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Incoherent Doppler Processing for Doppler Moment and Noise Estimation for Precipitation

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Abstract

The challenge of processing incoherent sets of radar echoes to estimate the Doppler moments and noise standard deviation from precipitation with a fast-scanning radar is addressed. The recently proposed maximum likelihood parametric Doppler spectrum estimator (PSE) is extended to accommodate the noise standard deviation along with the Doppler moments in the parametric model of the Doppler power spectral density (PSD). Its performance in estimating the Doppler moments and the noise standard deviation is compared with another maximum likelihood approach. The proposed approach has a smaller bias in the estimation of the noise standard deviation for a wide range of spectral widths on simulated data. The algorithm is verified on experimental data from a fast-scanning weather radar.

1 Introduction

Doppler weather radars sensitive to precipitation are used to estimate a few parameters of the Doppler spectrum from each radar resolution volume, known as the Doppler moments. The Doppler moments are further used to determine the severity of the storms. The zeroth Doppler moment (the total power) signifies the strength of the precipitation field, the mean Doppler velocity (the first Doppler moment) as a function of space can be used to determine the 3D wind fields, and the Doppler spectrum width (square root of the second central Doppler moment) is a measure of the velocity dispersion which can be resulted from many statistical factors such as turbulence, antenna beam shape, and the range weighting function.

The state-of-the-art Doppler moment estimators can be categorized into parametric and non-parametric methods. The non-parametric techniques are usually biased and require long observation intervals, but they are computationally efficient [1], [2], [3], [4], [5], [6]. On the other hand, the parametric techniques have lower bias than the non-parametric ones (provided the parametric models adequately describe the echo samples). There are two types of parametric methods; one is based on the parametric formulation for the covariance matrix of the radar echoes in the time domain, and the other is based on the power spectral density (PSD) of the echo samples in the frequency domain. Although the covariance-based methods are usually more accurate than the PSD-based methods (because they use both the real and imaginary parts of the echo samples) [7], they rely on the computation of the inverse of the covariance matrix and, therefore, are computationally more expensive. On the other hand, the PSD-based methods [8] (will be referred to as Levin's approach), [9] (will be referred to as parametric spectrum estimator (PSE)) only use the one-dimensional PSD and are computationally more efficient than the covariance-based methods.

This paper focuses on the PSD-based Doppler moment estimators. In addition to the Doppler moments, the PSE includes the Doppler resolution in the PSD model as a summation over the observation interval. However, Levin's approach uses a closed-form model of the PSD with parameters of interest (like the Doppler moments). It has been shown that PSE has a smaller bias for the Doppler spectrum width and requires a smaller coherent observation interval than Levin's approach (for a wide range of spectrum widths). Both estimators have the potential to include PSD measurements that are incoherent. An example of such measurements is realized in the fast scanning radar, where the incoherent PSD measurements are collected from several azimuthal scans.

In this paper, we extend the PSE to include the noise standard deviation as a parameter in the estimation along with the Doppler moments. The noise standard deviation was considered a known quantity in the previous work of [9] and it was manually estimated as the square root of the 15th percentile of power level in the PSD for the application to real scanning radar data.

The main body of the paper is organized as follows. Section 2 explains the model of the Doppler PSD model for the echo samples. Section 3 presents the theoretical performance analysis on simulated echo samples. Section 4 presents the application to the real scanning radar data. The conclusions are presented in section 5.

2 Doppler PSD Model

The signal model of the radar echoes is explained in [9, eq. (1)-eq.(3)]. If we normalize the Doppler moments with the Nyquist interval (unambiguous velocity interval $2V_a = \lambda/(2T)$) for simplicity and add zero-mean white Gaussian noise in the echo signal model (measurement model [9, eq. (6)]), the expected value of the PSD of the measurement echo signal model is given by:

$$\chi(f) = \mathbb{E}\left[\frac{1}{N}|Z(f)|^2\right] \tag{1}$$
$$= P\left[1 + 2\sum_{q=1}^{N-1}\left(1 - \frac{q}{N}\right)\exp\left(-2\pi^2\sigma_{fn}^2 q^2\right)\right.$$
$$\left.\times\cos\left(2\pi q(\mu_{fn} - f)\right)\right] + \sigma_n^2,$$

where Z is the measurement model of the PSD, $\mu_{fn} = \mu_v/(2V_a)$ is the normalized mean Doppler velocity, $\sigma_{fn} = \sigma_v/(2V_a)$ is the normalized Doppler spectrum width, σ_n^2 is the noise standard deviation, and f is the normalized Doppler frequency. The subscript $_v$ (under the Doppler moments) represents velocity, and a subscript $_{fn}$ represents the normalized quantity.

A maximum likelihood estimator is formulated considering the fact that the PSD of the measurements is exponentially distributed [8], [9], [10]. If we consider the measurements $\mathbf{Z}$ as a $L \times N$ matrix having L different measurements of PSDs with N coherent echo samples each, the likelihood probability $p(\mathbf{Z}|\Theta)$ is given by:

$$p(\mathbf{Z}|\Theta) = \prod_{i=1}^{N}\prod_{l=1}^{L}\frac{1}{\pi(\chi(f_i,\Theta))}\exp\left(-\frac{Z_l(f_i)}{\chi(f_i,\Theta)}\right), \tag{2}$$

where Θ represents the parameter set ($\Theta = [\mu_{fn}, \sigma_{fn}, \sigma_n]$), and f_i are normalized Doppler frequencies having a range of $[-0.5, 0.5]$. The log-likelihood takes the following form:

$$\log\left(p(\mathbf{Z}|\Theta)\right) = -\sum_{i=1}^{N}\left[L\log\left(\pi\chi(f_i,\Theta)\right) + \frac{\sum_{l=1}^{L} Z_l(f_i)}{\chi(f_i)}\right]. \tag{3}$$

The maximum likelihood estimation problem is defined as follows:

$$\hat{\Theta} = \max_{\Theta}\log\left(p(\mathbf{Z}|\Theta)\right). \tag{4}$$

3 Performance Analysis on Simulated Data

The maximum likelihood estimation is performed with the active-set and the Limited Memory Broyden-Fletcher-Goldfarb-Shanno (L-BFGS) algorithms [11] [12]. The proposed approach (PSE) is compared with another maximum likelihood approach (Levin's approach [8]). The assumption that the PSD is exponentially distributed for the log-likelihood formulation is the same for Levin's approach, but the model of the expectation of the PSD used is different. Levin's approach uses a complete closed-form model of $\chi(f)$ without considering the finite observation interval.

The simulated data used to assess the performance is generated by using [9, eq.(6)] by adding zero-mean white Gaussian noise to the echo samples with the help of an input signal-to-noise ratio (SNR) of 12dB [13]. The bias and standard deviation in the estimation of the parameters are shown with respect to the true normalized Doppler spectral width in Fig. 1. It can be observed that the bias in the estimates of normalized spectrum width $\hat{\sigma}_{fn}$ (a hat is used to signify that it is an estimated quantity) and noise standard deviation $\hat{\sigma}_n$ for PSE is lower than that of Levin's approach lower than a normalized spectral width of around $\sigma_{fn} = 0.16$. Doppler PSDs having a normalized spectral width of $\sigma_{fn} = 0.2$, or more can be considered "flat," and therefore, the estimates are increasingly biased [7]. Therefore, typically, weather radars should be designed with a suitable PRT to have a sufficient maximum unambiguous velocity that can contain the useful spectrum from a wide range of atmospheric events (such that the σ_{fn} remains lower than 0.1).

The theoretical standard deviations are derived by taking the diagonal elements of the inverse of the Fisher information matrix [9]. We have extended the formulation of the Fisher information matrix of [9, eq. (31)] for three parameters instead of two (3×3 matrix). The theoretical variance doesn't represent the unbiased CRB unless the estimator is unbiased. However, the proposed estimator is biased due to several factors, such as the finite observation interval and constrained optimization. We did not derive a biased CRB as it requires a functional form of the bias gradient for the estimator, which is difficult to achieve. Nonetheless, it can be concluded that as the observation interval reaches infinite $N \to \infty$, the estimator achieves unbiasedness, and the derived theoretical variance converges to the unbiased CRB. The theoretical standard deviations deviate from the numerical results for higher σ_{fn} due to the increasing biases.

4 Application to Real Radar Data

The proposed approach is applied to the data acquired from an X-band scanning radar at the Delft University of Technology in the Netherlands from a rainy event. The specification of this particular measurement set-up can be found in [9, Tab. I]. The scan speed of the radar was five rotations per minute (rpm) in the azimuthal direction. The number of echo samples for each resolution cell was 100 (with a PRT of $813.2\mu s$, the time on target per scan was 81.32ms). The pre-processing, including the range Fast Fourier Transform (FFT) and the clutter removal processes, is explained in [9, Sec. VII]. The maximum unambiguous velocity for this radar is $V_a = 9.8$m/s. The number of radar scans used in this experiment is five, and the number of PSD measurements is $L = 10$ (with two PSDs obtained from one radar scan using 50 echo samples each).

One resolution volume is chosen to show the variation in the estimated parameters as a function of L. The results are shown in Fig. 2. The location of this resolution cell can be referred from [9, Fig. 11, Tab. III] (It is marked with a label (2)). This region was chosen because the useful Doppler spectrum from the precipitation is aliased at this resolution volume, making it challenging for typical Doppler moment

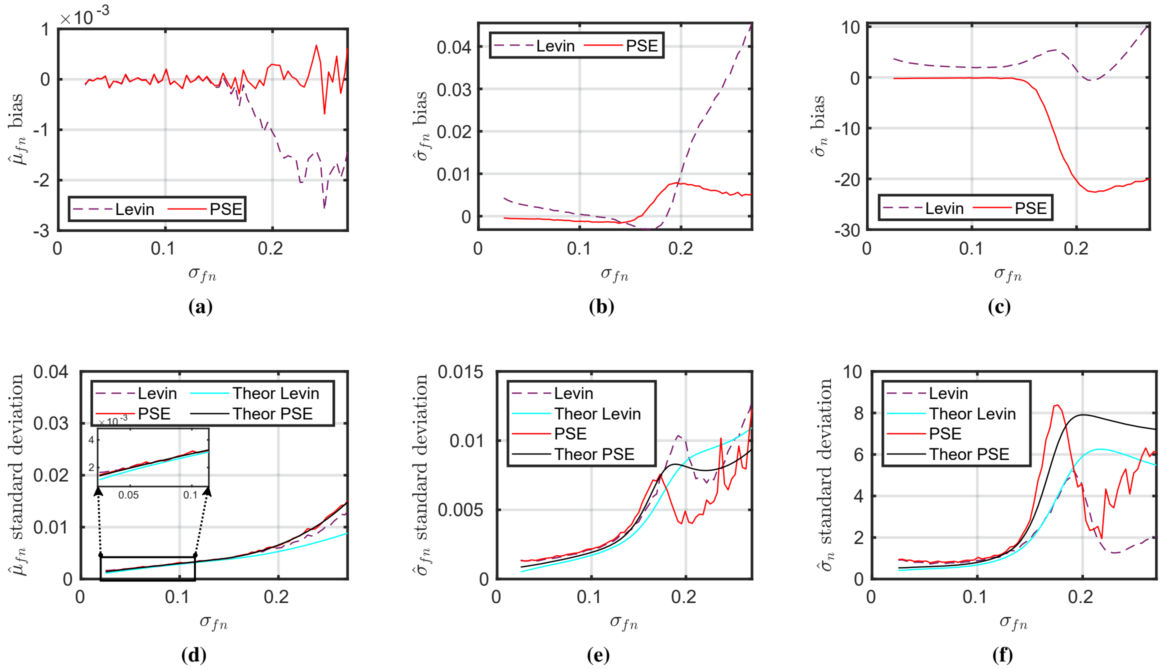

Figure 1. Estimation performance with σ_{fn} with an input SNR of 12dB, N = 64, L = 16 (a) Bias of $\hat{\mu}_{fn}$ (d) Standard deviation of $\hat{\mu}_{fn}$ (b) Bias of $\hat{\sigma}_{fn}$ (e) Standard deviation of $\hat{\sigma}_{fn}$ (c) Bias of $\hat{\sigma}_n$ (f) Standard deviation of $\hat{\sigma}_n$. The legend with "Theor" refers to theoretical plots.

estimators that use PSD measurements.

The results show that the PSE estimated Doppler spectrum width and noise standard deviation are smaller than that of Levin's. This behavior is also verified in the theoretical analysis in Fig. 1 for normalized Doppler spectrum widths (σ_{fn}) less than 0.25. The reconstruction in Fig. 2d is performed by replacing (for both PSE and Levin) the estimated parameters in the PSD model of (1). It can be observed that the reconstruction based on PSE outperforms Levin's approach. The smaller bias in the case of PSE is because it considers the Doppler resolution in its PSD model with a semianalytical form.

5 Conclusions

A semianalytical parametric Doppler moment estimator has been further extended by adding noise standard deviation as an estimation parameter. The performance analysis (bias and variance) of the proposed approach is presented with simulated radar echoes with respect to the true normalized spectral width. It has been shown that the proposed approach has a lower bias than Levin's approach. Furthermore, the proposed approach is robust towards the limited time on target because the model PSD includes the Doppler resolution in a semianalytical form. The proposed approach is applied to real radar data acquired from a fast-scanning X-band weather radar. It is shown that the estimation of the proposed approach is immune to spectrum aliasing, resulting in reliable estimates. In this paper, the signal is assumed to be stationary in terms of the spectral content for a certain period of time. Further studies should be conducted when the parameters of the Doppler spectrum change over time, assuming the processes are dynamic.

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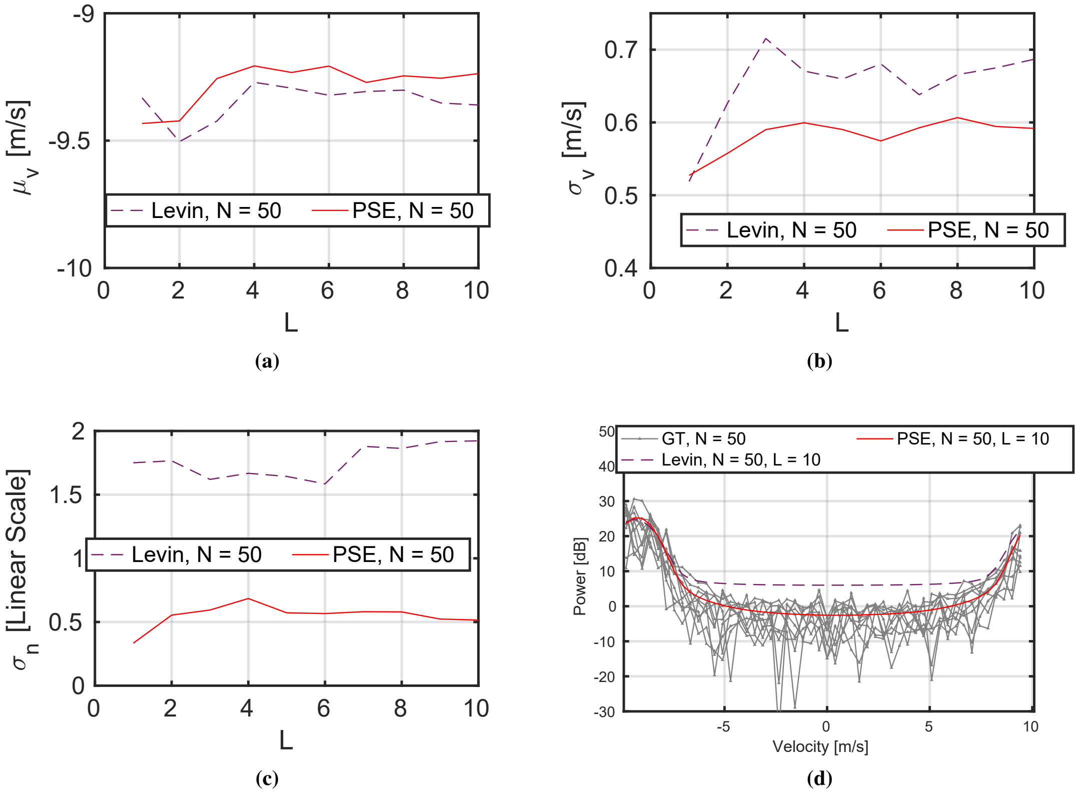

Figure 2. Parameter estimation and Doppler PSD reconstruction with real radar data collected from five consecutive scans of a fast scanning radar from the voxel located at range $R = 1.24$ km, azimuth $\phi = 264^{\circ}$ from the north in a clockwise direction, and an elevation of $\theta = 30^{\circ}$. (a) $\hat{\mu}_v$ m/s (b) $\hat{\sigma}_v$ m/s (c) $\hat{\sigma}_n$ (linear scale) (d) Reconstruction of the PSD. The abbreviation "GT" stands for "ground truth" PSD measurements.

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