0, \infty]$ for repeating cycles with a preventive replacement interval T . i is the discount rate. The integral represents the sum of the yearly discounted costs of one cycle. The factor e^{-it} approximates the present worth factor ($P/F, i, t$) and Q represents yearly costs. It is observed in the literature that Chen and Savits (1988) discount the preventive replacement costs C_p at T , the end of a preventive replacement interval. The initial investment is not taken into account in this mathematical model.

A nearly similar mathematical relationship is established by Mazzuchi et al. (2007):

$$\lim_{t \rightarrow \infty} E(K(T, i)) = \frac{C_f \cdot E(N(T, i)) + C_p \cdot e^{-iT}}{1 - e^{-iT}}, \quad (29)$$

where $E(K(T, i))$ is the expected total discounted costs over an infinite time horizon of a continuous repeating cycle, $E(N(T, i))$ is the expected number of discounted failures in a preventive replacement interval $[0, T]$, T is the preventive replacement time and i is the discount rate. In the terminology used in this paper, $E(N(T, i))$ in the numerator of Equation (29) is explained as $\sum_{t=1}^T e^{-it} \cdot h(t)$, where $h(t)$ is the renewal density function. From a LCC perspective, it is not common to use the term discounted failures, as the term discounting is reserved for monetary values. However, from a mathematical perspective, there is no difference, as in this case $C_f \cdot E(N(T, i)) = C_f \cdot \sum_0^t e^{-it} \cdot h(t) = \sum_0^t e^{-it} \cdot C_f \cdot h(t)$.

It is again noticed that Mazzuchi et al. (2007) discount the preventive replacement costs at the end of a cycle. The initial investment is not included in this model.

Chen and Savits (1988) and Mazzuchi et al. (2007) have in common that the numerators of Equations (28) and (29) calculate the present value of one cycle, and the denominator transforms this present value into the present value over an infinite time horizon. In the following subsection, the LCC approach is

compared to the mathematical equations for interval replacement optimisation by means of an example.

4.4. Practical example comparing discounted interval replacement calculations

The example of the maintenance of a hydraulic cylinder from subsection 3.4 is used. A practical application is the interval replacement optimisation of hydraulic cylinders at the Dutch Eastern Scheldt storm surge barrier. For the example and demonstration purposes, the number of assets (m) in Equation (19) is set at one at 1 as this will not influence the optimised replacement interval. The costs of preventive and corrective replacements are, respectively, € 30,000 and € 100,000. The initial investment is equal to the costs of a preventive replacement. The failure of the cylinder is modelled with a normal probability distribution with a mean of 15 years and standard deviation of 1.5. The inflation-free discount rate is 5% per year.

Discrete computations are made on a yearly basis (Table 2) and checked with more accurate discrete computations on a monthly basis. In this example, the results do not differ much from a practical point of view. Only slight differences in the order of magnitude of a few months were found. The renewal density function $h(t)$ is calculated as the sum of the first to tenfold convolution of $f(t)$ and shown in Figure 11. The k -fold convolution of a normal probability density function is obtained using mathematical rules (DasGupta, 2010, p. 203). For a probability density function $f(t)$ with a normal distribution having a mean μ_1 and a standard deviation σ_1 , the twofold convolution $h^{(2)}(t)$ of $f(t)$ with itself is again a normal distribution with a mean $\mu_2 = \mu_1 + \mu_1$ and a standard deviation $\sigma_2 = \sqrt{\sigma_1^2 + \sigma_1^2}$. Even so, let $h^{(3)}(t) = h^{(2)}(t) \cdot f^{(1)}(t)$ then $h^{(3)}(t)$ is a normal distribution function with a mean $\mu_3 = \mu_2 + \mu_1$ and a standard deviation $\sigma_3 = \sqrt{\sigma_2^2 + \sigma_1^2}$.

The interval cost optimisation graphs on a yearly basis are presented in Figures 12–14 for the approaches: (1) discounting with the use of LCC techniques and ending an interval with a preventive replacement, (2) discounting with use of the LCC techniques and beginning an interval with a preventive replacement and (3) discounting with the use of mathematical Equation (29) with a correction for the initial investment. The mathematical Equation (29) gives the present value over an infinite time horizon. This is transformed to EAC using the capital equivalent worth approach (Equation (3)).

4.5. Discussion of results

A first observation is that the differences between discounting with mathematical Equation (29) and the LCC approach are marginal. There is no need for a sensitivity analysis here, as these findings are not surprising. The continuous discount function e^{-it}

Table 2. Discounted interval replacement model: results calculated on a yearly basis (discrete).

Approaches to discounted interval replacement models	Optimum [y]	EAC [€]	CW = P_{∞} [€]	$H(T)$
Discounting with LCC techniques alternative 1	12	€ 3,669	€ 73,376	0.04
Discounting with LCC techniques alternative 2	12	€ 3,669	€ 73,376	0.04
Discounting with mathematical Equation (29)	12	€ 3,660	€ 73,197	0.04

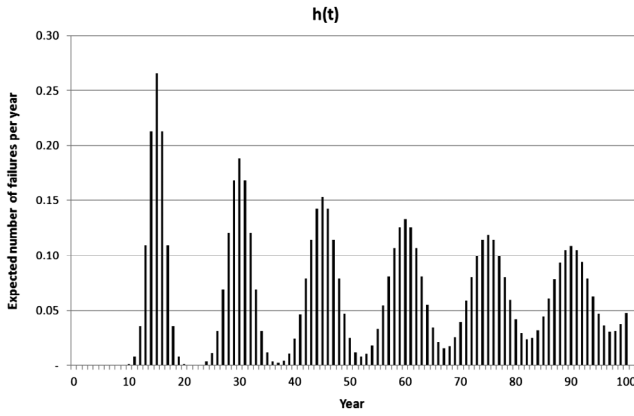


Figure 11. Renewal density function $h(t)$: expected number of failures per year. The time axis is stretched to show the impact of the convolutions.

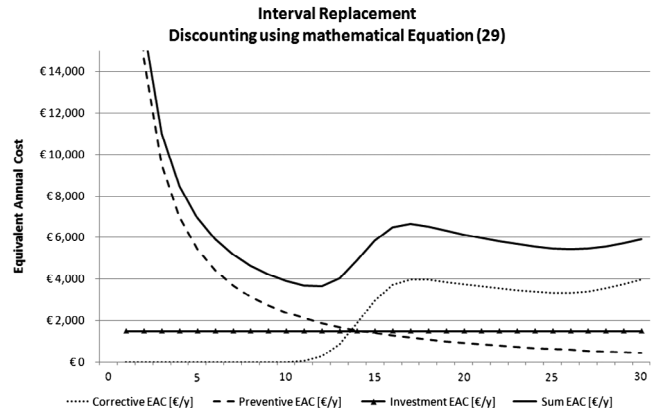


Figure 14. Interval replacement with discounting by the use of mathematical Equation (29) (Mazzuchi et al., 2007) with a correction for the initial investment and transformation to EAC conform Equation (3).

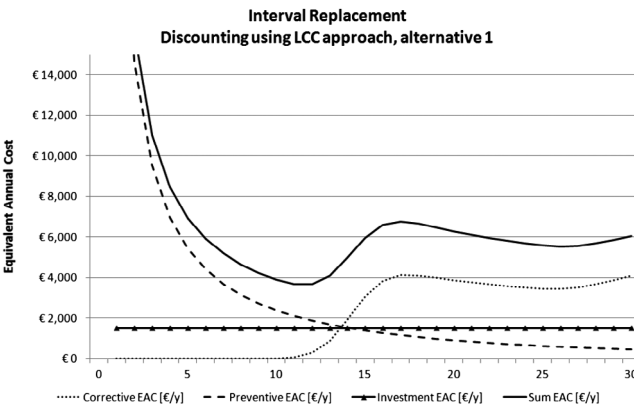


Figure 12. Interval replacement with discounting using LCC techniques and ending an interval with a preventive replacement (subsection 4.2.1).

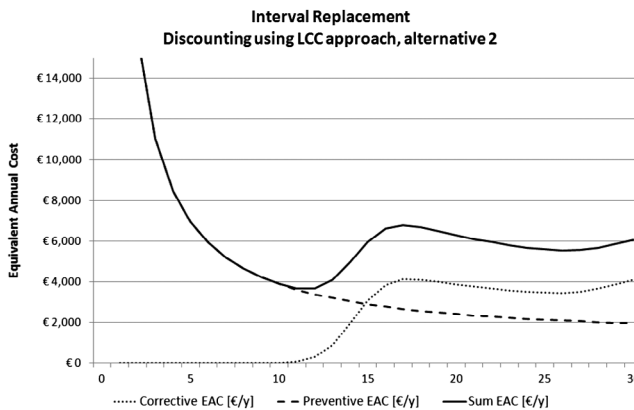


Figure 13. Interval replacement with discounting using LCC techniques and beginning an interval with a preventive replacement (subsection 4.2.2).

used in Equation (29) is comparable to the discrete present worth factor $(P/F, i, t)$. For normally used interest rates, the following approximation is valid:

$$e^{-it} \approx \frac{1}{(1+i)^t}. \quad (30)$$

The factor $\frac{1}{1-e^{-it}}$ in Equation (29) represents a continuous function for the transformation of the present value of one cycle to the present value of an endless stream of these cycles. Using the discrete LCC techniques, this transformation is achieved by combining the capital recovery factor $(A/P, i, t)$ and the capitalised equivalent worth (CW). The continuous transformation and the discrete transformation approximate each other:

$$\frac{1}{1-e^{-it}} \approx \frac{(A/P, i, t)}{i} = \frac{i(1+i)^t}{(1+i)^t - 1} \cdot \frac{1}{i} = \frac{(1+i)^t}{(1+i)^t - 1}. \quad (31)$$

Therefore, there is not much difference in computations in using the mathematical formula 29 or the LCC techniques.

A second observation is that LCC alternatives 1 and 2 give the same results. In scenario 1, an interval ends with a preventive replacement, and the initial investment $I_0 = C_p$ is compensated for afterwards. In scenario 2, an interval begins with a preventive replacement, and there is no need to compensate for an initial investment, as it is already incorporated in the first cycle. The total cash flows, however, are identical, and there is no distortion due to an expected cycle length as was seen in the age replacement modelling in Section 3. Therefore, it is not surprising that the results of scenarios 1 and 2 are identical.

The last observation concerns $H(t)$ for the optimised preventive replacement interval. $H(t)$ approximates 0.04 failures per interval of 12 years. From an economical point of view, one should not accept more expected failures. This is explained by the relatively high corrective replacement costs and the characteristics of the probability density function $f(t)$. $H(t)$ is the cumulative density function of $h(t)$, which is shown in Figure 11, while $h(t)$ is constructed by calculating the 10-fold convolution of $f(t)$ with itself. It is observed that in the previous example, the second and higher convolutions of $f(t)$ do not influence the economic optimum, which suggests that situations exist where $h(t)$ can well be approximated by $f(t)$. However, a higher standard deviation of $f(t)$ would increase the influence of the renewal density function.

Compared to age replacement models, discounting in interval replacement models is not difficult because of the absence of an expected cycle length. In the case that the initial investment equals the cost of a preventive replacement, there is no need to distribute an initial investment over an infinite time horizon because one can start a calculation with a preventive

replacement. The difficulty in the interval replacement model lies in the determination of the renewal density function $h(t)$ and/or the renewal function $H(t)$, irrespective of discounting.

5. Conclusions

The authors developed a stepwise and flexible LCC approach for discounting in age and interval replacement models for civil infrastructure assets and validated the new approach by comparing the results with case-specific formulas. Age and interval replacement optimisation strategies support the long-term asset and maintenance planning of organisations that operate and maintain these infrastructure assets. Some typical examples for the application of these models are the conservation of steel lock gates, the replacements of street light luminaires and the major overhauls or replacements of hydraulic cylinders.

Life cycles of civil infrastructure assets are often long. Therefore, the time value of money should be incorporated. Discounting in fundamental probabilistic maintenance optimisation models is hardly covered in the literature on engineering economy and reliability engineering. For instance, just a few authors developed dedicated mathematical formulas for the fundamental and popular age and interval replacement models (Chen & Savits, 1988; Fox, 1966; Mazzuchi et al., 2007; van Noortwijk, 2003). These mathematical formulas were used for validation of the developed LCC approach.

The LCC approach builds on well-known LCC techniques which are the present worth, the capital recovery and the capitalised equivalent worth. The LCC techniques are used in a specific order, and combined with reliability formula. The advantage of this stepwise LCC approach is that it enhances the understanding of discounting principles, their constraints and their field of applicability, for reliability and maintenance engineers in practice. In addition, the stepwise LCC approach explicitly takes the initial investment into account and allows for easy adaptation and extension when conditions change, for instance, changing cash flow patterns or reliability profiles.

Disclosure statement

No potential conflict of interest was reported by the authors.

References

- Almeida, J.O., Teixeira, P.F., & Delgado, R.M. (2015). Life cycle cost optimisation in highway concrete bridges management. *Structure and Infrastructure Engineering*, 11, 1263–1276.
- Asis, S., Subhash Chandra, P., & Bijan, S. (2011). Survey of maintenance policies for the last 50 years. *International Journal of Software Engineering & Applications*, 2, 130–148.
- Barlow, R.E., & Proschan, F. (1965). *Mathematical theory of reliability*. New York, NY: John Wiley.
- Blank, L., & Tarquin, A. (2012). *Engineering economy*. New York, NY: McGraw-Hill.
- Brealey, R.E., Myers, S.C., & Allen, F. (2011). *Principles of corporate finance*. New York, NY: McGraw-Hill/Irwin.
- Campbell, J.D., Jardine, A.K.S., & McGlynn, J. (2011). *Asset management excellence: Optimizing equipment life-cycle decisions*. Boca Raton, FL: CRC Press.
- Chen, C.S., & Savits, T.H. (1988). A discounted cost relationship. *Journal of Multivariate Analysis*, 27, 105–115.
- DasGupta, A. (2010). *Fundamentals of probability: A first course*. New York, NY: Springer.
- DOE. (2014). *Life cycle cost handbook. Guidance for life cycle cost estimation and analysis*. Washington, DC: U.S. Department of Energy.
- Feller, W. (1950). *An introduction to probability theory and its applications*. New York, NY: Wiley.
- FHWA. (2017). *Webportal Federal Highway Administration*. Retrieved from <https://www.fhwa.dot.gov/infrastructure/asstmgmt/lcca.cfm>
- Fox, B. (1966). Letter to the editor – Age replacement with discounting. *Operations Research*, 14, 533–537.
- Frangopol, D.M., Lin, K.Y., & Estes, A.C. (1997). Life-cycle cost design of deteriorating structures. *Journal of Structural Engineering*, 123, 1390–1401.
- Furuta, H., Frangopol, D., & Nakatsu, K. (2011). Life-cycle cost of civil infrastructure with emphasis on balancing structural performance and seismic risk of road network. *Structure and Infrastructure Engineering*, 7, 65–74.
- IAM. (2015). *Asset management – An anatomy – Version 3*. Bristol: Institute of Asset Management.
- ISO. (2008). ISO 15686-5:2008. *Buildings and constructed assets. Service-life planning. Part 5: Life-cycle costing*. Geneva.
- ISO. (2014). ISO 55000:2014. *Asset management – Overview, principles and terminology*. Geneva.
- Jardine, A.K.S., & Tsang, A.H.C. (2013). *Maintenance, replacement and reliability*. New York, NY: CRC Press.
- Korpi, E., & Ala-Risku, T. (2008). Life cycle costing: A review of published case studies. *Managerial Auditing Journal*, 23, 240–261.
- Kumar, R., & Gardoni, P. (2014). Renewal theory-based life-cycle analysis of deteriorating engineering systems. *Structural Safety*, 50, 94–102.
- Mazzuchi, T.A., van Noortwijk, J.M., & Kallen, M.J. (2007). *Technical Report TR-2007-9 Maintenance optimization*. Washington: The Institute for Integrating Statistics in Decision Sciences. Retrieved from <http://business.gwu.edu/wp-content/uploads/2014/03/TR-2007-9.pdf>
- NCHRP. (2003). *Report 483. Bridge life-cycle cost analysis*. Washington, DC: Transportation Research Board. Retrieved from http://onlinepubs.trb.org/onlinepubs/nchrp/nchrp_rpt_483.pdf
- Newman, D.G., Lavelle, J.P., & Eschenbach, T.G. (2016). *Engineering economy analysis*. New York, NY: Oxford University Press.
- Park, C.S. (2011). *Contemporary engineering economics* (5th ed.). Upper Saddle River, NJ: Pearson.
- PIARC. (2017). *Online library with best practices for LCCA*. Retrieved from <https://www.piarc.org/en/search.htm?q=life+cycle+costs>
- Rackwitz, R. (2001). Optimizing systematically renewed structures. *Reliability Engineering and System Safety*, 73, 269–279.
- Sullivan, W.G., Wicks, E.M., & Koeling, C.P. (2012). *Engineering economy*. Boston, MA: Pearson, Prentice Hall.
- van den Boomen, M., Schoenmaker, R., Verlaan, J.G., & Wolfert, A.R.M. (2016). *Common misunderstandings in life cycle costing analyses and how to avoid them. Paper presented at the Proceedings of the 5th International Symposium on Life-Cycle Civil Engineering: IALCCE 2016, Life-Cycle of Engineering Systems: Emphasis on Sustainable Civil Infrastructure*, Delft.
- Van der Weide, J.A.M., Pandey, M.D., & van Noortwijk, J.M. (2010). Discounted cost model for condition-based maintenance optimization. *Reliability Engineering and System Safety*, 95, 236–246.
- Van der Weide, J.A.M., Suyono, & van Noortwijk, J.M. (2008). Renewal theory with exponential and hyperbolic discounting. *Probability in the Engineering and Informational Sciences*, 22, 53–74.
- van Noortwijk, J.M. (2003). Explicit formulas for the variance of discounted life cycle cost. *Reliability Engineering and System Safety*, 80, 185–195.
- van Noortwijk, J.M., & Frangopol, D.M. (2004). Two probabilistic life-cycle maintenance models for deteriorating civil infrastructures. *Probabilistic Engineering Mechanics*, 19, 345–359.