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DOI

[10.1109/OJITS.2024.3349635](https://doi.org/10.1109/OJITS.2024.3349635)

Publication date

2024

Document Version

Final published version

Published in

IEEE Open Journal of Intelligent Transportation Systems

Citation (APA)

Siebinga, O., Zgonnikov, A., & Abbink, D. A. (2024). Human Merging Behavior in a Coupled Driving Simulator: How Do We Resolve Conflicts? *IEEE Open Journal of Intelligent Transportation Systems*, 5, 103-114. <https://doi.org/10.1109/OJITS.2024.3349635>

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Received 17 July 2023; revised 13 October 2023 and 8 December 2023; accepted 30 December 2023. Date of publication 4 January 2024; date of current version 25 January 2024.

Digital Object Identifier 10.1109/OJITS.2023.3349635

Human Merging Behavior in a Coupled Driving Simulator: How Do We Resolve Conflicts?

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This work was supported in part by Nissan Motor Company Ltd. and in part by Rijksdienst voor Ondernemend Nederland under Grant TKI2012P01. This work involved human subjects or animals in its research. Approval of all ethical and experimental procedures and protocols was granted by the TU Delft's Human Research Ethics Committee.

ABSTRACT Traffic interactions between merging and highway vehicles are a major topic of research, yielding many empirical studies and models of driver behaviour. Most of these studies on merging use naturalistic data. Although this provides insight into human gap acceptance and traffic flow effects, it obscures the operational inputs of interacting drivers. Besides that, researchers have no control over the vehicle kinematics (i.e., positions and velocities) at the start of the interactions. Therefore the relationship between initial kinematics and the outcome of the interaction is difficult to investigate. To address these gaps, we conducted an experiment in a coupled driving simulator with a simplified, top-down view, merging scenario with two vehicles. We found that kinematics can explain the outcome (i.e., which driver merges first) and the duration of the merging conflict. Furthermore, our results show that drivers use key decision moments combined with constant acceleration inputs (intermittent piecewise-constant control) during merging. This indicates that they do not continuously optimise their expected utility. Therefore, these results advocate the development of interaction models based on intermittent piecewise-constant control. We hope our work can contribute to this development and to the fundamental knowledge of interactive driver behaviour.

INDEX TERMS Road transportation, human in the loop, human Factors, vehicle driving.

I. INTRODUCTION

INTERACTIONS between vehicles, such as in highway merging, play a major role in everyday traffic. Therefore, driving behaviour in these interactions is an essential aspect of many transportation technologies. Empirical data and microscopic traffic models of human driving behaviour are thus essential tools for transportation engineers. These models and data are used in the design and safety assessment of highway on-ramps [1], [2] and urban intersections [3]. Microscopic traffic models can be used to evaluate traffic management systems [4]. And finally, autonomous vehicle designers are interested in these interactions to

develop socially acceptable and human-like autonomous behaviour [5], [6]. Particularly for the last use case, a good understanding of the individual negotiations and the continuous reciprocal actions of the drivers during interactions is essential.

Many recent studies have investigated interactive merging behaviour by modelling this behaviour or by conducting empirical investigations. Most of these studies use naturalistic data, i.e., data recorded in real-world scenarios. For example, Daamen et al. [7] and Marczak et al. [8] performed empirical analysis on traffic data which they recorded with helicopters. Wang et al. [9] and Srinivasan et al. [10] used existing open datasets to evaluate driver behaviour on merge ramps. Others have modelled interactive driver behaviour using naturalistic data to gain insights, e.g.,

The review of this article was arranged by Associate Editor Chongfeng Wei.

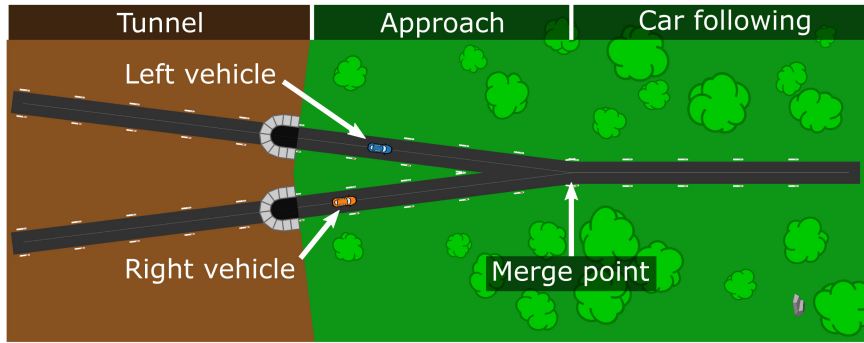


FIGURE 1. The simplified merging scenario used in the experiment. Two vehicles approach a pre-defined merge point at which their lanes merge into one. The track consists of three sections of equal length (50 m, total track length 150 m). The vehicle dimensions are 4.5 m x 1.8 m. In the tunnel, participants could observe both vehicles, but not control their vehicles. During the approach, the participants could control the acceleration of their vehicles to resolve the merging conflict. During the car-following section, the vehicles follow each other in the same lane.

using game theory [11], [12], [13], acceleration models comparable to car-following models [14], or machine-learned models [10], [15].

The usage of naturalistic data has the advantage that real-world behaviour can be studied. However, this approach has two main drawbacks. First, it is challenging to investigate the interacting drivers' operational behaviour and control inputs from naturalistic data. The previously mentioned studies use naturalistic data that was recorded with cameras on helicopters, quad-copters, or high buildings. Therefore, only sequential positions are recorded. Velocities and accelerations are reconstructed from this position data and control inputs are not included. Other naturalistic datasets that are recorded from within a vehicle (e.g., [16], [17], [18]). They do contain these signals for the ego vehicle. However, these datasets do not provide the same signals for the surrounding vehicles, which complicates the study of interactions.

The second drawback is that although the kinematic differences between situations can be observed, they cannot be controlled. This makes it difficult to investigate the relationship between the initial kinematics of the vehicles and the outcome of the merging conflict (e.g., who merges first and who yields). To gain a deeper understanding of individual reciprocal interactions, controlled experiments are needed.

However, only a very limited number of studies in a controlled environment (i.e., in a driving simulator) targeted interactions during merging (i.e., excluding studies of autonomous control strategies, gap acceptance, or traffic flow). Stoll et al. investigated human decision-making in merging scenarios based on videos of a controlled simulation [19]. Participants had to select their preferred reaction (e.g., accelerate or decelerate) after watching videos of vehicles they were "interacting with". Shimojo et al. used a driving simulator to investigate how the merging behaviour of drivers is affected by their perception of other drivers [20]. They used predetermined controls for one of the vehicles in the interaction, to influence this perception in a controlled way. In both experiments, the behaviour of one of the drivers was predetermined. Thus, there was no interaction

or dynamic negotiation between two human drivers. We conclude that the existing literature misses studies that investigate the reciprocal merging interactions between at least two human drivers in a controlled environment.

To address this gap, we conduct an experiment in a top-down view, coupled driving simulator in which we investigate reciprocal merging interactions between two human drivers. We investigate the operational behaviour of the drivers in terms of inputs (acceleration and velocity profiles). Furthermore, we examine the influence of different initial kinematics (both position and velocity) on the outcome of the interaction. Both on a high level in terms of which driver merges first, and in more detail through the metric Conflict Resolution Time (CRT) [21]. The focus of our work is on the dynamics of interactive behaviour. We hope this experiment advances the fundamental knowledge about vehicle-vehicle interactions in traffic and contributes to the development of interaction-aware intelligent transportation systems.

II. METHODS

We conducted an experiment in a coupled, top-down view driving simulator with 9 pairs of participants (6 female, 12 male, mean age: 25, std: 2.6). All participants met their "opponent" before the experiment and most participant pairs knew each other before the experiment. The details of this experiment (including Figures 1 and 2), and the analysis tools we developed to gain insight into the merging behaviour, have been previously published in [21]. This experiment was approved by TU Delft's Human Research Ethics Committee (HREC). All participants gave their consent before participating in the experiment.

The experiment regarded a symmetric simplified merging scenario (Figure 1) in which participants could control the acceleration of their vehicle using the gas and brake pedal of a steering-wheel game controller (Logitech Driving Force GT). The headings of the vehicles were always equal to the heading of the road, so no steering was involved. Participants could see the simulation on a computer screen (Figure 2). However, they could not see the other participant, who was



FIGURE 2. The experimental setup as seen from a participant's view. The other participant in the pair used an identical setup. The participants could not see each other.

seated in the same room behind a curtain. To prevent auditory communication, participants wore noise-cancelling headsets (Sony WH-1000XM3) with ambient music. All gathered data, the information letter we provided to participants before the experiment, and the informed consent form we used were published in the 4TU data repository [22]. The software needed to reproduce the experiment can be found on GitHub.¹ Interactive plots of all our results can be found in the online supplementary materials.²

To investigate the effects of the initial vehicle kinematics on the outcome of the merging conflict we varied the initial positions and initial velocities of the vehicles. Participants were instructed to “maintain their initial velocity yet prevent a collision”. To ensure a merging conflict, all conditions were chosen such that if both drivers would maintain their initial velocity, they would collide. Furthermore, participants were instructed to “remain seated, use one foot on the gas or brake pedal, keep both hands on the steering wheel, and not to communicate by making sounds or noise.” Finally, participants were told that “this is a scientific experiment, not a game or a race” and that “no vehicle has the right of way.”

The participants received visual feedback on their computer screens. Their visuals were randomly mirrored such that they appeared to approach the merge point from the left or the right side randomly. While in the experimenter's view, and in all results discussed here, we refer to the same participant in a pair as the left or right driver. If participants deviated from their initial velocity, their steering wheel provided vibration feedback, increasing with the deviation and with a dead band around the initial velocity. The vibration was implemented to facilitate speed perception. If the vehicles collided, the participants got a time penalty of 20 seconds. This was longer than the duration of a single trial, which took approximately 16 seconds. During this time, the experiment was paused and the participants had to wait and watch an animation on the screen. This increased the total duration of the experiment and therefore provided an incentive not to collide.

1. <https://github.com/tud-hri/simple-merging-experiment>

2. <https://tud-hri.github.io/simple-merging-experiment>

The vehicles started in a tunnel where participants could observe the initial velocities of both vehicles, but they could not control their vehicles yet. The drivers gained control when both vehicles exited the tunnel. The tunnel, with its different background colour, served merely as a visual representation of the possibility of controlling the vehicles. This section of the track had two purposes. First, it ensured that drivers could perceive their velocity relative to the velocity of the other vehicle before starting the interaction. In a pilot study, we tested a setup where a driver could only see their own vehicle in the tunnel. However, in this pilot, drivers accelerated directly at the tunnel exit to anticipate the appearance of another vehicle, which is a unilateral decision and thus prevents an interaction. Therefore, we decided to make both vehicles visible in the tunnel. Second, the tunnel exit marked an unambiguous moment when the interaction started (i.e., the start of the interaction).

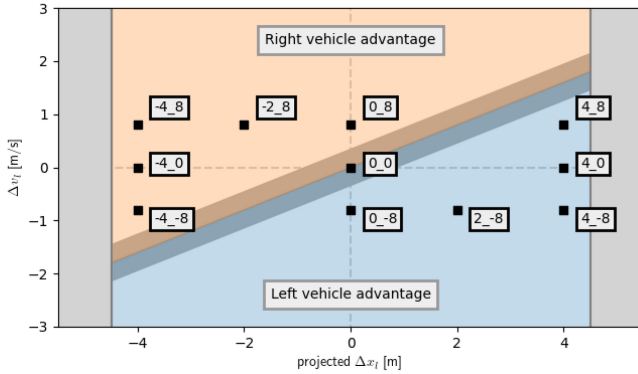
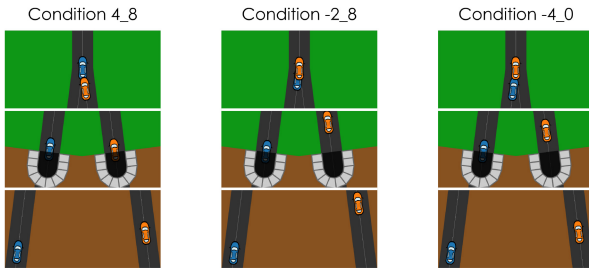
The vehicles' initial kinematics were varied to create 11 experimental conditions. We investigated both differences in velocity and headway (distance from front bumper to front bumper). For the differences in headway, we used the projected headway at the merge point as the underlying metric to design the conditions and determine the initial positions for a given velocity difference. The projected headway is the headway at the merge point if both drivers would maintain their initial velocity. We chose this metric because it does not depend on track dimensions or a snapshot of the vehicle state at an arbitrary point along the track (e.g., at the tunnel exit). A combination of relative velocity and projected headway fully defines the positions and velocities of both vehicles at the start of the experiment and at the tunnel exit because the drivers have no control over their vehicles in the tunnel (see Figure 4).

To visualise the differences between conditions, we plotted them in a 2D projected-headway - relative-velocity plane (Figure 3). This figure shows the conflict space. If the projected headway is larger than the vehicle length, there is no conflict. These areas are shown in grey on the left and right side of Figure 3. The figure also shows in which areas we expected the right or the left driver to have an advantage. This expectation was based on a (shorter) pilot experiment with the same experimental setup but different kinematic conditions.

We used this expectation to design and spread the conditions evenly over the conflict space. The diagonal darker area represents the area in which the (kinematic) advantage changes from the left to the right driver. We decided not to investigate this area but to (first) focus on driver behaviour in cases where the outcome is more distinct. Our aim here is to gain insight into the interactions and negotiations between the two drivers in these situations. However, we did include a baseline condition where neither driver has a position or velocity advantage. With these conditions, we aim to obtain a quantitative description of the most likely outcome (who merges first) based on the initial

TABLE 1. The number of observed collisions per condition. The total number of trials per condition was 90. Most collisions occurred between 4 and 6 seconds after the vehicles exited the tunnels.

Condition	-4_-8	-4_0	-4_8	-2_8	0_8	0_0	0_-8	2_-8	4_-8	4_0	4_8
Number of Collisions	3	1	3	2	4	5	3	2	1	2	2

**FIGURE 3.** The experimental conditions in their two-dimensional space. The x-axis shows the projected headway at the merge point if both drivers would keep their initial velocity. If the headway is larger than the vehicle length (4.5 m) there is no projected collision, this is indicated by the grey areas on the left and right side. The y-axis shows the initial velocity differences. Positive values mean that the left vehicle is (projected to be) ahead or moving faster. The diagonal darker area divides the space into areas where the left or right driver has the advantage of passing the merge point first. This line was estimated by interpolating the results of a pilot experiment (with different kinematic conditions) to find a 50% distribution between left and right going first. Note that this does not simply divide the plane into areas where one driver has the velocity or projected headway advantage.**FIGURE 4.** Three visualisations of experimental conditions. The figures show the relative positions of the vehicles and the start point, tunnel exit, and merge point. These merge point positions would occur if both vehicles would maintain their initial velocity. In most conditions, the slower vehicle has a position advantage at the tunnel exit. The exceptions are conditions 4_8 and -4_-8, where the vehicles exit the tunnel at the same time.

kinematics. We used the Python package Pymer4 [23] for all statistical models in this work.

We named the conditions based on the two dimensions that define them: the projected headway in meters and the velocity difference in decimetres per second. Positive numbers indicate that the left driver has an advantage. For example, in condition -2_8, the right driver has a projected headway advantage of 2 m, but the left driver drives 0.8 (m/s) faster. For more visual examples of conditions and their names, see Figure 4. In our experiment, every condition was repeated 10 times in a random order for every pair of participants.

We used the Conflict Resolution Time (CRT) [21] to analyse the conflict resolution behaviour of the pairs of

participants. The CRT denotes the time from the start of the interaction until the first moment at which the vehicles are no longer on a collision course (assuming constant velocity). To calculate the CRT, we post-process the data and determine for every time step if a collision would occur on the remaining track if both vehicles would continue their velocity. The time between the tunnel exit and the first moment where no collision would occur is the CRT. Drivers had limited time to resolve the conflict after exiting the tunnel; they reached the merge point (where they would collide if they take no action) in 4.9 seconds on average. CRT is a measure of the amount of time needed to resolve the conflict and, therefore, can be used as a measure of the difficulty of the merging conflict.

III. RESULTS

We structure our investigation of driver conflict resolution behaviour into two parts. First, we present the analysis of the joint behaviour of two drivers, to analyse the outcome of the conflict (who gives way) and how quickly each pair of drivers resolved the merging conflict. Metrics that capture the joint behaviour for each pair under different conditions include a percentage of who merged first, as well as the Conflict Resolution Time (CRT). Second, we investigate the contributions of each individual driver in a pair to resolve the conflict. This includes the actions the individual drivers took in terms of accelerations and the resulting velocity profiles.

A. JOINT BEHAVIOUR

1) WHO MERGED FIRST?

The high-level outcome of a merging conflict can be summarised by which driver reached the merge point first, except for the trials where the vehicles collided. However, collisions were rare across all conditions (Table 1). We plot the proportion of left and right vehicles that went first as a function of initial conditions in Figure 5. In the “neutral” 0_0 condition this proportion is almost evenly distributed. For the other 10 conditions with kinematic differences between the drivers, 5 conditions show a consistent outcome over all pairs and trials. This indicates that the outcome in these conditions is entirely defined by kinematics, with no variation between participant pairs. In one other condition (2_-8), only a single trial deviated from the outcome norm. Four conditions (-4_-8, 4_8, 0_-8, and 0_8) show a large majority of the outcomes where a particular driver merges first and a minority of the other driver merging first.

To investigate the relationship between the initial conditions (i.e., the kinematics at the start of each scenario) and the outcome (which driver merges first), we fitted a mixed-effects logistic regression model to the data. The model

TABLE 2. Mixed-effects logistic regression model describing the effect of projected headway and relative velocity on which driver reached the merge point first. Collisions were excluded, the left vehicle going first was labelled as 1, right first as 0. The model includes a random intercept for participant pairs to account for between-pair differences.

	Estimate	SE	Z	P-value	Confidence interval	
					0.025	0.975
Intercept	-0.32	0.212	-1.50	1.326×10^{-1}	-0.73	0.10
Projected headway	1.15	0.080	14.4	6.966×10^{-47}	0.99	1.31
Relative velocity	-3.4138	0.321	-10.6	2.858×10^{-26}	-4.04	-2.78

TABLE 3. Fixed effects estimates of the random intercept values per pair for the mixed-effects logistic regression model (Table 2).

Participant Pair	1	2	3	4	5	6	7	8	9
Intercept	-0.54	-0.42	-1.17	0.06	-0.13	-0.51	0.16	-0.22	-0.13

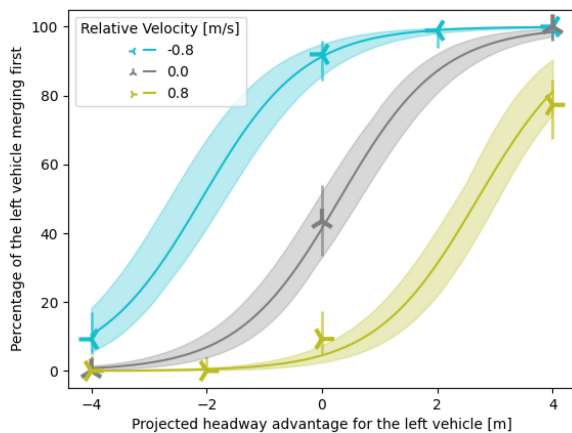


FIGURE 5. An overview of the high-level outcome per condition: which driver went first? Every condition was repeated 10 times for all 9 participant pairs. Therefore, the total number of trials per condition is 90. The markers show the measured data as the percentage of the left driver merging first, with the vertical line representing the 95% binomial proportion confidence intervals. Collisions were omitted from these results (see Table 1). The lines and shaded areas represent the (population) predictions of the mixed-effects logistic regression model (Table 2) with the 95% confidence interval.

parameters are shown in Table 2, and the model outcome is visualised in Figures 5 and 6. These results show that increasing the projected headway advantage increases the chances of a driver merging first ($z = 14.4, p < 10^{-46}$). The relative velocity on the other hand has a negative effect on the probability the driver merges first ($z = -10.6, p < 10^{-25}$). This means that, for equal projected headways, a driver with a higher initial velocity tends to merge behind the driver with a lower initial velocity. The explanation for this is that drivers with a higher initial velocity exit the tunnel later than the slower vehicle in most conditions (Figure 4). An important side-note to these effects is that we found these in a symmetric scenario with no right of way for either of the drivers.

The population level intercept had a negative estimated value that is not significant ($z = -1.5, p = 0.13$). This could be explained by the fact that the intercept explains a bias in the data towards the left or the right driver. This effect is clearest in the neutral condition (0_0), where we found that the right driver merged first in a small majority of the cases. Table 3 shows the estimated intercept values for the individual participant pairs. We expect that with more

participants, the bias on the population level will disappear and the intercept value will approach 0.

To visualise at which locations in the conflict space the left or right driver is more likely to merge first, we have created a top-down view heat map of the regression model. This heat map is shown in Figure 7 and closely resembles Figure 3.

2) CONFLICT RESOLUTION TIME

Besides how the conflict was resolved (which driver merged first) we investigated how quickly the conflict was resolved by examining the Conflict Resolution Time (CRT). This is a measure of the time it took the drivers to resolve the conflict and therefore resembles the difficulty of the conflict in a specific trial. Figure 8 shows the CRT distributions we found for all experimental conditions. The median CRT is highest for the neutral condition 0_0. In this condition, no driver has a headway or velocity advantage. Drivers have to negotiate a solution without a “most-likely” candidate solution. The lowest median CRT was found for the conditions where one driver only had a projected headway difference but the velocity was the same for both drivers. The conditions with velocities differences but no projected headway difference had high median CRTs. Thus conflicts where one driver has a pure projected headway advantage are easier to resolve than conflicts where one driver has a pure velocity advantage.

But besides these high-level observations, Figure 8 reveals no clear relationship between the initial kinematics and the CRT of the merging conflicts. We expected that the high-level outcome of the conflict (who merged first) might partly explain the CRT of that trial. More concretely, we expected trials where the driver with the kinematic advantage went first, to be resolved more quickly than trials where the driver with a disadvantage went first. To investigate this, we analysed CRT as a function of the kinematic advantage from the perspective of the first merging driver (Figure 9, Table 4). The projected headway and velocity differences in this figure are positive if the first merging driver had the advantage. We found that trials with a larger headway advantage for the driver that merged first had a lower CRT ($t = -15.3, p < 10^{-46}$). Trials with a velocity advantage for the first merging driver had a higher CRT ($t = 5.02, p <$

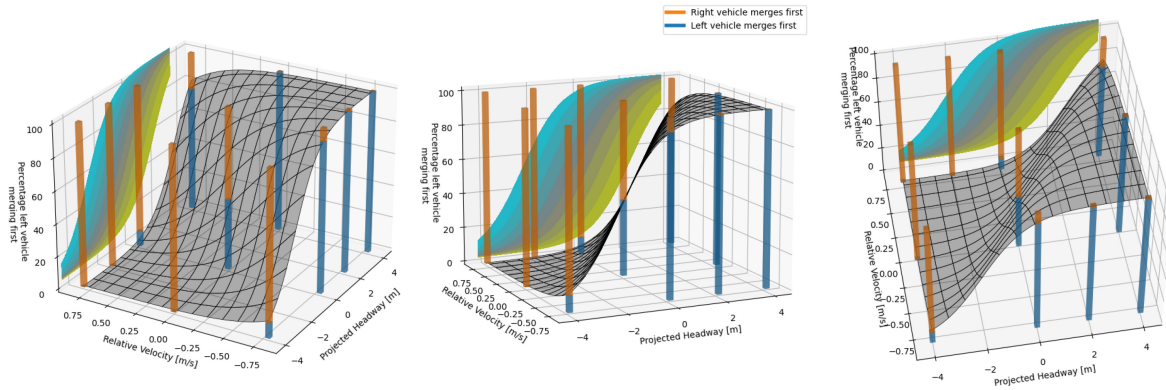


FIGURE 6. A 3-dimensional visualisation of a (population) prediction of the logistic regression model on the data. All three subplots show the same data for different angles. The model predictions are shown as the black surface and the background projections. The coloured bars show the data from the experiment. The x and y-axis represent the condition kinematics. The z-axis shows the percentage of trials where the left driver merged first. Collisions were excluded from this data (see Table 1). An interactive version of this plot can be found in the online supplementary materials.

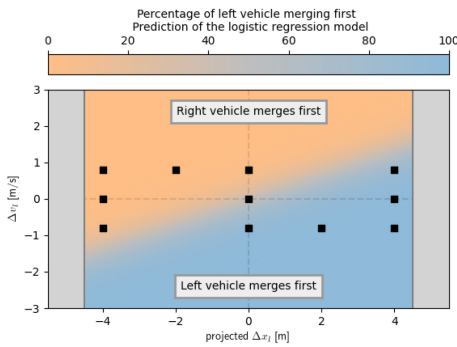


FIGURE 7. A heat map of a logistic regression model prediction for the driver that will merge first. The conditions where data was gathered are marked with black squares.

10^{-6}). Moreover, we found that the association between the CRT and the projected headway advantage was stronger for larger velocity advantage ($t = -6.09, p < 10^{-8}$). One important side note is that drivers with a higher initial velocity have a headway disadvantage in the approach section, i.e., they are approaching the merge point behind the other driver.

B. INDIVIDUAL BEHAVIOUR

To gain insight into the operational behaviour of the drivers, we investigated the aggregated velocity traces of all drivers (Figure 10). We choose to show the velocity traces for the neutral condition (0_0) here because this condition has the widest variety of solutions (in terms of who merges first). Because of this spread, this velocity plot is easier to read than the same plot for other conditions. However, the key aspects identified in this plot are representative of the other conditions (for the raw data, including plots, see [22]). Interactive versions of these plots are available in the supplementary material.

One of the striking characteristics of the velocity traces in Figure 10 are the triangular patterns that can be observed in many traces. Such triangular-shaped velocity patterns indicate two things. First, it shows that drivers use blocks

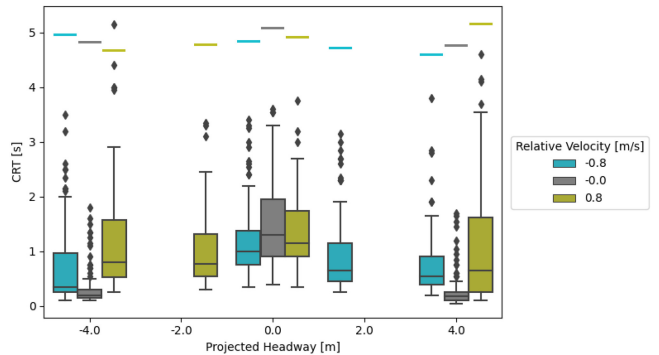


FIGURE 8. Distribution of the Conflict Resolution Time (CRT) for all conditions. The CRT is the time from the moment at which the drivers gain control until the first moment when they are no longer on a collision course (assuming constant velocities). The coloured horizontal bars indicate the average time at which the first vehicle reached the merge point in that condition. A figure that shows the same CRT distribution placed in the 2-dimension conflict space on the locations of the corresponding conditions is available in the online supplementary material.

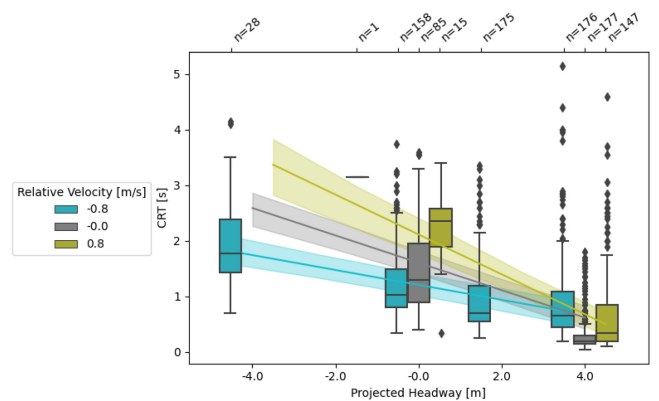


FIGURE 9. Distribution of the Conflict Resolution Time (CRT) from the perspective of the first merging driver. In this plot, positive numbers for headway and velocity differences indicate an advantage for the driver that merged first in that trial. This results in a different number of trials per box (see the labels at the top of the figure). The lines and shaded areas visualise predictions of the mixed effects model (Table 4) and its 95% confidence interval.

of constant acceleration (step inputs on gas/brake) to control their vehicle during an interaction. Second, in between these step inputs, or straight lines in the velocity trace, the input

TABLE 4. Mixed-effects linear regression model analysing the Conflict Resolution Time (CRT) as a function of the kinematic conditions. Positive headways and relative velocities indicate an advantage for the driver who merged first. Collisions were excluded.

	Estimate	SE	T-stat	P-value	Confidence interval	
					0.025	0.975
Intercept	1.61	0.107	15.2	4.97×10^{-10}	1.41	1.83
Projected headway	-0.25	0.016	-15.3	2.16×10^{-47}	-0.28	-0.22
Relative velocity	0.40	0.080	5.02	6.07×10^{-7}	0.25	0.56
Relative velocity : projected headway	-0.14	0.023	-6.09	1.68×10^{-9}	-0.18	-0.09

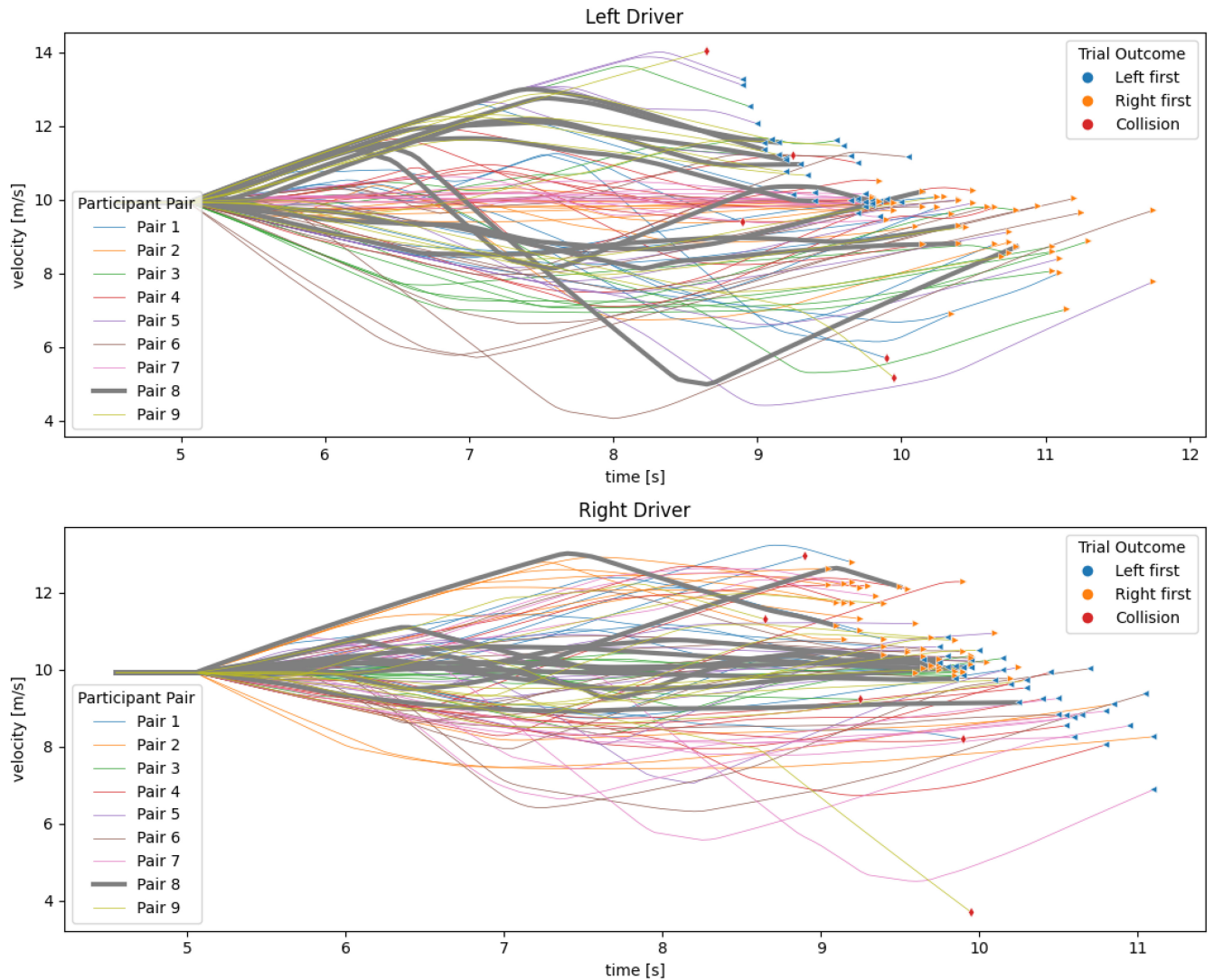


FIGURE 10. Velocity traces of the left and right drivers for all trials in the neutral condition 0.0, from the tunnel exit up until the merge point. The trials of a representative pair are highlighted to provide more insight into individual traces. The markers at the end of the trials indicate the final outcome of the trial. These plots show that drivers use triangular velocity patterns while interacting. These triangular patterns indicate that drivers use blocks of constant acceleration input with key decision moments in between. Interactive versions of these plots for all conditions are available in the online supplementary material.

changes rapidly, causing a sharp angle in the velocity trace. This indicates that drivers select an input level and stick to that until something triggers a new decision resulting in a new input level. We refer to this combination as *intermittent piecewise-constant control*, where intermittent refers to the observed decision moments, and piecewise-constant to the constant acceleration levels in between.

With this intermittent piecewise-constant control, drivers use key decision moments at which they determine a plan. After this decision, they stick with this plan until something triggers a new decision. Therefore, Figure 10 provides evidence that drivers do not continuously optimise their acceleration input while interacting in traffic. Thus, the assumption of continuous utility maximisation that is made

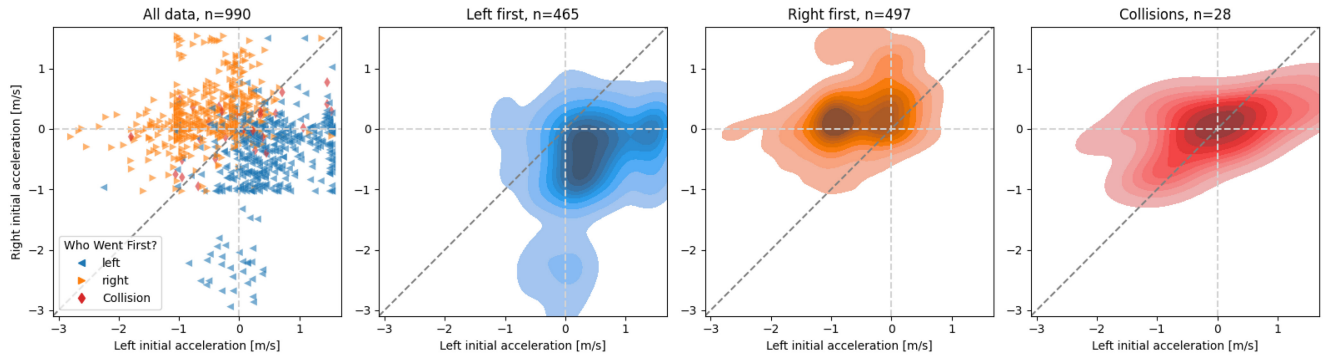


FIGURE 11. The outcome of the merging conflict plotted versus the initial acceleration input at tunnel exit for the left (x-axis) and right (y-axis) drivers for all conditions.

in many models of driver behaviour (e.g., [6], [24], [25], [26], [27]) does not hold for these interactions.

Another aspect shown in Figure 10 is that in many cases, the drivers immediately accelerate or decelerate at the moment they gain control. This indicates, that even in this purely symmetrical condition, drivers exit the tunnel with an intended solution in mind (i.e., they plan to go first or yield). To further investigate if drivers start the interaction with a mutual solution in mind, and if this solution is also reached, we plotted the outcome of the merging conflict versus the initial drivers' actions in Figure 11.

Figure 11 shows that in the majority of the interactions that do not end in a collision, the drivers initially cooperate. In most interactions that end in the left vehicle reaching the merge point first, the left driver's initial input was to accelerate and the right driver's initial input was to decelerate. This indicates two things. First, it shows that if drivers share the same perspective and observations of a merging situation, they form compatible ideas about who will merge first before they even start interacting (in that trial), i.e., drivers use a shared mental model [28]. Second, even though there are cases where the conflict is resolved by only one of the drivers (i.e., where the other driver's input is 0), in most cases, both drivers initially act simultaneously to prevent a collision.

IV. DISCUSSION

In this paper, we investigated the conflict-resolving behaviour of pairs of drivers in a simplified merging scenario. Our four most important findings are: 1) both the relative velocity and projected headway have a significant effect on which driver merges first; 2) the time it takes drivers to resolve the conflict (CRT) can be explained by the kinematics from the perspective of the driver that merges first; 3) drivers used a shared mental model about which driver merges first based on observations before the start of the interaction; and 4) drivers use intermittent piecewise-constant control to resolve the conflict, suggesting they do not constantly optimise some utility function. Rather the observed control behaviour is in line with satisficing (see [29]): in our experiment drivers seem to search for a plan that is good enough and stick to that plan until it no longer suffices. At

this key decision moment, they re-plan to find a new input that is good enough, and act accordingly.

A. RELATION TO THE EXISTING LITERATURE

Our study indicated for the first time that both the relative velocity and the projected headway significantly influence which driver merges first. When drivers are on a collision course, a velocity advantage decreases the probability of a vehicle merging first while a projected headway advantage increases that probability. Earlier studies mostly used naturalistic data, where these kinematics can not be controlled (e.g., [7], [8], [9], [30]), or reduced the analysis of kinematics to one dimension by studying time to arrival (e.g., [10]).

The finding that humans do not constantly optimise their behaviour corresponds to previous findings in simple economic games [31], velocity choice for isolated drivers [32], and high-level skill switching (between manual braking and using cruise control) during driving [33]. The key-decision moments with constant inputs in-between have previously been observed in individual truck driver behaviour in real traffic [34], and in steering behaviour in high-fidelity driving simulators [35]. However, our results are the first to show that these operational aspects of human driving are also present in merging interactions in a controlled experiment.

Previous empirical studies on merging behaviour used naturalistic data [7], [8], [9], [10], [30], in which these operational aspects are not included. Most of these studies focus on evaluating gap acceptance behaviour and were inspired by an interest in the effects of merging behaviour on traffic flow [7], [8], [30]. Among the existing studies of naturalistic merging conflicts, two—in particular—had a goal similar to ours: to understand the dynamics of drivers' conflict-resolving behaviour.

Wang et al. [9] studied social interactions on congested highways in the INTERACTION dataset [36]. They divided merges based on the positions of the vehicles at (what they define as) the start of the interaction. They label situations based on the through-lane vehicle initially being ahead or behind the merging vehicle. Through-lane drivers who overtake a merging car before they merge were labelled "rude", while drivers who let the merging vehicle merge in front of them were labelled "courteous". Thereby, the

authors attribute the outcome of who goes first purely to driving style. However, our results indicate that the outcome (who goes first) strongly depends on the vehicles' kinematic states at the start of the interaction. In our experiment, both the relative velocity and the size of the initial gap are important indicators of who merges first; we did not find substantial individual differences based on driving style. Although driving styles play an important role in real traffic, we interpret our results as a call for cautiousness when referring to drivers as rude or courteous purely based on the fact that they overtake each other.

Srinivasan et al. [10] used naturalistic data to evaluate a machine-learned model of human merging behaviour. They concluded that this machine-learned model can successfully predict the trajectories shown by drivers in scenarios where one of the vehicles has a large kinematic advantage. Compared to our work, they reduced the kinematic differences to a single dimension: time-to-arrival. A 0.0 s time-to-arrival difference corresponds to a 0.0 m projected headway in our work, but other time-to-arrival differences can be obtained with multiple combinations of projected headway and relative velocity. Our results show that these both have a significant impact on the outcome of the conflict in terms of the driver that merges first (Table 2) and on the CRT (Table 4). An important difference between our work and [10] is that we only regarded situations where the drivers are on a collision course from the start of the interaction while [10] regards large(r) kinematic differences. Nevertheless, we advocate using both relative velocity and projected headway for the kinematic analysis, because they have different effects on the outcome of the interaction. Besides that, we expect no major implications for machine-learned models of human behaviour based on our results.

B. IMPLICATIONS

However, when regarding approaches that are not purely data-driven, our results could have major implications for models and control strategies. Many driver models make the assumption that humans behave as rational utility maximisers (e.g., [12], [24], [27], [37]). And because these models make this assumption, many control strategies for autonomous vehicles in mixed traffic were proposed that make the same assumption (e.g., [5], [6], [38], [39], [40], [41]).

Roughly, two kinds of rational utility maximisation are used in driver models. First, there are the models that regard merging as a single high-level decision about who merges first, such as Kita already proposed in 1999 [37]. Second, there are models that assume drivers continuously optimise some reward function to determine their current input (Naumann et al. showed many examples of reward functions used for this approach in 2020 [27]). Our results have major implications for both assumptions.

For models that regard merging as a single decision, our exploration of different kinematic conditions provides valuable insights into driver behaviour. Our results confirm that the vehicles' kinematics at the start of the interaction

have a major impact on which driver merges first. This is in line with the model proposed by Kita [37]. However, our results also show that the individual differences in outcomes between pairs of drivers are restricted to a limited range of kinematic scenarios. In most scenarios, the same driver merges first for all driver pairs. This would indicate that modelling the decision of who merges first based on individual preferences (differences in reward function) is only valuable for a limited set of conditions where the kinematic differences are small.

For models that assume continuous optimisation, our results have more far-reaching implications. The aggregated velocity plot (Figure 10) shows that drivers do not continuously optimise, but re-plan at specific decision moments. This indicates that the assumptions that drivers **continuously** either: optimise, approximately optimise (up to a threshold), or noisily optimise their inputs are not consistent with driver behaviour. Instead, drivers seem to be triggered to change their behaviour at a certain point (at which they might partially optimise to find a new plan). Besides the key-decision moments, Figure 10 also shows piecewise-linear velocity patterns. This indicates that the assumption that drivers aim to minimise a squared difference between their current and desired velocity (as used in many models, e.g., [27], [42]) is also inconsistent with driver behaviour because that would lead to non-linear velocity profiles.

In general, our findings imply that the mathematical convenience related to main assumptions in game-theoretic models comes at a serious cost to their descriptive power. Thus, although game-theoretic approaches can be very valuable to determine optimal control decisions between rational agents (e.g., in vehicle-to-vehicle communication approaches [43], [44], [45]), we advise caution in applying them to predicting driver behaviour (either in driver models or in AV control).

C. RECOMMENDATIONS, LIMITATIONS, AND FUTURE WORK

Therefore, we interpret our results as an encouragement to develop new types of traffic interaction models that do allow for intermittent piecewise-constant control in operational behaviour. Siebinga et al. previously proposed a model framework that could describe intermittent control in traffic interactions [46]. But there are other (existing) lines of research that also hold potential for application to interactive scenarios, such as evidence accumulation models (e.g., [47], [48], [49]). Besides the intermittent control, new interaction models should use piecewise-constant acceleration as control inputs. Furthermore, they should be able to describe the most likely outcomes for different initial kinematics, independent of individual driver differences (Figure 5).

Although our work might provide inspiration for the development of novel interaction models, it also has some limitations. The main limitation is the simplification of the merging scenario. We started our investigation into

interaction dynamics in driving simulators with a simplified symmetric merging scenario in a top-down view simulator. We chose to use this simplified scenario because of the complexity of real-world merging. Our scenario does not include lateral control (i.e., steering), right-of-way or surrounding traffic. This allowed us to focus on the longitudinal control dynamics of two drivers who are on a collision course. We chose to use the top-down-view simulator because setting up a high-fidelity coupled simulator is a complicated and costly endeavour. Driver behaviour in our simulator has not been validated on naturalistic data yet. A detailed investigation into the relationship between the behaviour in our simulator and real-world merging is left for future work.

Such validation is complex because, as discussed in the introduction, the available naturalistic datasets lack insight into control inputs of multiple vehicles and control over kinematics. Some datasets do include many (uncontrolled) kinematic examples (e.g., [50]), which could allow validation of high-level outcomes (i.e., who goes first). Others include the control behaviour of individual drivers (e.g., [16]) enabling validation of the low-level control behaviour observed. However, it is likely that a custom naturalistic dataset needs to be collected for full validation of the simulator. A possible intermediate step could be an experiment with real vehicles on a test track. This would allow for a controlled but real test environment.

But nonetheless, we are confident that our conclusions will generalise to real-world driving. To explain why, we will discuss the three major differences between our simulated scenario and real-world driving and their potential effects on the results. The first major difference between our scenario and the real world is the absence of traffic rules and customs such as the right of way. These rules govern who can go first. Therefore, they probably don't greatly affect the operational behaviour we found. However, their absence may have affected the results regarding who goes first, conflict resolution time, and initial actions. Because the traffic rules effect is absent, the kinematic effects we found in our experiment may have been exaggerated. However, there is no reason to assume that the effects we found will not be present in a situation with traffic rules.

The second major difference is the simplification of the control inputs to acceleration and deceleration only. This design choice decreased the possible actions a driver can take as well as the difficulty of the task. This will have reduced the variability in our results. The same holds for the third (and maybe largest) major difference with driving on real roads: the top-down view of the situation. This perspective makes it easier for participants to estimate relative velocities and distances. Such a decrease in the inaccuracies in human perception could decrease the variability in the results, increasing the statistical power of our model. However, these factors are unlikely to affect the nature of the acceleration inputs (intermittent piece-wise constant control). The fact that the same input behaviour was previously found in real

traffic [34] strengthens our belief that the operational driver behaviour in our simulator resembles that of the real world.

Finally, the interactions in the experiment were not as risky and anonymous as real highway interactions. Participants knowingly executed 110 merging manoeuvres against the same opponent with a name and a face, while the consequences of a collision were not as severe in real life. This could have influenced the outcome because the participants could have learned the other driver's behaviour. However, we found no evidence of learning effects for any participant pair beyond the familiarisation trials (plots can be found with the online supplementary materials). Furthermore, participants could have changed how risky they behaved. The decreased severity of a collision could have caused more risky behaviour (which would explain the large number of collisions we observed) while the identifiable opponent could have led to more courteous behaviour. However, because we only draw conclusions on the operational behaviour and the differences in behaviour between conditions, it is unlikely that this influenced our conclusions. In the future, an experiment with more than two drivers and an experimental setup with random pairing could be used to verify this.

Another limitation of our work lies in the experimental construct of "the start of the interaction". In our experiment, we control this moment by giving participants control over their vehicle at a certain point in time when we are sure they have had enough time to observe the kinematics of the situation. This provided us with the opportunity to investigate a situation where both drivers observe and start acting at the same time. However, in real traffic, this is mostly not the case. There will be differences in when drivers see each other and consequently in when they act. How to extend metrics such as CRT, and thus how to leverage some of our findings in the real world, is not trivial. More work is needed to thoroughly investigate this.

V. CONCLUSION

In this paper, we investigated how drivers resolved merging conflicts in a coupled, top-down view driving simulator. We used a simplified merging scenario that only includes longitudinal control. We investigated driver behaviour under initial conditions with varying relative velocities and projected headways. We used mixed effects regression models, the concept of Conflict Resolution Time (CRT), and aggregated velocity plots to gain insight into driver behaviour. For the experimental conditions studied, we conclude:

- Drivers used intermittent control (modifying acceleration only at key decision moments) to resolve merging conflicts. This suggests that drivers do not behave as continuous rational utility maximisers in merging interactions.
- Drivers use piecewise-constant acceleration control (blocks of continuous acceleration) resulting in triangular velocity patterns to control their vehicle.

- Relative velocity and projected headway are good predictors of which driver is most likely to merge first. They have different effects and are thus both needed for a reliable prediction (instead of reducing the kinematics to a single time-to-arrival value).
 - We used a metric to describe the amount of time the drivers need to resolve a merging conflict (CRT). We found CRT is associated with the outcome of the interaction combined with the initial kinematic differences (projected headway and relative velocity).
 - Conditions where one driver has a pure projected headway advantage are resolved faster than conditions with a pure velocity advantage.
 - Drivers used shared mental models and observations before the start of the interaction to determine which driver will merge first.
- ACKNOWLEDGMENT**
- The authors thank Alexis Derumigny for the advice on statistical modelling.
- REFERENCES**
- [1] Y. Hassan, M. Sarhan, and M. Salehi, "Probabilistic model for design of freeway acceleration speed-change lanes," *Transp. Res. Record*, vol. 2309, no. 1, pp. 3–11, 2012. [Online]. Available: <https://doi.org/10.3141/2309-01>
 - [2] D. Lord and J. A. Bonneson, "Calibration of predictive models for estimating safety of ramp design configurations," *Transp. Res. Record*, vol. 1908, no. 1, pp. 88–95, 2005. [Online]. Available: <https://doi.org/10.3141/1908-11>
 - [3] C. Wang, C. Xu, J. Xia, Z. Qian, and L. Lu, "A combined use of microscopic traffic simulation and extreme value methods for traffic safety evaluation," *Transp. Res. Part-C, Emerg. Technol.*, vol. 90, pp. 281–291, May 2018.
 - [4] Q. Yang and H. N. Koutsopoulos, "A microscopic traffic simulator for evaluation of dynamic traffic management systems," *Transp. Res. Part-C, Emerg. Technol.*, vol. 4, no. 3 pp. 113–129, 1996.
 - [5] W. Schwarting, A. Pierson, J. Alonso-Mora, S. Karaman, and D. Rus, "Social behavior for autonomous vehicles," in *Proc. Nat. Acad. Sci.*, 2019, pp. 24972–24978.
 - [6] D. Sadigh, N. Landolfi, S. S. Sastry, S. A. Seshia, and A. D. Dragan, "Planning for cars that coordinate with people: Leveraging effects on human actions for planning and active information gathering over human internal state," *Auton. Robots*, vol. 42, pp. 1405–1426, Oct. 2018.
 - [7] W. Daamen, M. Loot, and S. P. Hoogendoorn, "Empirical analysis of merging behavior at freeway on-ramp," *Transp. Res. Record*, vol. 2188, no. 1, pp. 108–118, 2010. [Online]. Available: <https://doi.org/10.3141/2188-12>
 - [8] F. Marczak, W. Daamen, and C. Buisson, "Merging behaviour: Empirical comparison between two sites and new theory development," *Transp. Res. Part-C, Emerg. Technol.*, vol. 36, pp. 530–546, Nov. 2013.
 - [9] H. Wang, W. Wang, S. Yuan, X. Li, and L. Sun, "On social interactions of merging behaviors at highway on-ramps in congested traffic," *IEEE Trans. Intell. Transp. Syst.*, vol. 23, no. 8, pp. 11237–11248, Aug. 2022.
 - [10] A. R. Srinivasan et al., "Comparing merging behaviors observed in naturalistic data with behaviors generated by a machine learned model," in *Proc. IEEE Conf. Intell. Transp. Syst., (ITSC)*, 2021, pp. 3787–3792.
 - [11] A. Ji and D. Levinson, "A review of game theory models of lane changing," *Transportmetrica A, Transp. Sci.*, vol. 16, no. 3, pp. 1628–1647, 2020.
 - [12] H. Liu, W. Xin, Z. Adam, and J. Ban, "A game theoretical approach for modelling merging and yielding behaviour at freeway on-ramp section," *Transp. Traffic Theory*, vol. 3, pp. 197–211, Aug. 2007.
 - [13] K. Kang and H. A. Rakha, "Game theoretical approach to model decision making for merging maneuvers at freeway on-ramps," *Transp. Res. Record, J. Transp. Res. Board*, vol. 2623, pp. 19–28, Jan. 2017. [Online]. Available: <https://doi.org/10.3141/2623-03>
 - [14] X. Wan, P. J. Jin, F. Yang, J. Zhang, and B. Ran, "Modeling vehicle interactions during merge in congested weaving section of freeway ramp," *Transp. Res. Record, J. Transp. Res. Board*, vol. 2421, no. 1, pp. 82–92, 2014. [Online]. Available: <https://doi.org/10.3141/2421-10>
 - [15] C. Dong, J. M. Dolan, and B. Litkouhi, "Smooth behavioral estimation for ramp merging control in autonomous driving," in *Proc. IEEE Intell. Veh. Symp.*, 2018, pp. 1692–1697.
 - [16] J. F. Antin, S. Lee, M. A. Perez, T. A. Dingus, J. M. Hankey, and A. Brach, "Second strategic highway research program naturalistic driving study methods," *Safety Sci.*, vol. 119, pp. 2–10, Nov. 2019.
 - [17] R. Eenink, Y. Barnard, M. Baumann, X. Augros, and F. Utesch, "UDRIVE: The European naturalistic driving study," in *Proc. Transp. Res. Arena*, 2014, pp. 14–17.
 - [18] T. A. Dingus et al., "The 100-car naturalistic driving study. Phase 2, results of the 100-car field experiment," U.S. Dept. Transp., Virginia Tech Transp. Inst., Blacksburg, VA, USA, Rep. DOT HS 810 593, 2006.
 - [19] T. Stoll, L. Weihrauch, and M. Baumann, "After you: Merging at highway on-ramps," *Proc. Human Factors Ergonom. Soc. Annu. Meet.*, vol. 64, no. 1, pp. 1105–1109, 2020.
 - [20] A. Shimojo et al., "How impressions of other drivers affect one's behavior when merging lanes," *Transp. Res. Part-F, Traffic Psychol. Behav.*, vol. 89, pp. 236–248, Aug. 2022.
 - [21] O. Siebinga, A. Zgonnikov, and D. Abbink, "Interactive merging behavior in a coupled driving simulator: Experimental framework and case study," in *Proc. 13th Int. Conf. Appl. Human Factors Transp.*, 2022, pp. 516–525.
 - [22] O. Siebinga, A. Zgonnikov, and D. A. D. Abbink, "Data underlying the publication: Interactive merging behavior in a coupled driving simulator: Experimental framework and case study," Dataset, 4TU.ResearchData, Jul. 19, 2022, doi: [10.4121/19550377.V1](https://doi.org/10.4121/19550377.V1).
 - [23] E. Jolly, "Pymer4: Connecting r and python for linear mixed modeling," *J. Open Source Softw.*, vol. 3, no. 31, p. 862, 2018.
 - [24] O. Siebinga, A. Zgonnikov, and D. Abbink, "A human factors approach to validating driver models for interaction-aware automated vehicles," *ACM Trans. Human Robot Interact.*, vol. 11, pp. 1–21, Dec. 2022.
 - [25] A. Ji and D. Levinson, "Estimating the social gap with a game theory model of lane changing," *IEEE Trans. Intell. Transp. Syst.*, vol. 22, no. 10, pp. 6320–6329, Oct. 2021.
 - [26] W. Schwarting, J. Alonso-Mora, and D. Rus, "Planning and decision-making for autonomous vehicles," *Annu. Rev. Control, Robot., Autonom. Syst.*, vol. 1, no. 1, pp. 187–210, 2018.
 - [27] M. Naumann, L. Sun, W. Zhan, and M. Tomizuka, "Analyzing the suitability of cost functions for explaining and imitating human driving behavior based on inverse reinforcement learning," in *Proc. IEEE Int. Conf. Robot. Autom. (ICRA)*, 2020, pp. 5481–5487.
 - [28] B. Jonker, C. M. Riemsdijk, M. Birna van, and B. Vermeulen, "Shared mental models: A conceptual analysis," in *Proc. Int. Workshop Coord. Org., Inst., Norms Agent Syst.*, 2010, pp. 132–151.
 - [29] H. A. Simon, "Rational choice and the structure of the environment," *Psychol. Rev.*, vol. 63, no. 2, pp. 129–138, 1956.
 - [30] L. Klitzke, K. Gimm, C. Koch, and F. Koster, "Extraction and analysis of highway on-ramp merging scenarios from naturalistic trajectory data," in *Proc. IEEE 25th Int. Conf. Intell. Transp. Syst. (ITSC)*, 2022, pp. 654–660.
 - [31] C. F. Camerer, *Behavioral Game Theory: Experiments in Strategic Interaction*. Princeton, NJ, USA: Princeton Univ. Press, 2003.
 - [32] M. Schmidt-Daffy, "Prospect balancing theory: Bounded rationality of drivers' speed choice," *Accid. Anal. Prevent.*, vol. 63, pp. 49–64, Feb. 2014.
 - [33] M. A. Goodrich, W. C. Stirling, and E. R. Boer, "Satisficing revisited," *Minds Mach.*, vol. 10, no. 1, pp. 79–110, 2000.

- [34] G. Markkula, "Modeling driver control behavior in both routine and near-accident driving," *Proc. Human Factors Ergonom. Soc.*, vol. 58, no. 1, pp. 879–883, 2014.
- [35] G. Markkula, E. Boer, R. Romano, and N. Merat, "Sustained sensorimotor control as intermittent decisions about prediction errors: Computational framework and application to ground vehicle steering," *Biol. Cybern.*, vol. 112, no. 3, pp. 181–207, 2018.
- [36] W. Zhan et al., "Interaction dataset: An international, adversarial and cooperative motion dataset in interactive driving scenarios with semantic maps," 2019, *arXiv:1910.03088*.
- [37] H. Kita, "A merging-giveway interaction model of cars in a merging section: A game theoretic analysis," *Transp. Res. Part-A, Policy Pract.*, vol. 33, nos. 3-4, pp. 305–312, 1999.
- [38] J. F. Fisac, E. Bronstein, E. Stefansson, D. Sadigh, S. S. Sastry, and A. D. Dragan, "Hierarchical game-theoretic planning for autonomous vehicles," in *Proc. IEEE Int. Conf. Robot. Autom.*, 2019, pp. 9590–9596.
- [39] S. Coskun, Q. Zhang, and R. Langari, "Receding horizon Markov game autonomous driving strategy," in *Proc. Amer. Control Conf. (ACC)*, 2019, pp. 1367–1374.
- [40] M. Garzón and A. Spalanzani, "Game theoretic decision making for autonomous vehicles' merge manoeuvre in high traffic scenarios," in *Proc. IEEE Intell. Transp. Syst. Conf. (ITSC)*, 2019, pp. 3448–3453.
- [41] D. Isele, "Interactive decision making for autonomous vehicles in dense traffic," in *Proc. IEEE Intell. Transp. Syst. Conf. (ITSC)*, 2019, pp. 3981–3986.
- [42] O. Siebinga, "TraViA: A traffic data visualization and annotation tool in python," *J. Open Source Softw.*, vol. 6, p. 3607, Sep. 2021.
- [43] A. Talebpour, H. S. Mahmassani, and S. H. Hamdar, "Modeling lane-changing behavior in a connected environment: A game theory approach," *Transp. Res. Part-C, Emerg. Technol.*, vol. 59, pp. 216–232, Jul. 2015.
- [44] M. Elhenawy, A. A. Elbery, A. A. Hassan, and H. A. Rakha, "An intersection game-theory-based traffic control algorithm in a connected vehicle environment," in *Proc. IEEE Conf. Intell. Transp. Syst. (ITSC)*, 2015, pp. 343–347.
- [45] L. Banjanovic-Mehmedovic, E. Halilovic, I. Bosankic, M. Kantardzic, and S. Kasapovic, "Autonomous vehicle-to-vehicle (V2V) decision making in roundabout using game theory," *Int. J. Adv. Comput. Sci. Appl.*, vol. 7, no. 8, pp. 1–7, 2016.
- [46] O. Siebinga, A. Zgonnikov, and D. A. Abbink, "Modelling communication-enabled traffic interactions," *Roy. Soc. Open Sci.*, vol. 10, no. 5, 2023, Art. no. 230537.
- [47] G. Markkula et al., "Explaining human interactions on the road by large-scale integration of computational psychological theory," *PNAS Nexus*, vol. 2, pp. 1–13, May 2023.
- [48] A. Zgonnikov, D. Abbink, and G. Markkula, "Should I stay or should I go? Cognitive modeling of left-turn gap acceptance decisions in human drivers," *Human Factors*, 2022, to be published. [Online]. Available: <https://doi.org/10.1177/00187208221144561>
- [49] U. Durrani, C. Lee, and D. Shah, "Predicting driver reaction time and deceleration: Comparison of perception-reaction thresholds and evidence accumulation framework," *Accid. Anal. Prevent.*, vol. 149, Jan. 2021, Art. no. 105889.
- [50] T. Moers, L. Vater, R. Krajewski, J. Bock, A. Zlocki, and L. Eckstein, "The exid dataset: A real-world trajectory dataset of highly interactive highway scenarios in Germany," in *Proc. IEEE Intell. Veh. Symp. (IV)*, 2022, pp. 958–964.