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Community Detection for Temporal Weighted Bipartite Networks

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Abstract Community detection of temporal (time-evolving) bipartite networks is challenging because it can be performed either on the temporal bipartite network, or on various projected networks, composed of only one type of nodes, via diverse community detection algorithms. In this paper, we aim to systematically design detection methods addressing both network choices and community detection algorithms, and to compare the community structures detected by different methods. We illustrate our methodology by using a telecommunications network as an example. We find that three methods proposed identify evident community structures: one is performed on each snapshot of the temporal network, and the other two, in temporal projections. We characterise the community structures detected by each method by an evaluation network in which the nodes are the services of the telecommunications network, and the weight of the links between them are the number of snapshots that both services were assigned to the same community. Analysing the evaluation networks of the three methods reveals the similarity and difference among these methods in identifying common node pairs or groups of nodes that often belong to the same community. We find that the two methods that are based on the same projected network identify consistent community structures, whereas the method based on the original temporal bipartite network complements this vision of the community structure. Moreover, we found a non-trivial number of node pairs that belong consistently to the same community in all the methods applied.

Keywords Community detection · Temporal networks · Bipartite networks

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1 Introduction

Networks [10] have been used to represent complex systems. In a network, nodes represent the elements of a system, and their interactions or relations are represented by links. Community detection has been a fundamental network characterisation method to discover communities of nodes where nodes within a community are more similar or more strongly connected, whereas two nodes from different communities are less similar or weakly connected.

The detection of disjoint communities has been broadly studied, especially for static networks [4–6, 12]. Modularity [7], defined by Newman and Girvan, is one classic quantification of the quality of a partition of network nodes into disjoint groups, among many other possibilities. A partition of network nodes that maximises the modularity is recognised as the community structure of the network, and the corresponding maximal modularity is called the modularity of the network. Algorithms to detect communities that optimise the modularity have been widely proposed and applied; e.g., the greedy techniques proposed by Newman [8], and Blondel et al. [3]. These algorithms do not require the number of communities as an input.

Many real-world networks evolve over time. In a physical (virtual) contact network, two individuals are connected only when there is a face-to-face (email) contact instead of constantly. Community detection algorithms for static networks could be applied to detect the communities at each snapshot of the temporal (time evolving) network independently. Algorithms have been further developed for temporal networks to enhance the stability of the community structure over time, especially between two consecutive time steps [13]. Many real-world networks are static bipartite networks, where the nodes can be divided in two disjoint sets (such as authors and papers), and links (authorship relations) can only connect nodes from different sets. Bipartite graphs have been projected to networks composed of only one set of nodes in various ways, and classic static network community detection algorithms can be applied to the projected networks. Moreover, the definition of modularity has been further updated for static bipartite networks [2]. Correspondingly, algorithms to detect communities in a static bipartite network that optimised the bipartite network modularity have been designed [14].

A challenging problem is the community detection of a temporal weighted bipartite network [11] (e.g., a telecommunications network that records the data transfer between services and base stations, over time). For such networks, communities can be detected by diverse combinations of the network (original network or projected ones) and community detection algorithms (to detect the community structure per snapshot independently, or stably overtime). Each detection method identifies the communities, with possibly a specific community definition. The foundational questions are two. First, how to systematically design detection methods that utilise existing network projection methods and community detection algorithms, and second, and most importantly, how to compare the community structures detected by different methods, so that we can have an integrated overview.

In this work, we develop methodologies to address these two questions, illustrated by using a telecommunications network as an example. We introduce a basic framework to design community detection methods that systematically consider diverse network or network projection choices, and, correspondingly, various community detection algorithms. Three of the proposed methods recognise relatively evident community structures, at least in a fraction of network snapshots. To compare the community structures identified by these algorithms, we propose to construct an evaluation network that characterises the evident community structures detected by a method. By analysing the evaluation networks of these three methods, we obtain insights regarding, e.g., when different methods are applied, whether the frequency that a node pair belong to the same community is consistent, and whether the group of nodes that frequently belong to the same community differ. Our work may shed light on how to utilise existing community detection and network projection algorithms to obtain a multi-perspective vision of the community structure(s) of a network.

This paper is organised as follows. In Sect. 2, we design community detection methods. In Sect. 3, we evaluate and compare the community structures found by these methods. Finally, we present our conclusions in Sect. 4.

2 Methods

In this section, we propose methods to detect the community structure of a temporal bipartite weighted network from different perspectives. We start by introducing the temporal bipartite network. Second, we propose methods to project a temporal bipartite network to one or multiple networks composed of only one type of nodes. Finally, we briefly review the community detection algorithms that will be applied to the temporal bipartite network and to the projected networks, respectively.

2.1 Weighted Temporal Bipartite Network

Static bipartite networks are a type of networks in which the nodes can be divided in two disjoint sets, $\mathbb{S}$, of size S, and $\mathbb{U}$, of size U, and links ($\mathcal{L}$) can only connect nodes from different sets. A weighted bipartite network can be represented by its biadjacency matrix R, an $S \times U$ rectangular matrix in which each element $R_{s,u}$ represents the weight between nodes s and u.

Take the data transference between services and base stations in a telecommunications network as an example. It could be represented as a temporal weighted bipartite network (see Fig. 1). A temporal bipartite network observed or measured at discrete time $\mathbb{T} = [1, 2, ..., T]$, and composed of a set $\mathbb{S}$ of S services and a set $\mathbb{U}$ of U base stations can be represented by a $S \times U \times T$ temporal biadjacency matrix

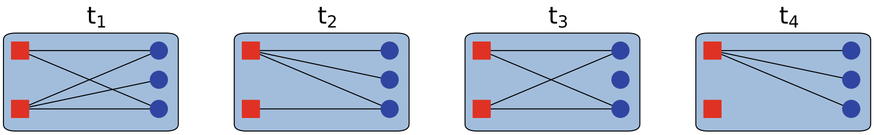


Fig. 1 Example of a temporal bipartite network at $T = 4$ time steps

Table 1 Basic properties of the bipartite telecommunications network

Number of services (S)	253
Number of base stations (U)	5166
Time window length (T) in steps	1440
Time window length in days	60
Time resolution per step	1 h

$\mathcal{R}$. Each element $\mathcal{R}_{s,u,t}$ represents the amount of data that has been transferred from service s to base station u at time t, where $s \in [1, S]$, $u \in [1, U]$ and $t \in [1, T]$. Basic properties of this telecommunications network can be found in Table 1.

2.2 *Projections of Weighted Temporal Bipartite Network*

Static bipartite networks have been projected to networks that contain only one type of nodes. The projected network, resulted from a given projection method, captures a specific relation among the same type of nodes. Such projection is motivated by the following. First, we might be interested in detecting communities within one type of nodes. Second, classic community detection methods can be further applied to a projected network. Projected networks are usually weighted networks, with the weights representing, e.g., a given kind of similarity between nodes. In this section, we will introduce diverse ways of projecting a temporal bipartite network, either per snapshot or as a whole, resulting in T projected networks or one projected network respectively. To illustrate our method, we project the temporal telecommunications network to networks among the services.

Static projection based on average cosine similarity. First, we explain a basic method that projects the temporal network as a whole to a static network of services. The volume of data transfer between a service i_1 and a base station j per step over time can be represented as a time series $\mathbf{w_{i_1,j}}$, where each element $\mathbf{w_{i_1,j}(t)} = \mathcal{R}_{\mathbf{i_1,j,t}}$ describes the volume of the data transfer between service i_1 and station j at time t. In this projection, the weight $\widehat{w}_{i_1,i_2}$ between two services i_1 and i_2 is the average cosine similarity between the two services' data transfer $\mathbf{w_{i_1,j}}$ and $\mathbf{w_{i_2,j}}$ with a base station j. Mathematically,

$$\widehat{w}_{i_1,i_2} = \frac{1}{U} \sum_{j \in \mathbb{U}_{i_1,i_2}} \frac{\mathbf{w_{i_1,j}} \cdot \mathbf{w_{i_2,j}}}{||\mathbf{w_{i_1,j}}||_2 \cdot ||\mathbf{w_{i_2,j}}||_2}, \tag{1}$$

where $\mathbb{U}_{i_1,i_2}$ represents the set of base stations that have transferred data from both nodes i_1 and i_2 at least once in $\mathbb{T}$. A large weight $\widehat{w}_{i_1,i_2}$ between two services implies that they are demanded by a base station over time in term of traffic in a similar way.

Temporal projection based on number of common neighbours. Temporal projection refers to methods that project each snapshot $\mathcal{G}(t)$ of the temporal bipartite network $\mathcal{G}$ to a network of the services. The first temporal projection method is defined as follows. In the projected network of $\mathcal{G}(t)$, two services are connected if they share any common neighbours in $\mathcal{G}(t)$, and the corresponding weight $\widehat{w}_{i_1,i_2}(t)$ is the number of common neighbours they have in $\mathcal{G}(t)$. That is,

$$\widehat{w}_{i_1,i_2}(t) = \sum_{j \in \mathbb{U}} \mathbf{1}_{\mathcal{W}(i_1,j,t)\mathcal{W}(i_2,j,t)>0}, \tag{2}$$

where the indicator function $1_{\mathcal{W}(i_1,j,t)\mathcal{W}(i_2,j,t)>0}$ equals one when the amount of data transfer $\mathcal{W}(i_1, j, t)$ and $\mathcal{W}(i_1, j, t)$ are both positive, or equivalently when j is a common neighbour for i_1 and i_2 at time t. A large weight $\widehat{w}_{i_1,i_2}(t)$ between two services indicates that both services have traffic with a large number of stations in common at time t.

Temporal projection based on the average geometric mean. At each time step t, we may wonder whether two services tend to have a large amount of data transfer with a common station, beyond their number of common neighbours. Hence, in the second temporal projection method, the weight $\widehat{w}_{i_1,i_2}(t)$ between two services projected from $\mathcal{G}(t)$ is defined as the geometric mean $\sqrt{w_{i_1,j}(t) \cdot w_{i_2,j}(t)}$ of their traffic with a common neighbour j, averaged over all common neighbours. Specifically,

$$\widehat{w}_{i_1,i_2}(t) = \frac{\sum_{j \in \mathbb{U}, \mathcal{W}(i_1,j,t)\mathcal{W}(i_2,j,t)>0} \sqrt{w_{i_1,j}(t) \cdot w_{i_2,j}(t)}}{\sum_{j \in \mathbb{U}} \mathbf{1}_{\mathcal{W}(i_1,j,t)\mathcal{W}(i_2,j,t)>0}}. \tag{3}$$

A large weight $\widehat{w}_{i_1,i_2}(t)$ between two services indicates that they tend to have a large amount of traffic with a station in common at time t.

2.3 Community Detection Methods

We adopt the concept of community and community detection algorithms that originated from modularity optimisation proposed by Newman [9] for networks of one type of nodes. We will illustrate how classic concepts and algorithms can be applied to detect the community structure of a weighted temporal bipartite network systematically.

2.3.1 Community Detection of Projected Networks

Classic community detection algorithms for static networks can be applied to the static projected network and the temporal projected network at each time step, to detect the communities of projected networks.

Consider first an undirected weighted network G that is composed of one type of nodes. It can be represented by a weighted adjacency matrix A. Given a weighted network and a partition of all the nodes into non-overlapping communities, the quality of this community partition can be measured by the modularity

$$Q = \frac{1}{2L} \sum_{i,j} \left[A_{i,j} - \frac{k_i \cdot k_j}{2L} \right] \delta_{c_i,c_j}, \tag{4}$$

where $k_i = \sum_j A_{i,j}$ is the sum of the weights of all the links connected to node i, so-called node strength; c_i is the label of the community to which node i belongs; the Kronecker delta $\delta_{c_i,c_j} = 1$, if $c_i = c_j$, and 0 otherwise; and $L = \frac{1}{2} \sum_{i,j} A_{i,j}$ is the total weight in the network.

The modularity of a partition describes the extent to which the weight of links within each community is bigger than the weight of those between communities. The modularity $Mod(G) \in [0, 1]$ of a network is the maximal modularity that could be obtained via community detection. Computing the modularity of a network is an NP-hard problem. We adopt the classic Louvain method [3] to obtain the approximate optimal modularity of a static network and its corresponding community partition.

The Louvain method [3]. This method starts with every node in its own community. For each node, it checks whether the modularity increases or not when changing its community to that of one of its neighbours. If there is an increase in modularity, then the community of that node is changed. This assignment step is repeated until there is no increase in modularity. The final community structure is considered as the optimal partition and the corresponding modularity is the modularity of the network. We will apply the Louvain method to detect the community structure of the static projected network and of the temporal projected network at each time t independently.

Stabilised Louvain method. To maintain the consistency of the community structures at two consecutive snapshots, we will also apply the stabilised Louvain method [1] to the temporal projected networks. Aynaud et al. modified the Louvain method, such that it considers the resulting community partition from the previous snapshot as the initialisation, whereas the modularity optimisation procedure remains the same.

2.3.2 Community Detection of a Temporal Bipartite Network

In the previous section, we have shown how to detect the communities of a temporal bipartite network by applying classic community detection methods to its projected

Table 2 Summary of the community detection methods proposed that combine the network and community detection algorithm differently. The methods that find evident community structures are highlighted in bold

Network	CD algorithm	Method name
Bipartite network	*Bi-Louvain*	*BiLouvain*
Cosine similarity static projection	Louvain	CS-Louvain
Common neighbours temporal projection	Louvain	CN-Louvain
	Stabilised Louvain	CN-stabilised
Geometric mean temporal projection	*Louvain*	*Geometric-Louvain*
	Stabilised Louvain	*Geometric-Stabilised*

networks. However, we can also apply a community detection algorithm for static bipartite networks to each snapshot $\mathcal{G}(t)$ of the temporal bipartite network.

The modularity definition for a static bipartite weighted network has been adapted by Barber [2] by redefining the null model to which we compare the weights within each community. We can express it as

$$Q = \frac{1}{L} \sum_{i=1}^{S} \sum_{j=1}^{U} \left[R_{i,j} - \frac{k_i \cdot d_j}{L} \right] \delta_{c_i, c_j}, \qquad (5)$$

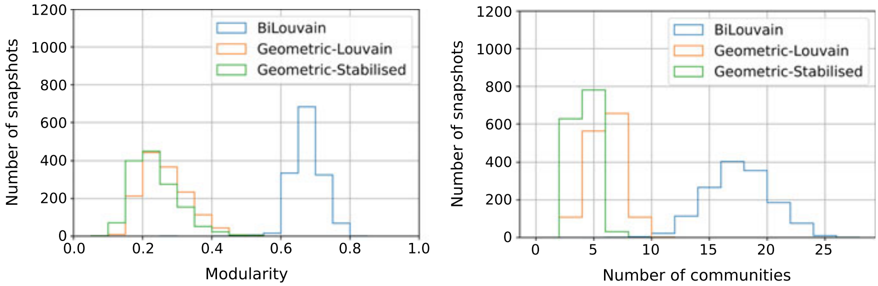
which considers the random weighted bipartite network with the same node strength as the given bipartite network as the null model.

The Bi-Louvain method [14]. Zhou et al. have proposed this community detection algorithm for static bipartite networks based on the Louvain method and modularity definition (5).

In summary, combinations of the aforementioned network choices, projected or not, and community detection algorithms lead to in total six community detection methods, as shown in Table 2.

3 Results

In this section, we evaluate the communities of services detected by the methods that we have proposed. First, we study to what extent the community structures found are evident through their modularity. Second, we investigate how the evident community structures (partition of services) detected by diverse methods provide a complementary or consistent vision.



(a) Modularity. (b) Number of communities.

Fig. 2 Representation of the **a** modularity and **b** number of communities of each of the methods

3.1 Modularity

Only the BiLouvain, Geometric-Louvain and Geometric-Stabilised methods have found evident community structure, i.e., the modularity is higher than 0.3 in, at least, a portion of the snapshots. Hence, the other methods will not be discussed further. Each of the three considered methods, partitions the nodes (services) into communities for each snapshot of the bipartite temporal network, or of the geometric mean temporal projection. In Fig. 2, we show the distribution of the modularity in a snapshot. The BiLouvain method shows the largest modularity of the three methods. For each of the three methods, we will further analyse the community structures in snapshots when the corresponding modularity is larger than 0.3.

3.2 Community Structure Comparison

We aim to compare the evident community structures found by these methods. In order to do that, we define an evaluation network to characterise the evident community partitions detected by a method.

3.2.1 Evaluation Network

An evaluation network contains the set $\mathbb{S}$ of services as nodes, and is constructed based on the community structures detected by a given method in every snapshot. Two nodes are connected if they have been assigned to the same community in, at least, one snapshot in which the modularity is larger than 0.3. The weight of the link is the total number of snapshots in which both nodes belong to the same community and the modularity is larger than 0.3. We build a weighted static evaluation network for each of the three methods. The weight distributions of the three evaluation networks are shown in Fig. 3. The average link weight in BiLouvain evaluation network

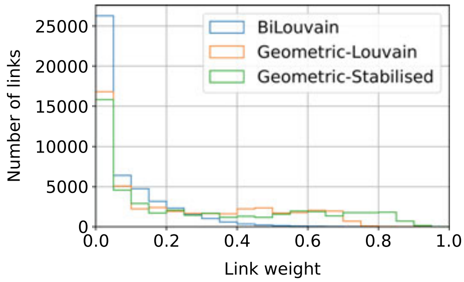


Fig. 3 Link weight distribution in each evaluation network

is evidently smaller than that in Geometric-Louvain and Geometric-Stabilised evaluation networks. This could be due to the larger number of communities detected by BiLouvain. This could also imply that the community structure detected by BiLouvain changes more significantly over time. The average link weight in the Geometric-Stabilised evaluation network is slightly larger than that in Geometric-Louvain evaluation network, supporting that Stabilised Louvain detects more stable community structure over time than Louvain.

3.2.2 Recognition Rate

First, we aim to understand whether two nodes that more frequently belong to the same community according to one method, or, equivalently, have a high weight in the corresponding evaluation network, also tend to belong to the same community more often according to another method. This is evaluated via the recognition rate between two methods, defined as follows. We rank the links in each evaluation network according to their weights. The set of fL links, with f being the ratio of links considered, with the highest link weights in the evaluation network derived from, e.g., the BiLouvain (Geometric-Louvain) method can be represented as J_f^{BL} (J_f^{GL}), where $L = \binom{S}{2}$ is the maximal possible number of links among S services, and $f \in [0, 1]$. The top f fraction recognition rate between, e.g., (the evaluation networks of) BiLouvain and Geometric-Louvain methods is defined as $r_{BL,GL}(f) = \frac{|J_f^{BL} \cap J_f^{GL}|}{|J_f^{BL}|}$, which measures the number of links in common between the two sets J_f^{BL} and J_f^{GL} normalised by the number of links fL in each set.

The link densities of the evaluation networks are all slightly above 0.7. Therefore, we compute the recognition rate for $f \in (0, 0.7]$. The top f recognition rate between random ranking of links and any ranking of links is f. As we can see in Fig. 4, the top f recognition rate between any two community detection method is higher than f, suggesting that all the evaluation networks share similarity in identifying similar set of links with a large weight. Moreover, the co-occurrence between the top links in Geometric-Louvain and Geometric-Stabilised is the highest. This is in line with the fact that Geometric-Louvain and Geometric-Stabilised use the same network

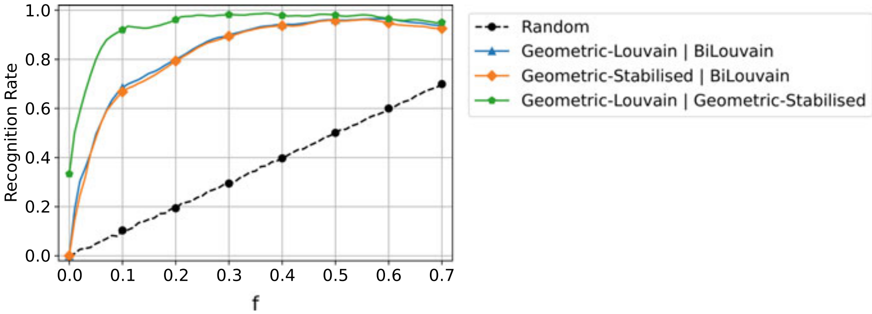


Fig. 4 Top f recognition rate between the proposed methods

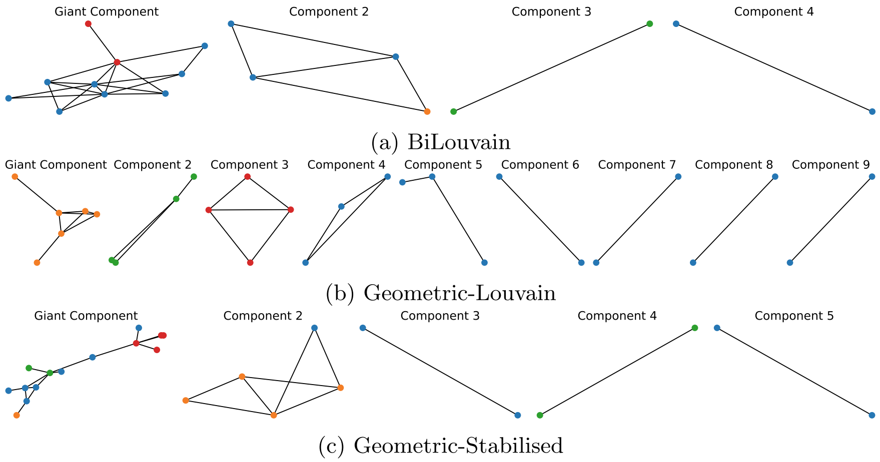


Fig. 5 Sub-evaluation network composed of 50 links with the largest weights derived by **a** BiLouvain, **b** geometric-Louvain and **c** geometric-stabilised respectively. All nodes are coloured in blue except those in the largest, second largest, and third largest component in the geometric-Louvain sub-evaluation network, which are coloured in orange, green and red, respectively

projection for community detection. The visualisation of the sub-evaluation network composed of the top 50 links with the largest link weight derived by each method in Fig. 5 reveals the same. For example, nodes in the largest, second largest, and third largest components of the Geometric-Louvain sub-evaluation network (coloured in orange, green and red respectively) are more likely to appear, or to be connected, in the Geometric-Stabilised sub-evaluation network in comparison to BiLouvain.

3.2.3 Persistent Community Component

Besides the similarity of two evaluation networks in identifying links with large weight, measured by the recognition rate, we explore further the similarity between

Table 3 Number of snapshots in which all the nodes in the indicated component (or clique) in the sub-evaluation network with 50 links belong to the same community

Method	BiLouvain	Geometric-Louvain	Geometric-stabilised
Giant component	11	174	95
Second largest component	124	372	336
Largest clique within giant component	141	311	472

two methods in identified groups of nodes that frequently belong to the same community, so-called persistent community component. Finding the persistent groups of size m requires the counting of the number of snapshots in which each of the $\binom{S}{m}$ groups belong to the same community, according to a given community detection method. Its computational complexity is high and it is difficult to be simplified when the community structure changes over time.

Identifying whether components in the aforementioned sub-evaluation network, composed of links with the highest weight, are persistent, could be an intuitive and insightful start. The motivation is that a group of nodes may frequently belong to the same community if pairs of them often belong to the same community.

The number of snapshots in which all the nodes in the largest (second largest) component of a sub-evaluation network fall into the same community is shown in Table 3. We find that nodes in the largest component of the BiLouvain sub-evaluation network belong to the same community less frequently compared to that of other sub-evaluation networks, although the largest component of the BiLouvain sub-evaluation network is denser. This difference in frequency is evident, especially in view that the total number of snapshots that have a modularity larger than 0.3 is far larger when BiLouvain is applied. For the Geometric-Louvain and Geometric-Stabilised methods, nodes in the biggest component and, especially, in the second biggest component belong to the same community in up to almost a quarter of the snapshots that have an evident community structure. The same observation holds when examining whether nodes in the second largest component and the largest clique within each giant component are persistent community components. This difference could be due to the lower average link weight in the BiLouvain sub-evaluation network, and the highly dynamic community structure detected by BiLouvain over time.

We find that each component in Fig. 5 tends to be persistent and composed of a specific type of services, e.g., related to social networks or provided by the same brand. The biggest component of the Geometric-Stabilised method, though persistent, is an exception, containing various types of services.

4 Conclusions

In this paper, we define multiple methods to detect community structures of a temporal weighted bipartite network. We study how the partitions found by different community detection methods align or complement each other, illustrated via a telecommunications network. The three community detection methods that find evident community structures are performed either on the original bipartite temporal network or on a temporal projection; i.e., projecting each temporal network snapshot independently. To compare them beyond their difference in community definition, we define an evaluation network to characterise the community structures found by each method, in which the nodes are the services of the telecommunications network, and the weight of the links between them is the number of snapshots in which both services belong to the same community. Then, we compare which nodes are the ones that are most commonly clustered together, first in terms of node pairs through the recognition rate, and then in terms of groups of nodes by studying the components of the sub-evaluation network with the highest-weight links. The two methods that partition the network based on the same temporal projection, using Louvain and stabilised Louvain, respectively, identify consistent community structures, whereas the third method, based on the original temporal bipartite network, provides a complementary perspective of the community structure. Moreover, we find that all three methods share a non-trivial number of common node-pairs that are often in the same community.

Our methodology, exemplified by a limited choice of candidate algorithms and one network, is the starting point to explore the multi-perspective vision of the community structure of a temporal bipartite network. It could be further improved by investigating, e.g., the time series associated to each link of an evaluation network that records the time stamps when two nodes belong to the same community, and networks with known ground truth community structure.

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