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Water System Examples for Control Education

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Abstract: Management of water systems is becoming more and more complex; this creates opportunities for the application of control theory. These opportunities are the subject of a course on operational water management given to students of the water management department, Delft University of Technology, over the past 15 years. Traditional examples in control theory courses are taken from industry and do not easily map to water systems, so examples were developed that use water systems to illustrate control theory concepts. This provided the students with a link between control theory and water management practice.

Keywords: Control education, Teaching curricula developments for control and other engineers, Water quality and quantity management, Systems with delay, Sampled data systems

1. INTRODUCTION

Water is an essential resource for society. It is used in households, in industry, in irrigation, and for power generation; it provides transport routes in the form of rivers and canals. At the same time water is a potential natural hazard. City streets should not flood, and agricultural land should not turn into a marsh, so transport of water away from given areas (drainage) is as important as the supply side. The demands placed on water systems are increasing due to growing scarcity and because of the rising value that is at risk in case of flooding. Shortages on the one hand and flooding on the other hand are less and less acceptable. As a result, management of water systems is becoming more and more complex. Control theory is one way to manage this complexity. It is being used in the management of sewer systems (van Nooijen et al., 2012; van Nooijen and Kolechkina, 2013, 2014, 2018b,a; García et al., 2015), in irrigation systems (Weyer, 2008), for inland waterways (Kasper et al., 2018), and for drainage (Hadid et al., 2019; Nederkoorn et al., 2012). These systems are quite interesting from a control point of view: they usually contain multiple delays; they are sampled data systems with relatively long control time steps; saturation occurs regularly; reservoirs act as integrators and produce phase shifts; non-linear components are common. However, these systems also have properties not usually encountered in other fields of control:

- Water systems contain moving water and are continuously supplied with more water. This means large amounts of kinetic energy are present in the system. Quickly closing a sluice gate or abruptly shutting down a pump station may result in something similar to an irresistible force meeting an, up to that time, immovable object. A 10 km irrigation or drainage canal delivering $50\,\mathrm{m}^3/\mathrm{s}$ with a conservative flow velocity of $1\,\mathrm{m/s}$ has 0.5×10^9 kg (or about 2700 classical Boeing 747 planes) of mass in motion.
- While it is almost never possible to take control time steps arbitrarily small, the lower bound on control time steps in water management tends to be very large indeed. A $260\,\mathrm{m}^3/\mathrm{s}$ pump station, such as found at IJmuiden (Netherlands), takes time and energy to start up and time to stop; this is not something that can be done every 30 seconds or even every 5 minutes. The same applies when adjusting an 80 year old moveable steel gate that is 23 meters wide; three such gates are installed at a sluice complex in the river Meuse at Borgharen (Netherlands).
- The capacity of the actuators to change the system behaviour tends to be limited. While the actuators tend to be big in an absolute sense, usually their ability to affect the state of the system is small. Typical examples are a polder system or a combined sewer system in a city in flat terrain. Both systems depend on pumps to transport water, but economic principles limit pump size. In both systems, storage is used to give the pumps time to deal with large precipitation events.
- Water systems tend to have orders of magnitude lower investment in control simply because the relatively low income per unit ‘product’. Even small systems cover many square kilometres. Let us take the water board of Delfland (Netherlands) as an example. It is responsible for surface water management, waste water treatment, and drainage in an area of $406\,\mathrm{km}^2$. The area contains 4293 km of canals, 1834 weirs of which 151 are automated, 194 small and 6 large pump stations. The 6 large stations have a total capacity of $100\,\mathrm{m}^3/\mathrm{s}$. The area has 1.2 million inhabitants. The income of the waterboard is approximately 250 million Euro per year (Hoogheemraadschap van Delfland, 2018). In the Netherlands, water boards,

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municipalities, water companies, provinces, and the national government spent a total 7×10^9 Euro (around one percent of the Gross Domestic Product) on water related tasks (Ligthart and Dekking, 2017). In many industries with a high degree of automation, turn-over per company is on the order of 10^{11} Euro. Hopefully, these numbers illustrate that the budget for automation and control in water management is somewhat lower than in the corporate world, while the systems to be controlled are somewhat bigger.

These aspects make it hard to use standard control theory examples. Moreover, a course on automatic control for students with a civil, hydraulic or environmental engineering background cannot assume prior exposure to control theory, but at the same time the water systems they need to deal with involve complex control theoretic concepts and methods. In response to these problems, an existing course on operational water management was redesigned to accommodate the more prominent role of automatic control in water management. Over a period of 15 years the course was refined, the treatment of control theory was extended, new examples were developed, and some scheduling changes were made: examples now precede associated theory where possible, and lectures and computer labs alternate in shorter blocks.

Students travel along different routes towards understanding; they use qualitative reasoning based on physics, examine the equations, and experiment with simulated or real water systems. One thing they have in common is that they learn best by doing. That is they learn by doing calculations and observing and modifying the behaviour of simulated systems in computer or laboratory experiments.

In the remainder of this paper, a description of the course will be given, the choice of subject matter will be motivated, examples will be presented that are used in the course and in the associated computer assignments, and observations related to the role of the examples in the course will be discussed.

2. GENERAL DESCRIPTION OF THE COURSE

The course 'operational water management' is an elective course that is part of the master's degree programme in Civil Engineering of Delft University of Technology, mainly for those students that specialize in water management. Please note that in the Dutch academic system most if not all students go on to a M.Sc. degree. There were about 25 to 30 students each year. After completing the course the students should be able to:

- Describe the interaction between controllers and processes qualitatively.
- Describe, quantify, and explain process behaviour for simple controlled water systems on the basis of observations.
- Describe and apply the Model Predictive Control (MPC) algorithm.
- Perform calculations needed to determine whether or not a given combination of controller and process is stable for a number of simple water resource systems. Use the results to design a controller that renders the controlled system stable.

The course corresponds to 4 credits in the European Credit Transfer and Accumulation System. It consists of 28 lectures of 45 minutes and 14 computer lab sessions of 2 hours. The students need to complete 7 assignments and hand in reports on the first 6. The final assignment is discussed during the final computer lab. The computer lab is based on Simulink (2017). The assignments are done in pairs, and students are allowed to consult other pairs. Each pair hands in a report which is used for formative assessment. Feedback is given, and students may be required to make improvements. A final written exam functions as a summative assessment. Grading rules for the exam are established before grading starts, and absolute grading is used. Usually about 70% of the students pass the written exam the first time. Interactions with the students have shown that the labs are essential to the learning process. The labs allow students to gain a practical understanding of the concepts presented in the lectures by interacting with block diagrams of water systems that illustrate the consequences of concepts such as sampling, delays, and feedback.

As the course is an elective, the students taking the course are usually highly motivated. While the students are quite pleased to see that the mathematics they studied can be put to good use, they are less enthusiastic about long theoretical discourses. Therefore it is essential to have water related examples for all aspects of control that are relevant to water systems. Currently the examples used in the course cover:

- Continuous, discrete, and hybrid systems.
- Treatment of Single-Input-Single-Output (SISO) and Multi-Input Multi-Output (MIMO) systems.
- Effects of delays, phase shifts, controller saturation, and sampling.
- Aspects of MPC.

Some of these examples will be presented in the next section.

3. DESIGN OF THE COURSE

For civil or environmental engineering students, a link to practical systems is essential. In the process of translating learning goals into course materials and activities, there were several choices to be made:

- Use only the time domain or use both time and frequency domain? The advantage of the frequency domain is that many calculations connected with the system response can be done by following a recipe. The disadvantage is that the Laplace transform (for continuous systems) or the z-transform (for discrete systems) need to be either taught or included in the course prerequisites. For our course we elected to use the frequency domain only for the continuous systems.
- Concentrate on SISO Systems or give equal time to MIMO Systems? Almost all water systems are MIMO, so these needed to get attention. Especially given the increasing importance of managing water quantity and water quality at the same time.
- Analyse non-linear systems: yes or no? If yes, then which method to use for stability analysis: the indi-

rect (first) method of Lyapunov or the direct (second) method of Lyapunov? While the direct method is more powerful in principle, the indirect method allows the use of linear systems theory, which may be easier to use for students that do not plan to specialize on non-linear systems theory. For our course we elected to teach linear systems theory, show how to linearise systems, and explain how this is linked to stability of non-linear systems.

Most standard textbook examples used to teach the concepts that would have to be taught based on these choices do not easily map to water systems. As a result of the first choice, the Laplace transform needed to be in the course because most students had no experience with it. This in turn meant that a simple water system was needed to illustrate its application. For this a simple reservoir was used. Combination with a PID controller then allows successively more complex systems as exercise and illustration material. The second choice was easily supported by basing examples on sequences of canals or reservoirs. Given the third choice, it was essential to have water management examples of non-trivial linear systems related to water systems that could be used to discuss linear systems theory. In this case non-trivial means systems that are unstable for certain values of the parameters. The examples should also illustrate phase shifts and the effects of time delays. Systems that could be linearised were, of course, also needed, but this presented less of a problem, as gates and weirs contain non-linearities.

4. EXAMPLES

4.1 A simple reservoir with continuous level control

The first example is intended to introduce the language of control theory in the context of water management. It consists of a reservoir with fixed surface area where inflow and outflow act on the stored volume without delay, and the water level in the entire reservoir is approximately the same. Care should be taken to present this to the students as simplified model of a physical system, for instance, a pond or a small lake fed by a stream and discharging over a broad crested weir with fixed crest level in free flow to another stream. Such a pond can be modelled as a simple integrator

$$\dot{h}(t) = \frac{q_{\text{in}}(t) - q_{\text{out}}(t)}{a} \tag{1}$$

where a is the surface area; h is the water level; q_{in} is the inflow, and q_{out} is the outflow. The flow rate q_{w} over the weir is modelled by

$$q_{\text{w}} = b c_{\text{w}} \sqrt{\mathfrak{g}} \left(\frac{2}{3} (h_{\text{up}} - z) \right)^{3/2} \tag{2}$$

where c_{w} is a constant that depends on the construction of the weir; h_{up} is the water level upstream of the weir; z is the crest level, and $\mathfrak{g}$ is the gravitational acceleration (approximately 9.8m/s^2). A typical broad crested weir is shown in Fig. 1. The fixed weir acts as a non-linear proportional controller. Aspects such as run-off directly into the pond, open water evaporation from the pond, precipitation directly into the pond, and exchange of water with the ground water can be incorporated by adjusting q_{in} to include them.

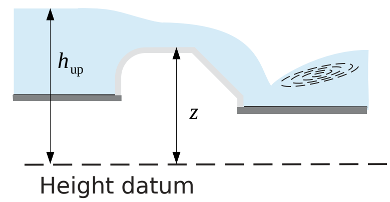

Fig. 1. A broad crested weir

Once the fixed weir is understood, it is possible to introduce a moveable crest and add a simple linear proportional controller. If it is desirable to focus more on the controller, then it is possible to replace the weir by a variable speed pump. Given the simple nature of the system, the role of the different terms of a Proportional Integral Differential (PID) controller can be demonstrated easily. A time varying setpoint can be brought in by changing the pond or lake to a lagoon that represents an area where tides are to be simulated. The one didactic disadvantage of this example is that even very strong proportional feedback does not cause stability problems.

4.2 A simple reservoir with delay

By changing the context of the reservoir, a delay can be introduced. The new context is that of a reservoir for on-demand supply of irrigation water that is fed by a transport canal. The free surface flow in the transport canal causes a delay between a change in inflow into the canal and the arrival of the effects of that change at the reservoir. Two research directions are being pursued at the moment for control of open channel flow: on the one hand direct study of control of the partial differential equations through their boundary conditions (Gugat and Leugering, 2003; Hayat and Shang, 2019; Bastin and Coron, 2016), and the approximation of a canal by a simpler model on the other hand (Schuurmans et al., 1995; Litrico and Fromion, 2009). In the course the second approach is used.

The transport delay depends on the flow state and the magnitude of the change, but this can be neglected in a first rough approximation. The model for this system is

$$\dot{h}(t) = \frac{q_{\text{in}}(t-\tau) - q_{\text{out}}(t)}{a} \tag{3}$$

where τ is the delay. If the inflow is assumed to be controlled directly by a PI(D) controller, for instance, when the inflow is through a pump where the flow rate can be set, then

$$q_{\text{in}}(t) = -\frac{a}{\tau} c_{\text{P}} (h(t) - h^*) - \frac{a}{\tau^2} c_{\text{I}} \int_{\hat{t}=0}^{t} \left(h(\hat{t}) - h^* \right) d\hat{t} \tag{4}$$

where h^* is the (constant) setpoint and the coefficients c_{P} and c_{I} are dimension free. The stability can now be derived using the Laplace transform (Silva et al., 2005). With state $x(t) = h(t) - h^*$, disturbance $d(t) = q_{\text{out}}(t)/a$, and initial value $x(0) = 0$, the transfer function is

$$\frac{\mathcal{L}x(s)}{\mathcal{L}d(s)} = -\frac{\tau^2 s}{(s\tau)^2 - (s\tau)\exp(-s\tau) c_{\text{P}} - \exp(-s\tau) c_{\text{I}}} \tag{5}$$

For $c_{\text{P}} = c_{\text{I}} = 0$ the system is stable. For $c_{\text{P}} > 0, c_{\text{I}} = 0$ the system is asymptotically stable if and only if

$$0 < c_{\mathrm{P}} \frac{\tau}{a} < \pi/2$$

This example introduces a delay and shows that stability is something that you need to design for.

4.3 Simple reservoirs in series, one way interaction

A simple reservoir acts as an integrator. If three or more are combined in series, then the resulting system can be used to show the effects of phase changes that exceed π. The outflow of the reservoirs is controlled by a proportional (P) controller based on the deviation from a desired water level in such a way that the separate reservoirs are stable systems. An additional P controller is then added to regulate the inflow into the first reservoir based on the level in the last reservoir. This brings into play the phase changes.

One way interaction occurs when the actuator is a sluice gate or weir in free flow or a pump. In case of identical reservoirs, actuators, and controllers, and after appropriate definition of the state x and disturbance d, and appropriate scaling of the variables, the system will be of the form

$$\begin{aligned} \dot{x}_1 &= -cx_1 + d_1 - c_{\mathrm{o}} x_n \\ \dot{x}_2 &= cx_1 - cx_2 + d_2 \\ &\vdots \\ \dot{x}_n &= cx_{n-1} - cx_n + d_n \end{aligned} \tag{6}$$

or in matrix form for a four reservoir system

$$\dot{x} = c \begin{bmatrix} -1 & 0 & 0 & -c_{\mathrm{o}}/c \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} + d \tag{7}$$

Stability can now be studied either by using the Laplace transform and the rules for combining transfer functions, or by determining the eigenvalues of the corresponding matrix either numerically or analytically. Moreover, the system is an example of a MIMO system, but can still be studied as SISO, if some variables are eliminated.

Use of weirs in free flow (downstream water level always below the crest) to separate the reservoirs allows the introduction of the concept of saturation as such weirs cannot implement negative flows. Use of pumps with limited capacity between the canals also allows experiments with saturation.

4.4 Simple reservoirs in series, two way interaction

In water management, flow rate is often controlled by sluice gates in submerged flow. The flow is then governed by

$$\begin{aligned} &q_{\mathrm{g}}\left(h_{\mathrm{up}}, h_{\mathrm{down}}, w\right) = \\ &b c_{\mathrm{g}} w \,\mathrm{sgn}\left(h_{\mathrm{up}} - h_{\mathrm{down}}\right) \sqrt{2\mathfrak{g}\left|h_{\mathrm{up}} - h_{\mathrm{down}}\right|} \end{aligned} \tag{8}$$

where h_{up} is the water level upstream of the gate; h_{down} is the water level downstream of the gate; $w \geq 0$ is the height of the gate opening; b is the width of the opening; c_{g} is a gate dependent constant, and μ is the contraction coefficient. Drowned flow occurs only when the gate opening, the upstream and downstream water level meet specific conditions. If we now consider reservoirs in series separated by drowned gates (Fig. 2), then both upstream and downstream water level affect the flow through a gate between two reservoirs. If we linearise the sluice gates around the operating point $h_{\mathrm{up}} = h^*_{\mathrm{up}}$, $h_{\mathrm{down}} = h^*_{\mathrm{down}}$ and $w = w^*$ where

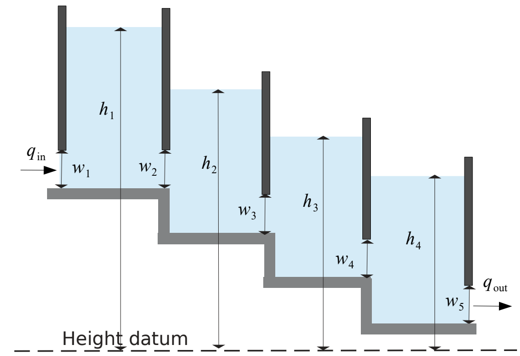

Fig. 2. Reservoirs in series separated by drowned gates

$$q_{\mathrm{g}}\left(h^*_{\mathrm{up}}, h^*_{\mathrm{down}}, w^*\right) = q^* > 0 \tag{9}$$

then

$$\begin{aligned} q_{\mathrm{g}}\left(h_{\mathrm{up}}, h_{\mathrm{down}}, w\right) &= q^* + \frac{q^*}{w^*} u \\ &+ \frac{q^*}{h^*_{\mathrm{up}} - h^*_{\mathrm{down}}}\left(x_{\mathrm{up}} - x_{\mathrm{down}}\right) + O\left(x^2_{\mathrm{up}}, x^2_{\mathrm{down}}, u^2\right) \end{aligned} \tag{10}$$

where $x_{\mathrm{up}} = h_{\mathrm{up}} - h^*_{\mathrm{up}}$, $x_{\mathrm{down}} = h_{\mathrm{down}} - h^*_{\mathrm{down}}$, $u = w - w^*$. The result is a linear system that can again be analysed either in the time domain using matrix eigenvalues or in the frequency domain. Now assume identical canals and gates, and controllers of the form

$$u = c_{\mathrm{P}} x_{\mathrm{up}} \tag{11}$$

on the gates. After appropriate definition of the state x and disturbance d, and appropriate scaling of the variables, the linearised system will be of the form

$$\begin{aligned} \dot{x}_1 &= c_{\mathrm{s}}\left(0 - x_1\right) - c_{\mathrm{s}}\left(x_1 - x_2\right) - c_{\mathrm{s}} c_{\mathrm{c}} x_1 + d_1 \\ \dot{x}_2 &= c_{\mathrm{s}}\left(x_1 - x_2\right) + c_{\mathrm{s}} c_{\mathrm{c}} x_1 - c\left(x_2 - x_3\right) - c_{\mathrm{s}} c_{\mathrm{c}} x_2 + d_2 \\ &\vdots \\ \dot{x}_n &= c_{\mathrm{s}}\left(x_{n-1} - x_n\right) + c_{\mathrm{s}} c_{\mathrm{c}} x_{n-1} - c_{\mathrm{s}} x_n - c_{\mathrm{s}} c_{\mathrm{c}} x_n + d_n \end{aligned} \tag{12}$$

where the level upstream of the first gate and downstream of the last gate are fixed at the operating point. This gives as matrix form for a four reservoir system

$$\dot{x} = c_{\mathrm{s}} \begin{bmatrix} -\left(2 + c_{\mathrm{c}}\right) & 1 & 0 & 0 & 0 \\ 1 + c_{\mathrm{c}} & -\left(2 + c_{\mathrm{c}}\right) & 1 & 0 & 0 \\ 0 & 1 + c_{\mathrm{c}} & -\left(2 + c_{\mathrm{c}}\right) & 1 & 0 \\ 0 & 0 & 1 + c_{\mathrm{c}} & -\left(2 + c_{\mathrm{c}}\right) & 1 \end{bmatrix} + d \tag{13}$$

Stability can now be studied by determining the eigenvalues of the corresponding matrix either numerically or analytically.

4.5 Quality and Quantity management

In Fig. 3 a polder is depicted where seepage into the polder contains a variable concentration of salt. A polder pump controls the water level, and an automated inlet is used to manage the salt concentration. A simple model is

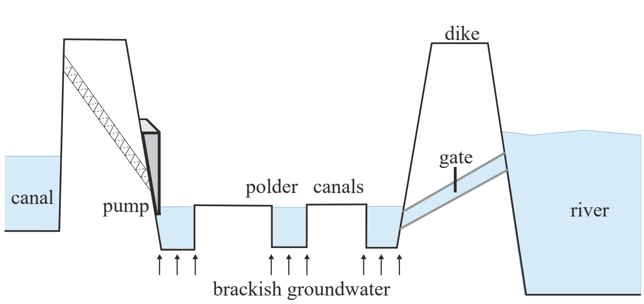

Fig. 3. Polder with salt intrusion

$$\frac{dv}{dt} = q_{\text{seep}}(t) + q_{\text{flush}}(t) - q_{\text{out}}(t) \quad (14)$$

$$\frac{dm}{dt} = q_{\text{seep}}(t)\, c_{\text{seep}}(t) - q_{\text{out}}(t)\, c_{\text{out}}(t) \quad (15)$$

where v is the volume in the polder canals; m is the total mass of salt in the polder; q_{seep} is the seepage of brackish ground water into the polder; q_{flush} is the flow rate of water through the gate used to lower the salt concentration in the polder canals; q_{out} is the water pumped out to remove salt and maintain an acceptable water volume v^* in the polder canals; c_{seep} is the concentration of salt in the groundwater, and c_{out} is the concentration of salt in the water that is pumped out. Full mixing is assumed, so c_{out} equals the concentration in the polder canals. If c^* represents the maximum allowed salt concentration then a simple controller model would be

$$q_{\text{flush}}(t) = c_{\text{P,flush}} \max\left(\frac{m(t)}{v(t)} - c^*, 0\right) \quad (16)$$

$$q_{\text{out}}(t) = c_{\text{P,out}}(v - v^*) \quad (17)$$

The example is used in one of the first assignments. Students are asked what type control is applied to (a) the volume and (b) the salt concentration. Options are: open loop, feed forward, or closed loop. It also serves to show the students a non-trivial MIMO system.

4.6 A simple reservoir with discrete-time control

In most if not all water systems, the actuators are operated at given time intervals. There are three main reasons for this:

- The actuators are large and need time to move to a new setting.
- Changing the actuator setting too often leads to excessive wear and tear.
- There are operational limits on how often commands can be sent to the actuator.

It turns out that this limitation makes the simple reservoir system much more interesting. To provide a first introduction to the problem, a discrete system is derived from the simple reservoir problem as follows. The actuator is replaced by a pump. This allows a fully discrete treatment. A control time step of τ_{s} is used. The discrete system state $x_{\text{p}}(k)$ is the deviation from setpoint $h(k\tau_{\text{s}}) - h^*$ at time $k\tau_{\text{s}}$. Its time evolution is given by

$$h([k+1]\tau_{\text{s}}) = h(k\tau_{\text{s}}) + \int_{t=k\tau_{\text{s}}}^{(k+1)\tau_{\text{s}}} \frac{q_{\text{in}}(t)}{a} - \frac{q_{\text{out}}(t)}{a} dt \quad (18)$$

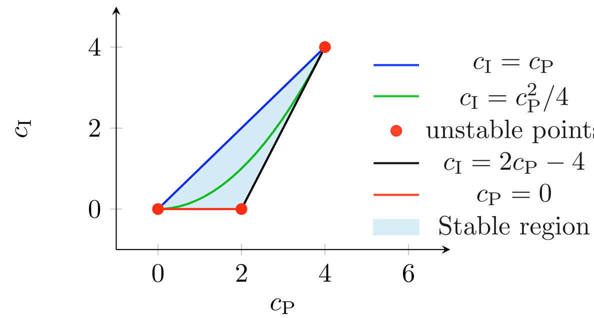

Fig. 4. Stability region for PI controlled discrete reservoir

If $q_{\text{out}}(t)$ is constant on the time interval $(k\tau_{\text{s}}, (k+1)\tau_{\text{s}})$, as it would be for a discrete time controller, then this can be simplified. As discrete process, we get

$$x_{\text{p}}(k+1) = x_{\text{p}}(k) + \frac{\tau_{\text{s}}}{a} d_{\text{p}}(k) - \frac{\tau_{\text{s}}}{a} u_{\text{p}}(k) \quad (19)$$

$$y_{\text{p}}(k) = x_{\text{p}}(k) \quad (20)$$

where the disturbance is

$$d_{\text{p}}(k) = \frac{1}{\tau_{\text{s}}} \int_{t=k\tau_{\text{s}}}^{(k+1)\tau_{\text{s}}} q_{\text{in}}(t)\, dt \quad (21)$$

and the input is

$$u_{\text{p}}(k) = \frac{1}{\tau_{\text{s}}} \int_{t=k\tau_{\text{s}}}^{(k+1)\tau_{\text{s}}} q_{\text{out}}(t)\, dt \quad (22)$$

The discrete approximation of the PI controller as presented in Åström and Wittenmark (2012) is used,

$$x_{\text{c}}(k+1) = x_{\text{c}}(k) + y_{\text{p}}(k) \quad (23)$$

$$y_{\text{c}}(k) = \frac{\tau_{\text{s}}}{a} c_{\text{P}} x_{\text{p}}(k) + \frac{\tau_{\text{s}}^2}{a} c_{\text{I}} x_{\text{c}}(k) \quad (24)$$

where in the controller output a scale factor was put in front of the coefficients to keep the dimensions of c_{P} and c_{I} consistent with the continuous case. The resulting system is

$$\begin{bmatrix} x_{\text{p}}(k+1) \\ x_{\text{c}}(k+1) \end{bmatrix} = \begin{bmatrix} (1 - c_{\text{P,s}}) & -c_{\text{I,s}} \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_{\text{p}}(k) \\ x_{\text{c}}(k) \end{bmatrix} + \frac{\tau_{\text{s}}}{a} \begin{bmatrix} 1 \\ 0 \end{bmatrix} d_{\text{p}}(k) \quad (25)$$

where

$$c_{\text{P,s}} = \frac{\tau_{\text{s}}}{a} c_{\text{P}}\,;\; c_{\text{I,s}} = \frac{\tau_{\text{s}}^2}{a} c_{\text{I}} \quad (26)$$

The characteristic polynomial of the matrix is

$$p(\lambda) = [(1 - c_{\text{P,s}}) - \lambda][1 - \lambda] - (-c_{\text{I,s}}) \quad (27)$$

or

$$\lambda^2 - (2 - c_{\text{P,s}})\lambda + 1 - c_{\text{P,s}} + c_{\text{I,s}} \quad (28)$$

with roots

$$\lambda_{1,2} = \frac{2 - c_{\text{P,s}} \pm \sqrt{(c_{\text{P,s}})^2 - 4c_{\text{I,s}}}}{2}$$

Standard discrete time system theory can now be used to determine stability in either time or frequency domain. In the lectures the set of pairs of $(c_{\text{P,s}}, c_{\text{I,s}})$ for which the scaled system is stable is derived (Fig. 4). In the computer lab, students are asked to use the material from the lecture to adjust the parameters of a discrete PI controller to create both an asymptotically stable system and an unstable system. The area of the reservoir and the time step can be varied. While the system is completely discrete, inflows are specified in m^3/s rather than m^3 per time step, so the time step enters through (18).

The same system is used to illustrate 'wind-up' by adding a constraint on the capabilities of the actuator. This is done by restricting $u_{\mathrm{p}}(k)$, but not $y_{\mathrm{c}}(k)$, to positive values and adding an additional disturbance representing open water evaporation q_{evap} in the reservoir. For q_{in} below q_{evap}, the water level will drop below setpoint, and the naive implementation of the controller will exhibit wind-up, leading to an interesting level excursion once inflow exceeds evaporation again. A discrete PI controller with wind-up protection is also provided. The computer lab environment allows easy access to the controller state which facilitates examination of the wind-up phenomenon.

4.7 An example of MPC applied to a reservoir

In countries like the Netherlands where drainage in low lying areas is limited by pump capacity, it can be advisable to anticipate on heavy precipitation events that exceed the installed capacity and temporarily lower the level of the open water in the network of canals and lakes used to transport the water pumped out of polders to a river or to the sea. Lowering the level unnecessarily is undesirable, as it can hinder shipping, damage foundations, and even endanger the polder dikes. It is therefore desirable to find a compromise between level deviations downward in anticipation of heavy precipitation and upward level deviations during heavy precipitation events. This can be used as a simple example of the principle of MPC. In practice, the uncertainty in the precipitation forecast will need to be dealt with, but it can be argued that the delay due to the rainfall-runoff process reduces that uncertainty for the first few hours. If we return to the discretized simple reservoir model, then MPC with a two to five step prediction horizon N and a simple quadratic objective function J is easy to implement, and can be used to illustrate what makes MPC different from standard feed-forward and feedback controllers.

Another important point that needs to be addressed is the fact that MPC is not a magic cure-all. The following simple example is used in the lab to illustrate this point. For $N = 2$ the controller actions can in principle be calculated by hand. If the system is a standard discrete reservoir with time evolution given by

$$x(k+1) = x(k) + d(x) - u(k) \tag{29}$$

with state x, inflow d, and outflow u, and we take an N step time horizon with objective function

$$J(u(.)) = \sum_{j=0}^{N-1} \ell(x(j), u(j)) \tag{30}$$

then a cost function ℓ of the form

$$\ell(x, u) = x^2 + 2u^2 \tag{31}$$

will work (Fig. 5), but

$$\ell(x, u) = x^2 + 2|u| \tag{32}$$

will not; the system may get 'stuck' in a state away from the setpoint (Fig. 6). Note that (32) needs to be treated with care; the jump in the derivative for $u = 0$ can present a problem for some optimizers and some starting points. The same example, this time with a constraint on

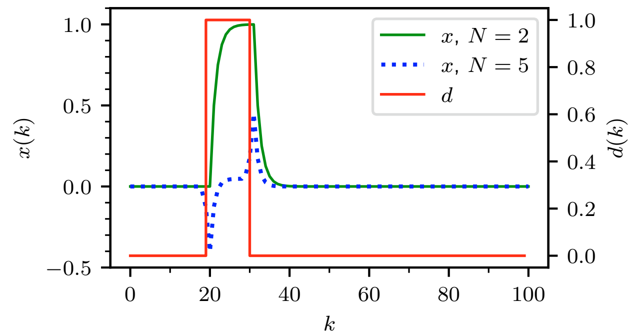

Fig. 5. MPC with cost function $\ell(x, u) = x^2 + 2u^2$

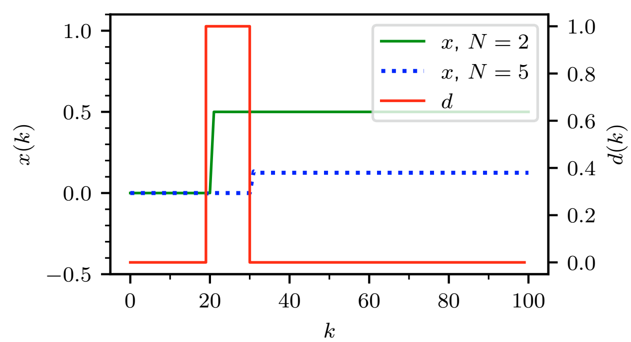

Fig. 6. MPC with cost function $\ell(x, u) = x^2 + 2|u|$

the control action, can be used to illustrate the use of MPC in the presence of constraints. The students were provided with full Matlab and Python implementations of the example.

5. CONCLUSIONS

During the last 15 years of the course, labs and examples were always present, but initially theory dominated the first part of the course. Lectures and labs were interleaved in 4 hour blocks. However, both student feedback and performance in the lab clearly indicated that the majority of the students needed links to real water systems from the start. As a result, the examples used to illustrate the theory were embedded in recognizable water systems. In addition, about five years ago, a switch was made to interleaving lectures and labs in two hour blocks to move theory closer to 'hands-on' work in the labs. Both changes improved student perception of the course. Effects on grades are not clear-cut, but both student and teacher happiness definitely increased as the examples improved. One reasonably clear measure of the quality of an example and the explanation of the linked theory was the time needed by students to perform a lab exercise based on it and the quality of their lab report. As individual examples were adapted, students needed less time in the lab and wrote better reports.

REFERENCES

Åström, K.J. and Wittenmark, B. (2012). *Computer-Controlled Systems*. Dover Publications Inc.

Bastin, G. and Coron, J.M. (2016). *Stability and boundary stabilization of 1-d hyperbolic systems*. Springer.

García, L., Barreiro-Gomez, J., Escobar, E., Téllez, D., Quijano, N., and Ocampo-Martinez, C. (2015). Modeling and real-time control of urban drainage systems: A review. *Advances in Water Resources*, 85, 120–132.

Gugat, M. and Leugering, G. (2003). Global boundary controllability of the de St. Venant equations between steady states. *Ann. I. H. Poincaré*, 20, 1—11.

Hadid, B., Duviella, E., Chiron, P., and Archiméde, B. (2019). A flood mitigation control strategy based on the estimation of hydrographs and volume dispatching. *IFAC-PapersOnLine*, 52(23), 17–22.

Hayat, A. and Shang, P. (2019). A quadratic Lyapunov function for Saint-Venant equations with arbitrary friction and space-varying slope. *Automatica*, 100, 52 – 60.

Hoogheemraadschap van Delfland (2018). Begroting 2019 en meerjarenraming 2020-2023. Technical report, Hoogheemraadschap van Delfland.

Kasper, J., Pranner, G., Simons, F., Denhard, M., and Thorenz, C. (2018). Enhancing automated water level control at navigable waterways by high-resolution weather predictions. *EPiC Series in Engineering*, 3, 1022–1029.

Ligthart, C. and Dekking, W. (eds.) (2017). *WATERSCHAPSBELASTINGEN 2017: Het hoe en waarom (Waterboard taxes 12017: how and why)*. Unie van Waterschappen. (in Dutch).

Litrico, X. and Fromion, V. (2009). *Modeling and Control of Hydrosystems*. Springer, London.

Nederkoorn, E., Schuurmans, J., Grispen, J., and Schuurmans, W. (2012). Continuous nonlinear model predictive control of a hybrid water system. *Journal of Hydroinformatics*, 15(2), 246–257.

Schuurmans, J., Bosgra, O.H., and Brouwer, R. (1995). Open-channel flow model approximation for controller design. *Applied Mathematical Modelling*, 19, 525–530.

Silva, G.J., Datta, A., and Bhattacharyya, S.P. (2005). *PID controllers for time-delay systems*. Control Engineering. Birkhäuser Boston Inc., Boston, MA.

Simulink (2017). *Simulink 2017b*. The Mathworks, 1 Apple Hill Drive Natick, MA USA.

van Nooijen, R.R.P. and Kolechkina, A. (2018a). A controlled sewer system should be treated as a sampled data system with events. *IFAC-PapersOnLine*, 51(16), 61 – 66.

van Nooijen, R., Kolechkina, A., and Campisano, A. (2012). Stability of digital discrete time controllers for water systems. In *Proceedings of the 6th International Congress on Environmental Modelling and Software (iEMSs)*, 1–8.

van Nooijen, R.R. and Kolechkina, A. (2013). Speed of discrete optimization solvers for real time sewer control. *Urban Water Journal*, 10(5), 354–363.

van Nooijen, R.R.P. and Kolechkina, A. (2018b). Graph theory algorithms for real time control of a sewer network. In G.L. Loggia, G. Freni, V. Puleo, and M.D. Marchis (eds.), *HIC 2018. 13th International Conference on Hydroinformatics*, volume 3 of *EPiC Series in Engineering*, 2127–2135. EasyChair.

van Nooijen, R.R.P. and Kolechkina, A.G. (2014). Room for automatic control in combined sewer systems. In E. Boje and X. Xia (eds.), *Proceedings of the 19th IFAC World Congress*, 5315–5320.

Weyer, E. (2008). Control of irrigation channels. *IEEE Transactions on Control Systems Technology*, 16(4), 664–675.