Stability Analysis of Mixed Traffic Flow using Car-Following Models on Trajectory Data

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Stability Analysis of Mixed Traffic Flow using Car-Following Models on Trajectory Data

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Abstract— **There is an increase in the need to understand traffic flow behavior through multiple fronts, especially in developing countries like India, where a high degree of heterogeneity, non-lane-discipline, and non-compliance to traffic rules prevail in the road sections. As a result, it can impact the stability of the traffic stream. Four car-following equations to analyze six vehicle categories using various stability criteria are evaluated for the mixed traffic conditions. With the help of processed trajectory data of the western expressway, consisting of multiple types of flows, the parameters of four car-following equations are calibrated for all vehicle categories separately to address heterogeneity. Final stability equations are derived from the available criteria in the literature and analyzed accordingly. To address non-lane-discipline, a straightforward methodology to examine the stability of a model which considers lateral movement is developed and validated, and proven to provide more stability to the traffic. Comparisons between vehicle categories and between car-following models are made. Various interpretations about the impact of parameters in car-following models on the stability of traffic are provided.**

Keywords— car-following model, local stability, string stability, optimum velocity functions, mixed traffic flow

I. INTRODUCTION

Traffic stream stability is one of the regulating parameters in assessing the discipline of the traffic stream. The fundamentals of traffic engineering demonstrate that the traffic stream's safety and efficiency are inversely related. Nevertheless, traffic stability will act as a pivot in balancing efficiency and safety. Increasing the traffic stream stability will enhance both safety and efficiency in the traffic stream. The car-following models (CFMs) are the most critical indications of microscopic traffic flow models. In the presence of multiple types of interactions with surrounding vehicles, car-following models describe driver-vehicle behavior. Research studies on car-following models and stability Analysis have been going on since the early 1950s. In sensing the stability, the above studies graded the stability in two types: local and string/ asymptotic stability. Local stability explains the vehicles' disturbances under the "following" conditions involving two to ten vehicles. Whereas the string stability expresses the interactions over a series of following vehicles in the traffic stream.

Further, to understand traffic stability, researchers across the world uncovered numerous findings. Numerous studies are made from the past to assess the stability in a traffic stream. Over the years, the need for a better understanding of traffic flow has only been increasing. Since the idea and execution of Connected

Autonomous Vehicles [CAVs] into traffic streams, such studies have only gotten more popular, especially to understand more about mixed Human Driven Vehicles [HDVs] and CAVs [1]. Given the limitation of high precision field data, the empirical stability studies are limited.

On the other hand, in mixed traffic, the vehicles are with varying geometric characteristics. As a result, traffic tends to display weak lane discipline. The driving behavior studies [2, 3] from mixed traffic demonstrates smaller vehicles' lateral behavior impacting mixed traffic performance. Simultaneously, given the variation in the vehicles' physical properties, earlier studies [4, 5] reported the underperformance of automated traffic tools in monitoring mixed traffic conditions. Due to the mixed traffic data constraints, the driving behavior studies from mixed traffic have not taken proper shape. Few studies [6, 7] attempted to model the vehicles' following and lateral behavior. It can be noted that, with the lateral behavior and weak lane discipline, the stability of the traffic stream can be impacted by any means.

Considering the research gaps, it is planned to understand mixed traffic stream's stability over varying flow conditions. On these lines, initially, trajectory data over different flow conditions are developed. The mixed traffic's driving behavior is modeled based on the trajectory data with the help of car-following models from both heterogeneous and mixed traffic perspectives. Based on the calibrated car-following models, both the local stability and the string stability scenarios are analyzed mathematically for each vehicle category.

II. OBJECTIVES

- To calibrate the parameters of various CFMs using trajectory data for mixed traffic.
- To derive stability equations for CFMs.
- To develop a methodology to analyze the stability of 2-dimensional traffic by considering lateral behavior.
- To understand various CFMs in terms of stability and their behavior concerning Indian mixed traffic.

III. METHODOLOGY

In understanding the stability of mixed traffic, the entire research work is carried out in four stages. In the initial stage, the study section's vehicular trajectory data is developed at different flow conditions. Trajectory data of a mid-block section in Western Expressway is extracted for 35 mins, including low, medium, and congested traffic states. Smoothening techniques are applied, and data is processed in MATLAB software, velocities, accelerations are computed. Further, leader-follower

pairs are extracted based on logic given in [2]. In the second stage, the car-following models are identified and calibrated with trajectory data to replicate the driving behavior. Three of the most widely used CFMs are selected because of their plausibility. To analyze stability by including the lateral movement of vehicles, a newly modeled customized-IDM (IDMM) [8] is used. To address heterogeneity, each vehicle category is calibrated separately for each CFM. The criteria at local and string stabilities are evaluated in the third stage based on the calibrated car-following models. Final stability equations for various stability criteria are derived for each CFM. The calibrated parameters are assessed in these equations to analyze the stability of all six vehicle categories using OVM, FVDM, and IDM, which assumes traffic to be lane-based. To include the non-lane-discipline, a straightforward methodology to analyze the stability using IDMM is developed, then this process is validated. Finally, in the last stage of the work, the stability in the traffic stream is analyzed. Then, physical interpretations of stability graphs are discussed, evaluated the impacts of individual parameters of CFMs on stability, compare the four models considered in the study, and assess each model's performance on all six vehicle categories. The terminologies are given in Table 1.

TABLE I Abbreviations for all the Terms Used in this Study

v	The velocity of subject vehicle (m/s)
a	Acceleration of the subject vehicle (m/s^2)
S	Spacing between the subject vehicle and its leader (m)
Δv	The speed difference between the subject vehicle and its leader (m/s)
v_f	Free-flow speed (m/s)
A	Desired maximum acceleration (m/s^2)
B	Comfortable deceleration (m/s^2)
S_0	Spacing at congested state between leader and follower (m)
$V(S)$	Optimum velocity as a function of spacing (m/s)
$S_e, V(S_e)$	Spacing and its corresponding speed in equilibrium steady state
T	Time gap/ reaction time of driver (s)
λ	Sensitivity parameter (/s), also referred to as inverse of time lag τ (s) in some literature
κ	Sensitivity to speed difference (m/s)
ΔS	Transition width (m)
β	Form factor
α	A sensitivity parameter (/s)
δ	Acceleration exponent
η	Exponent factor
ω	A unit step function
L	Lateral shift (m)
L_C	Lateral clearance (m)
$f_v, f_S,$ and $f_{\Delta v}$	Partial derivatives of control equations for respective car-following models for speed, spacing, and speed difference, respectively

IV. Data Collection and Extraction

Video-graphic data of Western Expressway in India; of a mid-block section of length 120 m and width 17.5 m is collected for 12 hours using a High-Definition (HD) video camera. By visual inspection, traffic flow is divided into three types, namely low, medium, and congestion volume. Trajectory data for every 0.5 s is extracted using Traffic Data Extractor (TDE) [9] for a total of 35 minutes. Here, the longitudinal and lateral positions of vehicles are obtained. Vehicles are divided into six categories, namely Motorised Three-Wheeler (3W), Motorised Two-Wheeler (2W), Bus, Car, Truck/Heavy Commercial Vehicle (HCV), and Light Commercial Vehicle (LCV). The composition

of which is 19.5 %, 11.3%, 4.7%, 45%, 6.7% and 12.8% respectively. The complete details of the trajectory data can be obtained from the authors' previous work [10]. To correct the errors in the extraction process, a 5-point moving average method is used to smoothen the data [11]. From raw trajectory data, speed and acceleration are calculated, and leader-follower pairs are extracted. The identification of leader and follower is based on the study for heterogeneous traffic conditions in a weak lane-based environment [2, 3].

V. Car-Following Models

The CFMs are the mathematical representatives of microscopic traffic behavior and are also a bridge connecting microscopic traffic to macroscopic. In the presence of multiple types of interactions with surrounding vehicles, CFMs describe driver-vehicle behavior. There are mainly three models that are most popular and are widely used, including in this study, namely, Optimum Velocity Model (OVM) [12], Full Velocity Difference Model (FVDM) [13], and Intelligent Driver Model (IDM) [14]. Many of such existing CFMs focus on lane-based, homogeneous traffic flow. The present study is focused on non-lane-based, heterogeneous traffic flow. Hence necessary precautions such as calibrating different parameter values for different vehicle categories are implemented. However, the discrete lane changing aspects are missing in those models, which is common in smaller vehicles in mixed traffic streams. With giving due weightage, an extended model of IDM (IDMM) [8] is considered additionally. The control equations of the CFMs considered in the study are as follows,

$$\text{OVM} \qquad a = \lambda\,(V(S) - v) \qquad (1)$$

$$\text{FVDM} \qquad a = \lambda\,(V(S) - v) + \frac{\kappa}{S} * \Delta v \qquad (2)$$

$$\text{IDM} \qquad a = A * \left[1 - \left(\frac{v}{v_f}\right)^\delta - \left(\frac{S^*}{S}\right)^2\right] \qquad (3)$$

$$S^* = S_0 + \max\left(0, vT + \frac{v*\Delta v}{2\sqrt{AB}}\right) \qquad (4)$$

$$\text{IDMM} \qquad a = A * \left[1 - \left(\frac{v}{v_f}\right)^\delta - \left(\frac{S^*}{S}\right)^2 + \omega * \left(\frac{L}{L_C}\right)^\eta\right] \qquad (5)$$

The S-shape and exponential type optimum velocity functions are considered for OVM and FVDM, respectively.

$$V(S) = \frac{v_f}{1+\tanh\beta} * \left[\tanh\left(\frac{S}{\Delta S} - \beta\right) + \tanh(\beta)\right] \qquad (6)$$

$$V(S) = v_f * \left[1 - \exp\left(-\frac{\alpha}{v_f} * (S - S_0)\right)\right] \qquad (7)$$

VI. Stability Criteria

Stability is an important parameter regulating traffic flow efficiency and safety in the traffic stream. The instability in traffic can generate a series of shockwaves, and this instability can be easily triggered due to a multitude of reasons, especially the nature of mixed traffic. In mirroring the vehicles' sensitivity, the car-following models can explain the stability of a traffic stream. Considering this, and in line with the literature [12-14], stability criteria are given for three models: OVM, FVDM, and IDM. Accordingly, the stability criteria of those models are given by the respective developers are as follows.

$$\text{OVM: } V'(S) < \frac{\lambda}{2} \qquad (8)$$

$$\text{FVDM: } V'(S) < \frac{\lambda}{2} + \kappa \qquad (9)$$

$$\text{IDM}: V'(S) < -\frac{f_v}{2} - f_{\Delta v} \quad \Rightarrow G - Value \tag{10}$$

If these conditions satisfy, the traffic stream is graded as stable. Further, the generalized stability criteria in line with the literature [15] are as follows,

$$F = \frac{f_v^2}{2} - f_v * f_{\Delta v} - f_s > 0 \Rightarrow String\ Stable \tag{11}$$

$$L = -f_v - f_{\Delta v} > 0 \qquad \Rightarrow Locally\ Stable \tag{12}$$

The parameters in Eq. (11) and (12) are given as follows,

$$f_v = \left. \frac{\partial f(v, S, \Delta v)}{\partial v} \right|_{(V(S_e), S_e, 0)}$$

$$f_s = \left. \frac{\partial f(v, S, \Delta v)}{\partial S} \right|_{(V(S_e), S_e, 0)} \tag{13}$$

$$f_{\Delta v} = \left. \frac{\partial f(v, S, \Delta v)}{\partial \Delta v} \right|_{(V(S_e), S_e, 0)}$$

It must be noted that the above stability conditions are tested for lane-based traffic flow conditions. On the other hand, in heterogeneous traffic, to apply the same methodologies, one can use different parameters for different vehicle categories. This study planned to calibrate the driving behavior for each vehicle category, and the stability can be evaluated accordingly. IDMM is used to represent non-lane discipline in our analysis. Since this considers 2-Dimensional flow, the above criteria cannot be applied directly. So, a straightforward methodology to analyze stability is developed and validated, explained in later sections.

Accordingly, the OVM, FVDM, and IDM's local stability and string stability conditions are derived. Note that in OVM, since there is no velocity difference term in its control equation, $f_{\Delta v}$ is zero in Eq. (11) and (12). The stability curves obtained by Eq. (8) and (9) are referred to as sensitivity curves. The stability values obtained from Eq. (10) are referred to as G-values, and the corresponding stability curve is called G-curves. Similarly, Eq. (11) and (12) are called F-value and L-value and corresponding curves as F-curve and L-curve, respectively, as seen in Table II. To demonstrate the mathematical computations for the stability, the derivation for FVDM, Eq. (2) using the optimum velocity function, Eq. (7), is shown as an example. On these lines, the control equation can be written as,

$$a = \lambda * v_f - \lambda * v_f * \exp\left(-\frac{\alpha}{v_f} * (S - S_0)\right) - v * \lambda + \frac{k}{S} * \Delta v \tag{14}$$

$$f_v = -\lambda \tag{15}$$

$$f_{\Delta v} = \frac{k}{S} \tag{16}$$

$$f_s = \lambda * \alpha * \exp\left(-\frac{\alpha}{v_f} * (S - S_0)\right) - \frac{k}{S^2} * \Delta v \tag{17}$$

At steady-state equilibrium, $\Delta v = a = 0$. Also, refer to Eq. (13), which shows the condition at which partial derivations are to be carried out. Therefore, Eq. (14) becomes,

$$0 = \lambda * v_f - \lambda * v_f * \exp\left(-\frac{\alpha}{v_f} * (S - S_0)\right) - v * \lambda + 0$$

$$\exp\left(-\frac{\alpha}{v_f} * (S - S_0)\right) = \frac{v_f - v}{v_f}$$

$$S = S_0 - \ln\left(1 - \frac{v}{v_f}\right) * \frac{v_f}{\alpha} \tag{18}$$

By substituting Eq. (18) to Eq. (16) and Eq. (17), we get,

$$f_{\Delta v} = \frac{k * \alpha}{\alpha * S_0 - v_f * \ln\left(1 - \frac{v}{v_f}\right)} \tag{19}$$

$$f_s = \lambda * \alpha * \left(1 - \frac{v}{v_f}\right) \tag{20}$$

On similar lines, all the models' stability criteria are computed in terms of F, L, and G values. Equations (15), (19), and (20) are the partial derivatives with respect to speed, speed difference, and spacing, respectively, which are used in stability equations, as shown in Table II. Partial derivates for all other models are obtained using the same procedure.

TABLE II Stability Criteria Equations for Selected Car-Following Models

Model	Stability Criteria Final Equation
OVM	$\lambda > 2 * \dfrac{v_f}{1 + \tanh\beta} * \dfrac{\text{sech}^2\left(\frac{S}{\Delta S} - \beta\right)}{\Delta S}$
	$F = \dfrac{\lambda^2}{2} - \dfrac{\lambda * v_f}{(1 + \tanh\beta) * \Delta S}$ $* \left(1 - \left(\frac{v}{v_f} * (1 + \tanh\beta) - \tanh\beta\right)^2\right)$
FVDM	$\lambda > 2 * \left(\alpha * \exp\left(-\frac{\alpha}{v_f} * (S - S_0)\right) - \kappa\right)$
	$F = \dfrac{\lambda^2}{2} + \dfrac{\lambda * \kappa * \alpha}{\alpha * S_0 - v_f * \ln\left(1 - \frac{v}{v_f}\right)} - \lambda * \alpha * \left(1 - \frac{v}{v_f}\right)$
	$L = \lambda - \dfrac{\kappa * \alpha}{\alpha * S_0 - v_f * \ln\left(1 - \frac{v}{v_f}\right)}$
IDM and IDMM	$G = \left(AT + v\sqrt{\dfrac{A}{B}}\right) * T * \dfrac{\sqrt{1 - \left(\frac{v}{v_f}\right)^\delta}}{(S_0 + vT)} - 1 \geq 0$
	$F = \dfrac{1}{2} * \left(-\dfrac{A\delta}{v_f} * \left(\frac{v}{v_f}\right)^{\delta-1} - \dfrac{2AT}{(S_0 + vT)}\right)$ $* \left(1 - \left(\frac{v}{v_f}\right)^\delta\right)\right)^2 - \left(-\dfrac{A\delta}{v_f} * \left(\frac{v}{v_f}\right)^{\delta-1} - \dfrac{2AT}{(S_0 + vT)}\right)$ $* \left(1 - \left(\frac{v}{v_f}\right)^\delta\right)\right) * \left(-\dfrac{v}{S_0 + vT} * \sqrt{\dfrac{A}{B}}\right)$ $* \left(1 - \left(\frac{v}{v_f}\right)^\delta\right)\right) - \dfrac{2A}{S_0 + vT} * \left(1 - \left(\frac{v}{v_f}\right)^\delta\right)^{\frac{3}{2}}$
	$L = \dfrac{A\delta}{v_f} * \left(\frac{v}{v_f}\right)^{\delta-1} + \dfrac{2AT}{(S_0 + vT)} * \left(1 - \left(\frac{v}{v_f}\right)^\delta\right)$ $+ \dfrac{v}{S_0 + vT} * \sqrt{\dfrac{A}{B}} * \left(1 - \left(\frac{v}{v_f}\right)^\delta\right)$

VII. CALIBRATION

To test the model in mixed traffic conditions, the models have to be calibrated for each vehicle category. The goal in calibrating any model is to obtain optimal parameters to have

the best model predictions of field observations. Initially, descriptive statistics of trajectory data are used to obtain v_f, A, and B, which are evaluated as to its 95th percentile values, S_0 as 2nd percentile and L_C is computed as average lateral clearance with surrounding vehicles. The values are given in Table III.

TABLE III Basic Parameters of Western Expressway

Parameters	3W	2W	Bus	Car	HCV	LCV
v_f (m/s)	14.1	19.2	14.1	15.9	13.1	14.5
S_0 (m)	1.86	2.27	3.77	1.63	3.91	1.37
A (m/s²)	3.44	4.48	3.34	2.69	3.20	2.94
B (m/s²)	3.47	4.25	3.37	2.83	3.23	3.21
L_C (m)	0.65	0.42	1.46	1.21	1.68	0.97

In line with the literature [2], all other parameters of car-following equations are calibrated using the genetic algorithm (GA). MATLAB software is used to run the optimization process since it has a built-in package of GA. To calibrate these model parameters, parameter constraints are chosen based on the simple logic that time or velocity cannot be negative. Therefore, the lower bound of sensitivity parameter, λ in OVM, and FVDM are set to zero; κ in FVDM is also set to zero. In optimum velocity functions 2 and 3, for β and ΔS, and α, the lower bounds are set to zero as well with no upper constraint. Same for δ and T in IDM. The parameters for IDMM are taken from Raju et al. [8]. The calibrated parameters are shown in Table IV.

TABLE IV Calibrated Parameters of the Car-Following Models

Model	Parameters	3W	2W	Bus	Car	HCV	LCV
OVM	β	3.5	3.2	3.16	2.62	3.5	3.1
	ΔS	14.6	12.5	17.3	15.6	16.9	15.6
	λ	0.05	0.06	0.04	0.05	0.04	0.05
FVDM	α	0.3	0.5	0.3	0.4	0.3	0.2
	λ	0.03	0.02	0.04	0.02	0.03	0.03
	κ	0.27	0.14	0.59	0.43	1.13	0.15
IDM	δ	0.49	0.59	0.23	0.38	0.21	0.29
	T	1.2	1.0	1.3	1.1	1.4	1.0
IDMM	δ	0.72	0.45	0.91	0.65	0.97	0.84
	η	0.65	1.12	0.31	0.94	0.23	0.51
	T	1.2	1.1	1.2	1.1	1.2	1.0

VIII. STABILITY ANALYSIS

A. OVM and FVDM

Based on the calibrated parameters, with the stability criteria, the stability is evaluated over the traffic stream in terms of F, L, and G values for the respective models. The L-value corresponds to the local stability, F-values to the string stability in OVM, FVDM, and IDM. On the other hand, G-value corresponds to the additional generalized stability in the IDM models. It can be noted that, given the OVM formulation, it always corresponds to be locally stable. Further, the stability analysis of OVM and FVDM is presented in Fig. 1.

From the plots in Fig. 1, it can be noted that the smaller vehicles, mainly 2W, tends to impact the string stability with a major portion of F-value below zero. This kind of trend is observed in both models. Simultaneously, unlike the string stability, the data patterns are inconsistent with physical
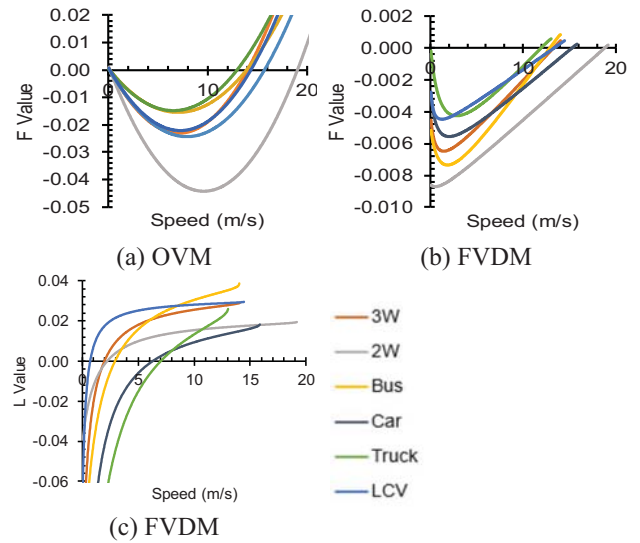

Fig. 1. Stability graphs of OVM and FVDM for different vehicle categories

characteristics in L-curves. This shows that, on a local level, the impact of vehicle category on the local stability is minimum over the traffic stream. From this analysis, it is witnessed that, at a global level, given the lateral freedom, the smaller vehicles tend to be less stable, which can be attributed to their lateral freedom. Whereas, in the case of local stability, numerous factors cause the turbulence under the "following" conditions. The impact can be witnessed by all the vehicles in the traffic stream. Due to this nature, the variation due to vehicle categories is limited in the L value plots.

Interpretation of plots in Fig. 1 as an influence of values in Table III and Table IV based on formulations given in Table II are made. From the L-value equations, it is clear that λ has a positive impact, and κ has negative implications on local stability. Safe minimum spacing (S_0) and free-flow speed (v_f) also affect the curve positively and negatively, respectively. In FVDM, α corresponds to lesser local stability. Moving on to interpreting graphs of string stability (F-curves), the OVM graph shows that vehicles attain a stable state beyond a certain speed. This is likely because, at lower speeds, the overreaction due to less spacing is higher, causing instability. The same applies to the graph of FVDM as well, but at very low speeds, the vehicles are stable. Because FVDM has a speed difference term with κ as its sensitivity, making the graphs stable at very low speeds. Unlike in local stability, κ corresponds to an increase in string stability but has no effects after a certain speed. This observation is evident in Fig. 1 (b), the FVDM graph. If the plots of OVM and FVDM are compared, it is clear that the curves of both graphs are reaching a stable state at nearly the same speed. This is because, if the speed difference term is taken out of the equation of FVDM, it reduces to OVM itself, which is valid at high spacing, corresponding to a higher speed. But since different optimum velocity functions are used for OVM and FVDM, they are not matching perfectly.

B. IDM and IDMM

With the help of calibrated IDM and IDMM, the stability parameters are analyzed along with added general stability

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parameter G-value as reported in Fig. 2. From the string stability (F) on IDM, it is observed again that smaller vehicles, mainly 2W, tend to disrupt the traffic stream's stability. On the other hand, at a local level, given the continuous function, the stream is highly stable compared to the results of OVM and FVDM. Similarly, the general stability from the IDM is in the combination of local and string stability. Further, it is inferred that, more or less, the local stability and string stability results from IDM are in line with the OVM and FVDM.

(a) IDM

(b) IDMM

(c) IDM

(d) IDMM
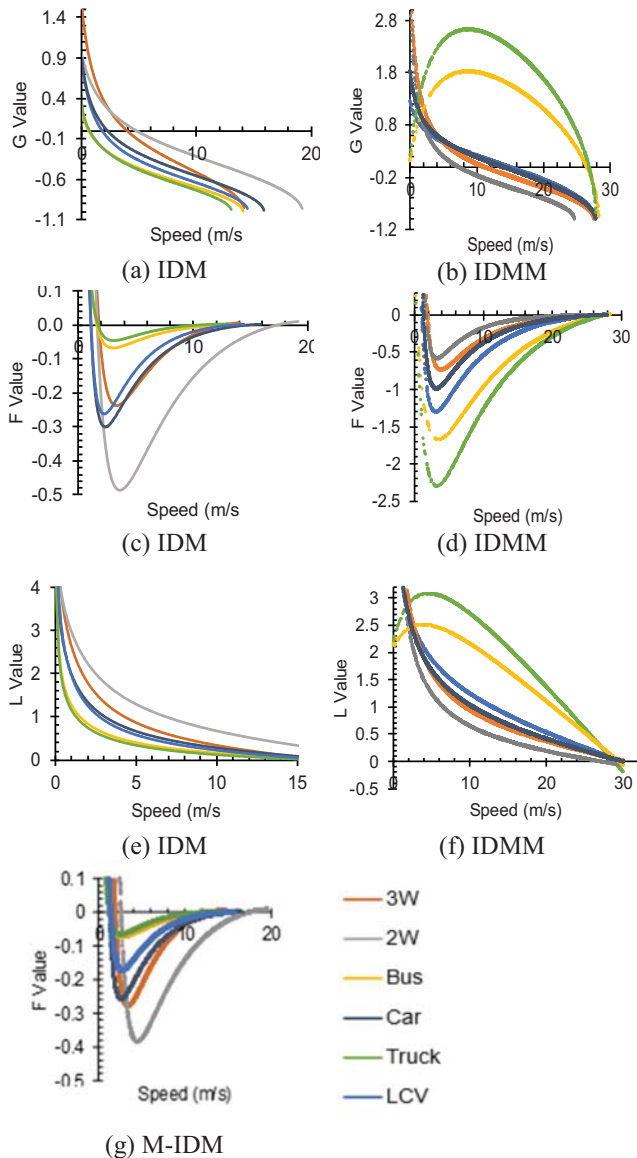
(e) IDM

(f) IDMM

(g) M-IDM

Fig. 2. Stability curves of IDM, IDMM and M-IDM

Customized-IDM (IDMM), as discussed earlier, includes discrete lateral movements that break the ordering of vehicles. Hence, the stability techniques of G, F, or L do not work directly. To overcome this, first, acceleration at every instant is calculated by using the control equation of IDMM. Then by using fundamental formulas of physics, $v = u + at$ and $S = ut + \frac{1}{2}at^2$ corresponding velocities and positions of each vehicle is calculated at every instant (0.5s interval) and hence

spacings between vehicles. The same stability criteria that are used to analyze classical-IDM are applied to this obtained or modelled data set. This same methodology is used to examine the stability of classical-IDM, shortly called modelled-IDM (M-IDM). It is found that the stability graphs of M-IDM (obtained by the same methodology as IDMM) are quite the same as stability graphs obtained by conventional methods (direct formulation). An example graph of M-IDM is shown in Fig. 3 (g), which can be compared with Fig. 3 (c).

From the analysis on 2D IDMM, the stability results across all three parameters are varied with the traditional IDM. It is observed that, unlike the other three models, the smaller vehicles (excluding truck and bus) impacted the string stability in a smaller manner compared to other vehicle categories. Similarly, the shape of the heavy vehicle categories' (Truck and Bus) curves varies in general and local stability curves. In understanding variation among the results better, an investigation is carried out. It can be noted that OVM, FVDM, and IDM are one-dimensional longitudinal models, by which the following behavior will be expressed.

On the other hand, in mixed traffic, the smaller vehicles in the traffic tend to be associated with a high degree of maneuverability. As a result, those vehicles can follow their leader with a high velocity in the mixed traffic stream and switch their lateral position for better movement. Nevertheless, with the one-dimensional longitudinal models, when the vehicles are nearing their leader vehicles, the relative velocity will be limited. As a result, the one-dimensional models exposed this phenomenon as unstable conditions. Whereas in the IDMM, the lateral movement of the vehicles is considered in the core formulation. As a result, IDMM can express the lateral movement and expressed the stability of those vehicles differently. Even the same phenomenon can be attributed to the general and local stability results from the IDMM.

In Table V, a brief outlook on each model's performance is provided based on string stability. Here, the stability of trajectory data is evaluated at every instant, and the ratio of stable points to the total number of points is taken. The critical observation we must make here is for IDM and IDMM. IDM shows very low to nil stability percentages for trucks and buses. Although lateral movements cannot affect stability mathematically in these 1-Dimensional models, it must account for some disturbance/ perturbation. It seems logical to see the order of percentage of stable points obtained by IDMM for all vehicle categories, with trucks and buses being the most stable.

Table V Percentage of Stable Points in Each Vehicle Category

Model	3W	2W	Bus	Car	Truck	LCV
OVM	26.2	12.5	27.2	32.8	29.7	28.8
FVDM	14.4	14.6	24.3	15.6	27.1	24.3
IDM	52.2	32.5	0.0	61.1	12.5	55.3
IDMM	29.6	11.3	95.0	43.6	97.2	49.5

IX. RESULTS AND DISCUSSION

Due to a high degree of non-lane discipline in mixed traffic, the smaller vehicles have more lateral freedom in the traffic

stream. As a result, under the "following" conditions, smaller vehicles in the traffic stream can follow their leaders with higher velocities and switch laterally in fewer durations. These lateral movements must account for some disturbance. Due to this phenomenon, the one-dimensional models (OVM and FVDM) expressed that smaller vehicles in the traffic stream are found to be highly unstable. However, from Table V, it is evident that G-value criteria correspond to much lower stability for larger vehicles (trucks and buses). In contrast, the IDMM considered this lateral movement phenomenon and showed more stability for larger vehicles since they do not correspond to additional disturbances to traffic.

From the local and string stability analysis across the models (Fig. 1 and Fig. 2), it is identified that, most of the time, the traffic stream is found to be locally stable. On the other hand, all considered models demonstrated unstable string stability conditions. This shows that vehicles tend to show proper responsiveness between the vehicles on a one to one vehicle following conditions. Whereas, given the variation in vehicle sizes and the lateral movement, the stability over the traffic stream is depleted. Parameters of car-following models assess the influencing factors in depleting the string stability.

The study shows that the stability results vary with a change in the model (Table V). This can be attributed to the model's core formulation in expressing driving behavior. At the same time, the stability analysis is performed with the established numerical models. However, in field conditions, driving behavior may not be an exact function of a mathematical model. As a result, at all times, the real stability in the traffic stream may not be related to the mathematical formulation. In this direction, even stability should be expressed as a function of empirical data. This can be extremely helpful in monitoring real-time stability from the traffic stream and hence forecast formation and dissolution of stop-and-go waves due to instabilities.

X. SUMMARY

Stability is key in sensing the orderliness of the traffic stream. With proper discipline in driving behavior among the vehicles, the traffic stream's orderliness increases, resulting in stable conditions. Given this, the safety and the efficiency in the traffic stream will be enhanced. Simultaneously, given the data constraints in the present context, the stability methodologies have not found the practitioners' path in decision-making. Furthermost, the stability studies are carried under the domain of mathematical simulation. With the availability of trajectory data in recent times, researchers heavily focused on understanding driving behavior and surrogate safety. Even in the present context, empirical stability studies are minimal.

On the other hand, in the case of mixed traffic conditions, given the nature of traffic movement, the traffic stream can be disturbed by any means, resulting in unstable conditions. In the present study, the stability formulations for seven car-following equations and the procedure to derive them are provided. Behavioral analysis of individual vehicle categories concerning each car-following model is done by physical interpretation of the graphs, by analyzing the impacts of all the parameters

governing these models. A methodology is developed to analyze stability for 2-Dimensional traffic flow. With the help of empirical trajectory data from mixed traffic conditions, the stability regimes are marked. Based on the methodology, the stability in the traffic stream are assessed. Further, the mitigation measures can be planned in a better manner on time. At the same time, stability can be used as a metric in investigating the drop in efficiency/ safety over the traffic networks.

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