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A Macroscopic Flow Model for Mixed Traffic using Two-Dimensional Speed Functions

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Introduction

Total travel time loss is a common metric for road network performance. Within an urban environment, streets are often shared by multiple user types such as cars, (motor)bikes and pedestrians. Although these entities share the same infrastructure, the experienced delay may differ. To better estimate mode-specific travel times, a model is required that can describe different traffic types. A model type that meets this requirement is a multi-class macroscopic model.

In this paper we model mixed traffic in an urban situation where cars and cyclists share the road. A specific example is a so called 'cycling street', which has multiple characteristics resulting from the property that bicycle traffic is prioritized over cars. There is no uniform design of a cycling street but it is typically wide enough for cars to overtake cyclists. However, cars are considered as guests on the cycling street and they have to slow down when cyclists are present. Furthermore, cars cannot overtake when the cyclist density exceeds a certain threshold and as a result, the cars have to match the cyclists' speed. When cars are moving slowly in a queue there is enough space for cyclists to carefully pass the queue and create their own queue closer to an intersection. Following these characteristics, the speed of both traffic types hence depends on the presence of the other type, and both traffic types can be the fastest class depending on the mixed density.

There are multi-class models that meet the requirement of different class-dependent desired speeds but these are subject to the condition that a 'fastest' class exists, i.e. one class always has a faster speed than the other irrespective of prevalent traffic conditions [1]. This limiting assumption should be adjusted for modeling mixed traffic on a cycling street, where the slower class (cyclists) can switch to being the faster class e.g. when the car density exceeds the jam density. Including this phenomenon in the model requires a different approach using class specific speed functions which depend on density of all classes as independent variables.

The main objective of this study is to introduce two-dimensional speed functions in a multi-class macroscopic model, together with the evaluation of the resulting model dynamics. Such a model allows to estimate travel times for cars and cyclists by describing their joint traffic dynamics, which in turn can be used to evaluate the overall performance of a mixed traffic road.

Model specification

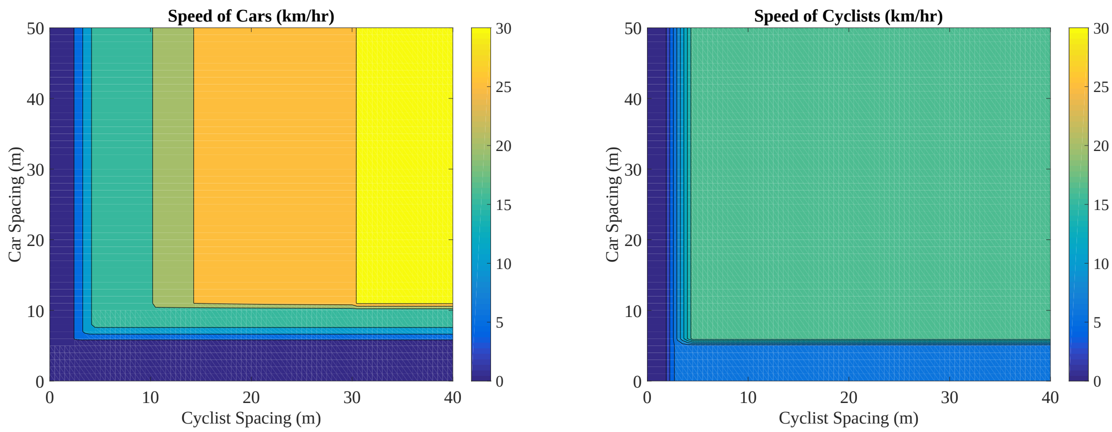
Macroscopic continuum models assume traffic to behave similar to a fluid. The starting point is the conservation of traffic participants, which can be expressed in Eulerian or Lagrangian coordinates. This study takes the Lagrangian approach, resulting in the class-specific expression for the conservation equation given in Eq. 1. The conservation equation holds for each class u and the variables are class-specific. Spacing is the main variable, denoted by s for class u, and is defined as the average distance between travelers belonging to the same entity. The spacing is inversely proportional to the class-specific density k_u. Furthermore, the equation is expressed in traffic units n which are the total traffic units that have passed a given point in space at a given time.

$$\frac{\partial s_u}{\partial t} + \frac{\partial v_u}{\partial n_u} = 0, \tag{1}$$

stating that the change in spacing s over an infinitesimal time unit t is equal to the change in speed v over an infinitesimal traffic unit n. In the numerical model, time and traffic units are discretised in finite steps of dt and dn respecively, while space remains a continuum. The positions of all traffic units are updated simultaneously in every timestep based on the speed, which depend on the spacing of both modes in the previous timestep.

The speed function in a macroscopic model is typically provided by a fundamental diagram, describing the relation between spacing, speed and flow rate. In the multi-class situation, the class-dependent fundamental diagram is based on the spacing distribution of both cars and cyclists, resulting in two-dimensional speed functions. To prevent confusion in abbreviation, the cyclists are referred to as 'b' for bicyclist from this point onward, resulting in v_b for speed of cyclists and v_c for speed of cars. The two-dimensional speed functions that are used in this study are shown in Figure 1.

The starting point for the two-dimensional speed functions is the triangular fundamental diagram for single class traffic flow [2], consisting of the cases $(a1$-$a3)$ and $(b1$-$b3)$ in Eq. (2) and (3) respectively. To connect the two classes, additional cases are introduced while trying to maintain a linear expression where possible. For cyclists, condition $(a4)$ is added that reduces the speed to $v_{b,\text{red}}$ when cyclists are passing a queue of cars. Two additional cases are introduced for cars. First, cars cannot overtake when there are too many cyclists on the road so they have to adapt their speed to match the cyclist' speed $(b2)$. Second, when cyclists are only sparsely present ($s_b > a$, with $a = 30$m), cars can overtake with reduced speed $(b4)$. Further characteristic values for jam spacing ($s_{u,\text{jam}}$), critical spacing ($s_{u,\text{crit}}$) and free flow speed ($v_{u,\text{f}}$) are presented in Table 1.



(a) (b)

Figure 1: Two-dimensional speed functions for cars (a) and bicyclists (b)

$$v_b(s_b, s_c) = \begin{cases} 0 & \text{if} \quad s_b = s_{b,\text{jam}}, \quad s_c > s_{c,\text{jam}} & (a1) \\ (s_b - s_{b,\text{jam}})w_b & \text{if} \quad s_{b,\text{jam}} < s_b \le s_{b,\text{crit}}, \quad s_c > s_{c,\text{jam}} & (a2) \\ v_{b,\text{f}} & \text{if} \quad s_b > s_{b,\text{crit}}, \quad s_c > s_{c,\text{jam}} & (a3) \\ \min(v_{b,\text{red}}, (s_b - s_{b,\text{jam}})w_b) & \text{if} \quad s_{b,\text{jam}} < s_b \le s_{b,\text{crit}}, \quad s_c = s_{c,\text{jam}} & (a4) \end{cases} \tag{2}$$

$$v_c(s_b, s_c) = \begin{cases} 0 & \text{if} \quad s_c = s_{c,\text{jam}} \quad s_b > s_{b,\text{jam}} & (b1) \\ \min(v_b, (s_c - s_{c,\text{jam}})w_c) & \text{if} \quad s_{c,\text{jam}} < s_c \le s_{c,\text{crit}} \quad s_b > s_{b,\text{jam}} & (b2) \\ v_{c,\text{f}} & \text{if} \quad s_c > s_{c,\text{crit}} \quad s_b > a & (b3) \\ v_{c,\text{red}} & \text{if} \quad s_c > s_{c,\text{crit}} \quad s_b \le a & (b4) \end{cases} \tag{3}$$

Face validation

As case to test the working of the model, we consider a situation with two platoons of five cars and three platoons of five cyclists. The initial spacing of both classes is 20 meters, and the cyclists have a 350-meter head start to

Table 1: FD characteristic values

	s_{jam}[m]	s_{crit}[m]	v_{f}[m/s]
cyclists	1.5	4.5	5.0
cars	5.0	10	9.0

allow for a situation where cars overtake cyclists. Furthermore, the cars are forced to stop after 190 seconds to illustrate how cyclists pass a queue of cars. The properties of the FDs used for this case study are given in table 1.

Figure 2 shows the resulting traffic operations. The color coding is linked to the desired speed of each class, resulting in no color when speed is uninfluenced, and in strongly colored when the speed is reduced for cars (purple) and cyclists (cyan).

Depending on the traffic situation, cars and cyclists experience a delay due to the presence of the other mode. In the first 40 seconds of the simulation, both cars and cyclists are separated in space and can travel without delay at their desired speed. When the cars catch up with the cyclists, they have to reduce speed but can still overtake the cyclists. As a result of the speed reduction, a delay is created and the car spacing reduces to 13 meters. Spacing and speed increase again after all cyclists are left behind. After 190 seconds, the first cars are stopped resulting in a speed reduction of the following platoon of cars and the car spacing reduces to 5 meters, which is equal to the jam density. The cyclists catch up with the cars and pass the queue at a reduced speed, resulting in a slight delay for cyclists.

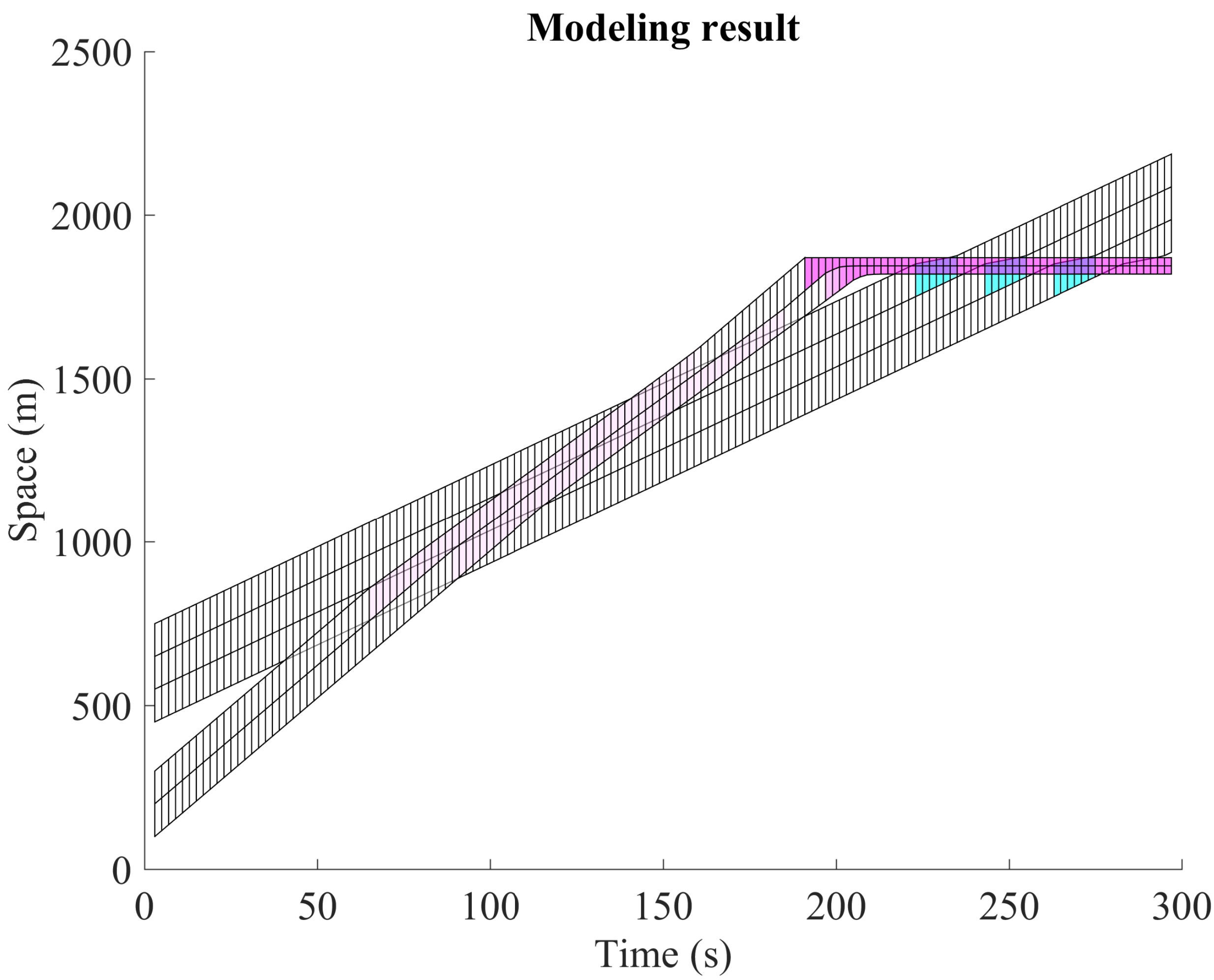


Figure 2: Simulation of the cycling street with a 350m head start for cyclists. The color represents the reduction of speed for cars (purple) and cyclists (cyan).

Conclusion and future work

The priliminary results indicate that introducing two-dimensional speed functions into a multi-class macroscopic model would allow mode-specific travel time estimation, and estimation of capacity in a mixed traffic situation.

Future work before the conference involves the actual estimation of mode-specific travel times for a cycling street situation and comparison of the results to empirical data.

The model presented in this work is specifically relevant for shared street situations, without a fastest class. Besides mixed car and bicycle traffic, the model could also be applied to other combination of modes, such as pedestrians and cyclists, or cars and pedestrians, as long as the class-dependent two-dimensional speed functions are updated to match the situation.

References

[1] F. van Wageningen-Kessels, H. van Lint, S.P. Hoogendoorn, K. Vuik, Transportation Research Record: Journal of the Transportation Research Board **2188**, 29 (2011). DOI 10.3141/2188-04

[2] C.F. Daganzo, Transportation Research Part B **28**(4), 269 (1994). DOI 10.1016/0191-2615(94)90002-7