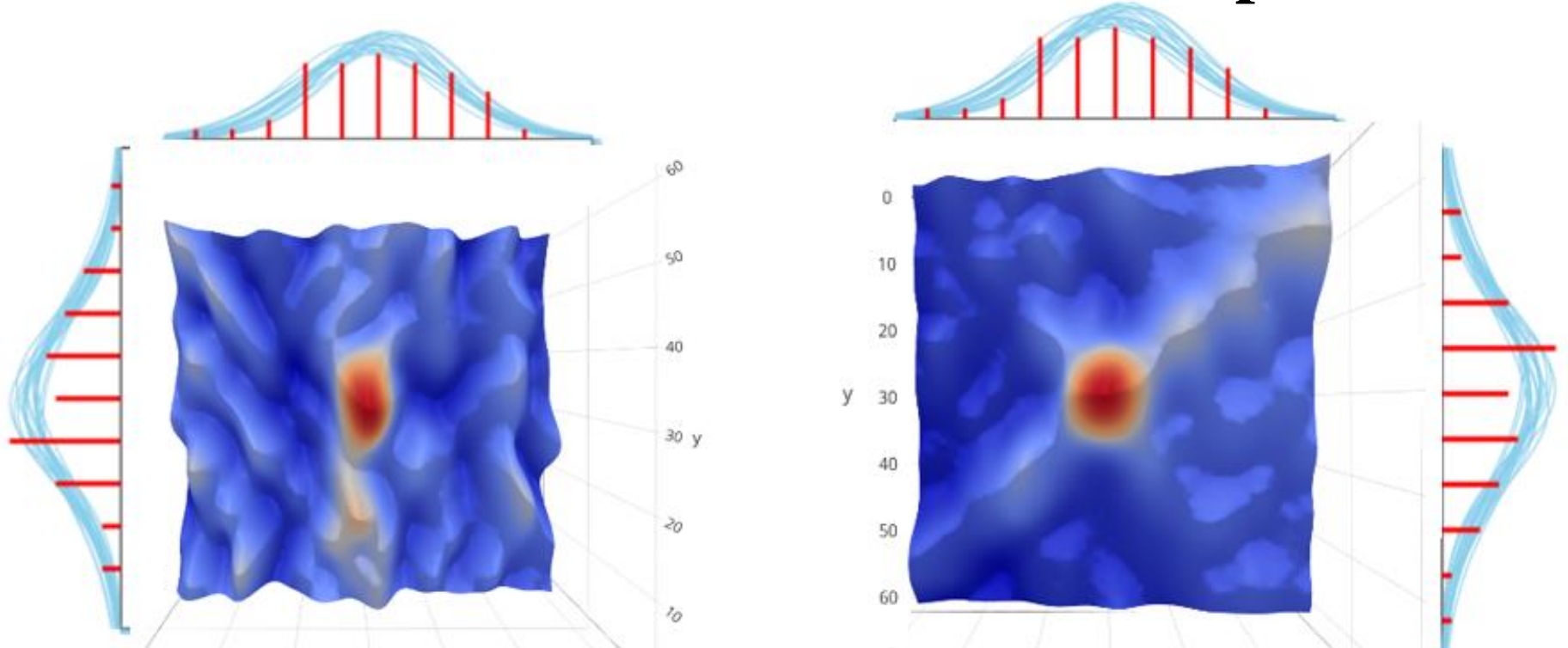


# A probabilistic analysis of results of co-registration between aerial and mobile laser scanned point clouds



Anastasia Anastasiadou



Prof. Dr. Ir. P.J.M van Oosterom, supervisor TU Delft

Ir. Edward Verbree , supervisor TU Delft

Dr. ir. Freek J. van Leijen, co-reader TU Delft

Ir. Peter Joosten, supervisor Cyclomedia

# Overview

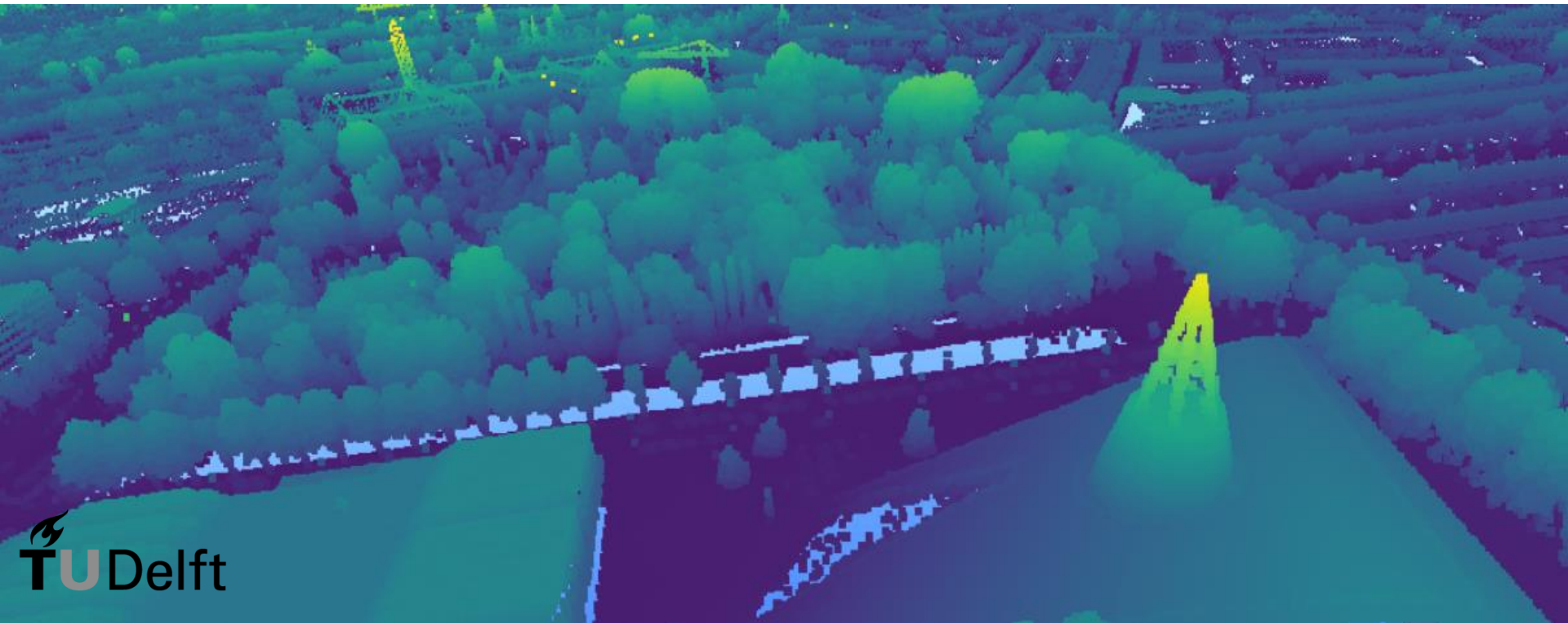
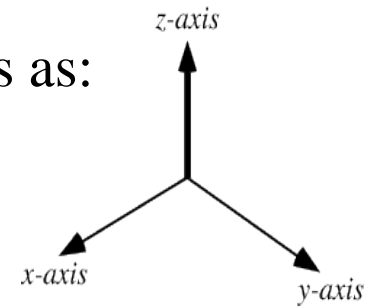
1. Point clouds
2. Differences in point clouds
3. Related work
4. Research objective
5. Methodology and results
6. Conclusions
7. Future recommendations

# 1. Point clouds

Set of points with 3D coordinates (x,y,z)

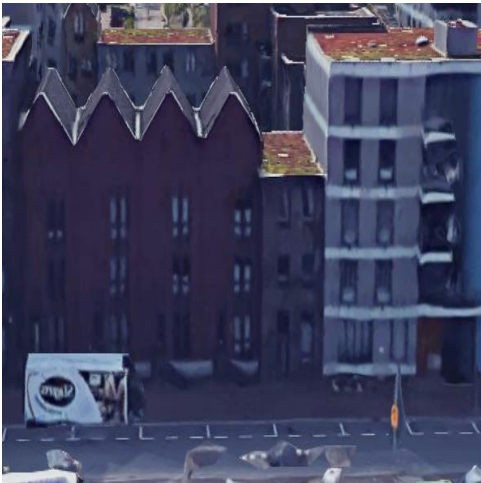
Significant source of 3D spatial info for purposes as:

- Object recognition
- Robotics
- Navigation systems
- Surveying purposes

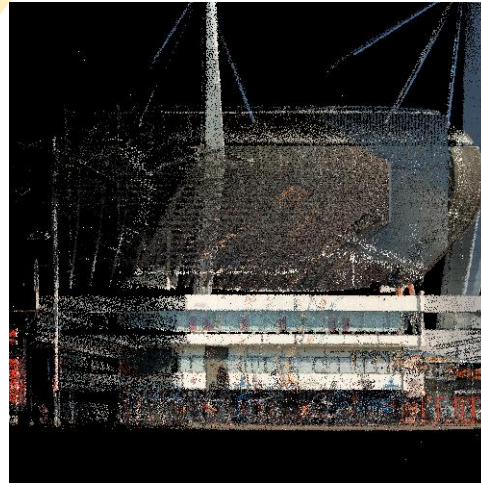


# Point clouds acquired from different sources

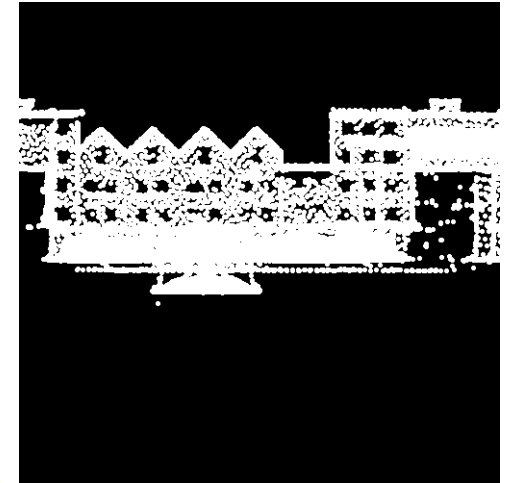
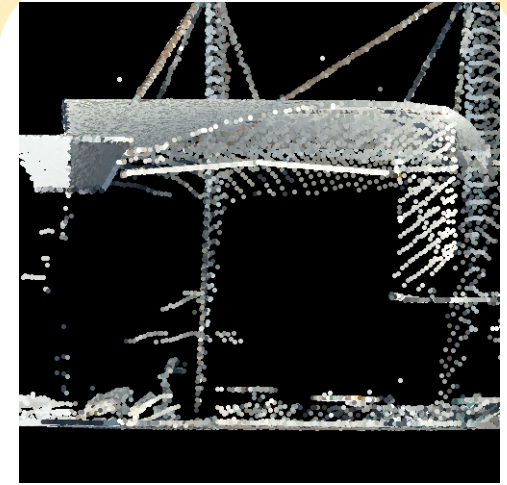
Images



Point clouds from **MLS**



Point clouds from **ALS**

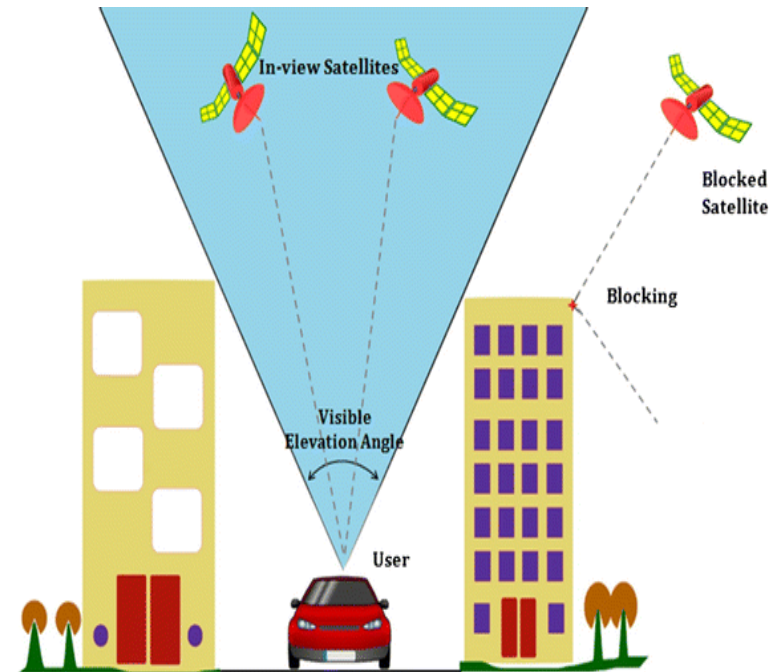


## 2. Differences in point clouds represent the same scene

- Different viewpoints
- Different point densities
- Different accuracies (sensor size or distance between source and object)
- Outliers
- Rotation errors
- Scale errors
- Lack of GPS signal



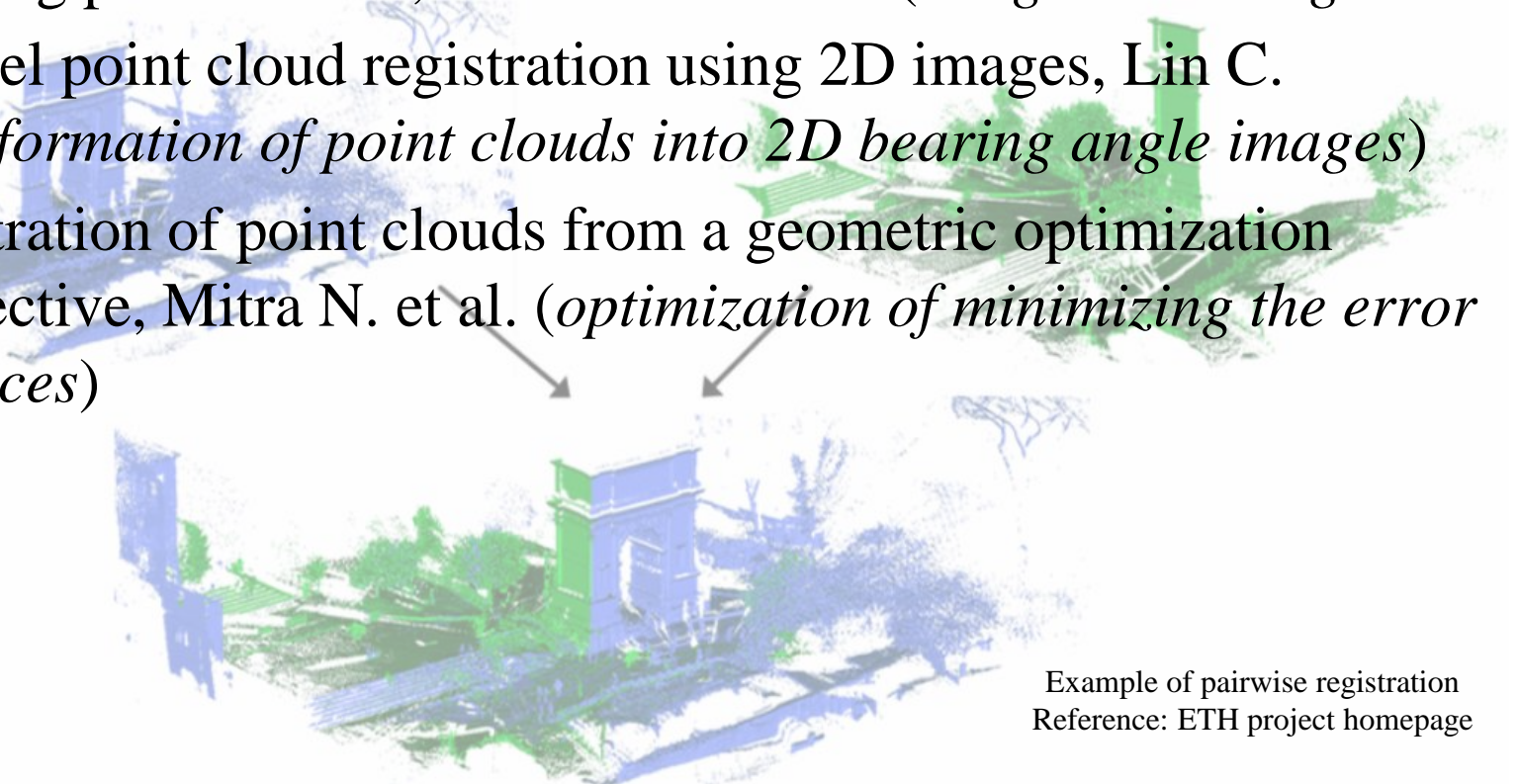
### Co-registration of point clouds



Tabatabaei, A., et al (2017). Reliable urban canyon navigation solution in gps and glonass integrated receiver.

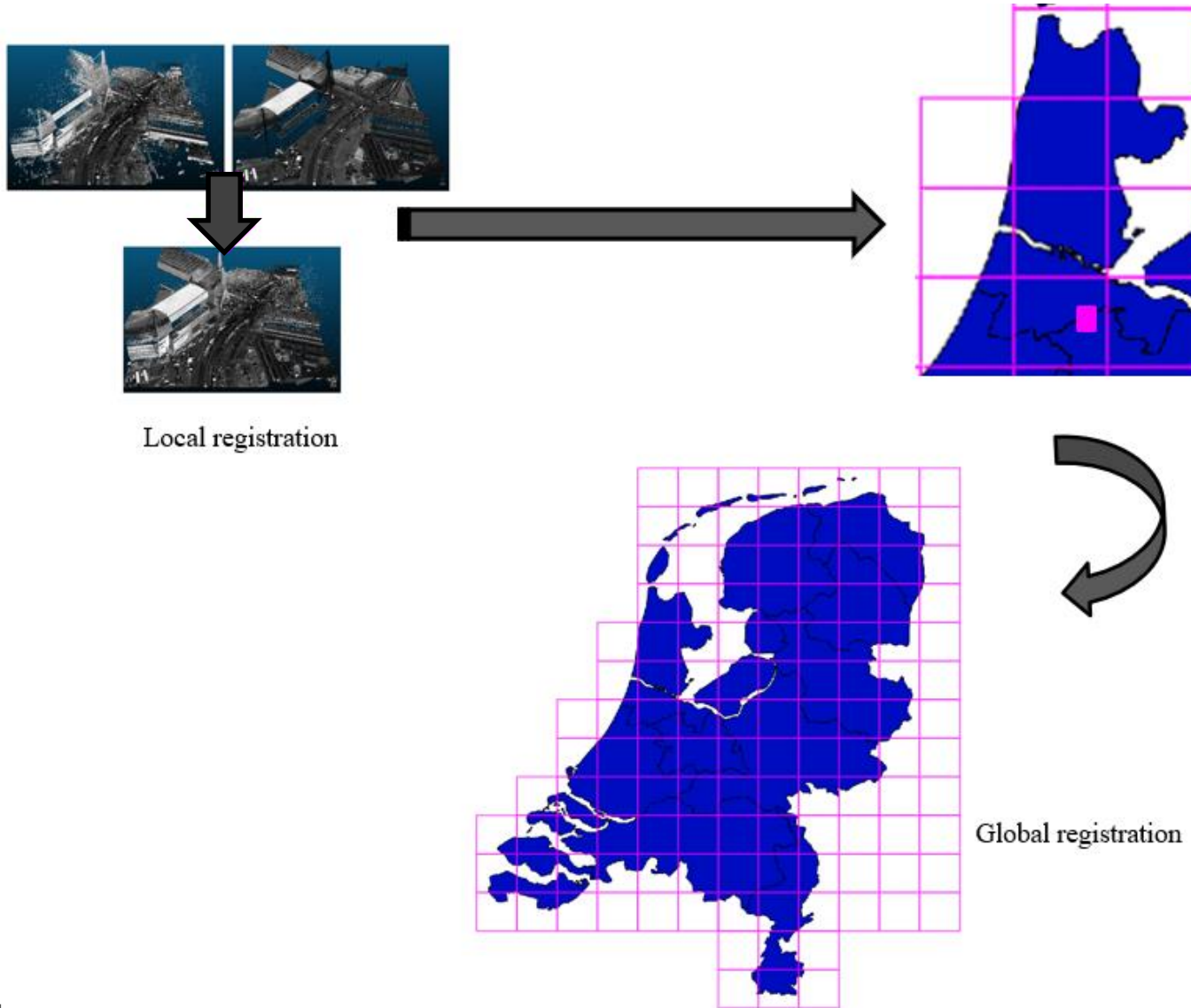
### 3. Previous related work

- Automated registration of terrestrial laser point clouds, P. Theiler (*key point matching registration*)
- An image-based method for the pairwise registration of mobile laser scanning point clouds , A. Christodoulou (*image-based registration*)
- A novel point cloud registration using 2D images, Lin C. (*transformation of point clouds into 2D bearing angle images*)
- Registration of point clouds from a geometric optimization perspective, Mitra N. et al. (*optimization of minimizing the error distances*)



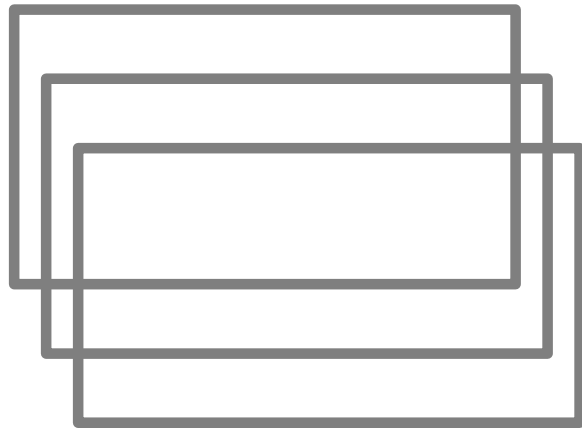
Example of pairwise registration  
Reference: ETH project homepage

# Global registration

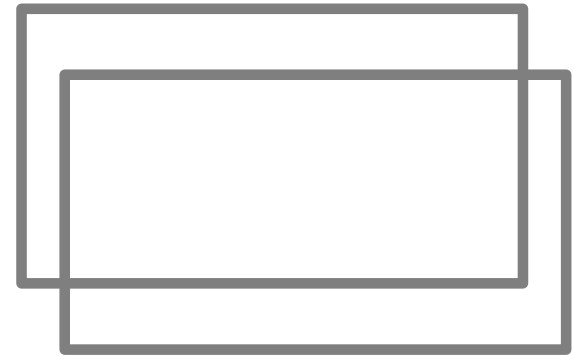


Local registration

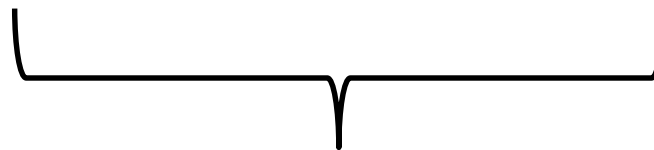
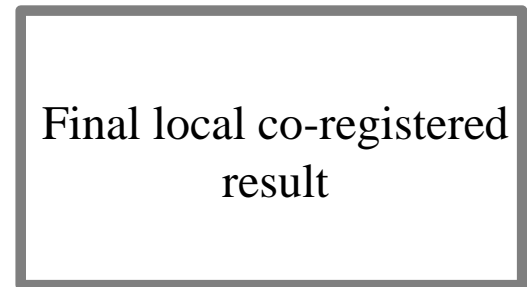
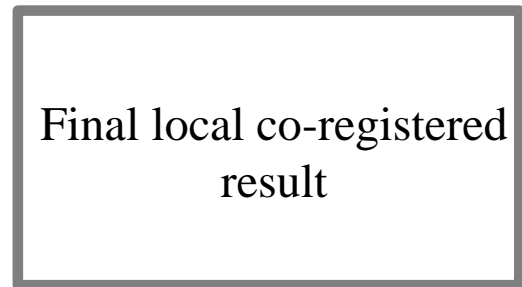
Global registration



Multiple tiles



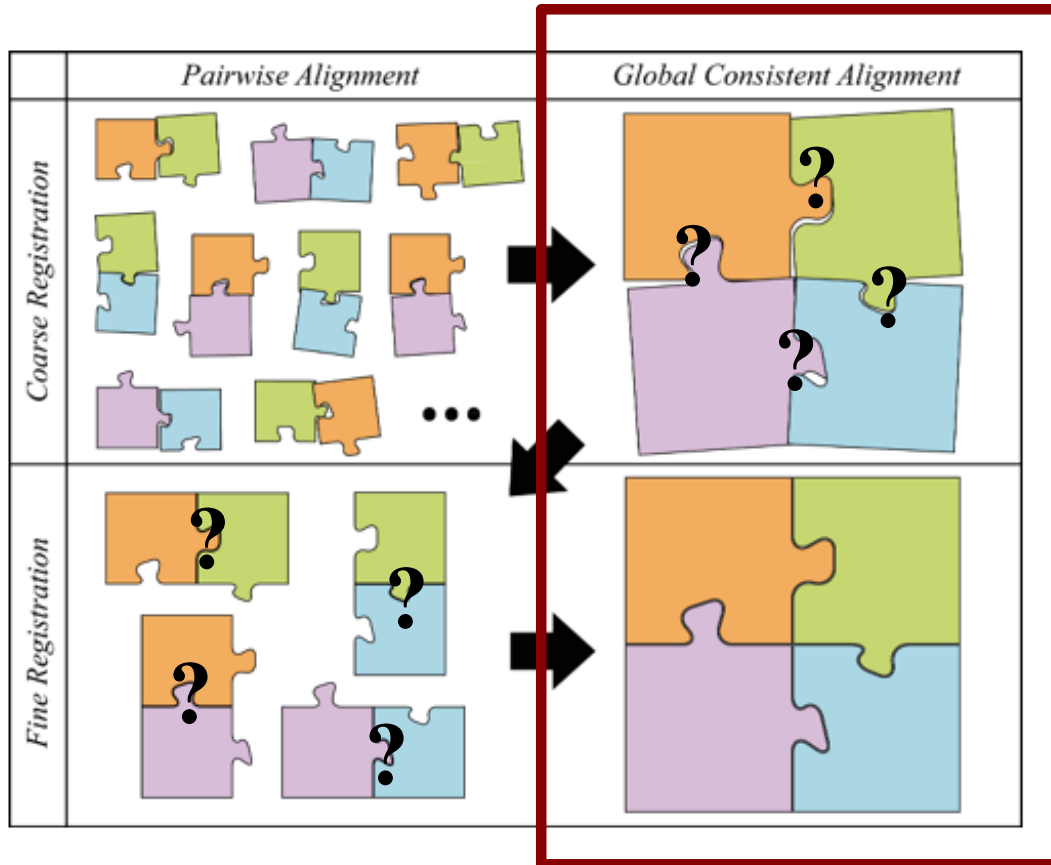
Co- registration



How to combine them  
based on their quality?



# Global registration



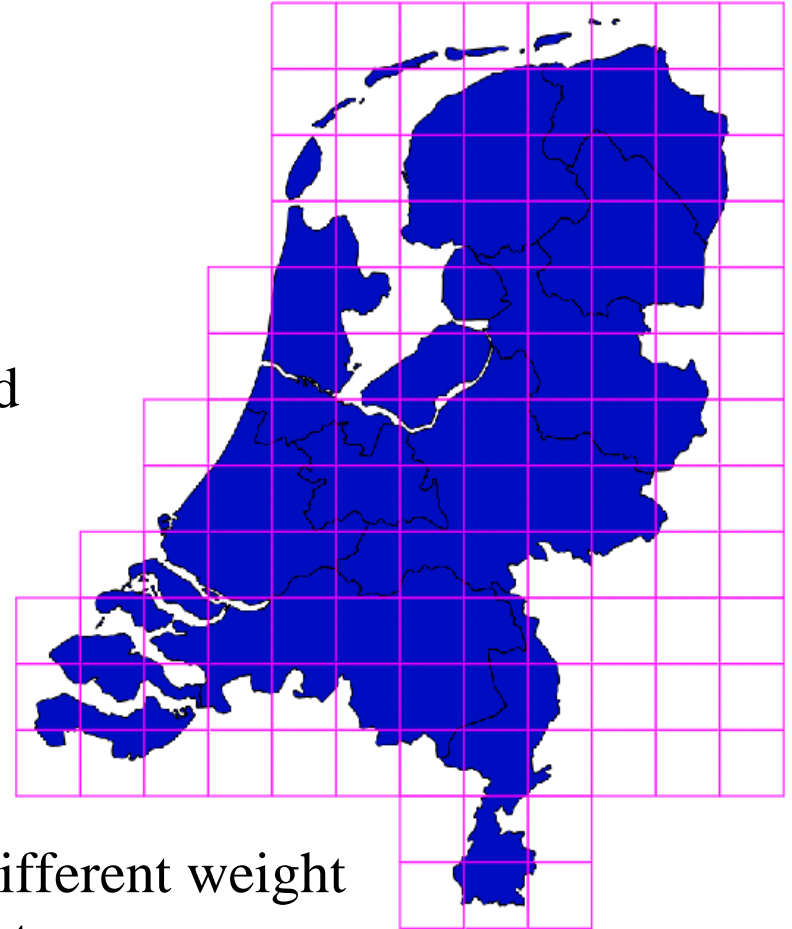
Global registration is only achieved when it is predefined how pieces of the puzzle has to be connected.

It is out of scope to implement the global registration

Theiler, P. W., Wegner, J. D., & Schindler, K. (2015). Globally consistent registration of terrestrial laser scans via graph optimization.

# Global registration

- Each square is a tile
- All tiles represent a large area
- Each tile should have a weight based on the quality of its co-registration



Important to have a different weight

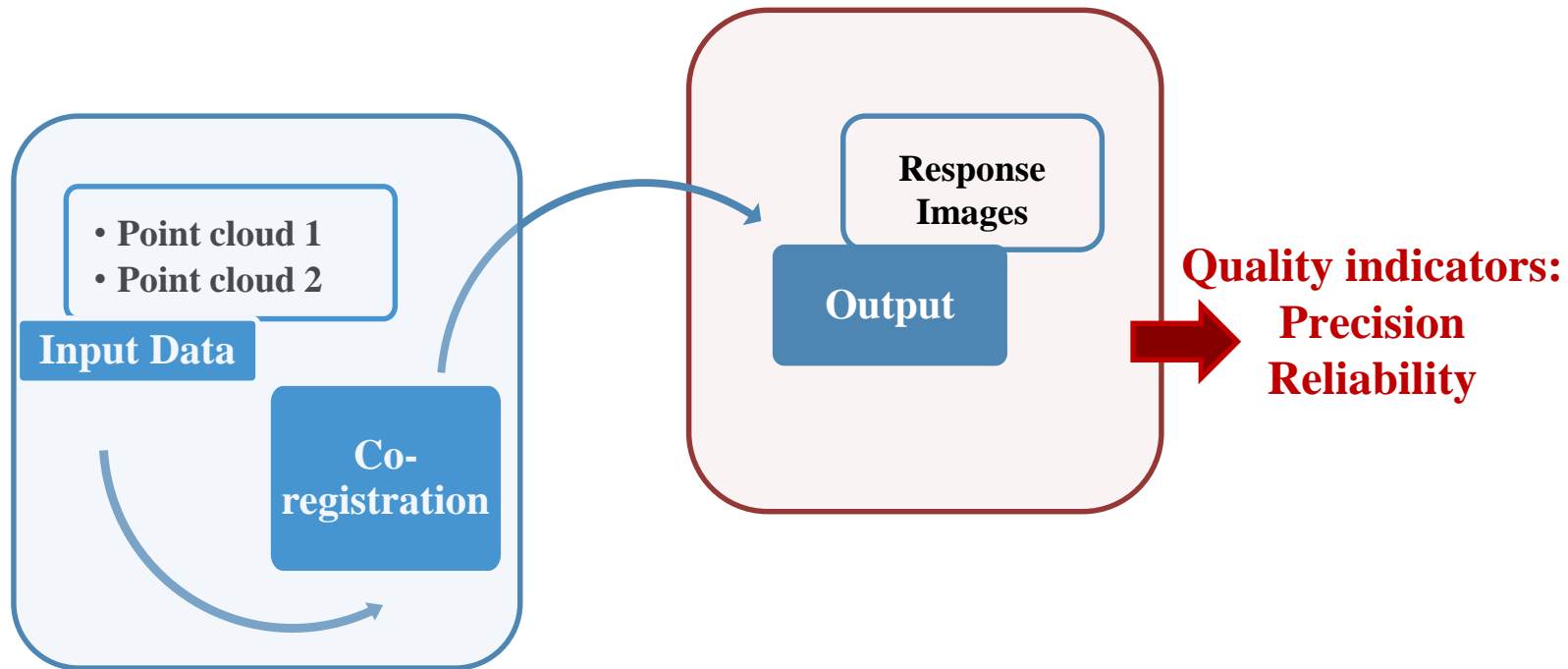
**In order to**

Know which result must contribute more and which less to the final result

# 4. Research Objective: Quality of the output

## Previous related work (functional part)

## Estimation of Quality (stochastic part)



# Quality

- ! exists in everyday life
- ! can be expressed as the degree of satisfying predefined demands
- ! determines our choices (products as cars or electrical gadgets or services as health, education)

**How about the **quality** of the co-registration of point clouds ?**  
**How important is to quantify the quality of a result?**



- More reliable and trustworthy results
- Results can be used for future analysis

# Quality of co-registration

**Quality is determined by stochastic variables and hence**

**It cannot be described by single values but from probabilistic distributions**

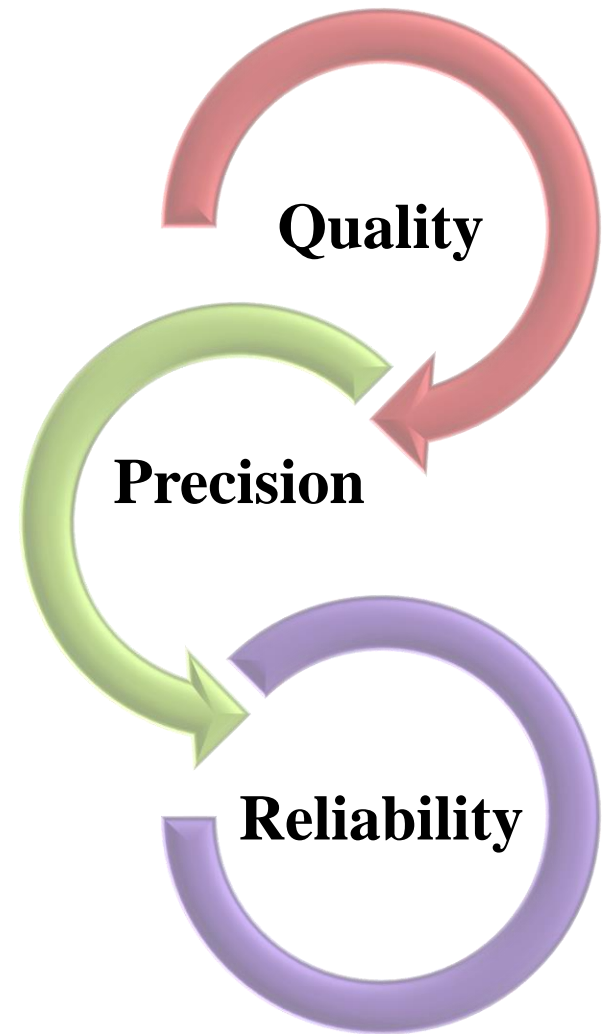
Focus on:

a probabilistic analysis,  
in terms of *precision* and *reliability*

How trustful is the result of the matching ?

How many errors are detected ?

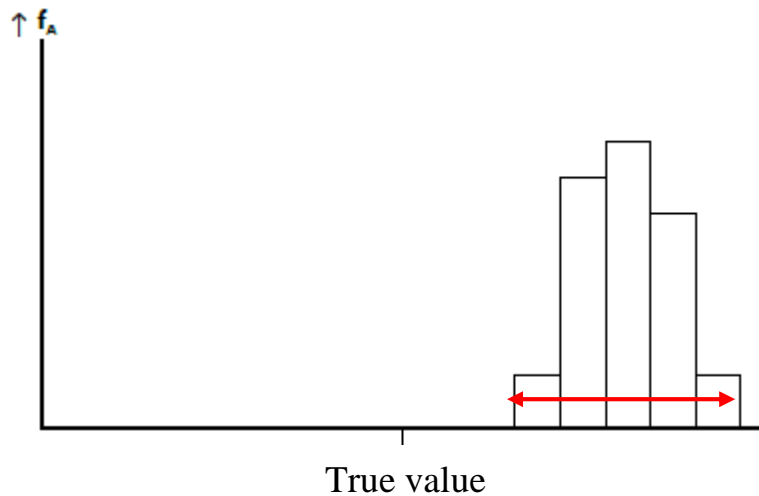
How many pairs of images are accepted ?



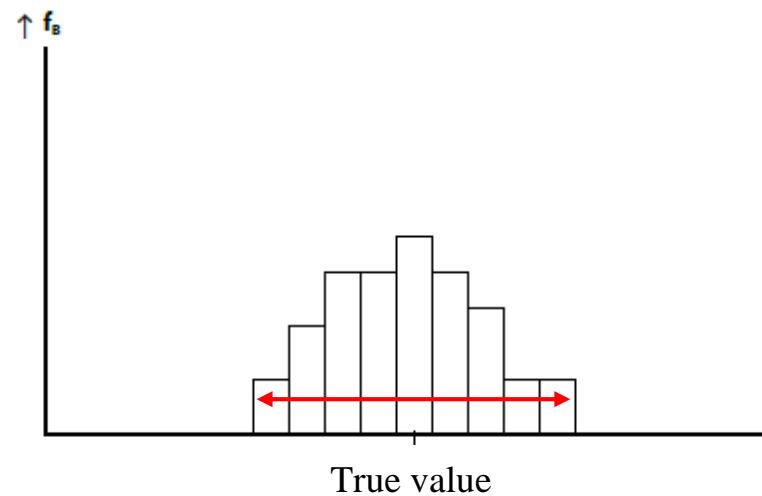
# Both **Precision** and **Reliability** are important

In relation to geodetic networks

Examples of frequencies of two measurements processes

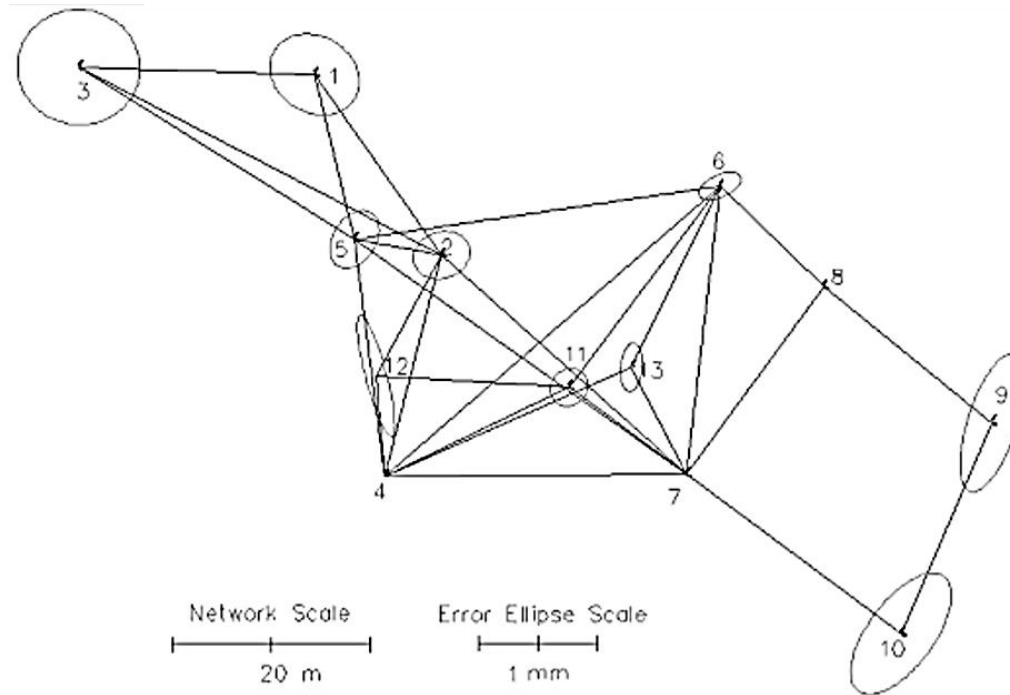


Precise but **not reliable**



**Not precise** but reliable

As *error propagation* exists in geodetic networks where **LSA** is used for minimizing the sum of squares of observational residuals



A *similar* weight adjustment can be implemented in results of co-registration based on the extracted quality

**Acquisition**

**Integration**

**Quality**

Datasets acquired  
from different  
sources  
(MLS, ALS)

Co-registration of  
different datasets

Quantify the quality  
by determining:  
Reliability &  
Precision



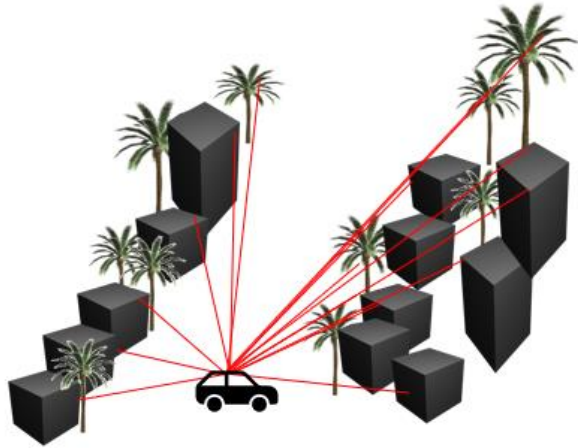
# Co-registration of point clouds

**Co-registration** is the *matching* of different point clouds with common characteristics referring to the same area

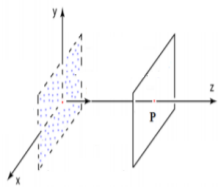
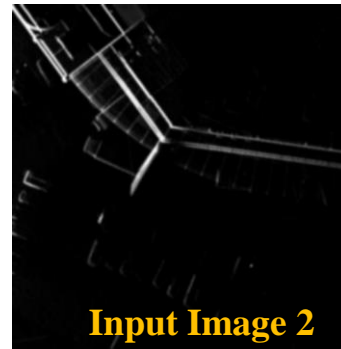
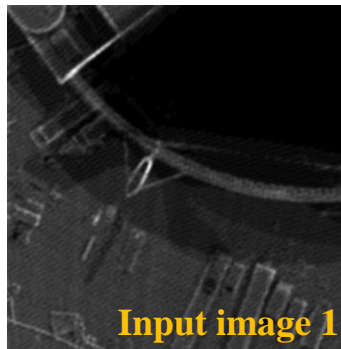
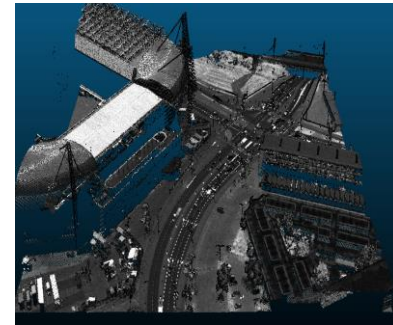
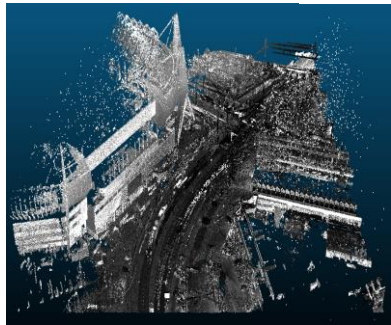
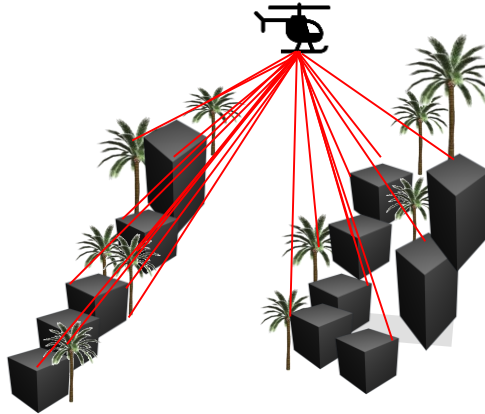
**Co-registration** can be achieved in pairs of point clouds either by using **features** or **images**:

- feature-based methods
- **2D image-based methods**

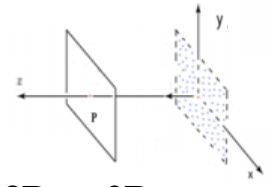
# Mobile laser scanned point cloud



# Aerial laser scanned point cloud



3D to 2D



3D to 2D

# Template matching with input images



## Response image

Using different attributes of points  
Different response images

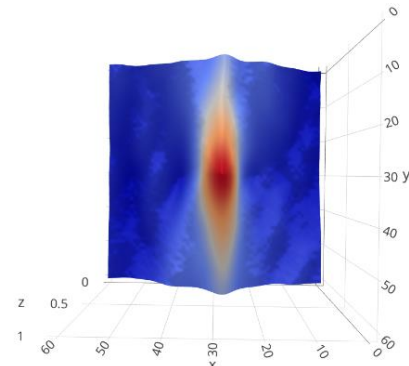
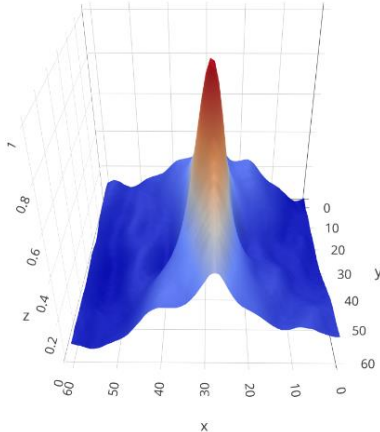
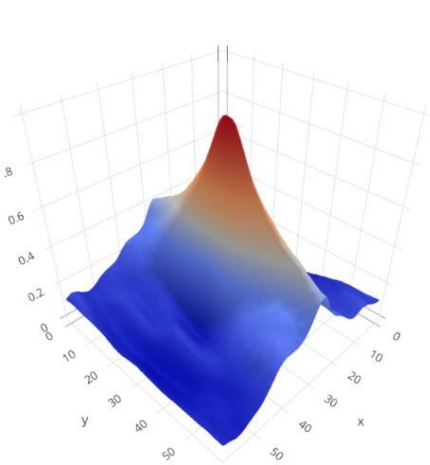
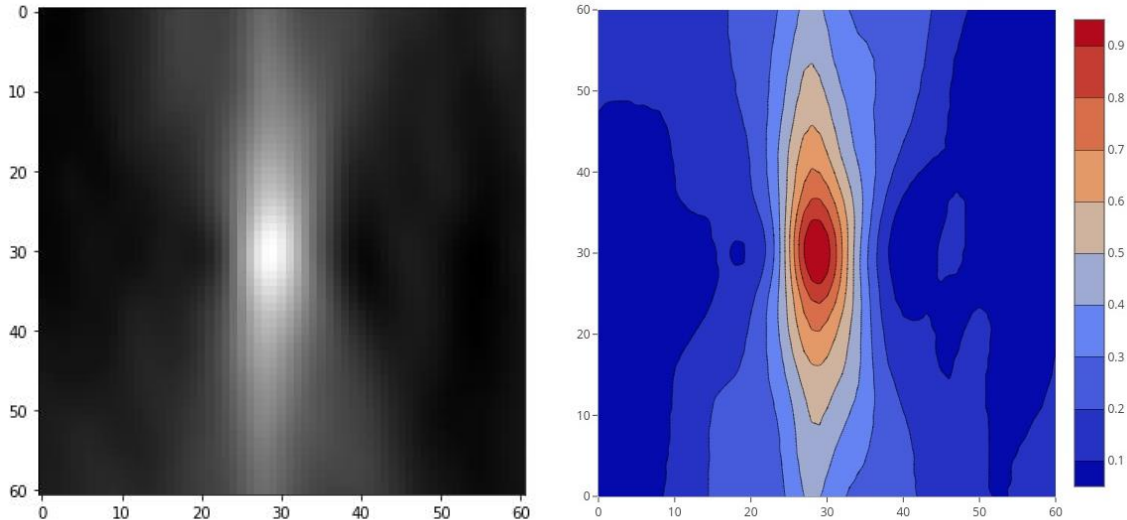


Images containing values related to the matching

Higher values indicate **better matching** & are the **brighter** parts of the image  
while

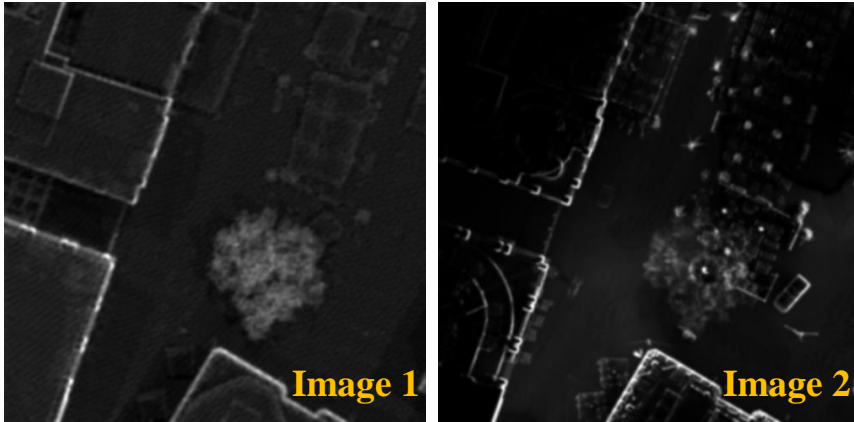
**Smaller** values indicate **less good matching** & are the **darker** parts of the image

# Response image in 2D and 3D

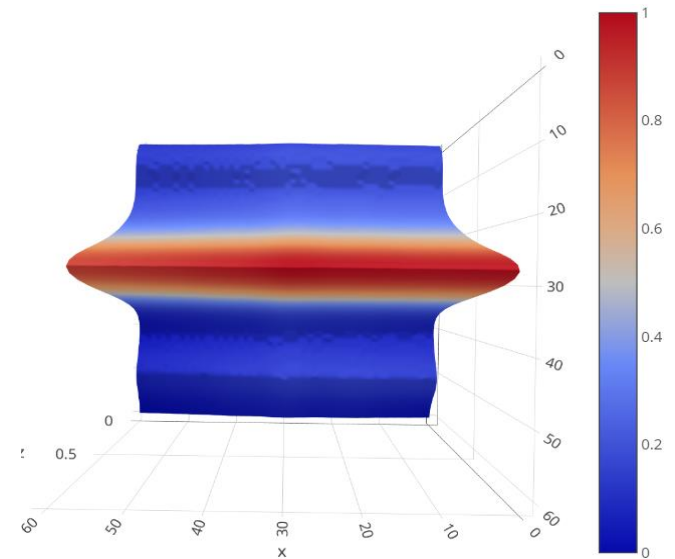
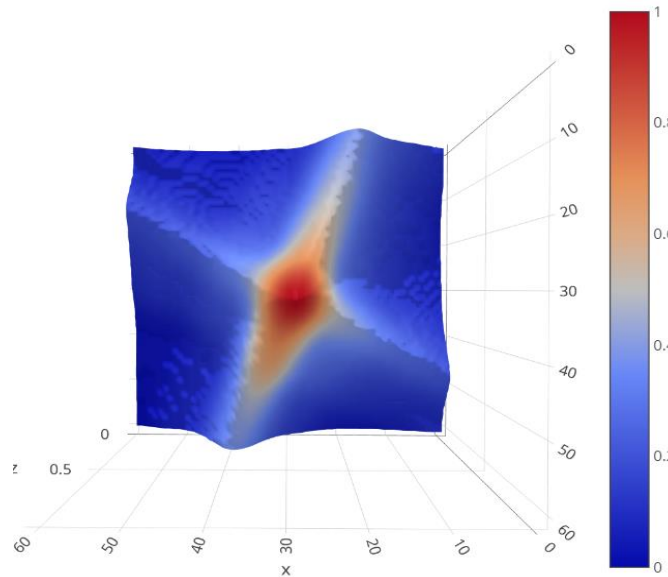
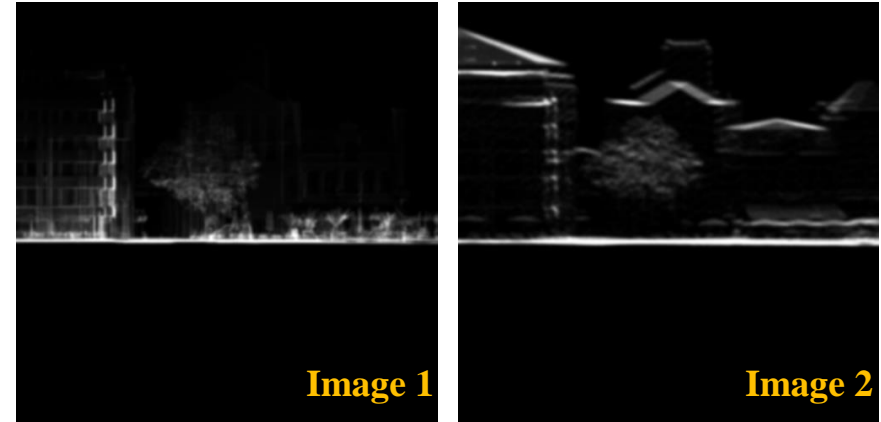


# What do response images indicate ?

Input images to template matching

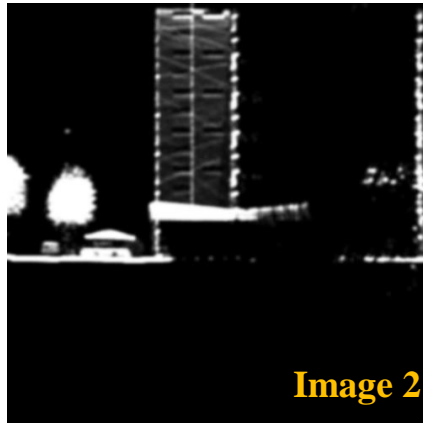


Input images to template matching

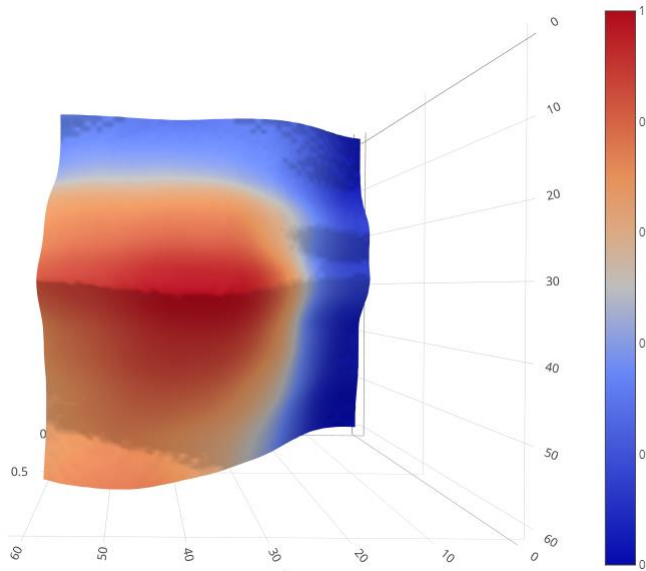
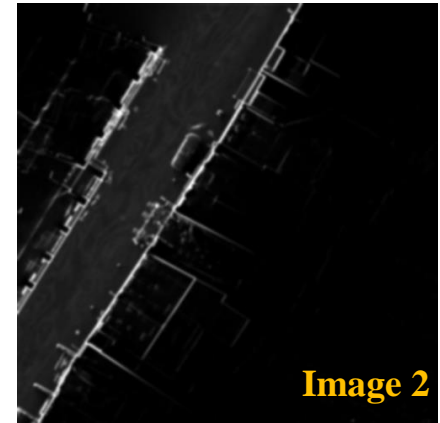
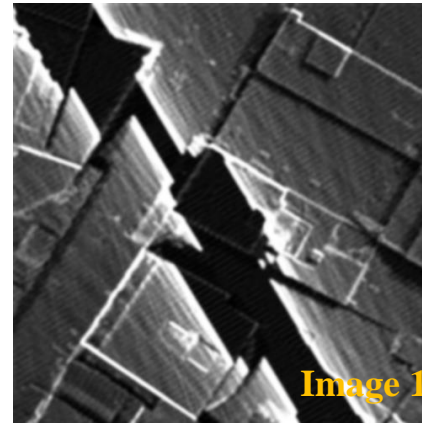


# What do response images indicate ?

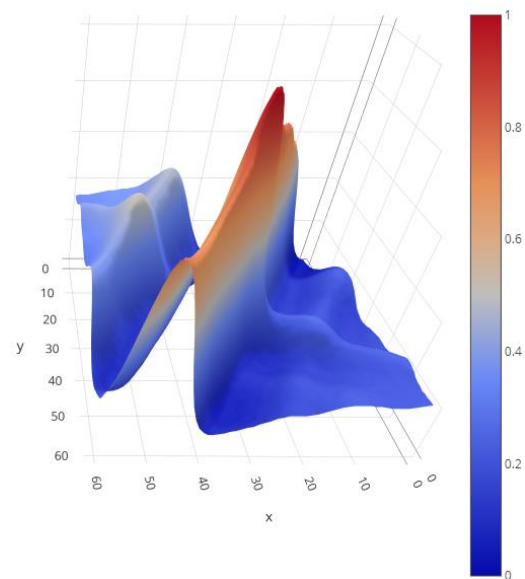
Input images to template matching



Input images to template matching



Response images



# 5. Methodology

Normalize values of images  
and focus only on significant  
part

Test response images for  
normal distribution in both  
dimensions

Fit a Gaussian line to  
individual pixel values

Calculation of standard  
deviation in pixel values

Calculation of correlation  
between axes of the image

Calculation of shift parameter

# 5. Methodology

Normalize values of images and focus only on significant part

Test response images for normal distribution in both dimensions

Fit a Gaussian line to individual pixel values

Calculation of standard deviation in pixel values

Calculation of correlation between axes of the image

Calculation of shift parameter



# Normalize image values

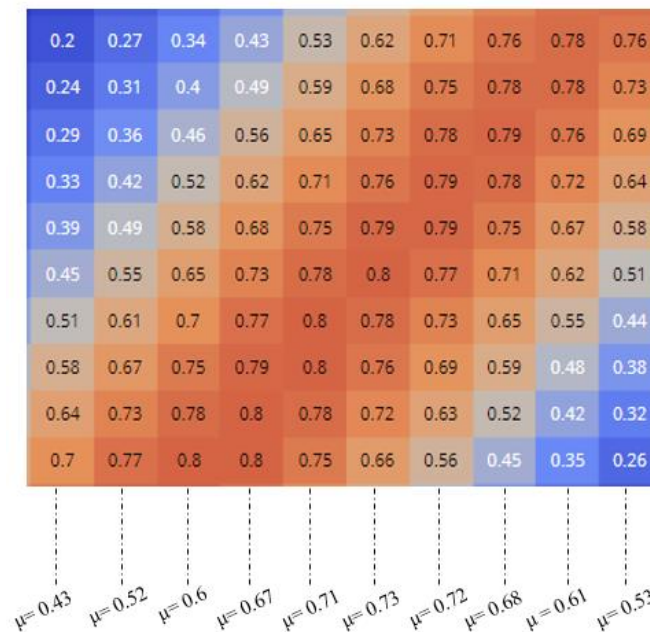
**Algorithm 5.1:** How to normalize pixel values for response images

```

create matrix for calculated values per row;
create matrix for calculated values per column;
for every response image do
  for every row of the response image do
    find mean value;
    append it to a new matrix for rows;
    find the mean values of matrix for rows;
    for every value in mean matrix for rows do
      subtract the mean value from each pixel value;
      if the value is positive then
        append the value to the matrix for rows;
      if the value is negative then
        replace the value with zero;
        append zero to the matrix for rows;
      end
    end
  end
end
for every column of the response image do
  find mean value;
  append it to a new matrix for columns;
  find the mean values of matrix for columns;
  for every value in mean matrix for columns do
    subtract the mean value from each pixel value;
    if the value is positive then
      append the value to the matrix for columns;
    if the value is negative then
      replace the value with zero;
      append zero to the matrix for columns;
    end
  end
end
end
end
  
```

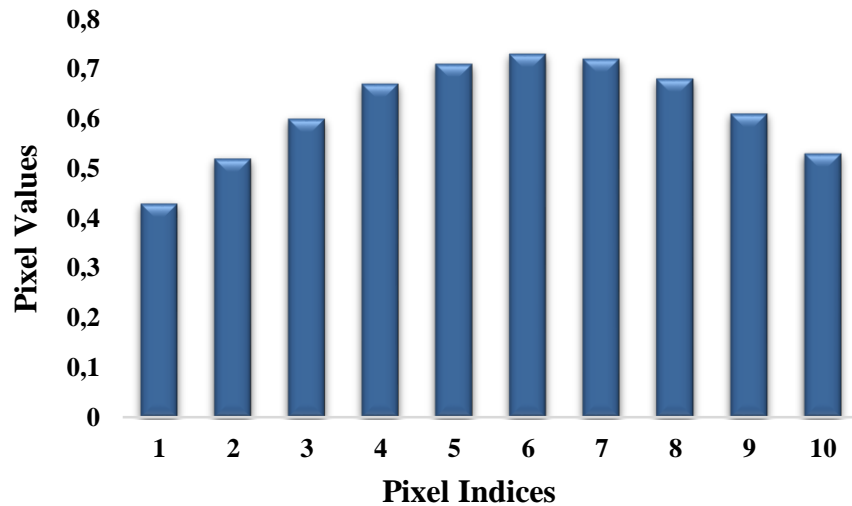


}  $\mu = 0.62$

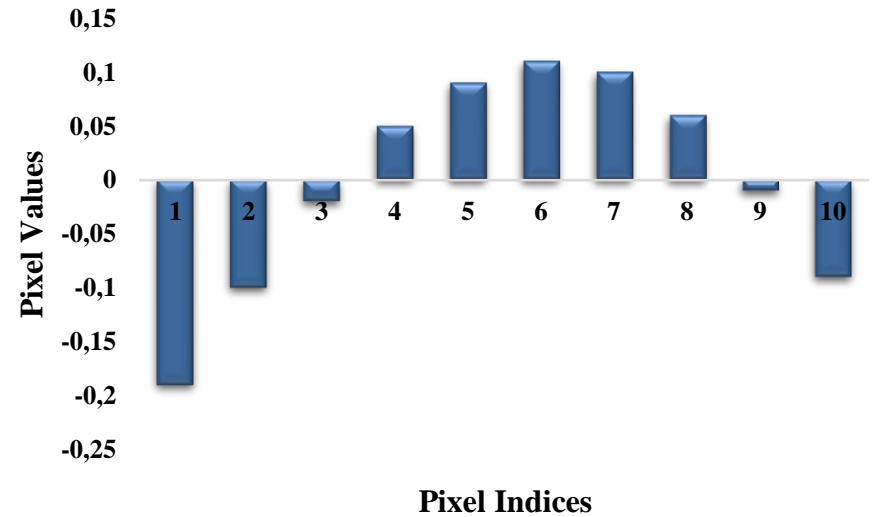


}  $\mu = 0.62$

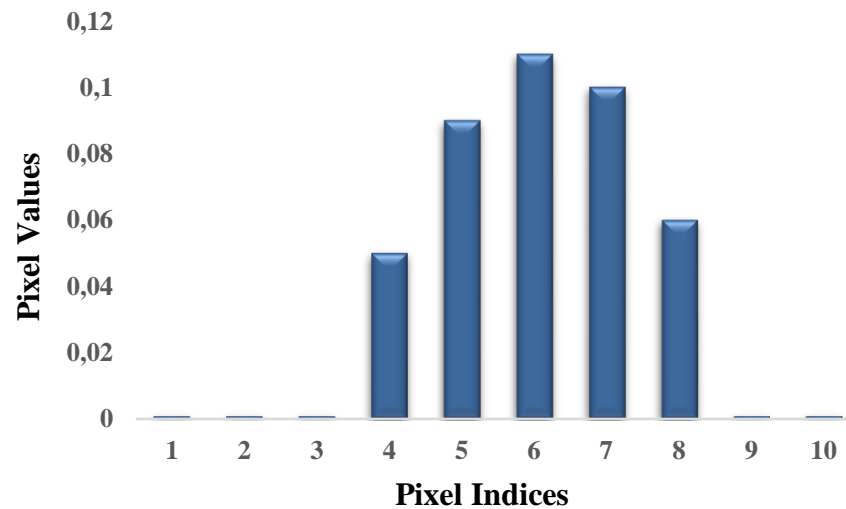
### Original values



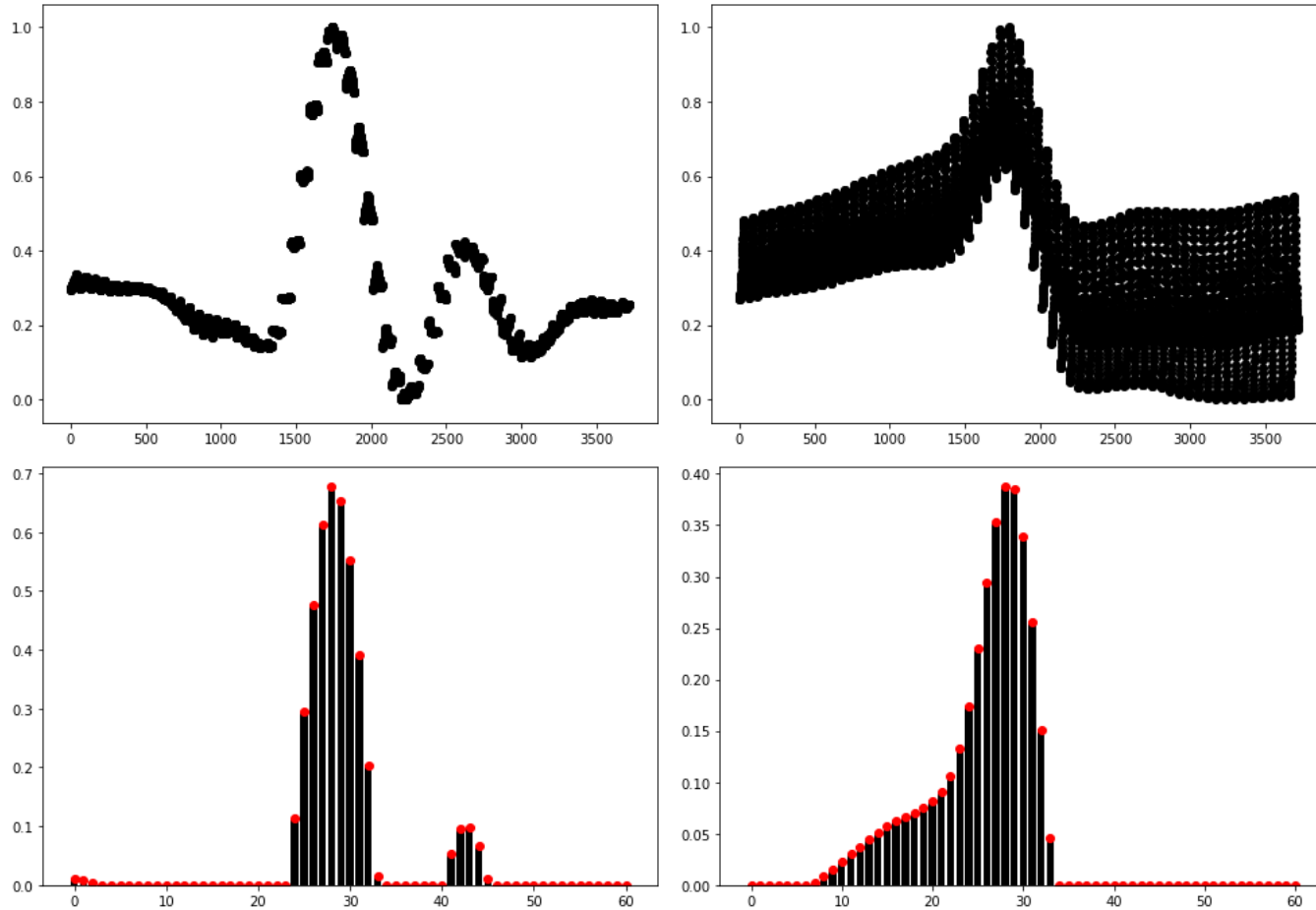
### Values after subtracting the mean



### Values after replacing the negative values with zeros



# Normalize image values



- Values are centralized
- Distributions remain the same
- Characteristics remain the same

Normalize values of images  
and focus only on significant  
part

Test response images for  
normal distribution in both  
dimensions

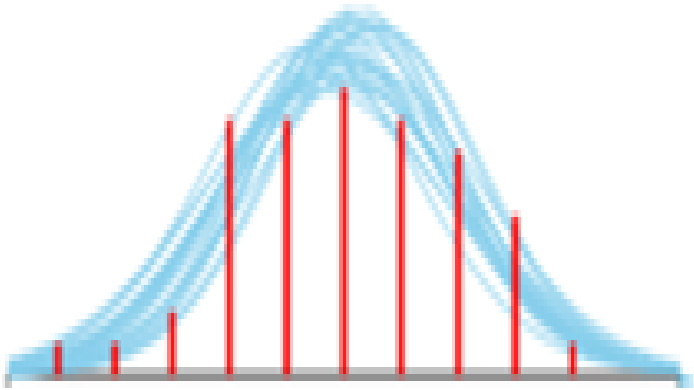
Fit a Gaussian line to  
individual pixel values

Calculation of standard  
deviation in pixel values

Calculation of correlation  
between axes of the image

Calculation of shift parameter

# Normal distribution



- ▶ It is the most dominant distribution
- ▶ It describes well various phenomena as error distribution
- ▶ Gives insight about quality of data with a probabilistic analysis
- ▶ It is often assumed by algorithms used for analysis

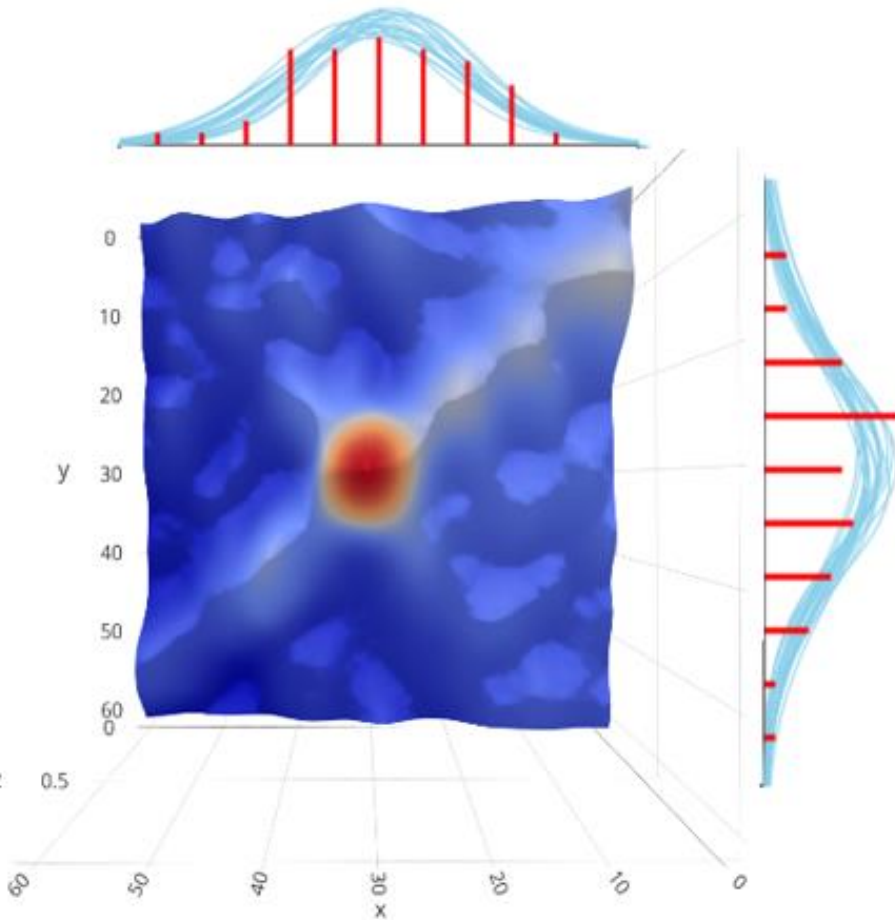
**Significance level** is set to 0.05%

**KS** test is applied &

**If** p-value is greater than 0.005 for **both axes**, the pixel values are considered statistically consistent

**While if** p-value is smaller, the pixel values are considered inconsistent and response image is rejected

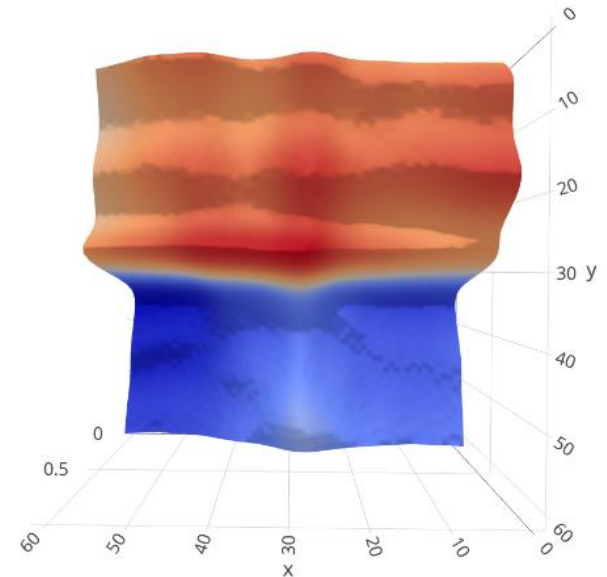
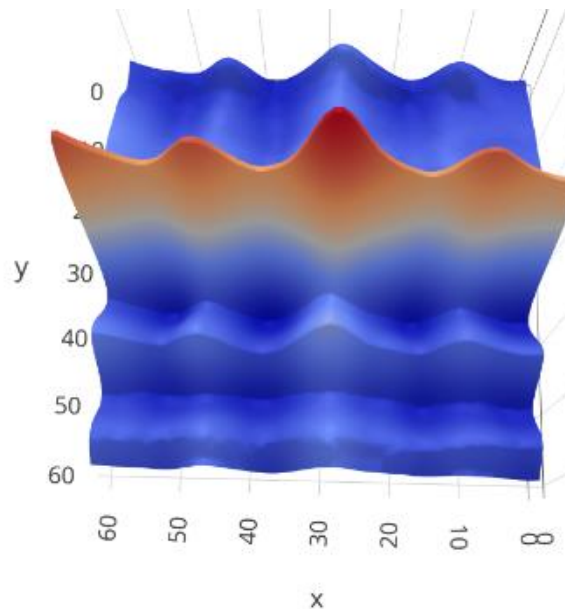
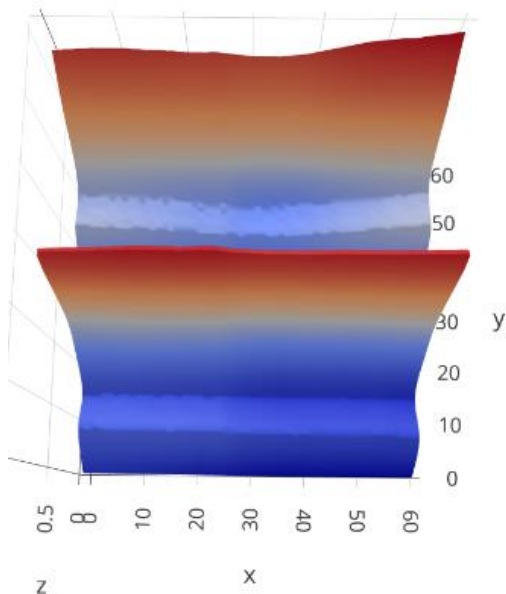
Only when **both axes** follow the same or similar distribution can give valuable result



## By testing the Normal distribution:

**As Unreliable** matchings do not pass the normality tests, they do not participate in global registration as:

- ▶ Multiple peaks
- ▶ Non clear peaks



Normalize values of images  
and focus only on significant  
part

Test response images for  
normal distribution in both  
dimensions

Fit a Gaussian line to  
individual pixel values

Calculation of standard  
deviation in pixel values

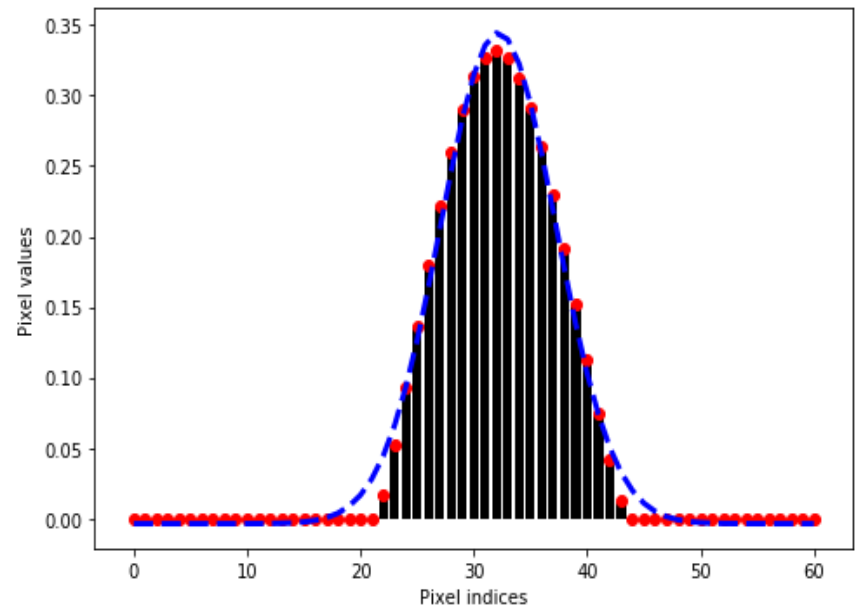
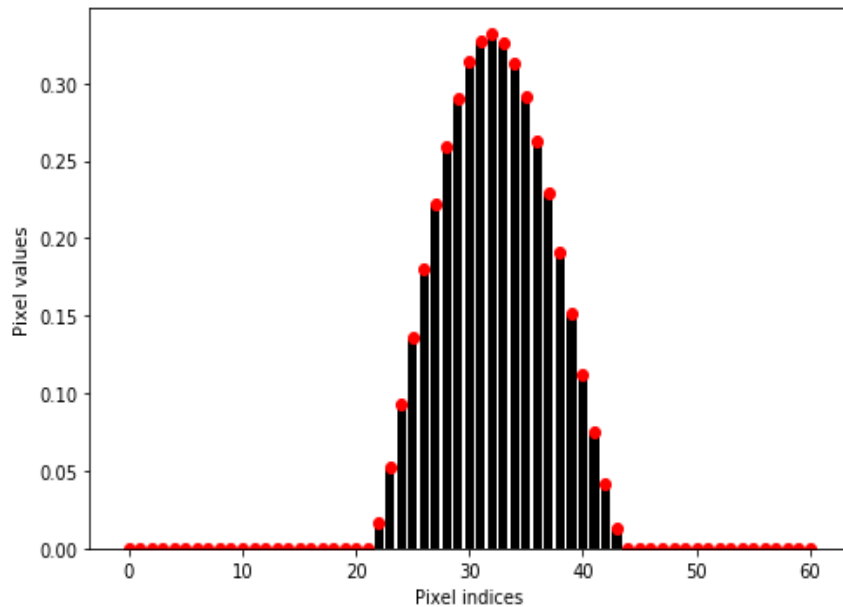
Calculation of correlation  
between axes of the image

Calculation of shift parameter



Instead of values of pixels, a line is fit to the each distribution  
**Polynomial** did not fit and a **Gaussian model** is selected

$$f(x; A, \mu, \sigma) = \frac{A}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x - \mu)^2}{2\sigma^2}\right]$$

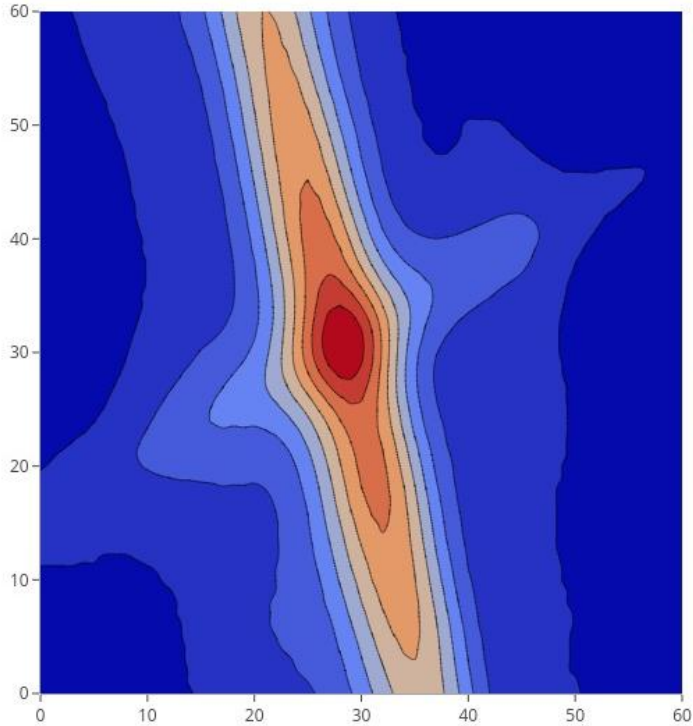


- values of pixels
- Gaussian line

# Why to fit a line?



## 1. Interpolation & Sub-pixel accuracy



0.29	0.361	0.447	0.545	0.639	0.722	0.78	0.796	0.773	0.71	0.624	0.533	0.451	0.384	0.341
0.282	0.349	0.435	0.529	0.631	0.722	0.792	0.824	0.812	0.761	0.682	0.592	0.502	0.431	0.376
0.275	0.341	0.424	0.522	0.624	0.725	0.804	0.851	0.859	0.82	0.745	0.651	0.557	0.475	0.408
0.275	0.337	0.42	0.518	0.624	0.729	0.824	0.886	0.906	0.875	0.804	0.71	0.604	0.51	0.431
0.278	0.337	0.42	0.518	0.627	0.741	0.843	0.922	0.949	0.929	0.863	0.761	0.647	0.537	0.443
0.286	0.349	0.427	0.525	0.635	0.753	0.863	0.945	0.984	0.969	0.902	0.796	0.671	0.549	0.443
0.302	0.361	0.439	0.533	0.643	0.757	0.871	0.957	1.0	0.988	0.922	0.816	0.682	0.549	0.435
0.322	0.376	0.451	0.537	0.643	0.753	0.859	0.945	0.992	0.984	0.922	0.816	0.682	0.545	0.424
0.341	0.392	0.455	0.537	0.631	0.729	0.827	0.91	0.957	0.957	0.902	0.804	0.675	0.537	0.416
0.357	0.4	0.455	0.522	0.604	0.69	0.78	0.859	0.91	0.918	0.875	0.784	0.663	0.533	0.412
0.361	0.396	0.439	0.494	0.565	0.639	0.722	0.796	0.851	0.867	0.839	0.765	0.659	0.533	0.416
0.357	0.38	0.412	0.459	0.518	0.584	0.659	0.733	0.792	0.82	0.808	0.749	0.655	0.541	0.427
0.337	0.353	0.38	0.416	0.467	0.529	0.604	0.675	0.741	0.78	0.78	0.741	0.659	0.557	0.447
0.314	0.325	0.345	0.376	0.424	0.482	0.553	0.627	0.698	0.745	0.761	0.733	0.667	0.573	0.467

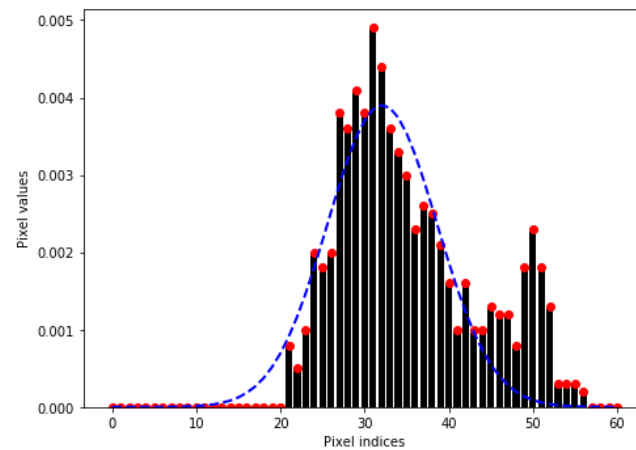
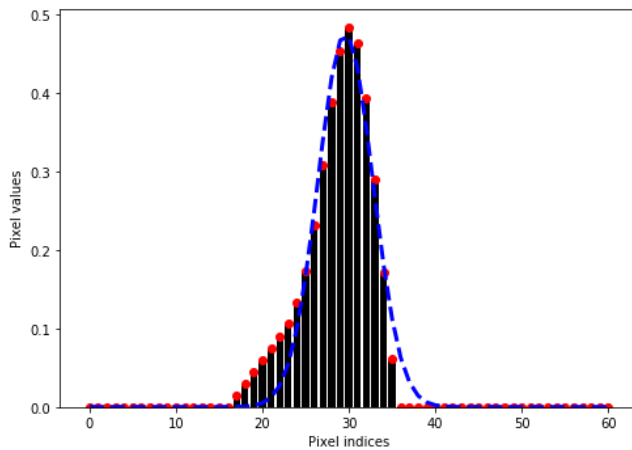
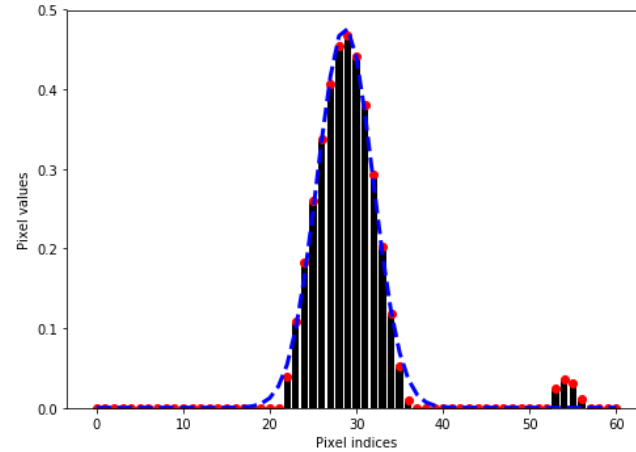
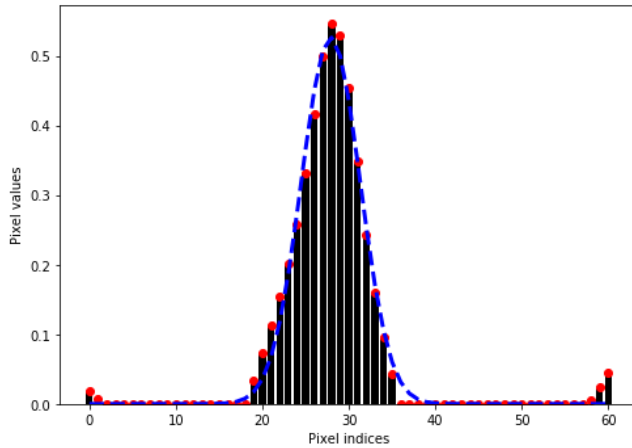
0.29	0.361	0.447	0.545	0.639	0.722	0.78	0.796	0.773	0.71	0.624	0.533	0.451	0.384	0.341
0.282	0.349	0.435	0.529	0.631	0.722	0.792	0.824	0.812	0.761	0.682	0.592	0.502	0.431	0.376
0.275	0.341	0.424	0.522	0.624	0.725	0.804	0.851	0.859	0.82	0.745	0.651	0.557	0.475	0.408
0.275	0.337	0.42	0.518	0.624	0.729	0.824	0.886	0.906	0.875	0.804	0.71	0.604	0.51	0.431
0.278	0.337	0.42	0.518	0.627	0.741	0.843	0.922	0.949	0.929	0.863	0.761	0.647	0.537	0.443
0.286	0.349	0.427	0.525	0.635	0.753	0.863	0.945	0.984	0.969	0.902	0.796	0.671	0.549	0.443
0.302	0.361	0.439	0.533	0.643	0.757	0.871	0.957	1.0	0.988	0.922	0.816	0.682	0.549	0.435
0.322	0.376	0.451	0.537	0.643	0.753	0.859	0.945	0.992	0.984	0.922	0.816	0.682	0.545	0.424
0.341	0.392	0.455	0.537	0.631	0.729	0.827	0.91	0.957	0.957	0.902	0.804	0.675	0.537	0.416
0.357	0.4	0.455	0.522	0.604	0.69	0.78	0.859	0.91	0.918	0.875	0.784	0.663	0.533	0.412
0.361	0.396	0.439	0.494	0.565	0.639	0.722	0.796	0.851	0.867	0.839	0.765	0.659	0.533	0.416
0.357	0.38	0.412	0.459	0.518	0.584	0.659	0.733	0.792	0.82	0.808	0.749	0.655	0.541	0.427
0.337	0.353	0.38	0.416	0.467	0.529	0.604	0.675	0.741	0.78	0.78	0.741	0.659	0.557	0.447
0.314	0.325	0.345	0.376	0.424	0.482	0.553	0.627	0.698	0.745	0.761	0.733	0.667	0.573	0.467

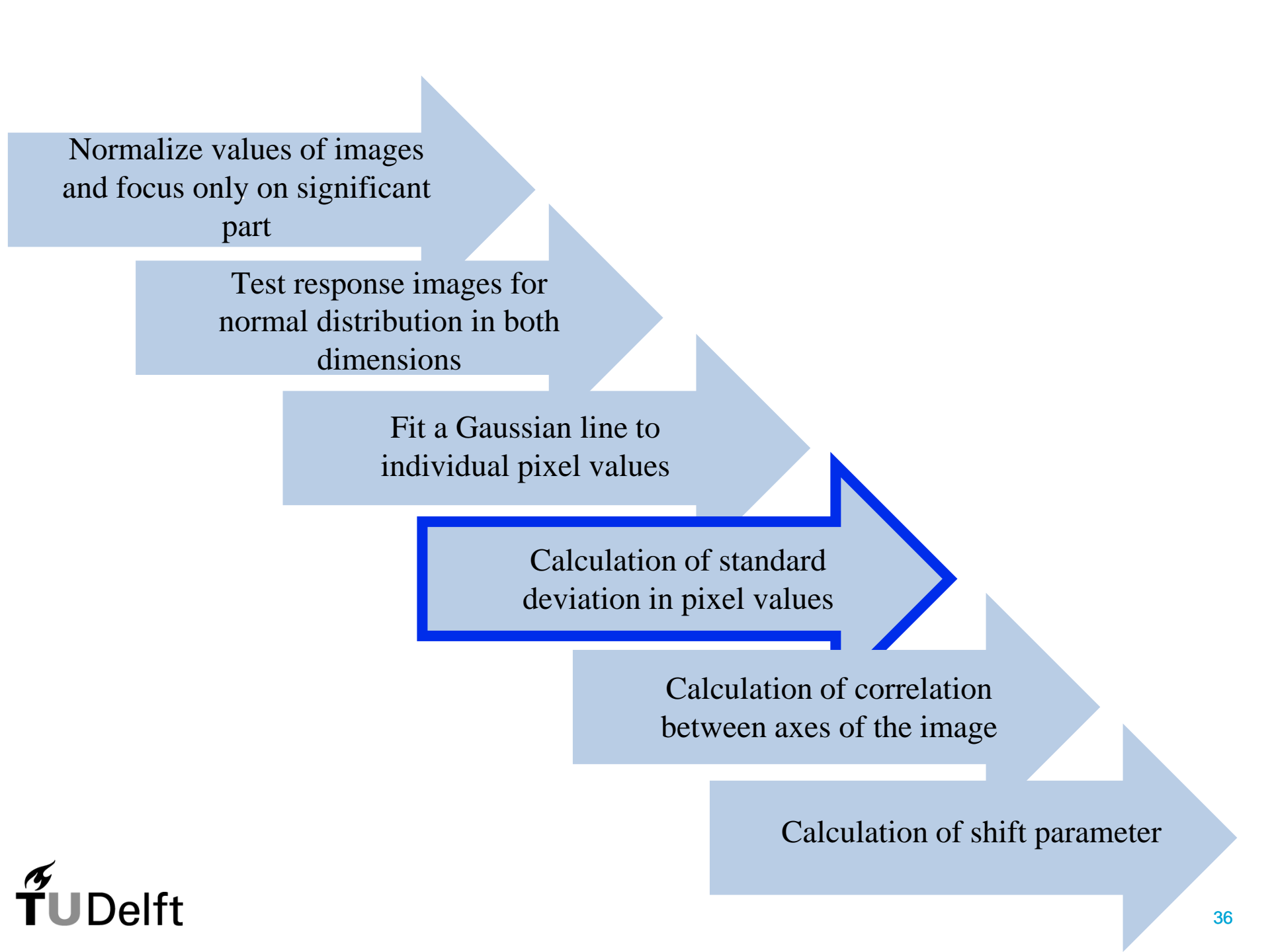
The peak of the matching is the Gaussian line peak  
and not the image peak

# Why to fit a line?



## 2. Minimizing the residuals





Normalize values of images  
and focus only on significant  
part

Test response images for  
normal distribution in both  
dimensions

Fit a Gaussian line to  
individual pixel values

Calculation of standard  
deviation in pixel values

Calculation of correlation  
between axes of the image

Calculation of shift parameter

By fitting a line?

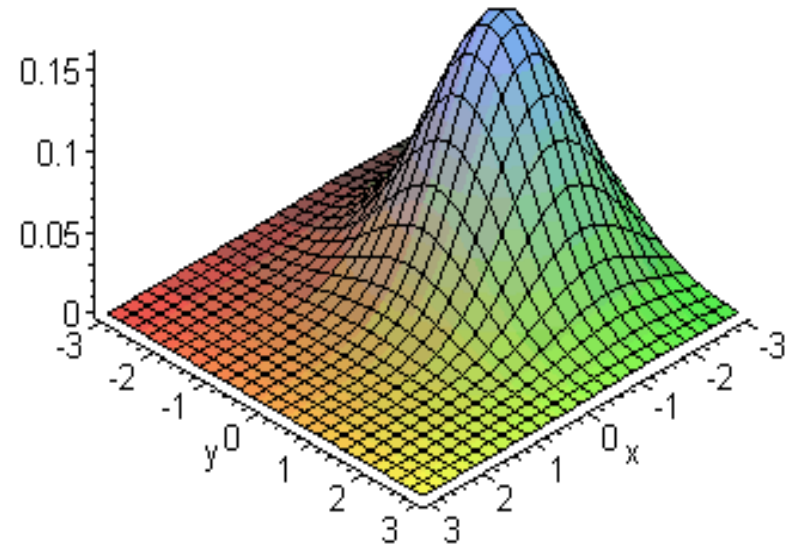
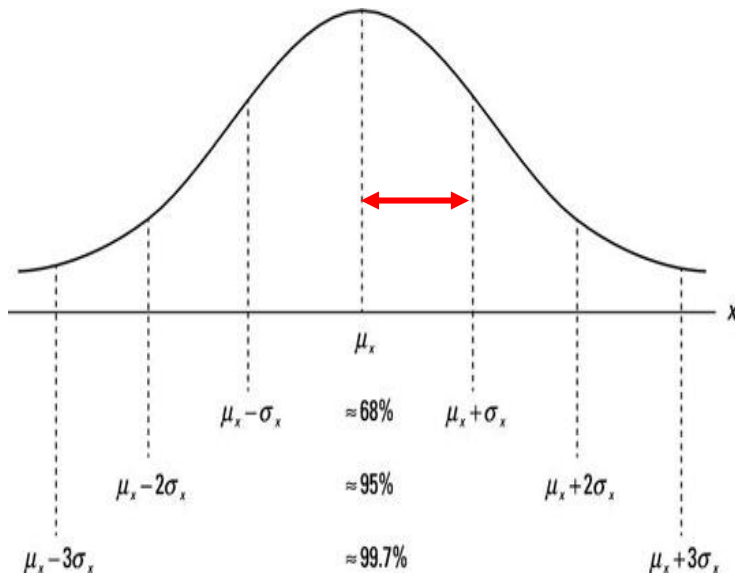


Calculate Standard deviation



In **1D** is a measure of the spread of probability

in **2D** is a 2D summary of 2D data, the standard deviation in x and y axes



## 2D analysis is required

x axis

### Multivariate Gaussian distribution

$$f(x, y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \exp\left[-\frac{1}{2(1-\rho^2)}\left(\left(\frac{x-\mu_x}{\sigma_x}\right)^2 + \left(\frac{y-\mu_y}{\sigma_y}\right)^2 - 2\rho\left(\frac{x-\mu_x}{\sigma_x}\right)\left(\frac{y-\mu_y}{\sigma_y}\right)\right)\right]$$

Parameters:

**Mean** value  $\mu_x$  and standard deviation  $\sigma_x$  in x axis

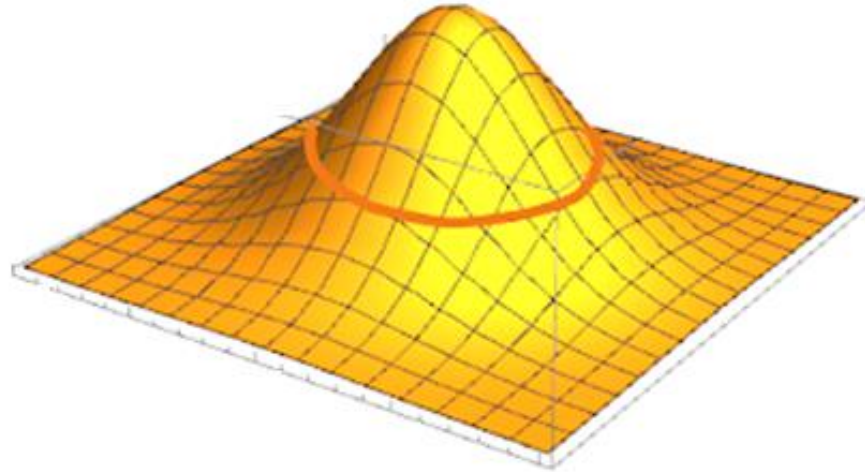
**Mean** value  $\mu_y$  and standard deviation  $\sigma_y$  in y axis

$\rho$  the correlation between the axes

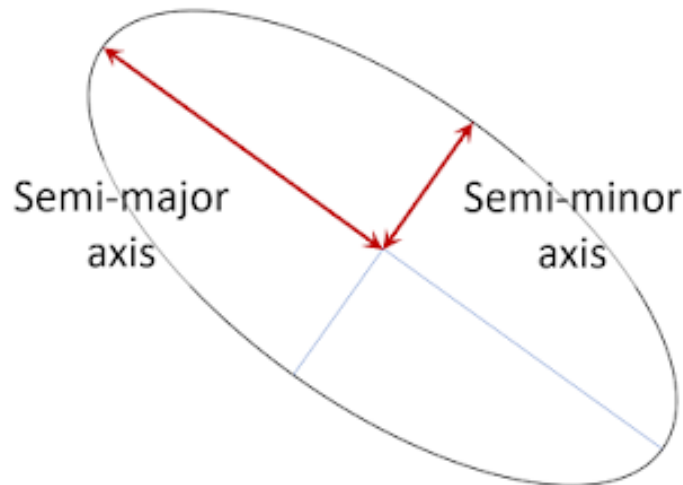
y axis

# Standard deviation in 2D:

- same mean values & same standard deviations: a **circle** that fits to the surface

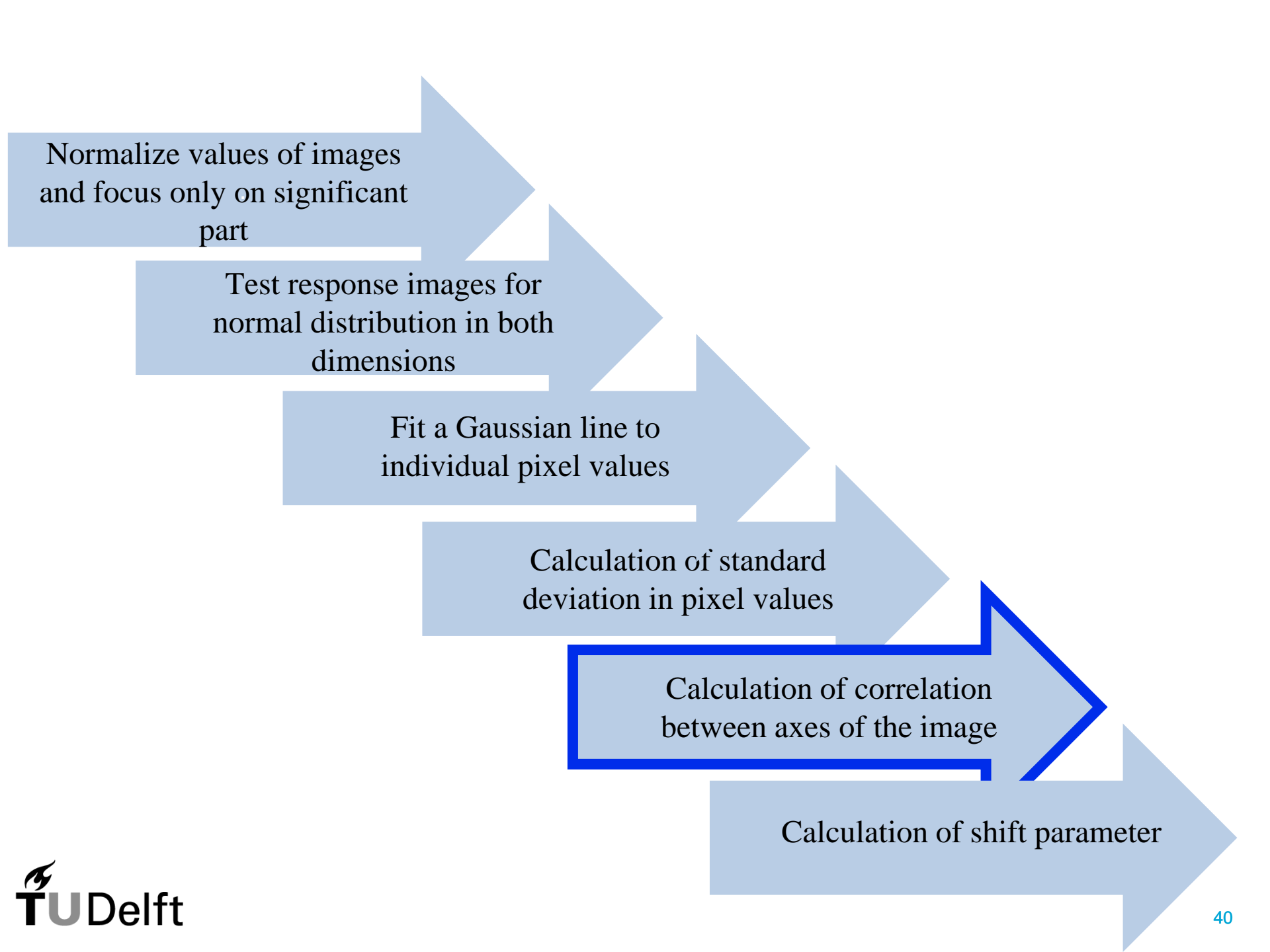


- different standard deviations: an **ellipse** that fits to the surface



Where :

Semi **major** axis: STD in **x axis**  
Semi **minor** axis: STD in **y axis**



Normalize values of images  
and focus only on significant  
part

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deviation in pixel values

Calculation of correlation  
between axes of the image

Calculation of shift parameter



*Axes* of ellipses are set by standard deviation in x and y axis  
*Orientation* of ellipses is set by the **correlation** between x and y  
**Pearson** correlation coefficient is calculated

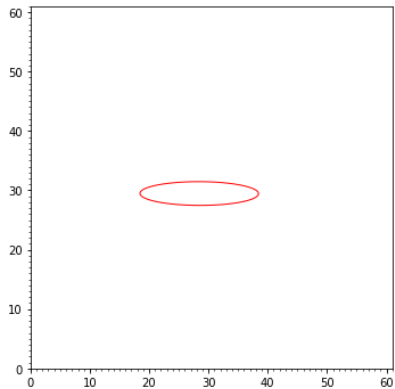
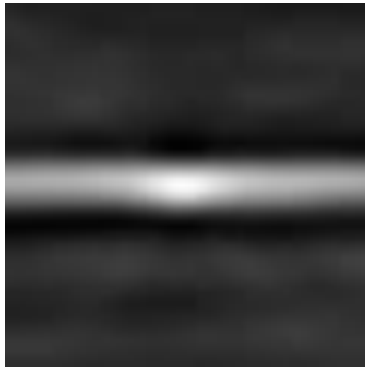
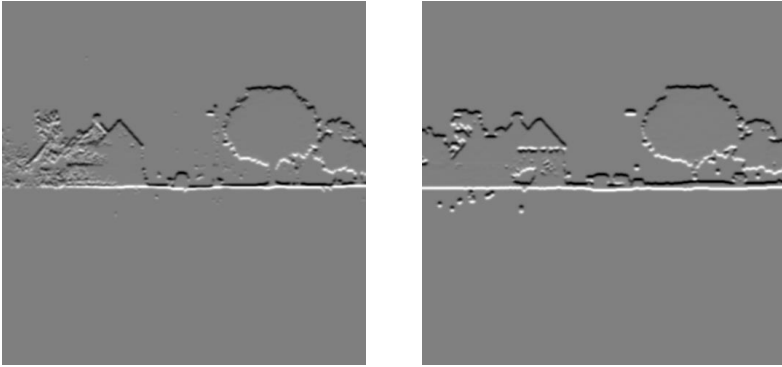
**Correlation ( $\rho$ ) is a measure of strength of association of values & has a range from -1 to +1**

$\rho = 0$  indicates no correlation

$\rho = -1$  indicates negative correlation

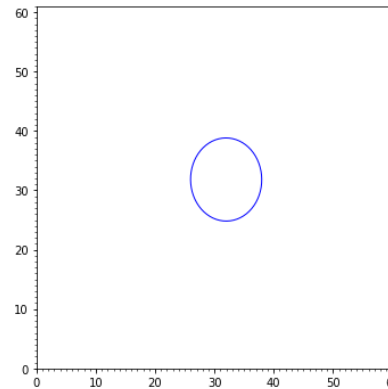
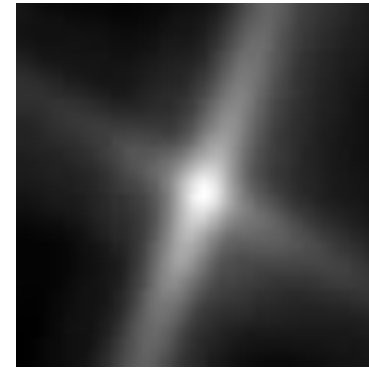
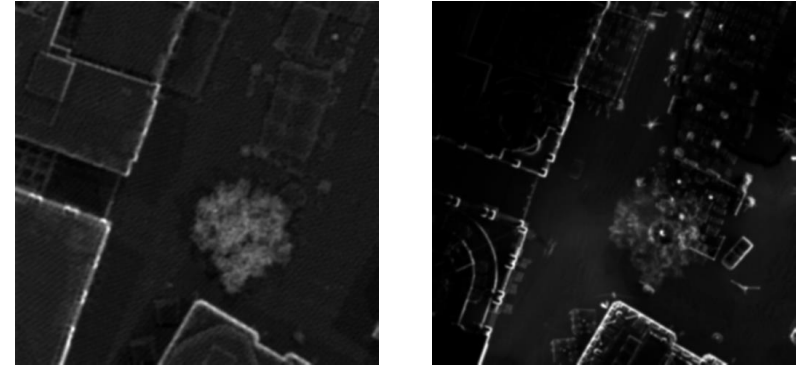
$\rho = 1$  indicates positive correlation

Correlation= 0.5



***Different***  
*transformations per x*  
*and y.*  
*The STD per row is 4*  
*& STD per column is 7*

Correlation= 0.98



***Similar*** transformations  
*per x and y*  
*The STD per row is 7 &*  
*STD per column is 6.3*

```
graph LR; A[Normalize values of images and focus only on significant part] --> B[Test response images for normal distribution in both dimensions]; B --> C[Fit a Gaussian line to individual pixel values]; C --> D[Calculation of standard deviation in pixel values]; D --> E[Calculation of correlation between axes of the image]; E --> F[Calculation of shift parameter];
```

Normalize values of images  
and focus only on significant  
part

Test response images for  
normal distribution in both  
dimensions

Fit a Gaussian line to  
individual pixel values

Calculation of standard  
deviation in pixel values

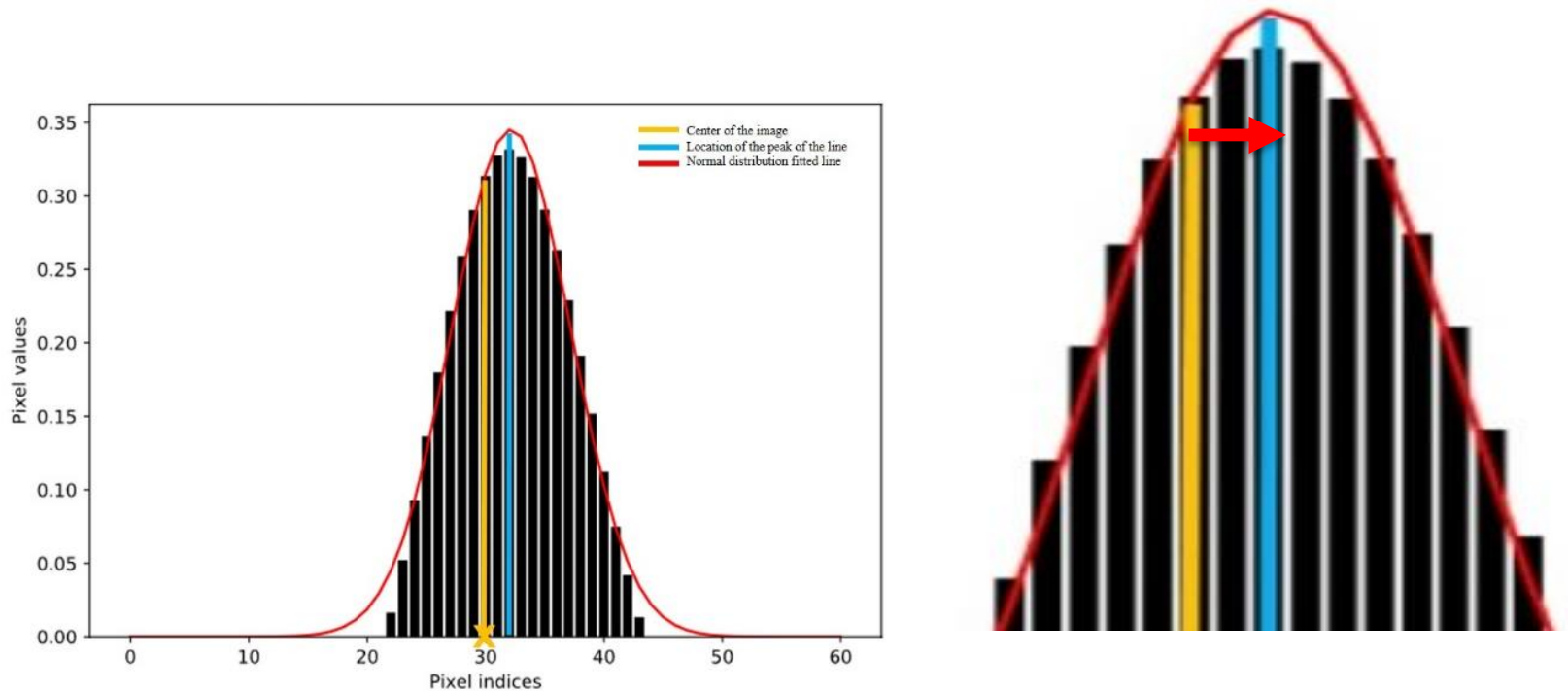
Calculation of correlation  
between axes of the image

Calculation of shift parameter

Perfect matching with **zero shift** is achieved when two input images are perfectly coincide at their centers

So

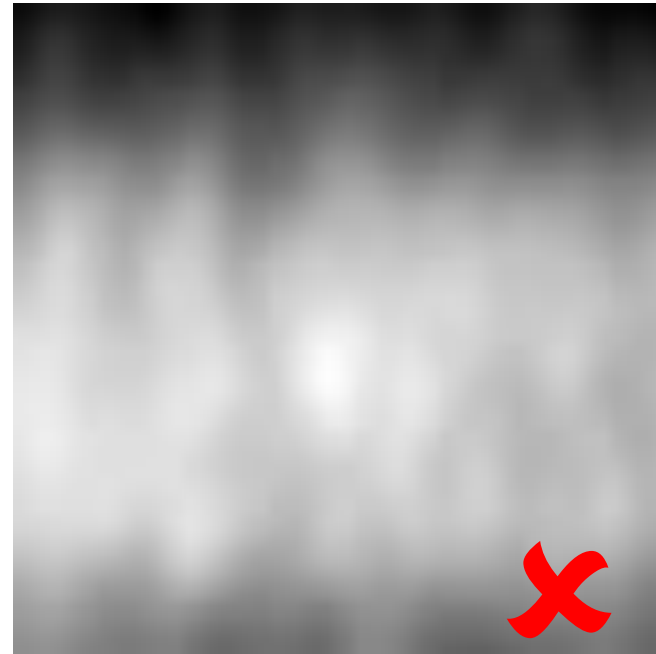
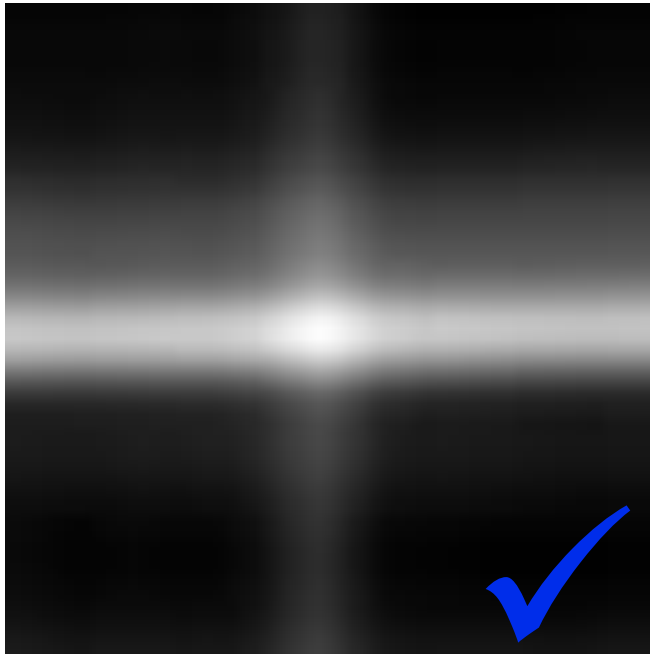
**Shift** = the **distance** between the **peak** of the line and the **center** of the image



+ shift : peak of the line is on the left side of center of the image

- shift : peak of the line is on the right side of center of the image

To what extent is it possible to estimate the **probabilistic aspects** of a result of a co-registration between aerial and mobile laser scanned point clouds?

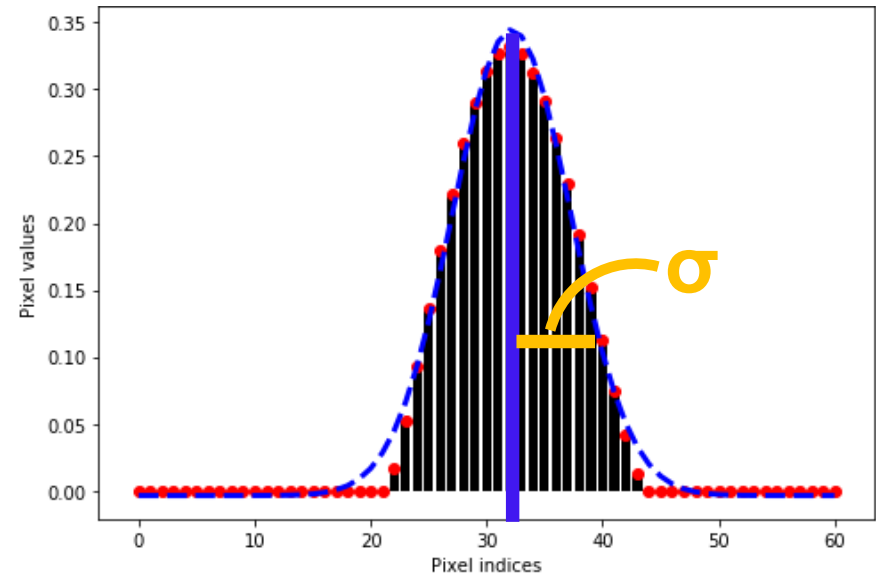
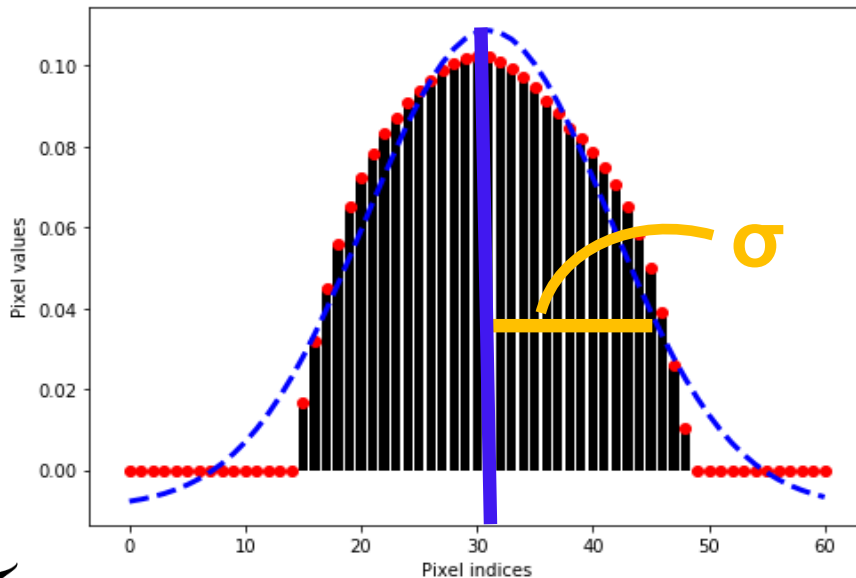


# 1. Control propagation of errors with **precision**

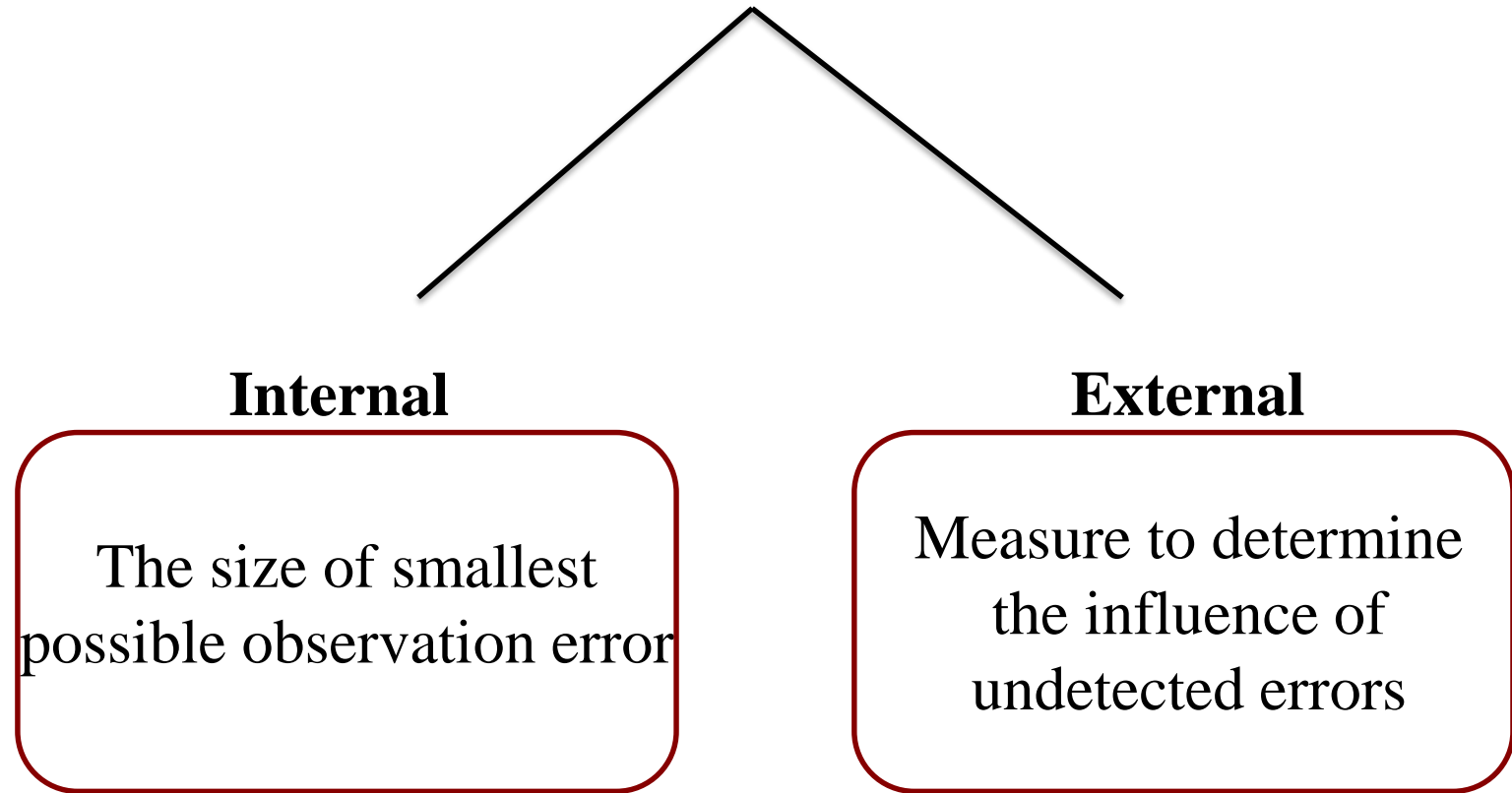
Normal distribution gives insights about how **STD is related to probabilities of errors** and:

**Large Standard deviation indicates a poor fit, low precision**

**Small standard deviation indicates a better fit, high precision**



## 2. Control sensitivity of data to errors with *Reliability*



With a 0.005 significance level, there is a 0.5% probability to have response images that do not satisfy the criteria

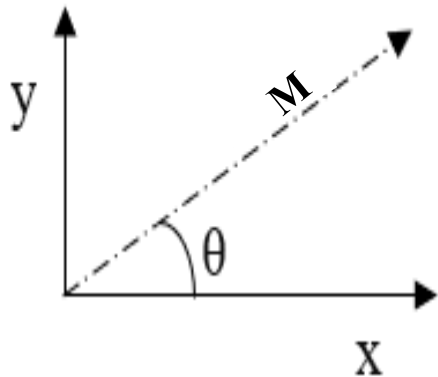
By fitting the line, the small errors are not taken into account as the residuals are minimized

# 6. Conclusions

Having the **STD**, the **reliability** and the **shift**

A local registration can be characterized.

**Arrows** are used for representing the 3 parameters.



**Length M** : *Magnitude* =  $\sqrt{x^2 + y^2}$

**Direction** :  $\theta = \text{atan}\left(\frac{y}{x}\right)$

**Thickness**: average of STDs

where:

x = shift parameter in x axis

y = shift parameter in y axis



# Results for 2 selected regions

## City center



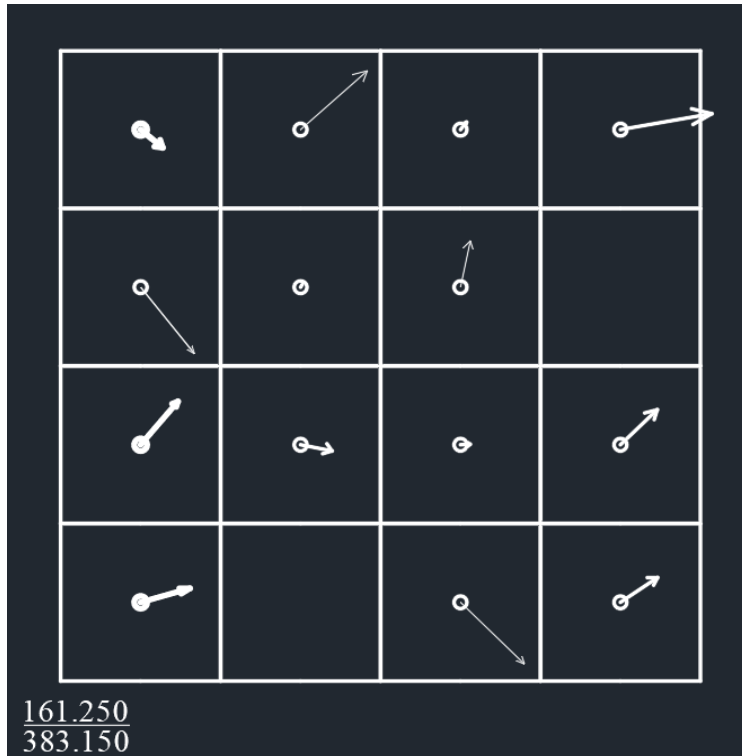
## Stadium



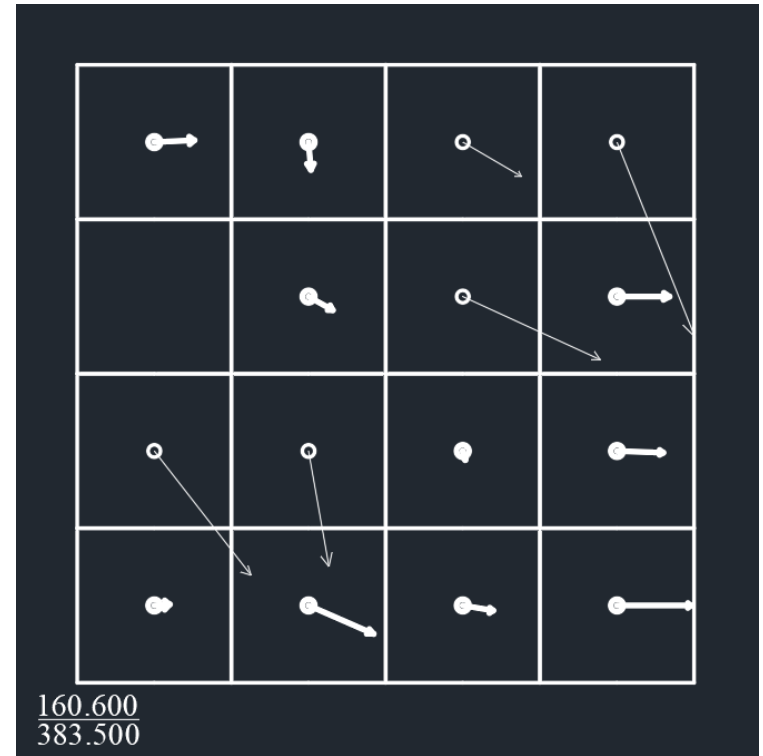
# Results for 2 selected regions

1<sup>st</sup> step

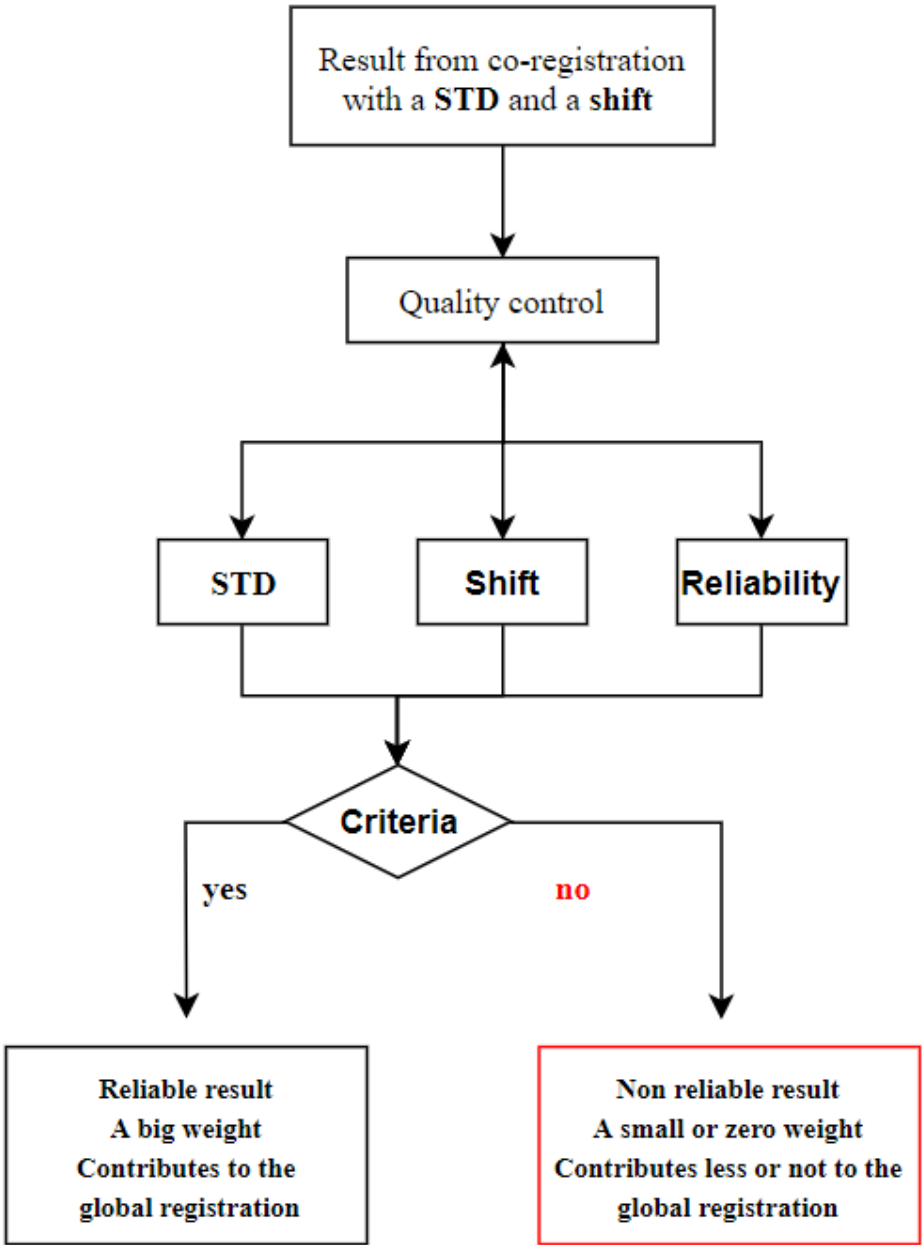
## City center



## Stadium



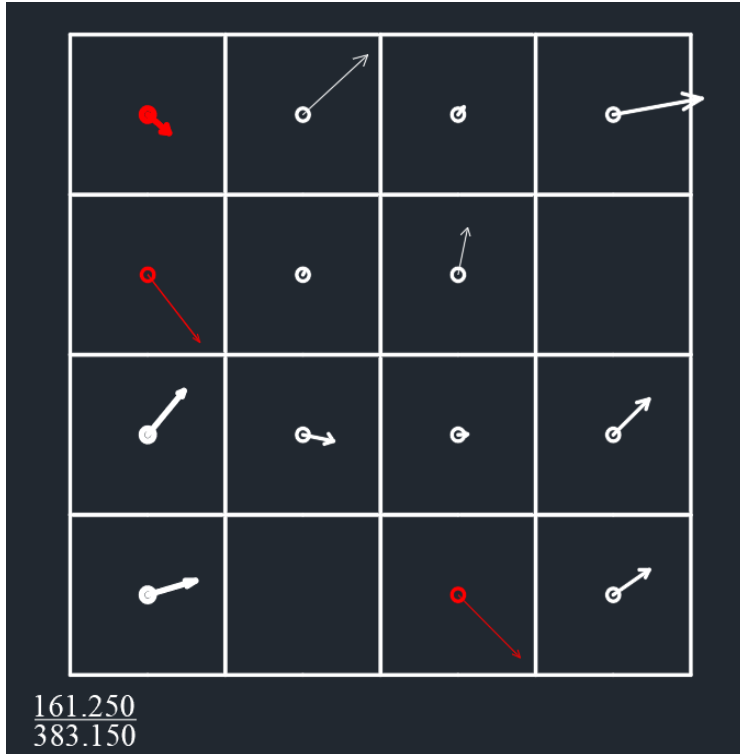
# Classify the results



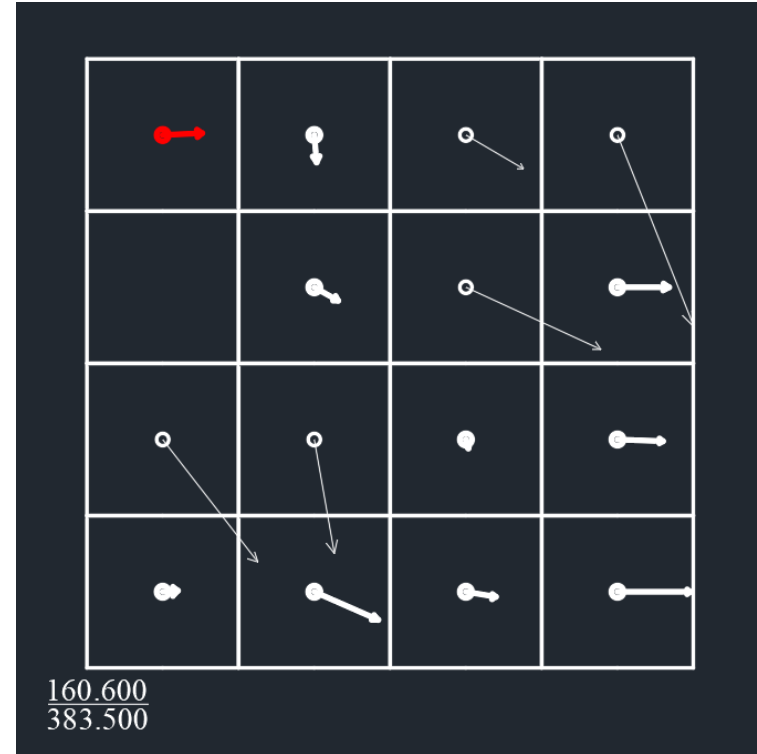
# Results for 2 selected regions

2<sup>nd</sup> step

## City center



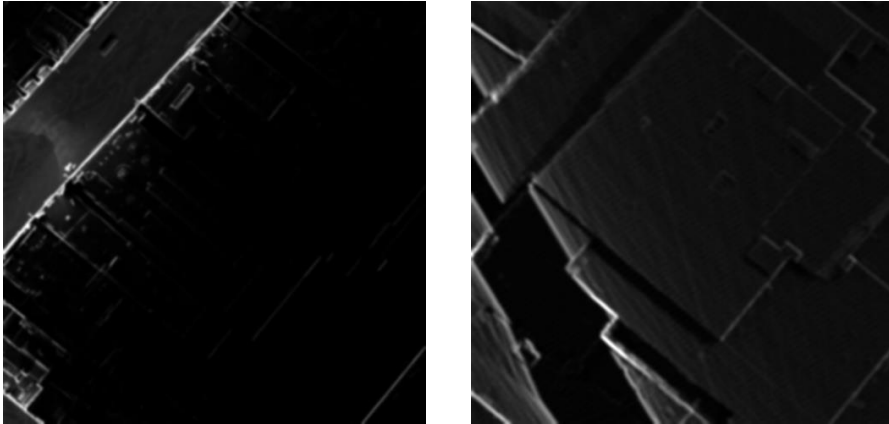
## Stadium



**Red arrows** may occur due to:

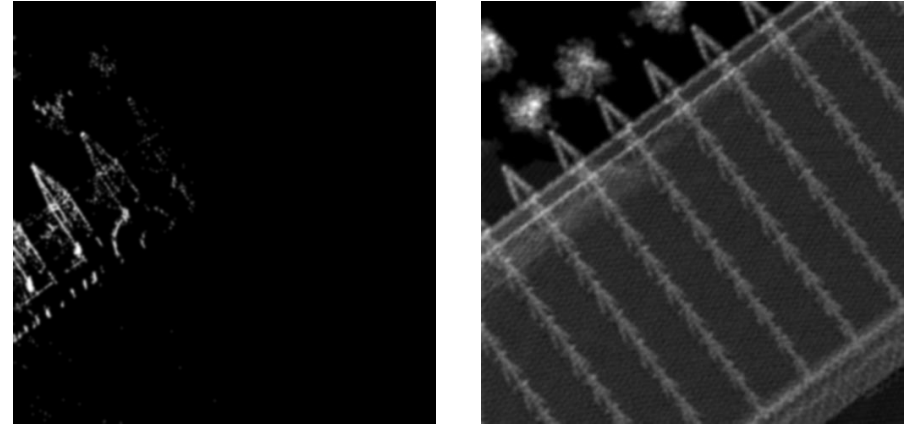
## City center

Errors in datasets



## Stadium

Limited similar information



# Summary

- ▶ A probabilistic analysis can reveal whether the requirements are met or not



Normally distributed values

Good results

Tiles are further used in analysis

Weight is assigned based on estimated quality

Result contributes to the global registration with a respective weight



Values are not normally distributed

Poor results

Tiles cannot be used for further analysis

Weight is small or zero

Result does not contribute to the global registration as it is not considered a reliable result

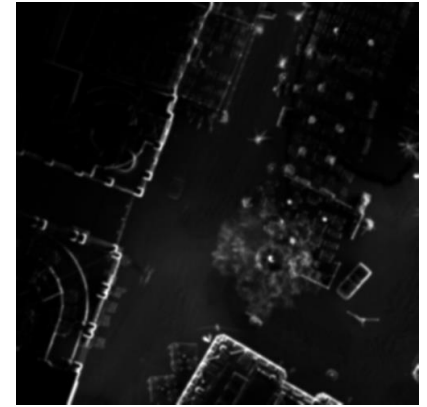
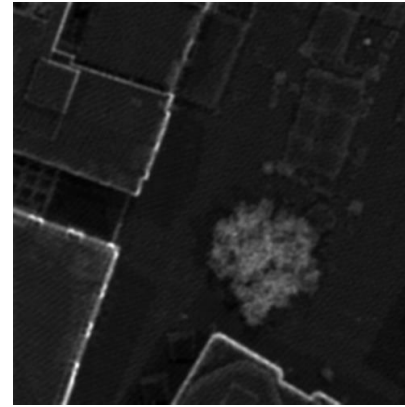
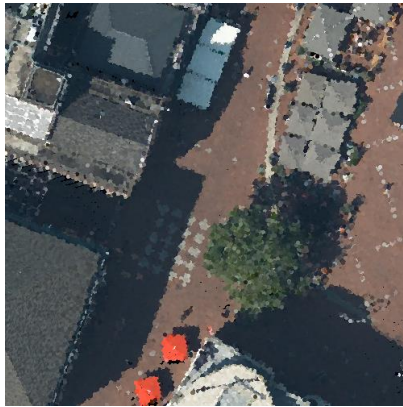
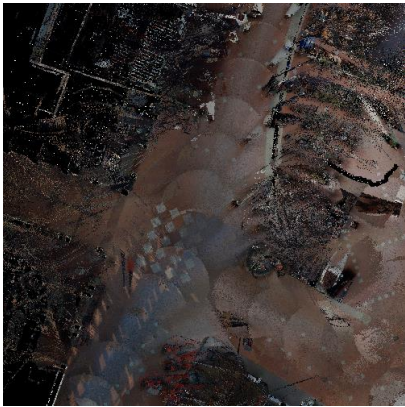
# Summary

- ▶ Set a **base** for a creation of a **reliable global registration**
- ▶ It is important to test not only the **functional** part of projects but also the **stochastic** that describes the quality and hence check if the result follows some demands
- ▶ It is a robust method as the response images are **classified** based on their quality and not on applied thresholds
- ▶ Ideal method for **large scale projects** (cities, countries) where one by one tests are not possible and automated processes are necessary
- ▶ It is a **generic approach** and it can be adopted for testing template matching between different point clouds or point clouds and topographic maps

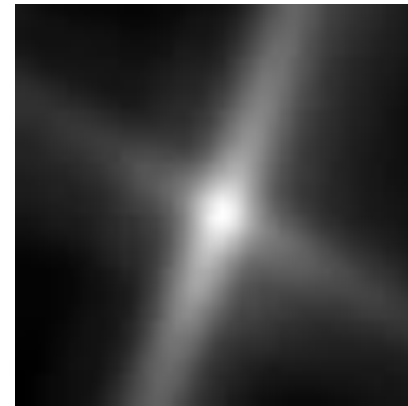
# Summary

- ▶ Different attributes are suitable for different environments

Scene with **building** structures



Color attribute



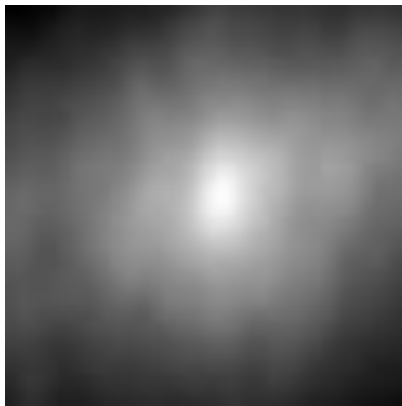
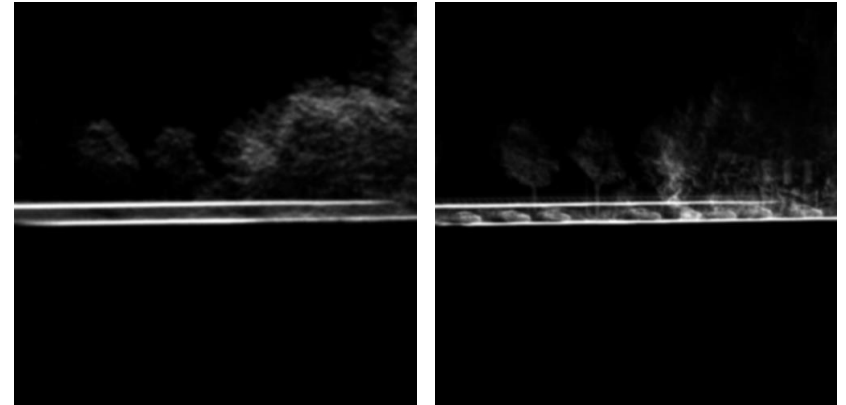
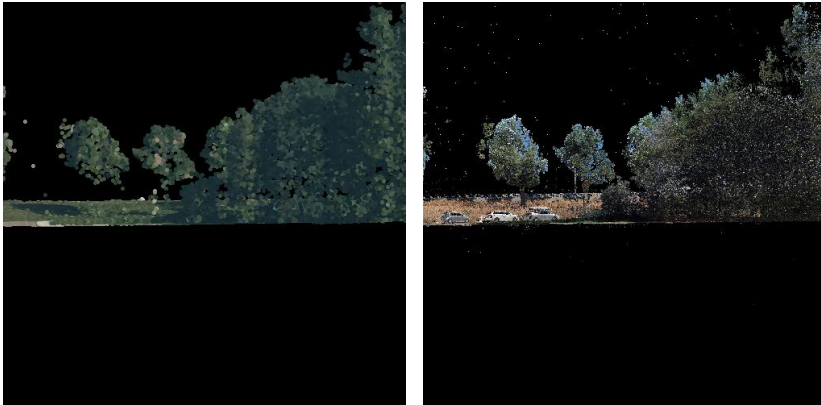
Density attribute



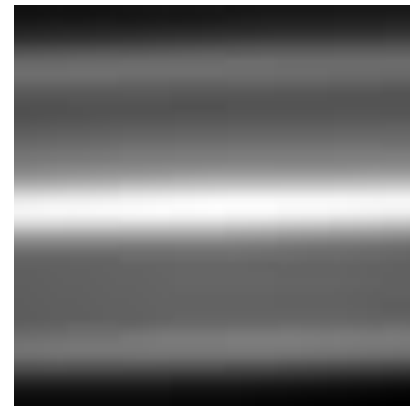
# Summary

- ▶ Different attributes are suitable for different environments

## Scene with **vegetation**



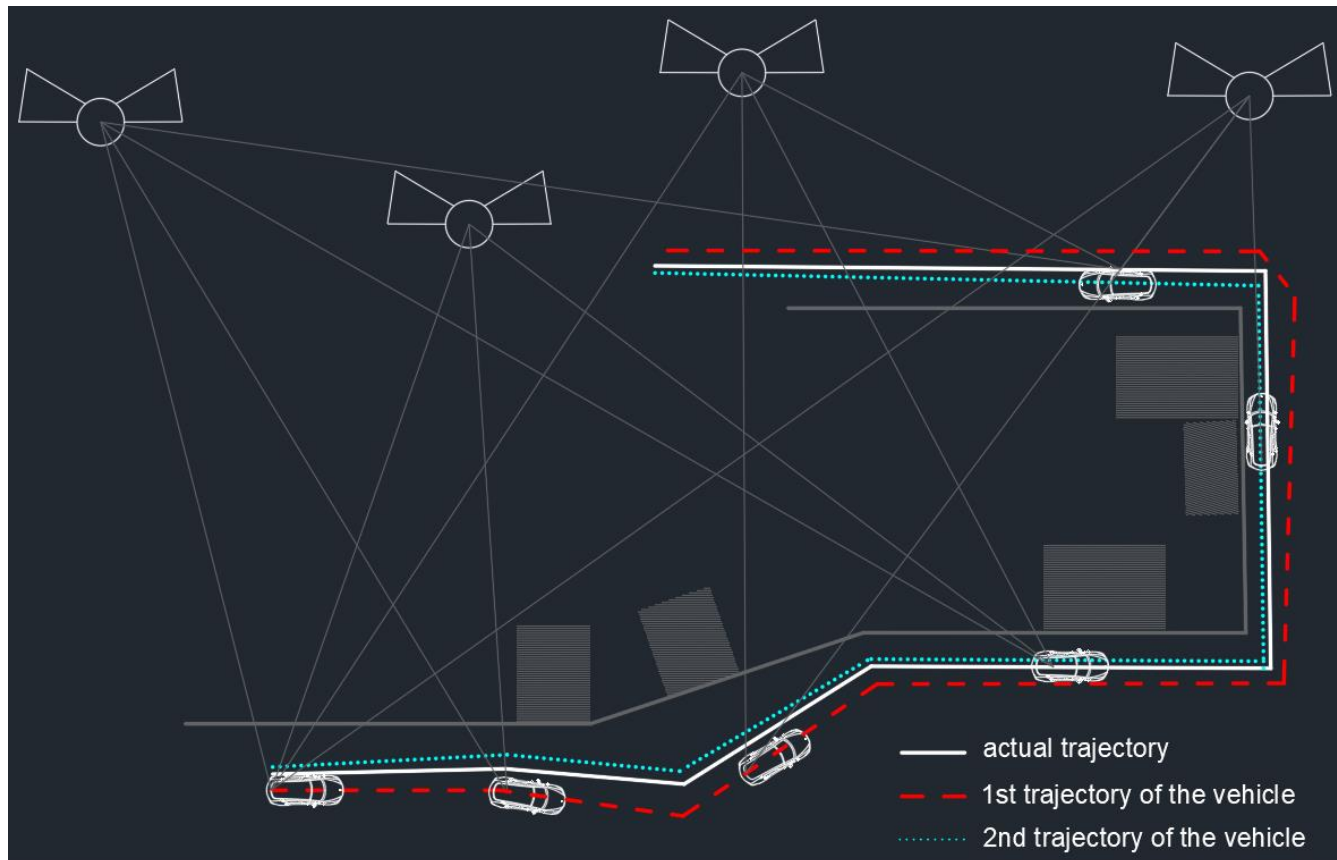
Color attribute



Density attribute

# Summary

- ▶ The calculated shift parameter for the local registrations can be compared to the calculated shift parameters for the vehicles' trajectories



# Quality matters especially in projects related to real world environment:

▶ **Navigation** tools, autonomous vehicles

▶ **Urban planning** for understanding the conditions and needs of cities (pavements, street furniture, New York City Street Tree Map) == **object recognition**

▶ **Transportation** ( assess, analyze existed infrastructure, check maintenance of roads) == **instead of traditional surveying works**

# Why Quality is important ?

**!! A result without a quality indicator is just a number without actual value**

**!! By knowing the quality, time, money & human power are saved**

# Future recommendations:

- ▶ *Consider weights based on 3 criteria for the global registration:*
  - ▶ *accuracy of the matching*
  - ▶ *quality of the matching*
  - ▶ *the accuracy of MLS and ALS data individually*

*By knowing the quality of MLS and ALS, the transformation parameters are applied to the dataset with the lowest quality*

- ▶ *Aggregate response image using different attributes for the same scene*
- ▶ *Check also how lines following different distributions fit to the pixel values*
- ▶ *Implement a model for simulating the different shift parameters and standard deviations in consecutive and neighboring tiles in order to **categorize** the tiles by following similar behaviors or not and **set respective weights***

# Thank you !

“

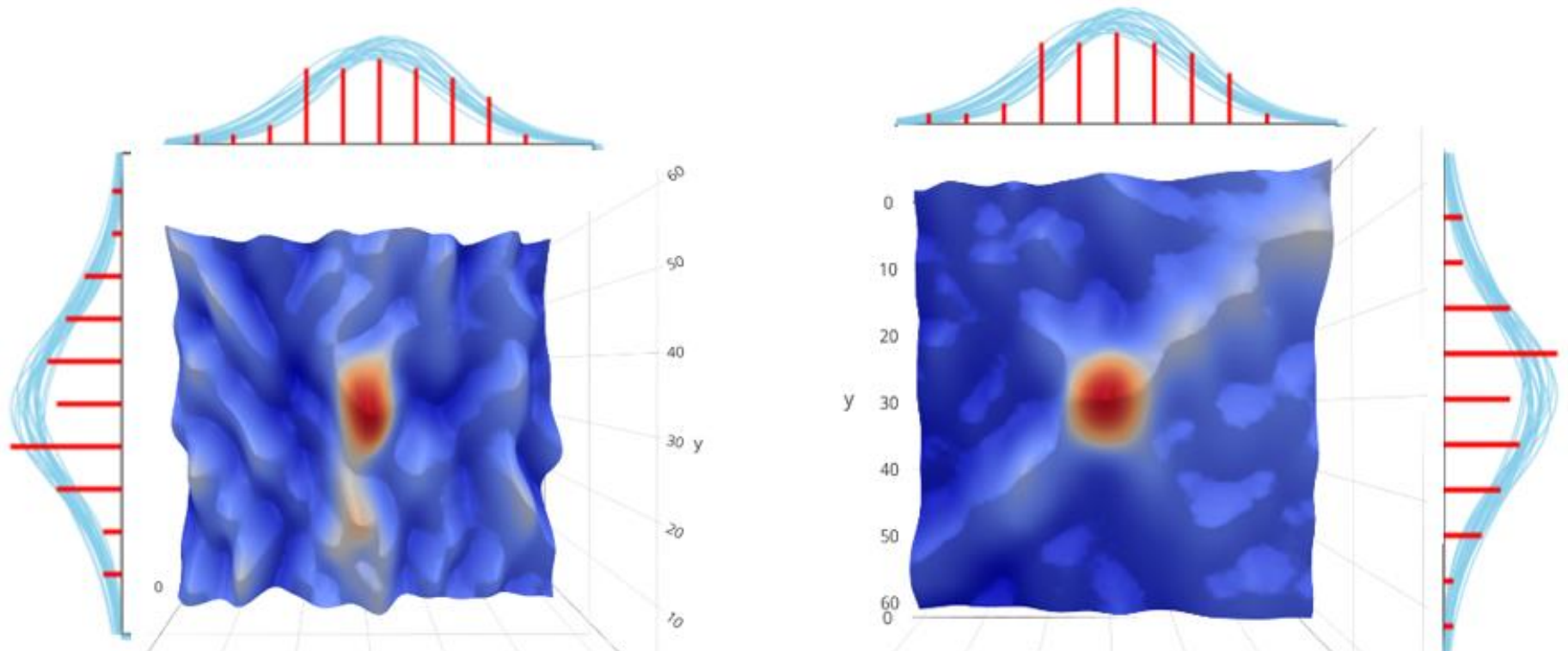
QUALITY  
IS NOT AN ACT,  
IT IS A HABIT.

”

— ARISTOTLE

# A probabilistic analysis of results of co-registration of aerial and mobile laser scanned point clouds

Anastasia Anastasiadou

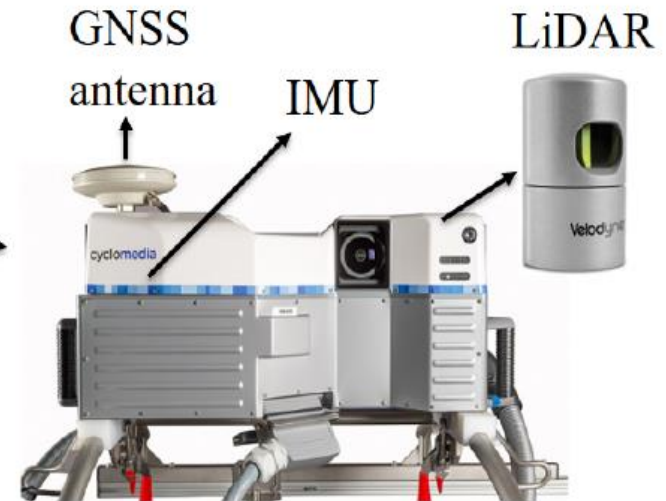


# External reliability

Consider <b>good</b> data as <b>errors</b>	lose precision
Consider <b>good</b> data as <b>good</b> data	gain precision
Consider <b>bad</b> data as <b>good</b> data	lose accuracy as errors are propagated into final result
Consider <b>bad</b> data as <b>errors</b>	lose precision but errors do not influence the good data

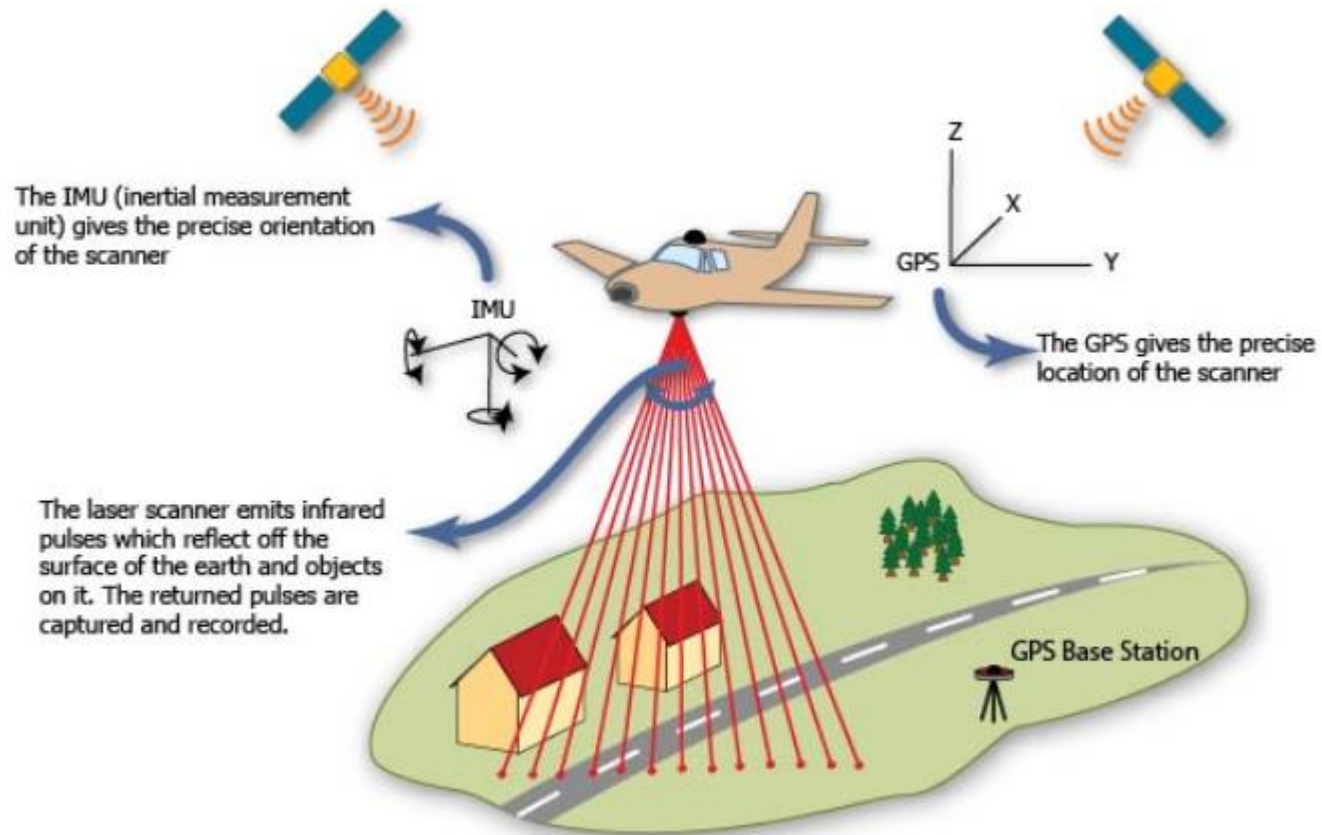


# Mobile laser scanned point clouds



A LiDAR sensor, a GNSS receiver and an IMU mounted on a mobile platform of the company Cyclomedia Technology B.V.

# Aerial laser scanned point clouds



A LiDAR sensor, a GNSS receiver and an IMU mounted on an aerial platform

Reference: University of Connecticut, Stormwise program