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Embodied Design versus Dynamic Visualization: Benefits for a Far Transfer Problem Solving in Trigonometry

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Abstract: The development of digital technologies enables visualizations of many scientific relations. In this paper, we question the efficiency of the visualizing tools from a theoretical perspective that combines cultural-historical and radical embodied approaches. Dynamic visualizations expose the target relations in a ready-made form, while embodied action-based design provides students with an opportunity to re-invent the target relations through active sensory-motor coordinations. In a multiple-case study, we compare these design genres for learning trigonometric relations. While dynamic visualizations were more efficient in simple tasks, embodied action-based design enhanced students' reasoning in a far transfer task.

Introduction

New technological tools provide unique opportunities for visualizing scientific relations, such as relativity theory principles in virtual reality or trigonometry relations in dynamic geometrical environments (e.g., DeJarnette, 2018). Putting the development of digital technologies in the broader context of technology evolution in society, researchers from cultural-historical tradition stress that technologies appear as a reification of cultural practices (Pea & Cole, 2019; Wenger, 1998). The most stable aspects of practice become fixated in a form of visual inscriptions and other artifacts. When providing students with a visualization of some mathematical or scientific relation, educators present a former scientific practice of establishing the target relation in a ready-made form. For example, a sine function was constituted in the history of mathematics as a practice of relating an arc on a circle to a chord (Toomer, 1974); a sine graph appeared later as depicting this relation (presumably by Albert Dürer). In many textbooks a sine graph is drawn as already connected with a unit circle. In this case, students come to learn about this relation in a ready-made form.

However, some approaches postpone involving ready-made scientific artifacts into teaching practice (e.g., Gravemeijer, 1999). The students are invited to re-invent the target mathematical or scientific relations—such as a relation between an arc and *y*-coordinate on a unit circle to *x*- and *y*-coordinates of a sign graph—in their own practice before embedding them into artifacts and technologies. These approaches are particularly meaningful from a radical embodied perspective that considers cognitive functions to be complex dynamic systems of perception-action loops (Abrahamson & Sánchez-García, 2016; Chemero, 2009). As students learn to use cultural artifacts, they incorporate artifacts into their extended cognitive functions by including them into perception-action loops. We suggest referring to this process as a genesis of *body-artifacts functional systems* (Shvarts, Alberto, Bakker, Doorman, & Drijvers, under review). In this study, we investigate if educational technology that invites students to re-invent a sine graph is beneficial for furthering students' reasoning compared to the technology that automatically generates a sine graph without students' active participation. We expect that the students who re-invent artifacts that reified mathematical practices—such as a sine graph—acquire perception-action loops for building these artifacts. Later, they might use the understanding of artifacts' constitution in reasoning. Contrary, students who encounter an artifact as automatically generated, might lack understanding of how it was built, similar to using a smartphone without knowing how it works.

Action-based embodied design versus dynamic visualizations

Various dynamic visualizations have been designed to show the students the relations between a point on the unit circle and a correspondent point on a sine graph (e.g., DeJarnette, 2018). In these technological solutions, educators invite students to *observe* mathematical relations, which are already embedded into technology. However, the demonstration of the relations does not automatically lead to their understanding by the students (Yerushalmy, 1991), questioning these designs' efficiency. As we combine a cultural-historical approach that states that perception develops (Radford, 2010; Vygotsky, 1997) and a radical embodied approach that claims perception serves action (Abrahamson & Sánchez-García, 2016; Maturana & Varela, 1987), we might wonder if the students are able to *notice the target relations* that designers are trying to show them.

Action-based embodied design (Abrahamson & Sánchez-García, 2016) provides a principal alternative to visualizing technologies. Mathematical relations are not explicitly exposed to the student, but delivered as a motor problem (Bernstein, 1967). Embodied interactive activities provide continuous feedback to students' hand-movements. Aiming to receive positive feedback, students maintain target distances on a visual display in a target mathematical relation. Thus students reinvent mathematical relations and artifacts in their sensory-motor practices. Instead of relying on students' existing perception abilities, an action-based embodied design fosters the development of new forms of perception by establishing sensory-motor coordinations.

The essence of the difference between dynamic visualizations and action-based embodied designs lies in the different answers to the question, "Who gets to constrain the student's interaction with the virtual objects: the software or the student?" (Abrahamson & Abdu, 2020, Context section). The embodied action-based design's research program assumes that emergent task constraints in interaction with a student might be beneficial for targeted conceptual learning. In this study, we collect empirical evidence to corroborate or reject this theoretical expectation. Thus we question: *What are the differences in reasoning about a novel trigonometry problem in learning with action-based embodied design and learning with interactive dynamic visualization?*

Methodology

We report the results of a contrasting multiple-case study (Miles & Huberman, 1994) that compares two design genres: embodied action-based design and interactive dynamic visualization.

Learning materials

Both types of interactive designs were programmed in the Numworx digital environment within broader design research on the embodied approach to trigonometry learning (see Alberto, Bakker, Walker-van Aalst, Boon, & Drijvers, 2019). There were four parts in the learning materials dedicated to (1) sine value in a unit circle; (2) relation between an arc of a unit circle and *x*-coordinate of a sine graph; (3) relation between a sine value in a unit circle and *y*-coordinate on a sine graph; (4) constructing a sine graph by relating an arc and a sine value on the unit circle to *x*- and *y*-coordinates of the sine graph (see Figure 1). Each part consisted of three sequential types of tasks: interactive sensory-motor task, reflection task, and mathematical task. In sensory-motor tasks, the students studied the relations between a point on a unit circle and a point on a sine graph. Then the students reflected on these relations in written form and through multiple-choice task. Finally, they solved mathematical tasks based on the studied relations.

Contrasting conditions

In the action-based embodied design condition (ED), the students studied the target relations in the form of a motor problem as they manipulated two points: a point on the unit circle and a point on the Cartesian coordinates. Continuous feedback from the system informed them when the points were in the corresponding positions by changing the color from red to green (see Figure 1). As students continuously maintained green feedback, they developed new coordinations, thus incorporated the target mathematical relations in their bodily functional system of perception-action loops. In the dynamic visualization condition (DV), the students moved only one point on the unit circle, while the point on the Cartesian coordinates was automatically moved in correspondence with the first point. So the experimental factor determined whether the students were maintaining the target relation in sensory-motor enactment based on the feedback or whether the target relation was maintained by the software and exposed to the students in a ready-made form. Figure 1 shows tasks from two conditions in Part 4. The reflection and mathematical tasks were the same in both conditions.

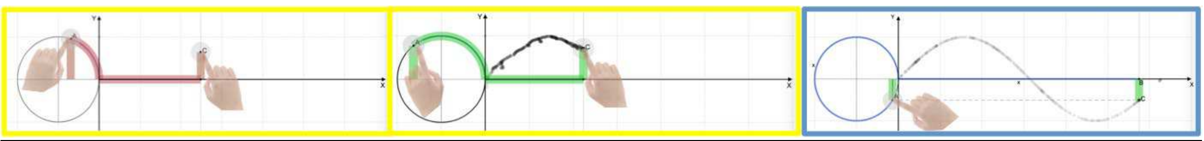


Figure 1. Embodied design (in a yellow frame, red and green feedback, ED): a sine graph emerged from active coordination of two hands. Dynamic visualization (in a blue frame, DV): a sine graph was automatically generated when moving one hand around the unit circle.

Research protocol and participants

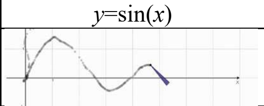
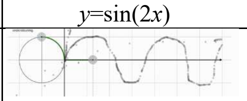
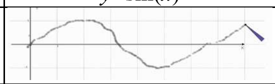
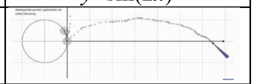
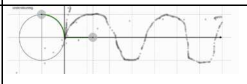
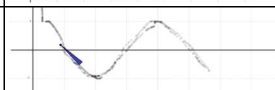
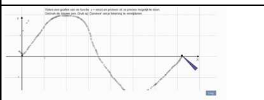
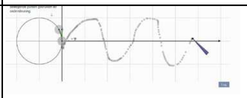
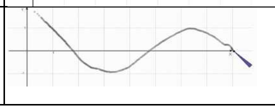
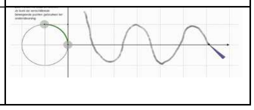
The procedure consisted of a pre-test, learning stage, a post-test, and an interview. The post-test included near transfer tasks similar to the tasks from the learning materials, such as estimating $\sin(\frac{7}{12}\pi)$ and drawing a graph $y=\sin(x)$, and far transfer tasks such as estimating sin(12) and drawing a graph of $y=\sin(2x)$. In the interview, we asked the students to explain how they solved the post-test tasks and scaffolded them towards accomplishing these tasks in a case of mistake, thus investigating their reasoning. All data collection was run online. The students (14-15 years old) had no pre-knowledge in trigonometry beyond trigonometric relations in a right triangle. Three students learned with embodied design and four students learned with dynamic visualizations.

Results

In solving the algebraic near transfer tasks in the post-test, the students from the DV condition performed better than the students from the ED condition: four DV students answered 100%, 57%, 50%, and 29% of the tasks, while there ED students answered 50%, 29%, and 14% of the tasks. In two cases, the ED students were still unsure how to quantify the sine value on a unit circle. However, as soon as the researcher in the interview would ask the students to recall their embodied experience, they could easily correct their mistakes. The disadvantage of ED might be explained by the need to communicate with a real tutor for a successful transition of embodied experiences into mathematical discourse when learning with embodied designs (e.g., Flood, 2018). This communication was provided only in the interviews, after the post-test.

For the drawing tasks, students from ED condition clearly benefited over students from VD condition. The vivid differences were observed in the far transfer task of drawing the graph *y*=sin(2*x*). The most challenging part of this task lies in the idea that the graph's period is two times smaller when *x* is multiplied by two. As Table 1 illustrates, all three students from the ED condition expressed this change in the period in their drawings. They could explain their drawings referring to the point moving around the unit circle two times quicker for the formula *y*= sin(2*x*), and, consequently, the waves of the graph appearing more frequently. Only one out of four students from the DV condition came up with this change in the graph; two students drew the graph extending in the horizontal direction, and one did not come up with any drawing (not shown in Table 1).

Table 1: Students' drawing of in the post test: initial sine graph and sin(2*x*) in the far transfer task.

Embodied action-based condition (ED)			Dynamic visualization condition (DV)		
	y=sin(*x*)	*y*=sin(2*x*)		*y*=sin(*x*)	*y*=sin(2*x*)
Ellie			Mark		
Janice			Bella		
Emma			Yana		

Emma, a student from the ED condition, explained: "*I assume that with 2x that [point] on the circle will travel twice the distance, instead of [the same distance as] on the x-axis."* While explaining, she doubled two distances in her gesturing around the unit circle: she stopped at the distance π/6 and then at the distance π/3, she then stopped at the distance π/2 and then at the distance π. Further, Emma moved synchronously two fingers in the air, one going around the circle and another going on the *x*-axis, thus repeating the enactment from the embodied learning. However, in this gesture, she moved the finger around the unit circle faster, in correspondence with the new formula. Emma dived into the process of constructing the sine graph from a unit circle as she gestured out the coordination of the distance along the unit circle and the distance on the graph bodily adjusting this sensory-motor coordination to the new situation. The other ED students gave similar explanations, referring to their experiences of coordinating a point on the Cartesian plane with the unit circle.

The students from the DV condition also recognized the unit circle as being useful for building a sine graph. However, they could not adjust the construction process to the new situation. Instead, they were trying out any option of enlarging or diminishing the sine graph two times in a vertical or horizontal direction. Mark referred to the unit circle explaining how the initial sine graph was built, but then concluded with enlarging the graph along the *x*-axis doubling the distance due to the multiplication: "*Now it says 2x so I think that now the arc of the graph that is above the x-axis is twice as long. Because the distance that is covered on the circle is twice covered in the graph."* Ada had drawn nothing, and when asked in the interview she supposed the graph should be two times longer. Later she changed her mind and suggested that graph "*Will become double as high."* Yana had a correct idea of diminishing the graph's period: "*I thought I am just going through that circle twice. […] That I was just going to draw it [a wave] twice.*" However, she was uncertain and changed her mind: She gestured along *y*-axis and said that she should adjust the graph along *y*-axis as sine is *y*-value.

The students from the DV condition reasoned about the sine graph as an outcome of the construction procedure (which was done by the machine during their learning). They tried out any possible transformation that would match a change from *x* to 2*x* and did not consider if these suggested transformations harm the relations between a sine graph and a unit circle. On the contrary, ED students had embodied experience of actively re-inventing a sine graph based on a unit circle. This experience helped maintain the entire operation of graph building coherent and implement the formula's changes into this operation.

Conclusions

The embodied experience of actively coordinating distances on a unit circle and Cartesian plane helped the students from the embodied design condition (ED) realize how the sine graph was constructed. The students could apply this understanding in further reasoning about a composite function graph without being ever taught this new material. Thus, an embodied design might facilitate creative problem-solving in far-transfer tasks. Remarkably, students' reasoning involved adapting the sensory-motor coordinations, thus it run at the embodied level. For the students from the dynamic visualizations condition (DV), a sine graph was not constrained by their own enactment but by the software (Abrahamson & Abdu, 2020). The students could not flexibly adjust the procedure of sine graph construction to the new situation. Instead, they tried to adjust an outcome of constructing procedure—the sine graph per se. Without caring about relations between a unit circle and a sine graph, the students tried to enlarge y=sin(x) graph in *some* direction, matching the multiplication in the formula.

Ready-made mathematical relations—exposed in dynamic visualization of a sine graph—presented mathematical practice in a reified form (Wenger, 1998), thus an artifact—a sine graph—extended bodily functional system (Shvarts et al., under review) in a rigid way and fostered unwarranted manipulations. The embodied action-based design provided an opportunity to actively constrain the target relations. The students appropriated trigonometric relations into perception-action loops of their flexible body-artifacts functional systems. Once the mathematical relations were included in the systems of perception-action loops, the students could adjust them to a novel situation in further reasoning. While this small-scale study results are well explained from the chosen theoretical perspective, further scaling up is needed to corroborate the conclusion of the embodied design effectiveness over dynamic visualizations in far-transfer tasks.

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