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# Multi-Trace Multi-Screen Methods for Mixed Transmission/Scattering Problems

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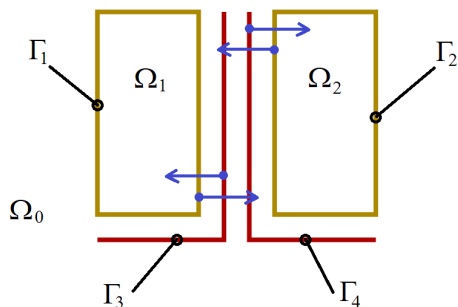


Fig. 1. Two penetrable boxes partially coated by a T-shaped perfectly conducting multiscreen. Normals  $n_i$ ,  $i = 1, \dots, 4$  are in blue.

**Abstract**—In this contribution a novel fast-converging integral equation method is introduced that can be used to solve the mixed transmission/scattering problems by composite structures including multiple domains and metallic coatings that can contain junctions. This is achieved by combining the global multi-trace method with the quotient space discretisation of the multi-screen boundary integral equation. The method is illustrated by means of a sufficiently general geometry, the discretisation is discussed, and an effective Calderón multiplicative preconditioner is introduced. Numerical results corroborate the correctness and efficiency of the method.

## I. INTRODUCTION

Boundary element methods are an efficient means to model the scattering and transmission by piece-wise homogeneous devices. It is common for such devices to contain metallic coatings, which typically reside at the interface between two of the comprising domains. In all but the simplest cases, these devices contain junctions, i.e. curves along which three or more interfaces meet. Depending on the location of any metallic coatings, this gives rise to a rich and hard to classify variety of junction types [6].

Modellers have been successful in writing down integral equations and their discretisations for the field traces on the interfaces. Initially, focus lied on the minimisation of degrees of freedom, leading to the development of so-called single-trace methods. Single-trace methods give rise to the smallest possible linear systems and continuity of tangential fields across domain interfaces is enforced exactly, but no preconditioners are available for these systems, limiting their applicability to moderately sized systems only.

Multi-trace formulations use a different approach: a priori independent field traces are assigned to each domain, and

interdomain continuity is only enforced implicitly. Careful analysis [3] and application in a variety of situations [5] have proven the strength of this approach. Traces are defined on manifolds and the construction of effective preconditioners is no longer thwarted. The increase in DoFs is reasonable given near-linear matrix-vector multipliers and an essentially unlimited amount of computing nodes.

Another challenge are multi-screens: infinitely thin conductors that may contain junctions, i.e. curves along which three or more manifolds meet. Through the careful definition of so called jump spaces to represent the total induced current as quotients of multi-trace and single trace spaces, it is possible to write down well-posed integral equations [2]. Discretisation is done in the parent multi-trace space, yielding singular systems, whose non-unique solutions all correspond to the same physical information [1], [4].

This contribution is concerned with the modelling of scattering/transmission by systems that contain both: multiple penetrable domains, and perfectly conducting multi-screens. At the basis of the formulation is (i) the gap idea used in the multi-trace methods, and (ii) the insight of “inflating” multi-screens, enabling the description of the geometry in terms of junction-free manifolds. The orientation of some of the manifolds and the domains in which they are embedded is to some extent a matter of arbitrary choice. Nevertheless, it is possible to write down a consistent system of integral equations for the field traces and induced currents. The construction of this system for a sufficiently general example will be demonstrated, and the discretisation using spaces of RWG functions, and the construction of an effective Calderón preconditioner using dual BC basis functions will be elucidated. It will be shown that with preconditioning GMRES quickly converges to one of an infinite family of solution for the resulting linear systems.

## II. FORMULATION AND DISCRETISATION

The geometry in Fig. 1 comprises two penetrable domains. Their shared interface and bottoms are coated by a perfect electric conductor. The coating is a multi-screen, with a junction at the bottom of the shared interface. To write down the multi-trace formulation for this setup, (i) a small virtual gap occupied by background material is introduced, and (ii) the front and back of the multi-screen are separated. This conceptually removes any junctions that were present in the geometry. A full description of the inflated multi-screen includes a horizontal plane, which is left out to reduce the number of DoFs [4]. For the geometry in Fig. 1 it is now

$$\begin{pmatrix} K_{11}^{(0)} + K_{11}^{(1)} & -\eta_0 T_{11}^{(0)} - \eta_1 T_{11}^{(1)} & K_{12}^{(0)} - \frac{1}{2} & -\eta_0 T_{12}^{(0)} & -\eta_0 T_{13}^{(0)} & -\eta_0 T_{14}^{(0)} \\ \frac{1}{\eta_0} T_{11}^{(0)} + \frac{1}{\eta_1} T_{11}^{(1)} & K_{11}^{(0)} + K_{11}^{(1)} & \frac{1}{\eta_0} T_{12}^{(0)} & K_{12}^{(0)} - \frac{1}{2} & K_{13}^{(0)} - \frac{1}{2} & K_{14}^{(0)} - \frac{1}{2} \\ K_{21}^{(0)} - \frac{1}{2} & -\eta_0 T_{21}^{(0)} & K_{22}^{(0)} + K_{22}^{(2)} & -\eta_0 T_{22}^{(0)} - \eta_2 T_{22}^{(2)} & -\eta_0 T_{23}^{(0)} & -\eta_0 T_{24}^{(0)} \\ \frac{1}{\eta_0} T_{21}^{(0)} & K_{21}^{(0)} - \frac{1}{2} & \frac{1}{\eta_0} T_{22}^{(0)} + \frac{1}{\eta_2} T_{22}^{(2)} & K_{22}^{(0)} + K_{22}^{(2)} & K_{23}^{(0)} - \frac{1}{2} & K_{24}^{(0)} - \frac{1}{2} \\ K_{31}^{(0)} - \frac{1}{2} & -\eta_0 T_{31}^{(0)} & K_{32}^{(0)} + \frac{1}{2} & -\eta_0 T_{32}^{(0)} & -\eta_0 T_{33}^{(0)} & -\eta_0 T_{34}^{(0)} \\ K_{41}^{(0)} + \frac{1}{2} & -\eta_0 T_{41}^{(0)} & K_{42}^{(0)} - \frac{1}{2} & -\eta_0 T_{42}^{(0)} & -\eta_0 T_{43}^{(0)} & -\eta_0 T_{44}^{(0)} \end{pmatrix} \begin{pmatrix} m_1 \\ j_1 \\ m_2 \\ j_2 \\ j_3 \\ j_4 \end{pmatrix} = - (e^{inc} \times n_1, \quad n_1 \times h^{inc}, \quad e^{inc} \times n_2, \quad n_2 \times h^{inc}, \quad e^{inc} \times n_3, \quad e^{inc} \times n_4)^T \quad (1)$$

possible to write down the representation formulas in  $\Omega_i$ ,  $i = 0, 1, 2$ , using  $e \times n_3 = e \times n_4 = 0$ . Adding the interior and exterior representation formulas gives equation (1) for the electric and magnetic traces. Single and double layer operators between surfaces  $\Gamma_p$  and  $\Gamma_q$  w.r.t. material  $r$  are denoted  $T_{qp}^{(r)}$  and  $K_{qp}^{(r)}$ , respectively. The signs of the off-diagonal *geometric* identities require extra care and depend on the orientation and ordering of the interfaces [5]. The order and signs of equations in (1) is chosen such that the matrix structure and its link to exterior and interior representation formulas is most manifest. For the benefit of iterative solvers the usual permutations and sign flips are performed to bring all single layer contributions to the diagonal with the same sign. This system is amenable to operator or Calderón preconditioner. Here a block diagonal preconditioner  $\text{diag}(T_{11}^{(0)}, T_{11}^{(0)}, T_{22}^{(0)}, T_{22}^{(0)}, T_{33}^{(0)}, T_{44}^{(0)})$  is proposed. Stable discretisation requires the use of both primal RWG/RT and dual BC/CW boundary element spaces according to the standard procedure.

### III. NUMERICAL RESULTS

The structure from Fig. 1 is illuminated by a wave with signature  $e^{inc}(x) = (0, 1, 0)^T \exp(-i\kappa_0 x_1)$ . The domains  $\Omega_1$  and  $\Omega_2$  are occupied by materials characterised by  $(\kappa_1, \eta_1) = (2.4\kappa_0, \eta_0)$  and  $(\kappa_2, \eta_2) = (2.4\kappa_0, \eta_0)$ . The simulation is performed for different values of the background medium wavenumber  $\kappa_0$  and the target mesh size  $h$ . The number of iterations required for GMRES convergence up to  $2 \cdot 10^{-4}$  is summarised in Table I. An asterisk indicates failure to converge within  $\#\text{DoFs}/5$  iterations. In Fig. 2 it can be seen that the tangential electric field is continuous at transmissive interfaces and is zero at conducting interfaces.

TABLE I  
ITERATION COUNTS [UNPRECONDITIONED VS PRECONDITIONED]

wavenumber	mesh size	iterations unprecond.	iterations precond.
1.0	0.125	586	36
	0.1	702	37
	0.075	2630*	37
	0.05	5432*	37
3.0	0.125	938*	107
	0.1	1416*	106
	0.075	2630*	106
	0.05	5432*	99

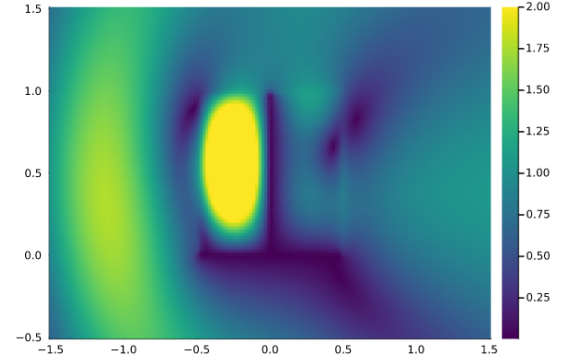


Fig. 2. The absolute value of the  $y$ -component of the electric field at  $y = 0.5$  meter is continuous at transmissive interfaces and tends to zero conducting interfaces. ( $\kappa_0 = 3.0, h = 0.05$ )

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