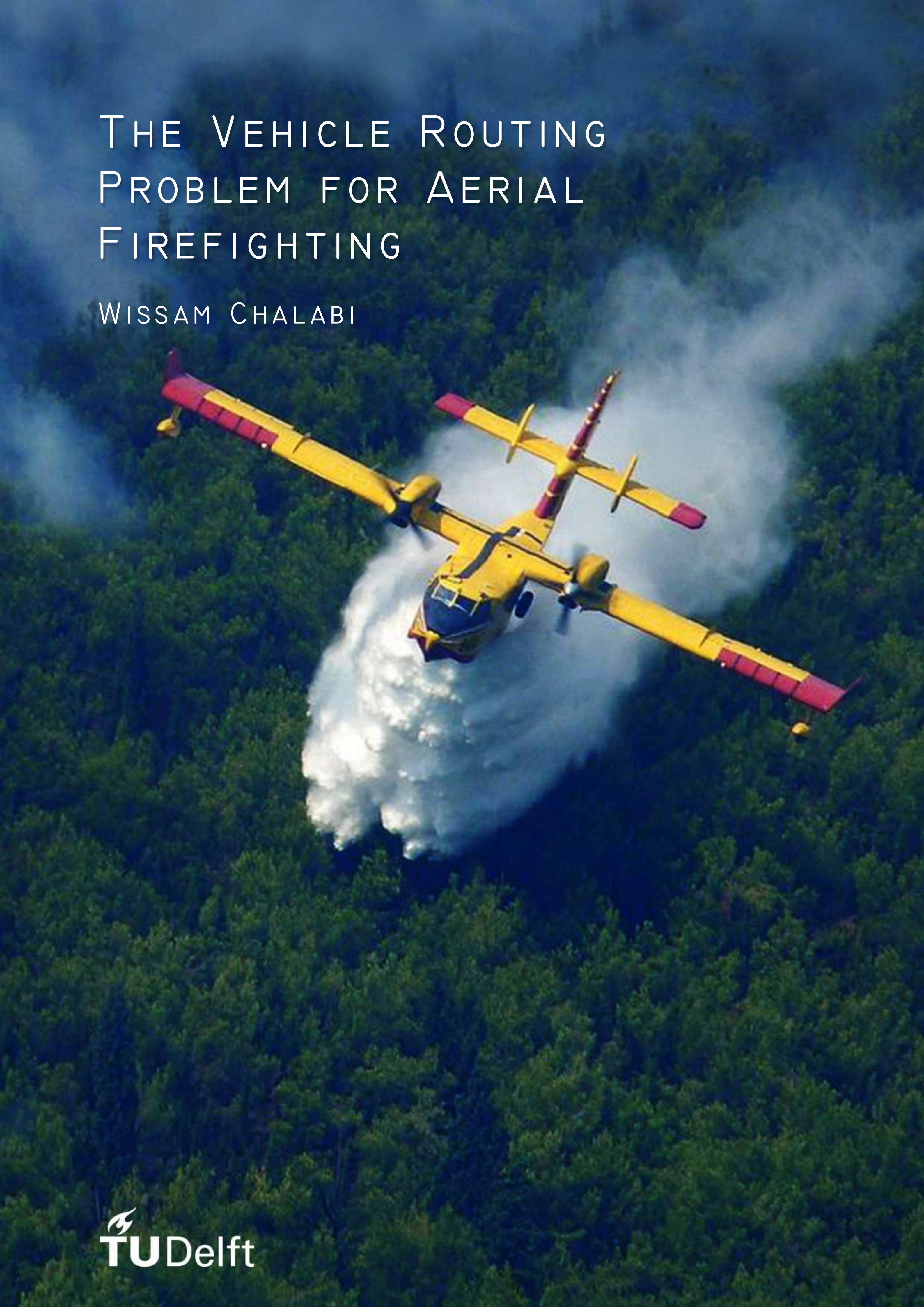
The Vehicle Routing Problem for Aerial Firefighting

Wissam Chalabi



TU Delft

The Vehicle Routing Problem for Aerial Firefighting

by

Wissam Chalabi

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TU Delft

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Wissam Chalabi
Delft, March 2023

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List of Abbreviations

AA	Air Attack
ACO	Ant Colony Optimization
AFF	Aerial Firefighting
AFUE	Aerial Firefighting Use and Effectiveness
ASM	Aerial Supervision Module
ATGS	Air Tactical Group Supervisor
CVRP	Capacitated Vehicle Routing Problem
DA	Deterministic Annealing
DARP	Dial-a-Ride Problem
DCVRP	Distance Constrained Vehicle Routing Problem
DOBJ	Drop Objective
FEIS	Fire Effects Information System
FOFEM	First Order Fire Effects Model
GA	Genetic Algorithms
HFVRP	Heterogeneous Fleet Vehicle Routing Problem
ILS	Iterated Local Search
LAT	Large Air Tanker
MTVRP	Multi Trip Vehicle Routing Problem
MVCTP	Multi Vehicle Covering Tour Problem
OR	Operations Research
PDP	Pickup and Delivery Problem
PR	Path Relinking
PTP	Profitable Tour Problem
SA	Simulated Annealing
SDVRP	Split Delivery Vehicle Routing Problem
SEAT	Single Engine Air Tanker
SS	Scatter Search
TOP	Team Orienteering Problem
TS	Tabu Search
USDA	United States Department of Agriculture

VLAT Very Large Air Tanker

VNS Variable Neighborhood Search

VRP Vehicle Routing Problem

VRPSPD Vehicle Routing Problem with Simultaneous Pickup and Delivery

VRPTW Vehicle Routing Problem with Time Windows

VSP Vehicle Scheduling Problem

WFAS Wildland Fire Assessment System

Introduction

As the climate crisis worsens, unwanted wildfires are occurring and getting out of control more often. While prevention efforts are the most effective way to protect the environment from wildfires, the role of Aerial Firefighting (AFF) is crucial when fires do spread beyond the control of ground crews.

A conversation with firefighting pilot Glen Purdam at an AFF conference in Estonia prompted the idea for this project. The need for operations research (OR) based methods to plan the routes of AFF aircraft is apparent. Following conversations with experts and a literature study confirmed the need for such a tool. As a result, a Vehicle Routing Problem (VRP) approach was chosen to create a decision-making tool to aid in the deployment of AFF aircraft during wildfires. This tool can ultimately be used to choose the most optimal routing of AFF aircraft, as a supportive tool to decision-makers, either on the operational level or on a strategic level of fleet planning, or as a training or assessment tool of previous fires.

This thesis report is organized as follows : In Part I, the scientific paper is presented, and Part II contains the literature study that preceded the research.

I

Scientific Paper

The Vehicle Routing Problem for Aerial Firefighting

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Abstract

Optimizing the routes of firefighting aircraft can reduce the time it takes to contain wildfires and make sure they remain within control. In this paper, a novel formulation of the Vehicle Routing Problem (VRP) is developed to improve aerial firefighting operations by optimizing aircraft routes. The formulation is a capacitated split delivery multi-trip VRP with time windows and hierarchical objectives. The primary objective is to minimize the time of carrying out all requested drops, and the secondary objective is to minimize the total flight time. Two types of aircraft are used: Scoopers and tankers. The main difference is that scoopers can refill their water tank from a water body. By easily adjusting the capacity and speed of the aircraft, most firefighting aircraft can be modelled using these two types, including helicopters. The program allows the user to input the number and types of aircraft available, the locations of airfield, fires, and nearest water body, intensity of each fire, and more. Several random cases and case studies were solved within the expert-recommended time limit of 5 minutes, yielding reasonable optimized routes. The problem is scalable and sizes ranging from one to 80 drops were tested and solved within 22 minutes. Furthermore, given a certain fire situation, the model can be simulated with various aircraft combinations to gain insights into fleet optimization. In one case study, it was demonstrated that replacing a scooper with a tanker can result in halving the total operation time. Strategic fleet planning is also demonstrated in a case study with the use of a Monte Carlo simulation, in order to compare the performance of different fleet options for a given setting. Therefore, the model is not only applicable in live situations, but can also be used as a supportive tool in planning for upcoming fire seasons, or reviewing and learning from past fires.

1 Introduction

As climate change worsens, the likelihood and severity of forest fires increase due to the changes in temperature, precipitation patterns, and vegetation [Dennison, 2009]. For instance, the number of fires in European countries has significantly increased in 2022 compared to the average of the previous 10-year period, as mapped by the European Forest Fire Information System (EFFIS), which is shown in Figure 1 [EFFIS, 2023].

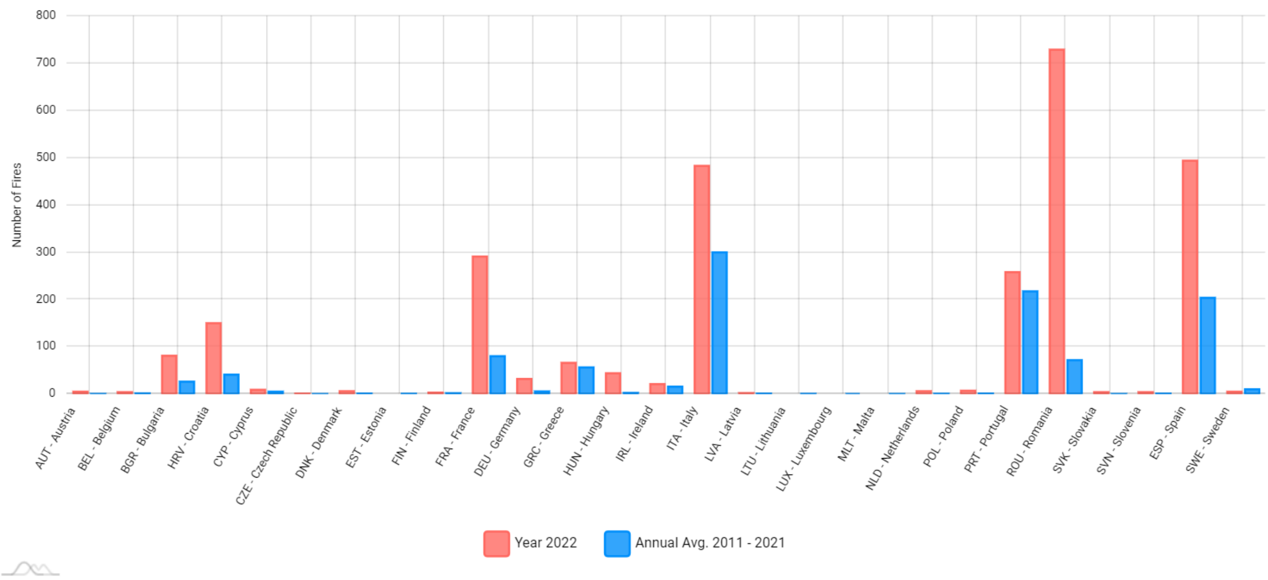

Figure 1: Comparison between the number of wildfires in 2022 and the average during the 2011-2021 period.

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An important principle in fire suppression is that the success of the initial attack can largely affect the ultimate outcome of a wildfire, by controlling the fire while it is still small. Most fires are indeed controlled and contained before they become large. The minority that do escape and become large require significantly more time and resources to control, as well as inflict far more damage onto the environment [Wollstein et al., 2022]. Aerial firefighting (AFF) can be an important resource in the initial attack, by reaching aggressive fires early and helping to contain them before they become large.

While computer programs are used to support authorities in firefighting, the routing of aerial resources is mostly human-managed. As of 2023, the process is managed mostly through a dynamic operation where ground-based operators communicate continuously with aerial supervisors. The latter relay information about the situation they oversee via radio communication, and possibly request firefighting aircraft. The ground-based operators then grant and deploy those aircraft. This is a simplified description of the process, which can be found in Air Attack manuals of fire management authorities such as the National Wildfire Coordinating Group [NWCG, 2023]. This means that the process is prone to errors, or at the very least sub-optimal choices can be made for the routing of aerial resources, especially when there are multiple targets and multiple aircraft to be assigned.

The Vehicle Routing Problem (VRP) is a type of transportation problem that aims to optimize the routing of vehicles that need to complete some routes to fulfill their tasks. In the VRP, transportation requests and a fleet of vehicles are given, and the task is to then find the set of routes that should be executed by the vehicles in order to incur the minimum cost. The cost is not only monetary, but can be defined for different situations, for example as the travel time or route length [Irnich et al., 2014].

An Aerial Firefighting VRP (AFFVRP) model has the potential for several helpful use cases. Besides the primary application of live use by authorities during complex fire scenarios, to deploy and allocate aerial firefighters to different fires, a few other uses can be proposed. It can be used for training operators to grow their intuition for efficient routing. After large fires, it can be used in hindsight for comparison and assessment of the operation, learning lessons about what can be improved in future operations. It can also be used for fleet selection. If a fire scenario is run with different kinds of fleets, a comparison can be made and insights into the advantages and disadvantages of different combinations of aircraft can be gained.

The purpose of this work is to support firefighting authorities in containing wildfires by optimizing the routing of aerial firefighting aircraft. This research paper presents the novel adaptation of the VRP specifically to AFF. It considers two different types of aircraft, a type that can collect water from water bodies and go on to another fire, and another type that must return to the airfield before it can visit another fire. A VRP model is formulated and programmed in Python to be optimized using the commercial solver Gurobi. It is therefore important to find the balance between the size of the problem (number of fires and aircraft), and the computation time. The model allows the user to modify inputs such as the number of fires, fire locations, fire intensity, aircraft capacity and speed. It is worth noting that for this approach to be useful in live situations, it must provide good solutions within a recommended amount of time.

This paper is structured as follows: Section 2 presents the findings of the literature review. Section 3 presents the AFFVRP model. Section 4 presents the main results and the performance of the model. The case studies are then presented in Section 5. A validation test was done with an expert and is presented in Section 6. Finally, Section 7 contains the conclusions and recommendations for further work.

2 Literature Review

To design a routing model for AFF, it is first important to lay the foundation of understanding firefighting operations, such as the types of aircraft used and the kinds of routes performed. This is presented in section 2.1. Then, a review of the VRPs that are considered relevant for this paper is presented in section 2.2.

2.1 Firefighting Operations

2.1.1 Types of Aircraft

Table 1 and Table 2 show some of the commonly used aircraft for AFF. This paper considers three important characteristic distinctions:

1. **Ability to collect water during flight:** Some aircraft are able to collect water during the flight and go on to the next fire that requires a drop. These can either be helicopters, which use a bucket or a hose to collect and drop water, or fixed wing amphibious aircraft, which can scoop up water from a larger water

body and continue their flight. Ultimately, two types of aircraft shall be used, referred to as scoopers and tankers. Helicopters can be modeled under the scooper category by simply changing the input of their speed and capacity.

2. **Capacity:** The capacity of aerial firefighters is an important characteristic. To simplify matters, tankers are assumed to have a larger capacity than scoopers, but this is ultimately possible to change as an input to the model, to reflect capacity differences of various fleets.

3. **Speed:** The cruise speed of the aircraft can determine how quickly they reach the fire. For this paper, and based on the data in Table 1 and Table 2, tankers will be considered faster than scoopers. This will also be possible to change as an input to the model, to reflect speed differences of various fleets.

Table 1: Overview of fixed-wing firefighting aircraft [USDA, 2020].

Aircraft Model	**Aircraft Type**	**Capacity (Gallons)**	**Cruise speed (mph)**
DC-10	VLAT	9400	350
Boeing 747-400	VLAT	18000	450
BAe-146-200	LAT	3000	350
RJ-85	LAT	3000	350
Lockheed EC-130Q	LAT	4000	350
Boeing MD-87	LAT	3000	350
Lockheed C-HC-130H/J	LAT	3000	230
Boeing 737-300	LAT	4000	350
Viking Q-400	LAT	2600	375
Air Tractor 802	SEAT	800	200
Viking CL-415	Scooper	1620	200
Air Tractor Fire Boss	Scooper	700	170

Table 2: Overview of firefighting helicopters [USDA, 2020].

Helicopter Model	**Helicopter Type**	**Bucket Capacity (Gallons)**	**Cruise speed (mph)**
Sikorsky/Erickson S-64E/F	Type 1	2500	132
Boeing BV-234	Type 1	3000	175
Boeing CH-47 "Chinook"	Type 1	3000	175
Boeing CH-46E "Sea Knight"	Type 1	1100	138
Kaman K-1200	Type 1	680	90
Airbus H215/225 NG	Type 1	1000	160
Bell 205/210, Bell UH-1	Type 2	500	121
Bell 212 HP	Type 2	350	132
Bell 412	Type 2	360	140
Airbus H125 NG	Type 3	260	157
Bell 407 NG	Type 3	270	152
Bell 206 L3/4	Type 3	225	127

2.1.2 Types of Drops

The United States Department of Agriculture (USDA) carried out a detailed study over the period 2015-2018 and produced a report in 2020 titled "Aerial Firefighting Use and Effectiveness" (AFUE) [USDA, 2020]. The report summarizes the drop objectives of all recorded drops as shown in Figure 2.

For helicopters and scoopers, the most common objectives are to reduce fire intensity and to delay fire spread. For the airtankers, the most common objectives are to delay fire spread and to halt the advance of the fire line. A more elaborate description of the different objectives is provided in Table 3[USDA, 2020].
To further simplify this for the purposes of this paper, a categorization of drops is narrowed down to two main types of drops: Drops that should fall directly on a certain fire location, and drops that form some sort of line outside the actual fire, referred to as direct drops and laying lines respectively. A summary of this categorization is visualized in Figure 3.

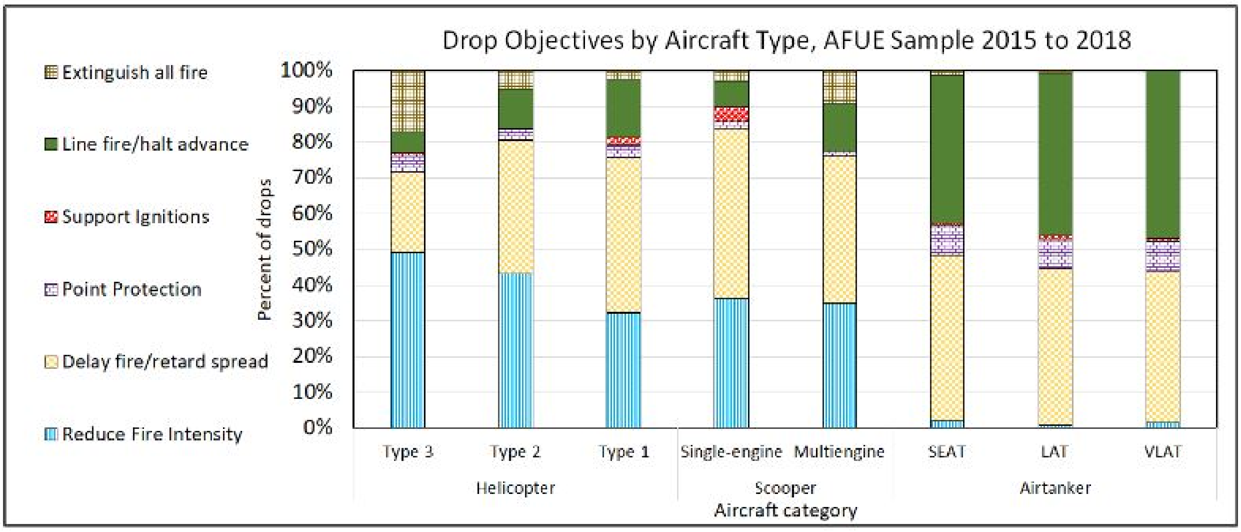


Figure 2: Drop objective per aircraft type, according to AFUE sample 2015-2018 [USDA, 2020].

Table 3: Examples of possible drop objectives [USDA, 2020].

ID	Drop Objectives	Description
DOBJ-1	Reduce fire intensity/ flame length	The intent of the drop(s) is to cool an area of fire activity. This may be needed so ground personnel can work closer to the fire activity (e.g., begin or continue to go direct). Examples include knocking down crown fire, torching, and preventing spotting, etc.
DOBJ-2	Delay fire spread/ retard growth	The intent of the drop(s) is to delay the fires rate of spread in the same location (head, heel, specific flank, spot, etc.) of the fire. Examples include buying time for ground resources to construct line or for evacuations
DOBJ-3	Point protection	The intent of the drop(s) is to protect a value(s) at risk (VAR). These drops should be within the immediate area of the VAR or be executed primarily to reduce the probability of fire reaching the VAR or to reduce damage to the VAR.
DOBJ-4	Line fire/ halt advance	The intent of the drop(s) is to construct aerial line to halt fire spread. These drops are used to halt the spread of a section of the fires edge before, during, or after ground engagement or without the aid of ground personnel
DOBJ-5	Extinguish fire/ spot fire	The intent of the drop(s) is to fully extinguish the entire portion of the fire or spot fires (generally a rare occasion, usually a small area, and likely a fine/flashy fuel).

In conclusion, drop objectives can be summarized as either direct drops onto a particular spot, or laying lines ahead of the fire. The former is better achieved by helicopters due to their hovering ability, and the latter by fixed wing aircraft due to their ability to drop while flying at a higher speed. Two main types of aircraft are identified: scoopers and tankers. Scoopers have the ability to collect water from a water body and go on to another fire, while tankers must return to the depot in order to refill before visiting a second fire. However, scoopers tend to be slower and have lower capacity than tankers. For the purposes of this paper, helicopters are considered under the category of scoopers. The most important inputs that should be possible to change in order to model different kinds of aircraft are capacity and speed.

2.2 Applicable Vehicle Routing Problems

The VRP can be useful in optimizing the routes of AFF aircraft. However, it has not been used and applied in this specific context. There is some work on the use of the VRP for planning routes of Unmanned Aerial Vehicles (UAVs), which includes some AFF surveillance applications [Khoufi et al., 2019]. There are also efforts to use the VRP in the coordination of ground-based and aerial assets in firefighting [Shahidi et al., 2022]. However, a direct adaptation of the VRP for the routing of aerial firefighters is not found at the time of this work.

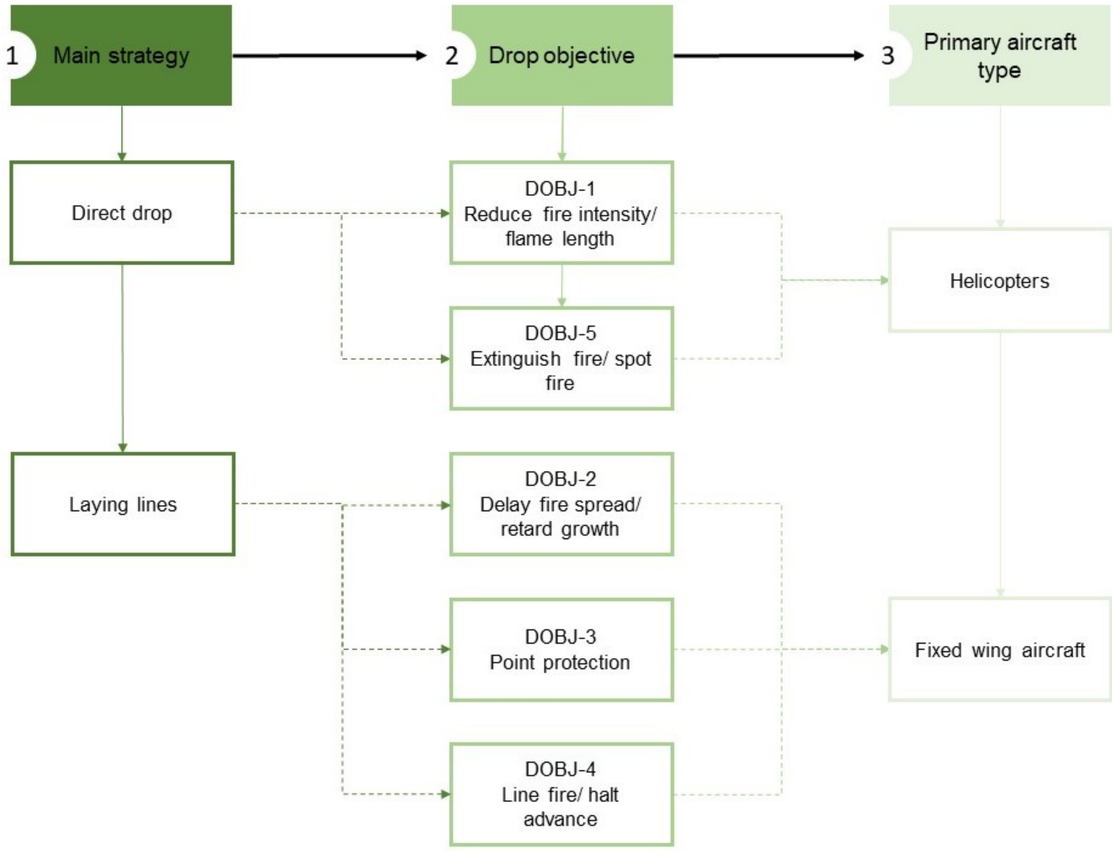


Figure 3: Overview of firefighting strategies based on drop objectives.

While literature on the use of the VRP in AFF is scarce, there are many versions of the VRP, applied in different contexts, which can be useful and applicable to the problem of AFF. Examples include Dynamic VRPs, which involve changing situations and the recalculation of routes as the problem evolves, a version of which was presented by Psaraftis [H.N.Psaraftis, 1988], and then with time windows by Haghani and Jung [A.Haghani and S.Jung, 2005].

1. **Capacitated VRP (CPVRP):**
   Transportation requests in the CVRP originate from a single depot denoted by O. Goods are distributed from O to a given set of N customers, $N = \{1, 2, ..., n\}$. The customer's demand is the amount that should be delivered to customer $i \in N$, which is given by a scalar $q_i \leq 0$, for example the mass of the goods to be delivered. An assumption is made that the fleet $K = \{1, 2, ..., |K|\}$ is homogeneous. This means that the capacity $Q > 0$ and operating costs of the vehicles $|K|$ available at the depot are the same. Furthermore, Let S be a customer subset such that $S \subseteq N$. A vehicle that is assigned to this subset will start from the depot, visit each customer within the subset once, and return to the depot. The travel cost for a vehicle moving from i to j is denoted as c_{ij}. For directed graphs $G = (V, A)$, the in-arcs of S are defined as $\delta^-(S) = (i, j) \in A : i \notin S, j \in S$, while the out-arcs are defined as $\delta^+(S) = (i, j) \in A : i \in S, j \notin S$. A basic formulation of the problem can then be presented as [Laporte et al., 1986]:

$$\text{Minimize} \sum_{(i,j)\in A} c_{ij} x_{ij} \tag{1}$$

subject to

$$\sum_{j\in\delta^+(i)} x_{ij} = 1 \qquad \forall i \in N \tag{2}$$

$$\sum_{i\in\delta^-(j)} x_{ij} = 1 \qquad \forall j \in N \tag{3}$$

$$\sum_{j\in\delta^+(0)} x_{0j} = |K|, \tag{4}$$

$$\sum_{(i,j)\in\delta^+(S)} x_{ij} \geq r(S) \qquad \forall S \subseteq N, S \neq \emptyset \tag{5}$$

$$x_{ij} \in \{0,1\} \qquad \forall (i,j) \in A. \tag{6}$$

2. **Split Delivery VRP (SDVRP):**
   There are good reasons to split a service to reach a more optimal operation. For example, using multiple smaller vehicles may be cheaper than fewer larger vehicles that are more expensive to operate. It can also be necessary to split the delivery if the demand of a particular customer is too large to be satisfied by a single vehicle, and this is likely the case in AFF. If the fire is to represent a customer, and the demand is the amount of water that needs to be dropped for the operation to succeed, then many fires will require multiple aircraft to make drops, and any single aircraft, even with large capacity, may not be able to satisfy the need. The SDVRP is a VRP relaxation that allows each demand to split into several smaller demands, which can be delivered by multiple vehicles. The formulation of the SDVRP was introduced by [Dror et al., 1994] as the following:

$$\text{Minimize} \quad \sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{v=1}^{m} c_{ij}x_{ijv} \tag{7}$$

   subject to

$$\sum_{i=0}^{n} x_{ikv} - \sum_{j=0}^{n} x_{kjv} = 0 \quad (k = 0,\ldots,n; v = 1,\ldots,m) \tag{8}$$

$$\sum_{v=1}^{m} y_{ij} = 1 \quad (i = 1,\ldots,n) \tag{9}$$

$$\sum_{i=1}^{n} q_i Y_{iv} \leqslant Q_v \quad (v = 1,\ldots,m) \tag{10}$$

$$\sum_{j=0}^{n} x_{ijv} \geqslant y_{iv} \quad (i = 1,\ldots,n; v = 1,\ldots,m) \tag{11}$$

   where x_{ijv} is a binary variable equal to 1 if and only if vehicle v travels from node i to node j in the optimal solution, and y_{iv} is the part of the demand of customer i that is delivered by vehicle v.

3. **VRP with Time Windows (VRPTW):**
   In cases where customers require service within a certain time window, the VRPTW may be a suitable approach. In this case, customer i specifies an earliest time of service a_i and a latest time of service b_i. The time of servicing this customer T_i must be within the interval $[a_i, b_i]$. If a vehicle arrives to customer i too early it must wait [Desrochers et al., 1992]. This is applicable to AFF as drops will be requested at certain locations with a time limit. If the aircraft is too late to make the drop, the fire will have advanced and the drop may no longer be required at that particular location.

4. **The Multi Trip VRP (MTVRP):**
   So far, all operations and routing have been discussed within the context of a single trip, meaning that all routes are carried out after initially leaving the depot, and concluded by ultimately returning to the depot. The MTVRP involves expanding the operation timeline to make vehicles consider multiple trips, such that they can return to the depot and leave again to perform another route [Irnich et al., 2014]. This is useful for tankers which will often need to go to the airfield to refill retardant or water before flying on to another fire.

In conclusion, the VRP has not been applied precisely to AFF with the goal of optimizing the routes of aircraft. However, some well established versions and features of the VRP can be used to design a representative AFF VRP. The CVRP is relevant since the different aircraft will have different capacities, and the model should ensure that capacities are not exceeded. The SDVRP is relevant as fires can require large amounts of water to be dropped, which may only be achievable by several aircraft performing the drops at the same location. The VRPTW includes time windows which can be useful to set a time limit for a requested drop at a particular location. Beyond that time limit, the fire will advance and that particular location will no longer require a drop. The MTVRP contributes a useful feature for tankers, which need to return to the airfield after each drop in order to refill the tank, before going on the subsequent trip as requested.

2.3 Literature Gap and Research Motivation

The lack of literature where the VRP is directly applied to AFF reveals an opportunity to create a novel formulation of the VRP that takes into account the unique challenges of AFF. Discussions with experts in the field of AFF have further confirmed the need for such a mathematical approach to efficient aircraft routing while fighting forest fires. Since the importance of quick initial attacks is well established, the possible gain of even a few seconds by using an optimized approach can make a valuable difference to the efforts of forest fire containment.

The contribution of this work is to define a novel VRP formulation that represents Aerial Firefighting realistically and lead to more effective containment of forest fires. The question requires a two-way approach. Firstly, AFF needs to be studied carefully, in order to transform the characteristics of this dynamic operation into useful parameters and inputs for the model. Secondly, the VRP and the wide range of techniques used within it should be combined consistently to establish a routing problem that can generate meaningful solutions.

3 Methodology

3.1 Problem Definition

To design a VRP formulation that represents the real life needs of AFF, several assumptions and decisions are made, with regards to how certain aspects of AFF can be transferred into a mathematical environment. In this section, the most important of those decisions are presented and discussed. These decisions define the problem and are the guiding principles behind the model development that follows.

3.1.1 Types of aircraft

Two types of aircraft are considered. The parameters associated with these two types can be changed to eventually represent most kinds of AFF aircraft. The two types are called scoopers $p \in P$ and tankers $k \in K$. The key difference is that scoopers can pick up water from a body of water, and continue to perform subsequent requested drops, while tankers must return to the airfield after a drop, and refill before they can head out again to another fire. The other differences are: scoopers are slower and have lower capacity. However, the speed and capacity are input parameters in the model, meaning that they can be altered easily to represent different kinds of aircraft. For example, helicopters can be represented by the scooper type.

3.1.2 Types of nodes

There are three types of nodes: airfield, water body, and fire. To be precise, the fire node represents the drop location. In reality, this may not be a fire if the mission is to lay lines ahead of the fire front. However, the node is simply called fire to offer an easy-to-understand visualization. The airfield is assumed to be the location of take-off and landing of all available aircraft, and the water body is the nearest useful water body, which is assumed to be used for all scooper refilling needs during the operation. These assumptions represent reality well for most cases, according to expert feedback. Exceptions could be that multiple water bodies are used, if there are several suitable ones near the fire. For the airfields, it is possible that the aircraft are not initially based in the nearest airfield, although they would still likely operate from the nearest one once in the area. It is also possible to use multiple airfields for two reasons: to avoid congestion if multiple airfields are available, and when the fires are so large that they require a large fleet, which needs more than one airfield to operate from.

3.1.3 Fire intensity

Fire intensity, for the purposes of this research, is defined as the number of scooper (because they have the lower capacity) drops required. This is a simple and efficient way to define the intensity. The more intense the fire, the more drops it requires.

3.1.4 Subfires

In VRP formulations, a typical constraint is that each node is visited only once. Creating a model without this constraint can increase the complexity of the model. The SDVRP offers an approach to do this, but a simpler and less complex approach is to split the nodes themselves. Each fire is split into several smaller fires called subfires. For instance, if a fire has intensity 5, it is split into 5 subfires. Instead of one node, 5 nodes near each other are visualised. These are the 5 drop locations for that fire, and each drop location has a demand equal to the capacity of scoopers.

3.1.5 Tanker multi trips and subfire drops

Tankers do not have the ability to scoop up water from a water body. Once they make a drop, they need to return to the airfield to refill the tank with water or retardant. The planning horizon for the model assumes a single trip for the scoopers (although multiple drops), but it does provide the option of multiple trips for the tankers. The maximum number of trips can be set as a user input. Tankers need to return to the airfield after a drop. However, their capacity is larger than scoopers. Hence, making one drop on one subfire would be wasteful. For example, if the tanker capacity is 10, and the demand of each subfire is 5, then the tanker would waste half the capacity if it only drops on one subfire. To resolve this, tankers are allowed to make several drops as long as their capacity can cope, but only within the same main fire. In reality, this is just one single drop, which is able to satisfy the demand of multiple subfires.

3.1.6 Time windows

When a drop is needed at a certain location, it is usually needed urgently. Depending on the dynamics of the fire and how fast it is spreading, after some time the drop will no longer be required at that location. It will instead move and be required at a different location where the fire has progressed. Therefore, time windows are defined for each drop location. The lower limit is always zero, since the earlier the drop is made, the better. An upper limit can be defined as a user input. This is done for each subfire.

3.1.7 Hierarchical objectives

Two objectives are identified based on expert advice. Firstly, the drops need to be carried out as soon as possible. Secondly, the total flight time should be minimized. The first objective is more important than the second, because it is tied to the success of the mission, and every second that a fire is allowed to burn, causes more damage and increases the risk of the fire getting out of control and becoming much more costly. The second objective is desirable by operators, to minimize the operation costs. A hierarchical structure is used where the first objective is given priority and solved for. Once that objective is solved, the second objective is minimized as much as possible under the result of the first.

3.1.8 Minmax objective

In the primary objective, the time of performing the last drop needs to be minimized. However, the time at which the drops are carried out is only computed as a result of the model. This creates a challenge where the object to be minimized is not calculated yet, but it cannot be calculated properly if it is not minimized. Minimax is a decision rule used in game theory and optimization which minimizes the maximum loss. In this case, the latest time of a drop is the maximum, which needs to be minimized. This is done by setting the time of the drops equal to a new variable, and then introducing a constraint that ensures the time of the drops is less than or equal to the new variable, while the new variable is minimized in the objective function.

3.1.9 Scooper time distance and water bodies

Scoopers go to a water body to scoop up more water, then move on to the next drop location. Modeling this and plotting it can cause long computation times because scoopers will have many node visits. One way to avoid this while still maintaining the concept is implemented as follows: For the time matrix of scoopers, when the distance between drop locations is calculated, the distance to the water body from the first location, and the distance from the water body to the second location are added up. This way, the trip to the water body is accounted for, but does not need to be explicitly modeled. This will give the same results but skip many node visits which could save some computation time. The only disadvantage is that the plots will not show the scoopers going to the water body, even though in reality and in the calculations, they do. This is seen as a worthwhile trade-off because computation time is important in the case of live application. Furthermore, the visualization can be tweaked to show the water body visits if necessary or required.

3.2 AFFVRP: A Novel Formulation of the VRP for Aerial Firefighting

The AFFVRP model is presented in this section through the sets in Table 4, the parameters in Table 5, and the variables in Table 6, after which the mathematical formulation is presented followed by a description of each objective and constraint. The problem is formulated as:

Table 4: List of sets of the model.

Symbol	Description
F	Set of all fires, indexed by f
H	Set of main fires, indexed by h
N_f	Set of subfires, indexed by n_f
P	Set of scoopers, indexed by p
K	Set of tankers, indexed by k
U	Set of trips, indexed by u
N	Set of all nodes: $\Delta \cup F$, indexed by n
A	Set of all arcs: $A = \{(i,j) \in N^2, i \neq j\}$
A_K	Set of tanker arcs

Table 5: List of parameters of the model.

Parameter	Description
n	Number of main fires
$Tankers$	Number of tankers K
$Scoopers$	Number of scoopers P
T_{ij}^k	Time distance between nodes i and j by aircraft type k (tanker)
T_{ij}^p	Time distance between nodes i and j by aircraft type p (scooper)
D_i	Demand quantity of node i
R_i	Processing time at node i , the time it takes for the full dropping manoeuvre
R_D	Processing time at airfield, the time it takes tankers to refill or scoopers to refuel
C_k	Capacity of tanker k
C_p	Capacity of scooper p
k_{cruise}	Cruise speed of tanker k
p_{cruise}	Cruise speed of scooper p
$[T_f^0, T_f^1]$	Time window for fire f
M	Big arbitrary value
I	Intensity of a main fire, measured in how many drops of C_p it requires

Table 6: List of variables of the model.

Variable	Description
x_{ij}^{ku}	Binary routing variable. =1 if vehicle k travels from i to j on trip u
x_{ij}^p	Binary routing variable. =1 if vehicle p travels from i to j
τ_i^{ku}	Continuous: Time when aircraft k starts processing node i on trip u
τ_i^p	Continuous: Time when aircraft p starts processing node i
τ_D^{ku}	Continuous: Time when aircraft k returns to depot on trip u. Trip $(u-1)$ ends and trip u starts at the moment of arrival to the airfield
τ_{D0}^p	Continuous: Time when scooper p leaves airfield to a fire
τ_{D1}^p	Continuous: Time when scooper p returns to airfield
Z	Continuous: Minmax variable: Time of last drop

$$\textbf{Minimize} \quad Z \tag{12a}$$

$$\textbf{Minimize} \quad \sum_{k\in K}\sum_{u\in U}\sum_{(i,j)\in A_K} T_{ij}^k x_{ij}^{ku} + \sum_{p\in P}\sum_{(i,j)\in N} T_{ij}^p x_{ij}^p \tag{12b}$$

Subject to:

$$\tau_j^{ku} \leq Z \qquad \forall k \in K, \forall u \in U, \forall j \in F \tag{13}$$

$$\tau_j^p \leq Z \qquad \forall p \in P, \forall j \in F \tag{14}$$

$$\sum_{k\in K}\sum_{u\in U}\sum_{(i,j)\in A_K} x_{ij}^{ku} + \sum_{p\in P}\sum_{i\in N\setminus j} x_{ij}^p = 1 \qquad \forall j \in N_f, \forall h \in H \tag{15}$$

$$\sum_{p\in P}\sum_{i\in N\backslash j} x_{ij}^{p} \le 1 \qquad \forall j \in F \tag{16}$$

$$\sum_{p\in P}\sum_{i\in F} x_{ij}^{p} = 0 \qquad j = i \tag{17}$$

$$\sum_{h\in H}\sum_{j\in N_f} x_{0j}^{ku} \le 1 \qquad \forall k \in K, \forall u \in U \tag{18}$$

$$\sum_{j\in F} x_{0j}^{p} \le 1 \qquad \forall p \in P \tag{19}$$

$$\sum_{j\in F} x_{0j}^{k(u+1)} \le \sum_{i\in F} x_{i0}^{ku} \qquad \forall k \in K, u = 1, \dots, U-1 \tag{20}$$

$$\sum_{j\in F} x_{0j}^{p} - \sum_{j\in F} x_{j0}^{p} = 0 \qquad \forall p \in P \tag{21}$$

$$\sum_{i\in N\backslash\{j\}} x_{ij}^{p} - \sum_{i\in N\backslash\{j\}} x_{ji}^{p} = 0 \qquad \forall j \in F, \forall p \in P \tag{22}$$

$$\sum_{(i,j)\in A_K} x_{ij}^{ku} - \sum_{(j,i)\in A_K} x_{ji}^{ku} = 0 \qquad \forall k \in K, \forall u \in U, \forall j \in N_f, \forall h \in H \tag{23}$$

$$\sum_{j\in F} x_{0j}^{ku} - \sum_{j\in F} x_{j0}^{ku} = 0 \qquad \forall k \in K, \forall u \in U \tag{24}$$

$$\sum_{k\in K}\sum_{u\in U}\sum_{(i,j)\in A_K} C_k x_{ij}^{ku} + \sum_{p\in P}\sum_{i\in N\backslash\{j\}} C_p x_{ij}^{p} \ge D_j \qquad \forall j \in N_f, \forall h \in H \tag{25}$$

$$\sum_{j\in N_f} D_j x_{0j}^{ku} + \sum_{i\in N_f\backslash\{j\}} D_j x_{ij}^{ku} \le C_k \qquad \forall k \in K, \forall u \in U, \forall h \in H \tag{26}$$

$$\tau_D^{ku} + R_D + T_{0j}^{k} - (1 - x_{ij}^{ku})M \le \tau_j^{ku} \qquad \forall k \in K, \forall u \in U, \forall j \in N_f, \forall h \in H \tag{27}$$

$$\tau_{D0}^{p} + R_D + T_{0j}^{p} - (1 - x_{0j}^{p})M \le \tau_j^{p} \qquad \forall p \in P, \forall j \in F \tag{28}$$

$$\tau_i^{p} + R_i + T_{ij}^{p} - (1 - x_{ij}^{p})M \le \tau_j^{p} \qquad \forall p \in P, \forall i \in F, \forall j \in F \tag{29}$$

$$\tau_j^{p} + R_j + T_{j0}^{p} - (1 - x_{j0}^{p})M \le \tau_{D1}^{p} \qquad \forall p \in P, \forall j \in F \tag{30}$$

$$\tau_j^{ku} + R_j + T_{j0}^{k} - (1 - x_{j0}^{ku})M \le \tau_D^{k(u+1)} \qquad \forall k \in K, u = 1, \dots U-1, \forall j \in F \tag{31}$$

$$\tau_i^{ku} + T_{ij}^{k} - (1 - x_{ij}^{ku})M \le \tau_j^{ku} \qquad \forall i \in N_f\backslash j, \forall k \in K, \forall u \in U, \forall j \in N_f, \forall h \in H \tag{32}$$

$$T_j^0 \le \tau_j^{ku} \le T_j^1 \qquad \forall k \in K, \forall u \in U, \forall j \in F \tag{33}$$

$$T_j^0 \le \tau_j^{p} \le T_j^1 \qquad \forall p \in P, \forall j \in F \tag{34}$$

The objectives are given by Equation 12a and Equation 12b. The former is the Minmax objective with higher priority, which minimizes the Minmax variable Z in order to minimize the time of the latest drop. The latter minimizes the total flying time by all active aircraft.

Constraints (13) and (14) are the Minmax constraints for the tankers and scoopers respectively. Constraint (15) ensures that every subfire is visited only once. Constraint (16) ensures that every fire is visited at most once by scoopers. Constraint (17) ensures that scoopers do not return to the same subfire. Constraint (18) ensures that on a given trip, tankers leave the airfield at most once. Constraint (19) ensures that every scooper leaves the airfield at most once. Constraint (20) is for the order precedence of tanker trips. It ensures that a new tanker trip cannot begin unless the previous trip has ended. Constraint (21) is for the scooper conservation of flow around the airfield. It ensures that the scoopers leaving the airfield return to it. Constraint (22) is for the scooper conservation of flow around subfires. it ensures that the scoopers visiting a subfire leave that subfire. Constraint (23) is for the tanker conservation flow around subfires. It ensures that tankers entering a subfire on a certain trip, also leave that subfire during that trip. Constraint (24) does the same but for the airfield. Tankers leaving the airfield return to it for each trip they make. Constraint (25) makes sure that fire demand is satisfied. More specifically, it ensures that the demand of each subfire is at least met, or exceeded, by any scoopers or tankers that visit it. Constraint (26) makes sure that tankers can visit several subfires within the same fire, as long as the total demand of those subfires does not exceed the tanker capacity.

Constraints (27) - (32) are time precedence constraints. Constraint (27) ensures that the time of a tanker drop on a certain trip is later than the time of leaving the airfield, the processing time at the airfield, and the travel time from the airfield to the drop location. Constraint (28) does the same for scoopers. Constraint (29) ensures that the time of a scooper drop is later than the previous drop operation, in addition to the processing time and travel time. Constraint (30) does the same for the time of a scooper returning to the airfield after a drop at a subfire. Constraint (31) is for time precedence of trips for a given tanker. It ensures that a trip only starts, and starts at the airfield, after the previous trip has concluded. Constraint (32) is for time precedence of tankers between subfires. Note that no processing time is included here, because when tankers visit multiple subfires, in reality, that represents just one drop that the tanker is performing on a larger distance of the same fire. Constraints (33)-(34) are for time windows, where tankers and scoopers must visit subfires within their given time windows.

4 Results

In this section, a discussion on the results and how they were obtained is presented. In section 4.1, a high-level description of the implementation and outputs is provided. Then, the computational settings used are described in section 4.2. Finally, computational insights on the performance of the model, namely the time it takes to obtain useful results are presented in section 4.3.

4.1 Implementation

The model described in section 3.2 was programmed in such a way that the desirable inputs can easily be altered at the beginning of the code. Inputs include scooper and tanker capacity and speed, number of fires, intensity of fires, and locations of all nodes. The output of the model includes the arcs per aircraft, that is per scooper and tanker. For the tankers, the arcs for each trip are given, if multiple trips are used. The model runs in a loop for fleet combinations, so the solutions are given for each combination of the specified number of scoopers and tankers. It also includes a plot that shows the arcs used, and the solution to the objectives. The plot also shows the location of the nearest water body. However, it does not show the scoopers going to the water body. As described in section 3.1, the trip to the water body is included in the time distance matrix of scoopers, so when the plot shows a scooper going from one fire to another, it is in fact, and in the calculations, going to the water body and then to the second fire. An example of the output plot is shown in Figure 4.

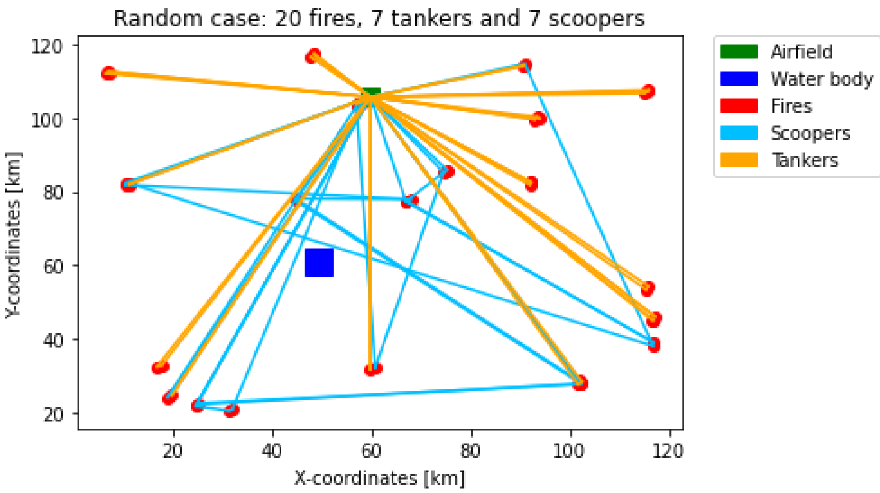


Figure 4: Example of a plot from a run using 20 random fires of varying intensity, 7 tankers and 7 scoopers.

4.2 Computational Settings

The model was programmed in Python and solved with the commercial optimization solver Gurobi, which uses the branch and bound method. Specifically, the Gurobi version used is Gurobi Optimizer version 9.5.1 build v9.5.1rc2 (win64). The computer used for all solutions has the following specifications: Processor type is Intel64 Family 6 Model 158 Stepping 10 GenuineIntel ca 2592 Mhz. With 6 cores and 12 logical processors. The RAM size is 16 GB. The Operating System (OS) is Microsoft Windows 11 Home, and the OS version is 10.0.22621 N/A Build 22621.

Using these specifications, most of the results were possible to obtain within the 5 minute time limit. However, to investigate the computational capabilities and limits of the model, problems of different sizes (varying number of fires and subfires) are attempted. This is elaborated on further in section 4.3.

4.3 Computational Insights

In principle, there is no limit to the number of fires or subfires that can be solved. However, a computation time of 5 minutes is preferred because this is the point in time when an operator would start communicating routes to the aircraft. Although it takes about 20 minutes before the aircraft are in the air. Fires including up to 20 fronts or about 70 subfires were tested and logical results were obtained within the 5 minute computation time.

The number of fires has been varied from 1 to 25. The intensity of each fire (number of subfires) has been varied from 1 to 10. All fleet combinations from 1 scooper and 1 tanker up to 10 scoopers and 10 tankers have been tested. Various aircraft characteristics were tested, by adjusting their capacity and speed. Trying all these parameters did not affect the feasibility of solutions. At times, the model exceeded the time limit before finding a solution, but this was for extreme instances with unrealistically large numbers of fires. Up to 70 subfires was possible to solve within the 5 minute time limit.

To gain more insight into the performance of the model and the time it takes to obtain useful results, several instances of the simulation with various numbers of fires and subfires were tested, and the performance parameters were recorded and are presented in Table 7. Note that these are single instances to give an indication regarding the performance for problems of various sizes. Other instances may vary from the ones recorded here. This investigation used a time limit of 21600 seconds, or 6 hours, at which point the simulation was stopped and the computational insights recorded. The model limitations are observed here as the 5 minute optimality gap is higher than 50% in problems of 15 subfires, which are medium sized. For larger problems the gap increased further, and remains high even after 6 hours.

Table 7: Computational insights on the time it takes to obtain useful results for problems of various sizes.

Number of main fires H	**Number of subfires** F	**Computation time [seconds]**	**Optimality Gap [%]**	**Gap after 5 minutes [%]**
1	5	1	0	0
3	10	69.5	0	0
5	15	21600	39.7	58.1
7	20	21600	60.7	73.2
8	25	21600	77.4	81.7
9	30	21600	85.0	86.9
10	35	21600	79.2	81.2
10	40	21600	75.2	89.9
10	45	21600	78.3	92.1
12	50	21600	99.9	100

5 Case Studies

The case studies are based on input from an expert, Mr. Glen Purdam, an active firefighting pilot who has worked in different regions of the world. A summary of the expert insights is given here before the case studies representing the realistic scenarios are presented.

5.1 Expert Judgement

Fire size
Most of the fires requiring AFF are still relatively small, the maximum number of fires dealt with by one airbase is around 5. Those fires require a few drops (2 to 4) in order to be kept small (up to about 2 hectares) such that the ground firefighters can control them. However, sometimes fires can get out of control and become medium or large fires that require more AFF involvement. These are classified as medium fires to keep a simple categorisation, although in reality they may be considered large. A medium fire may have up to 20 fire fronts, requiring about 15 to 20 drops. The extremely large fires are referred to as campaign fires. These fires are very large and the main purpose of AFF is to protect property by laying lines ahead of areas with valuable assets, such as homes or residential areas. These fires require hundreds of drops.

Number of aircraft
Usually, an AFF base has about 2 aircraft on standby, in case a fire starts in the area. If there are already fires in the area, the airfield can have up to 6 aircraft (tankers) on standby. More than 6 is unusual because it can cause traffic issues, and delays when the aircraft need refuelling, as well as taking turns to take-off and

land. If scoopers are available, up to 6 scoopers can also join the airbase in addition to the maximum 6 tankers mentioned previously. Scoopers are less of a concern for airfield operations since they do not need to be at the airfield much, because they can scoop up water from water bodies, and they can even refuel at another airfield if necessary. This is because many airfields have the infrastructure for refueling aircraft, but the main airbase is particularly prepared for AFF operations, and tankers need that infrastructure to refill the retardant or water tanks. Furthermore, if helicopters are available, up to about 6 helicopters can also operate out of the same airfield. In summary, the maximum number of aircraft can be up to 6 tankers, 6 scoopers, and 6 helicopters. For the purposes of this paper, that means up to 6 tankers and 12 scoopers (since helicopters are treated as scoopers).

Scooper operations

There are two relevant insights concerning the operation of scoopers. Firstly, scoopers tend to work in pairs or groups of 3. The reason is to increase efficiency. In a pair, double the capacity is achieved for the same operational effort. Secondly, scoopers can deliver 100 or more drops if the water body is nearby. They can manage the scooping operation quickly (under a minute) and return to the fire. Depending on the location of the water body, this could mean up to a drop every 5 minutes. Furthermore, aircraft can operate for up to 12 hours a day, limited by darkness and weather conditions among other factors.

Computation time requirements

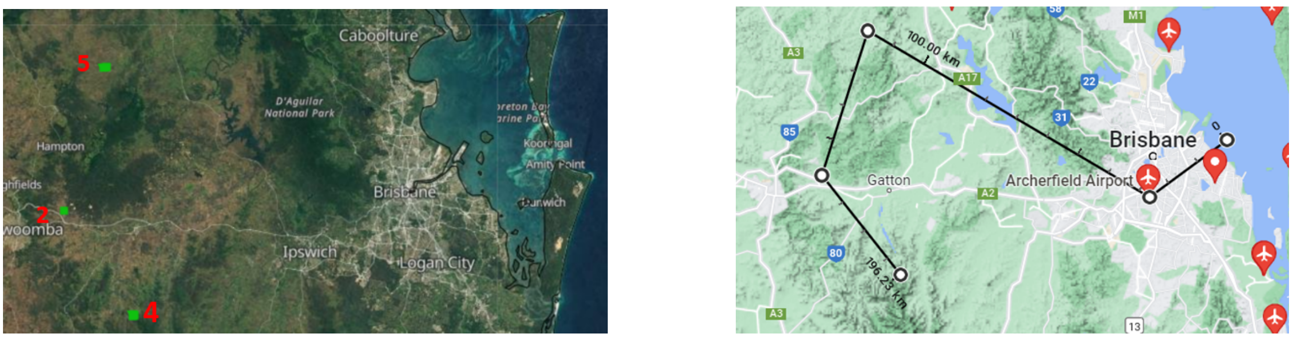
The model presented in this paper can have multiple use cases. One of the uses is live during a fire, to determine the routes for the available aircraft. This use case is time sensitive. From the moment that a fire is reported and known to the airbase, it usually takes about 20 minutes before the aircraft are in the air. Hence, the maximum computation time should not exceed this. However, having preliminary results after 5 minutes is preferable, so that this information is available at the right time in the decision-making process for operators.

5.2 Case 1: Small Fires Near Brisbane, Australia

The European Commission has made a situation viewer available to the public where satellite images and data of forest fires can be viewed either live or for a particular date through EFFIS [European Commission, 2023]. This is used to spot a couple of fires near Brisbane, Australia. An assumption is made about the intensity of the fires based on their relatively small size. It is assumed that the first fire, the most northern, requires 5 scooper drops, the middle fire 2 scooper drops, and the most southern fire 4 scooper drops.

Next, an airfield in the area is selected, and the distances to the fires and the nearest body of water are measured using Google maps, shown in Figure 5.

Finally, the locations of the nodes are duplicated and used as input to the model, with an assumption of a reasonable available fleet, and the model is optimized to produce the plot seen in Figure 6.



(a) 3 fires near Brisbane, Australia [European Commission, 2023].

(b) Google maps measurements between fires, airfield, and water body.

Figure 5: Extraction of model inputs for the Brisbane case.

The results also show that the time of the latest drop is 39.93 minutes, and the resulting routes are summarized in Table 8. The nodes were defined as follows: The northernmost fire contains nodes: 2,5,6,7,8. The middle fire contains nodes: 3,9. The southernmost fire contains nodes: 4,10,11,12.

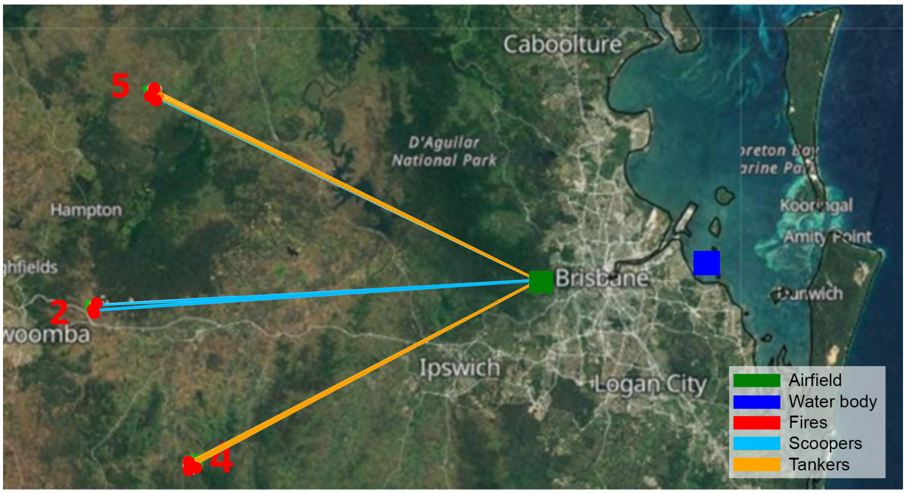


Figure 6: Routing plot for the Brisbane case.

Table 8: Resulting routes of the optimization for the Brisbane case study.

Scoopers	Routes	Tankers	Trips	Routes
Scooper #1	0,9,0	Tanker #1	Trip #1	0,7,6,0
Scooper #2	0,3,0		Trip #2	0,11,12,0
Scooper #3	0,2,0	Tanker #2	Trip #1	0,10,0
Scooper #4	0,4,0		Trip #2	0,5,8,0

5.3 Case 2: Medium Fire Near Taza, Morocco

Following expert judgement, the category of medium sized fires encompasses fires that get out of control and require drops in the range of 15 to 20. In this case, a case of fires that burned near the town of Taza, Morocco in July of 2022 is selected. The main reason for choosing this case, besides the media coverage which can provide more insights, is that this area is far from the coast. Assuming that nearby lakes are dry during the summer and inaccessible for amphibious aircraft, this offers some variation in the problem and may show the limitations of using scoopers or helicopters when natural bodies of water are not easily accessible.

The Moroccan authorities used 3 scoopers to help fight this fire (4 in total including fires in other regions at the same time)[Alarabiya News, 2022]. Assuming 20 subfires and using 3 scoopers and no tankers in the model results in a last drop time of 265.1 minutes. Since the geographical situation suggests that tankers can be more effective in this case, the model is run with other fleet combinations that include a tanker. The combinations chosen are only slightly different than the fleet used in reality. For example, adding a tanker to the fleet (1 tanker and 3 scoopers), or replacing a scooper with a tanker (1 tanker and 2 scoopers). Such results can be insightful for authorities who are looking to improve their AFF fleets. Results show that with only a small change to the fleet, the time of carrying out the drops can be significantly improved. All simulation results on this case study are obtained with a computation time limit of 5 minutes.

As explained in section 3, the trips made by scoopers to the water body and back to a fire are included in calculations, but not in the plot. Therefore, a scooper going from a fire to the water body and back to another fire, will appear as a simple line from the first fire to the second. This is important in this case because the water body is far from the fires. It is expected that scoopers will be used less than tankers.

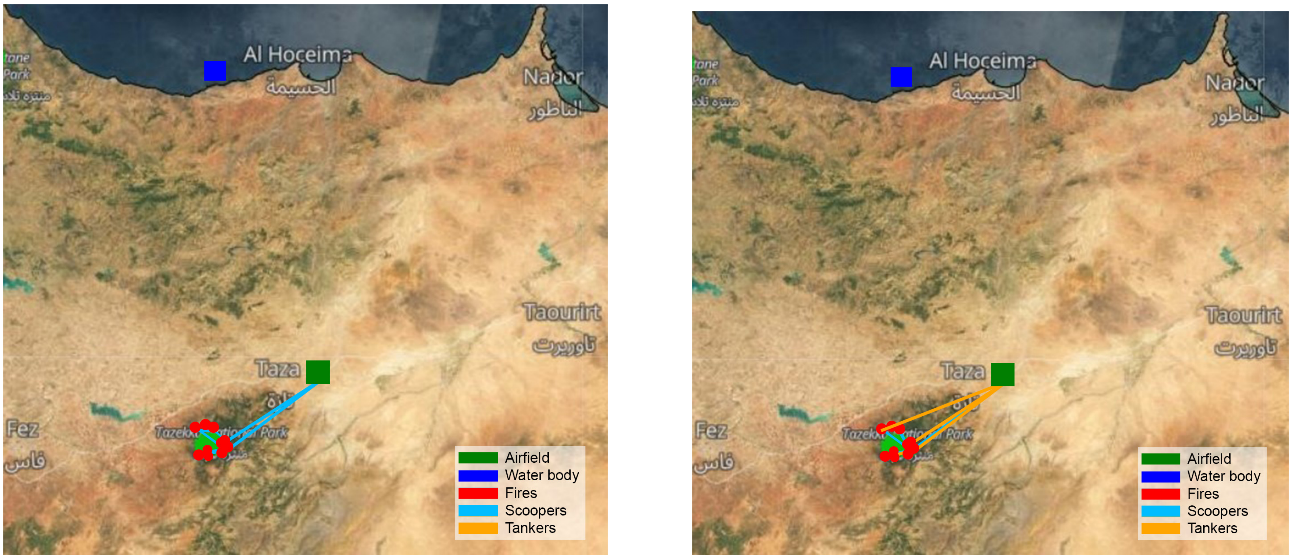
The routing plots for the first two options are shown in Figure 8. The plots for the remaining two options are shown in Appendix A. It can be observed that even though there is only 1 tanker compared with 3 scoopers, the tanker is used extensively over multiple trips, 6 to be precise. This is a logical outcome given the long distance scoopers need to travel to the water body, and the lower speed they have compared to the tanker.
Table 9 shows the effect of adding a tanker to the fleet. The time it takes to make the requested drops is reduced from about 265 minutes to only around 100 minutes with an additional tanker. This is a large improvement that satisfies the same requests in less than half the time. However, adding a tanker to the fleet may be expensive. The other fleet combinations may be more relevant when costs are considered. Replacing one of the scoopers with a tanker also reduces the time from 265 minutes to around 117 minutes. Furthermore, replacing 2 scoopers with only 1 tanker still leads to a significant improvement, reducing the time to 150 minutes. In this case, it

was assumed that scoopers need to travel to the coast to refill water. It is recommended that decision makers thoroughly assess their area of operation and verify the presence or absence of bodies of water or refilling possibilities which are faster than this. This case relied on this assumption to demonstrate a case where scooper refilling is not easily accessible.

(a) Satellite data showing the fire area south of Taza on July 15 2022 [European Commission, 2023].

(b) Google maps measurements between fire, Taza airport (pinned), and the coast.

Figure 7: Extraction of model inputs for the Taza fire.



(a) Routing plot for the Taza case study with 3 scoopers and no tankers.

(b) Routing plot for the Taza case study with 3 scoopers and 1 tanker.

Figure 8: The difference in routing plots when one tanker is added to the used fleet.

Table 9: Time of last drop at the Taza fire for different fleet combinations.

Number of Tankers	**Number of Scoopers**	**Time of last drop [min]**
0	3	265.1
1	3	100.5
1	2	116.7
1	1	150.0

5.4 Case 3: Large Fires Near Bordeaux, France

This case study is on the very large fires, which burn thousands of hectares. This is the type of fire that most commonly receives news coverage and media attention. In this case, we study the fires that took place in July 2022 south of Bordeaux, France. First, a description of the fire situation is outlined in section 5.4.1. Then, the approach to modeling such large fires is described in section 5.4.2. Finally, the results of the case study using the proposed model are presented in section 5.4.3

5.4.1 Fire progression near Bordeaux, July 2022

In July 2022, two large fires south of Bordeaux, in the department of Gironde in France, burnt more than 20800 hectares (ha). One of the fires was in La Teste-de-Buch and the other in Landiras, the two areas are about 50 km apart.

The fires started in the afternoon of 12 July 2022, and progressed aggressively for the following days as shown in Figure 9. The x-axis in the figure shows the morning state, denoted by M, and the afternoon state, denoted by A, on every day from the 13th until the 20th of July 2022. The aircraft types used to assist in fighting the fires were the Canadair CL-415 and CL-215 as scoopers (Capacity of 6140 and 4800 liters respectively)[Viking Air, 2023], and the Bombardier Dash 8 400-AT (Capacity of 10000 litres) [Conair, 2021].

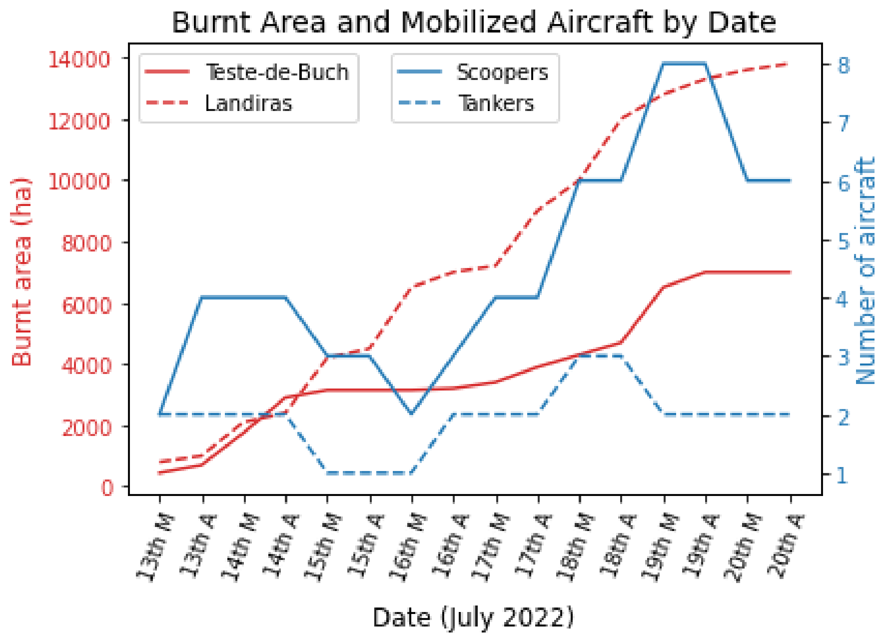


Figure 9: Fire progression at Teste-de-Buch and Landiras and mobilized aircraft.

5.4.2 Modeling large fires

Fires of this scale require several firefighting aircraft, and hundreds of drops per aircraft per day. Thus, modeling the problem can involve thousands of nodes if approached in this way. However, in reality, the fire is dynamic and the drop requests keep changing. The number and location of drops can change on an hourly basis. Hence, a more realistic approach to represent and apply the problem is to optimize a snapshot of the problem at a certain moment, and then run it again with the updated information at certain time increments.

As an example, consider a large fire that lasts for three days. At the first hour of the first day, the fire is still small, perhaps only one or two aircraft are available and deployed, and only a few drop requests are identified. By the first hour of the third day, reinforcements would have arrived and the fleet has expanded to several aircraft. The wind may have changed direction and the fire may have advanced towards a residential area, so the requested drop locations change accordingly.

This means that large fires cannot and should not be modelled at once with a full fleet and all the drop requests at once. An incremental approach is more useful with several runs of the model, where the inputs are continuously updated and the model rerun. The result is updated with optimized routes at every increment. This also has the benefit of solving the computation time issue. The initial problem with thousands of nodes would take too long to solve if it is to be used live. In contrast, the incremental approach divides this problem into much smaller problems, which can each be solved in minutes or seconds, making them useful in the live setting.

5.4.3 Results

Two instances from Figure 9 are selected to demonstrate the use of the model for large fires. The instances are selected to show the results using different fleet combinations as shown in Table 10. Furthermore, the assumed inputs are summarized in Table 11. Most notably, a planning horizon of roughly one hour is assumed for the operation. This is part of the incremental approach to obtain fast results and continuously update the results as the situation in real life evolves. The user can then easily alter any inputs that have changed, and run the simulation again every hour to quickly obtain updated optimized routes. The number of required drops is based on what is reasonably possible in the planning horizon, given the number of mobilized assets and total burnt area.

Table 10: Selected fire situations for the Bordeaux case study.

Simulation number	**Date (July 2022)**	**Total burnt area at Teste-de-Buch (ha)**	**Total burnt area at Landiras (ha)**	**Mobilized Scoopers**	**Mobilized Tankers**
#1	16th morning	3150	6500	2	1
#2	19th morning	6500	12800	8	2

Table 11: Inputs of the Bordeaux case study simulations.

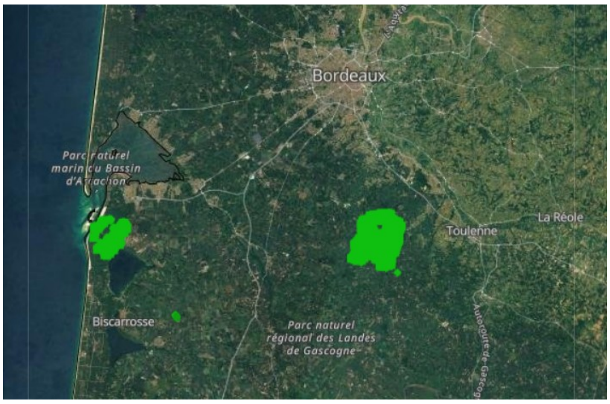
Parameter	**Value**
Scooper capacity	5000 [litres]
Tanker capacity	10000 [litres]
Scooper speed	6 [km/min]
Tanker speed	10 [km/min]
Planning horizon	ca 1 [hour]
Required number of scooper drops for simulation #1 [Teste-de-Buch, Landiras]	[7, 13]
Required number of scooper drops for simulation #2 [Teste-de-Buch, Landiras]	[30, 50]
Preferred Time limit for each simulation	5 [minutes]

Simulation #1
The fire situation on July 16, 2022 is found through the EFFIS system, as seen in Figure 10(a), and mapped accordingly as seen in Figure 10(b). The number and types of mobilized aircraft are obtained from the press release of the local authority [Préfète de la Gironde, 2022a]. The subfires, which can be seen as the drop locations, are split among the two large fire areas. The Teste-de-Buch (west) fire has 7 drop requests and the Landiras (east) has 13. The resulting time carrying out all requested drops is 76.05 minutes. The resulting routes are presented in Table 12 and visualized in Figure 11. In the latter, it is clear that the tanker is carrying out the drops at Landiras, which are further from the water body, while the scoopers mainly tend to the drops at Teste-de-Buch, which are near the water body. This verifies the model since it is a logical assignment, as scoopers can make use of the water body, and the tanker has a higher cruise speed. Therefore, this routing makes use of the strengths of the different aircraft types.

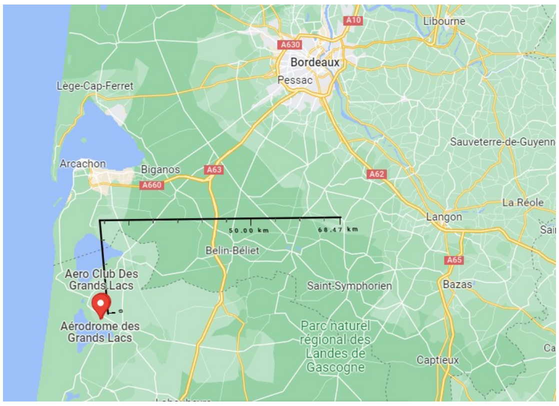
Table 12: Resulting routes from simulation #1 of the Bordeaux case.

<table>
<tr><th>Scoopers</th><th>Routes</th><th>Tankers</th><th>Trips</th><th>Routes</th></tr>
<tr><td rowspan="2">Scooper #1</td><td rowspan="2">0,8,13,7,11,
16,14,0</td><td rowspan="4">Tanker #1</td><td>Trip #1</td><td>0,5,19,0</td></tr>
<tr><td>Trip #2</td><td>0,4,0</td></tr>
<tr><td rowspan="2">Scooper #2</td><td rowspan="2">0,12,20,9,
2,3,10,15,0</td><td>Trip #3</td><td>0,6,21,0</td></tr>
<tr><td>Trip #4</td><td>0,17,18,0</td></tr>
</table>

Fleet selection insights:
Since the number of aircraft mobilized for this simulation case is relatively small (3 in total), it is of interest



(a) Satellite data showing the fire situation at both Teste-de-Buch (west) and Landiras (east) on July 16 2022 [European Commission, 2023].



(b) Google maps measurements between fires, airfield, and water body for July 16, 2022.

Figure 10: Extraction of model inputs for simulation #1 of the Bordeaux fires.

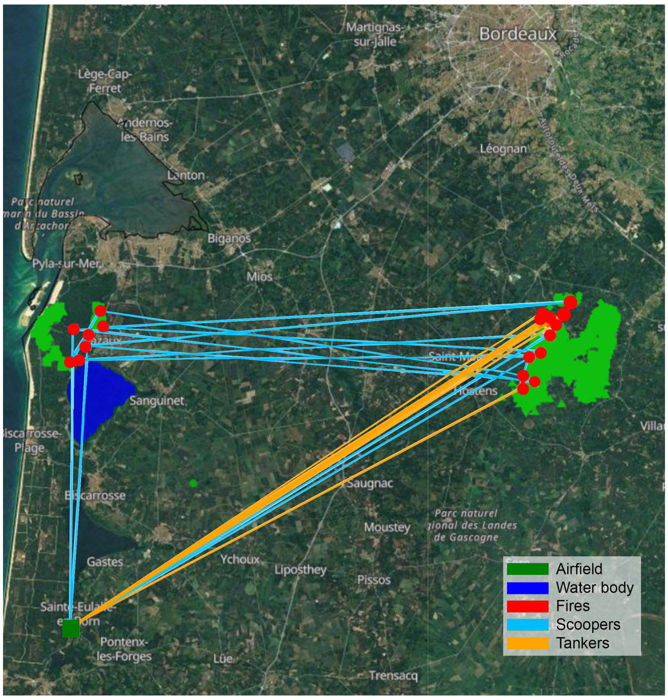


Figure 11: Routing plot for simulation #1 of the Bordeaux case with a total of 20 subfires.

to find out how much efficiency is gained when the fleet is expanded. The question is, by how much can the result of 76.05 minutes be reduced when the fleet is expanded? The answer to this question can provide insight into better fleet selection and allocation. The model is run in a loop with various combinations of tankers and scoopers. The results of this fleet study are presented in Figure 12. Two conclusions are drawn from these findings:

1. There is a logical general trend that more available aircraft lead to a reduced time to complete all requested drops.
2. Scoopers have a more significant impact on reducing the total operation time. This is because in this case, the water body is located conveniently near the fires. For each given number of available scoopers, there is a maximum number of tankers, beyond which the operation time does not reduce significantly. For instance, given one scooper, the second tanker helps, but the third and beyond do not. Given two scoopers, the third tanker helps, but the fourth and beyond do not. This is due to the longer processing time of tankers at the base. Hence, the scoopers manage to make many drops possibly before the tankers take off or make their first drop. This was verified by removing the tank-filling time from the model, and observing an increase in tanker activity and a reduction of operation time when more tankers are assigned.

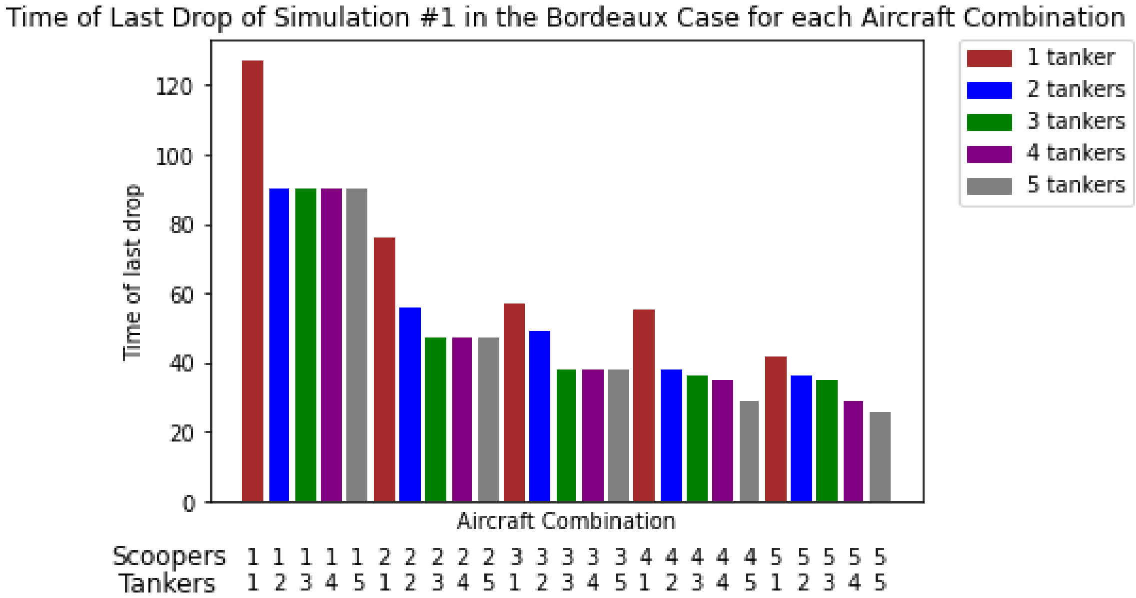
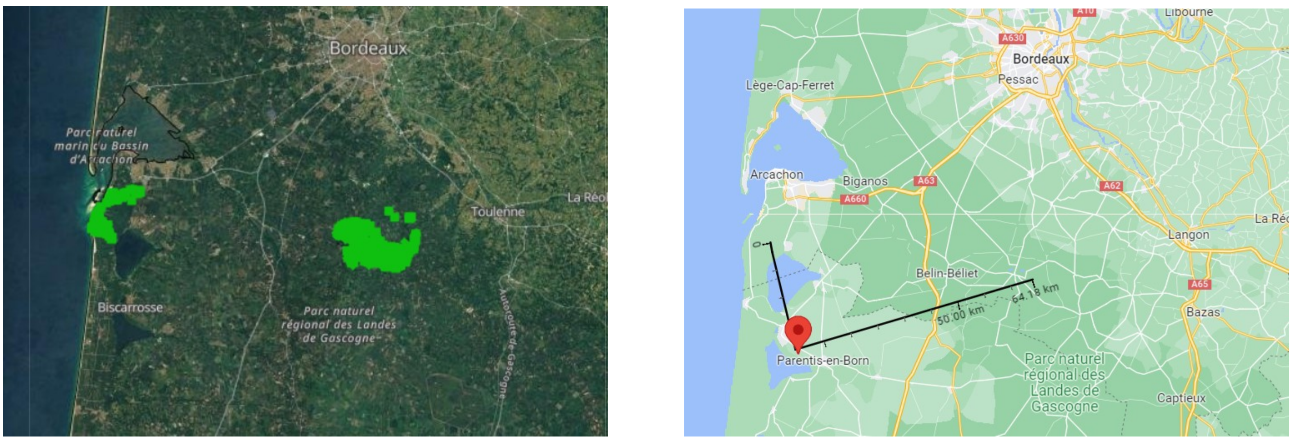


Figure 12: Time of last drop for several fleet combinations

Simulation #2
The fire situation on July 19, 2022 is similarly mapped by obtaining satellite images through EFFIS, as seen in Figure 13(a), and duplicated on Google maps to measure the distances as seen in Figure 13(b). At this time, the fires are extremely large and out of control. The wind is moving south and so the fires are spreading in that direction, and the aerial assets are deployed to lay lines south of the fires to slow them down. This case will be the largest instance of case studies with a total of 80 drop requests. The fire at Teste-de-Buch (west) is estimated to have 30 drop requests, while the fire at Landiras (east) is estimated to have 50. According to the local press release ([Préfète de la Gironde, 2022b]), the number of mobilized scoopers on this day was 8, and the number of mobilized tankers was 2.



(a) Satellite data showing the fire situation at both Teste-de-Buch (west) and Landiras (east) on July 19 2022 [European Commission, 2023].

(b) Google maps measurements between fires, airfield, and water body for July 19, 2022.

Figure 13: Extraction of model inputs for simulation #2 of the Bordeaux fires.

Computational insights
For this fleet combination, the resulting time to carry out all requested drops is 98.7 minutes. The routes are shown in Figure 14. With a large instance of the simulation, involving 80 subfires and 10 aircraft, the model shows some limitations. A feasible solution was not possible within the preferable time limit of 5 minutes. This solution was obtained with a time limit of 1300 seconds, meaning in just under 22 minutes. This may still be acceptable as it takes at least about 20 minutes before the aircraft are in the air. Nonetheless, this demonstrates the size of the problem at which the model reaches the limit of the acceptable time limit range. For problems

of this size or larger, a different approach may be used when computation time is critical. For example, the problem can be divided into smaller problems. For instance, two problems of 40 subfires each, and half the aerial assets available for each. The sub problems can then be solved simultaneously and obtain results faster.

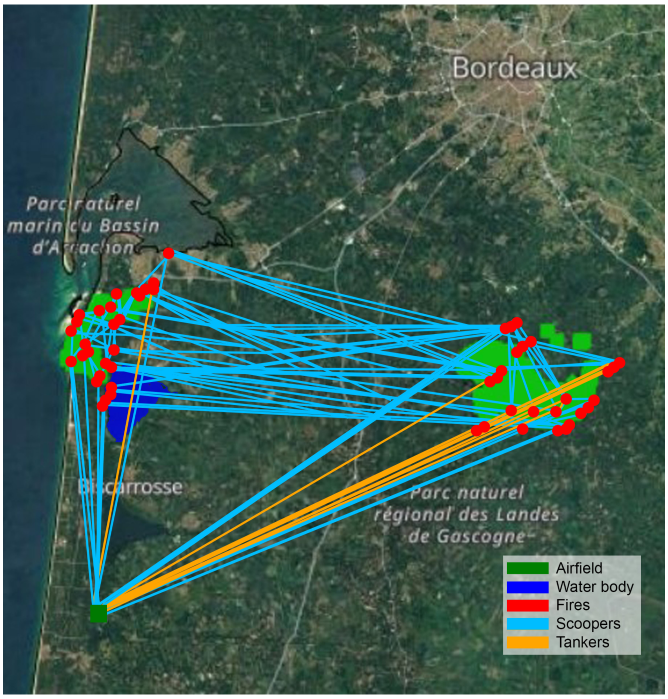


Figure 14: Routing plot for simulation #2 of the Bordeaux case with a total of 80 subfires.

5.5 Case 4: Monte Carlo Simulation for Fleet Optimization

While the previous cases have been focused on the operational level of firefighting, this case will demonstrate the use of the model for tactical or strategic decisions. This case will be based on a fictitious environment and the purpose is to decide what kind of fleet an at-risk area should prepare. Let us consider an area of 100 km by 100 km. Let the airbase be located at [50,0] and the nearest water body at [0,50]. Consider that the firefighting authority in charge has an existing fleet of 1 tanker and 1 scooper. In preparation for hotter summers and potentially more intense fire seasons, they are planning to acquire one more aircraft. The decision that needs to be made is whether they should acquire an additional scooper or tanker. Resulting in two fleet options:

- Fleet A: 2 scoopers and 1 tanker
- Fleet B: 1 scooper and 2 tankers

To make this decision, they need to know which additional aircraft will contribute more significantly to the objective of containing fires as quickly as possible. It is also important to quantify the contributions of the two options, in order to perform a cost-benefit analysis. Tankers typically cost more than scoopers, and so the tanker may need to show a significant advantage to justify its cost.

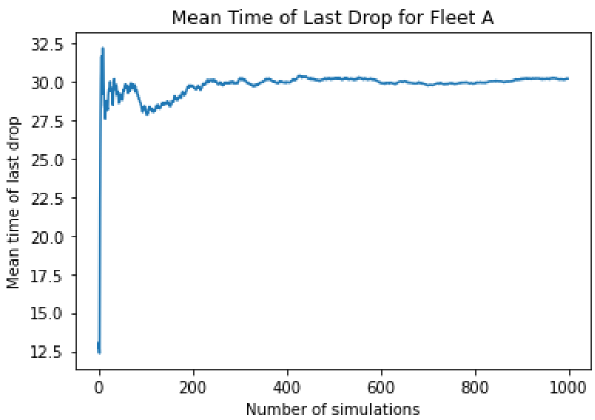
To achieve this, a Monte Carlo simulation is used, where 1000 simulations are performed for each fleet option, and the results are compared to decide which fleet option is recommended. For the simulations, the number, intensity, and locations of the fires are randomized for each run. However, the total number of subfires is limited to 10 for two reasons: Firstly, the goal of this aircraft acquisition is to have a fleet that can get to fires as soon as possible and make drops to stop them spreading before they become large. Thus, the relevant fires require up to 10 drops. In cases of fires that are out of control, the situation becomes special and reinforcements may be employed from other regions, so that is not of interest in this exercise. Secondly, to run a total of 2000 simulations, keeping the number of subfires under 10 can be beneficial as optimal results can be obtained within the available time. This is supported by the findings in Table 7.

Number of Simulations
According to the law of large numbers [Eberhardt and Glymour, 2011], to draw valid conclusions from a Monte Carlo simulation, the number of simulations used must be large enough to demonstrate that the simulation

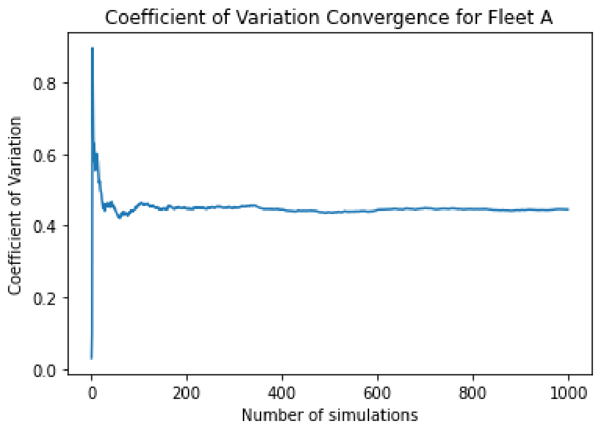
results converge. This is verified by investigating the mean and the Coefficient of Variation (CV) and checking at which number of simulations they converge and stabilise. The CV is calculated as follows:

$$CV = \frac{\sigma}{\mu} \tag{35}$$

where σ is the standard deviation and μ is the mean.

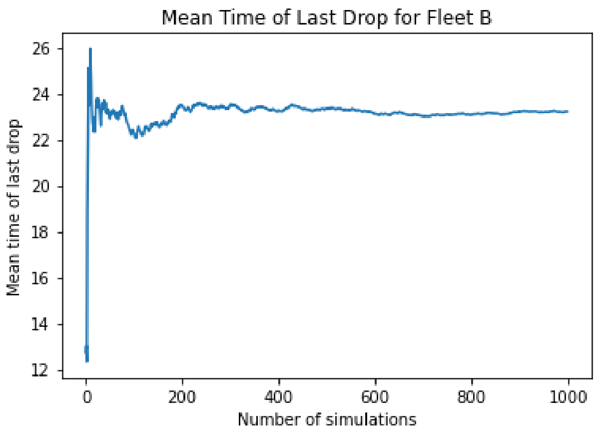


(a) Convergence of the mean time of last drop for fleet A over 1000 simulations.

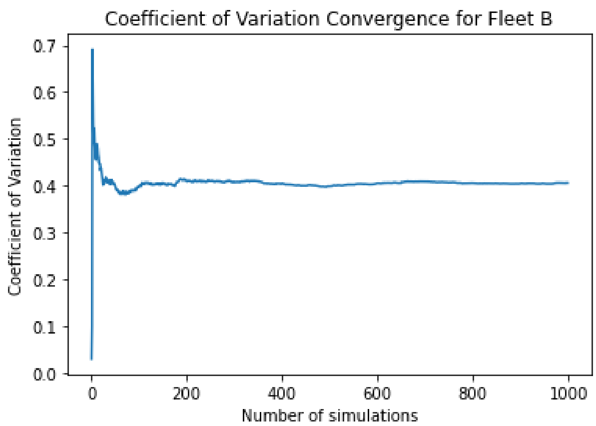


(b) Convergence of the coefficient of variation for fleet A over 1000 simulations.

Figure 15: Convergence of mean and coefficient of variation for fleet option A.



(a) Convergence of the mean time of last drop for fleet B over 1000 simulations.



(b) Convergence of the coefficient of variation for fleet B over 1000 simulations.

Figure 16: Convergence of mean and coefficient of variation for fleet option B.

Figure 15 and Figure 16 show that after about 400 simulations, the mean time of last drop and the CV for both fleet A and B start to converge and stabilise. This demonstrates that using 1000 simulations for each case is sufficient to draw conclusions about the time of last drop in the given scenario. It also demonstrates the importance of this statistical verification, as using under 400 simulations could have yielded unreliable results.

Fleet Optimization Results

After running 1000 simulations for each fleet option, the distribution of the time of last drop is shown in Figure 17 using a kernel distribution estimation, and further statistical results are summarized in Table 13. The main result is that the mean time of the last drop for fleet option A is higher than fleet option B. This means that the acquisition of an additional tanker is more favorable than an additional scooper. In Figure 17, it is clear that the additional tanker is not always favorable, but out of all 1000 simulations, it is clear that fleet B with an additional tanker has an advantage. The mean time of last drop with an additional scooper was 30.18 minutes, while with an additional tanker it was 23.21 minutes, giving a significant difference of around 7 minutes. Gaining this much time while trying to contain fires before they become large and get out of control is valuable. It is then up to the authorities to decide if this advantage justifies the cost difference between the scooper and the tanker.

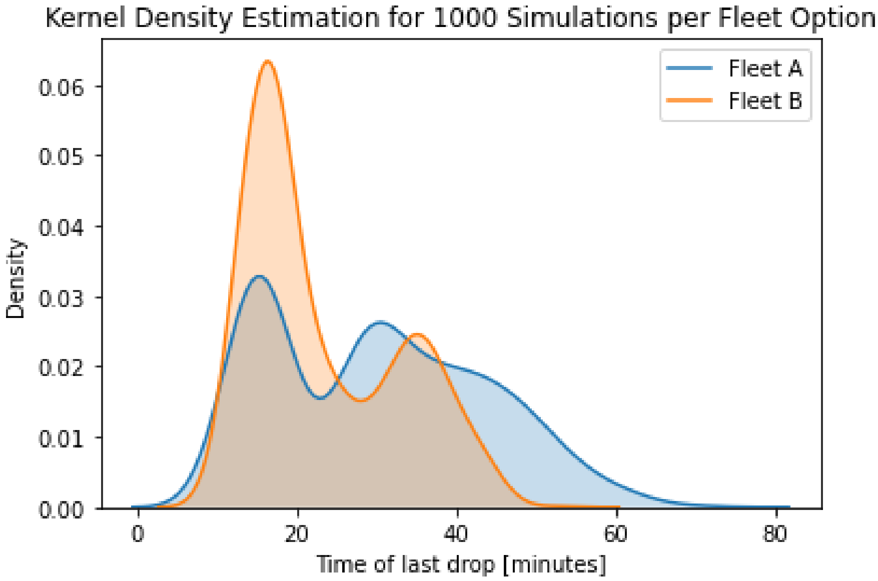


Figure 17: Time of last drop distribution for fleet A and fleet B using 1000 simulations per fleet.

Table 13: Statistical results of the Monte Carlo simulations comparing two fleet options.

Fleet Option	**Number of Scoopers**	**Number of Tankers**	**Mean Time of Last Drop [minutes]**	**Standard Deviation [minutes]**	**Coefficient of Variation**
A	2	1	30.18	13.43	0.44
B	1	2	23.21	9.42	0.41

6 Validation Test

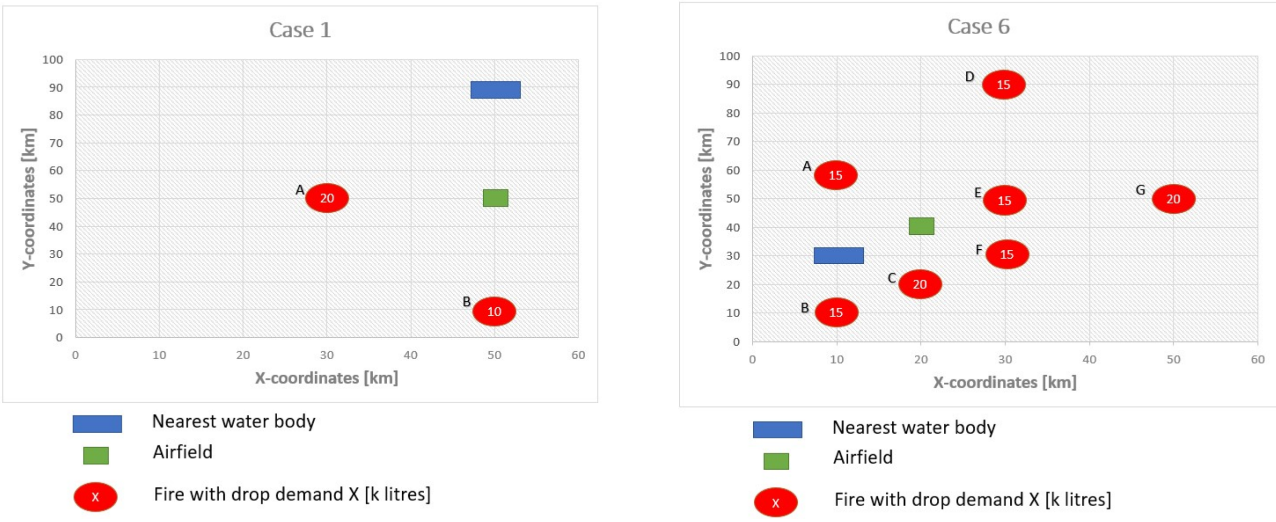
The ultimate aim of the model, and the project, is to create a tool that can help firefighting authorities contain fires more efficiently. The AFFVRP model has been formulated and programmed to produce efficient AFF routes, but the question that needs to be answered is: are these routes more efficient than what human operators can come up with, within the short time available during an emergency. To investigate this, a validation test is designed and taken by an expert in the field (an active firefighting pilot), and the results are compared with the model's results. The hypothesis is that the model will outperform the human, especially as the cases increase in complexity. In this section, the test design is described in section 6.1 and the results are discussed in section 6.2.

6.1 Validation Test Description

The validation test is comprised of six cases of fire situations, and the test taker is asked to assign four aircraft to the best routes they can find. The test was designed while making sure there is no unnecessary added complexity for the human. Mostly using easy numbers to calculate, because if the model outperforms, it should not be due to the human spending time on doing unnecessarily complicated arithmetic. Therefore, the following assumptions were used:

The available fleet is composed of two scoopers and two tankers, denoted by p_1, p_2 for scoopers and k_1, k_2 for tankers. The scoopers have a capacity of 5 [k litres] and a speed of 5 [km/min]. The tankers have a capacity of 10 [k litres] and a speed of 10 [km/min]. All aircraft need 10 minutes at the airfield before take-off, and 1 minute for each drop manoeuvre. The time for scoopers to collect water from the water body is considered negligible. All subfires are considered to have equal urgency (thus no time windows are used), and the only objective considered is the primary objective, that is to minimize the time of the last drop, or to satisfy all drop requests at the earliest time possible. The graphics used to represent the situation are simple and clear. Furthermore, the test layout and some examples are explained thoroughly to the test taker before the test takes place. The test taker is also asked to use a timer and attempt to solve each case within 5 minutes. The same time limit is used when the model is run to solve the given cases.

The test contains 6 cases of varying complexity, the simplest of which contains 2 main fires and 6 subfires, and the most complex contains 7 main fires and a total of 23 subfires. These cases are shown in Figure 18. The full test can be found in Appendix B.



(a) Fire situation in case 1 of the validation test.

(b) Fire situation in case 6 of the validation test.

Figure 18: Fire situations of the least and most complex cases of the validation test.

6.2 Validation Test Results and Discussion

The main hypothesis was that the model will provide more efficient routes, and earlier last drop times, than the human expert. This is shown to be true in all cases except case 1. The reason for this exception is that case 1 was a simple case, and the optimal solution was fairly obvious, therefore both human and model found the optimal solution and gave identical routes. The time of last drop by the expert was 14 minutes, while the model gave 14.1 minutes, despite selecting the same routes. The reason for the small discrepancy is that the test that the expert took had some simplifying assumptions, namely that the fire is a single point. The model is more specific and maps the fires more realistically, with spotting around an area. Therefore, when the expert calculates a route, it is a straight line to a point and then back, while the model has a straight line to a point, but then another small line to another nearby point if necessary, and then back to the airfield. The resulting routes are therefore the same for case 1, and more efficient by the model in cases 2-6.

Another hypothesis is that the advantage of the model will grow in correlation with case complexity. As the cases become more complex, it was expected that the human will have a harder time finding good routes, resulting in a larger difference in time of last drop. In this validation test, this hypothesis does not seem correct. An explanation for this is that with growing complexity, the solution space increases as well, and the model algorithm takes longer to find superior solutions. Hence, the hypothesis is still expected to hold if no time limits are imposed (or more relaxed ones are used), but for the used time limit of 5 minutes, the model also struggles to find good solutions as the optimality gap is still large. This is best seen in case 6, the most complex case of the test, involving 23 subfires and 4 aircraft. The resulting time of last drop by the expert was 61.3 minutes and by the model 58.4 minutes, as shown in Table 15. This difference is smaller than the difference observed in the less complex cases. However, if the time limit is extended, the solution does improve further, yielding a last drop time of 54.9 minutes for case 6 when the time limit is extended to 10 minutes.

Table 14: Validation test results comparing performance of expert human and AFFVRP model for cases 1-3.

	Time of last drop [minutes]					
	Case 1		**Case 2**		**Case 3**	
	Expert	**Model**	**Expert**	**Model**	**Expert**	**Model**
Overall	**14**	**14.1**	**30.9**	**26.9**	**31.1**	**27.8**
p_1	14	14	30.9	26.9	23	23
p_2	14	14	25.9	23.2	23	18.9
k_1	12	14	12	27	14.5	14.5
k_2	14	12	14	14	31.1	27.8

Table 15: Validation test results comparing performance of expert human and AFFVRP model for cases 4-6.

	Time of last drop [minutes]					
	Case 4		Case 5		Case 6	
	Expert	Model	Expert	Model	Expert	Model
Overall	**53.9**	**42.2**	**48.7**	**43.6**	**61.3**	**58.4**
p_1	53.9	37.7	48.7	41.3	61.3	52.5
p_2	40.9	38.3	43.7	39.0	61.0	52.8
k_1	43.7	34.9	40.5	43.6	49.9	58.4
k_2	29.5	42.2	29.5	33.7	49.9	46.8

7 Conclusions and Recommendations

The aim of this work is to support firefighting authorities in containing wildfires by optimizing the routing of aerial firefighting aircraft. Reaching a fire and carrying out drops earlier can ensure that the fire is contained before it becomes large and out of control. A gap in the literature was found in the use of the Vehicle Routing Problem (VRP) to define and solve realistic Aerial Firefighting (AFF) problems. Therefore, the contribution of this work is a novel VRP formulation that represents AFF realistically, referred to as the AFFVRP. In addition, a python program was developed to model the problem and solve it using the commercial solver Gurobi.

To represent AFF problems realistically, an active AFF pilot was consulted and their input was the basis for the assumptions made throughout the work. The following assumptions were the most influential: Fires are categorized in three sizes: small fires require under 10 drops, medium fires require around 15 to 20 drops, and large fires require hundreds of drops. Tankers take about 10 minutes to refill at the airfield, while scoopers need about one minute to refill from a water body. It takes about 20 minutes before the aircraft are in the air, from the moment that a fire is reported. However, the flight plans are made about 5 minutes after the report. Hence, model results would be most useful in live situations if produced within 5 minutes.

The AFFVRP model makes use of a combination of features from various types of well-studied VRPs. It is a capacitated split delivery multiple trip VRP with time windows and hierarchical objectives. Two types of aircraft are used which can represent most firefighting aircraft by simply changing the capacity and speed. The two types are tankers and scoopers. The main difference is that scoopers can make use of natural bodies of water to refill their tank. Helicopters can also be represented by this type. The VRP definition is capacitated as it takes into account the capacity of aircraft used and ensures it is not exceeded. It is split delivery as one fire may be visited by several aircraft. It is multi-trip as tankers are defined with the ability to make multiple trips, meaning they can return to the airfield, refill, and make a subsequent trip to another fire. Time windows are defined for each fire to represent the reality that drop requests will change when the fire has progressed. Two objectives are defined in a hierarchical way. The prioritized objective is to minimize the time of the last drop of the mission, while the second objective is to minimize the total flight time of all aircraft.

Several random simulations were performed, and four case studies, one for a small fire, one for a medium, and one for a large fire, in addition to a strategic fleet planning case study. Feasible and optimal results were obtained within the recommended 5 minutes for most of these cases. The exception was the largest scenario performed with 80 drop requests as part of the large fire case study. This took about 22 minutes which is just too long for live fire applications. However, in such large fires, it is unlikely to take such an approach, since the fire progression is dynamic and can change with the weather conditions. In that case, it is more sensible to establish and address fewer drop requests, and then repeat the process incrementally as new information about the drop locations continues to emerge.

The validation test demonstrates that the model can be used to improve AFF routing compared to a human operator. An expert human and the model solved 6 cases of varying complexity, and the model yielded superior results in all cases except one. The only exception is the simplest case where both human and model found the optimal routing. Besides the expected advantage by the model, a hypothesis was that this advantage would increase when the fire situation increases in complexity. However, the test shows that this advantage is smaller in the most complex case, compared to the others. This is explained by the fact that complex cases have a larger solution space, and the model does not necessarily find great solutions within the imposed time limit of 5 minutes, since at this time, the optimality gap is still quite large (around or above 90%). Nonetheless, the model demonstrates overall superior performance compared to the human, validating that the AFFVRP fulfills its purpose, of helping to contain wildfires more efficiently.

Besides the live situation application, the model can be useful in several other ways. It can be used as a supporting tool in training for various possible fire situations, as well as an assessment tool to review how a fire was handled and what could have been done more efficiently, in order to learn for future situations. One of the important applications demonstrated in the case studies is fleet selection. When assessing a particular fire situation, the model can be simulated with different kinds of fleets, i.e. different aircraft combinations, and the resulting time of operation can be compared. Conclusions can then be drawn on how the fleet may be changed to improve firefighting capabilities. For example, in the medium fire case study, it was demonstrated that replacing a scooper with a tanker could result in more than halving the total operation time. Furthermore, the fourth case study was focused on fleet optimization, and showed how a Monte Carlo simulation can be used to make fleet planning decisions. In a fictional scenario, it was shown how adding a tanker instead of a scooper to an existing fleet, can result in a 7 minute advantage when performing all requested drops.

To improve this work and develop it further, a few recommendations are proposed. Firstly, the problem setting can be expanded. In its current form, the AFFVRP makes use of a single airfield or airbase, from which all aircraft depart, and only one (nearest) water body is considered. Both of these can be extended. For large fires, it can be the case that several airbases are used to tackle the same fires. Further, even though the nearest water body is the most likely to be used, there are situations where several water bodies are accessible, and it can be sensible to make use of them. Therefore, adding this to the model will add to the realism of its parameters. Secondly, the aircraft selection can be expanded. In this model, two types of aircraft are considered, and most firefighting aircraft can easily be modeled by these two, by simply changing the capacity and speed. However, it is not yet possible to use multiple types of aircraft within the same type. This means that for a particular simulation, it is only possible to use one type of scooper and one type of tanker. Expanding this to include the ability to model multiple types of scoopers and helicopters, and different types of tankers, for the same fire simulation, would also add more flexibility to include and study all kinds of fleets. Thirdly, to create a split delivery VRP, a discretization approach is used. Large fires are split into several subfires, such that an aircraft could visit each subfire once. The disadvantage of this approach is that aircraft with larger capacity than the smallest subfire, risk wasting part of their capacity. For example, if the smallest subfire created has a demand of 5 [k litres], and the aircraft visiting it has a capacity of 8 [k litres], 3 [k litres] are wasted. Further discretization into smaller fires can improve this aspect. For example if the smallest demand is 2 [k litres], and the aircraft with a capacity of 8 [k litres] is allowed to visit 4 subfires, the full capacity can be used.

Besides further developments of the problem definition, improving the solution process is also recommended. As shown in Table 7, the optimality gap with the 5 minute time limit was larger than 80% for problems comprising of 25 subfires and more. For non-urgent use of the model, this is acceptable as better solutions, and eventually an optimal solution, are found when more time is afforded. However, for the live application where the 5 minute time limit is a constraint, better solutions are desired. Using suitable heuristics to find better solutions within this time limit, can make this work more useful in real life applications.

Lastly, two further recommendations can be made to make the model more accessible to interested parties. Firstly, automating the import of data such as airfield, water body, and fire locations can make the model faster to use. In its current state, coordinates are typed in manually. Bringing the program to a state where a map area can be selected or imported from a different program is recommended to save time and obtain the results quicker. Secondly, developing a user friendly interface is recommended because it allows people who are unfamiliar with programming languages to use it. This can be helpful because the model can then be tested by various experts and operators in the field of aerial firefighting, and offer more opportunities for validation.

The AFFVRP model created during this project can model many types of fire situations and fleets, and yields good results that outperform a human expert within a time limit of 5 minutes. When the time limit is of no concern, the model can give optimal results and help with strategic fleet optimization. Implementing the discussed recommendations can improve the model by including the possibility to represent even more types of fire situations and fleets, as well as improve the solutions in live applications, and make it more accessible to operators and other experts who can offer more feedback or further validate it.

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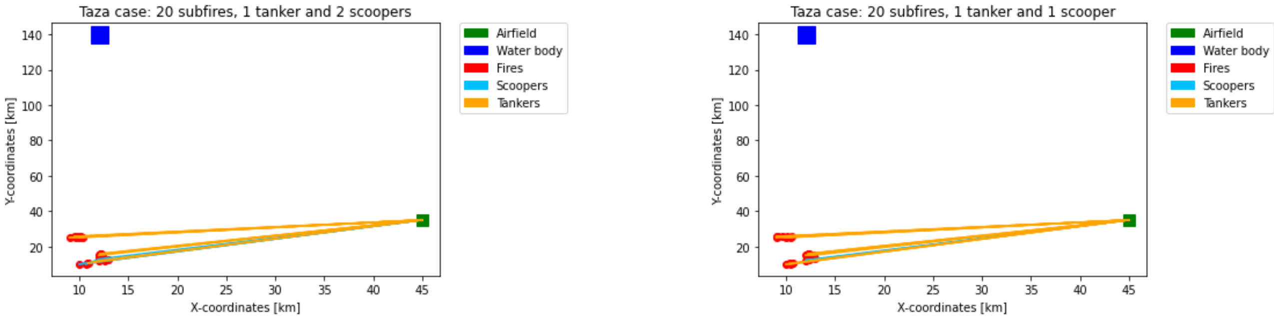
Appendices

A Appendix 1

This appendix contains the remaining plots for the medium case study of Taza, Morocco. These plots are of the same case study but with different fleets than those shown in the paper.



(a) Routing plot for the Taza case study with 3 scoopers and no tankers.

(b) Routing plot for the Taza case study with 2 scoopers and 1 tanker.

Figure 19: Taza medium fire case study: The difference in routing plots when a scooper is replaced with a tanker, and when 2 scoopers are replaced by a tanker.

B Appendix 2

This appendix contains the full validation test as presented to the test-taker. The answers of the expert, which were used for validation, are also filled in. Table 16 shows the comparison between expert and model solutions.

Table 16: Comparison of routes generated by expert and model for validation test cases.

	Aircraft	p_1	p_2	k_1	k_2
Case 1	**Expert**	A	A	A	B
	Model	A	A	A	B
Case 2	**Expert**	ABB	AB	A	C
	Model	AB	BB	AA	C
Case 3	**Expert**	BB	BB	A	CA
	Model	BB	A	A	BC
Case 4	**Expert**	CAAB	CAE	CDE	CB
	Model	CAD	DCB	BA	CEE
Case 5	**Expert**	EBAA	EBBA	DC	DC
	Model	BBA	BC	EDD	AC
Case 6	**Expert**	EAAAC	FBBBCCC	DGE	DGF
	Model	EBFBA	FCCBD	GEFG	CAD

Test setup and assumptions
It is advised to print this test and take it with a pen and paper, but it can also be taken on screen. No calculator or computing tools are needed. Please set a timer and make sure to solve each case within 5 minutes. There are 6 cases, so the full test is expected to take a maximum of 30 minutes.

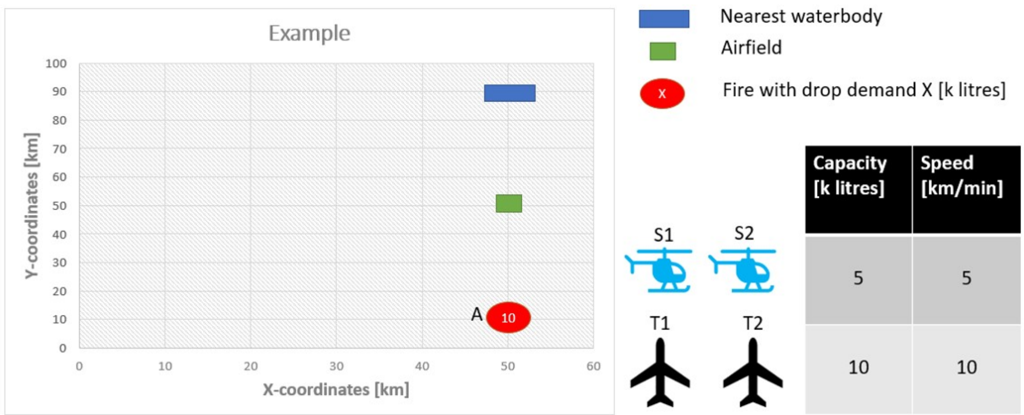
For each case, you have a fleet of 2 scoopers and 2 tankers. Scoopers go from the airfield to a fire, then to the water body before going to another fire, and so on. Tankers go to a fire and then back to the airfield. They can then go to another fire, but it takes 10 minutes to refill before they take off again. For the first departure, all aircraft (scoopers and tankers) take 10 minutes of processing time before take-off. Every scooper has a speed of 5 (km/min) and a capacity of 5 (k litres). Every tanker has a speed of 10 (km/min) and a capacity of 10 (k litres). Aircraft do not have the ability to portion their drops. That means that if they visit a fire, they must drop their full load and depart the fire with an empty (water/retardant) tank. The time of the drop manoeuvre is 1 minute, and the time of the scooping manoeuvre is negligible. Assume that all fires have equal urgency.

Assume that all aircraft can complete all missions without the need to refuel.

The main objective is to satisfy the demand of all fires as soon as possible. Meaning to make the last drop of the mission as soon as possible. Assume that no other objectives matter. For example, wasting water: If a fire has a demand of 5 (k litres), and you believe that the tanker will get there first, it is ok for the tanker to drop 10 (k litres), wasting 5.

Answer by filling in the routes of every aircraft in the highlighted cells of the given table. Two examples are provided first. Then the test cases begin.

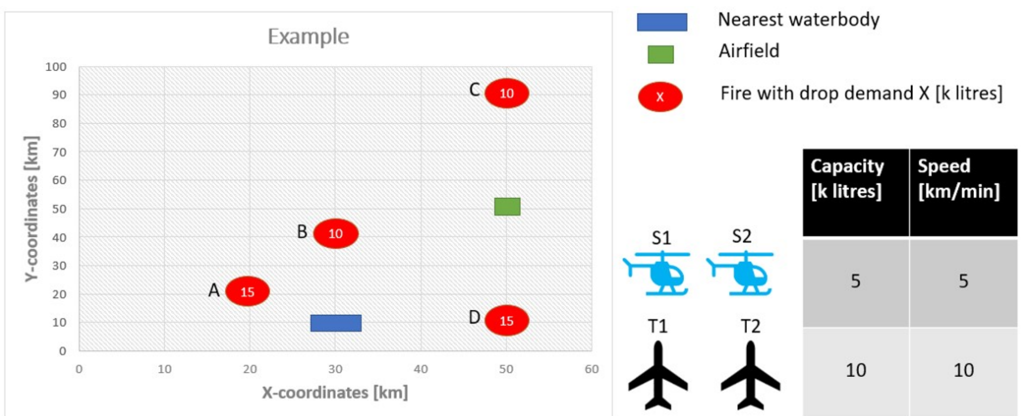
Example 1:



Aircraft	Route
S1	
S2	
T1	A
T2	

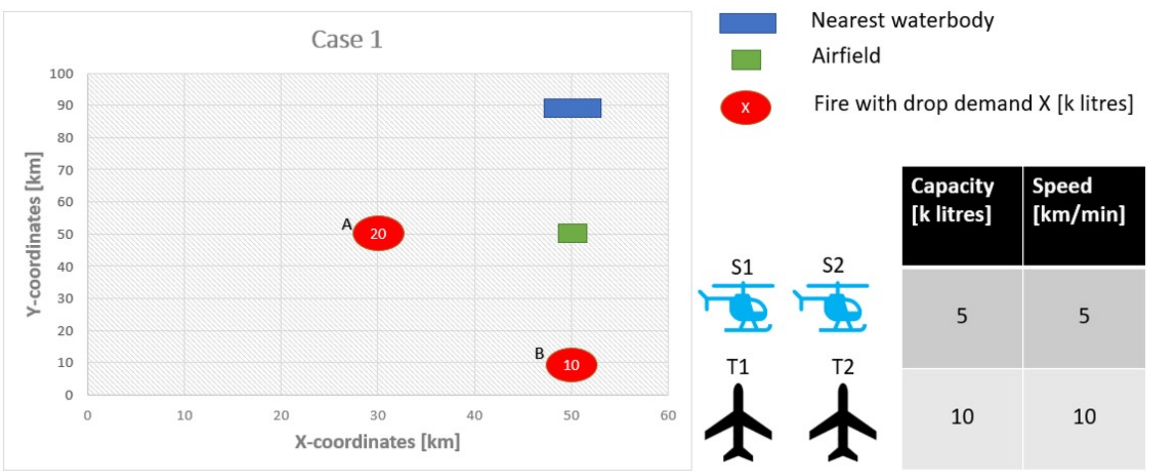
Figure 20: Example 1 in the validation test

Example 2:



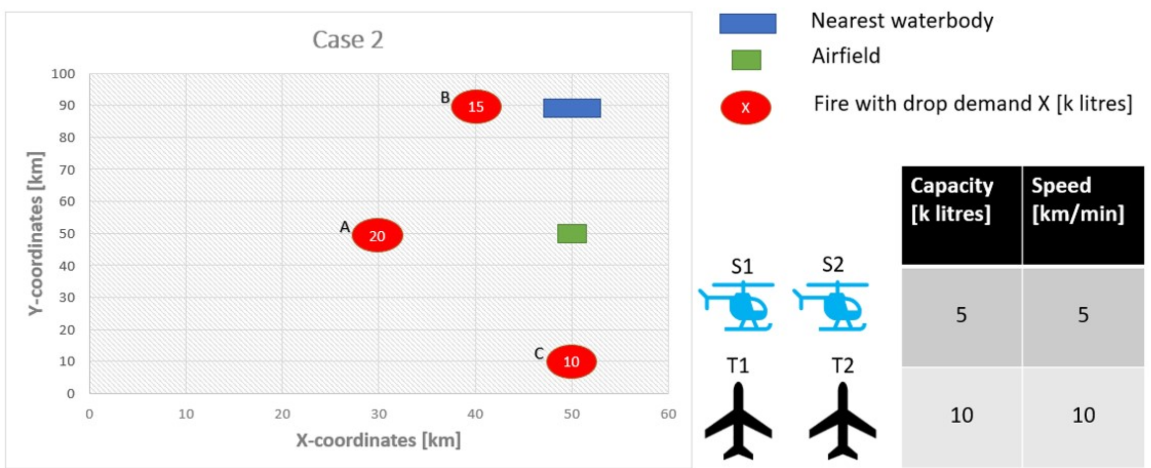
Aircraft	Route
S1	B-A-D (Airfield-B-waterbody-A-waterbody-D)
S2	B-A-A
T1	C
T2	D

Figure 21: Example 2 in the validation test.



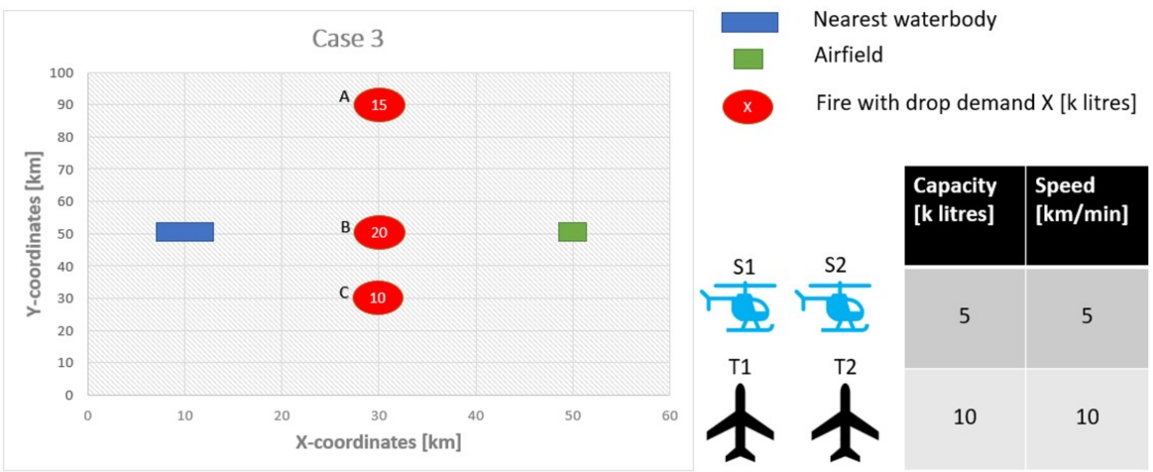
Aircraft	Route
S1	A
S2	A
T1	A
T2	B

Figure 22: Case 1 in the validation test.



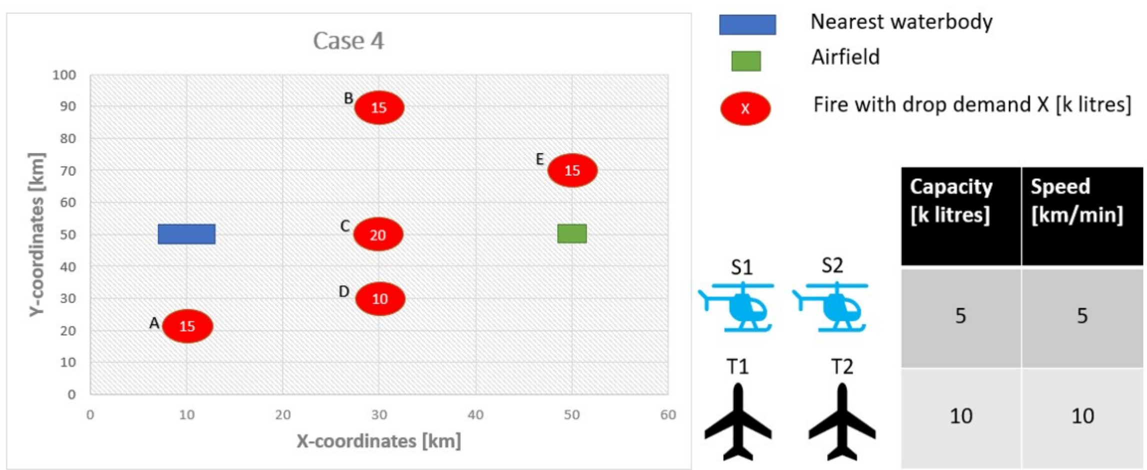
Aircraft	Route
S1	ABB
S2	AB
T1	A
T2	C

Figure 23: Case 2 in the validation test.



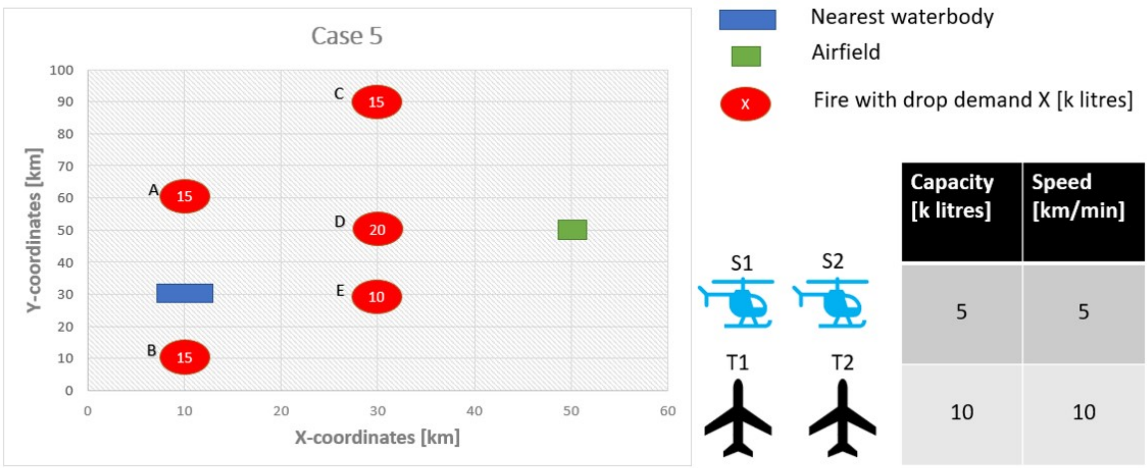
Aircraft	Route
S1	BB
S2	BB
T1	A
T2	CA

Figure 24: Case 3 in the validation test.



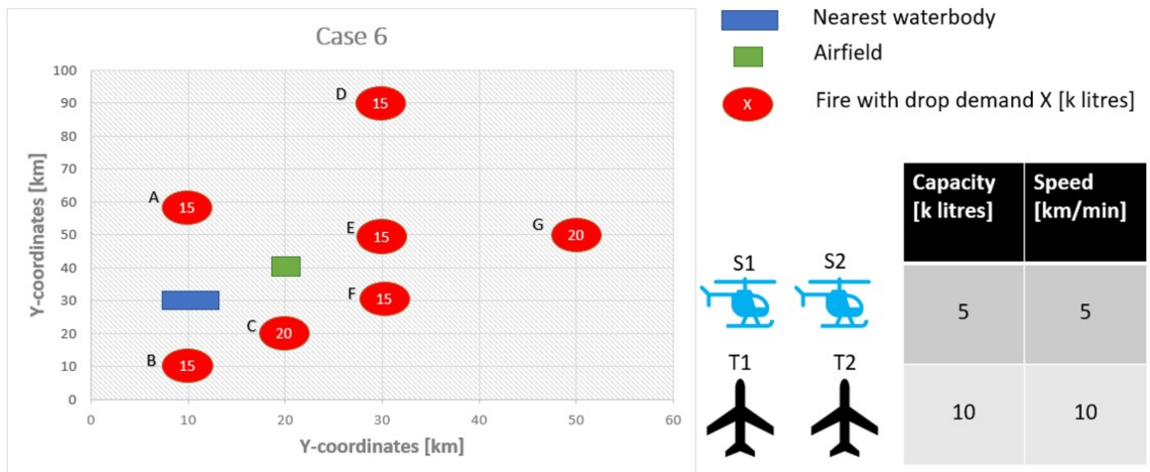
Aircraft	Route
S1	CAAB
S2	CAE
T1	CDE
T2	CB

Figure 25: Case 4 in the validation test.



Aircraft	Route
S1	EBAA
S2	EBBA
T1	DC
T2	DC

Figure 26: Case 5 in the validation test.



Aircraft	Route
S1	EAAAC
S2	FBBBCCC
T1	DGE
T2	DGF

Figure 27: Case 6 in the validation test.

II

Literature Study
previously graded under AE4020

1 Introduction

Climate change can have a significant impact on the frequency and severity of forest fires. Higher temperatures and changes in precipitation patterns can increase the risk of forest fires by creating drier, more combustible conditions. Drought, which can be exacerbated by climate change, can also increase the risk of forest fires by reducing the moisture content of vegetation and making it more prone to ignition.

In addition, climate change can also affect the behavior of forest fires by making them more intense and longer lasting. Higher temperatures and drier conditions can create more favorable conditions for the spread of fires, leading to larger and more destructive fires.

Climate change can also have indirect effects on forest fires by increasing the likelihood of lightning strikes, which can start fires, or by increasing the likelihood of human activities that can start fires, such as campfires or cigarettes.

The cost of forest fires can vary widely depending on a variety of factors, such as the size of the fire, the duration of the fire, the resources required to fight the fire, and the value of any assets that are lost or damaged. Some of the costs associated with forest fires include:

- Suppression costs: These are the costs associated with fighting the fire, including the costs of personnel, equipment, and supplies.
- Property damage: Forest fires can damage or destroy homes, businesses, and other structures, leading to significant financial losses.
- Economic impacts: Forest fires can disrupt local economies by disrupting transportation, tourism, and other industries.
- Environmental impacts: Forest fires can have long-term environmental impacts, such as soil erosion, water quality degradation, and the loss of habitat for wildlife.
- Health impacts: Forest fires can have negative health impacts on humans, including respiratory problems and other health issues related to smoke exposure.

The overall cost of forest fires can be significant, and it is important to invest in prevention and management efforts to minimize the risk of fires and their impacts. In general, prevention is a more favorable way to combat forest fires. However, preparations still need to be in place in case a forest fire has started despite the prevention efforts. In this case, there are several methods to fighting forest fires, for example suppression, containment using fire breaks, or fire control by guiding the fire to a safer area. In some cases, aerial efforts are needed, and different kinds of approaches and aircraft are required depending on the situation.

There are several types of aircraft used for firefighting. These can be largely categorized into fixed-wind aircraft and helicopters. Helicopters have the advantage of hovering over a small body of water to pick up more water with the help of a bucket, but also the disadvantage of usually lower speed. Fixed-wing aircraft are faster and can also be divided into two main types: Scoopers and tankers. Scoopers are aircraft capable of scooping from a body of water such as a river or a lake to then return to the fire and make another drop. Tankers tend to be larger aircraft that lack the ability to land on water. These are usually filled with water or retardant at the airfield, and have to return to the airfield after making their drop, and possibly refilled before heading out to the fire again if necessary.

Fire simulations can be used to study and learn about the ways in which forest fires spread. They can also be used to predict fire behavior in certain areas, in order to prepare properly for fires in areas with specific characteristics such as high moisture levels tree density.

1.1. Problem Statement

When aerial firefighting is used, multiple aircraft may be deployed. The management of these aircraft can be a complicated operation. In the case of multiple simultaneous fires, it may be needed to perform water or retardant drops at several locations as soon as possible. The importance of fighting the fire as soon as possible is paramount, as the fire can get out of control if not quickly addressed after ignition. In this scenario, the authorities have an available fleet of possibly different types of aircraft, and need to decide which aircraft goes to which fire to perform which drop.

The allocation of resources in such a scenario can be inefficient or sub-optimal because it is an overwhelming problem for the human perspective. In order to solve this, I propose the use of the Vehicle Routing Problem (VRP). The VRP is a well established transportation problem usually used to solve logistical challenges such as transportation of goods to customers using a fleet of vehicles. In order to apply the VRP to aerial firefighting, both problems need to be studied carefully and the VRP needs to be adjusted appropriately to support the operation of aerial firefighting.

Thus, the problem statement is to develop the VRP to optimize the routing of a fleet of aerial firefighters in order to fight forest fires.

1.2. Research Objective and Context

The goal of the research is to create a tool that can support authorities in their decision-making with regards to the deployment of aerial firefighting assets to fight and contain wildfires. To achieve this goal, a vehicle routing model will be adapted to reflect the unique combination of challenges faced by firefighting aircraft. The decisions made by these authorities are motivated by different factors. The main goal is usually to minimize the burned area. However, decision-makers may opt to allow a larger burned area, if it means stopping the fire from reaching a residential area for example. Thus, there are different ways to measure the potential damage a fire can cause, and therefore different approaches to optimize the containment of the fire. A general principle however, is that early attack and containment of fires is the most important indicator of success. Hence, reaching the fires as fast as possible will be a high priority.

The research objective can be defined as follows:

"Minimize the spread of wildfires by choosing optimal routing for the available fleet of firefighting aircraft "

Every wildfire is unique and every authority fighting it has a different set of firefighting aircraft with different capabilities. This formulation incorporates the consideration of these differences and will allow a potential user to use their specific fleet as an input in the model.

In order to pursue this research objective, a few sub-objectives may be defined:

- Develop an understanding of fire behavior and the simulation tools available to model this behavior.
- Develop an overview of AFF operation principles.
- Define the roles played by different types of AFF aircraft.
- Develop a VRP model that takes into account the unique challenges of AFF and solves for the most optimal routing.

The context of this project is not tied to any particular company or organisation. The idea of the project started developing during an international conference on aerial firefighting, mainly in a conversation with Australian firefighting pilot Glen Purdam. Mr. Purdam, acting independent of any organisation he may fly for, is an external supervisor for the project. His expertise in the field sheds light on the necessity for an optimized decision-making tool to aid authorities in the efficient deployment of aerial firefighting assets. Ideally, the project would also see the development of a more accurate fire simulation, but this is beyond the determined scope. This project will focus on creating a vehicle routing model that specifically incorporates the unique challenges of aerial firefighting.

1.3. Research Questions

The main research question can be formulated as follows:

"How can the vehicle routing problem be applied to aerial firefighting and lead to more effective containment of wildfires?"

This main research question can be divided into sub-questions, which sequentially reflect the research required in order to answer the main research question:

1. What methods can be used to model fire spreading behavior?
   - What are the main factors that influence the direction and speed of fire spreading in wild areas?
   - What fire data or simulations are used by official authorities?
2. How do the responsible authorities make decisions with regards to the deployment of their aerial firefighting fleet during wildfires?
   - What are the main priorities when containing wildfires?
   - How is the organisation of the decision-making process during a fire?
   - What kinds of strategies are used to effectively contain fires?
   - What kinds of aircraft are used and what are their advantages and disadvantages?
3. How can the vehicle routing problem (VRP) be applied to the aerial firefighting problem?
   - What kinds of standard vehicle routing problems are there?
   - What unique aspects of the aerial firefighting problem need to be incorporated into the VRP model?
   - What aspects of the various existing VRP models can be useful for the aerial firefighting problem?

1.4. Report Structure

In order to prepare for answering these questions, the literature study will summarize the necessary knowledge. The study will be divided into three separate topics corresponding to the research sub-questions. chapter 2 will focus on describing fire behavior and fire simulations. chapter 3 will look at the perspective of decision-makers, and how they usually make decisions with regards to aerial firefighting. Finally, chapter 4 will study the vehicle routing problem and its adaptability to the aerial firefighting problem.

2

Fire: Behavior, Data, and Simulations

In order to solve any problem, it is important to have a fundamental understanding of it. This thesis project deals with the special field of aerial firefighting, it is therefore important to develop some understanding of fire behavior, in order to know how fire spreads and how it can be effectively contained in different scenarios. This chapter focuses on the basics of fire behavior, before diving into the available fire simulation tools.

2.1. Fire Behavior

Fire ire behacan occur in many different contexts. The way fire behaves can be unpredictable and depends on the context. For example, a bush fire in open terrain behaves differently than a fire spreading through a closed building. Although some of the principles hold across different situations, to be clear, when the term fire behavior is used in this project, reference is made to fire behavior in wildfires. The interesting characteristics then are mainly the direction and speed of spread of the fire. There are several factors that influence these characteristics, and thereby fire behavior. The following important factors are briefly introduced in this section: Weather, topography, and fuel characteristics. Fuel means anything that can burn because of the fire, for example vegetation and wooden logs from trees.

The three factors are widely considered to be the most influential on fire behavior. Reference is often made to the "fire triangle", quoting these three factors as the three sides of this notable triangle, as shown in Figure 2.1.

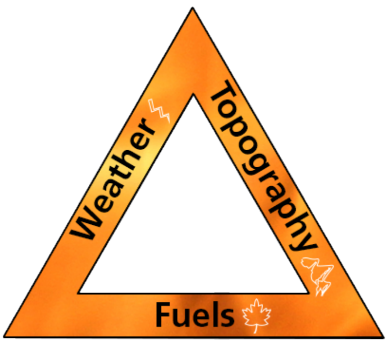


Figure 2.1: The fire triangle [63].

2.1.1. Wind speed/direction

Weather influences the behavior of fire. Wind speed and direction can be driving influences on the speed and direction of the spread of the fire. Wind can affect fire in different ways, not only moving the fire in its direction. Firstly, wind can carry moist air away from fuels. This causes the fuels to dry out faster and be more likely to ignite. Secondly, wind can carry burning embers and bring them ahead of the fire, causing spot fires further ahead and making it more difficult to contain the current fire. Furthermore, wind continues to bring more oxygen to the fire, giving it the supply it needs to burn longer and more intensely [63].

Winds take place at different altitudes above the surface. Firstly, there are surface winds, which occur near the earth surface and have a direct effect on the surface. Secondly, local winds, which occur within a few hundred meters above the surface. These winds are impacted by the temperature differences in the area. Finally, general or high winds, which are created by large pressure differences in the climate and move from one region to another. These winds are on a much higher level, literally and figuratively, and are therefore unaffected by the terrain conditions [63].

In the absence of wind and terrain variation, fire burns in a circular shape. The presence of wind will carry the fire in its prominent direction, and make it spread in a more elliptical shape. This is assuming that the density of burnable fuels is equal in all directions, and that the terrain is level. The effect of those factors is of course also relevant and will be discussed in the next points.

2.1.2. Fuel parameters

A fuel can be anything that the fire can burn through. In the context of wildfires, this is mainly vegetation and wood from trees and bushes. The characteristics of a fuel determine how flammable it is. The most important characteristics are moisture levels, chemical makeup, and density [65].

Higher moisture means that the fire must first eliminate the moisture before it can burn through the fuel. This means that higher moisture level is desirable to slow down a fire or to prevent it. Living trees usually contain higher moisture levels than dead ones. This is another area where prevention efforts matter. Forest services often remove dead or low moisture fuels from forests in order to prevent the ignition or fast spread of potential fires [65].

The chemical makeup of fuels refer to the physical contents of the fuel. For example, some plants contain oils, which can cause faster and more intense burning. This is why some areas with certain types of trees may be more vulnerable to wildfires than others [65].

Finally, the density of the fuel is directly related to its flammability. Denser fuels will burn faster and more intensely. This is because the particles of the fuel are more closely packed together and will heat and ignite each other easier than less dense fuels with more distance between particles [65].

2.1.3. Topography

Topography comprises of physical characteristics of the land on which the fire is burning. The main consideration with regards to topography is the slope of the terrain. Namely, fire burns faster going up a slope, and slower going down a slope. This is logical since while going up a slope, the top of the flames already reaches an unburnt fuels and pre-heats them, making them catch fire quicker. Furthermore, the updraft caused by the fire provides more oxygen and wind to the unburnt part, making it more ignitable. As a rule of thumb, a slope of 20% results in twice as fast fire speed. A 30% slope in thrice the spread, and so on [63].

Another topography characteristic that influences the fire is "aspect". The aspect means the direction that the slope is facing, for example a southern aspect means that the slope is facing the southern direction. This gives information on how much sunlight the slope is exposed to. The more sunlight exposure, the more likely it is that the fuels on the slope are dry and pose a higher risk of ignition or faster spread [63].

2.2. Fire Simulations

Fire simulations can be useful for different purposes. In order to effectively fight the spread of wildfires, it is important to understand how fire is expected to behave. As discussed in section 2.1, fire can behave very differently based on the environment and weather. Therefore, simply using data from past fires to predict fire behavior may be insufficient. Simulation tools allow for low cost predictions of how fires may spread in particular areas and times of the year. The user can input characteristics of the area they are concerned with, and can even input different weather conditions, to get an estimation of how a fire may spread if ignition occurs, and can therefore prepare properly.

Nonetheless, simulating fire spread is not a simple task. Once fire ignites, it can create its own weather, because of the indraft that comes from the fire, and so these simulations are used as tools to better understand wildfires and gain from that understanding. For example preposition firefighting assets in dangerous areas or train local firefighters. However, it is not expected that the simulations are so accurate as to give a completely realistic prediction of how a fire will spread in reality. Therefore, there are several simulations used by different authorities. These simulations each give certain outputs and require certain inputs, as well as have their limitations. Several simulations are discussed in this section, and some of the most widely used ones are summarised in Table 2.1 and Table 2.2.

- BehavePlus: This is the most commonly used fire simulation. The program is made up of many mathematical models that describe fire behavior and effects. Originally, the system was available in 1984, and has since been updated several times to the current Version 6, which comprises of many mathematical modules. The modules and their corresponding calculations are shown in Figure 2.2. The modules and their corresponding mathematical models and references are shown in Figure 2.3.Applications of the BehavePlus tool include predicting the behavior or an ongoing fire, planning fire treatments, assessing fuel hazard, and general understanding of fire behavior in training of personnel [69].
- FlamMap: A 2 dimensional fire growth simulation modeling system. It includes the previously separate simulation program FARSITE and includes the following behavior models: Rothermel's (1972) surface fire spread model, Van Wagner's (1977) crown fire initiation model, Rothermel's (1991) crown fire spread model, Albini's (1979) spotting model, Finney's (1998) or Scott and Reinhardt's (2001) crown fire calculation method, and Nelson's (2000) dead fuel moisture model. "The FARSITE (Fire Area Simulator) is a fuel growth model that spatially projects fuel perimeters and behavior over complex landscapes. The model uses spatial data themes from a geographic information system (GIS) along with weather and wind data to propagate fuel as a spreading wave front." [52]. The simulation tool uses an expanding tree structure on the left hand side of the interface and shows the results on a map. An example from the program is shown in Figure 2.4.
- FireFamilyPlus: or FireFamily+ (FF+) "a software package used to calculate fuel moistures and indices from the US National Fire Danger Rating System (NFDRS) using hourly or daily fire weather observations primarily from Remote Automated Weather Stations (RAWS)" [44]. FF+ is used for several applications in addition to computing NFDRS indices. For instance, it can compute climatology breakpoints for fire management decision makers, present the historical relationship between weather conditions and fire risks, which can be used to monitor seasonal progression of fire risk, and even analyse weather information to produce estimates for ongoing fires and their potential growth pattern.
- FireMON: "Fire Effects Monitoring and Inventory System is an agency independent plot level sampling system designed to characterize changes in ecosystem attributes over time" [51]. FireMON is dissimilar to other simulations in that while it provides a data analysis program, it does not provide any form of visual simulation, it is rather a sampling system to analyse fire related data.
- FOFEM: A First Order Fire Effects Model used for predicting fire effects rather than simulating fire spread. The fire effects considered include tree mortality, fuel consumption, smoke production, and soil heating caused by wildfires. First order effects means effects that are immediate consequences of fire, in contrast to secondary effects such as tree regeneration [37].

- FEIS: Fire Effects Information System is not a simulation but a worthwhile mention since it can be used for similar applications. "It is an online collection of reviews of the scientific literature about fire effects on plants and animals and about fire regimes of plant communities in the United States"[70]. It includes reviews on species, fire studies, and fire regime syntheses giving updated information on fire regimes of ecosystems. This provides data on important fire prediction factors such as historical fire frequency and historical ignition sources [70].

- ArcFuels: "A toolbar implemented in ArcMap which creates a trans-scale (stand to large landscape) interface to apply pre-existing forest growth (e.g., Forest Vegetation Simulator) and fire behavior models (e.g., FlamMap) to aid in vegetation management, fuel treatment planning, wildfire behavior modeling, and wildfire risk assessments." [1].

- WFAS: Wildland Fire Assessment System is a tool by the United States Forest Service (USFS) that classifies regions on a map by their Fire Danger Rating (FDR). FDR takes into account current and past weather, fuel types, and live and dead fuel moisture.[71].

Module name	Calculations	Possible linkages
SURFACE	Surface fire rate of spread Fireline intensity and flame length Reaction intensity and heat per unit area Intermediate values: heat source, heat sink, characteristic dead fuel moisture, relative packing ratio, etc. Standard, custom and special case fuel models Wind adjustment factor	
CROWN	Transition from surface to crown fire Crown fire rate of spread Crown fire area and perimeter Fire type: surface, torching, conditional crown or crowning Crown fire intensity and flame length Power of the fire, power of the wind	Surface fireline intensity or flame length can come from SURFACE Surface fire heat per unit area can come from SURFACE
SAFETY	Safety zone size based on flame height Area, perimeter and separation distance	Head fire flame length can come from SURFACE
SIZE	Elliptically shaped point source fire Area, perimeter and shape	Head fire rate of spread and effective wind speed can come from SURFACE
CONTAIN	Fire containment success for single or multiple resources given line construction rate, arrival time, resource duration, head or rear attack, and direct or parallel attack Final area and perimeter, fire size at initial attack and fireline constructed	Head fire rate of spread can come from SURFACE Length-to-width ratio and fire size at report can come from SIZE
SPOT	Maximum spotting distance from torching trees, burning piles or wind-driven surface fire	Head fire flame length can come from SURFACE for wind-driven surface fire
SCORCH	Crown scorch height from surface fire flame length	Surface fireline intensity or flame length can come from SURFACE
MORTALITY	Probability of mortality from bark thickness and crown scorch	Scorch height can come from SCORCH
IGNITE	Probability of ignition by firebrands Probability of ignition from lightning strikes	

Figure 2.2: The mathematical modules and corresponding calculations in BehavePlus [6].

Module	Model	Reference
SURFACE	Surface head fire rate of spread, reaction intensity, characteristic dead fuel moisture, live fuel moisture of extinction, etc.	Rothermel (1972); Albini (1976*b*)
	Fireline intensity, flame length	Byram (1959); Albini (1976*a*)
	Surface fire flame residence time	Anderson (1969)
	Fire characteristics chart; relationship among rate of spread, heat per unit area, fireline intensity and flame length	Andrews and Rothermel (1982); Andrews *et al.* (2011)
	Direction of maximum spread	Finney (1998); Rothermel (1983)
	Spread in direction from ignition point of a point source fire	Andrews (1986)
	Effective wind speed	Albini (1976*a*)
	Wind adjustment factor	Albini and Baughman (1979); Baughman and Albini (1980); Rothermel (1983); Andrews (2012)
	Wind speed at 10 m adjusted to 20 ft	Turner and Lawson (1978); Lawson and Armitage (2008)
	13 standard fire behaviour fuel models	Rothermel (1972); Albini (1976*a*); Anderson (1982)
	40 standard fire behaviour fuel models	Scott and Burgan (2005)
	Custom fire behaviour fuel models	Burgan (1987); Burgan and Rothermel (1984)
	Dynamic fuel load transfer	Burgan (1979); Andrews (1986), Burgan and Rothermel (1984); Scott and Burgan (2005)
	Two fuel models, weighted rate of spread	Rothermel (1983)
	Two fuel models, harmonic mean	Fujioka (1985)
	Two fuel models, two-dimensional expected spread	Finney (2003)
	Palmetto-gallberry special case fuel model	Hough and Albini (1978)
	Western aspen special case fuel model	Brown and Simmerman (1986); Brown and Debyle (1987)
CROWN	Critical surface intensity needed for transition from surface to crown fire	Van Wagner (1977)
	Transition to crown fire, relationship of surface fire intensity and critical surface fire intensity	Van Wagner (1989; 1993); Finney (1998); Scott and Reinhardt (2001)
	Crown fire rate of spread, area and perimeter	Rothermel (1991)
	Critical crown fire rate of spread, needed for an active crown fire	Van Wagner (1977)
	Active crown fire condition	Van Wagner (1989, 1993); Finney (1998); Scott and Reinhardt (2001)
	Fire type: surface, torching, conditional crown or crowning	Van Wagner (1993); Finney (1998); Scott and Reinhardt (2001)
	Crown fire flame length	Thomas (1963)
	Crown fire intensity	Rothermel (1991)
	Power of the fire, power of the wind	Byram (1959); Rothermel (1991)
SAFETY	Safety zone size, separation distance, radius	Butler and Cohen (1996, 1998*a*, 1998*b*)
SIZE	Elliptical fire size and shape, area, perimeter, length-to-width ratio	Anderson (1983); Andrews (1986)
CONTAIN	Fire containment	Albini *et al.* (1978); Fried and Fried (1996)
SPOT	Spotting distance from torching trees	Albini (1979); Chase (1981)
	Spotting distance from a burning pile	Albini (1981)
	Spotting distance from a wind-driven surface fire	Albini (1983*a*, 1983*b*); Chase (1984); Morris (1987)
SCORCH	Crown scorch height	Van Wagner (1973)
MORTALITY	Tree mortality	Ryan and Reinhardt (1988); Ryan and Amman (1994); Reinhardt and Crookston (2003); Hood *et al.* (2007)
	Bark thickness	Ryan and Reinhardt (1988); Reinhardt and Crookston (2003); Lutes (2012)
IGNITE	Probability of ignition from firebrand	Schroeder (1969)
	Probability of ignition from lightning	Latham and Schlieter (1989)
Fine Dead Fuel Moisture Tool	Fine dead fuel moisture tables	Rothermel (1983)

Figure 2.3: The mathematical modules and corresponding mathematical models in BehavePlus [6].

Table 2.1: Overview of fire simulation outputs and inputs [27].

Software	Simulation output	Simulation requirements
BehavePlus	fire spread, spotting distance, scorch height, tree mortality, fuel moisture, wind adjustment factor, Rothermel surface model.	Experienced users who can estimate reasonable values for inputs such as fuel moisture levels in a given area and time, as well as interpret whether results are reasonable and make necessary adjustments.
FlamMap	Spread rate, fire intensity.	Landscape file, can be created by choosing the following parameters: - Topography (Elevation, slope, aspect) - Forest Canopy cover - Forest canopy height - Forest canopy base height - Forest canopy bulk density
WFAS	Fire danger level, fire danger maps (color coded)	current and antecedent weather, fuel types, and both live and dead fuel moisture
FireFamily+	- Fuel moisture and fire danger indices for the NFDRS 1978, 1988 and NFDRS2016 and the Canadian Forest Fire Danger Rating System and the Fosberg Fire Weather Index. - Climatology tools to explore and display seasonal variations in fire danger.	hourly or daily fire weather observations primarily from Remote Automated Weather Stations (RAWS)
FireMON	sampling system designed to characterize changes in ecosystem attributes over time, including: - a sampling strategy manual - standardized sampling methods - field forms - a data analysis program	Whatever data is to be analysed, e.g. vegetation, fuel, fire info..etc.

Table 2.2: Overview of fire simulation limitations [27].

Software	Limitations
BehavePlus	-It is a point system: conditions are constant for each calculation, although output tables and graphs can cover multiple conditions. - Requires expert insight to get meaningful results - System difficult to expand since it is still basedon the work done in 1977 and subsequent updates.
FlamMap	- It is a point system: conditions are constant for each calculation. - Does not simulate temporal variations in fire behavior caused by weather and diurnal fluctuations. - Does not display spatial variations caused by backing or flanking fire beavior.
WFAS	- Maps not specific enough (pixelated appearance) - Collects data from fire stations (difficult to manage/ unreliable consistency). - Values between stations are estimated with an inverse distance-squared technique on a 10-km grid. This works pretty well in areas of relatively high station density, but has shortcomings otherwise.
FireFamily+	Mainly gives fuel moistures/climate conditions, not fire spread values.
FireMON	It is a sampling methodology framework, not truly a simulation program.

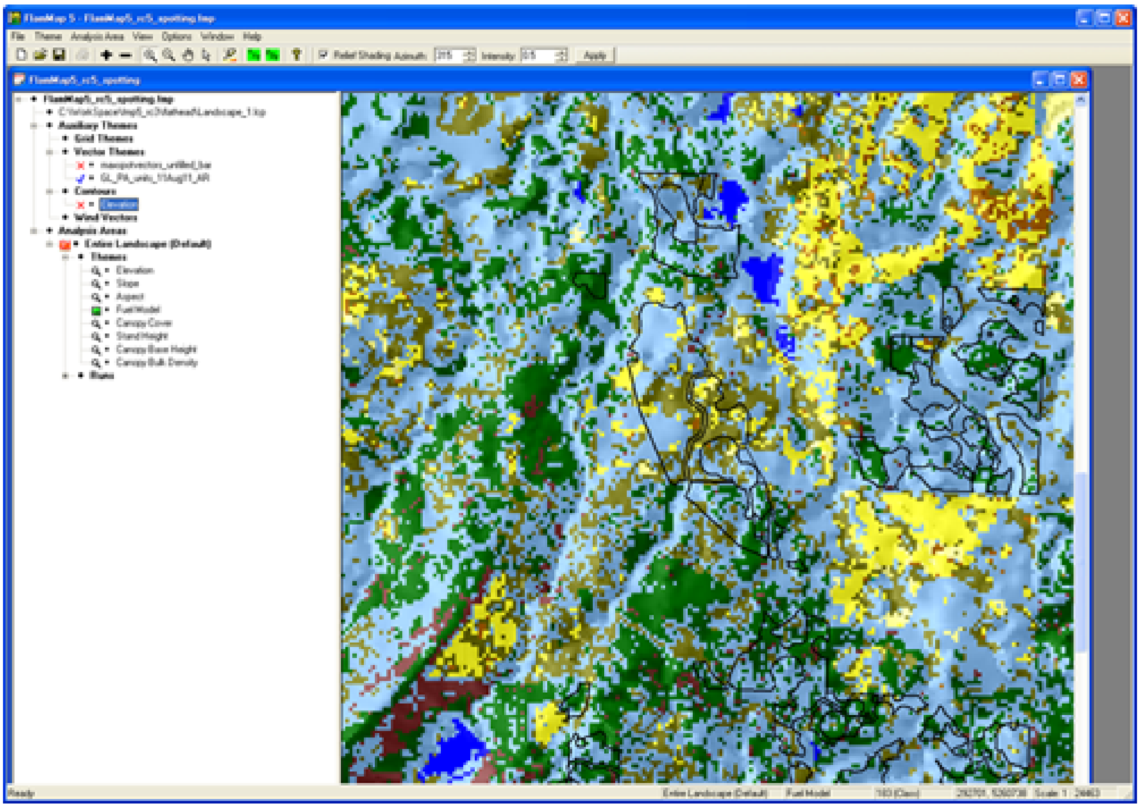


Figure 2.4: A screenshot of the FlamMap interface [45].

3

Aerial Firefighting Operations

Decision making when it comes to deploying aircraft for wildfires can vary greatly depending on the authorities involved. Decisions pertaining to small fires may differ from large fires. Fires in the United States may be handled differently than in Europe, or Australia, given the cultural differences and systems that have been in place for a long time in those regions. There can even be a difference among states or regions within the same country. Sometimes, the fires are out of control and the national authority will ask for help from neighboring countries or alliances. For example, the forest fires in Greece in 2019 were fought by aircraft from multiple countries, especially EU allies. The coordination between all these involved parties could understandably lead to largely different practices in terms of how aircraft routing decisions are made. However, there are some common practices on how fires are managed, which can be useful for the purposes of this project. Understanding some basic universal principles can lay the foundation for this work, even though there may be some deviations in particular cases.

3.1. Firefighting strategies

There are various strategies used to fight wildfires. Firstly, it is important to state the fact that prevention efforts are the most effective strategy to fight wildfires. Most wildfires are started by humans. According to data from the U.S. Forest Service Research Data Archive, nearly 85% of wildfires in the period 2000-2017 in the United States were started by humans [65]. Prevention campaigns have proven effective in countries like Portugal, where a comprehensive prevention campaign in communities with higher risk for wildfires is adopted to decrease the number of started fires [13], despite the rising temperatures due to global warming. Besides education campaigns, there are tactical prevention methods such as building fire lines. A fire line is a line created through the forest by clearing a path of all flammable fuel, for instance by cutting trees and clearing all wood and leaves. If a fire starts in this area, it stops when it reaches this line, as it does not find more fuel to burn through. A fire line can also be made by digging a line-long hole, since fire also slows down when going down a terrain.

Furthermore, when wildfires do occur, aerial resources are not always available or necessary. In certain conditions, the fires can be contained completely by ground firefighters. However, as temperatures rise, and dry weather conditions become more common for longer periods of time, the risk for intense fires that get out of control is increasing. Those are the cases where AFF is needed.

3.1.1. AFF strategies

In order to define AFF strategies, let us consider the possible intentions behind AFF drops. Drop objectives are summarized in Table 3.1 taken from the United States Department of Agriculture's (USDA) 2020 report on Aerial Firefighting Use and Effectiveness (AFUE).

Table 3.1: Examples of several possible drop objectives [62].

ID	Drop Objectives	Description
DOBJ-1	Reduce fire intensity/ flame length	The intent of the drop(s) is to cool an area of fire activity. This may be needed so ground personnel can work closer to the fire activity (e.g., begin or continue to go direct). Examples include knocking down crown fire, torching, and preventing spotting, etc.
DOBJ-2	Delay fire spread/ retard growth	The intent of the drop(s) is to delay the fires rate of spread in the same location (head, heel, specific flank, spot, etc.) of the fire. Examples include buying time for ground resources to construct line or for evacuations
DOBJ-3	Point protection	The intent of the drop(s) is to protect a value(s) at risk (VAR). These drops should be within the immediate area of the VAR or be executed primarily to reduce the probability of fire reaching the VAR or to reduce damage to the VAR.
DOBJ-4	Line fire/ halt advance	The intent of the drop(s) is to construct aerial line to halt fire spread. These drops are used to halt the spread of a section of the fires edge before, during, or after ground engagement or without the aid of ground personnel
DOBJ-5	Extinguish fire/ spot fire	The intent of the drop(s) is to fully extinguish the entire portion of the fire or spot fires (generally a rare occasion, usually a small area, and likely a fine/flashy fuel).

To simplify things and get a good overview on firefighting strategies, the objectives in Table 3.1 can be categorized into two main approaches: Direct drops and laying lines:

- Direct drops:
  Water or retardant is dropped directly on the fire. This could either be to help directly extinguish the fire, or to reduce the intensity of the fire, in order to help the ground crews get close enough to extinguish it. This kind of drop is usually performed by helicopters, since they can hover at a low altitude to do precise drops. This encompasses DOBJ-1 and DOBJ-5 as outlined in Table 3.1.
- Laying lines:
  This is the more common approach simply because aerial assets are usually called when fires are quite large or are expected to spread to a large area. This means that the priority becomes limiting the spread of the fire. This is done by 'laying lines' of retardant or water a bit ahead of the fire front line, to stop the spread of the fire in that direction. This is usually carried out by fixed wing aircraft, since they have the cruise speed and capacity required to lay longer lines. This encompasses DOBJ-2, DOBJ-3, and DOBJ-4 as outlined in Table 3.1.

An overview of strategies is provided in Figure 3.1.

3.1.2. Air Attack

Contrary to what the name suggests, the role of the Air Attack (AA) is not to attack the fire. The AA is a role fulfilled by an aircraft that arrives early to the fire, and flies at a higher altitude, typically around 2500 feet above ground level. The AA aircraft typically carries two important people. The AA pilot, whose role is only to pilot the aircraft, including maintaining communications with air traffic control for example. The other person is the Air Tactical Group Supervisor (ATGS). Their role is to gain an overview on the fire, and coordinate the aerial firefighting efforts. The ATGS coordinates with the ground supervisor, the ground crews, and the firefighting aircraft.

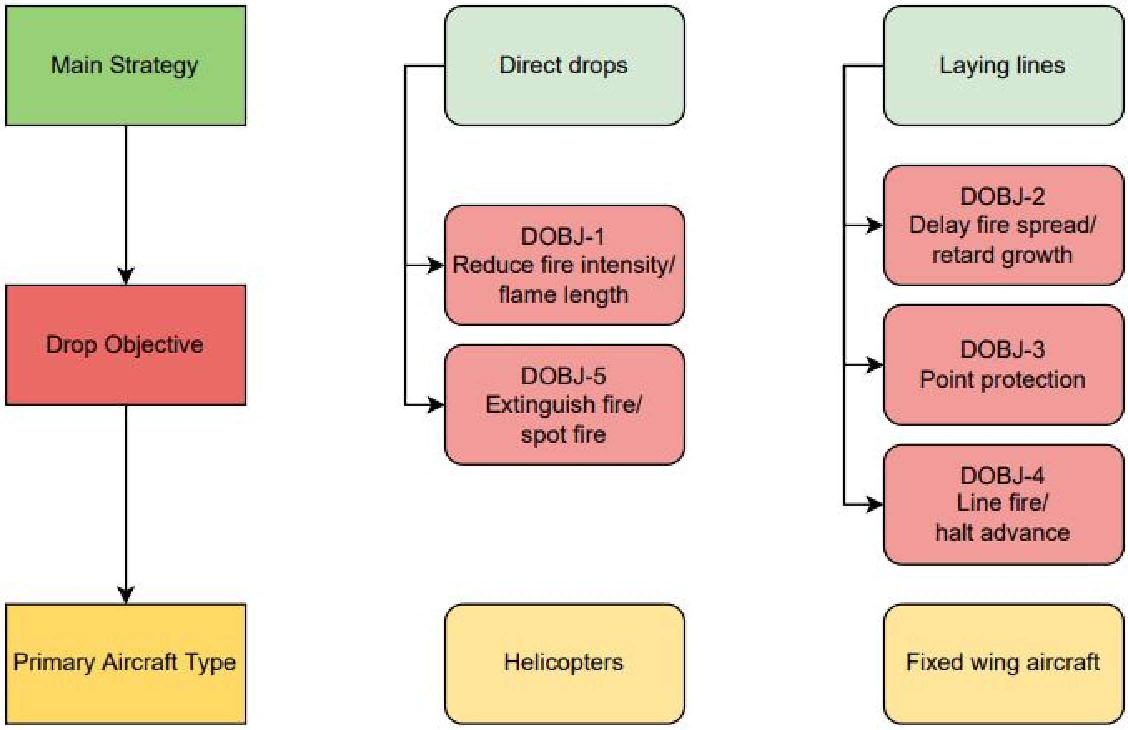


Figure 3.1: Overview of firefighting strategies based on drop objectives

3.1.3. Lead plane

The lead plane is an aircraft that leads Large Air Tankers (LATs) and Very Large Air Tankers (VLATs) quite literally. The lead plane will fly over the desired drop area while communicating with the standby LATs or VLATs, who will be loitering at a higher altitude and watching, and show them where the drop should take place. LATs and VLATs drop a large amount of retardant to lay lines and stop the spread of large fires. These aircraft can also take quite some time to return to base and refill and return to the fire for their second drop. Therefore, the accuracy of their drops is crucial. For this reason, the lead plane guides them to make sure the LAT and VLAT pilots understand exactly how and where to perform the drop.

3.1.4. Aerial Supervision Module

The Aerial Supervision Module (ASM) is a combination of the AA and the lead plane. It is an aircraft with a pilot who can perform the tasks of the lead plane, and a ATGS who performs the role of the AA.

3.1.5. Aircraft Stack

The 'stack' is the way in which AFF aircraft are organised during a mission. Every aircraft type is assigned a certain altitude. Usually larger aircraft are assigned a higher altitude. Aircraft enter a circular path near the fire at the assigned altitude and loiter clockwise until given instructions to make their drop. When it is their turn, they descend to the necessary altitude and make the drop. The AA aircraft is at the highest altitude in order to keep an overview and coordinate between the firefighting aircraft and the ground crew and the ground supervisor. The AA aircraft is also the only one that flies in its circular path anti-clockwise. This makes it easier for the AA to see all other aircraft in the stack. If the AA flies in the same direction, it may have blind spots to other aircraft who may be flying directly under it. By flying in the opposite direction, this cannot happen for more than a few moments and so the AA maintains its overview.

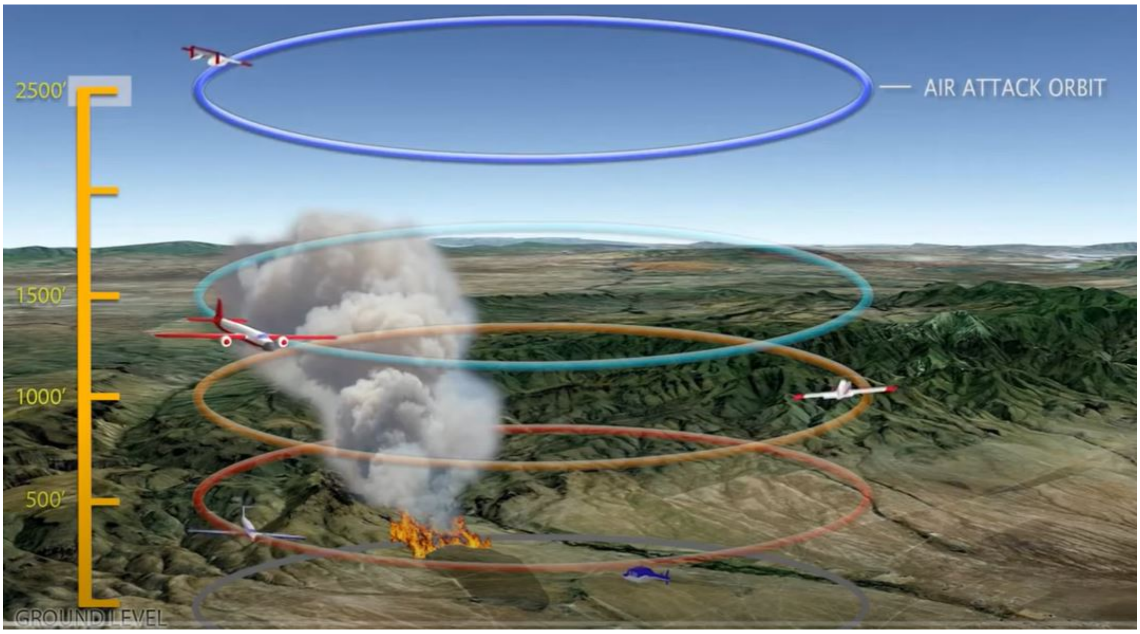


Figure 3.2: Visual animation of a stack showing the AA at the highest level [34].

3.2. Aircraft Types

There are many types of aircraft that are used for AFF. They can be categorized into three main categories: fixed wing tankers, helicopters, and scoopers. Scoopers are amphibious aircraft that can land on a body of water to scoop up a load of water into their tank, and directly return to make more drops on the fire. Although scoopers are also fixed wing aircraft, let us consider them a separate type since their mission profile differs significantly from the other aircraft types. In this section, the three types are described in more detail and examples are presented.

In Figure 3.3, the distribution of drop objective per aircraft type is shown. This sheds light on which aircraft types are mostly used for which kind of mission/drops. It also shows what kinds of drops are overall mostly used. This shows indeed that the tankers spend most of their drops laying lines, the scoopers perform both main strategies, while the majority of helicopter drops are direct drops.

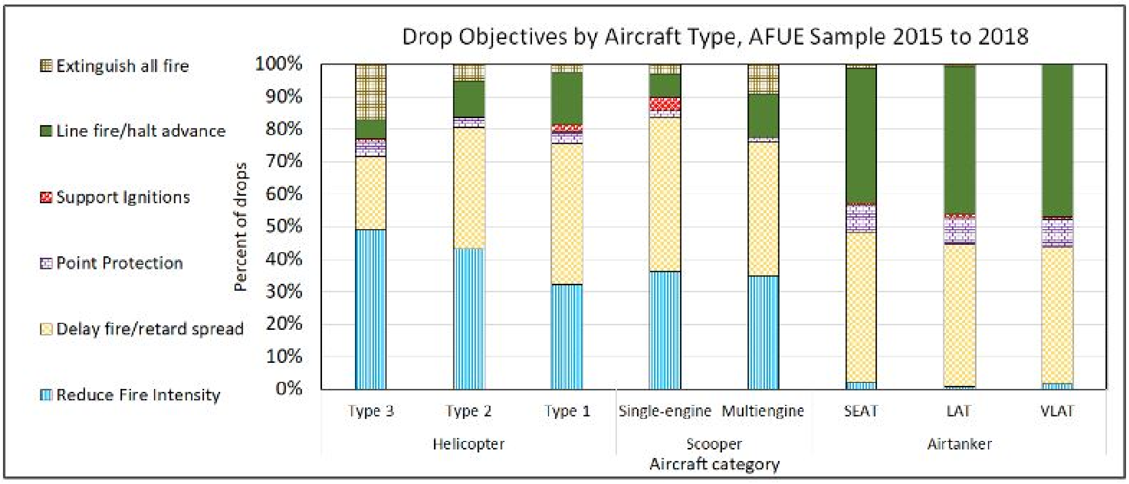


Figure 3.3: Drop objective per aircraft type, according to AFUE sample 2015-2018 [62].

3.2.1. Fixed Wing Tankers

Three main types of fixed wing tankers are considered:

- Single Engine Air Tankers (SEATs) are smaller single engine aircraft that have a lower load capacity. Their advantage is low cost which means that an authority can pre-position and deploy several such aircraft in a high risk area, and they can therefore arrive early to the fire. An example is the Air Tractor 802, which has a capacity of 800 gallons, or 7200 pounds, and a cruise speed of about 320 km/h [62].
- Large Air Tankers (LATs) are larger aircraft, usually ex-military or passenger aircraft that have been repurposed for AFF. Their capacity varies from around 2600 gallons / 23000 pounds to about 4000 gallons / 36000 pounds, and their cruise speed is in the range of 370 to 550 km/h [62]. Examples include Boeing 737-300, Lockheed EC-130Q, Lockheed C-HC-130H/J, and Viking Q-400.
- Very Large Air Tankers (VLATs) are the largest AFF tankers and are used for laying lines at very large fires. Their capacity can be over 10000 gallons. Prominent examples are the DC-10 tanker, which has a capacity of 9400 gallons and a cruise speed of 560 km/h, and the Boeing 747-400, which has a remarkable capacity of 18000 gallons and a cruise speed of 724 km/h [62].

3.2.2. Helicopters

Firefighting helicopters use either a tank and a hose, or a bucket, or possibly a combination of the two. The advantage of helicopters is that they can hover over a precise area. This means that they can refill their tank/bucket from a local body of water, even quite small ones, for example a swimming pool. Helicopters are categorized into three types, aptly named Type 1, Type 2, and Type 3.

- Type 1 are heavy helicopters with a higher capacity, usually over 1000 gallons. Examples include the Sikorsky/Erickson S-64E/F with a 2500 gallons capacity, the Boeing CH-47 "Chinook" with a 3000 gallon capacity, and the Airbus H215/225 with a 1000 gallons capacity [62].
- Type 2 are medium helicopters, with a lower capacity than Type 1, usually in the 300-500 gallon range. Examples are theBell 212HP with a 350 gallon capacity, and the Bell 210 with a 500 gallon capacity [62].
- Type 3 are light helicopters with a relatively low capacity. Examples are the Airbus H125 with a 260 gallon capacity, and the Bell 206 B3 with a 160 gallon capacity [62].

3.2.3. Scoopers

Scoopers are amphibious aircraft capable of landing on water as well as land. In firefighting operations, they can find a suitable body of water like a lake or a sea nearby the fire, and they can perform a manoeuvre where they scoop up water within a few seconds without stopping, similar to a touch-and-go manoeuvre. There are mainly two well known aircraft that are used as scoopers by authorities around the world:

- Viking CL-415 and its predecessor the CL-215 is a twin engine fixed wing scooper with a capacity of 1620 gallons and a cruise speed of 320 km/h [62]. The updated CL-515, which will contain newer instruments and some other minor updates is in development.
- Air Tractor Fire Boss is a smaller but popular single engine scooper with a capacity of 700 gallons [62].

Note that the aircraft discussed in this section have been limited to those who can actually make drops as they are the most relevant to this project. However, there are several other kinds of aircraft that may be part of a firefighting fleet, but with other missions than making drops. These could be lead planes or AA aircraft, or even passenger aircraft that simply transport ground firefighters, or evacuate people from dangerous fire areas. There are also passenger helicopters used for rescue missions, and aircraft that transport and drop smokejumpers. It is determined that these aircraft hold less relevance for this project, or in case they do become relevant, they do not need to be mapped so thoroughly as they do not have a drop load capacity. A more comprehensive list of fixed-wing firefighting aircraft is provided in Table 3.2 and of helicopters in Table 3.3.

Table 3.2: Overview of some fixed-wing firefighting aircraft [62].

Aircraft Model	Aircraft Type	Capacity (Gallons)	Cruise speed (mph)
DC-10	VLAT	9400	350
Boeing 747-400	VLAT	18000	450
BAe-146-200	LAT	3000	350
RJ-85	LAT	3000	350
Lockheed EC-130Q	LAT	4000	350
Boeing MD-87	LAT	3000	350
Lockheed C-HC-130H/J	LAT	3000	230
Boeing 737-300	LAT	4000	350
Viking Q-400	LAT	2600	375
Air Tractor 802	SEAT	800	200
Viking CL-415	Scooper	1620	200
Air Tractor Fire Boss	Scooper	700	170

Table 3.3: Overview of some firefighting helicopters [62].

Helicopter Model	Helicopter Type	Bucket Capacity (Gallons)	Cruise speed (mph)
Sikorsky/Erickson S-64E/F	Type 1	2500	132
Boeing BV-234	Type 1	3000	175
Boeing CH-47 "Chinook"	Type 1	3000	175
Boeing CH-46E "Sea Knight"	Type 1	1100	138
Kaman K-1200	Type 1	680	90
Airbus H215/225 NG	Type 1	1000	160
Bell 205/210, Bell UH-1	Type 2	500	121
Bell 212 HP	Type 2	350	132
Bell 412	Type 2	360	140
Airbus H125 NG	Type 3	260	157
Bell 407 NG	Type 3	270	152
Bell 206 L3/4	Type 3	225	127

Figure 3.4: The Viking Canadair CL-415 scooper performing a water drop [4].

4

The Vehicle Routing Problem

The term Vehicle Routing Problem (VRP) describes a type of transportation problem that was initially introduced in 1959 by Dantzig and Ramser [29], then called the truck dispatching problem. Five years later, in 1964, Clarke and Wright [28] proposed an effective greedy heuristic for the approximate solution of the VRP. More than half a century has passed since, and a large number of publications have been made in the field, presenting many adjusted versions of the VRP, and proposing algorithms for exact and approximate solutions. The practical applications of the VRP in real life problems may be credited for the great interest by the scientific and commercial communities.

"A generic verbal definition of the family of vehicle routing problems can be the following:

Given: A set of transportation requests and a fleet of vehicles.

The problem is then to find a plan for the following:

Task: Determine a set of vehicle routes to perform all (or some) transportation requests with the given vehicle fleet at minimum cost; in particular, decide which vehicle handles which requests in which sequence so that all vehicle routes can be feasibly executed"[64].

4.1. The Capacitated Vehicle Routing Problem

The Capacitated Vehicle Routing Problem (CVRP) is a well-studied version of the VRP. It is a relatively basic variant and therefore deemed a good starting point for understanding the family of VRPs. The CVRP will be introduced in this section with the mathematical notation, which should be lay the foundations for later understanding other variants.

4.1.1. CVRP Problem Statement

Transportation requests in the CVRP originate from a single depot denoted by O. Goods are distributed from O to a given set of N customers, $N = \{1, 2, ..., n\}$. The customer's demand is the amount that should be delivered to customer $i \in N$, which is given by a scalar $q_i \leq 0$, for example the mass of the goods to be delivered. An assumption is made that the fleet $K = \{1, 2, ..., |K|\}$ is homogeneous. This means that the capacity $Q > 0$ and operating costs of the vehicles $|K|$ available at the depot are the same. Furthermore, Let S be a customer subset such that $S \subseteq N$. A vehicle that is assigned to this subset will start from the depot, visit each customer within the subset once, and return to the depot. The travel cost for a vehicle moving from i to j is denoted as c_{ij}. [64]

4.2. Types of Transportation requests

The CVRP is a basic type of VRP, and as described in section 4.1, it deals with the distribution of goods from a depot to customers. In this section, other types of transportation requests are listed and briefly described.

4.2.1. Delivery and Collection

The contrasting service to delivery is collection, or as often described "pick-up". VRPs dealing with pick-ups can either occur at the beginning of the supply chain, for example collecting milk from a dairy producer [41], or at the end of the supply chain, for example in waste collection where empty containers need to be collected [10].

Within the delivery and collection problem, or the pickup and delivery, there are several variants. The important distinction to note is that some problems separate the delivery and pickup parts. Meaning that customers who request a delivery are separate than customers who request a pickup. The VRP with Simulataneous Pickup and Delivery (VRPSPD) involves two transportation requests per customer, one for delivery from the depot to the customer, and the other for pickup from the customer to the depot. [56].

4.2.2. Simple Visits and Vehicle Scheduling

Some problems do not involve the pickup nor the delivery of goods from customers. Simple visits include for example a repair person or a caregiver who must visit several customers to provide a service, rather than picking up or delivering any tangible goods. This problem is however unlikely to be relevant for AFF as that concerns the dropping of water or retardant.

Vehicle Scheduling Problems (VSPs) concern routes that are to be followed via a given sequence and following a certain schedule, for example bus lines or train services. Surveys on VSPs are compiled by Desrosiers et al [19], but again not expected to have high relevance with regards to AFF as no drops are involved.

4.2.3. Alternative and Indirect Services

In some situations, the customer cannot be represented simply by one location or node. In this case, the request may be to the customer directly or to an alternative location. For example parcel where customers have the choice to receive the delivery at home, at the office, or at some collection point. The Multi-Vehicle Covering Tour Problem (MVCTP) [35] deals with this by visiting certain locations that are close to the customer. The application for the MVCTP is useful in the creation of optimal collection routes to the pickup locations within a given region.

4.2.4. Point-to-Point Transportation

Point-to-point transportation is a characteristic of pickup-and-delivery problems. It can be the case that neither the delivery point nor the pickup are the depot. Therefore the problem is also referred to as a many-to-many VRP.

Point-to-Point transportation can involve the transportation of goods or people. When involving goods, the problem is called Pickup-and-Delivery Problem (PDP) [18]. When involving people, the problem is called Dial-a-Ride Problem (DARP)[64].

4.2.5. Repeated Supply

Two interesting VRPs are noteworthy when dealing with repeated supply. Firstly, the Periodic VRP (PVRP) [15], where a periodic delivery is maintained with a feasible visiting pattern. The customers may give their availability and the visiting pattern can be selected within those bounds. For example, a visit every monday and friday. The second repeated supply VRP to consider is the Inventory Routing Problem (IRP) [5]. In IRPs, the customer does not make a request with regards to the timing of the delivery. It is the delivery company instead who decides when to perform deliveries and even how much to deliver, the objective being to make sure the customer does not run out of stock. For example, this could be the refilling of coffee vending machines at multiple locations.

4.2.6. Non-split and Split Services

Non-split service means that a delivery or pickup is performed by one vehicle, meaning for example that the amount that needs to be delivered is not split amongst multiple vehicles. However, there can be good reasons to split a service in order to reach a more optimal operation, for example, using multiple smaller vehicles may be cheaper than fewer larger vehicles that are more expensive to operate. The Split Delivery VRP (SDVRP) [21] allows each demand to split into several smaller demands, which can be delivered by multiple vehicles.

4.2.7. Combined Shipment and Multi-modal Service

Combined shipment also involves using several vehicles to deliver the same shipment. However, the shipment remains non-split. This means that each vehicle transports the shipment for a portion of the route, and then another vehicle picks it up and continues on the next route segment. This problem works where different kinds of vehicles are used for different parts of the routes for efficiency. For example where a ship is used to cross an ocean, then a truck travels long distance, and finally vans deliver shipments to a customer.

Some of the problems within combined shipment and multi-modal service are hub-and-spoke or cross-docking. A survey of these problems is done by Guastroba et al [32].

4.2.8. Routing with Profits and Service Selection

In the cases where it is impossible to meet all demand, choices must be made and certain deliveries must be prioritized. The problems of this type can be classified into three categories:

- Routing costs and profits can be combined into one objective, then the single-vehicle routing problem is a Profitable Tour Problem (PTP) [64], which is described as Capacitated PTP (CPTP) by Archetti et al [8]
- The route can be upper-bounded and the objective can be set to profit maximization, then the problem is called the Team Orienteering Problem (TOP) [7].
- The goal can be to find a least cost routing, while a lower bound is set for the profit, then the problem is referred to as a Prize-Collecting CRP (PCVRP) [50].

4.2.9. Dynamic and Stochastic Routing

When taking into account changing conditions and uncertainty, Dynamic and stochastic routing are involved. "A problem is:

- Dynamic: if parts or all relevant information about the system conditions become available during operation;
- Stochastic: if system conditions are uncertain, but uncertainty is described by a given probability distribution." [64]

Information about the customers' locations and demands becomes known over time in dynamic VRPs [23] [36]. This is certainly the case in AFF as fire spreads unpredictably and demand may increase or emerge in a new area during the operation, as winds change and laying lines in a new location becomes critical. Some of the customers or areas to drop may be known in advance, but others only have probability-based information.

There are two other types of dynamic VRPs. Firstly, Haghani and Jung [3] considered the time dependency of travel duration. This takes into account different reasons why a vehicle may take longer to arrive at the destination, for example due to congestion or hazards. This may also be relevant in AFF as some routes have hazards such as powerlines or reduced visibility due to smoke, which can be unsafe to fly through. In the second problem, the dynamic part is the availability of vehicles [48], which means that a vehicle may become unavailable during operations, for example due to a technical problem with the vehicle or an unaccounted for delay. This may certainly be relevant in AFF as AFF aircraft perform under rough conditions and can face maintenance needs during a mission.

When uncertainty surrounding certain problem aspects is involved, and these problem aspects can then be described as random variables, we speak of stochastic VRP[20] [55]. Due to this uncertainty, planned routes may have delays or end early when a vehicle capacity is reached. Stochastic VRP can therefore be used to analyse the impact of uncertainty on the objectives.

4.3. Intra-route Constraints

Different types of constraints can play a role in determining whether a route is feasible. These are referred to as intra-route constraints or local constraints. This section will discuss route length, multiple use of vehicles, and time windows and scheduling.

4.3.1. Route Length

The length of the route, or the distance travelled by the vehicle(s) may be constrained. This could be relevant in case the vehicle has a maximum range or endurance, which could be the case for some kinds of aerial firefighters. This leads to a special kind of VRP referred to as the Distance-constrained CVRP (DCVRP) [12]. Let $d_{ij} > 0$ be the distance between nodes i and j for all $(i,j) \in A$, where A is the set of arcs. If $L > 0$ is the upper bound of the route length, then the distance constraints could be expressed as:

$$\sum_{(i,j)\in A} d_{ij}x_{ijk} < L \qquad \forall k \in K \tag{4.1}$$

Where K is the set of vehicles, and x_{ijk} is the binary variable that equals 1 if vehicle k is travelling from i to j, and 0 otherwise.

4.3.2. Multiple Use of Vehicles

There is an assumed time horizon when setting up or solving a VRP. Usually, it is assumed that over time horizon T, every vehicle will perform one route and return to the depot. There is a version of the VRP that makes use of vehicles' ability to perform multiple routes. Taillard et al. lay out the VRP with Multiple use of vehicles (VRPM) where a vehicle may perform p routes as long as $T_1 + T_2 + ... + T_p \leq T$ is satisfied [67]. This type of VRP is relevant in the case of firefighting especially for scoopers, which could be seen as performing several routes when they go to pick up water and return to the fire, or go on to visit a new fire.

Another approach to this issue is the use of the Multi-Trip VRP (MTVRP) which relies on the same concept. In the case of tankers or non-amphibious aircraft, this could be a useful feature. If it is expected that tankers would return to the airfield to refill their water or retardant tank, and are again needed at the fire, a MTVRP approach could be employed. An algorithm for the MTVRP has been proposed by Mingozzi et al. [57].

4.3.3. Time Windows and Scheduling

Most VRP variants have some constraints relating to scheduling. Given that the real life applications are related to pick up and delivery of goods from and to customers, it is sensible that certain time preferences can be indicated.

Cordeau et al. [14] present the VRP with Time Windows (VRPTW) where the travelling time t_{ij} for each arc $(i,j) \in A$ and a time window $[a_i, b_i]$ for each node $i \in V$ are given. Then, the time of visiting a node, that is the start time of the visit is denoted by T_{ik} when performed by vehicle k. For feasibility, the following constraint must be satisfied:

$$a_i \leq T_{ik} \leq b_i \qquad \forall i \in V, k \in K \tag{4.2}$$

Furthermore, time precedence constraints are also useful. This means that the time of arrival at node j must be later than the time of arrival at the previous node i, in addition to the travel time from i to j, plus any other operation time such as processing time at node i. This can be expressed as:

$$T_{ik} + T_{i,processing} + t_{ij} \leq T_{jk} \qquad \forall (i,j) \in A, k \in K \tag{4.3}$$

Fires are a dynamic phenomenon, they do not remain burning at the same spot. Due to weather conditions, the fire front will move. If a water drop is not made at a certain location by a certain time, the fire front may have moved further and that drop at that location may no longer be required, but rather requested at a new location. For this reason, the approach of time windows could be useful in the aerial firefighting application. An upper bound could be set for each fire, which is the latest time at which a drop of water or retardant must be carried out, where a drop at a later time is no longer required at that specific location.

Another justification for the usefulness of time windows in aerial firefighting is simply the principle of urgency. Fighting fires as early as possible is crucial to containing and ultimately stopping them before they are out of control. Setting an upper bound of a time window to potentially aggressive fires, could help prioritize those fires in the model and lead to a more successful firefighting operation.

4.4. Fleet Characteristics

In this section, the fleet and concept of the depot are addressed. It is not necessarily the case that there is only one depot for a given VRP. It is also rarely the case that all vehicles have exactly the same characteristics, some may be faster, or available for longer periods of time, or even have different capabilities.

4.4.1. Multiple Depot VRP

The Multi Depot VRP (MDVRP) considers a homogeneous fleet of vehicles that end their trips at a different depot than the one where they started, as discussed by Renaud et al. [43]. Depots can also be considered along the route, for example for replenishment. This case is considered by Crevier et al. [17] and Tarantilis et al. [68]. This is particularly interesting for AFF as scoopers can pick up water along their path to the next fire.

4.4.2. Heterogeneous or mixed Fleet VRP

Heterogeneous or mixed fleet refers to the fleet consisting of different types of vehicles. These vehicles may differ in capacity, costs, speeds, or even the set of customers they can access. There may be problems where a certain type of vehicle can only satisfy certain types of customers. The simplest example of this is the delivery of large items, where only larger vehicles should be visiting customers requesting these big items that cannot fit in smaller vehicles. The Heterogeneous Fleet VRP (HFVRP) is considered by Baldacci et al. [9].

In AFF, it is at the very least important to use a mixed fleet in order to distinguish between aircraft that can refill from water bodies and those who need to return to the airfield to refill. Furthermore, distinctions can also be made in capacity, speed, and costs.

4.5. Inter-route Constraints

Inter-route constraints or global constraints are considered for problems where the feasibility of the solution depends not only on the local constraints described above, but also on the relationship between the routed themselves and how they are combined. For example, when there is an objective to balance the workload amongst vehicle drivers, a constraint could be that the difference between the maximum length and minimum length of routes should not exceed a certain threshold [47]. These kinds of constraints are not likely to be relevant for AFF. Therefore, they will not be described in further depth. Nonetheless, the concept is worth including for completeness of the overview.

4.6. Objectives

One of the most common objectives of VRPs is minimizing cost, but many other types of objectives are possible. Minimizing travel time or distance, maximizing profit or fairness are also common objectives. It is also possible to model several objectives. In this section single objectives are first discussed before multiple objectives are also addressed. In the case of AFF, objectives like minimizing route length, travel time, cost, or time of servic can all be relevant.

4.6.1. Single Objective Optimization

The costs of performing a route can have several components, and so the single objective of minimizing cost could entail minimizing several things. For instance, there are fixed and variable costs. In the VRPTW, costs of driver compensation could depend on the route durations [59]. If a driver must drive past a certain time window, their rate might be higher for the extra time. In some industries, customer satisfaction can be important, and is correlated with the timely delivery of their goods. This can be incorporated into a single objective by adding a cost for delays. In AFF, or other humanitarian operations, such a latency cost can also be useful because it is usually highly desired to arrive to the area of crisis as soon as possible.

A particularly interesting case is when the objective attempts to achieve some sort of balance. For example when it is desired to be fair to the different drivers and assign them to roughly equivalent routes. This could mean that it is desired to minimize the maximum route, which is known as a min-max objective [72]. This can be useful in the AFF context because in the case of multiple fires that all require visits/drops, it is not only useful to minimize the total time, but also the time at which the last fire is visited. It is important to note that using such a balancing objective by itself is not a good idea, because a perfectly balanced solution can imply inefficient routes, since some efficient routes must be sacrificed in order to shorten the longer ones [64].

4.6.2. Hierarchical Objectives

It is often desired to minimize route length or total travelling time. Yet, minimizing the number of vehicles used can also be desirable, but these objectives are conflicting. The approach then is to use multiple objectives in a hierarchy. This means that one objective is prioritized and solved for, once that is fixed, the second objective is optimized. For example, we can minimize the number of vehicles used, and once the minimum number of vehicles is established, this is fixed and the second objective, such as route length, can be optimized for that number of vehicles [61].

4.7. Heuristics for the VRP

When it comes to solving VRPs, many advanced mathematical programming algorithms have been developed over the years. Despite these advancements, only relatively small scenarios with about 100 customers can be solved optimally, and the computing time can vary highly [46]. In AFF or other real life applications, the problem instances can be large and/or require a quick solution. Therefore, the use of efficient and flexible heuristics is not only desirable, but necessary in some situations. In this section we discuss some of the classic methods.

4.7.1. Constructive Heuristics

Constructive heuristics are used to obtain an initial solution for an improvement heuristic. Many metaheuristics have improved over the years and can now be initialized without a constructive heuristic. Nonetheless, this section will discuss two classical methods, the Clarke and Wright Savings heuristic, and petal algorithms.

The Clarke and Wright Savings Heuristic

The Clarke and Wright heuristic [28] starts by constructing routes that only go to and from a point i, as in $(0, i, 0)$ for $i = (1, ..., n)$. Then it merges this with another similarly constructed route such as $(0, j, 0)$ to obtain a single combined route: $(0, i, j, 0)$. The saving generated can then be expressed as $s_{ij} = c_{i0} + c_{0j} - c_{ij}$. In the algorithm by Laporte et. al. [33], the feasible combined route with the largest saving is implemented at each iteration, until there are no more feasible combinations. The advantages of this algorithm are its simplicity and intuitive process.

Petal Algorithms

Petal algorithms rely on the generation of a set S of feasible routes and their combination through the solution of a set partitioning problem. If d_k is the cost of the route k, and a_{ik} a binary coefficient equal to 1 if and only if customer i is in route k, and x_k a binary variable equal to 1 if and only if route k is in the solution. Then the problem is formulated as follows [46]:

$$minimize \sum_{k \in S} d_k x_k$$

$$s.t. \sum_{k \in S} a_{ik} x_k = 1 \qquad \forall i = 1, ..., n$$

$$x_k \in \{0, 1\} \qquad \forall k \in S$$

Christofides et. al. [12] used this heuristic to obtain quick solutions within a 2.38% cost margin from the best known solutions.

4.7.2. Classical Improvement Heuristics

In the case of intra-route moves, any improvement heuristic designed for the Traveling Salesman Problem (TSP) can be applied. One of these is the λ-OPT exchanges [49], where λ edges are removed and replaced.

In the case of inter-route improvement moves, the most common types are [46]:

- RELOCATE: Remove k consecutive customers from their current route and reinsert them somewhere else.
- SWAP: Swap consecutive customers between different routes.
- 2-OPT: Remove two edges from different routes and reconnect them differently.

4.7.3. Metaheuristics

Metaheuristics for the VRP can be categorized into two main types: Local search methods and population-based heuristics. Local search methods rely on the concept of moving from one solution to another (ultimately better) solution in the neighborhood, While population-based heuristics develop a population of solutions and aims to generate better solutions by combining them.

Within the local search methods, some of the main methods are: simulated annealing [66][2], deterministic annealing [30][31][26], tabu search [54][25], iterated local search[38][39], and variable neighborhood search [60].

Within the population-based heuristics, some well-known approaches are: ant colony optimization[58], genetic algorithms [16][40], scatter search, and path relinking [24][53].

In this section, a brief description of each of these is provided with the relevant literature overview, which is cited in this introduction.

Local search

If $N(x_t)$ is the neighborhood of solutions, then local search algorithms start with an initial solution x_1 and move with every iteration from x_t to x_{t+1} within $N(x_t)$. The cost of solution x_{t+1} is not necessarily better (less) than that of x_t, therefore cycling must be actively avoided.

Simulated Annealing (SA)

In SA, a solution x is randomly selected, and if the cost of this solution is more favorable than that of x_t, then $x_{t+1} = x$, otherwise x_{t+1} is set to either x or x_t with a certain probability p_t and $(1 - p_t)$ respectively. p_t is commonly defined as:

$$p_t = exp(-[f(x) - f(x_t)]/\theta_t) \tag{4.4}$$

where $f(x)$ is the cost function of solution x and θ_t is a decreasing function of t [46].

Deterministic Annealing (DA)

In DA, accepting a solution x is based on a deterministic rule. In a well known algorithm that implemented this by Li et. al. [26], a solution x is drawn from neighbourhood $N(x_t)$ and $x_{t+1} = x$ if $f(x) \leq \sigma f(x*)$, where σ is slightly larger than 1 and $x*$ is called a record and is the best known solution. if $f(x) > \sigma f(x*)$, then $x_{t+1} = x_t$ [46].

Tabu Search (TS)

To avoid cycling, TS blocks some solutions that share some attributes with x_t making them "tabu". The procedure then is to move from a solution t to the best non-tabu solution x_{t+1}. When a potential solution corresponds to a new best known solution, its tabu status, if it had one, is revoked. Many TS implementations have been proposed over the past years. A relatively new tabu search algorithm that can serve as a good reference is by Zachariadis and Kiranoudis [22].

Iterated Local Search (ILS)
In ILS, an embedded local search mechanism is performed until a stopping criterion is reached. At this point a new starting solution is generated by perturbing the current solution. Then, the embedded local search is applied again. This process of perturbing the current solution and then searching locally is repeated until the algorithm reaches the threshold of a stopping criterion.

One of the main benefits of ILS is that it can escape local optima by periodically perturbing the current solution and starting the search process again. This can allow the algorithm to find better solutions than it would if it were to just run the local search algorithm on its own.

Variable Neighborhood Search (VNS) In VNS, the search begins with a randomly generated initial solution, which is then improved upon using a local search algorithm. The algorithm then switches to a different neighborhood structure and repeats the local search process. This process is repeated until no further improvement is possible or a predetermined number of iterations has been reached.

The key idea behind VNS is that by using a variety of different neighborhood structures, the algorithm is able to escape local optima and explore a wider range of the search space, which can lead to the discovery of better solutions. It is a popular choice for problems where the structure of the optimal solution is not well understood or the search space is very large.

VNS was proposed as a general search strategy by Mladenovic and Hansen [60], and was successfully applied to the VRP by Kytojoki et al. [42], where high quality solutions on instances involving up to 20,000 customers were identified.

Population-Based Algorithms

Population-based methods derive their principles from natural phenomena. The behavior of social insects and the evolution process of species are some of the inspirations behind these methods. In this section we discuss ant colony optimization, genetic algorithms, and scatter search and path relinking.

Ant Colony Optimization (ACO)
One of the most successful ACO algorithms is that presented by Reimann et al. [58]. An attractiveness value $\chi = \tau_{ij}^{\alpha} - s_{ij}^{\beta}$ is used, where τ_{ij} is the pheromone value and measures how good of a combination i and j were in previous iterations, while α and β are user-controlled parameters. The combination of i and j is subject to probability $p_{ij} = \chi_{ij}/(\sum_{(h,l)\in\Omega_k} \chi_{hl})$, where Ω_k is the set of the feasible (i,j) combinations resulting in the k best savings [58][46].

Genetic Algorithms (GA)
The basic steps of a genetic algorithm are:

1. Initialize a population of candidate solutions.
2. Evaluate the fitness of each candidate solution.
3. Select the fittest candidate solutions to be parents.
4. Create offspring from the parents by applying genetic operators such as crossover and mutation.
5. Evaluate the fitness of the offspring.
6. Replace the least fit candidates in the population with the offspring.
7. Repeat steps 2-6 until a satisfactory solution is found or a predetermined number of iterations has been reached.

The first successful application to the VRP was by Prins [16][46]. The method uses a combination of genetic operators, selection, crossover, and an efficient local search. The solution is represented as a giant tour without trip delimiters. A strong disadvantage of this method is that it is not ideal for large problems. If there are many levels to the problem in question, the growth becomes exponential and quickly very large. At that point the problem may have to be divided up into several smaller problems.

Scatter Search (SS) and Path Relinking (PR)
In some GA methods, the crossover operator puts together sequences of visits from different solutions blindly and randomly [46]. This means that solutions obtained after a crossover can be of low quality. In order to address this, some research has attempted using more intentional and intelligent recombinations, for instance within the context of SS or PR [24][53]. A relatively recent PR algorithm was used successfully within the VRPTW by Tarantilis et al. [11].

5 Conclusions

This literature study has been performed to prepare for a thesis project seeking to use the Vehicle Routing Problem (VRP) in the field of Aerial Firefighting (AFF), in response to the increasing need for AFF due to the increasing number of unwanted forest fires.

The main research question is: "How can the vehicle routing problem be applied to aerial firefighting and lead to more effective containment of wildfires?". In order to answer this question, Three areas of study were identified and developed: Fire behavior and simulations, Decision-making in AFF operations, and last but not least the VRP and its adaptability to the AFF problem.

The three main influences on fire behavior are weather, fuels, and topography, which make up the so-called "fire triangle". Many fire behavior simulations are used by different authorities, but the most widely used one is called BehavePlus. It is made up of many mathematical modules, which are outlined in Figure 2.3.

AFF strategies have been broken down into two main approaches: Laying lines, and making direct drops. An overview was created of strategies and drop objectives, and corresponding aircraft types in Figure 3.1. In AFF, fixed wing aircraft as well as helicopters are used. The former have the advantage of speed, while helicopters have the advantage of hovering over a small body of water to pick up water and make direct drops. In general, all aircraft used for AFF can be categorized into tankers and scoopers. Tankers simply have a tank that carries water or retardant, and after making a drop, must return to the airfield to refill. Scoopers have the ability to land on water and scoop up more water to swiftly return to the fire and make more drops. These two types of aircraft abilities must be modelled for any realistic attempt to design an efficient AFF strategy.

The VRP has been researched thoroughly since its initial introduction in 1959 by Dantzig and Ramser [29]. It has since developed into a field with many VRP "versions". Some of the well-studied versions that could be useful for this project are:

- The Capacitated VRP (CVRP): involves taking into account the capacity that each vehicle can carry, and introducing constraints in the model to make sure that the overall capacity is not exceeded.
- The Split Delivery VRP (SDVRP): considers cases where delivery can be split up amongst several vehicles. This can be relevant for AFF as a fire might require several aircraft to "deliver" water or retardant.
- Dynamic Routing: is applied when parts or all relevant information about the system conditions become available during operation. As forest fires can create a quickly changing environment, employing a dynamic approach to an AFF VRP can be useful.

- The VRP with Time Windows (VRPTW): involves time windows within which the delivery may be performed. This can be useful since AFF drops should be made within a time limit, otherwise the fire would have advanced and the drop would no longer be required at that precise location.
- The Multi Trip VRP (MTVRP): involves expanding the operation horizon of vehicles to make them consider multiple trips, which is useful in AFF as tankers will often need to go to the airfield to refill retardant or water before returning to fires.

The VRP to be developed in this project will likely use a combination of elements from the types listed above. Furthermore, the use of multiple objectives will likely also be useful. The concept of hierarchical objectives allows the problem to solve for an important objective first, and then further optimize the problem after having fixed the solution of the prioritized objective. For example, the number vehicles needed to solve a problem can first be minimized, and then with that number of vehicles, the route length may be minimized.

The VRP can take a long time to solve if the problem involves a large number of customers (or fires in this case). Therefore, the use of heuristics is a common approach to solving VRPs, where good-enough solutions are obtained more quickly. Metaheuristics of the VRP can be categorized into two main types: Local search and population-based heuristics. Local search methods move from one solution to another within a given neighborhood, seeking better solutions or local optima. Population-based heuristics develop a population of solutions and combine them to obtain superior solutions. Some local search methods are simulated annealing, deterministic annealing, tabu search, iterated local search, and variable neighborhood search. Population-based heuristics include ant colony optimization, genetic algorithms, scatter search and path relinking.

In order to successfully develop a VRP for AFF applications, the literature on the different types of VRPs will be helpful, and the different relevant aspects from previous VRPs will be "borrowed" and combined effectively. Furthermore, the research on understanding fire behavior and how authorities make decisions in AFF operations should help to make sure that the defined problem is realistic and the solution applicable in real life operations.

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