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Train scheduling with flexible coupling and decoupling at stations for an urban rail transit line

Kangqi Zhao¹, Yihui Wang¹, Miaomiao Ding¹, Shukai Li¹, Egidio Quaglietta², Lingyun Meng³

Abstract—More and more people in big cities choose urban rail transit as the main means of public transportation. With the increasing unbalanced passenger flow in time and space, the traditional operation mode with fixed train formation (or composition) is difficult to satisfy the varying passenger demands. This paper distinguishes different train formations in urban rail transit, and specifies the definition and the operation process for flexible composition of trains. An integrated train scheduling problem with flexible train composition is proposed, where the key constraints for practical train operation and the utilization of rolling stocks are considered. These constraints involve turnaround constraints, flexible train formation constraints, headway constraints and passenger flow constraints. The resulting problem is a mixed integer nonlinear programming problem, which can be transformed into a mixed integer linear programming problem and then be solved using existing optimization solvers, e.g., CPLEX. Based on the practical infrastructure and passenger demand data of the Beijing Daxing International Airport Express, a set of case studies is carried out to demonstrate the effectiveness of the presented model and solution approach. The computational results show that the train schedule with flexible train compositions can largely reduce the number of waiting passenger when compared with the train schedules with fixed train compositions and with multiple train compositions.

I. INTRODUCTION

With the rapid urban expansion caused by economic development, urban rail transit has gradually become the first choice for many passengers' daily commuting. One of important roles of urban rail transit is to alleviate the congestion of urban traffics. Since urban rail transit is mainly dedicated to passengers, the rail operators generally design the train schedules according to the daily passenger flow, where the arrival and departure times of train services are specified in the scheduling. However, due to the functional settings of big cities, the passenger flow is often highly unbalanced in both temporal and spatial dimensions. The appropriate train schedules that can satisfy the unbalanced passenger demand and minimize the operation cost of rail

operators are important for the daily operation of urban rail transit lines.

The passenger-demand-oriented train scheduling has been a hot research topic for many years. Wang et al. [1] proposed an event-driven model including arrival events, departure events and passenger arrival rates changing events to describe the operation of the trains, where sequential quadratic programming (SQP) is used to solve the train scheduling problem. It is proved that the non-fixed headway train schedule is better than the traditional fixed-headway train schedule. Yin et al. [2] studied the over-crowdedness problem of the transfer stations in metro line, where a mixed integer linear programming model is proposed to improve an adaptive large neighborhood search (ALNS) algorithm. The passenger flow information is generally uncertain, Cacchiani et al. [3] proposed three mixed integer linear programming models by adopting the light robustness method to generate the robust train schedules. Moreover, Bešinović et al. [4] adjusted the train schedules and gate control factors to limit the number of passengers at the platform in case of disruptions.

Most existing research on passenger-demand-oriented train scheduling for urban rail transit assumes that the train formation is fixed for the whole operation period. However, the train schedules with fixed train composition cannot adapt to unbalanced passenger demands flexibly and precisely, which could result in wasting of capacity supply in off-peak hours and lack of capacity supply in peak hours. Similar trends could also happen if the passenger demand in the two operational directions of the urban rail transit line is highly unbalanced. Hence, the change of train formations during the operation attracts more and more attentions. Nold et al. [5] divided the train coupling technologies into four generations according to the coupling complexity. Similarly, Liu et al. [6] summarized the existing train formation techniques and defined four different train formations, i.e., fixed train formation, multiple train formation, flexible train formation and dynamic train formation. These four train formation technologies are illustrated in Fig 1. Specifically, multiple train formation means that the composition of trains can be changed at the depot via coupling and decoupling. Hence, there is no coupling and decoupling operations during the operations and trains with different compositions can be used during the peak and off-peak hours to satisfy passenger demands for multiple formation mode. Moreover, Hu [7] developed a multi-objective mixed integer nonlinear programming for the train scheduling with multiple formations, where the overall energy consumption was effectively reduced by 20.4% with the introduction of short and long

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Train formation mode	Diagrammatic presentation	Generation of train coupling technologies
Fixed formation		0
Multiple formation		1
Flexible formation		2
Dynamic formation		3

Fig. 1: Different train formation techniques via coupling and decoupling[5][6]

train compositions.

Since the multiple train composition cannot be adjust the composition of trains on the way, this may not be well suited for the highly spatial unbalanced passenger demands. The flexible composition is presented, where the train units are still physically connected via couplers. However, the coupling and decoupling of train units at stations take some time [8]. Recently, there are several practical experiments carried out by Shanghai metro operators, which demonstrates that the flexible coupling and decoupling of train units can be completed in a short time without affecting the operating efficiency of the urban rail transit system [9]. Hence, this paper proposes a mixed integer nonlinear programming model for the train scheduling with flexible train composition to better match the spatially unbalanced passenger flow demand. We also note that virtual coupling or dynamic coupling first presented by Bock et al. [10] is also a promising research direction. However, the practical experiments or application of virtual coupling is still not ready yet, so we focus on flexible train composition in this paper particularly.

The remainder of the paper is organized as follows. Section II discusses the statement of the scheduling problem with flexible train composition and presents the assumptions for the model formulation. In Section III, a mixed integer nonlinear programming model is presented, including all the required constraints and objective functions. In Section IV, a case study is used to verify the effectiveness of the proposed model. The conclusion is presented in Section V.

II. PROBLEM STATEMENT

A. Coupling and decoupling of trains at stations

The coupling and decoupling of trains can only occur at the specified platforms. In Fig 2, Train 1 arrives at the station first and it waits at this station to be coupled with Train 2. Since trains are allowed to couple and decouple with each other at stations, the signalling system allows two trains entering the same platform, which is different from the current setting of the metro signalling systems. When Train 2 enters the station, it is physically coupled with Train 1. Then these two trains form a convoy and depart from the station simultaneously. We note that the arrival and departure times of Train 1 and Train 2 would be the same until they

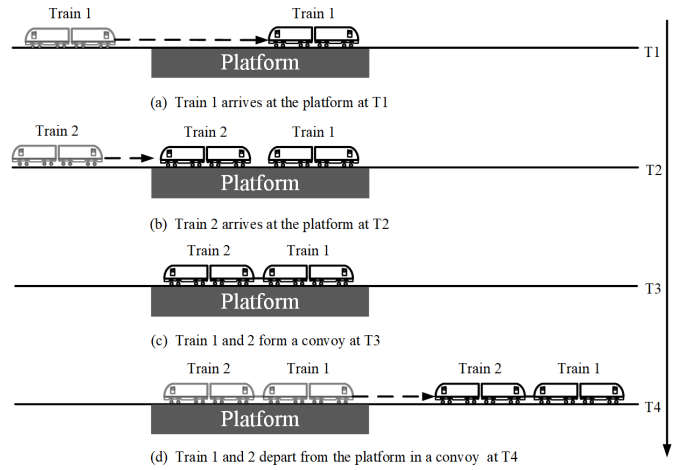


Fig. 2: The coupling process of train 1 and 2 at a platform

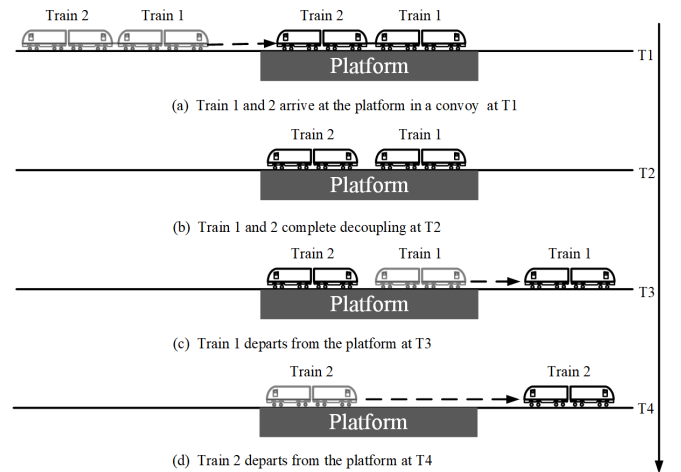


Fig. 3: The decoupling process of train 1 and 2 at a platform

are decoupled with each other. In a similar way, trains can be decoupled at platforms as shown in Fig 3, where Train 1 and Train 2 arrive at the platform simultaneously as a convoy. After the decoupling process, Train 1 would depart from the station first and Train 2 then leaves the platform after a certain time headway.

B. Assumptions

In order to formulate the problem of train scheduling under flexible train compositions in urban rail transit line, we have made the following assumptions:

- Trains could be coupled and decoupled at all stations.
- Overtaking is not allowed during the operations and trains stop at every station.
- The passenger arriving rates at platforms and the alighting ratios are constants in small time intervals.

III. MATHEMATICAL FORMULATION

A. Definition of parameters and variables

Table III in Appendix lists the parameters, decision variables and subscripts used in our formulation.

B. Train operation constraints

1) *Departure and arrival times constraints:* The arrival and departure time of train services can be calculated by

$$d_{i,j}^{\text{up}} = a_{i,j}^{\text{up}} + \tau_{i,j}^{\text{up}}, \quad (1)$$

$$\tau_{i,j}^{\text{up}} \geq \tau_j^{\text{up,min}}, \quad (2)$$

$$\tau_{i,j}^{\text{up}} \leq \tau_j^{\text{up,max}} + |c_{i-1,j}^{\text{a,up}} - c_{i-1,j}^{\text{d,up}}|D + |c_{i,j}^{\text{a,up}} - c_{i,j}^{\text{d,up}}|D. \quad (3)$$

Due to possible coupling and decoupling operations, the dwell time of the train services at the station is uncertain. If train services i and $i-1$ need to be coupled or decoupled at station j , the dwell time of the train services would be longer when compared with normal operations (i.e., without coupling and decoupling). It is worth to note that many equations are divided into up and down direction and have similar forms. For the sake of simplicity, this paper only lists the equations in the up direction. The equations with the down direction can be easily obtained by using the variables and parameters for the down direction in Table III. The relationship of the departure time and the next station arrival time can be calculated by

$$a_{i,j}^{\text{up}} = d_{i,j-1}^{\text{up}} + r_{i,j-1}^{\text{up}}. \quad (4)$$

2) *Turnaround operation constraints:* We only consider the turnaround operations at the end of the line (i.e., the station 1 and J). For station J, the turnaround operation do not change the train service index, but change the running direction of the train, which can be written as

$$r^{\text{turn,min}} \leq a_{i,J}^{\text{dn}} - d_{i,J}^{\text{up}} \leq r^{\text{turn,max}}. \quad (5)$$

For station 1, not only does the running direction of the train change, but also the train service index change. It can be calculated by

$$a_{i_2,1}^{\text{up}} - d_{i_1,1}^{\text{dn}} \leq r^{\text{turn,max}} + (1 - \gamma_{i_1,i_2})R, \quad (6)$$

$$a_{i_2,1}^{\text{up}} - d_{i_1,1}^{\text{dn}} \geq r^{\text{turn,min}} + (\gamma_{i_1,i_2} - 1)R. \quad (7)$$

It is worth to note that if service i_1 do not be connected with any other services, that service i_1 would go back to the depot directly after departing from the station 1 in the down direction. Each service can connect with at most one service, i.e.,

$$\sum_{i_1=1}^{i_1=I} \gamma_{i_1,i_2} \leq 1. \quad (8)$$

3) *Flexible train formation constraints:* When the arrival times of multiple adjacent train services at a station are taken the same value, while the departure times are different, it is considered that these train services are decoupled at this station. Similarly, when several train services arrive at this station at different times but depart from this station at the same time, these train services have been coupled at this station. Therefore, the convoy status at arrival time and departure time (i.e., $c_{i,j}^{\text{a,up}}$ and $c_{i,j}^{\text{d,up}}$) are taken into account to describe the composition operation of trains at stations.

The maximum number of train compositions in a convoy is denoted by n^{max} , there we have

$$\sum_{i'=i}^{i+n^{\text{max}}-1} c_{i',j}^{\text{a,up}} \leq n^{\text{max}} - 1, \quad (9)$$

$$\sum_{i'=i}^{i+n^{\text{max}}-1} c_{i',j}^{\text{d,up}} \leq n^{\text{max}} - 1. \quad (10)$$

The coupling status of trains cannot be changed between stations, i.e.,

$$c_{i,j}^{\text{a,up}} = c_{i,j-1}^{\text{d,up}}. \quad (11)$$

Moreover, if two trains are coupled, their arrival and departure time must be identical, which can be formulated as

$$c_{i,j}^{\text{a,up}}(a_{i+1,j}^{\text{up}} - a_{i,j}^{\text{up}}) = 0, \quad (12)$$

$$c_{i,j}^{\text{d,up}}(d_{i+1,j}^{\text{up}} - d_{i,j}^{\text{up}}) = 0. \quad (13)$$

4) *Headway constraints:* If two adjacent train services are not coupled, their arrival times and departure times should satisfy the following constraints:

$$(1 - c_{i,j}^{\text{a,up}})(a_{i+1,j}^{\text{up}} - a_{i,j}^{\text{up}} - h^{\text{min}}) \geq 0, \quad (14)$$

$$(1 - c_{i,j}^{\text{a,up}})(a_{i+1,j}^{\text{up}} - a_{i,j}^{\text{up}} - h^{\text{max}}) \leq 0, \quad (15)$$

$$(1 - c_{i,j}^{\text{d,up}})(d_{i+1,j}^{\text{up}} - d_{i,j}^{\text{up}} - h^{\text{min}}) \geq 0, \quad (16)$$

$$(1 - c_{i,j}^{\text{d,up}})(d_{i+1,j}^{\text{up}} - d_{i,j}^{\text{up}} - h^{\text{max}}) \leq 0, \quad (17)$$

$$(1 - c_{i,j}^{\text{a,up}})(1 - c_{i,j}^{\text{d,up}})(a_{i+1,j}^{\text{up}} - d_{i,j}^{\text{up}}) \geq 0. \quad (18)$$

Specifically, Equation (18) ensures that there is only one train in the platform at any time.

5) *Passenger flow constraints:* In order to describe the unbalanced time distribution of passengers, the operation period is divided into small intervals. The departure times of train services could fall into a particular time interval, which can be written as

$$\sum_{m=1}^M t_{m-1} x_{i,j,m}^{\text{up}} \leq d_{i,j}^{\text{up}} \leq \sum_{m=1}^M t_m x_{i,j,m}^{\text{up}}, \quad (19)$$

$$\sum_{m=1}^M x_{i,j,m}^{\text{up}} = 1. \quad (20)$$

We define the number of newly arrived passengers for service i at station j as the number of arrival passengers between the departure time of the preceding train and the departure time of the following train, which is calculated by Equation (21) presented in next page, and the meaning of the equation is demonstrated in Fig 4. $p_{i,j}^{\text{board,up}}$ is computed by Equation (22) presented in next page. The number of passengers waiting at the j station platform after the departure time of service i is formulated as

$$p_{i,j}^{\text{wait,up}} = \begin{cases} p_{i,j}^{\text{new,up}} - p_{i,j}^{\text{board,up}} & i = 1 \\ p_{i-1,j}^{\text{wait,up}} + p_{i,j}^{\text{new,up}} - p_{i,j}^{\text{board,up}} & i \geq 1 \end{cases}. \quad (23)$$

Moreover, the number of onboard passengers after the departure time of service i at station j is written as

$$p_{i,j}^{\text{onboard,up}} = \begin{cases} p_{i,j}^{\text{board,up}} & i = 1 \\ (1 - \phi_j^{\text{up}})p_{i,j-1}^{\text{onboard,up}} + p_{i,j}^{\text{board,up}} & i \geq 1 \end{cases}. \quad (24)$$

$$p_{i,j}^{\text{new,up}} = \begin{cases} \sum_{m=1}^M \lambda_{j,m}^{\text{up}} (d_{i,j}^{\text{up}} - t_m) x_{i,j,m}^{\text{up}} + \sum_{m=1}^M (\sum_{k=m}^M x_{i,j,k}^{\text{up}}) \lambda_{j,m}^{\text{up}} (t_m - t_{m-1}), & i = 1 \\ \sum_{m=1}^M \lambda_{j,m}^{\text{up}} (t_{m-1} - d_{i-1,j}^{\text{up}}) x_{i-1,j,m}^{\text{up}} + \sum_{m=1}^M \lambda_{j,m}^{\text{up}} (d_{i,j}^{\text{up}} - t_m) x_{i-1,j,m}^{\text{up}} + \\ \sum_{m=1}^M \lambda_{j,m}^{\text{up}} (t_m - t_{m-1}) \sum_{k=1}^m (x_{i-1,j,k}^{\text{up}} - x_{i,j,k}^{\text{up}}) + \sum_{m=1}^M \lambda_{j,m}^{\text{up}} (t_m - t_{m-1}) x_{i,j,m}^{\text{up}} & i > 1 \end{cases} \quad (21)$$

$$p_{i,j}^{\text{board,up}} = \begin{cases} \min\{p_{i,j}^{\text{new,up}}, T^{\text{cap}}\} & i = 1, j = 1 \\ \min\{p_{i-1,j}^{\text{wait,up}} + p_{i,j}^{\text{new,up}}, T^{\text{cap}}\} & i \neq 1, j = 1 \\ \min\{p_{i-1,j}^{\text{wait,up}} + p_{i,j}^{\text{new,up}}, T^{\text{cap}} - (1 - \phi_j^{\text{up}}) p_{i-1,j}^{\text{onboard,up}}\} & i = 1, j \neq 1 \\ \min\{p_{i,j}^{\text{new,up}}, T^{\text{cap}} - (1 - \phi_j^{\text{up}}) p_{i-1,j}^{\text{onboard,up}}\} & i \neq 1, j \neq 1 \end{cases} \quad (22)$$

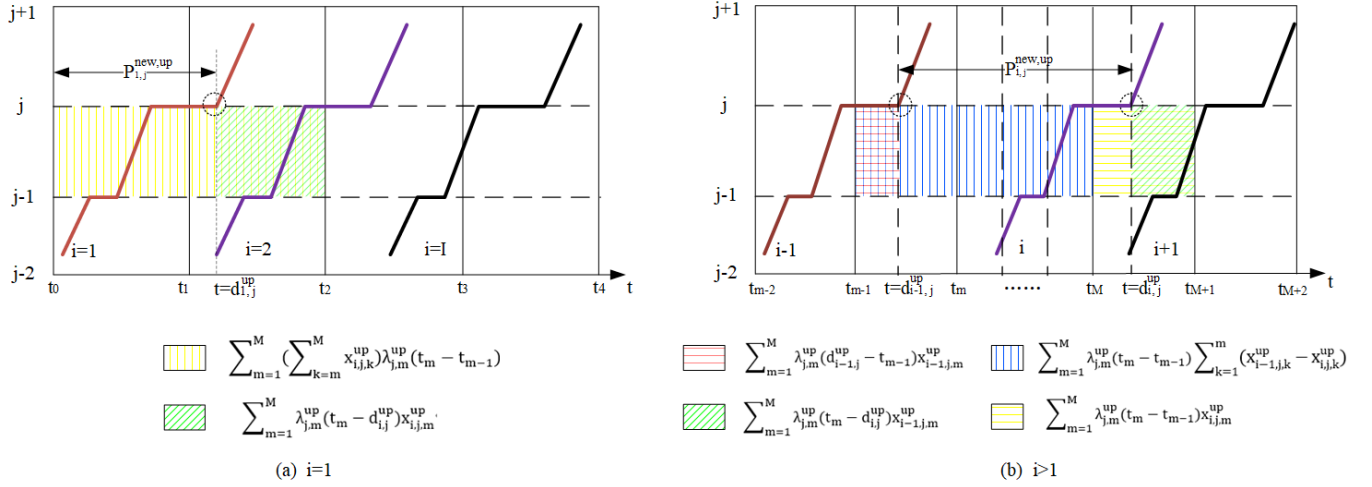


Fig. 4: Illustrative diagram for the calculation of newly arrival passengers

C. Objective function

Since passenger satisfaction and operation costs are both important for rail operators, the objective function here is to minimize the number of waiting passengers at platforms and to maximize the number of onboard passengers, which is formulated as follows:

$$\begin{aligned} \min Z = & w_1 \sum_{i \in \mathbf{I}} \sum_{j \in \mathbf{J}} ((1 - c_{i,j}^{\text{d,up}}) p_{i,j}^{\text{wait,up}} + (1 - c_{i,j}^{\text{d,dn}}) p_{i,j}^{\text{wait,dn}}) \\ & - w_2 \sum_{i \in \mathbf{I}} \sum_{j \in \mathbf{J}} (p_{i,j}^{\text{onboard,up}} + p_{i,j}^{\text{onboard,dn}}), \end{aligned} \quad (25)$$

where w_1 and w_2 are positive weights to indicate the relative importance of these two objectives. We note that minimizing the number of waiting passengers could improve passengers' travel experience. It is worth to note that $p_{i,j}^{\text{wait,up}}/p_{i,j}^{\text{wait,dn}}$ is not the real number of waiting passengers on the platform if the train service i is coupled with the following train service $i + 1$. Hence, only the waiting passenger for the last train service of the composition or the uncoupled train services are considered in the objective function. Moreover, maximizing the number of onboard passengers could enhance the transport capacity usage to save operation costs for rail operators.

IV. CASE STUDY

In order to illustrate the effectiveness of the train scheduling model with flexible coupling and decoupling, the practical data of Beijing Daxing International Airport Express is used for the computational experiments. The MINLP model is transformed into an MILP model by applying the properties presented in Bemporad and Morari[11], and is then solved by IBM ILOG CPLEX optimization studio. All experiments are carried out on a laptop with a 3.90GHz Intel Xeon w-2245 CPU.

A. Set-up

Beijing Daxing International Airport Express is the first urban rail transit line that realizes train operation with multiple formations in Beijing. Specifically, trains with 8 cars are employed for the peak hours and trains with 4 cars are used for the off-peak hours. The length of this line is 41.36 km and it has three stations, i.e., stations CQ, DXXC and DXJC. The considered time period for this case study is from 7:00 to 12:00, where the peak hour is between 8:00 and 9:00 and the rest are off-peak hours. The passenger arrival rate for the peak and off-peak hours are 52.2 and 8.8 passengers per minute at station DXXC, respectively. The passenger arrival rate for the peak and off-peak hours are 26.1 and 4.4 passengers per minute at station CQ in the up direction and at station DXJC in the down direction. The minimum

TABLE I: Values of model parameters

Parameter	Value	Parameter	Value
$r^{\text{turn,max}}$	8 min	$r^{\text{turn,min}}$	2 min
h^{max}	10 min	h^{min}	5 min
T^{cap}	244 passengers	n^{max}	2
w_1	1	w_2	1

TABLE II: Performance comparison among the operation of flexible formation, fixed formation and multiple formation for Daxing Airport Express

Train formations	Number of left passengers	Average load factor of trains	Operation duration (min)
Fixed composition with 8 cars	32	0.541	181
Fixed composition with 4 cars	14488	0.625	247
Multiple train compositions	231	0.623	243
Flexible train compositions	123	0.624	245

composition of trains is set as 4 cars. The maximum load factor of train services is set as 1.2. Moreover, the passenger alighting rate is 0.5. The number of train services is chosen as 30. The settings of other parameters are given in Table I.

B. Computational results

The performance of train schedules with different train compositions are given in Table II, where the train schedule with flexible train compositions is compared with the train schedules with fixed composition with 4 cars, fixed composition with 8 cars, and multiple compositions. The number of left passengers in Table II indicates the sum of waiting passengers at the platforms that cannot board the first coming train during the operation period. It can be observed that the number of left passengers for the case with fixed composition with 8 cars is the smallest, but the average load factor and the operation duration with this case are also the smallest, which means that if the train schedule with fixed composition with 8 cars transports the same passengers as the flexible train compositions, it will require more train services and rolling stocks. Except for the case with fixed compositions with 8 cars, the number of left passengers for the case with flexible

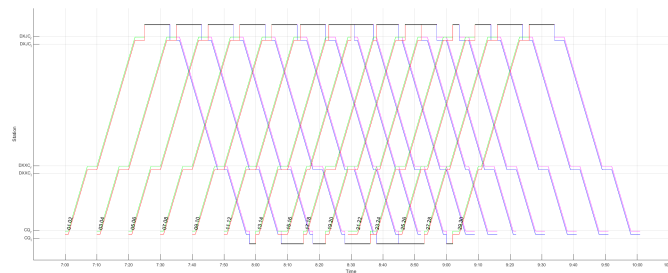


Fig. 5: Train scheduling solution for fixed composition with 8 cars in Beijing Daxing International Airport Express

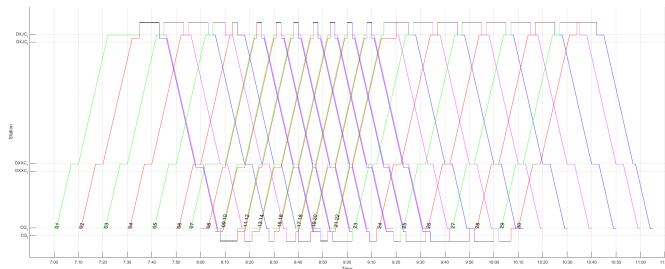


Fig. 6: Train scheduling solution for flexible train composition in Beijing Daxing International Airport Express

train compositions is the smallest. This indicates that the train schedules with flexible train compositions can satisfy the various passenger demand better.

Due to the fixed number of train services in the setup of this case study, the operation duration for the case with fixed composition of 8 cars is only 181 minutes, while the operation durations for the other cases are around 4 hours. Because the operation duration of the case with fixed composition with 8 cars is shorter and less passengers are involved, so the average load factor this case is smaller when compared with that for the other cases. The train schedules for the cases with fixed composition of 8 cars and flexible compositions are given in Figs 5 and 6. As can be observed in Fig 5, two train services are always operated as a whole composition in the operation duration. In Figs 6, the train services can be grouped flexibly according to the passenger demand. During the off-peak hours, train services are operated separately. However, train services are tend to be bounded in the peak hours to carry more passengers. Specifically, train services 01 and 02 depart from Station CQ separately, while they coupled together at Station DXXC, because the arrival time of the first train service in the down direction is nearly an hour later than that in the up direction and many passengers have accumulated on the down direction platform. The number of waiting passengers in the down direction can be reduced by coupling the train service 01 and 02 in a convoy. Moreover, train services 21 and 22 arrive at Station DXXC as a convoy and they decoupled with each other at Station DXJC. The reason is that the peak hours have ended, the passenger flow during the off-peak hours begin to decrease, and the headway between train services begin to increase, so it is not necessary to continue the operation using train convoy with 8 cars.

V. CONCLUSIONS

In this paper, a train scheduling model with flexible coupling and decoupling of trains at stations is presented to satisfy the unbalanced passenger demand and to minimize the operation costs of rail operators. The train operation constraints, especially the dynamic process of coupling and decoupling of trains at stations, are formulated explicitly. The computational results based on the Beijing Daxing International Airport Express have shown that the train schedules with flexible composition of trains outperforms the train

schedules with fixed train compositions and multiple train compositions. The current model could support transport planners in determining the train schedules for urban rail transit lines with multiple, flexible or even dynamic train formations. As future work, we would like to investigate more effective and efficient solution approaches to solve this complex problem.

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APPENDIX

Table III lists the parameters, decision variables and subscripts used in our formulation of the scheduling problem for flexible composition.

TABLE III: Parameters, decision variables and subscripts for the mathematical formulation

Symbol	Description
\mathbf{I}	Set of train services, $\mathbf{I} = \{1, 2, 3, \dots, I\}$
\mathbf{J}	Set of stations, $\mathbf{J} = \{1, 2, 3, \dots, J\}$
\mathbf{M}	Set of time intervals, $\mathbf{M} = \{1, 2, 3, \dots, M\}$
i	Train service index, $i \in \mathbf{I}$
j	Station index, $j \in \mathbf{J}$
m	Time interval index, $m \in \mathbf{M}$
$\tau_j^{\text{up,max}} / \tau_j^{\text{up,min}}$	Maximum/minimum dwell time of train services at station j in the up direction
$\tau_j^{\text{dn,max}} / \tau_j^{\text{dn,min}}$	Maximum/minimum dwell time of train services at station j in the down direction
$r_j^{\text{up}} / r_j^{\text{dn}}$	Running time between station j and $j+1$ in the up/down direction
$r^{\text{turn,max}} / r^{\text{turn,min}}$	Maximum/minimum turnaround operation time in the up/down direction
$\lambda_{j,m}^{\text{up}} / \lambda_{j,m}^{\text{dn}}$	Rate of passengers arriving at the up/down direction platform at station j in the interval m at station j in the up/down direction
$\phi_j^{\text{up}} / \phi_j^{\text{dn}}$	Rate of passengers alighting from the trains in the up/down direction at station j
D	Extra dwell time for coupling or decoupling operations at stations
R	Big enough positive number
t_m	Ending time of time interval m
$h^{\text{max}} / h^{\text{min}}$	Maximum/minimum headway in the same direction
T^{cap}	Passenger capacity of minimum formation units
n^{max}	Maximum number of formation
$a_{i,j}^{\text{up}} / a_{i,j}^{\text{dn}}$	Arrival time of service i at station j in the up/down direction
$d_{i,j}^{\text{up}} / d_{i,j}^{\text{dn}}$	Departure time of service i at station j in the up/down direction
$\tau_{i,j}^{\text{up}} / \tau_{i,j}^{\text{dn}}$	Dwell time of service i at station j in the up/down direction
$\gamma_{i,i'}$	Binary variable, $\gamma_{i,i'} = 1$ if service i is connected with service i' at station 1
$c_{i,j}^{\text{a,up}} / c_{i,j}^{\text{a,dn}}$	Binary variable, $c_{i,j}^{\text{a,up}} / c_{i,j}^{\text{a,dn}} = 1$ if service i arrives at station j in the state of coupling with service $i+1$ in the up/down direction
$c_{i,j}^{\text{d,up}} / c_{i,j}^{\text{d,dn}}$	Binary variable, $c_{i,j}^{\text{d,up}} / c_{i,j}^{\text{d,dn}} = 1$ if service i departs from station j in the state of coupling with service $i+1$ in the up/down direction
$x_{i,j,m}^{\text{up}} / x_{i,j,m}^{\text{dn}}$	Binary variable, $x_{i,j,m}^{\text{up}} / x_{i,j,m}^{\text{dn}} = 1$ if departure time of service i at station j in time interval m in the up/down direction
$p_{i,j}^{\text{new,up}} / p_{i,j}^{\text{new,dn}}$	Number of passengers newly arriving at the j station platform between service $i-1$ and i
$p_{i,j}^{\text{wait,up}} / p_{i,j}^{\text{wait,dn}}$	Number of passengers waiting at the j station platform after the departure time of service i
$p_{i,j}^{\text{board,up}} / p_{i,j}^{\text{board,dn}}$	Number of passengers boarding train service i at the j station platform after the departure time of service i
$p_{i,j}^{\text{onboard,up}} / p_{i,j}^{\text{onboard,dn}}$	Number of onboard passengers after the departure time of service i at station j